



Interpretation of data on $\eta \rightarrow \pi\pi\pi$ Dalitz plots distributions

A. Kupść

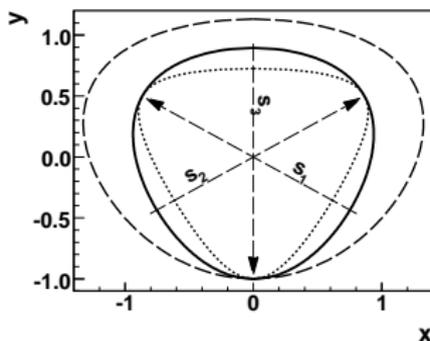
with C.-O. Gullström and A. Rusetsky

- Introduction
- Status of Dalitz plot parameters
- Amplitude from fits to the data
- Outlook

Decay $0 \rightarrow 1 + 2 + 3$

2 variables:

$$s_i \equiv (\mathcal{P}_0 - \mathcal{P}_i)^2 = (m_0 - m_i)^2 - 2T_i m_0 \quad s_3 \equiv s$$

For $m_1 = m_2 = m$:

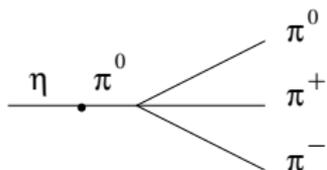
$$x \equiv \frac{1}{\sqrt{3}} \frac{T_1 - T_2}{\langle T \rangle}; \quad y \equiv \frac{T_3}{\langle T \rangle} - 1.$$

$$|A(x, y)|^2 \propto 1 + ay + by^2 + dx^2 + fy^3 + \dots$$

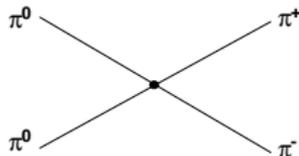


- Current Algebra (\equiv ChPT LO) (67)

$\eta - \pi^0$ mixing:



\Rightarrow



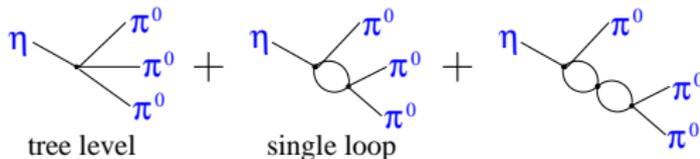
\Rightarrow elementary low energy QCD process – $\pi\pi$ scattering

- $\eta \rightarrow \pi^+\pi^-\pi^0$ $\Gamma=66$ eV (Exp 294 eV)
- NLO ChPT
- NLO ChPT + dispersive
- NNLO ChPT

Gasser, Leutwyler (84)

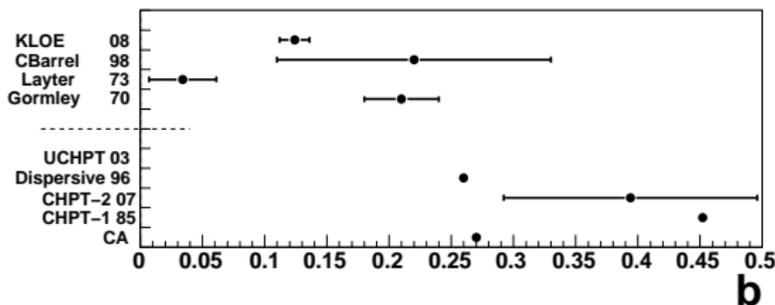
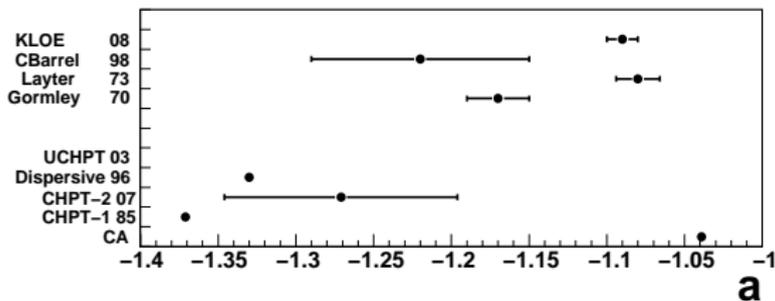
(96)

Bijnens, Ghorbani (07)





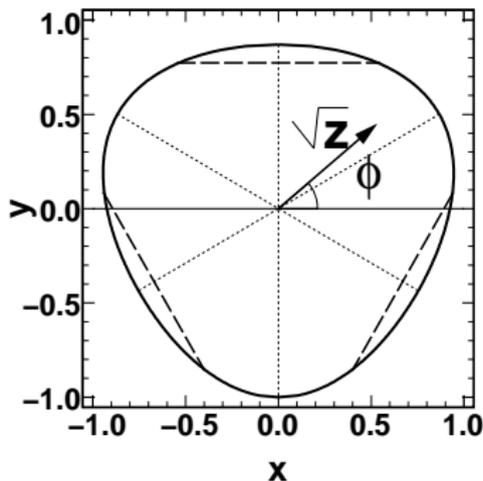
Dalitz parameters: $\eta \rightarrow \pi^+ \pi^- \pi^0$



$$|A(x, y)|^2 \propto 1 + ay + by^2 + dx^2 + fy^3 + \dots$$



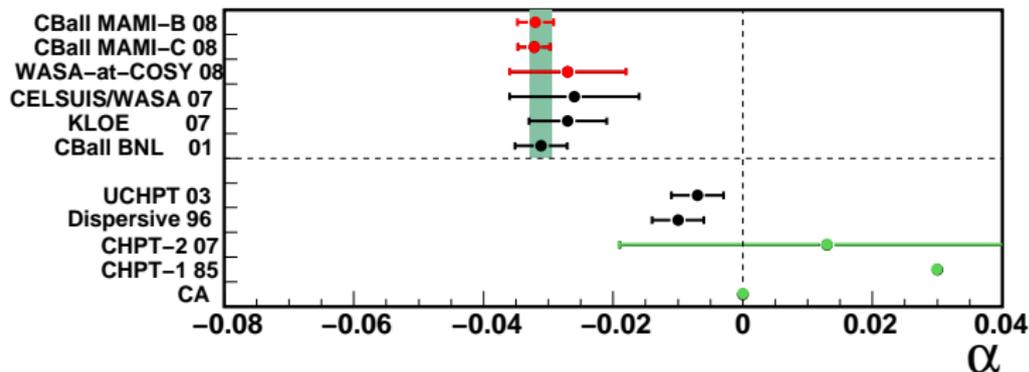
$$\eta \rightarrow 3\pi^0$$



- Symmetrized Dalitz plot
- $z = x^2 + y^2$,
 $-30 < \phi < 30^\circ$
- $|\bar{A}(z, \phi)|^2 \propto 1 + 2 \alpha z + \dots$



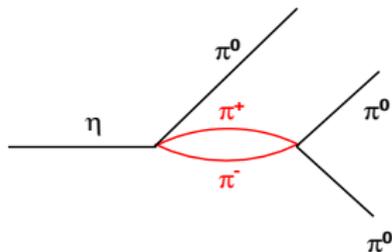
$$\eta \rightarrow 3\pi^0: \alpha$$



- Experiment: average $\alpha = -0.0312 \pm 0.0017$
- ChPT LO: $\alpha = 0$
- In ChPT one, two loop $\alpha > 0$



Non-relativistic Effective Field Theory



- ChPT is not precise enough:
 - ... usually $m_{\pi^+} = m_{\pi^0}$ in loops \Rightarrow no cusp
cusp: [Ditche, Kubis, Meißner \(08\)](#)
 - ... do not reproduce Dalitz plot parameters correctly
 - ... 2-loop large uncertainties (due to LEC)
- Non-relativistic Effective Field Theory:
 - [Colangelo, Gasser, Kubis, Rusetsky PLB638:187,2006](#)
 - ... NR approximation for $\pi^0\pi^0 \rightarrow \pi^+\pi^-$ threshold
 - ... Expansion in scattering lengths
 - ... ChPT propagators \Rightarrow additional LEC



NREFT for $K_L, \eta \rightarrow 3\pi^0$

Bissegger, Fuhrer, Gasser, Kubis, Rusetsky PLB659:576,2008

$$\begin{aligned} A_{\eta \rightarrow \pi^0 \pi^0 \pi^0} &\equiv \bar{A} = \bar{A}^{\text{tree}} + \bar{A}^{1\text{-loop}} + \bar{A}^{2\text{-loop}} + \dots \\ A_{\eta \rightarrow \pi^+ \pi^- \pi^0} &\equiv A = A^{\text{tree}} + A^{1\text{-loop}} + A^{2\text{-loop}} + \dots \end{aligned}$$

$$\begin{aligned} \bar{A}^{\text{tree}} &= K_0 + K_1(T_1^2 + T_2^2 + T_3^2) \\ A^{\text{tree}} &= L_0 + L_1 T_3 + L_2 T_3^2 + L_3(T_1 - T_2)^2 \end{aligned}$$

Example 1-loop result for $\bar{A} = 3A = \text{const}$:

$$\begin{aligned} \bar{A}^{1\text{-loop}} &= \frac{4i}{3}(a_2 - a_0)L_0(\sigma(s_3) + \sigma(s_2) + \sigma(s_1)) \\ &+ \frac{2i}{3}(a_2 + 2a_0)K_0(\sigma_0(s_3) + \sigma_0(s_2) + \sigma_0(s_1)) \end{aligned}$$

Predictions for $\eta \rightarrow \pi^0 \pi^0 \pi^0$

Gullström, AK, Rusetsky (08)

 $\Delta I = 1$ rule (Condon-Shortley phases):

$$\bar{A}(s_1, s_2, s_3) = -A(s_1, s_2, s_3) - A(s_2, s_3, s_1) - A(s_3, s_1, s_2)$$

$$K_0 = -(3L_0 + L_1 Q - L_3 Q^2),$$

$$K_1 = -(L_2 + 3L_3),$$

where $Q = M_\eta - 3M_{\pi^0}$.Fit L_j to $\eta \rightarrow \pi\pi\pi$ data:

- only KLOE data on $\eta \rightarrow \pi^+ \pi^- \pi^0$
- use also α result



Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$

KLOE paper fit amplitude using D'Ambrosio, Isidori $\pi - \pi$ rescattering matrix.

JHEP0805:006,2008

Amplitudes:

$$A(x, y) = A_c(1 + \alpha y + \beta y^2 + \gamma x^2 + \dots)$$

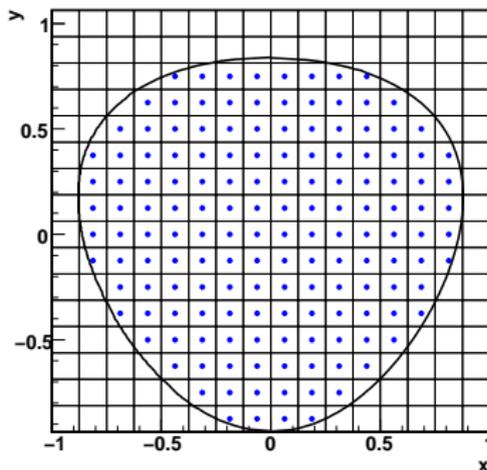
$$\bar{A}(z) = 3A_c(1 + (\beta + \gamma)x^2 + \dots)$$

$$a = 2\text{Re}\alpha \quad b = (2\text{Re}\beta + |\alpha|^2)$$

$$d = 2\text{Re}\gamma \quad f = 2\text{Re}\alpha\bar{\beta}$$



- Generate Dalitz plot with KLOE parameters
- KLOE bins + statistics
- ⇒ Statistical uncertainties + correlations
- Limitations:
- Efficiency, background, systematic uncertainties EVCL

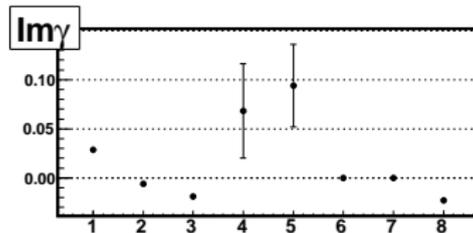
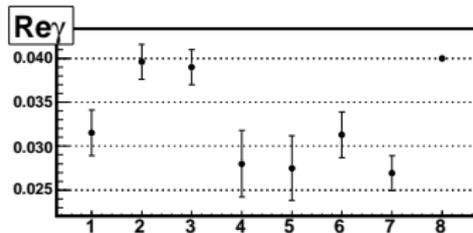
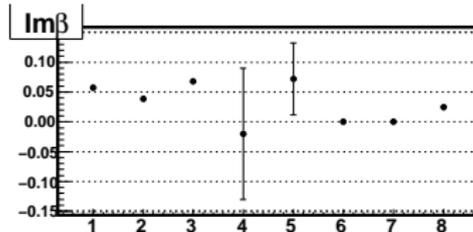
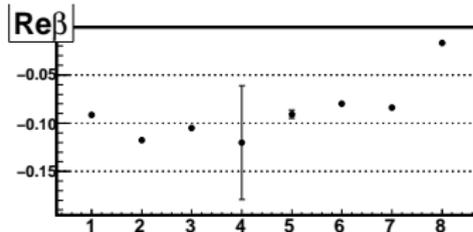
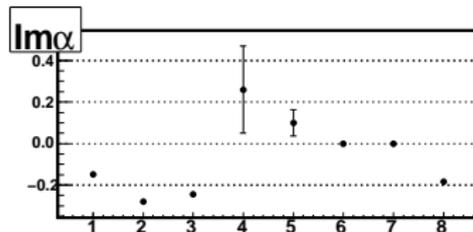
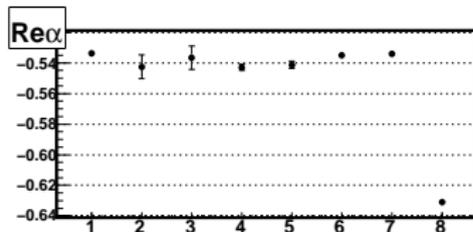




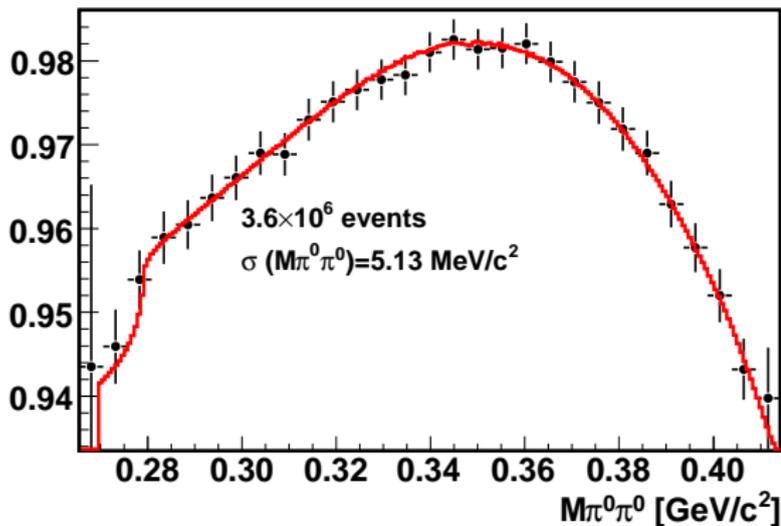
- 1 KLOE parameterization (w/o alpha)
- 2 2loop NREFT alpha Isospin $\chi^2 = 150/(153-4)$
- 3 2loop NREFT alpha $\chi^2 = 144/(153-4)$
- 4 Polynomial $\chi^2 = 126/(152-7)$ w/o alpha
- 5 Polynomial $\chi^2 = 126/(153-7)$ with alpha
- 6 Polynomial $\chi^2 = 136/(152-4)$ w/o alpha
- 7 Polynomial $\chi^2 = 143/(153-4)$ with alpha
- 8 2-loop ChPT result



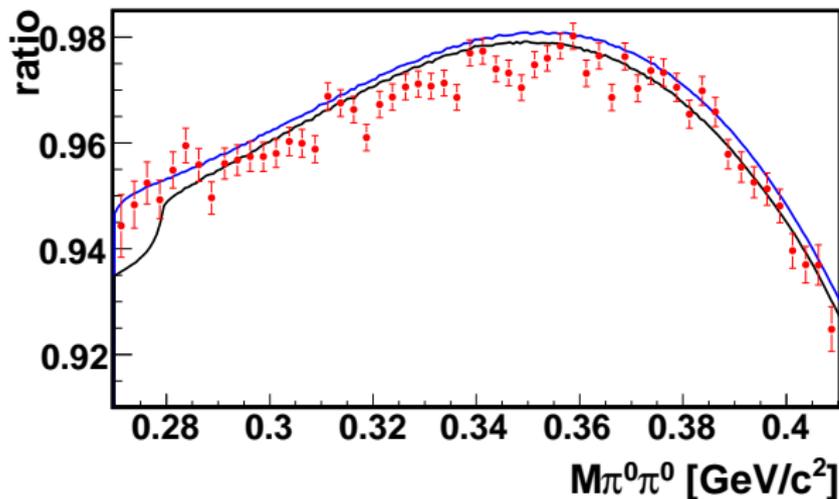
Preliminary results



1- KLOE, 2- 2Loop NREFT Isospin, 3- 2Loop NREFT α , 4-7 Direct fit, 8- 2Loop ChPT



min $M_{\pi^0\pi^0}$ or all combinations
(for $M_{\pi^0\pi^0} < 0.3 \text{ GeV}/c^2$ unique)



3×10^6 events; $\sigma(M_{\pi^0\pi^0}) = ?$

Crystal Ball arXiv:0812.1999



- Model independent Dalitz Plot densities
- $\alpha(\phi)$ dependence for $\eta \rightarrow \pi^0\pi^0\pi^0$
- Cusp in $\eta \rightarrow 3\pi^0$: statistics and $M_{\pi^0\pi^0}$ resolution
- *NREFT* a tool to parameterize the cusp region and to test consistency $\eta \rightarrow \pi^0\pi^0\pi^0$ vs $\eta \rightarrow \pi^+\pi^-\pi^0$