$(g-2)_\mu$: recent improvements and preliminary update

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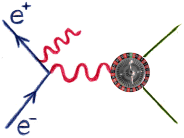
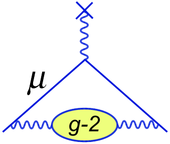


19^{th} - 20^{th} May 2016



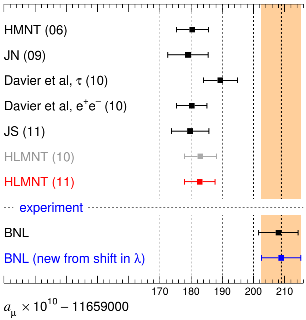
Overview

1. Introduction
   - $(g-2)_\mu$ analysis recap
   - Outstanding issues
2. $a_\mu^{\text{had, LOVP}}$ below 2 GeV
   - The basics
3. Data combination & fitting: systematic bias
   - Fitting global systematic normalisation uncertainties
   - Unbiased fitting
   - Results
4. New data & results
   - Overview
   - $e^+e^- \to K^+K^-$
   - $e^+e^- \to \pi^+\pi^-$
5. Conclusions

Where we left off... [HLMNT(11), J. Phys. G38 (2011), 085003]

QED contribution	11 658 471.808 (0.015) $\times 10^{-10}$	Kinoshita & Nio, Aoyama et al
EW contribution	15.4 (0.2) $\times 10^{-10}$	Czarnecki et al
Hadronic contribution		
LO hadronic	**694.9 (4.3)** $\times 10^{-10}$	HLMNT11
NLO hadronic	−9.8 (0.1) $\times 10^{-10}$	HLMNT11
light-by-light	10.5 (2.6) $\times 10^{-10}$	Prades, de Rafael & Vainshtein
Theory TOTAL	**11 659 182.8 (4.9)** $\times 10^{-10}$	
Experiment	**11 659 208.9 (6.3)** $\times 10^{-10}$	world avg
Exp − Theory	**26.1 (8.0)** $\times 10^{-10}$	**3.3 σ discrepancy**

(Numbers taken from HLMNT11, arXiv:1105.3149)



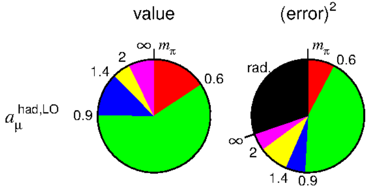
→ $a_\mu^{\text{had, LOVP}}$ still dominating uncertainty

→ Eagerly awaiting new hadronic cross section data

→ ...and new results from Fermilab

$\Longrightarrow$ **Hints at increased discrepancy,**

$$\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$$



Problems concerning data combination

⇒ All data is different

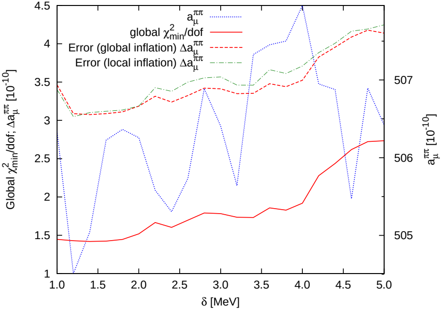
Does data include ISR, FSR, VP correction in δ_{rad}?

What forms do the uncertainties take: point-to-point, overall systematic percentage, covariance matrix...?

⇒ How do we re-bin/cluster data?

What cluster sizes do we allow?

How do we cluster points at resonance peaks as opposed to in a continuum region?



⇒ How do we fit data?

What function/method is the best?

How do we construct a trustworthy representation of the uncertainties as an overall uncertainty on a_μ?

How do we achieve this whilst avoiding potential bias through error propagation?

Existing/new data

⇒ Do we discard old, imprecise data?

Some older data sets (∼80s) are questionably inaccurate compared to newer sets?

Can we justify removing this data?

⇒ Differences between direct scan and radiative return data

More and more radiative return measurements...

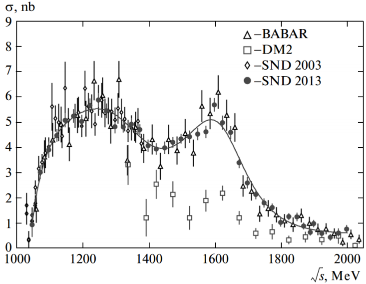
How do they compare?

⇒ BaBar problem

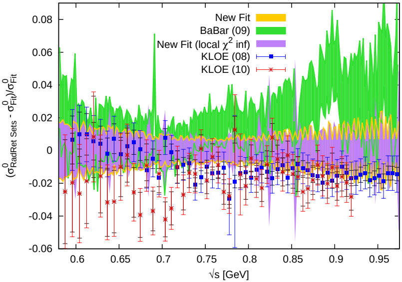
Why do BaBar measurements sit higher than all other data?

Causing tension between sets...

No longer just $\pi^+\pi^-$



$[e^+e^- \to \pi^+\pi^-\pi^0$, SND, JETP(2015), 121, 1, 27-34]

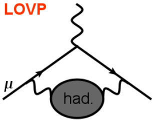


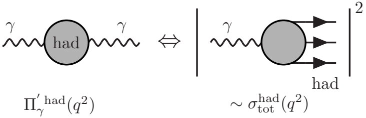
Calculating $a_\mu^{\text{had, LOVP}}$

Need to precisely determine $a_\mu^{\text{had, LOVP}} < 2$ GeV

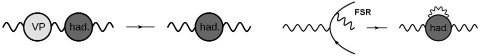
→ pQCD not useful

⟶ Use analyticity and the optical theorem





...and undress cross section (remove all VP effects, but including FSR)



...as input into dispersion relation:

$$a_\mu^{\text{had, LOVP}} = \frac{1}{4\pi^3} \int_{s_{\text{th}}}^{\infty} \mathrm{d}s \, \frac{\sigma_{\text{had}}^0(s)}{\sigma_{\text{pt}}(s)} \frac{4\pi\alpha^2}{3s} K(s)$$

- Sum ∼30 exclusive channels (isospin relate missing channels)
- Combine data from many experiments, different energy bins, uncertainties, correlations...
- Re-binning through *adaptive clustering* and minimise χ^2-function

Data combination: step-by-step

→ Apply appropriate radiative corrections to data: VP, FSR etc.

→ Re-bin data into energy-dependent clusters (piecewise constant R)

→ Determine cluster properties: energy, cross-section value, uncertainty...

$$R_m = \left[\sum_k \sum_{i=1}^{N^{(k,m)}} \frac{R_i^{(k,m)}}{\left(\mathrm{d}\tilde{R}_i^{(k,m)}\right)^2}\right] \left[\sum_k \sum_{i=1}^{N^{(k,m)}} \frac{1}{\left(\mathrm{d}\tilde{R}_i^{(k,m)}\right)^2}\right]^{-1}$$

→ Construct uncertainty covariance matrices for all data

→ Fit data according to χ^2 minimum
⟶ Determine local χ^2 per cluster and global χ^2_{min}/d.o.f.

→ Integrate over fitted clusters as input into dispersion integral to determine $a_\mu \pm \delta a_\mu$

→ Inflate error, δa_μ, according to local cluster χ^2

The f_k method

- Fit data and minimise a non-linear χ^2-function [HLMNT, 2012]
  $\longrightarrow$ Two fitting parameters: initial cluster values R_m, normalisation factor f_k of each experiment k

$$\chi^2(R_m, f_k) = \sum_{k=1}^{N_{exp}} \left\{ \left(\frac{1-f_k}{df_k} \right)^2 + \left[\sum_{m=1}^{N_{clu}} \sum_{i=1}^{N^{(k,m)}} \left(\frac{R_i^{(k,m)} - f_k R_m}{dR_i^{(k,m)}} \right)^2 \right]_{\text{w/o cov. mat}} + \left[\sum_{m=1}^{N_{clu}} \sum_{n=1}^{N_{clu}} \sum_{i=1}^{N^{(k,m)}} \sum_{j=1}^{N^{(k,n)}} \left(R_i^{(k,m)} - f_k R_m\right) C^{-1}(m_i, n_j) \left(R_j^{(k,n)} - f_k R_n\right) \right] \right\}$$

The f_k's are multiplicative re-normalisation factors for the data which vary as the χ^2-function is minimised

$\longrightarrow$ Penalty Trick Method

What is a biased result?

→ *D'Agostini Bias* = Fit favours more precise measurement

[D'Agostini, Nucl. Instrum. Meth. A **346** (1994) 306]

⇒ How do we include a global normalisation uncertainty whilst avoiding a D'Agostini bias? ⟶ f_k (penalty trick) method

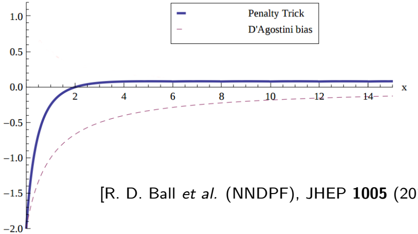
PENALTY TERM

NORMALISATION FACTOR

$$\chi^2(R_m, f_k) = \sum_{k=1}^{N_{exp}} \left\{ \left(\frac{1 - f_k}{df_k} \right)^2 + \left[\sum_{m=1}^{N_{clu}} \sum_{i=1}^{N^{(k,m)}} \left(\frac{R_i^{(k,m)} - f_k R_m}{dR_i^{(k,m)}} \right)^2 \right]_{\text{w/o cov. mat}} \right.$$

FIT PARAMETERS

Is the f_k method truly free from bias?



[R. D. Ball *et al.* (NNDPF), JHEP **1005** (2010) 075]

The f_k method: a toy example

Consider two measurements: $R_i^{(k,m)}$ and $R_j^{(l,m)}$ with equal (uncorrelated) errors...

$$\rightarrow \mathrm{d}f_k = \mathrm{d}f_l \equiv \mathrm{d}f \;\; ; \;\; \mathrm{d}R_i^{(k,m)} = \mathrm{d}R_j^{(l,m)} \equiv \mathrm{d}R$$

Unbiased solution $\longrightarrow R_m = \bar{R}_m = \frac{1}{2}(R_i^{(k,m)} + R_j^{(l,m)})$

Minimising w.r.t R_m and f_k and substituting, we find: $R_m = \bar{R}_m(1+\beta_m)$

$\Rightarrow \beta_m$ is the *bias contribution* to the cluster centre R_m

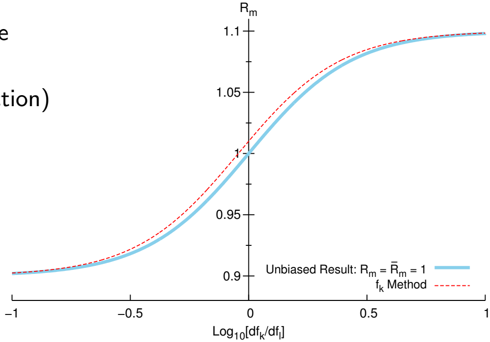
(due to non-linear nature of χ^2 function)

$\longrightarrow$ Now, let

$R_i^{(k,m)} = 0.9$ and $R_j^{(l,m)} = 1.1$

$\mathrm{d}R_i^{(k,m)} = \mathrm{d}R_j^{(l,m)} \equiv \mathrm{d}R = 0$

$\Rightarrow R_m = 1.0050125 \neq \bar{R}_m$



The R^I_m method

Following $\rightarrow$ NNPDF Collaboration, R.D.Ball *et al.* [JHEP **1005** (2010) 075]
$\rightarrow$ Benayoun *et al.* [Eur. Phys. J. C **75** (2015) no.12, 613]

$\Rightarrow$ Claim: fix covariance matrices during fitting

$$\mathbf{C}\left(i^{(k,m)}, j^{(l,n)}\right) = \mathsf{C}^{\text{stat}}\left(i^{(k,m)}, j^{(l,n)}\right) + \mathsf{C}^{\text{sys\%}}\left(i^{(k,m)}, j^{(l,n)}\right) R^0_m R^0_n$$

$\Rightarrow$ ...to prevent error propagation skewing theory value

$$\chi^2 = \sum_{m,n=1}^{N_{clu}} \sum_{i,j=1}^{N^{(k;m,n)}} \left(R_i^{(k,m)} - R_m\right) \mathbf{C}^{-1}\left(i^{(k,m)}, j^{(l,n)}\right) \left(R_j^{(k,n)} - R_n\right)$$

$\longrightarrow$ Allows for inclusion of full correlated normalisation uncertainties without renormalisation factors

$\longrightarrow$ Iterative routine forces convergence to unbiased solution

$\longrightarrow$ Observe one (maximum two) iteration to convergence

$\longrightarrow$ *Benayoun et al.* predict reduction in mean value and improved uncertainty

Channel	$A = m$	$A = M_0$	Exp. Value
$\pi^+\pi^-$	495.06 ± 1.43	494.59 ± 0.89	492.98 ± 3.38
$\pi^0\gamma$	4.53 ± 0.04	4.54 ± 0.04	3.67 ± 0.11
$\eta\gamma$	0.64 ± 0.01	0.64 ± 0.01	0.56 ± 0.02
$\pi^+\pi^-\pi^0$	40.83 ± 0.57	40.84 ± 0.57	43.54 ± 1.29
K_LK_S	11.56 ± 0.08	11.53 ± 0.08	12.21 ± 0.33
K^+K^-	16.79 ± 0.20	16.90 ± 0.20	17.72 ± 0.52
Total	569.41 ± 1.55	569.04 ± 1.08	570.68 ± 3.67

The R_m^I method: a toy example

Consider two measurements: $R_i^{(k,m)}$ and $R_j^{(l,m)}$ with equal (uncorrelated) errors...

$$\to \chi^2(R_m) = \sum_{i=1}^{N^{(k,m)}} \frac{\left(R_i^{(k,m)} - R_m\right)^2}{\left(\mathrm{d}R_{i;\ \mathrm{stat}}^{(k,m)}\right)^2 + \left(\mathrm{d}R_{i;\ \mathrm{sys\%}}^{(k,m)}\right)^2 \left(R_m^0\right)^2}$$

Minimising w.r.t R_m and reintroduce $\to \mathrm{d}f_k = \mathrm{d}f_l \equiv \mathrm{d}f \quad ; \quad \mathrm{d}R_i^{(k,m)} = \mathrm{d}R_j^{(l,m)} \equiv \mathrm{d}R$

$$R_m = \left[\frac{R_i^{(k,m)} + R_j^{(l,m)}}{(\mathrm{d}R_{\mathrm{stat}})^2 + (\mathrm{d}R_{\mathrm{sys}})^2 (R_m^0)^2}\right]\left[\frac{2}{(\mathrm{d}R_{\mathrm{stat}})^2 + (\mathrm{d}R_{\mathrm{sys}})^2 (R_m^0)^2}\right]^{-1}$$

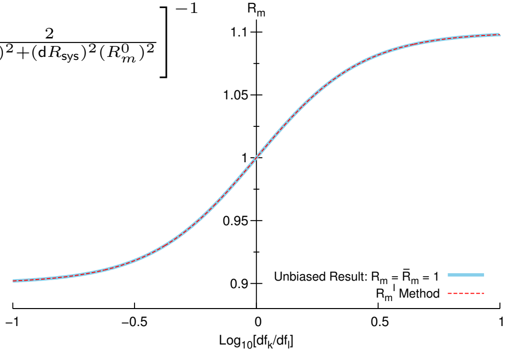
$$= \tfrac{1}{2}\left(R_i^{(k,m)} + R_j^{(l,m)}\right)$$

$\longrightarrow$ Again, let

$R_i^{(k,m)} = 0.9$ and $R_j^{(l,m)} = 1.1$

$\mathrm{d}R_i^{(k,m)} = \mathrm{d}R_j^{(l,m)} \equiv \mathrm{d}R = 0$

$\Rightarrow R_m = 1 = \bar{R}_m$



What effect will this have on our results?

Use dominant $\pi^+\pi^-$ channel as example of change to $a_\mu^{\text{had, LOVP}}$

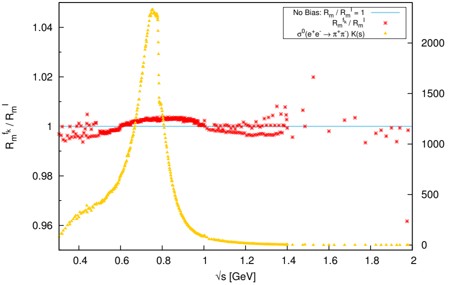
$\rightarrow$ Consider real cluster with one measurement:

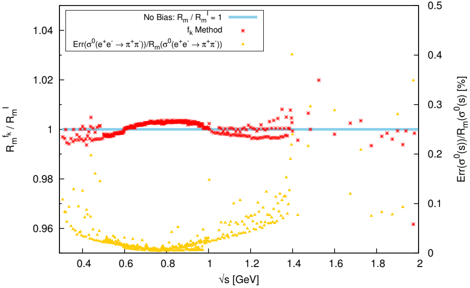
$E_{cm}(\text{GeV}) = 0.4600$, $\sigma^0(\text{nb}) = 123.6418 \pm 22.4855 \quad \Rightarrow \quad R_m^0 = 123.6418$

Experimental data includes three more measurements in different clusters.

$\Rightarrow$ Normalisation uncertainty provides weighting to cluster value through *fixed* covariance matrix correlations

f_k method $\rightarrow R_m^{f_k} = 130.81$; R_m^I method $\rightarrow R_m^I = 130.97$





Are previous results reliable?

Did the f_k method incur a bias?

Compare f_k method and R^I_m method with only multiplicative normalisation uncertainties.

→ If we see **differences** in mean value, then bias previously influenced the fit.
⟶ Previous results unreliable

→ If we see **no differences** in mean value, then bias did not influence fit (any change comes from improved treatment of systematics)
⟶ Previous results reliable

Example - $\pi^+\pi^-$
Set 1 - CMD-2(06) (0.7% Systematic Uncertainty), Set 2 - CMD-2(06) (0.8% Systematic Uncertainty), Set 3 - SND(04) (1.3% Systematic Uncertainty)

From $0.37 \rightarrow 0.97$ GeV

Fit Method:	f_k Method		R^I_m Method		
Channel	a_μ	χ^2_{min}/d.o.f.	a_μ	χ^2_{min}/d.o.f.	Difference
$\pi^+\pi^-$	481.42 ± 4.26	1.10	481.42 ± 4.05	1.02	0.00

What new data is available to us since 2011 analysis?

Since HLMNT(11)...

- $\pi^0\gamma$: SND (2016) (1 direct scan)
- $\pi^+\pi^-$: KLOE (2012), BESIII (2015) (2 radiative return)
- $\pi^+\pi^-\pi^0$: SND (2015) (1 direct scan)
- $\pi^+\pi^-\pi^+\pi^-$: BaBar (2012) (1 radiative return)
- $\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$: CMD-3 (2013) (1 direct scan)
- K^+K^-: BaBar (2013) (1 radiative return)
- $K_S^0K_L^0$: BaBar (2014), CMD-3 (2016) (1 radiative return, 1 direct scan)
- $K^+K^-\pi^+\pi^-$: CMD-3 (2015) (1 direct scan)
- $\eta\gamma$: SND (2014) (1 direct scan)
- $\eta\pi^+\pi^-$: SND (2015) (1 direct scan)
- $\omega\pi^0$: SND (2015) (1 direct scan)
- $p\bar{p}$: CMD-3 (2015) (1 direct scan)
- $n\bar{n}$: SND (2014) (1 direct scan)

No new data for $\pi^+\pi^-\pi^0\pi^0$, $\pi^+\pi^-\pi^+\pi^-\pi^0$, $\pi^+\pi^-\pi^0\pi^0\pi^0$, $K_S^0K^\pm\pi^\mp$, $K^*K\pi$, $K^+K^-\pi^0\pi^0$, $K^+K^-\pi^+\pi^-\pi^0$, $\eta\pi^+\pi^-\pi^+\pi^-$, $\eta\omega$, $\omega\pi^+\pi^-$, $\eta\phi$, $\phi\pi^0$, $\phi\pi^+\pi^-$

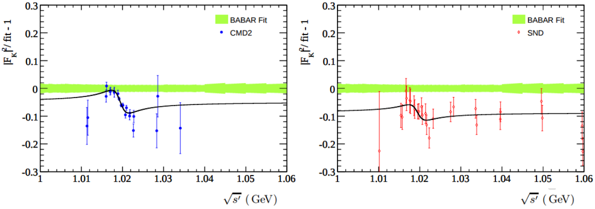
$e^+e^- \to K^+K^-$: m_ϕ calibration uncertainty study

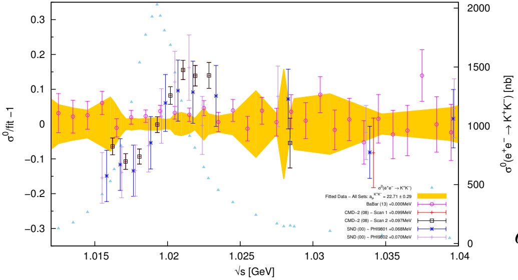
→ Tension in fitted Φ mass between BaBar, CMD-2 and SND

→ Study to shift energies of data points by differences in the ϕ-mass calibration to potentially reduce tension

→ Allow variation in δm_ϕ

[J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **88** (2013) no.3, 032013]





→ Marginal improvement in error and χ^2/d.o.f but still not resolved

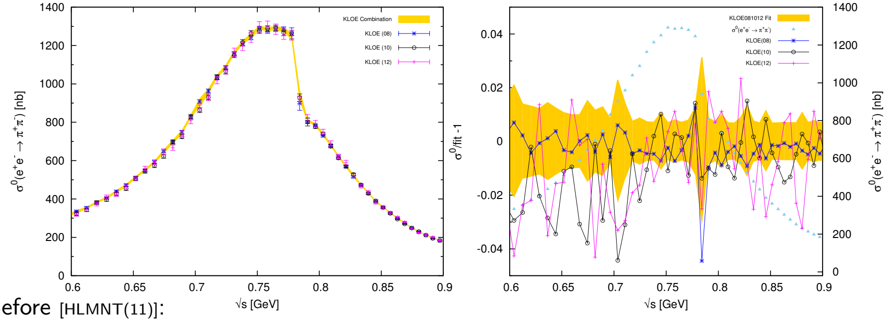
$\Longrightarrow$ **BaBar puzzle still remains**

$$a_\mu^{K^+K^-}(\leq 2.0\text{GeV}) = 22.75 \pm 0.27$$

$$\chi^2_{\text{min}}/\text{d.o.f.} = 2.09$$

$e^+e^- \to \pi^+\pi^-$: inclusion of KLOE(12) & KLOE combination matrix [[KLOE Collaboration], Phys. Lett. B **720** (2013) 336; arXiv:1406.4639 [hep-ph]]

Combination of KLOE(08), KLOE(10) & KLOE(12) radiative return data



Before [HLMNT(11)]:

$$a_\mu^{\pi^+\pi^-}(0.305 \leq \sqrt{s} \leq 2.0 \text{ GeV}) = (505.77 \pm 3.09) \times 10^{-10} \quad ; \quad \chi^2_{\text{min}}/\text{d.o.f.} = 1.35$$

After:

$$a_\mu^{\pi^+\pi^-}(0.305 \leq \sqrt{s} \leq 2.0 \text{ GeV}) = (505.05 \pm 2.19) \times 10^{-10} \quad ; \quad \chi^2_{\text{min}}/\text{d.o.f.} = 1.37$$

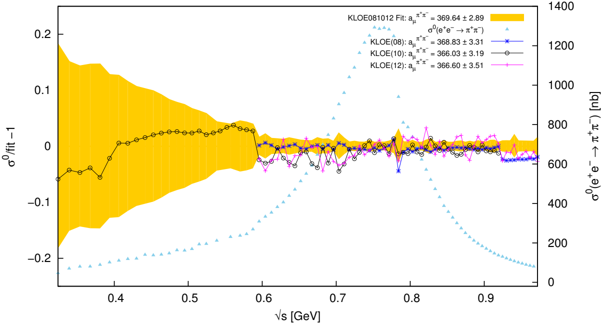
KLOE combination issues

Individual KLOE measurements give (in range $0.6 \le \sqrt{s} \le 0.9$ GeV):

		FJ03-VP	$\longrightarrow$	VP_HLMNT_v2_2
KLOE(08)	$\Rightarrow$	368.83 ± 3.31	$\longrightarrow$	368.27 ± 3.31
KLOE(10)	$\Rightarrow$	366.03 ± 3.19	$\longrightarrow$	365.48 ± 3.19
KLOE(12)	$\Rightarrow$	366.60 ± 3.51		

Combining using R^I_m method:

KLOE(081012) $\Rightarrow$ 369.94 ± 2.89 $\longrightarrow$ 369.64 ± 2.89



$\Rightarrow$ KLOE(08) & KLOE(12) correlated systematics above ~ 0.9GeV drag fit up

$\Rightarrow$ Removing these correlated systematics gives: $a_\mu^{\pi^+\pi^-} = 367.70 \pm 2.70$

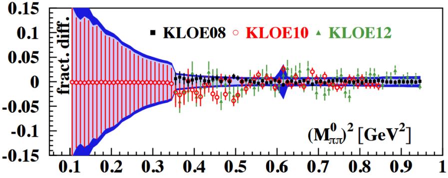
$\Rightarrow$ We believe R^I_m method is free from bias BUT if so... what is the cause?

KLOE combination: method comparison

$\Rightarrow$ KLOE preliminary combination [1406.4639; Acta Phys. Polon. B **46** (2015) 45] uses... "*the best linear unbiased estimator (BLUE) method... (whereby) the BLUE values are constructed using only the covariance matrix with statistical uncertainties.*"

$\longrightarrow$ Systematic uncertainties added separately to error contribution

$\Longrightarrow$ Equivalent to minimising χ^2 function??????



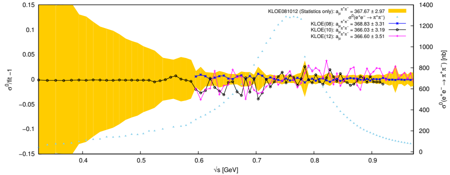
Courtesy of S. Müller

$\to$ Using BLUE method...

$a_\mu^{\pi^+\pi^-}(0.324 \le \sqrt{s} \le 0.975 \text{ GeV}) = 487.8 \pm 5.7$

$a_\mu^{\pi^+\pi^-}(0.592 \le \sqrt{s} \le 0.922 \text{ GeV}) = 378.1 \pm 2.8$

$\Rightarrow$ Open for discussion...



$\to$ Using R^I_m method

(restricted to only statistical errors only in the χ^2)

$a_\mu^{\pi^+\pi^-}(0.324 \le \sqrt{s} \le 0.975 \text{ GeV}) = 486.7 \pm 5.2$

$a_\mu^{\pi^+\pi^-}(0.592 \le \sqrt{s} \le 0.922 \text{ GeV}) = 378.1 \pm 3.0$

$a_\mu^{\pi^+\pi^-}(0.6 \le \sqrt{s} \le 0.9 \text{ GeV}) = 367.67 \pm 2.97$

KLOE combination: $\pi^+\pi^-$ implications

$\Rightarrow$ Full combination of $\pi^+\pi^-$ data remains stable

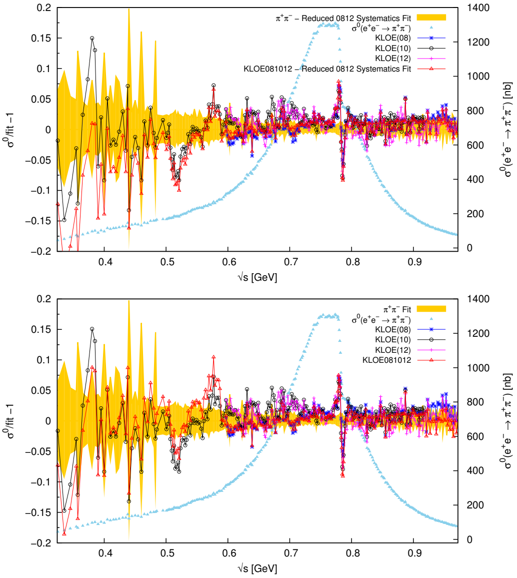
$\rightarrow$ Removing KLOE(08) & KLOE(12) correlated systematics > 0.9 GeV:

$$a_\mu^{\pi^+\pi^-}(\leq 2.0\text{GeV}) = 503.71 \pm 2.10$$

$\rightarrow$ Calculating a full $\pi^+\pi^-$ combination with all KLOE systematics:

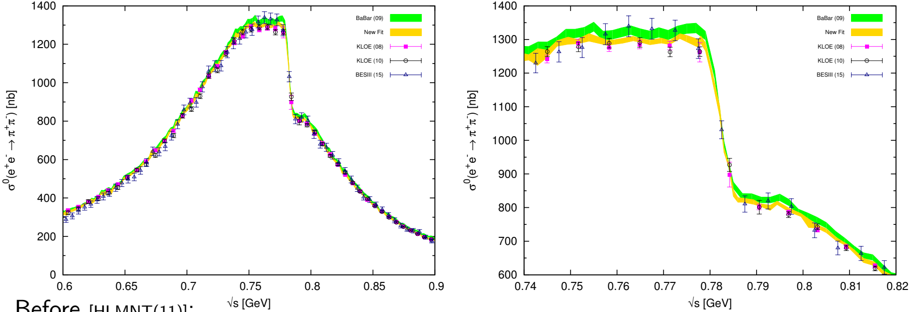
$$a_\mu^{\pi^+\pi^-}(\leq 2.0\text{GeV}) = 504.00 \pm 2.12$$

$\Rightarrow$ Effect is less than 14% of the error



$e^+e^- \to \pi^+\pi^-$: inclusion of BESIII data [M. Ablikim *et al.* [BESIII Collaboration],Phys. Lett. B **753** (2016) 629]

Measured in energy range of $0.6 \leq \sqrt{s} \leq 0.9$ GeV using the radiative return method



Before [HLMNT(11)]:

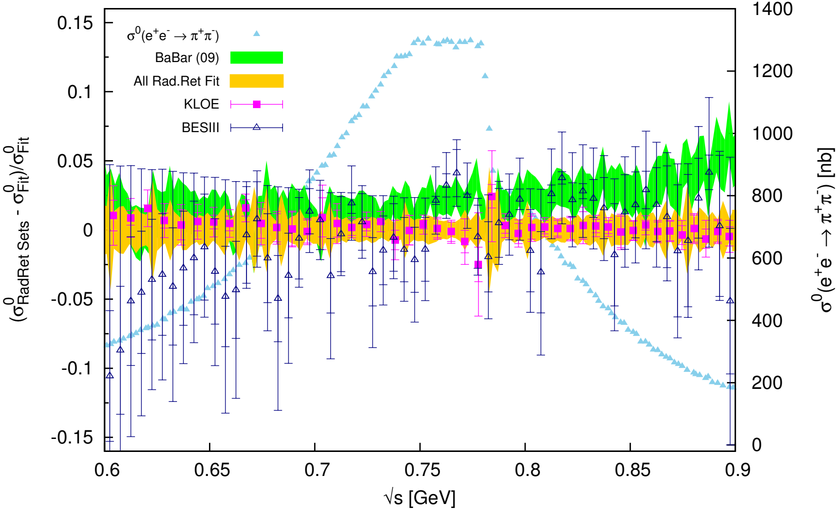
$$a_\mu^{\pi^+\pi^-}(0.305 \leq \sqrt{s} \leq 2.0 \text{ GeV}) = (505.77 \pm 3.09) \times 10^{-10} \quad ; \quad \chi^2_{\text{min}}/\text{d.o.f.} = 1.35$$

After (excluding new KLOE data):

$$a_\mu^{\pi^+\pi^-}(0.305 \leq \sqrt{s} \leq 2.0 \text{ GeV}) = (504.15 \pm 2.17) \times 10^{-10} \quad ; \quad \chi^2_{\text{min}}/\text{d.o.f.} = 1.26$$

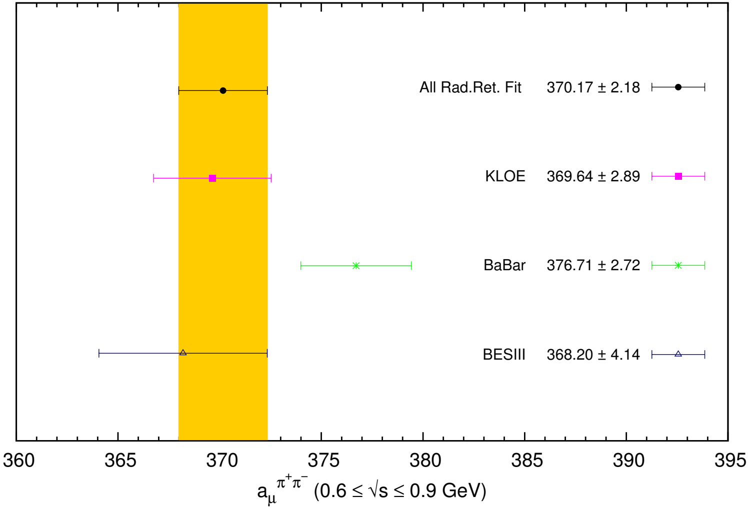
$e^+e^- \to \pi^+\pi^-$: radiative return comparison

⇒ Using full method with complete radiative return data...



$e^+e^- \to \pi^+\pi^-$: radiative return comparison

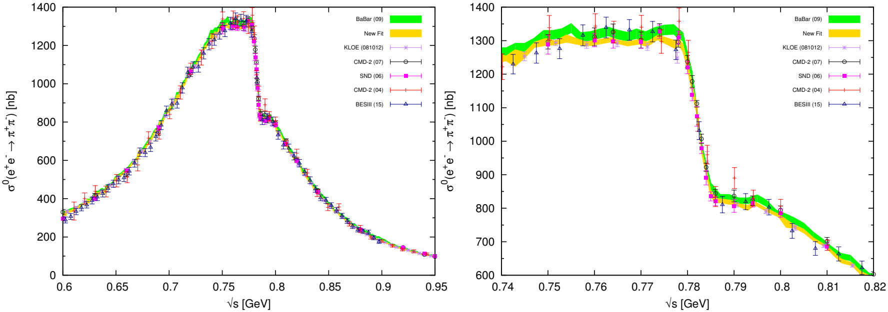
⇒ Using full method with complete radiative return data...



$a_\mu^{\pi^+\pi^-}$: preliminary result

$\pi^+\pi^-$ channel contributes $\sim 73\%$ to $a_\mu^{\text{had, LOVP}}$

$\longrightarrow$ Channel now combines 999 data points from 25 experimental measurements
(5 radiative return measurements with full uncertainty covariance matrices)



$\Rightarrow$ Full combination yields... $a_\mu^{\pi^+\pi^-}(0.6 \leq \sqrt{s} \leq 0.9 \text{ GeV}) = 370.88 \pm 1.52$

$a_\mu^{\pi^+\pi^-}(0.305 \leq \sqrt{s} \leq 2.0 \text{ GeV}) = (504.00 \pm 2.12) \times 10^{-10}$; $\chi^2_{\text{min}}/\text{d.o.f.} = 1.28$

Results comparison

Comparative (PRELIMINARY) results after introducing new fitting method and including new data in leading channels (in units 10^{-10}):

	HLMNT(11)		This work		
Channel	$a_\mu(\leq 2\text{GeV})$	$\chi^2_{\text{min}}/\text{d.o.f.}$	$a_\mu(\leq 2\text{GeV})$	$\chi^2_{\text{min}}/\text{d.o.f.}$	Difference
$\pi^+\pi^-$	505.77 ± 3.09	1.39	504.00 ± 2.12	1.28	-1.77
$\pi^+\pi^-\pi^0$	47.51 ± 0.98	3.04	47.68 ± 0.97	2.51	+0.17
$\pi^+\pi^-\pi^0\pi^0$	20.73 ± 1.28	1.29	20.42 ± 1.11	0.72	-0.31
$\pi^+\pi^-\pi^+\pi^-$	14.73 ± 0.48	1.81	14.91 ± 0.47	1.61	+0.18
K^+K^-	22.12 ± 0.41	1.95	22.75 ± 0.27	2.09	+0.63
$K^0_s K^0_l$	13.33 ± 0.17	1.10	13.09 ± 0.13	0.73	-0.24
$\pi^+\pi^-\pi^+\pi^-\pi^0$	1.42 ± 0.09	1.21	1.39 ± 0.09	1.05	-0.03
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	0.30 ± 0.01	1.67	0.28 ± 0.01	1.49	-0.02
Total:	625.91 ± 3.55		624.52 ± 2.63		-1.39

- Changes in almost each channel due to new method/new data
- Improved error estimate and goodness-of-fit
- Much more statistically reliable and trustworthy fitting method
- Reduction in overall mean value would mean increased $g-2$ discrepancy, δa_μ
- Room for improvement in clustering, new data etc. $\Rightarrow$ Results expected to change

Outlook and the future...

→ More data = improved precision

→ Looking forward to new measurements from KLOE, CMD-3, SND, BaBar, BESIII (& Belle)

→ AND new experimental measurements from Fermilab and J-PARC

⇒ As it stands (from ONLY the results in this talk)...

LO hadronic	694.9 (4.3)	⟶	693.5 (3.6)	this work
Theory total	11659182.8 (4.9)	⟶	11659181.4 (4.4)	this work
Experiment			11659208.9 (6.5)	world avg
Exp - Theory	26.1 (8.0)	⟶	27.48 (7.8)	
Δa_μ	3.3σ	⟶	3.5σ	

Summary

- Data has to be treated with intense care and scrutiny in combination
- f_k method shown to exhibit potential bias...
- R_m^I method avoids bias by fixing covariance matrix through iterative fit
- Method allows for more stable, robust and trustworthy treatment of uncertainties (covariance matrices)
- K^+K^- data tension unresolved through energy shift study
  $\Rightarrow$ BaBar problem
- Unresolved issues regarding combination of KLOE data and systematic uncertainties
  $\Rightarrow$ Full $\pi^+\pi^-$ combination remains stable
- Lower mean value and reduced uncertainty for $\pi^+\pi^-$ and $a_\mu^{\text{had, LOVP}}$ overall
- Results indicate slight increase to $(g-2)_\mu$ discrepancy

Thank You

Extra Slides

An iterated fit

Iterating the fit ensures an unbiased solution $\rightarrow$ Forces the fit to converge to an unbiased result

$$\chi^2_1(R_m) = \sum_{m=1}^{N_{clu}} \sum_{n=1}^{N_{clu}} \sum_{i=1}^{N^{(k,m)}} \sum_{j=1}^{N^{(k,n)}} (R_i^{(k,m)} - R_m)\mathbf{C}_0(i^{(k,m)}, j^{(l,n)})(R_j^{(k,n)} - R_n)$$

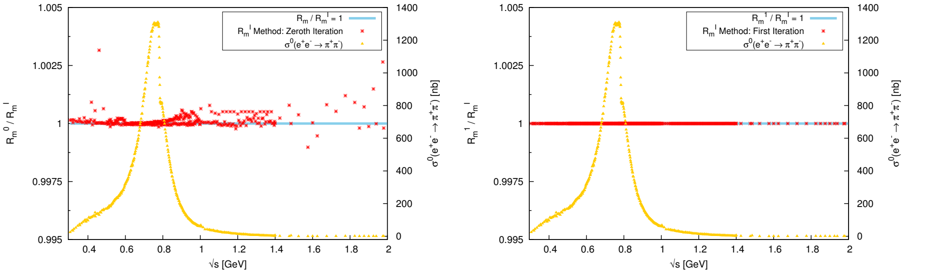
$$\mathbf{C}_0(i^{(k,m)}, j^{(l,n)}) = \mathsf{C}^{\mathsf{stat}}(i^{(k,m)}, j^{(l,n)}) + \mathsf{C}^{\mathsf{sys\%}}(i^{(k,m)}, j^{(l,n)}) R_m^0 R_n^0$$

$\Rightarrow$ Feed the fitted R_m values into the next iteration...

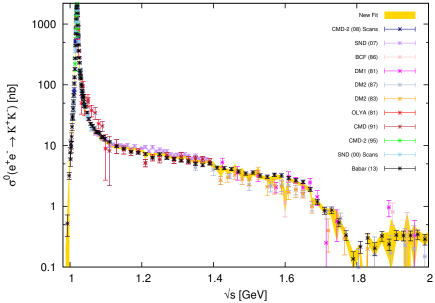
$$\chi^2_2(R_m) = \sum_{m=1}^{N_{clu}} \sum_{n=1}^{N_{clu}} \sum_{i=1}^{N^{(k,m)}} \sum_{j=1}^{N^{(k,n)}} (R_i^{(k,m)} - R_m)\mathbf{C}_1(i^{(k,m)}, j^{(l,n)})(R_j^{(k,n)} - R_n)$$

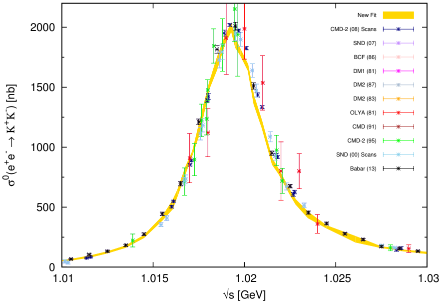
$$\mathbf{C}_1(i^{(k,m)}, j^{(l,n)}) = \mathsf{C}^{\mathsf{stat}}(i^{(k,m)}, j^{(l,n)}) + \mathsf{C}^{\mathsf{sys\%}}(i^{(k,m)}, j^{(l,n)}) R_m^1 R_n^1$$

Repeat until fit converges and returns final fitted values for clusters, $R_m = R_m^I$



$e^+e^- \to K^+K^-$: inclusion of BaBar data [J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **88** (2013) no.3, 032013]





Before [HLMNT(11)]:

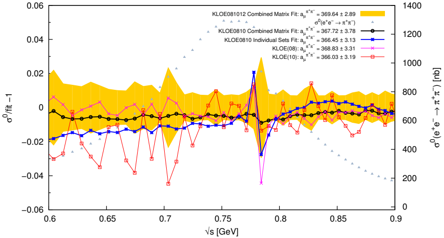
$$a_\mu^{\pi^+\pi^-} \sqrt{s} \leq 2.0 \text{ GeV}) = (22.12 \pm 0.41) \times 10^{-10} \quad ; \quad \chi^2_{\text{min}}/\text{d.o.f.} = 1.95$$

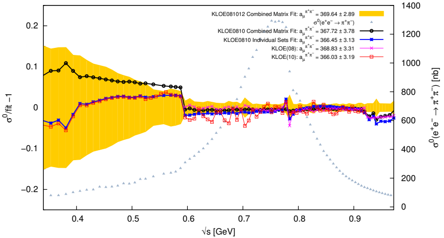
After:

$$a_\mu^{\pi^+\pi^-} (\sqrt{s} \leq 2.0 \text{ GeV}) = (22.75 \pm 0.27) \times 10^{-10} \quad ; \quad \chi^2_{\text{min}}/\text{d.o.f.} = 2.09$$

KLOE Combination: KLOE(0810)

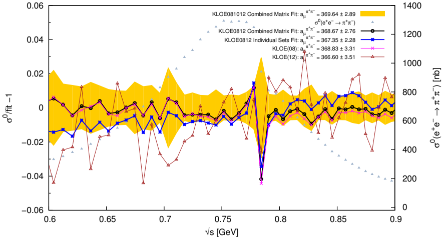
	Individual Sets		Combination Matrix		
KLOE Sets	a_μ	$\chi^2_{\text{min}}/\text{d.o.f.}$	a_μ	$\chi^2_{\text{min}}/\text{d.o.f.}$	Difference
KLOE(0810)	366.45 ± 3.13	7.65	367.72 ± 3.78	7.74	$+1.27$
KLOE(0812)	367.35 ± 2.28	1.01	368.67 ± 2.76	1.29	$+1.32$
KLOE(1012)	366.91 ± 2.29	3.06	366.49 ± 2.27	2.99	-0.42
KLOE(081012)	367.34 ± 2.06	1.21	369.64 ± 2.89	1.27	$+2.30$

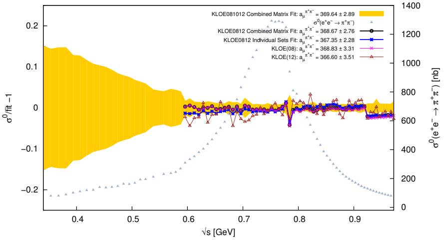




KLOE Combination: KLOE(0812)

	Individual Sets		Combination Matrix		
KLOE Sets	a_μ	χ^2_{min}/d.o.f.	a_μ	χ^2_{min}/d.o.f.	Difference
KLOE(0810)	366.45 ± 3.13	7.65	367.72 ± 3.78	7.74	$+1.27$
KLOE(0812)	367.35 ± 2.28	1.01	368.67 ± 2.76	1.29	$+1.32$
KLOE(1012)	366.91 ± 2.29	3.06	366.49 ± 2.27	2.99	-0.42
KLOE(081012)	367.34 ± 2.06	1.21	369.64 ± 2.89	1.27	$+2.30$





KLOE Combination: KLOE(1012)

	Individual Sets		Combination Matrix		
KLOE Sets	a_μ	χ^2_{min}/d.o.f.	a_μ	χ^2_{min}/d.o.f.	Difference
KLOE(0810)	366.45 ± 3.13	7.65	367.72 ± 3.78	7.74	+1.27
KLOE(0812)	367.35 ± 2.28	1.01	368.67 ± 2.76	1.29	+1.32
KLOE(1012)	366.91 ± 2.29	3.06	366.49 ± 2.27	2.99	-0.42
KLOE(081012)	367.34 ± 2.06	1.21	369.64 ± 2.89	1.27	+2.30