Deuteron Electromagnetic Structure in Holographic QCD

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Plan of the Talk

- Introduction
- Hadron Stucture in Holographic QCD
- Application to Deuteron Electromagnetic FF and Stucture Functions
- Conclusions

Introduction

- 1993 't Hooft   Holographic Principle

  Information about string theory contained in some region of space can be represented as "Hologram" (theory which lives on the boundary of that region)

- 1997-1998 Maldacena, Polyakov, Witten et al   AdS/CFT correspondence

  Duality of 4D conformal supersymmetric Yang-Mills and supersting theories

  - Matching partition functions gives relation between parameters

  Strings g_s – coupling, l_s – length, R – AdS radius

  SU(N) YM g_{YM} – coupling, 't Hooft coupling $\lambda = g_{YM}^2 N$

  $2\pi g_s = g_{YM}^2\,, \qquad \frac{R^4}{l_s^4} = 2\, g_{YM}^2\, N$

  - Symmetry arguments: Conformal group acting in boundary theory isomorphic to $SO(4,2)$ – the isometry group of AdS_5 space

Introduction

- AdS/CFT $\rightarrow$ AdS/QCD upon breaking conformal invariance

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space

- HQCD models reproduce main features of QCD at low and high energies: chiral symmetry, confinement, power scaling of hadron form factors

- Physical interpretation of extra 5th dimension as Scale

Introduction

- AdS metric Poincaré form

  $$ds^2 = g_{MN}(z)\, dx^M dx^N = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$$ R – AdS radius

- Metric Tensor $g_{MN}(z) = \epsilon^a_M(z)\epsilon^b_N(z)\eta_{ab}$

- Vielbein $\epsilon^a_M(z) = \frac{R}{z}\,\delta^a_M$ (relates AdS and Lorentz metric)

- Manifestly scale-invariant $x \to \lambda x,\ z \to \lambda z$.

- z – extra dimensional (holographic) coordinate;
  $z = 0$ is UV boundary , $z = \infty$ is IR boundary

- Five Dimensions: L = Length, W = Width, H = Height, T = Time, S = Scale

Introduction

- Action for scalar field

$$S_\Phi = \frac{1}{2} \int d^dx dz \sqrt{g}\, e^{-\varphi(z)} \Big(\partial_M \Phi(x,z) \partial^M \Phi(x,z) - m^2\, \Phi^2(x,z) \Big)$$

- Dilaton field $\varphi(z) = \kappa^2 z^2$
- $g = |\mathrm{det} g_{MN}|$
- m – 5d mass, $m^2 R^2 = \Delta(\Delta - 4)$, $\Delta = -3$ conformal dimension
- Kaluza-Klein (KK) expansion $\Phi(x,z) = \sum\limits_n \phi_n(x)\, \Phi_n(z)$
- Tower of KK modes $\phi_n(x)$ dual to 4-dimensional fields describing hadrons
- Bulk profiles $\Phi_n(z)$ dual to hadronic wave functions

Introduction

- Use $-\partial_\mu \partial^\mu \phi_n(x) = M_n^2 \phi_n(x)$

- Substitute
$$\Phi_n(z) = \left(\frac{R}{z}\right)^{1-d} \phi_n(z)$$

- Identify $\Delta = \tau = N + L$ (here $N = 2$ – number of partons in meson)

$$\left[-\frac{d^2}{dz^2} + \frac{4L^2 - 1}{4z^2} + \kappa^4 z^2 - 2\kappa^2\right] \phi_n(z) = M_n^2 \phi_n(z)$$

- Solutions: $\phi_{nL}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+1)}}\, \kappa^{L+1}\, z^{L+1/2}\, e^{-\kappa^2 z^2/2}\, L_n^L(\kappa^2 z^2)$

- $M_{nL}^2 = 4\kappa^2 \left(n + \frac{L}{2}\right)$

- Massless pion $M_\pi^2 = 0$ for $n = L = 0$ Brodsky, Téramond

Introduction

- "Positive dilaton": Brodsky, Téramond

$$S_\Phi^+ = \frac{1}{2}\int d^dx dz\,\sqrt{g}\,e^{\varphi(z)}\left[\partial_M\Phi_+\partial^M\Phi_+ - m^2\,\Phi_+^2\right]$$

- "Negative dilaton": Gutsche, Lyubovitskij, Schmidt, Vega PRD 85 (2012) 076003

$$S_\Phi^- = \frac{1}{2}\int d^dx dz\,\sqrt{g}\,e^{-\varphi(z)}\left[\partial_M\Phi_-\partial^M\Phi_- - (m^2 + U(z))\,\Phi_-^2\right]$$

Potential

$$U(z) = \frac{z^2}{R^2}\left(\varphi''(z) + \frac{1-d}{z}\varphi'(z)\right)$$

- "No-wall"

$$S_\Phi = \frac{1}{2}\int d^dx dz\sqrt{g}\left[\partial_M\Phi\partial^M\Phi - (m^2 + V(z))\Phi^2\right]$$

Potential

$$V(z) = \frac{z^2}{R^2}\left(\frac{1}{2}\varphi''(z) + \frac{1}{4}(\varphi'(z))^2 + \frac{1-d}{2z}\varphi'(z)\right)$$

- All 3 actions are equivalent upon the field rescaling $\Phi_\pm = e^{\mp\varphi(z)}\Phi_\mp = e^{\mp\varphi(z)/2}\Phi$

Introduction

- Extension to higher-spin AdS boson (mesons)

Fields $\Phi \;\to\; \Phi_{M_1 M_2 \cdots M_J}$

5d mass $m^2 R^2 \;\to\; m_J^2 R^2 = (\Delta - J)(\Delta + J - 4)$

Dilaton potential

$$U_J(z) = \frac{z^2}{R^2}\left(\varphi''(z) + \frac{1 + 2J - d}{z}\varphi'(z)\right)$$

Solutions

- $\phi_{nL}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L)}}\; \kappa^{L+1}\; z^{L+1/2}\; e^{-\kappa^2 z^2/2}\; L_n^L(\kappa^2 z^2)$

- $M^2_{nLJ} = 4\kappa^2\left(n + \frac{L+J}{2}\right)$

Introduction

- Scattering problem for AdS field gives information about propagation of external field from z to the boundary $z = 0$ — bulk-to-boundary propagator $\Phi_{\rm ext}(q, z)$ [Fourier-trasform of AdS field $\Phi_{\rm ext}(x, z)$]:
  $\Phi_{\rm ext}(q, z) = \int d^d x e^{-iqx} \, \Phi_{\rm ext}(x, z)$

- Vector field as example
  $$\partial_z \left(\frac{e^{-\varphi(z)}}{z} \partial_z V(q, z) \right) + q^2 \, \frac{e^{-\varphi(z)}}{z} V(q, z) = 0 \,.$$
  $$V(Q, z) \;=\; \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right)$$
  Consistent with GI, fulfills UV and IR boundary conditions :
  $V(Q, 0) = 1 \,, \;\; V(Q, \infty) = 0$
- Hadron form factors

$$F_\tau(Q^2) = \langle \phi_\tau | \hat{V}(Q) | \phi_\tau \rangle = \int\limits_0^\infty dz \, \phi_\tau^2(z) \, V(Q, z) = \frac{\Gamma(\tau) \, \Gamma(a+1)}{\Gamma(a+\tau)}$$

is implemented by a nontrivial dependence of AdS fields on 5-th coordinate

Introduction

- Power scaling at large Q^2

$$F_\tau(Q^2) \sim \frac{1}{(Q^2)^{\tau-1}}$$

Quark counting rules: Matveev-Muradyan-Tavhelidze-Brodsky-Farrar

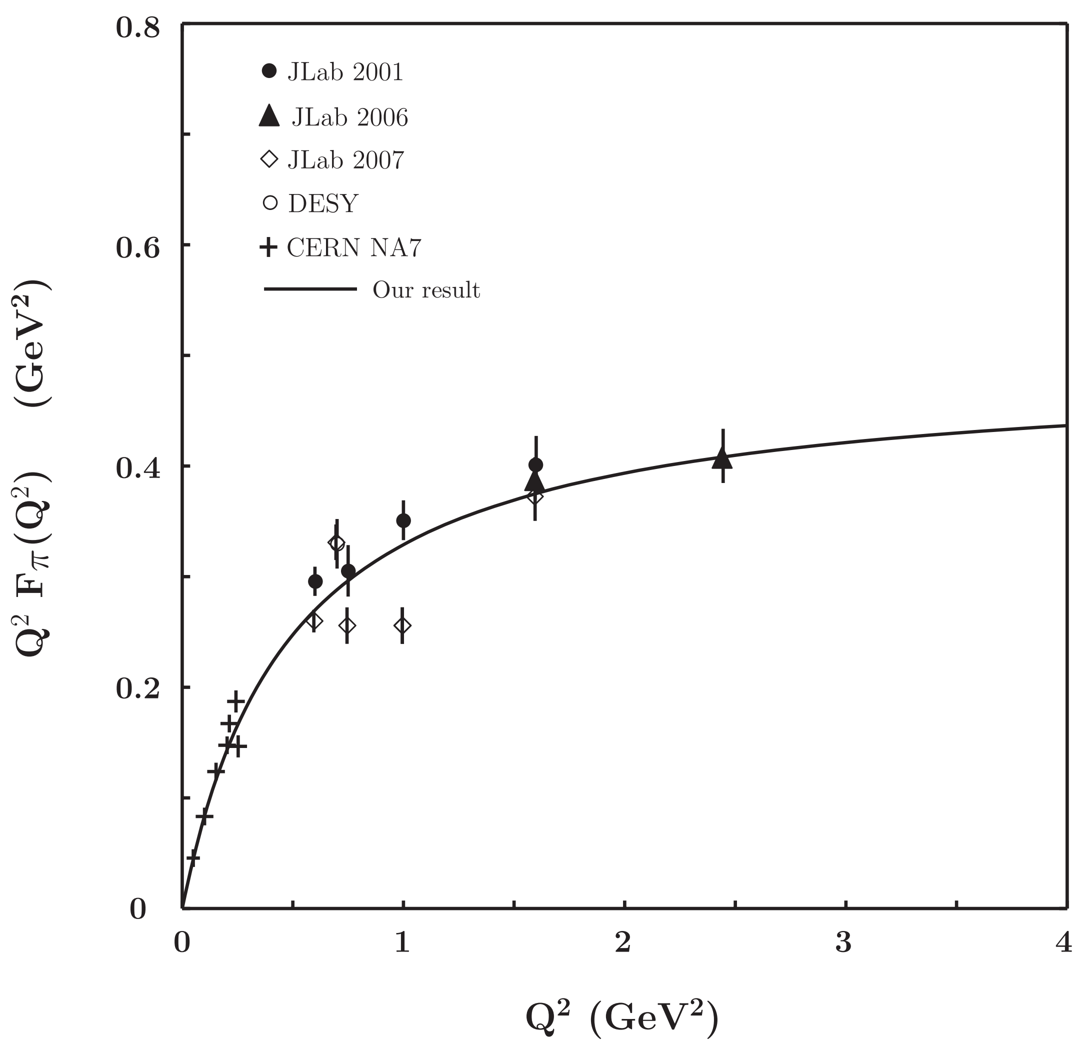
$$\text{Pion}: \quad \frac{1}{Q^2}$$

$$\text{Nucleon(Dirac)}: \quad \frac{1}{Q^4}$$

$$\text{Nucleon(Pauli)}: \quad \frac{1}{Q^6}$$

$$\text{Deuteron(Charge)}: \quad \frac{1}{Q^{10}}$$

Mesons: pion form factor



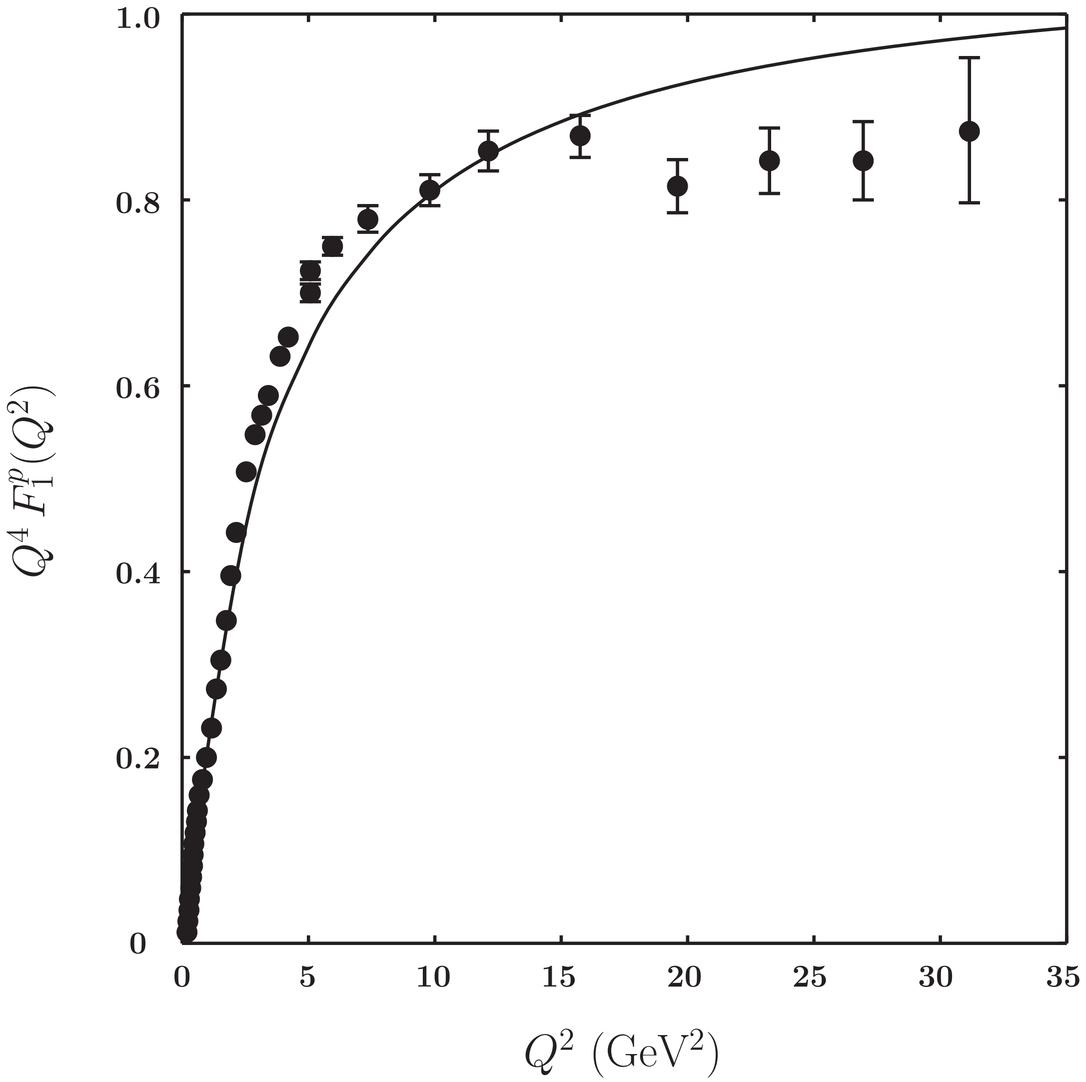
Electromagnetic stucture of nucleons

Scale parameter $\kappa = 383$ MeV

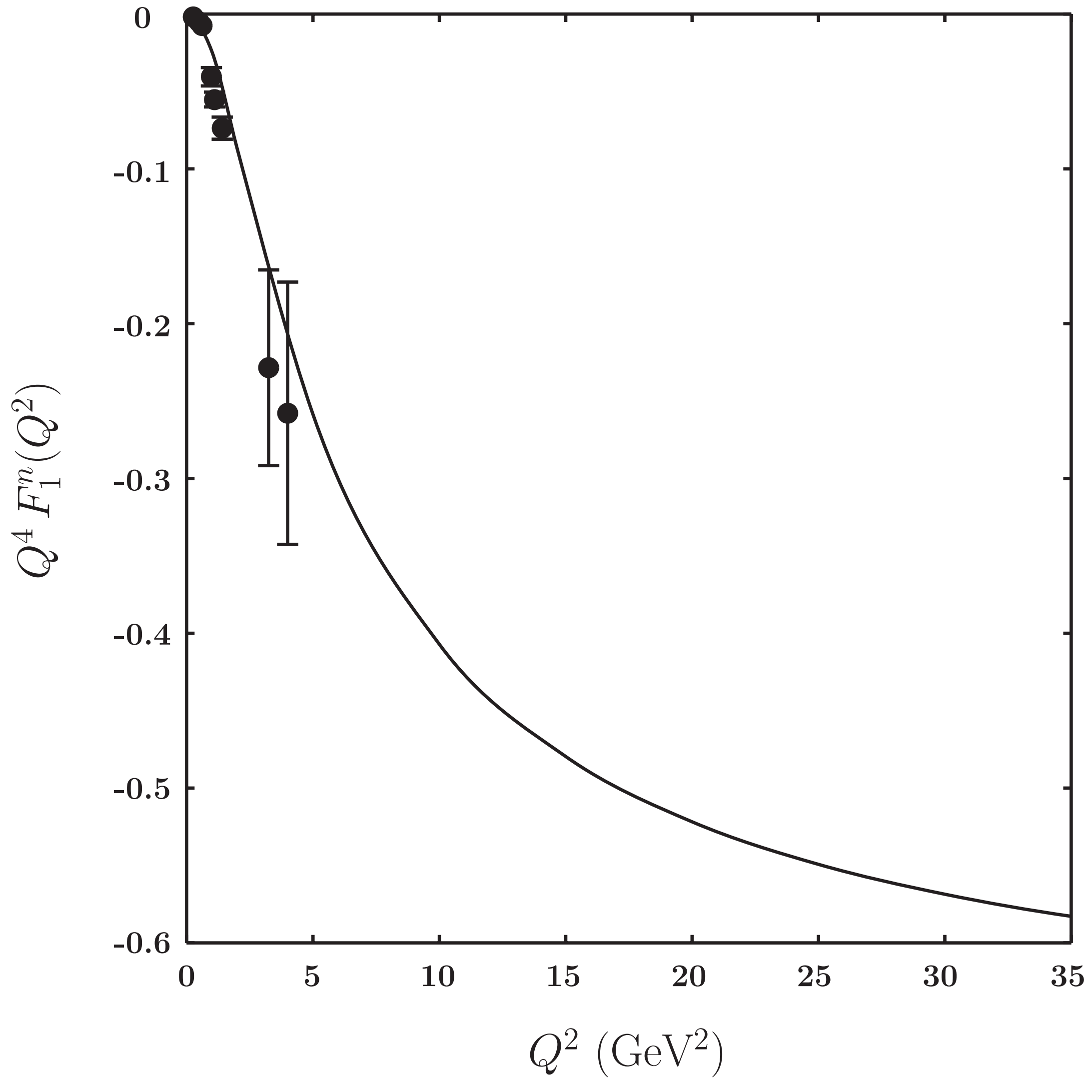
Mass and electromagnetic properties of nucleons

Quantity	Our results	Data
m_p (GeV)	0.93827	0.93827
μ_p (in n.m.)	2.793	2.793
μ_n (in n.m.)	-1.913	-1.913
r_E^p (fm)	0.840	0.8768 $\pm$ 0.0069
$\langle r_E^2 \rangle^n$ (fm^2)	-0.117	-0.1161 $\pm$ 0.0022
r_M^p (fm)	0.785	0.777 $\pm$ 0.013 $\pm$ 0.010
r_M^n (fm)	0.792	$0.862^{+0.009}_{-0.008}$
r_A (fm)	0.667	0.67$\pm$0.01

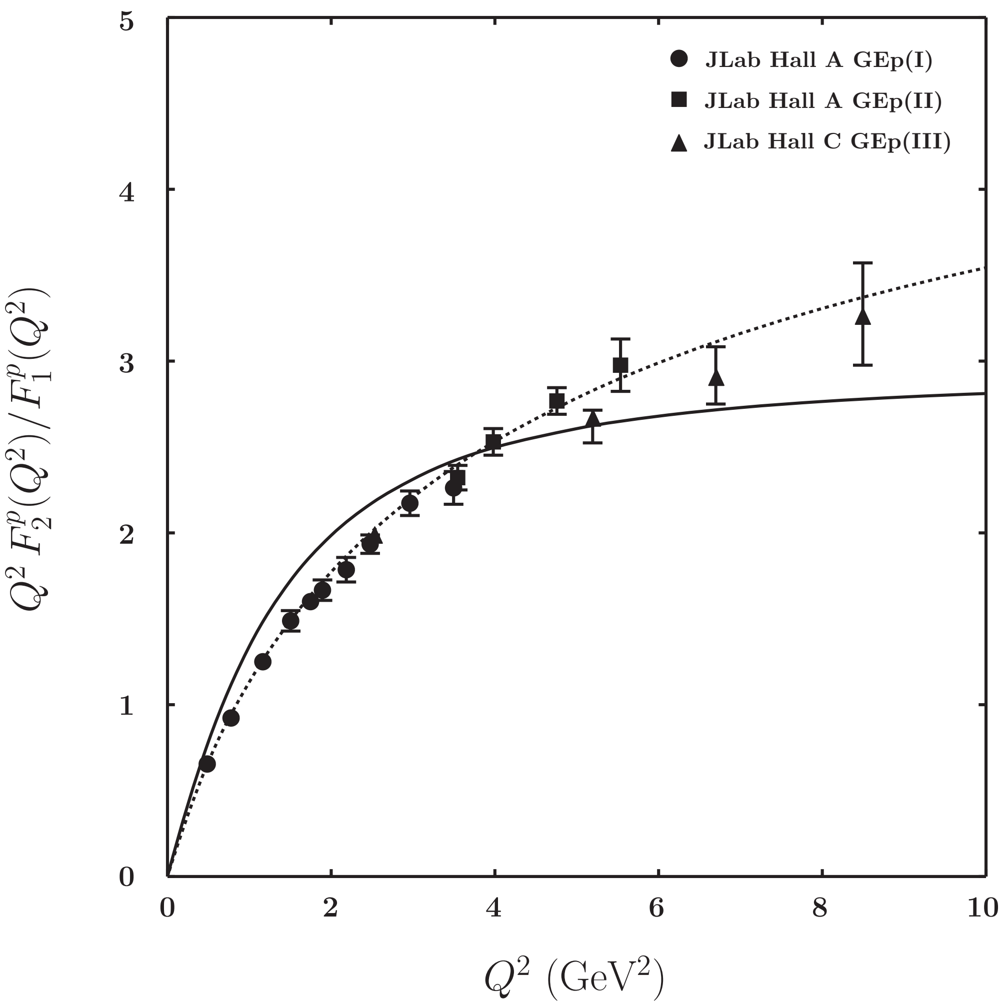
Electromagnetic stucture of nucleons



Electromagnetic stucture of nucleons



Electromagnetic stucture of nucleons



Dotted line: approximation by Stan Brodsky

Roper resonance $N(1440)$

- Put $n = 1$ and get solutions dual to Roper:

$$M_{\mathcal{R}} \simeq 1440 \text{ MeV}$$

- $N \to R + \gamma$ transition

$$M^\mu = \bar{u}_{\mathcal{R}} \left[\gamma_\perp^\mu \, F_1(q^2) \, + \, i\sigma^{\mu\nu} \frac{q_\nu}{M_{\mathcal{R}}} \, F_2(q^2) \right] u_N \,, \quad \gamma_\perp^\mu = \gamma^\mu - q^\mu \frac{\not{q}}{q^2}$$

- Helicity amplitudes

$$H_{\pm\frac{1}{2}0} = \sqrt{\frac{Q_-}{Q^2}} \left(F_1 M_+ - F_2 \frac{Q^2}{M_{\mathcal{R}}} \right)$$

$$H_{\pm\frac{1}{2}\pm1} = -\sqrt{2Q_-} \left(F_1 + F_2 \frac{M_+}{M_{\mathcal{R}}} \right)$$

- Alternative set [Weber, Capstick, Copley et al]

$$A_{1/2} = -b \, H_{\frac{1}{2}1} \,, \quad S_{1/2} = b \, \frac{|\mathbf{p}|}{\sqrt{Q^2}} \, H_{\frac{1}{2}0} \,,$$

$$Q_\pm = M_\pm^2 + Q^2 \,, \quad M_\pm = M_{\mathcal{R}} \pm M_N \,, \quad b = \sqrt{\frac{\pi\alpha}{2EM_{\mathcal{R}}M_N}}$$

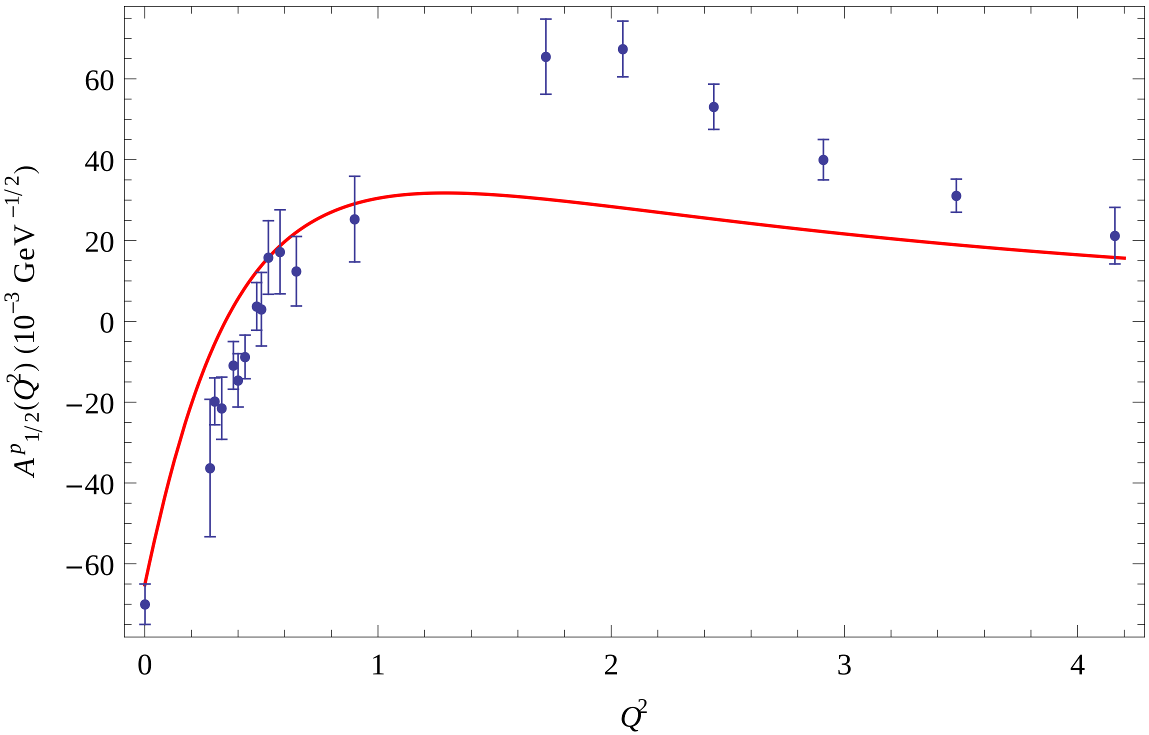
Roper resonance $N(1440)$

Helicity amplitudes $A^N_{1/2}(0)\,,\ S^N_{1/2}(0)$

Quantity	Our results	Data
$A^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.065	-0.065 $\pm$ 0.004
$A^n_{1/2}(0)$ (GeV$^{-1/2}$)	0.040	0.040 $\pm$ 0.010
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	0.040	
$S^p_{1/2}(0)$ (GeV$^{-1/2}$)	-0.040	

Roper resonance $N(1440)$

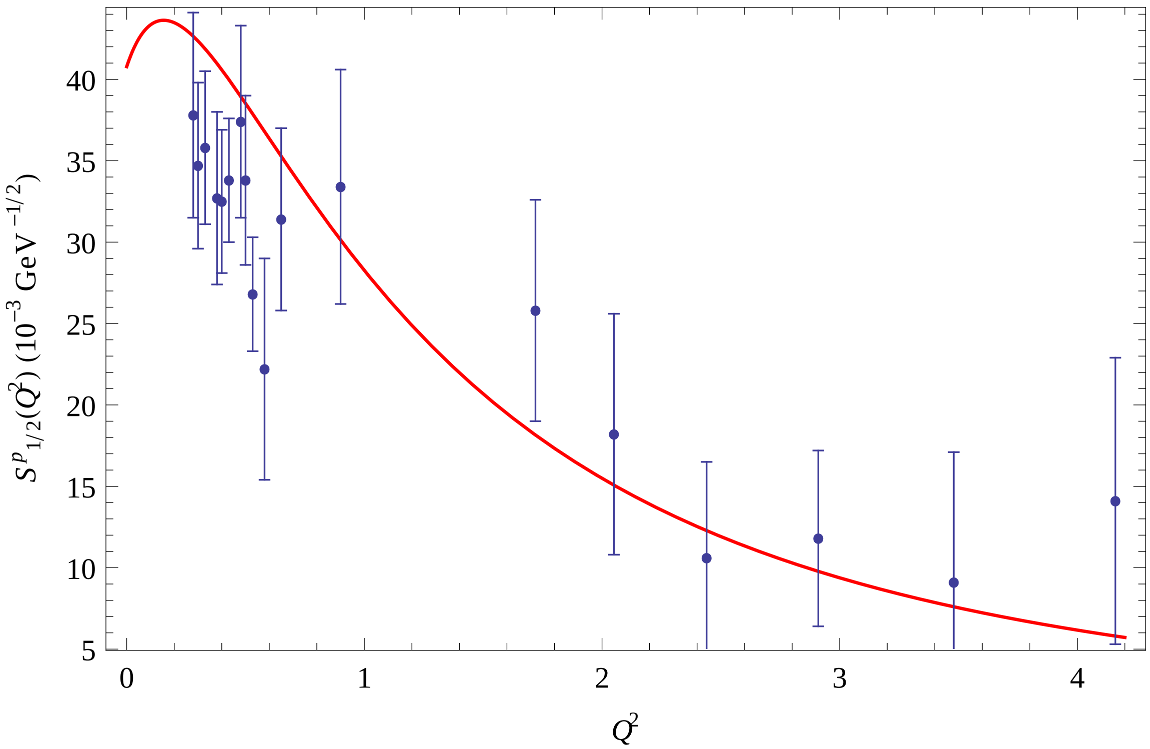
Helicity amplitude $A^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, PRC86 (2012) 035203

Roper resonance $N(1440)$

Helicity amplitude $S^p_{1/2}(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, PRC86 (2012) 035203

Deuteron in a soft-wall AdS/QCD approach

- Strategy
- We first propose an effective action describing the dynamics of the deuteron in the presence of an external vector field
- Based on this action the deuteron electromagnetic form factors are calculated, displaying the correct $1/Q^{10}$ power scaling for large Q^2 values.
- This finding is consistent with quark counting rules and the earlier observation that this result holds in confining gauge/gravity duals.
- The Q^2 dependence of the deuteron form factors is defined by a single and universal scale parameter κ, which is fixed from data.

Deuteron in a soft-wall AdS/QCD approach

- Effective action in terms of AdS fields $d^M(x,z)$ and $V^M(x,z)$
- $d^M(x,z)$ – dual to Fock component contributing to deuteron with twist $\tau = 6$
- $V^M(x,z)$ – dual to the electromagnetic field

$$
\begin{aligned}
S \quad &= \quad \int d^4x dz\, e^{-\varphi(z)} \Big[-\frac{1}{4} F_{MN}(x,z) F^{MN}(x,z) - D^M d_N^\dagger(x,z) D_M d^N(x,z) \\
&- \quad i c_2(z) F^{MN}(x,z) d_M^\dagger(x,z) d_N(x,z) \\
&+ \quad \frac{c_3(z)}{4M_d^2} \partial^M F^{NK}(x,z) \Big(-d_M^\dagger(x,z) \overleftrightarrow{D}_K\, d_N(x,z) + \mathrm{H.c.} \Big) \\
&+ \quad d_M^\dagger(x,z) \Big(\mu^2 + U(z) \Big) d^M(x,z) \Big]
\end{aligned}
$$

Deuteron in a soft-wall AdS/QCD approach

- $F^{MN} = \partial^M V^N - \partial^N V^M$ - stress tensor of vector field

  $D^M = \partial^M - ieV^M(x,z)$ - covariant derivative

  $\mu^2 R^2 = (\Delta - 1)(\Delta - 3)$ - five-dimensional mass

  $\Delta = 6 + L$ is the dimension of $d^M(x,z)$

  L is the maximum value orbital angular momentum

  $U(z) = U_0\, \varphi(z)/R^2$ is the confinement potential

  U_0 is constant fixed the deuteron mass.

  Use axial gauge for both vector fields $d^z(x,z) = 0$ and $V^z(x,z) = 0$

Deuteron in a soft-wall AdS/QCD approach

- First perform Kaluza-Klein (KK) decomposition for vector AdS field dual to deuteron

$$d^\mu(x,z) = \exp\Big[\frac{\varphi(z) - A(z)}{2}\Big] \sum_n d_n^\mu(x)\Phi_n(z)\,,$$

$d_n^\mu(x)$ is the tower of the KK fields dual to the deuteron fields with radial quantum number n and twist-dimension $\tau = 6$, and $\Phi_n(z)$ are their bulk profiles.
Then we derive the Schrödinger-type equation of motion for the bulk profile

$$\Big[-\frac{d^2}{dz^2} + \frac{4(L+4)^2-1}{4z^2} + \kappa^4 z^2 + \kappa^2 U_0\Big]\Phi_n(z) = M_{d,n}^2 \Phi_n(z)\ .$$

Deuteron in a soft-wall AdS/QCD approach

- The analytical solutions of this EOM read

$$\Phi_n(z) = \sqrt{\frac{2n!}{(n+L+4)!}}\, \kappa^{L+5}\, z^{L+9/2}\, e^{-\kappa^2 z^2/2}\, L_n^{L+4}(\kappa^2 z^2)\,,$$

$$M^2_{d,n} = 4\kappa^2 \left[n + \frac{L+5}{2} + \frac{U_0}{4} \right],$$

where $L_n^m(x)$ are the generalized Laguerre polynomials.

- Restricting to the ground state $(n = 0,\ L = 0)$ we get $M_d = 2\kappa \sqrt{\frac{5}{2} + \frac{U_0}{4}}$
- Using central value for deuteron mass $M_d = 1.875613$ GeV and $\kappa = 190$ MeV (fitted from data on electromagnetic deuteron form factors), we fix $U_0 = 87.4494$.

Deuteron in a soft-wall AdS/QCD approach

- We can compare this value for the deuteron scale parameter to the analogous one of κ_N defining the nucleon properties - mass and electromagnetic form factors. In description of nucleon case we fixed the value to $\kappa_N \simeq 380$ MeV, which is 2 times bigger than the deuteron scale parameter κ.

- Difference between the nucleon and deuteron scale parameters can be related to the change of size of the hadronic systems - the deuteron as a two-nucleon bound state is 2 times larger than the nucleon.

Deuteron in a soft-wall AdS/QCD approach

- The gauge-invariant matrix element describing the interaction of the deuteron with the external vector field (dual to the electromagnetic field) reads

$$
\begin{aligned}
M^{\mu}_{\rm inv}(p,p') \quad &= \quad -\Big(G_1(Q^2)\epsilon^*(p')\cdot\epsilon(p) - \frac{G_3(Q^2)}{2M_d^2}\,\epsilon^*(p')\cdot q\,\epsilon(p)\cdot q\Big)\,(p+p')^\mu \\
&- \quad G_2(Q^2)\,\Big(\epsilon^\mu(p)\,\epsilon^*(p')\cdot q - \epsilon^{*\mu}(p')\,\epsilon(p)\cdot q\Big)
\end{aligned}
$$

where $\epsilon(\epsilon^*)$ and $p(p')$ are the polarization and four-momentum of the initial (final) deuteron, and $q = p' - p$ is the momentum transfer.

Deuteron in a soft-wall AdS/QCD approach

- Three EM form factors $G_{1,2,3}$ of the deuteron are related to the charge G_C, quadrupole G_Q and magnetic G_M form factors by
- Expressions for the form factors

$$G_C = G_1 + \frac{2}{3}\tau_d G_Q\,, \quad G_M \;=\; G_2\,, \quad G_Q = G_1 - G_2 + (1+\tau_d)G_3, \quad \tau_d = \frac{Q^2}{4M_d^2}$$

These form factors are normalized at zero recoil as

$$G_C(0) = 1\,, \quad G_Q(0) = M_d^2\mathcal{Q}_d = 25.83\,, \quad G_M(0) = \frac{M_d}{M_N}\mu_d = 1.714$$

- $\mathcal{Q}_d = 7.3424$ GeV^{-2} and $\mu_d = 0.8574$ – quadrupole and magnetic moments of the deuteron.

Deuteron in a soft-wall AdS/QCD approach

- Structure functions

$$\begin{aligned} A(Q^2) &= G_C^2(Q^2) + \frac{2}{3}\tau_d G_M^2(Q^2) + \frac{8}{9}\tau_d^2 G_Q^2(Q^2)\,, \\ B(Q^2) &= \frac{4}{3}\tau_d(1+\tau_d) G_M^2(Q^2)\,. \end{aligned}$$

- Scaling at large Q^2

$$\begin{aligned} \text{Leading}: &\qquad A(Q^2) \sim B(Q^2) \sim G_C^2(Q^2) \sim G_1(Q^2) \sim 1/Q^{10} \\ \text{Subleading}: &\qquad G_M(Q^2) \sim G_Q^2(Q^2) \sim G_2(Q^2) \sim 1/Q^{12} \end{aligned}$$

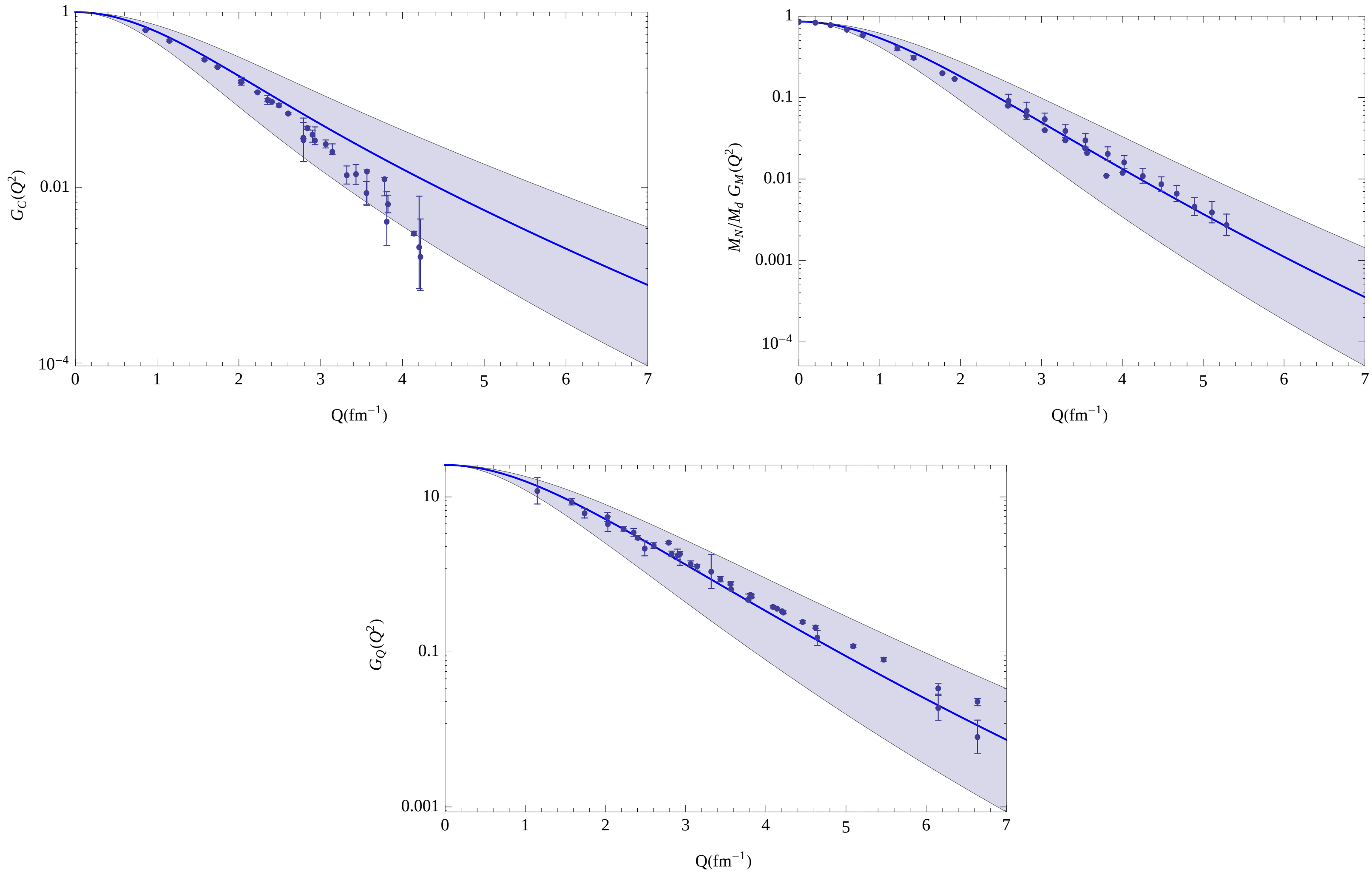
It fixes the z dependence of $c_2(z)$ and $c_3(z)$

$$\begin{aligned} c_2(z) &= \frac{M_d}{30 M_N} \mu_d\, \kappa^2 z^2 \\ c_3(z) &= \left(M_d^2 \mathcal{Q}_d - 1 + \frac{M_d}{30 M_N} \mu_d \right) \kappa^2 z^2 \end{aligned}$$

Deuteron in a soft-wall AdS/QCD approach

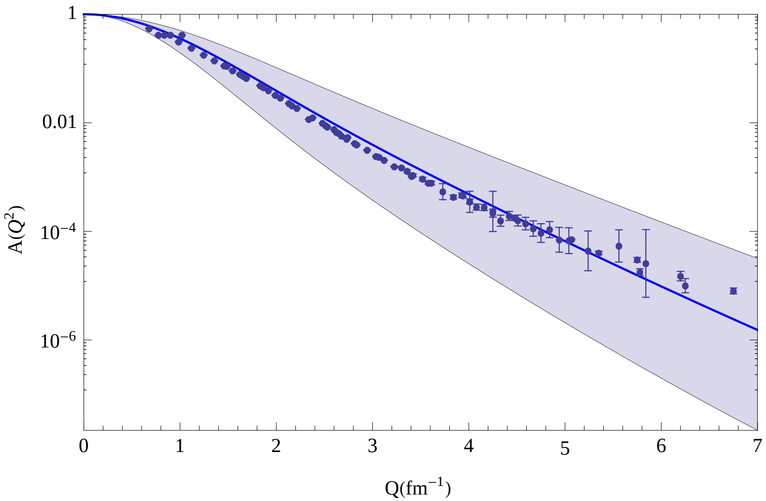
- Numerical results for the charge $G_C(Q^2)$, quadrupole $G_Q(Q^2)$ and magnetic $G_M(Q^2)$ FF
- Shaded band corresponds to values of κ in range of 150 MeV $< \kappa <$ 250 MeV.
- Increase of the parameter κ leads to an enhancement of the form factors.
- The best description of the data on the deuteron form factors is obtained for $\kappa = 190$ MeV and is shown by the solid line.

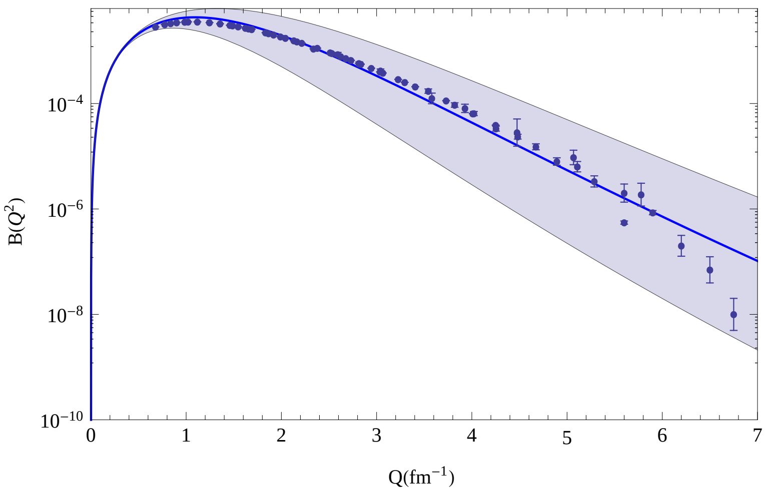
Deuteron in a soft-wall AdS/QCD approach



Deuteron form factors

Deuteron in a soft-wall AdS/QCD approach





Structure Functions $A(Q^2)$ and $B(Q^2)$

Deuteron in a soft-wall AdS/QCD approach

Charge radius
$r_C = (-6G'_C(0))^{1/2} = 1.85$ fm
Data: $r_C = 2.13 \pm 0.01$ fm

Magnetic radius $r_M = (-6G'_M(0)/G_M(0))^{1/2} = 2.29$ fm
Data $r_M = 1.90 \pm 0.14$ fm.

Summary

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- HQCD models reproduce main features of QCD at low and high energies
- Soft–wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances
- Mesons, baryons, exotics from unified point view and including high Fock states