Quantum Information Geometry: quantum metrics for finite dimensional systems

Patrizia Vitale[a]
with V.I. Man'ko, G. Marmo and F. Ventriglia

[a] Dipartimento di Fisica Università di Napoli "Federico II" and INFN

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On the occasion of the 70th birthday of Beppe Marmo

Outline

- Statistical manifold,

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- The tomographic picture
- Outlook

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Invariance : No information loss (information monotonicity) under coarse graining $x \to y(x)$

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$$g_{jk}(\xi) := -\frac{\partial^2 S}{\partial \xi \partial \xi'}|_{\xi=\xi'}$$

is the Fisher Rao metric for any q

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The tomographic metric for N=2
Spin tomography

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$$\mathcal{W}(+, u_{13}) = \frac{1+x_1}{2} \quad \mathcal{W}(+, u_{23}) = \frac{1+x_2}{2} \quad \mathcal{W}(+, Id) = \frac{1+x_3}{2}$$

- Once the sufficiency set chosen, we can replace

$$dx_k = 2d\mathcal{W}_k(+; u)$$

and get the quantum metric in terms of tomograms

$$g_q = g_{jk}(q, x)dx^j \otimes dx^k \longrightarrow g_q(\mathcal{W})$$

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