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$\pi - \pi$  rescattering  
in hadronic  $\eta$  decays

Carl-Oscar Gullstrom

Introduction

$$\eta \rightarrow 3\pi^0$$

$$\eta \rightarrow \pi^+ \pi^- \pi^0$$

# $\pi - \pi$ rescattering in hadronic $\eta$ decays

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$\eta \rightarrow 3\pi$  meeting, frascati 2009



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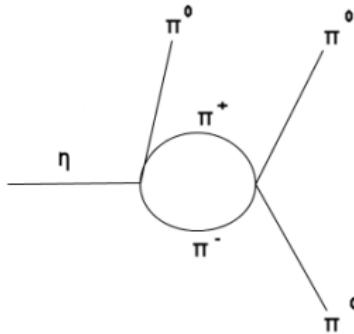
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"cusp effect in  $K_L \rightarrow \pi^0 \pi^0 \pi^0$ " (arXiv:0710.4456)  
Uses Effective field theory rather than CHPT





Has been used to measure the pion scattering length  $a_0$  and  $a_2$   
with high statistics in  $K \rightarrow \pi^+ \pi^0 \pi^0$

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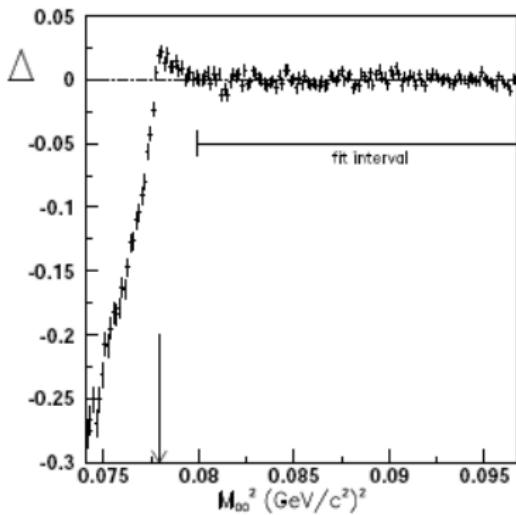


Figure 4:  $\Delta \equiv (data - fit)/data$  versus  $M_{00}^2$ . The point  $M_{00}^2 = (2m_+)^2$  is indicated by the arrow. Also shown is the  $M_{00}^2$  region used in the fit.



# Variables of theory

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$$M_{tot} = M_{tree} + M_{1-loop} + M_{2-loop} + \dots$$

rescattering parameters:  $a_2 = -0.044, a_0 = 0.22,$   
 $r_2 = -0.08, r_0 = 0.276$

Lagrangian couplings:  $\alpha, a, b, c, d$

Kinematics:  $x, y, z$  (Dalitz parameters)

Loop integrals:  $j(s, m_1, m_2), F(s, m_1, m_2, m_3, m_4)$



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Symmetry around dalitz plot  $z = x^2 + y^2$ , cusp effect predicted

$$|\bar{A}|^2 \propto 1 + 2\alpha z$$

fit is done by finding  $\alpha$  in

$$M_{tree}(\alpha_{exp}) = M_{tree}(\alpha) + M_{Neutralrescattering}\left(\alpha, (a, b, c, d)_{exp}\right) + M_{Chargerescattering}\left(\alpha, (a, b, c, d)_{exp}\right)$$



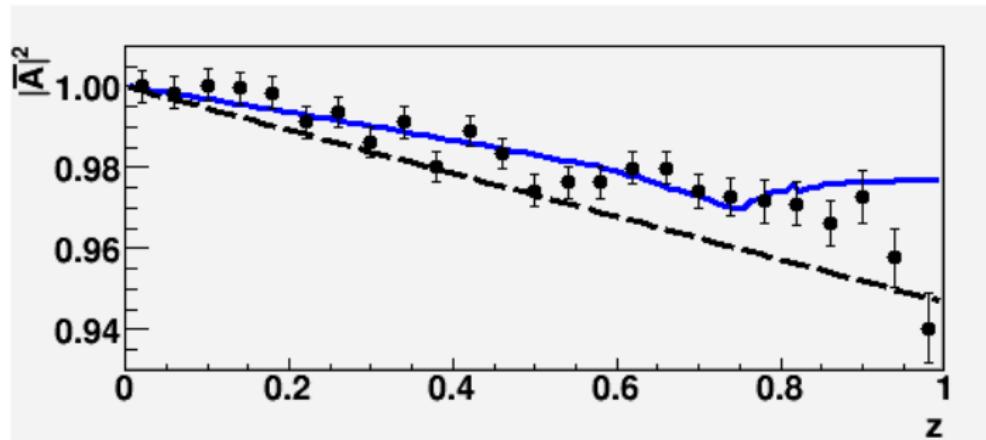
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$|\bar{A}|^2$ : dashed line exp tree amplitude, solid line 1-loop corrections,  
points kloe data



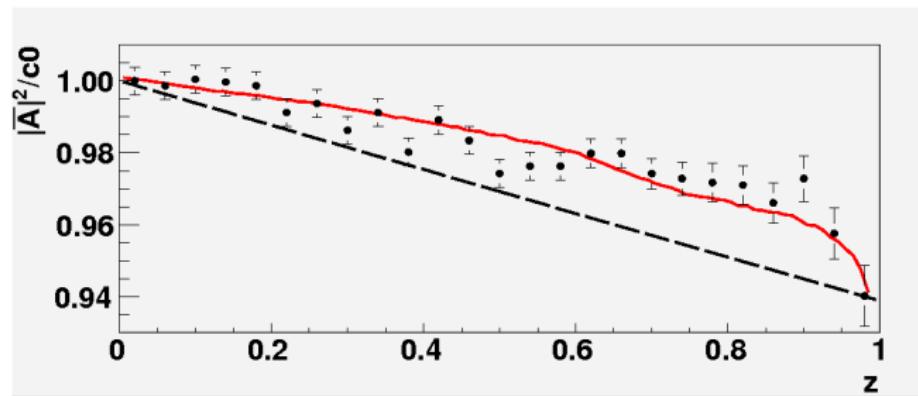
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## Tabular of parameter $\alpha$

$ \bar{A} _{tree}^2$	$\alpha$
Theory	0
Experimental	$-0.031 \pm 4$
1-Loop( $z < 0.7$ )	-0.004
2-Loop	0.01



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No cusp due to kinematic limits

$$|A|^2 \propto 1 + ay + by^2 + cx^2 + dy^3$$
$$M_{Tree} \propto 1 + v_0y + v_2y^2 + v_3x^2 + v_4y^3$$

connected to neutral channel by:  $\bar{A} = 3A \implies$   
 $\alpha = 0.569 * (v2(a, b) + v3(c))$



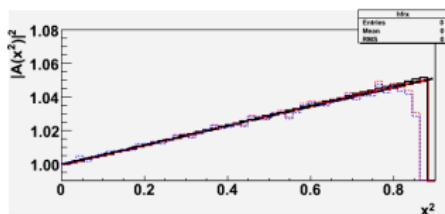
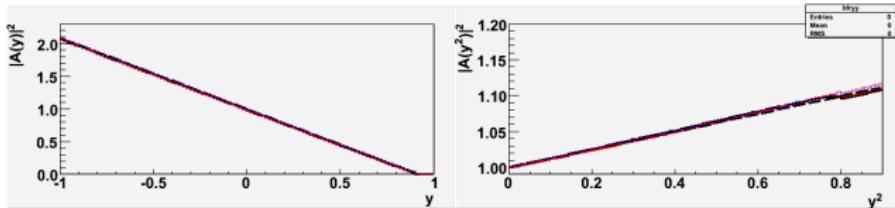
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### Plots of the cut

$|A(y, a)|^2 = 1 + a * y = |A(a, b, c, d)|^2 - |A(b, c, d)|^2$ : red line 1-loop calculation, blue line experimental amplitude, dotted lines cuts on  $x, y=0$ , solid line projection on  $x, y$  axis



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### Tabular for parameter a,b,c,d

	$ A _{tree}^2$	a	b	c	d	$\alpha = f(v_1, v_2)$
Theory	-1.0	0.25	0	0	0	0
Experimental	-1.09	0.125	0.057	0.13	-0.031	
1-Loop	-1.15	0.114	0.042	0.18	-0.046	
2-Loop	-1.18	0.13	0.056	0.21	-0.046	



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## Whats missing for the charge channel?

Theoretical:

complex part of  $M_{tree}$

Charge is dominate in neutral but neutral is not dominate in charge. Higher order of precision is required

Experimental:

Bining cut, dalitz boundary important for higher terms but it cut out for statistical reason.

Measurement of Effective range & Effective Shape. Only use  $a$ ; will give  $a=-1.35$  at 1-loop



# dalitz boundary

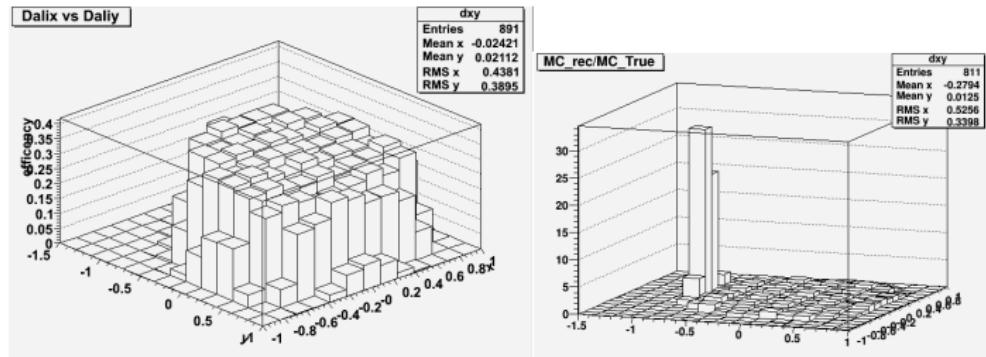
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MC on KLOE data with nbins $\sim$ 160 as used in the analysis



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# Measurement off effective Range parameters $A_i r_i$

The high statistical values  $r_0 = 0.276 \pm 4$   $r_2 = -0.0806 \pm 4$  is from inserting  $a_i$  in formulas from ChPT rather than an exact measurement. The data is from 70'th.

G. Colangelo et al. / Nuclear Physics B 603 (2001) 125–179

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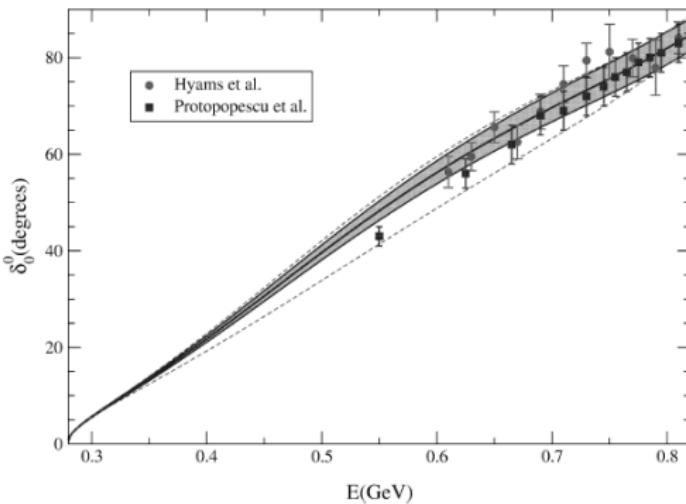


Fig. 7.  $I = 0$  S-wave phase shift. The full line results with the central values of the scattering lengths and of the experimental input used in the Roy equations. The shaded region corresponds to the uncertainties of the result. The dotted lines indicate the boundaries of the region allowed if the constraints imposed by chiral symmetry are ignored [7]. The data points are from Refs. [51,52].



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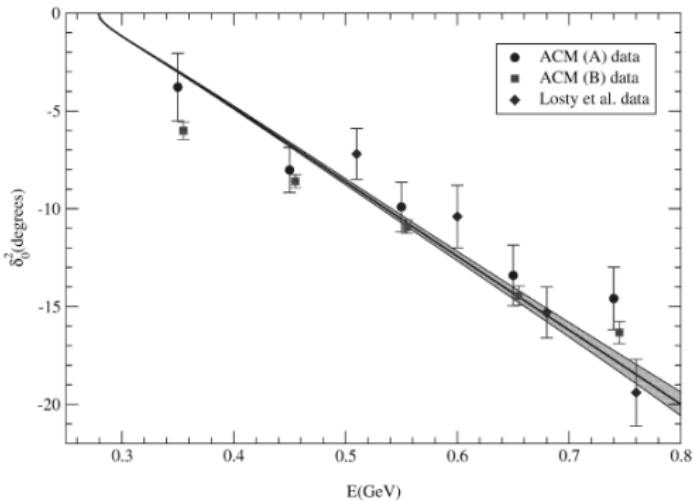


Fig. 9.  $I = 2$  S-wave phase shift. The full line results with the central values of the scattering lengths and of the experimental input used in the Roy equations. The shaded region corresponds to the uncertainties of the result. The data points represent the two phase shift representations of the Aachen-Cern-Munich collaboration [53] and the one of Losty et al. [54].