Fit of the Vacuum Polarization

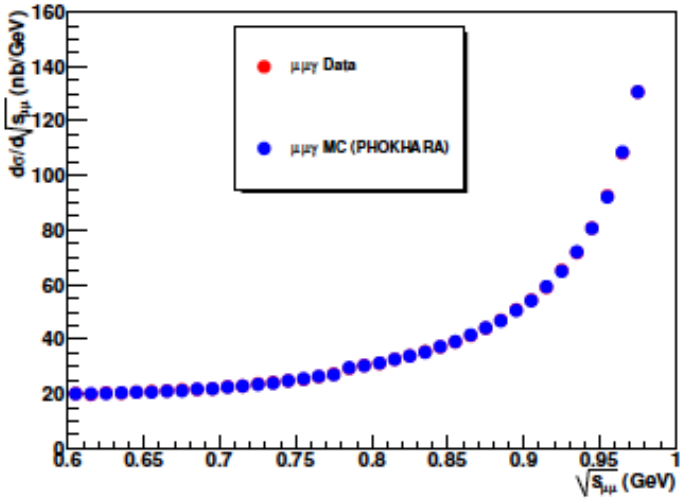
(Preliminary results)

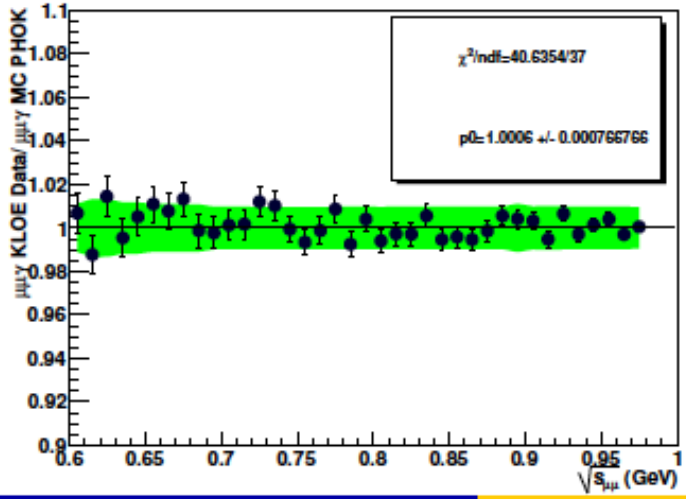
V. De Leo, G. Venanzoni

RMC meeting 20-5-16

Measurement of μμγ with 1.7fb-1

New measurement of the $\mu^+\mu^-\gamma$ cross section





$$\frac{d\sigma}{dM_{\mu\mu}} = \frac{N_{obs}-N_{bkg}}{dM_{\mu\mu}} \frac{(1-\delta_{FSR})}{\epsilon(\sqrt{s_\mu})L}$$

$$\frac{d\sigma^{DATA}_{\mu\mu\gamma}}{d\sigma^{MC}_{\mu\mu\gamma}} = 1.0006 \pm 0.0007$$

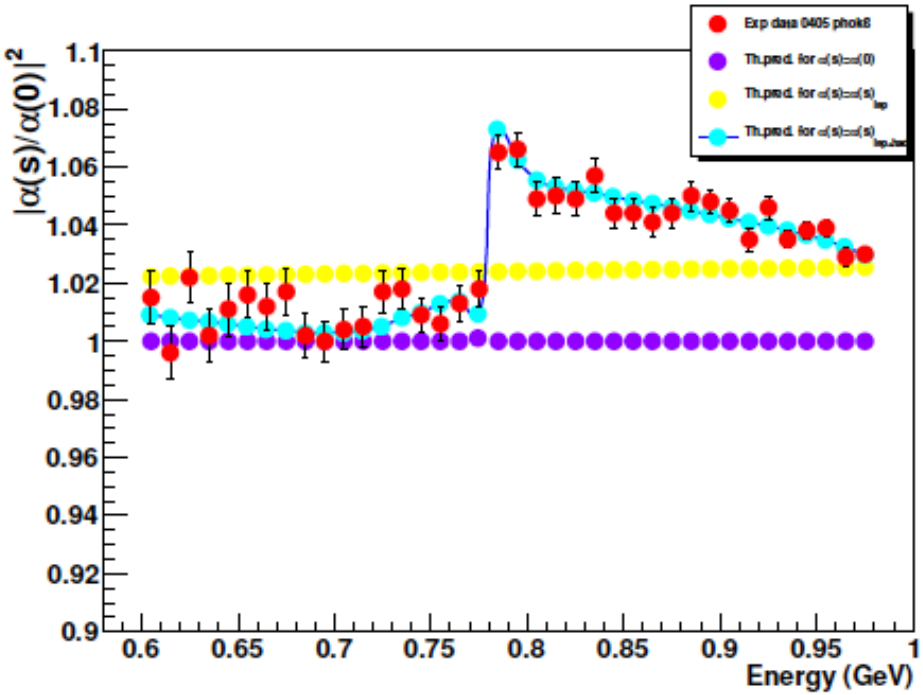
Excellent agreement with NLO theory (PHOKHARA MC) VP inside

H. Czyż, A. Grzelinska, J.H. Khn, G. Rodrigo, Eur. Phys. J. C 39 (2005) 411.

Total systematic error $\sim 1\%$.

Extraction of $|\alpha(s)/\alpha(0)|^2$

Measurement of the effective α_{QED} coupling constant between 600 and 980 MeV



$$\left|\frac{\alpha_{QED}(s)}{\alpha_{QED(0)}}\right|^2 = \frac{\frac{d\sigma^{ISR}}{dM_{\mu\mu}}}{\frac{d\sigma^{MC}}{dM_{\mu\mu}}}$$

$\frac{d\sigma^{MC}}{dM_{\mu\mu}}$ with the VP contribution removed.

$$\left|\frac{\alpha(s)}{\alpha(0)}\right|^2 = 1/(1-\Delta\alpha(s))$$

$$\Delta\alpha(s) = \Delta\alpha_{lep} + \Delta\alpha_{had}$$

(we neglect the top contribution)

- "Theoretical prediction" (provided by the alphaQED package of F. Jergerlehner) $\Delta\alpha_{lep}$ computed in QED with negligible error; $\Delta\alpha_{had}$ obtained by a compilation of data in time-like region (with 0.1% accuracy). $\rightarrow \Delta\alpha_{had}(s) = -(\frac{\alpha s}{3\pi})Re\int_{m_\pi^2}^{\infty} ds' \frac{R(s')}{s'(s'-s-i\epsilon)}$
- The red points show the KLOE data with statistical error bars.

Some results

Re$\Delta\alpha$

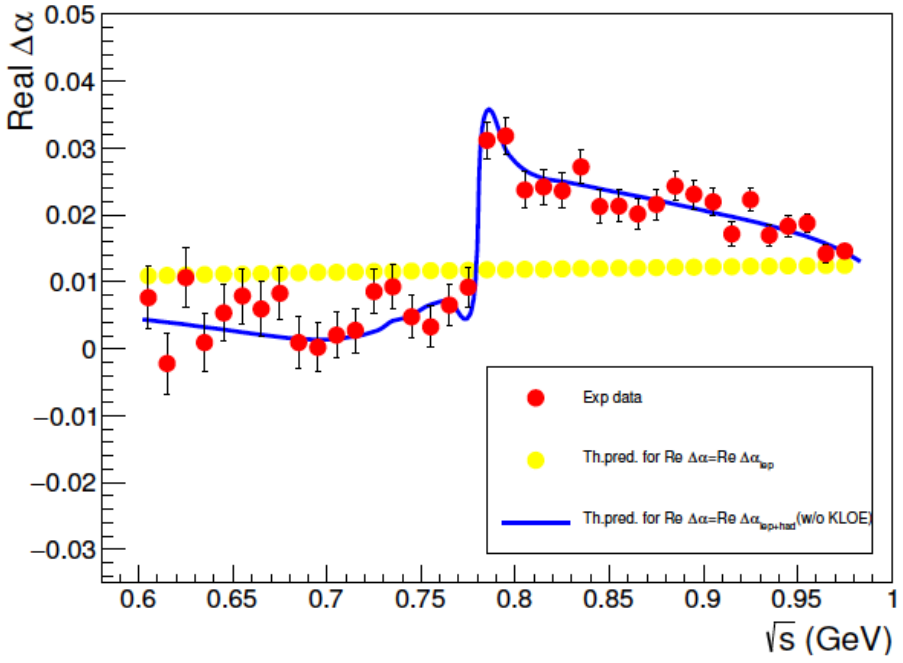


Fig. 8. $Re\,\Delta\alpha$ extracted from the experimental data with only the statistical error included compared with the `alphaQED` prediction when when $Re\,\Delta\alpha = Re\,\Delta\alpha_{lep}$ (yellow points) and $Re\,\Delta\alpha = Re\,\Delta\alpha_{lep+had}$ (blue solid line).

$$Re\,\Delta\alpha = 1 - \sqrt{|\alpha/\alpha(s)|^2 - (Im\,\Delta\alpha)^2}$$

Excellent agreement with the "theoretical prediction" (data compilation)

Fit of ReΔα

Formula* for the VP operator

$$P(s),\Pi(s)=\frac{s}{4\pi^2\alpha}\left[PV\int_{4m_\pi^2}^{\infty}\frac{\sigma^{bare,dressed}_{ee\to\gamma\to everything}(s')ds'}{s-s'}-i\sigma^{bare,dressed}_{ee\to\gamma\to everything}(s)\right]$$

$$P(s)=P_{leptons}(s)+P_{hadrons}(s)$$

narrow resonances

$$J/\psi(1S),\psi(2S),\Upsilon(1S,2S,3S,4S,10860,11020)$$

$$\sigma^{res}_x(s)=\frac{12\pi Br^{e^+e^-}_x}{m_x^2}\frac{m_x^2\Gamma_x^2}{|s-m_x^2-i\,m_x\Gamma_x|^2}\Rightarrow\Pi^{res}_x(s)=\frac{3Br^{e^+e^-}_x}{\alpha}\frac{s}{m_x^2}\frac{m_x\Gamma_x}{s-m_x^2+i\,m_x\Gamma_x}$$

***= F. Ignatov, talk at Radio Montecarlow meeting Oct 08**

Fit of ReΔα

A similar formula

Lettere al Nuovo Cimento Vol. 9, 17, 1974

Hadron Vacuum Polarization Effect for Muon Pair Production in Positron-Electron Collision.

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(ricevuto il 21 Gennaio 1974)

It is expected to have various information on the structure of hadrons from the electron-positron colliding-beam experiment [1,2].

The purpose of this paper is to understand the effect of vacuum polarization of hadrons for the muon pair production up to, say, 2000 MeV total energy by taking into account the well-established vector mesons ρ, ω, φ and ρ′(1600) [3].

In addition to a general investigation of the effect for the lowest-order QED mechanism we are especially concerned with the vacuum polarization due to ω-meson so as to determine the associated parameters including ρ-ω interference effect. Experimentally, the vacuum polarization effect due to φ-meson has already been studied and the parameters associated with this effect have been established at a certain degree [4], so that the problem under consideration will be studied in a comparative manner with that phenomenon. In what follows a careful study at and around the energy corresponding to the ω mass will clarify the problem under consideration.

The $e^+e^- \to \mu^+\mu^-$ cross-section, which takes into account the modification of the photon propagator due to vector mesons, is obtained by a Breit-Wigner formula for $\sigma(e^+e^- \to V)$

(1) $$\sigma'(e^+e^- \to \mu^+\mu^-) = \left|1 - \sum_V \exp[i\Phi_V] \frac{3B_V}{\alpha} \frac{M_V \Gamma_{V\,\mathrm{tot}}}{M_V^2 - 4E^2 - iM_V(\ldots_V)M_V/2E}\right|^2 \sigma(e^+e^- \to \mu^+\mu^-),$$

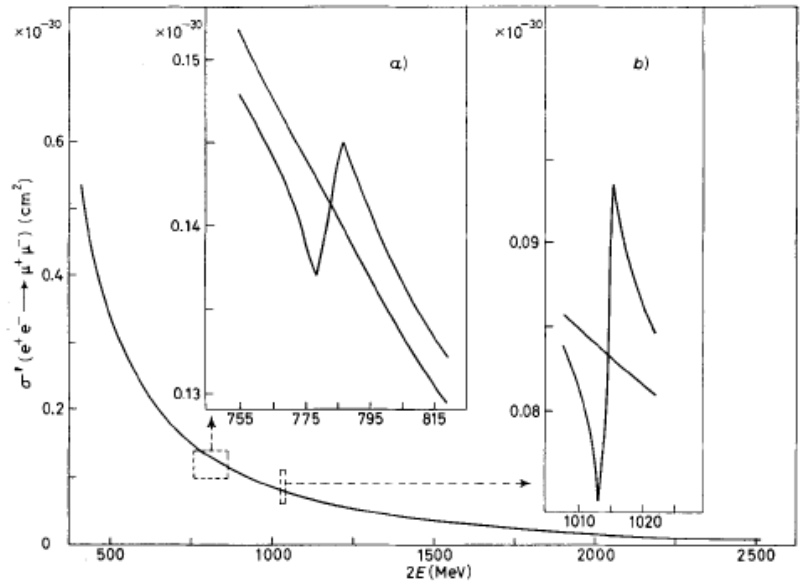
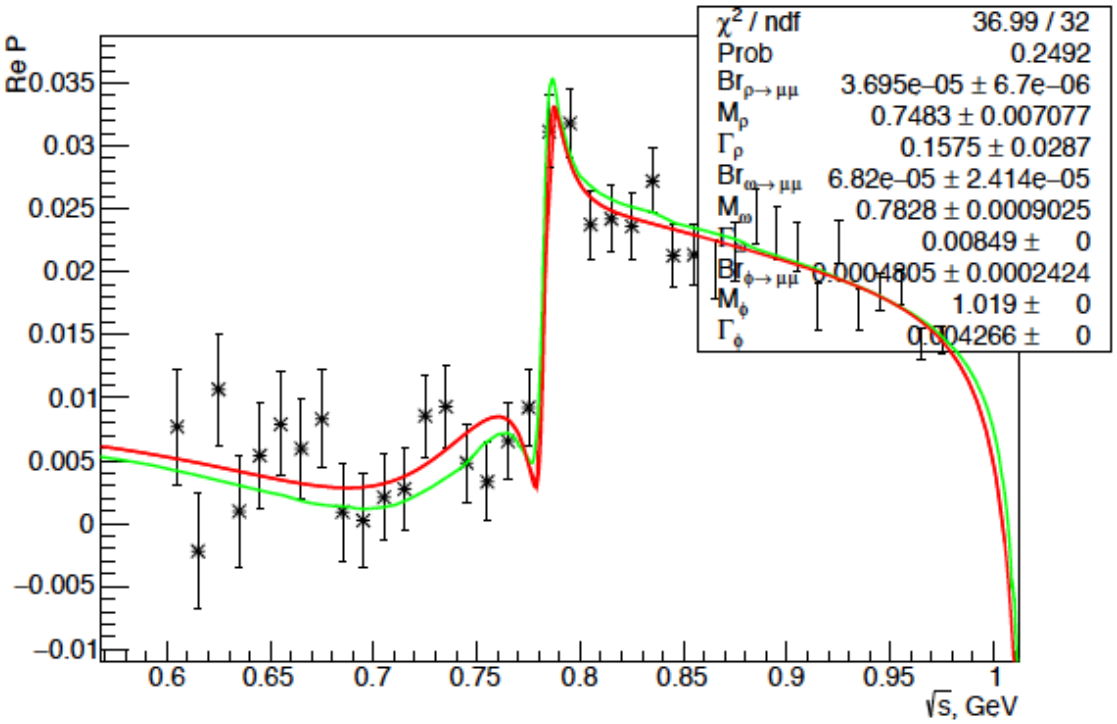


Fig. 1. – Cross-sections $\sigma'(e^+e^- \to \mu^+\mu^-)$ vs. total energy $2E$. Cross-sections at and around ω and φ masses are compared in *a*) and *b*).

TABLE I. – *Parameters used in the numerical estimate.*

States	ρ [5]	ω [3]	φ [4]	ρ′ [3]	
M_V (MeV)	775.4	783.8	1019.5	1600	
Γ_V (MeV)	149.6	9.8	4.00	500	$\Gamma_{\rho' \to \pi\pi} = 180$ [7]
					$\Gamma_{\rho' \to \rho\pi\pi} = 320$
B_V	$4.95 \cdot 10^{-5}$ [3]	$8.7 \cdot 10^{-5}$ [*]	$3.0 \cdot 10^{-4}$	$4.95 \cdot 10^{-5}$ [**]	

Fit of ReΔα



Red curve: overlap of three Breit-Wigner functions (ρ, ω, ϕ).
Green curve: Theoret. paramatrization of the $Re\Delta\alpha$

$$Re\Delta\alpha(s) = \frac{3}{\alpha}\sum_{x=\rho,\omega,\phi}\sqrt{(BR_x^{e^+e^-}\cdot BR_x^{\mu^+\mu^-})}\cdot\frac{s}{m_x}\cdot\frac{\Gamma_x(s-m_x^2)}{(s-m_x^2)^2+m_x^2\Gamma_x^2}$$

Output parameters of the fit: $BR_{\rho\to\mu^+\mu^-}$, M_ρ, Γ_ρ, $BR_{\omega\to\mu^+\mu^-}$, M_ω, $BR_{\phi\to\mu^+\mu^-}$

Fit of Re$\Delta\alpha$

Only statistical error

Parameter	Result from the fit	PDG
M_ρ*, MeV	750 ± 6	775.26 ± 0.25
Γ_ρ*, MeV	139 ± 17	147 ± 0.9
$BR(\rho \to \mu^+\mu^-)BR(\rho \to e^+e^-)$	$(16.9 \pm 3.2) \cdot 10^{-10}$	$(21.5 \pm 1.3) \cdot 10^{-10}$
M_ω, MeV	782.9 ± 0.9	782.65 ± 0.12
$BR(\omega \to \mu^+\mu^-)BR(\omega \to e^+e^-)$	$(4.9 \pm 1.8) \cdot 10^{-9}$	$(6.5 \pm 2.3) \cdot 10^{-9}$

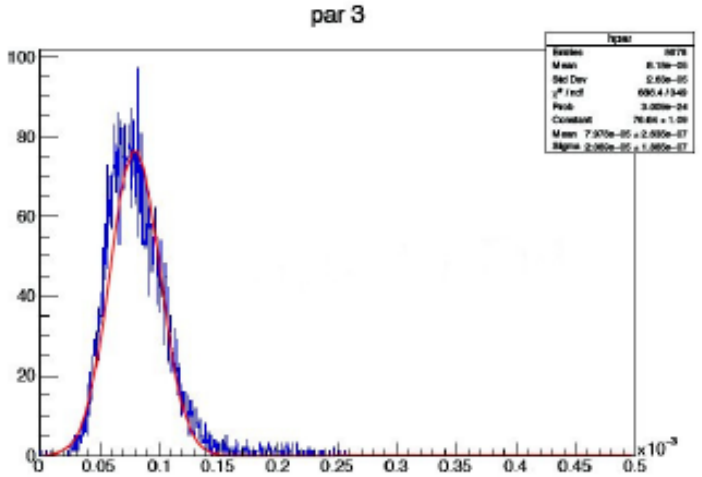
Table : Results from the fit of $Re\,\Delta\alpha$ compared with the world average values. Only statistical errors are reported for the fit values.

***Difference is due to different parametrization (KS our; GS PDG))**

Parameters fixed to the PDG values: $\Gamma_\omega = (8.49 \pm 0.08) MeV$, $M_\phi = (1019.461 \pm 0.019) MeV$, $\Gamma_\phi = (4.266 \pm 0.031)$ MeV, and $BR(\phi \to e^+e^-)BR(\phi \to \mu^+\mu^-) = 8.5^{+0.5}_{-0.6} \cdot 10^{-8}$.

Inclusion of syst error for BR(ω→μμ)

- Systematic error evaluated by Toy MC (5000 iterations)



$$BR(\omega \to \mu^+\mu^-)BR(\omega \to e^+e^-) = (4.9 \pm 1.8 \pm 1.5) \cdot 10^{-9}$$

(where the first error is statistical, the second is systematic)

By dividing for $BR(\omega \to e^+e^-) = (7.28 \pm 0.14) \cdot 10^{-5}$ from PDG we obtain:

$$BR(\omega \to \mu^+\mu^-) = (6.8 \pm 2.4 \pm 2.1) \cdot 10^{-5}$$

compared to $BR(\omega \to \mu^+\mu^-) = (9.0 \pm 3.1) \cdot 10^{-5}$ from PDG.

Conclusions

- Fit on Re$\Delta\alpha$ has been presented. Results on BR($\omega\rightarrow\mu\mu$)* BR($\omega\rightarrow$ee) competitive with PDG (obtained in a different way)
- Evaluation of syst. errors for BR still preliminary (but we don't expect big changes)
- Paper is in progress

Thanks!

Spares

Im$\Delta\alpha$ Calculation

The Imaginary part of $\Delta\alpha(s)$ can be related to the total cross section $\sigma(e^+e^- \to \gamma^* \to \textit{anything})$ ("anything" means any possible state).

$$Im\Delta\alpha = -\frac{\alpha}{3} R(s).$$

where:

$$R(s) = \sigma_{tot} / \frac{4\pi\alpha(s)^2}{3s}$$

- $R(s)$ take into account leptonic and hadronic contribution
- $R(s) = R_{lep}(s) + R_{had}$, where the leptonic part reads:

$$R_{lep} = \sqrt{1 - \frac{4m_l^2}{s}} \left(1 + \frac{2m_l^2}{s}\right), \quad (l = e, \mu, \tau).$$

In the energy region around the $\rho - meson$ we approximate the hadronic cross section to be given by the 2π dominant contribution

Im$\Delta\alpha$ Calculation

The hadronic contribution was calculated by using the KLOE12 measurement of the pion form factor $|F_\pi(s)|^2$ which is the dominant contribution in this region:

$$R_{had}(s) = \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi^0(s)|^2$$

where:

$$|F_\pi^0(s)|^2 = |F_\pi(s)|^2 |\frac{\alpha}{\alpha(s)}|^2.$$

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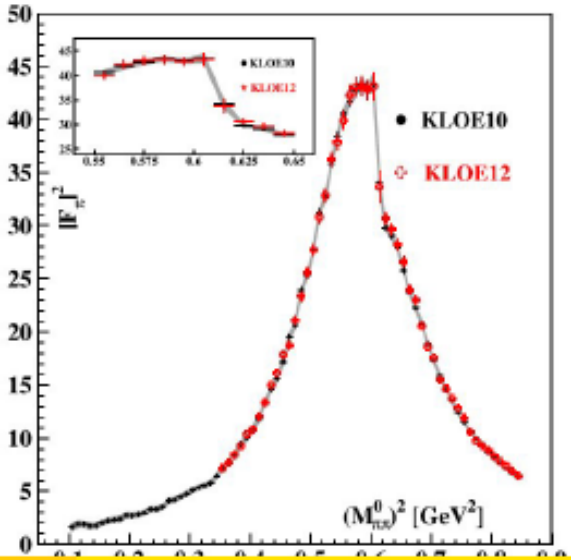
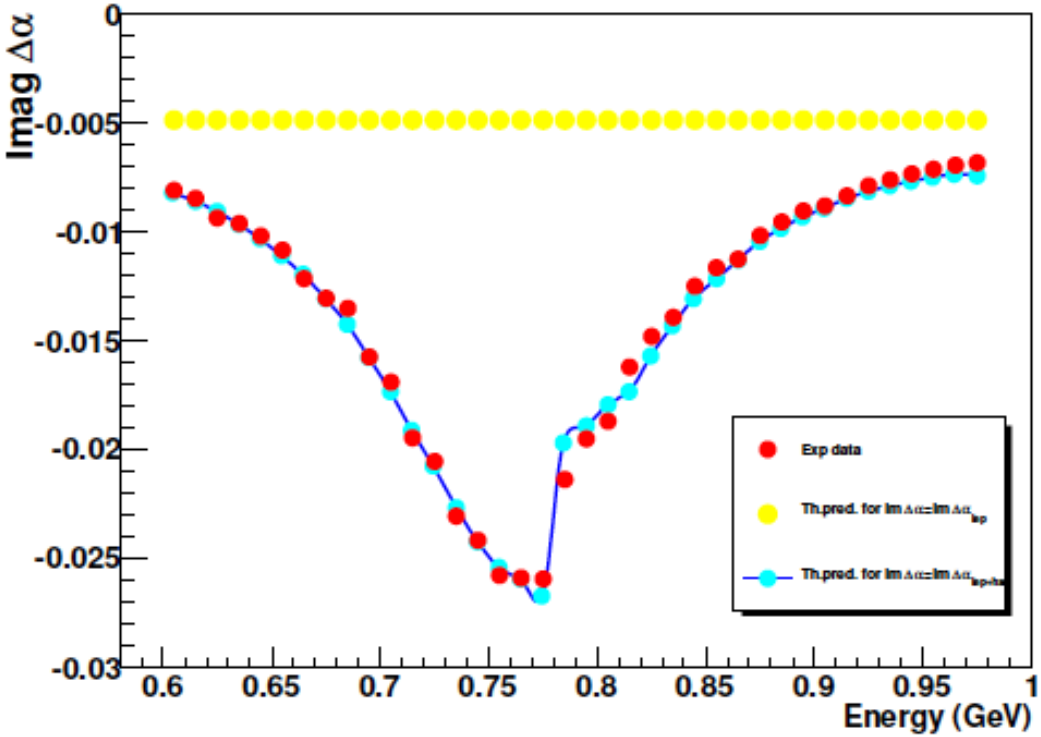


Im$\Delta\alpha$ Calculation

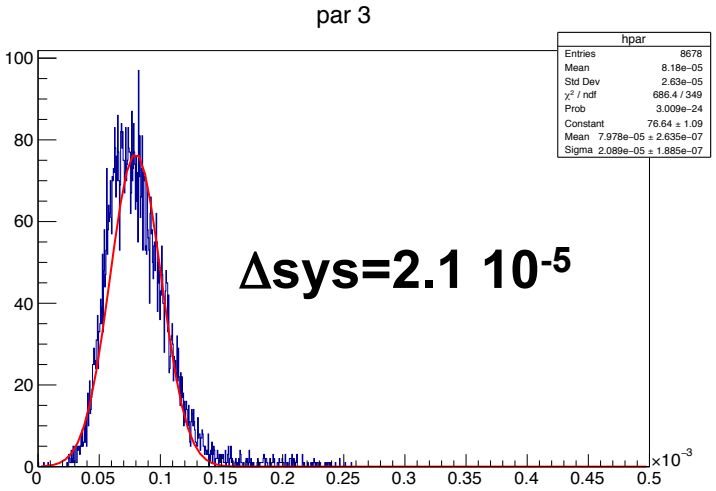


Good agreement with the prediction (based on data compilation)

Inclusion of syst error for BR(ω→μμ)

Systematic error evaluated by Toy MC

5000 iterations



KLOE2 results (preliminary):

$$BR(\omega \to \mu\mu) = (6.8 \pm 2.4_{stat} \pm 2.1_{syst} \pm 0.2_{PDG})10^{-5}$$

BR(ω→μμ)

$\Gamma(\mu^+\mu^-)/\Gamma_{total}$ **Γ_{15}/Γ**

VALUE (units 10^{-5})	EVTS	DOCUMENT ID		TECN	COMMENT
9.0±3.1 OUR FIT					
9.0±2.9±1.1	18	HEISTER	02C	ALEP	$Z \to \mu^+\mu^- + X$

Fit of Re$\Delta\alpha$

Only statistical error

Comparison with PDG

	Fit Output	PDG
$BR_{\rho\to\mu^+\mu^-}$	$(3.69 \pm 0.67) \cdot 10^{-5}$	$(4.55 \pm 0.28) \cdot 10^{-5}$
M_ρ*	$0.748 \pm 0.007\ GeV$	$0.775 \pm 0.0002\ GeV$
Γ_ρ*	$0.157 \pm 0.028\ GeV$	$0.149 \pm 0.0008\ GeV$
$BR_{\omega\to\mu^+\mu^-}$	$(6.8 \pm 2.4) \cdot 10^{-5}$	$(9.0 \pm 3.1) \cdot 10^{-5}$
M_ω	$0.7828 \pm 0.0009\ GeV$	$0.7826 \pm 0.0001\ GeV$
Γ_ω	$0.00849\ GeV$ (fixed)	$0.00849 \pm 8 \cdot 10^{-5}\ GeV$
$BR_{\phi\to\mu^+\mu^-}$	$(4.8 \pm 2) \cdot 10^{-4}$	$(2.87 \pm 0.19) \cdot 10^{-4}$
M_ϕ	$1.019\ GeV$ (fixed)	$1.019 \pm 1.9 \cdot 10^{-5}\ GeV$
Γ_ϕ	$0.00426\ GeV$ (fixed)	$0.000426 \pm 3.1 \cdot 10^{-5}\ GeV$

*Difference is due to different parametrization (KS our; GS PDG))