

Titolo nota

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To Beppe with friendship and admiration

① - An example :

: Euler-Poisson

$$\dot{L}_1 = \left(\frac{1}{A_2} - \frac{1}{A_3} \right) L_2 L_3 + a_2 y_3 - a_3 y_2$$

$$\dot{y}_1 = \frac{1}{A_3} L_3 y_2 - \frac{1}{A_2} L_2 y_3$$

: know

$$a_1 = 1 \quad a_2 = a_3 = 0$$

$$A_3 = 1 \quad A_2 = A_1 = 2$$

$$c_1 = y_1^2 + y_2^2 + y_3^2$$

$$c_2 = L_1 y_1 + L_2 y_2 + L_3 y_3$$

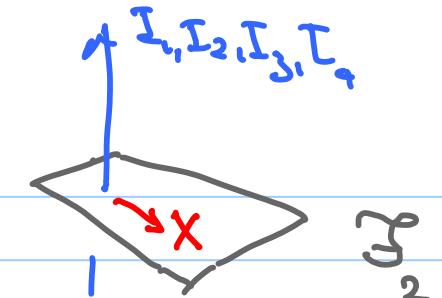
$$h_1 = \frac{1}{4} (L_1^2 + L_2^2 + 2L_3^2) - y_1$$

$$h_2 = \frac{1}{8} (L_1^2 - L_2^2 - 4y_1)^2 + \frac{1}{8} (2L_1 L_2 + 4y_2)^2$$

$$\left[\frac{(x_1 - x_2)^2}{4} \left(u - \frac{h_1}{6} \right)^2 - R(x_1, x_2) \left(u - \frac{h_1}{6} \right) - \frac{1}{4} R_{,u}(x_1, x_2) = 0 \right]$$

†

(u_1, u_2)



+

$$x_1 = \frac{1}{2} (L_1 + i L_2)$$

$$x_2 = \frac{1}{2} (L_1 - i L_2)$$

$$R(x_1, x_2) = -x_1^2 x_2^2 + 2h_1 x_1 x_2 + c_2 (x_1 + x_2) + c_1 - \frac{1}{2} h_2$$

$$\begin{aligned} R_1(x_1, x_2) = & -2h_1 x_1^2 x_2^2 - (c_1 - \frac{1}{2} h_2)(x_1 + x_2)^2 - 2c_2 (x_1 + x_2) x_1 x_2 + \\ & + 2h_1 (c_1 - \frac{1}{2} h_2) - c_2^2 \end{aligned}$$

$$\frac{du_1}{\sqrt{P(u_1)}} + \frac{du_2}{\sqrt{P(u_2)}} = 0$$

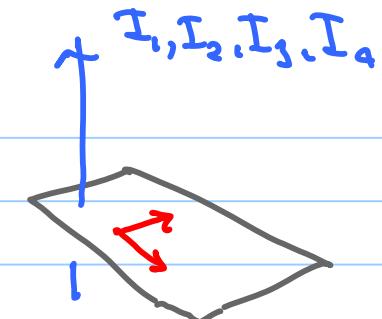
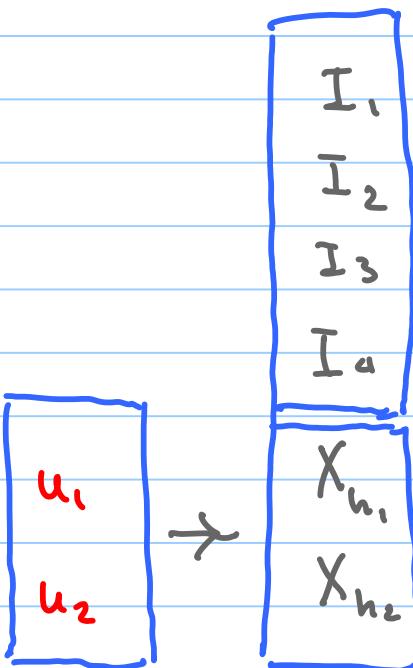
$$\frac{u_1 du_1}{\sqrt{P(u_1)}} + \frac{u_2 du_2}{\sqrt{P(u_2)}} = dt$$

$y^2 = P(u)$ spectral curve of
Kovalevski

② - A question :

What properties of the fundamental
equation of Kovalevski do explain
this miracle ?

③ - The role of symmetries



$$\{h_1, h_2\} = 0$$

$$[X_{h_1}, X_{h_2}] = 0$$

$$X_j(I_a) = 0$$

$$\frac{du_1}{\sqrt{P(u_1)}} + \frac{du_2}{\sqrt{P(u_2)}} = 0$$

$$H = h_1$$
$$k = ah_2 + b h_1^2$$
$$a, b \in \mathbb{R}$$

$$\frac{u_1 du_1}{\sqrt{P(u_1)}} + \frac{u_2 du_2}{\sqrt{P(u_2)}} = dt$$

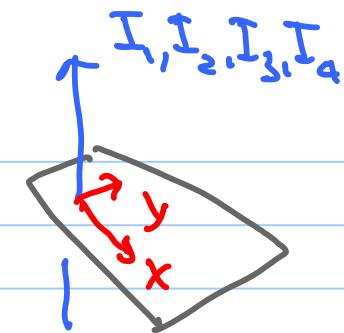
$$\frac{du_1}{\sqrt{P(u_1)}} + \frac{du_2}{\sqrt{P(u_2)}} = d\tau$$

$$\frac{u_1 du_1}{\sqrt{P(u_1)}} + \frac{u_2 du_2}{\sqrt{P(u_2)}} = 0$$

④ - The problem :

Find a second-order polynomial

$$k(u) = Eu^2 + Fu + G$$



$$[x, y] = 0$$

$$X(I_a) = 0$$

$$y(I_a) = 0$$

whose roots allow to put the
equations of motion of both X and y
in the Abel's form

⑤ - The condition :

$$EF' - FE' = \dot{EG} - G\dot{E}$$

$$EG' - GE' = \dot{FG} - G\dot{F}$$

These two differential conditions are necessary
and sufficient to get the Abel's form

