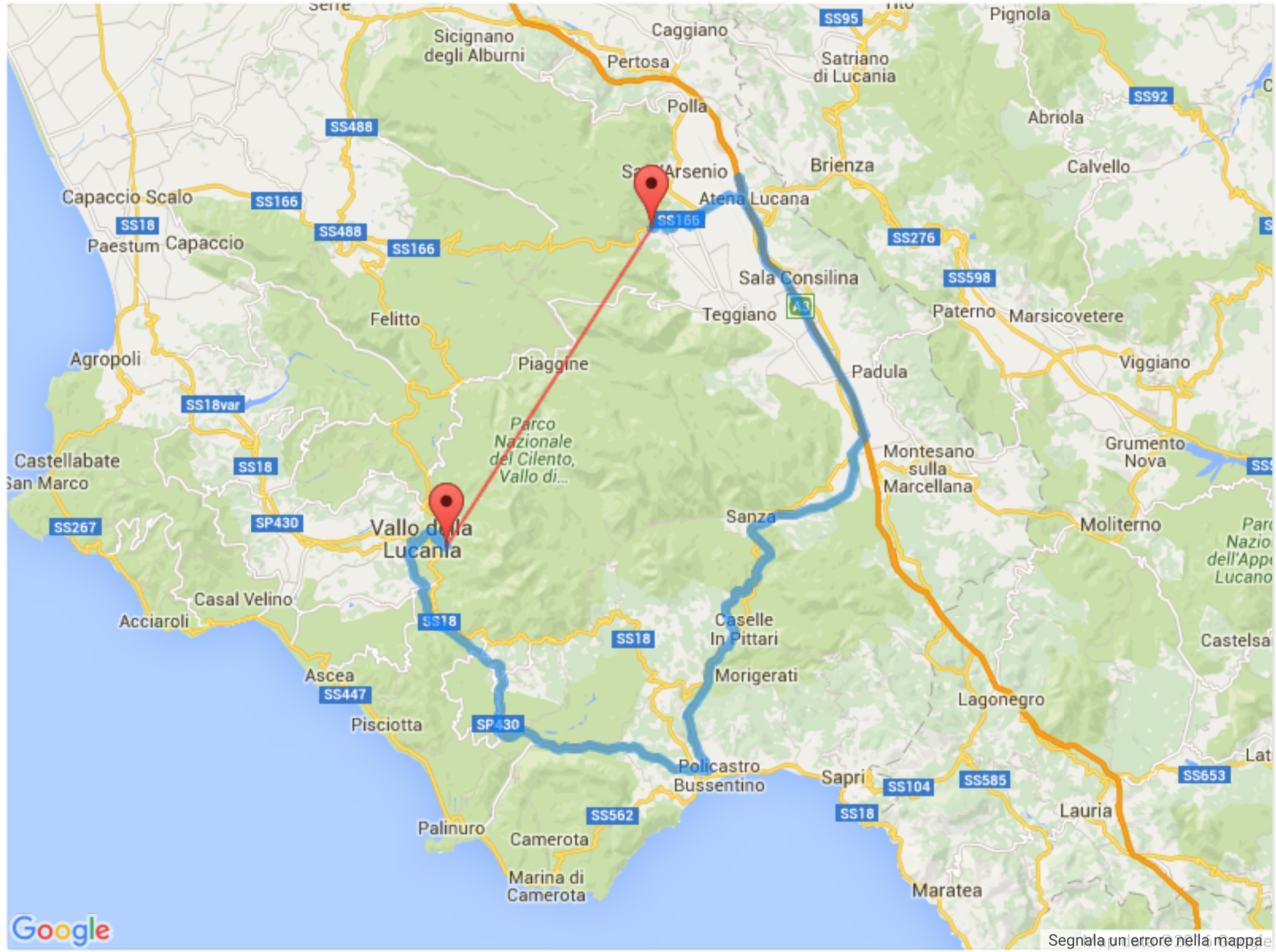
Zeno was born in Elea, 27 km from San Rufo

Saverio Pascazio
Dipartimento di Fisica and INFN
Bari, Italy

San Rufo, 11 July 2016


Sicignano
degli Alburni
Caggiano
Pertosa
Satriano
di Lucania
Pignola
Polla
Abriola
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Brienza
Atena Lucana
Capaccio Scalo
Paestum
Capaccio
Sala Consilina
Teggiano
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Marsicovetere
Felitto
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Padula
Viggiano
Agropoli
Parco
Nazionale
del Cilento,
Vallo di...
Montesano
sulla
Marcellana
Grumento
Nova
Castellabate
Vallo della
Lucania
Sanza
Moliterno
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Acciaroli
Caselle
In Pittari
Morigerati
Ascea
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Bussentino
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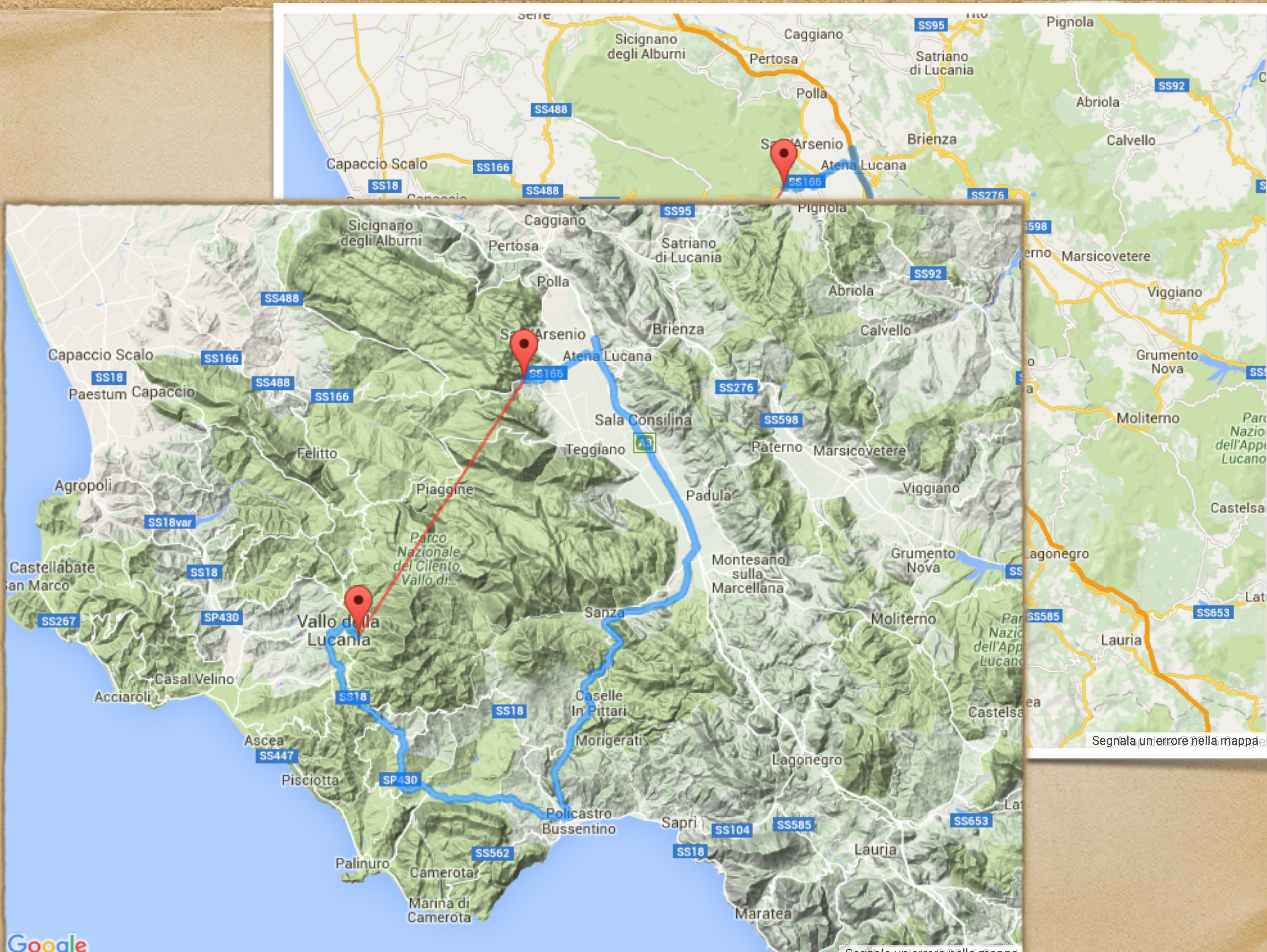
Segnala un errore nella mappa



Sicignano degli Alburni
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SS104
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Palinuro
Camerota
Marina di Camerota
Maratea
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Segnala un errore nella mappa


San Rufo
Velia (Elea)
Sicignano degli Alburni
Caggiano
Pertosa
Polla
Satriano di Lucania
Abriola
Calvello
Brienza
Atena Lucana
Capaccio Scalo
Paestum
Capaccio
Sala Consilina
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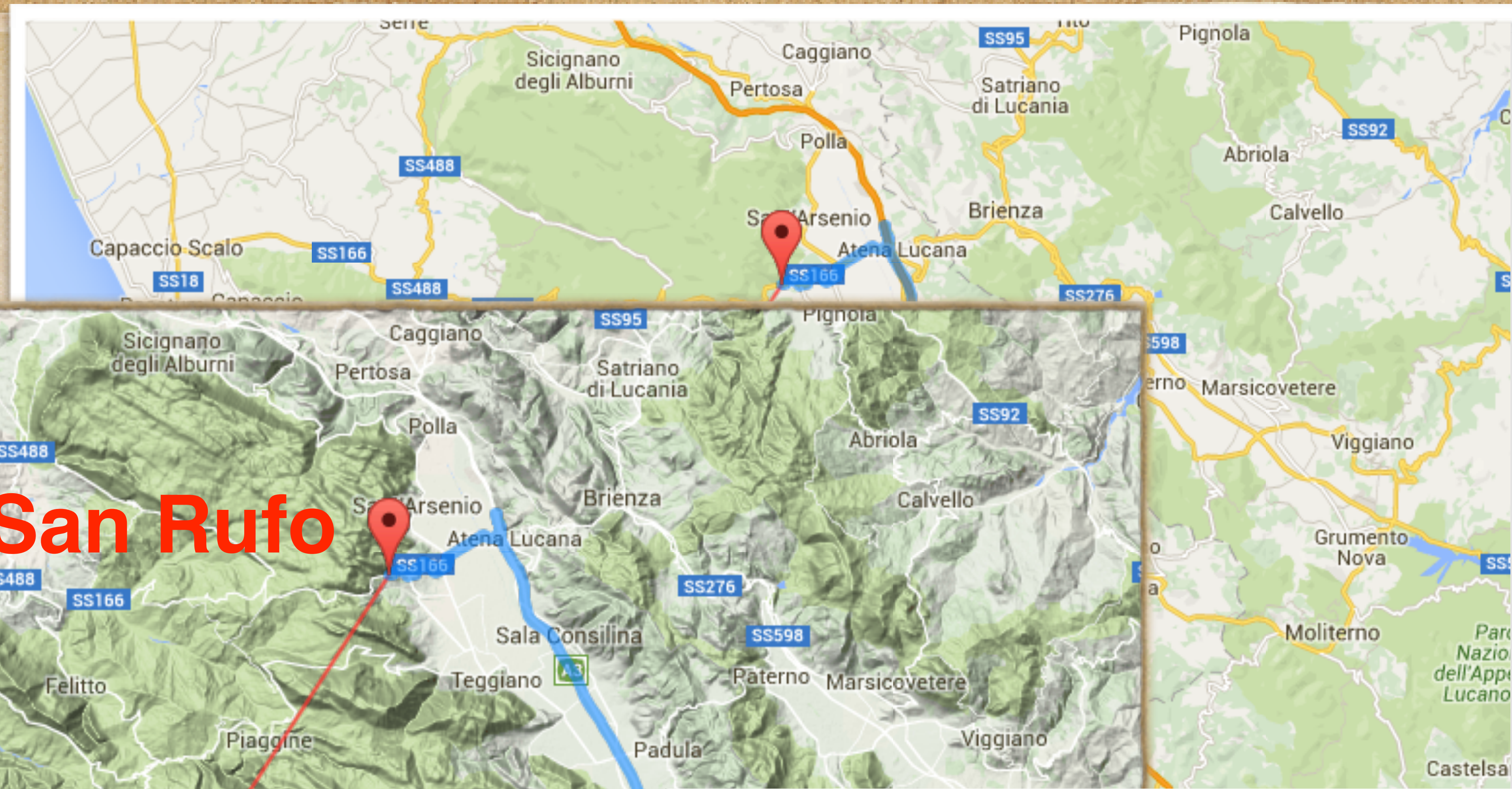
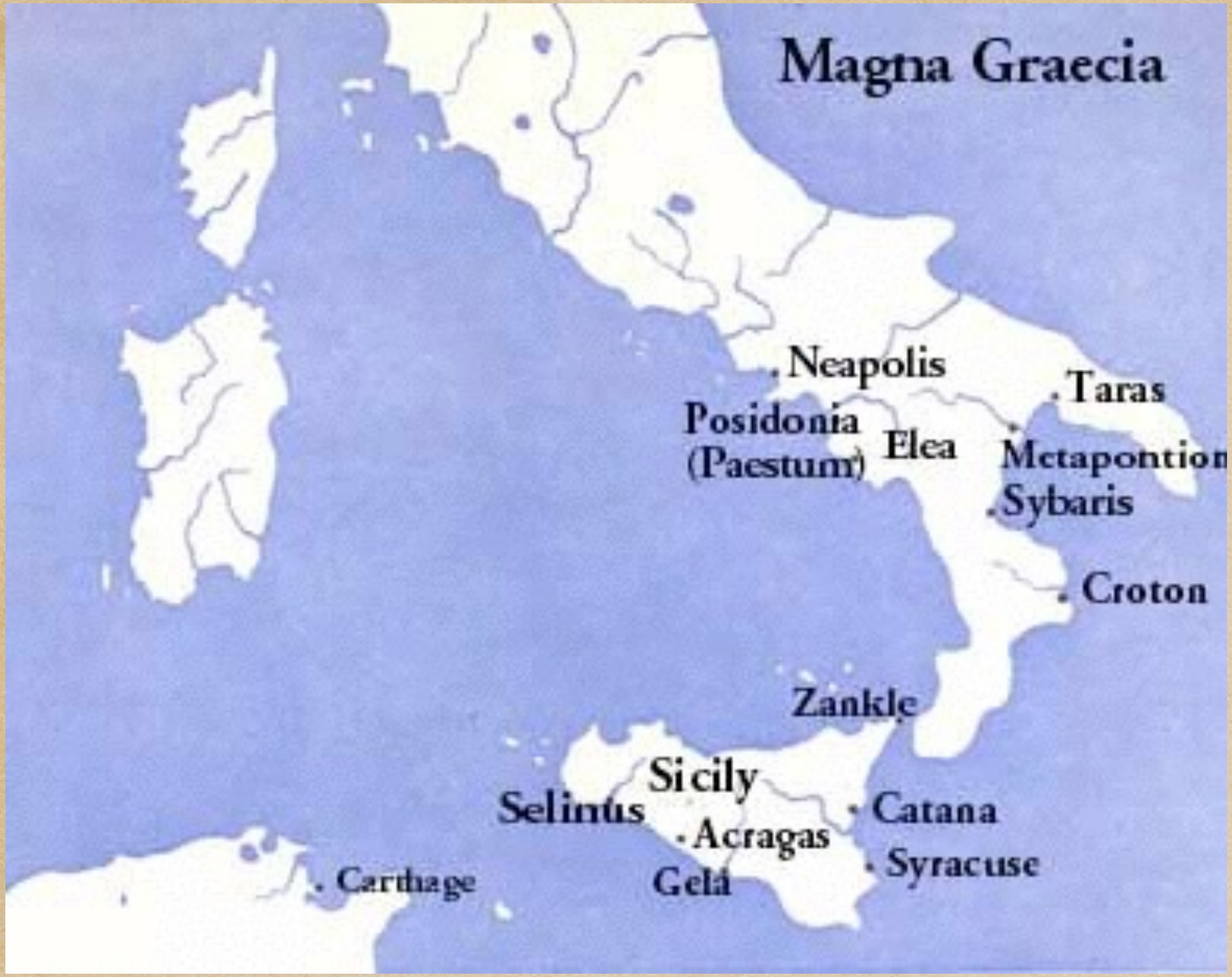
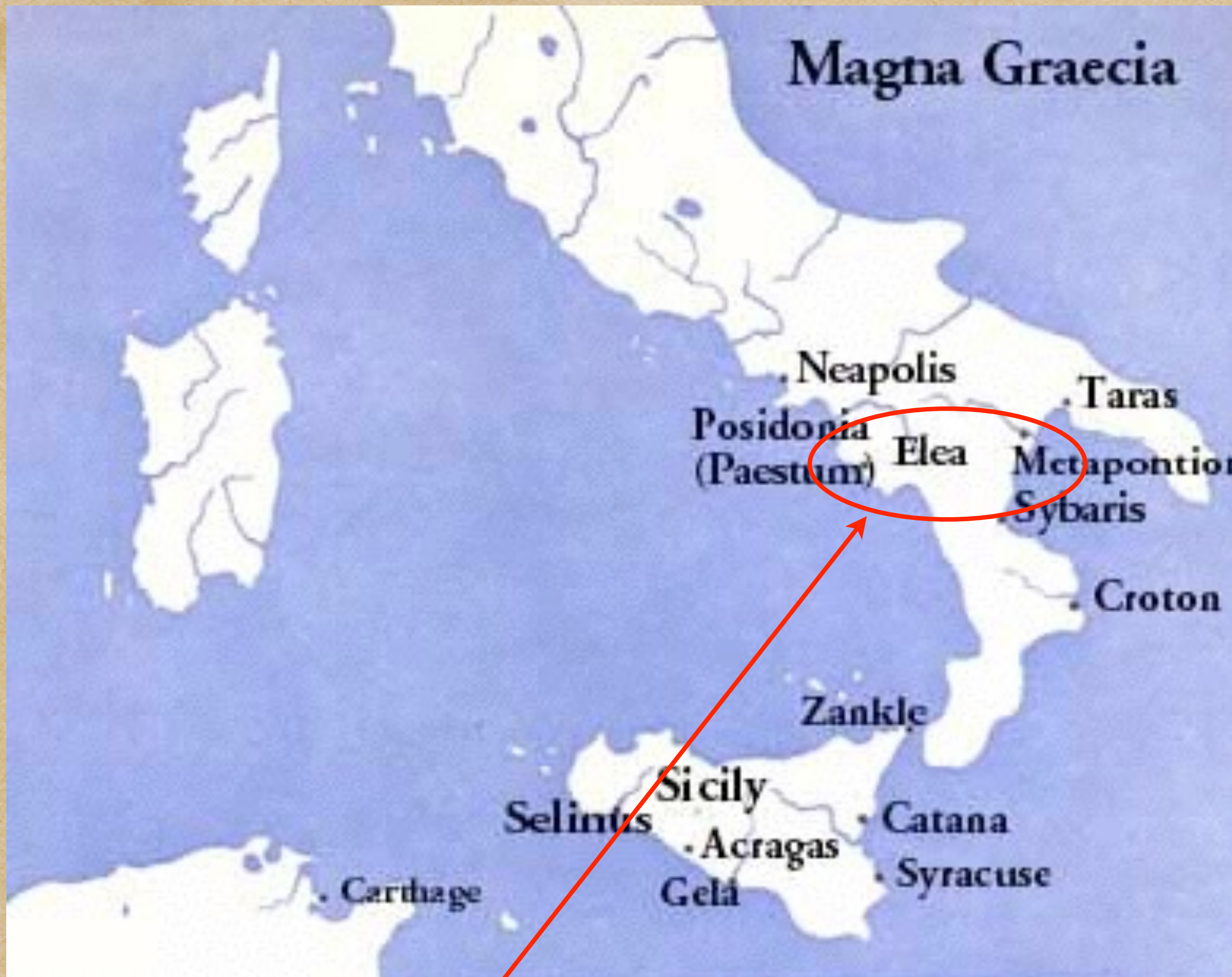


Calcolo della distanza da Novi Velia a San Rufo

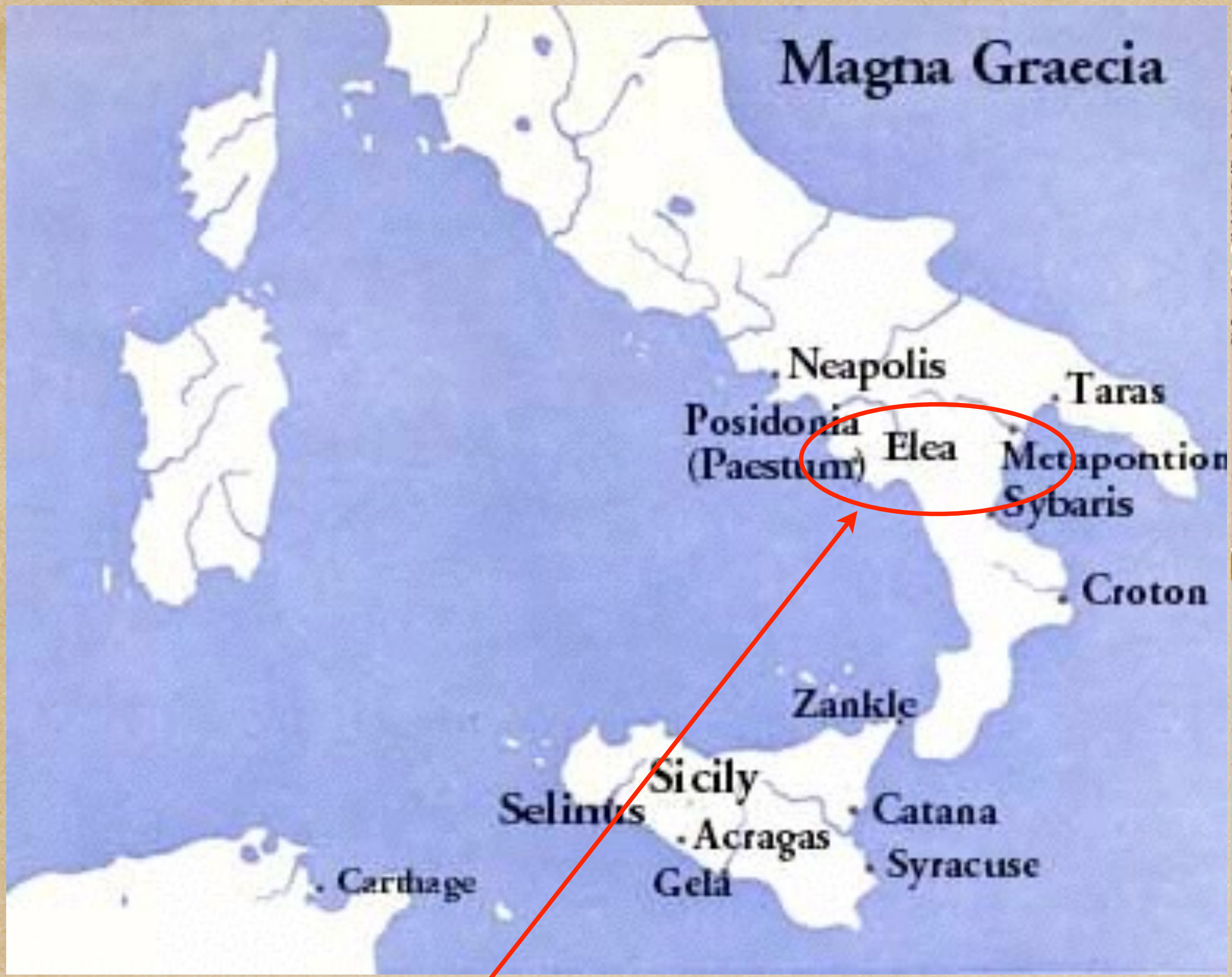
La distanza in chilometri sulla strada = 108 km
Distanza in miglia sulla strada = 66.96 miglia
Tempo di percorrenza = 1 ore 32 minuti
Consumo di carburante = 7.56 litri
Il prezzo del carburante = 12.1 euro

La distanza in linea retta in chilometri = 27.73 km
La distanza in linea d'aria in miglia = 17.2 miglia


Magna Graecia
Neapolis
Taras
Posidonia
(Paestum)
Elea
Metapontion
Sybaris
Croton
Zankle
Sicily
Selinus
Catana
Acragas
Syracuse
Carthage
Gela

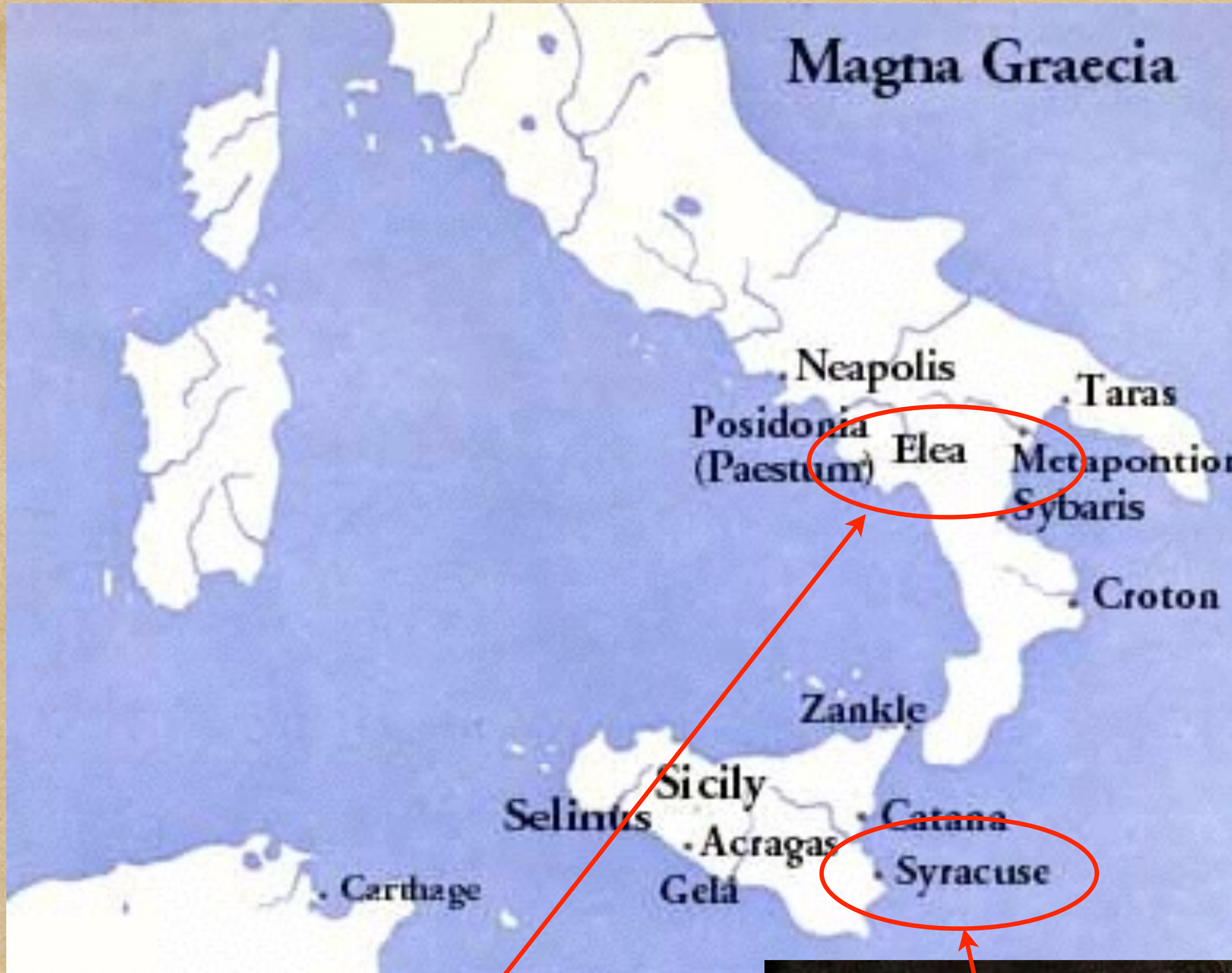


Zeno was born here

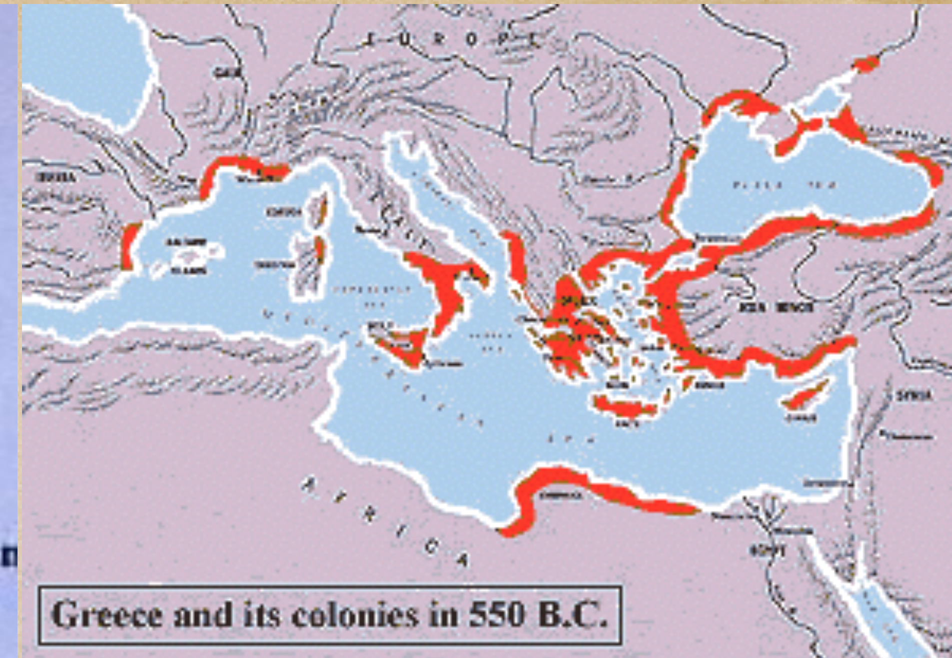


Greece and its colonies in 550 B.C.

Zeno was born here



Zeno was born here



Archimedes

Zeno of Elea

Zeno was an Eleatic philosopher, a native of Elea in Italy, son of Teleutagoras, and the favorite disciple of Parmenides. He was born about 488 BC, and at the age of forty accompanied Parmenides to Athens.

They where supposed to discuss politics but they spent most of the time discussing philosophy. They came back defeated.

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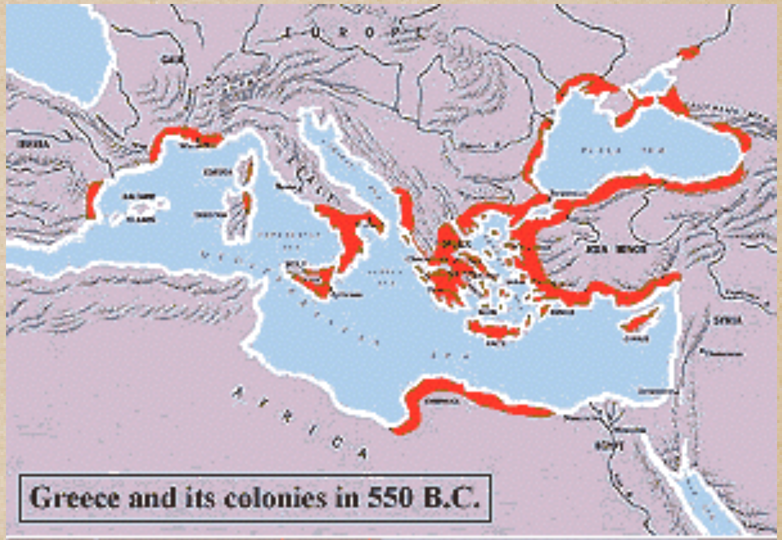
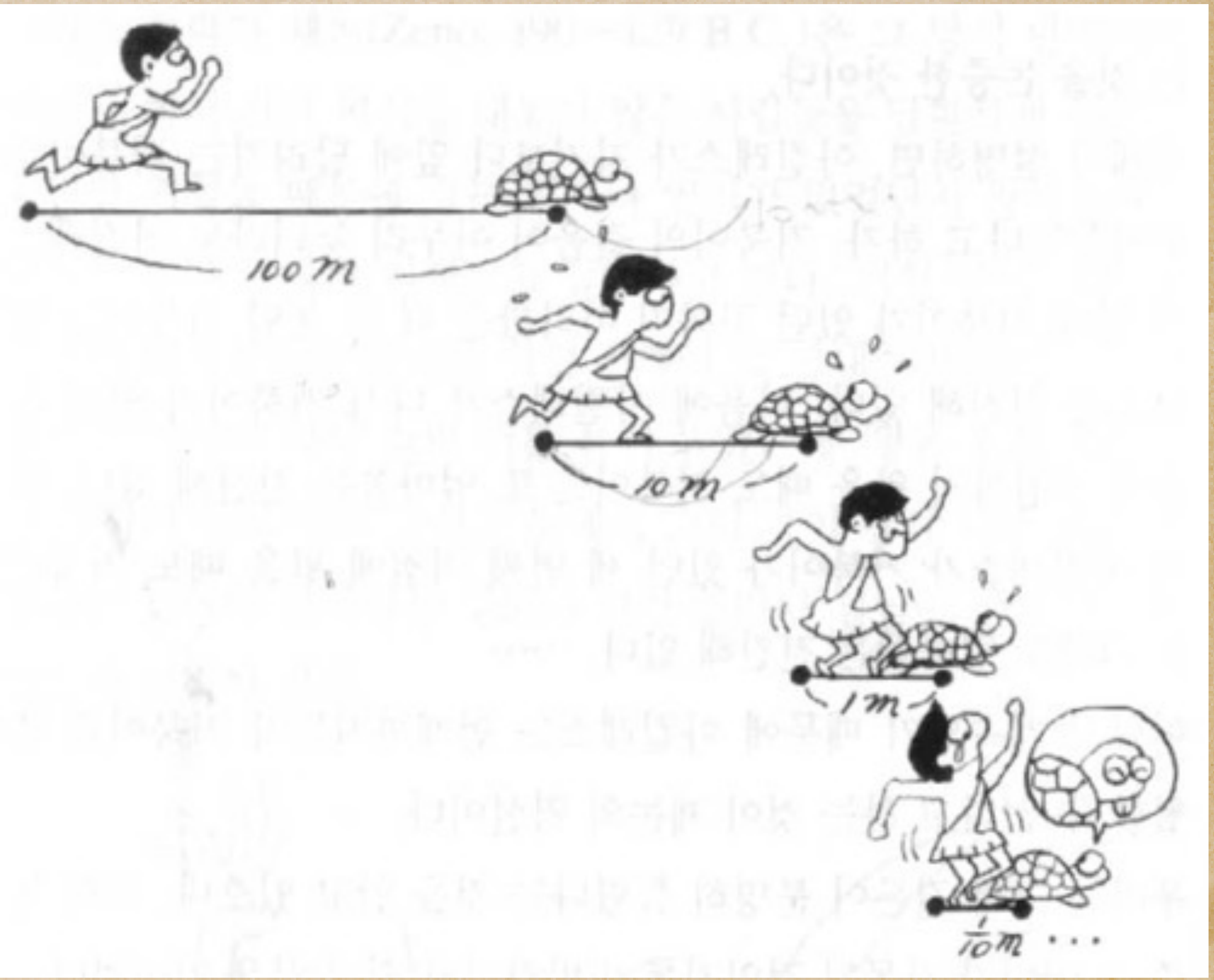


Greece and its colonies in 550 B.C.

Zeno and Parmenides

- the paradoxes:
- the most famous one: the tortoise



Zeno and Parmenides

- the paradoxes: the sped arrow

Zeno and Parmenides

- the paradoxes: the sped arrow

At any given moment the sped arrow is in a space equal to its own length, and therefore is at rest at that moment. So, it is at rest at all moments. The sum of an infinite number of these positions of rest is not a motion.

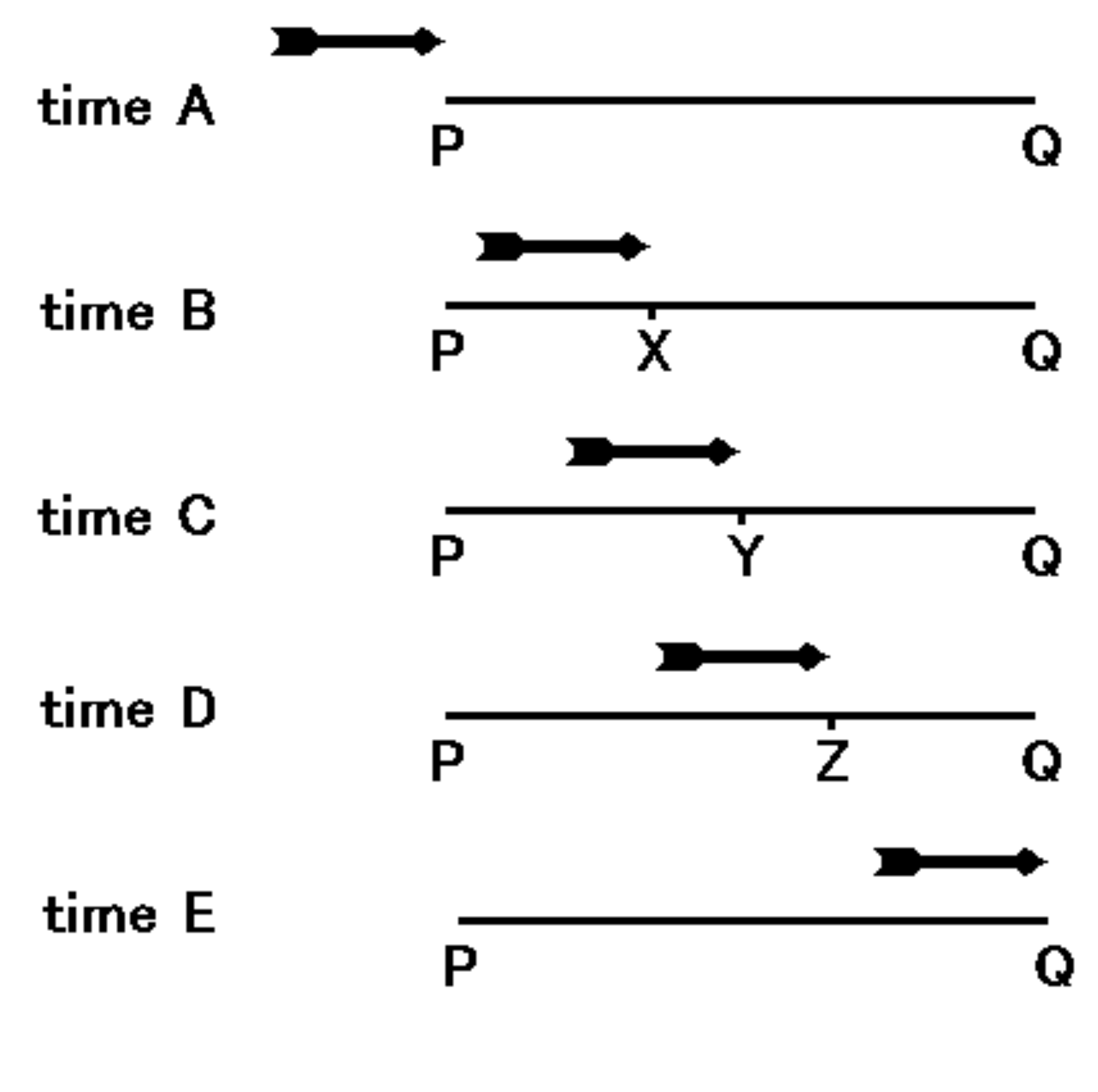


Figure 1

Zeno and Parmenides

- the paradoxes: the sped arrow

notice: act of observation not (explicitly) mentioned

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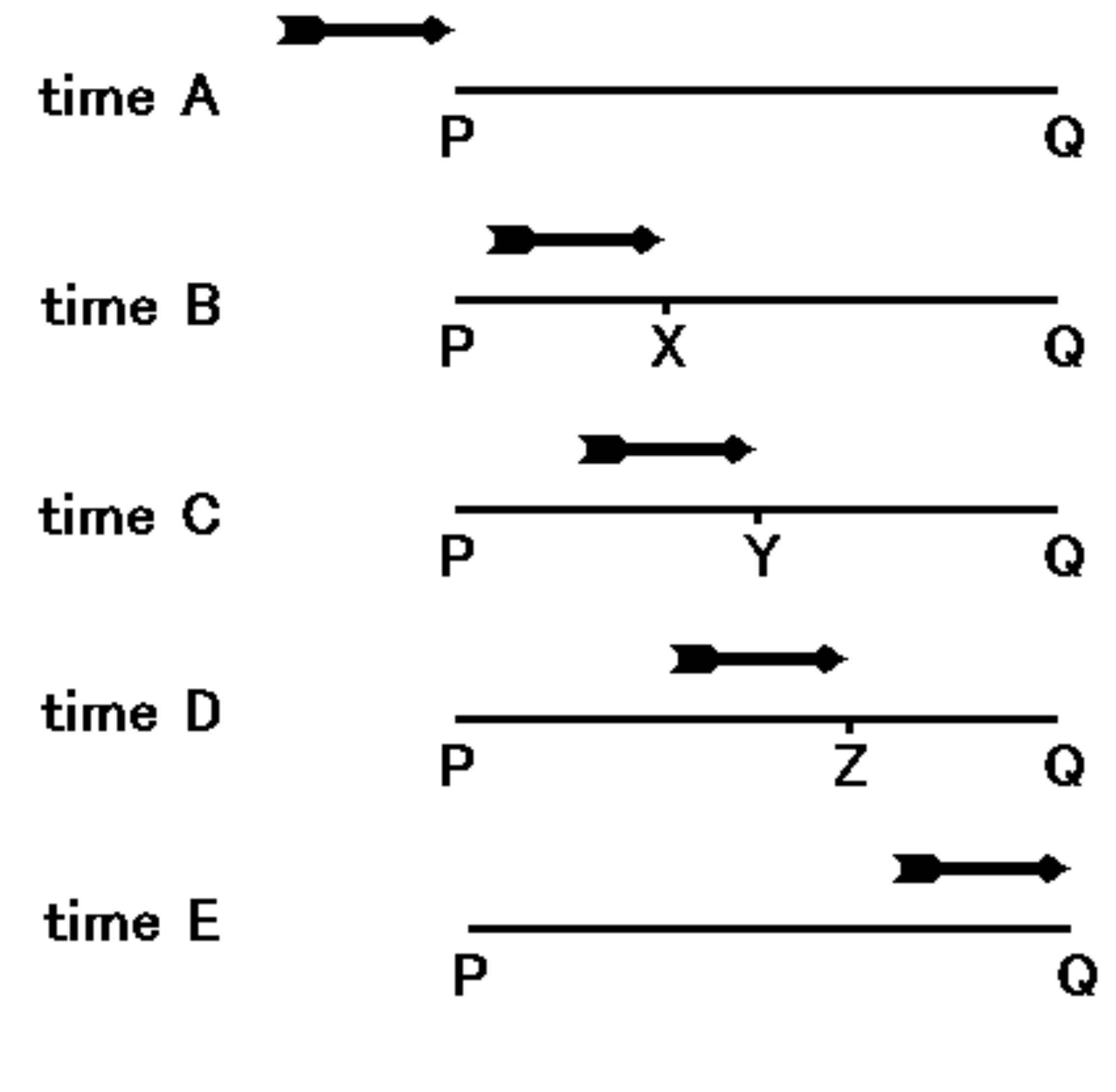

Figure 1

School of Athens
(Raphael's Room of the Signature, Vatican Museums)

School of Athens
(Raphael's Room of the Signature, Vatican Museums)

School of Athens
(Raphael's Room of the Signature, Vatican Museums)

Quantum Zeno effect

quantum system	ψ
Hamiltonian	H
Schrodinger equation	$\psi_t = e^{-iHt}\psi_0$

Quantum Zeno effect

quantum system ψ

Hamiltonian H

Schrodinger equation $\psi_t = e^{-iHt}\psi_0$

survival probability

$$p(t) = |\langle \psi_t | \psi_0 \rangle|^2 = 1 - t^2/\tau_Z^2$$

$$\tau_Z^{-2} \equiv \langle \psi_0 | H^2 | \psi_0 \rangle - \langle \psi_0 | H | \psi_0 \rangle^2$$

$$p(t) = |\langle \psi_t | \psi_0 \rangle|^2 = 1 - t^2/\tau_Z^2$$
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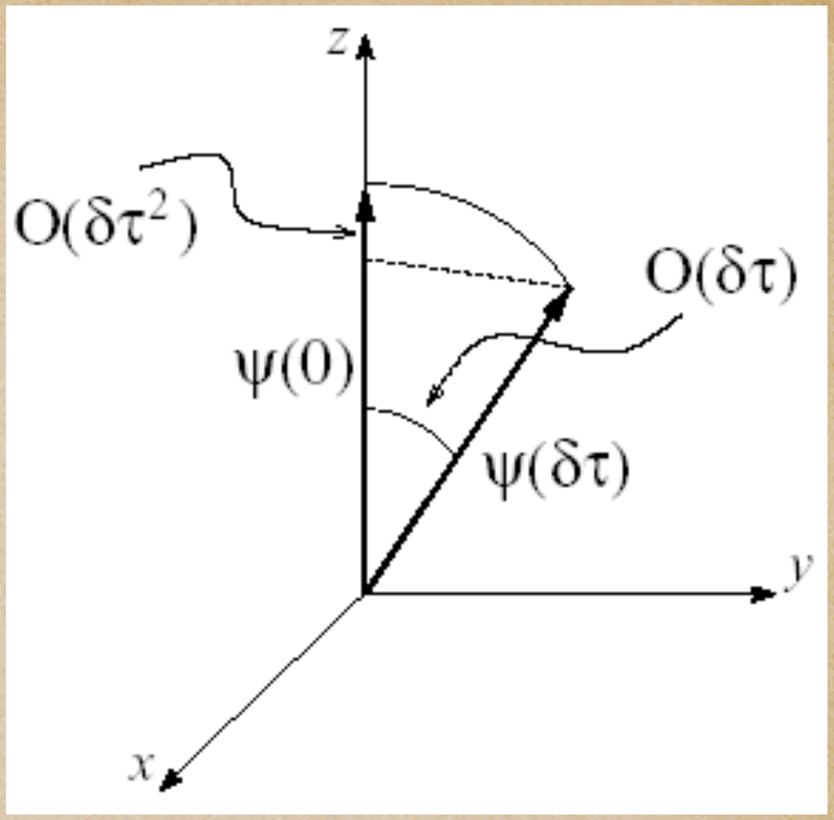

$$p(t) = |\langle\psi_t|\psi_0\rangle|^2 = 1 - t^2/\tau_Z^2$$
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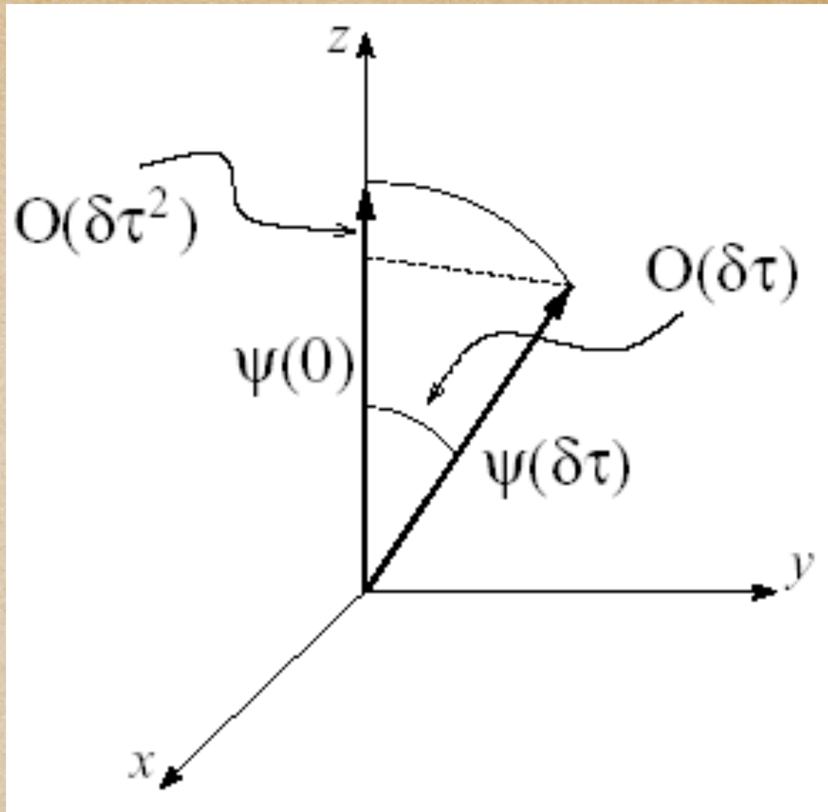


$$[p(t/N)]^N \simeq \left(1 - \frac{t^2}{\tau_Z^2 N^2}\right)^N \xrightarrow{N\to\infty} 1$$

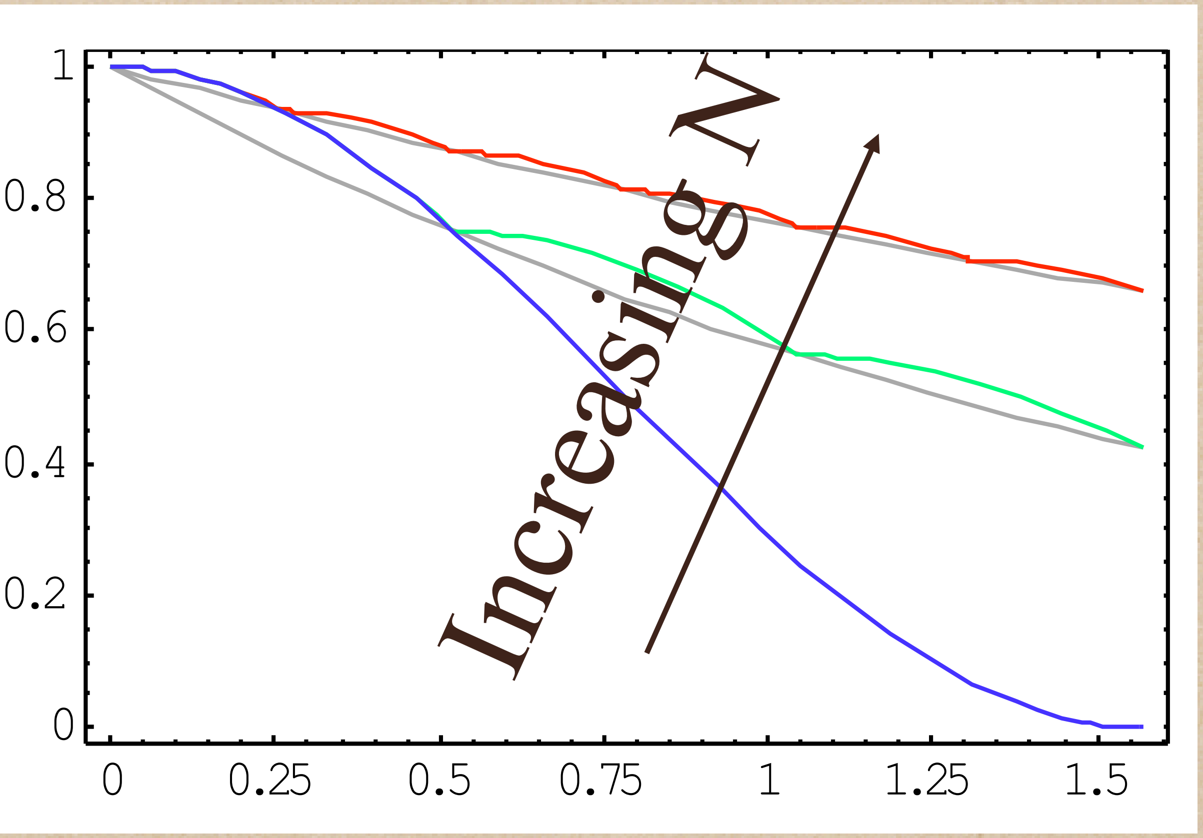
Frequent observations hinder dynamics

Quantum Zeno effect

$$[p(t/N)]^N \simeq \left(1 - \frac{t^2}{\tau_Z^2 N^2}\right)^N \stackrel{N\to\infty}{\longrightarrow} 1$$

MANY experiments on MANY physical systems

$$[p(t/N)]^N \simeq \left(1 - \frac{t^2}{\tau_Z^2 N^2}\right)^N \xrightarrow{N\to\infty} 1$$



$[p(t/6)]^6$

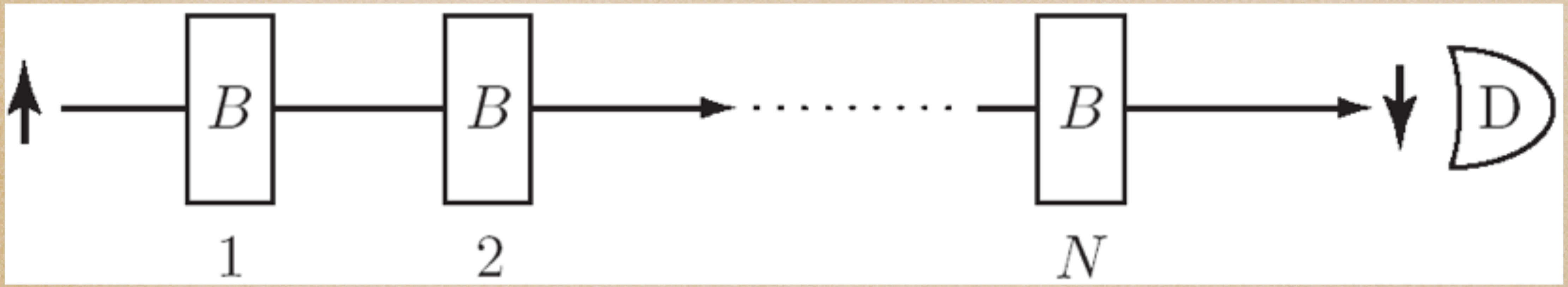
$[p(t/3)]^3$

$[p(t/1)]^1$

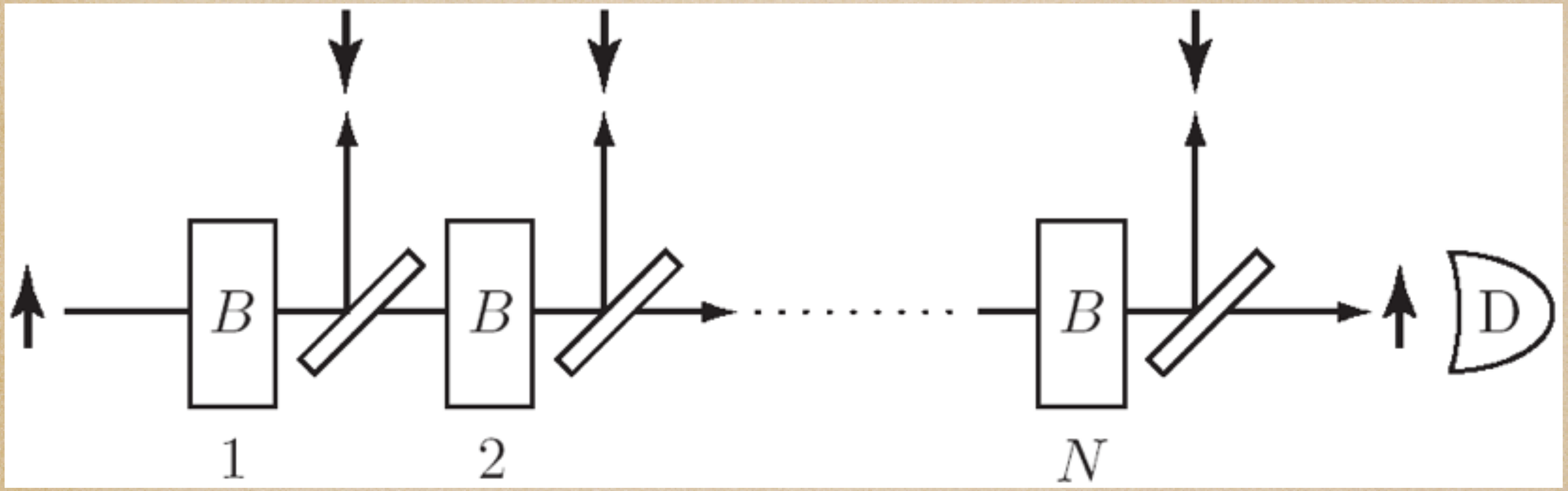
MANY experiments on MANY physical systems

one expt: neutron spin

one expt: neutron spin

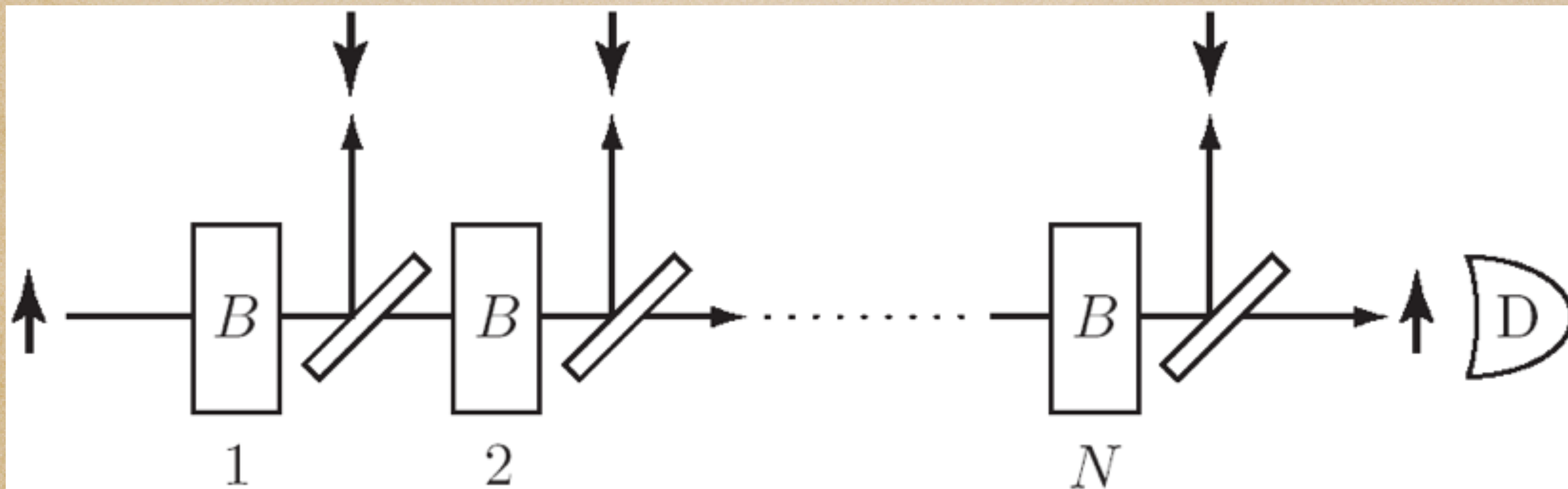


one expt: neutron spin



Pascazio, Namiki, Badurek, Rauch, Phys. Lett. A **169**, 155 (1993)

one expt: neutron spin



$$p(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \cos\left(\frac{t}{\tau_Z}\right)^2$$

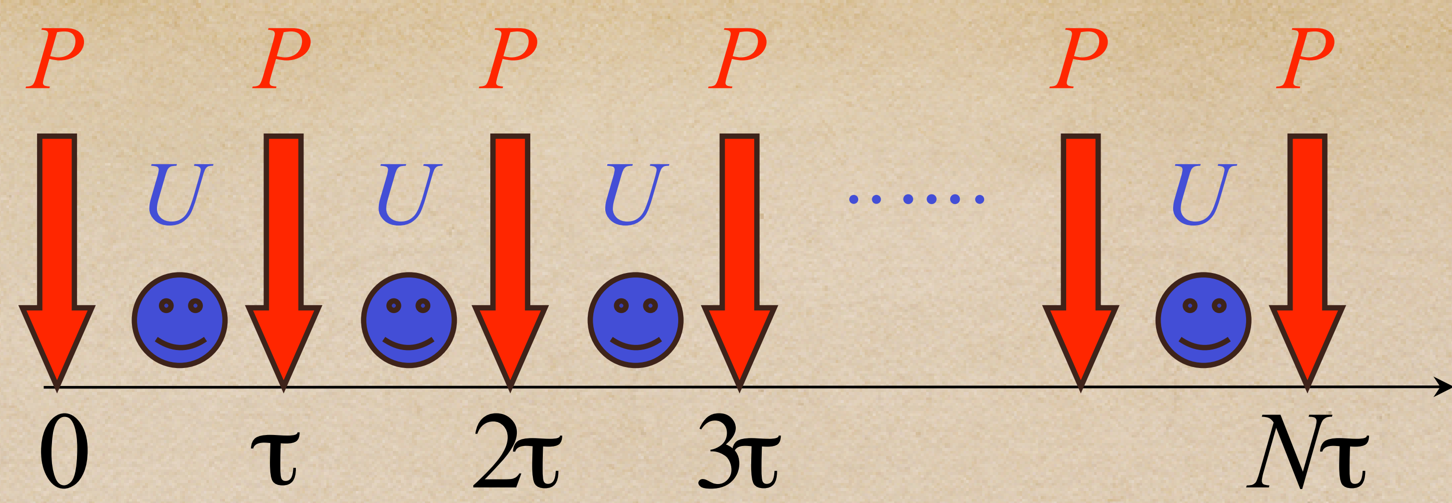
$$p^{(N)}(t) = p\left(\frac{t}{N}\right)^N = \cos\left(\frac{t}{N\tau_Z}\right)^{2N}$$

$$p^{(N)}(t) \xrightarrow{N \to \infty} 1$$

Pascazio, Namiki, Badurek, Rauch, Phys. Lett. A **169**, 155 (1993)

Repeated observation freeze system in its initial state

Formulation due to Misra and Sudarshan 1977



$$\rho^{(N)}(t) = V_N(t)\rho_0 V_N(t)^\dagger, \qquad V_N(t) = [PU(t/N)P]^N$$

The probability to find the system in $\mathcal{H}_P$ reads

$$p^{(N)}(t) = \mathrm{Tr}[V_N(t)\rho_0 V_N(t)^\dagger] \xrightarrow{N\to\infty} \mathrm{Tr}[P\rho_0] = 1$$

Repeated observation freeze system in its initial state

Formulation due to Misra and Sudarshan 1977

examples and theorems

- the value of proof
- Franco (+ myself)
- Beppe: time consuming

examples and theorems

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examples and theorems

- the value of proof
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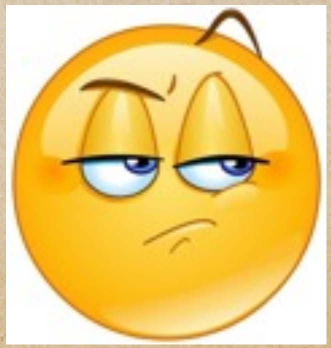







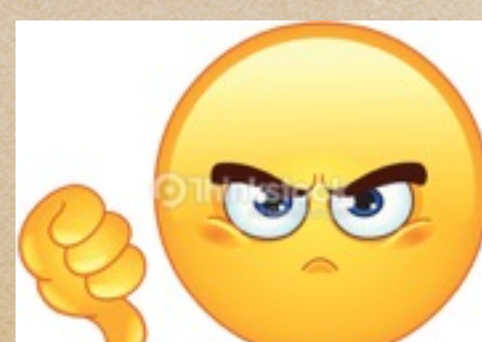
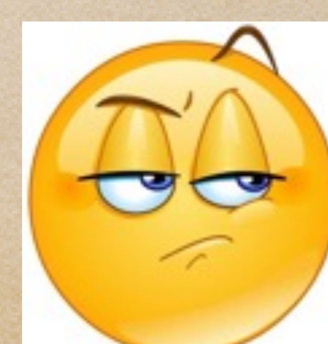


Very
Good


Very
Good

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THE GEOMETRY OF THE QUANTUM ZENO EFFECT

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3. Geometry of Quantum Mechanics

States are not vectors $|\psi\rangle \in \mathcal{H}$ but rather rays, elements of the Hilbert manifold $\mathbb{P}\mathcal{H}$, which can be conveniently parametrized as rank-one projection operators. This projection map enables one to identify on $\mathbb{P}\mathcal{H}$ a metric tensor, usually called the Fubini–Study metric, and a symplectic structure [9, 10]. Both of them define on $\mathbb{P}\mathcal{H}$ that is called a Kählerian structure. To work with this tensor on $\mathcal{H}$ instead of $\mathbb{P}\mathcal{H}$ is quite convenient for computational purposes.

We shall consider finite-dimensional systems. Let $\mathcal{H} \simeq \mathbb{C}^n$ and introduce the orthonormal basis $\{|e_1\rangle, \dots, |e_n\rangle\}$ and coordinates $z_k = (q_k + \mathrm{i} p_k)$. This entails

$$\mathcal{H} \to \mathbb{C}^n \to \mathbb{R}^{2n}, \tag{3.1}$$

with

$$|\psi\rangle \mapsto \begin{pmatrix} \langle e_1|\psi\rangle \\ \langle e_2|\psi\rangle \\ \vdots \\ \langle e_n|\psi\rangle \end{pmatrix} =: \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} =: \begin{pmatrix} q_1 + \mathrm{i}p_1 \\ q_2 + \mathrm{i}p_2 \\ \vdots \\ q_n + \mathrm{i}p_n \end{pmatrix} \mapsto \begin{pmatrix} q_1 \\ \vdots \\ q_n \\ p_1 \\ \vdots \\ p_n \end{pmatrix}. \tag{3.2}$$

The complex inner product of $\mathcal{H}$ induces the Hermitian structure

$$\mathfrak{h} = \sum_k \mathrm{d}\bar{z}_k \otimes \mathrm{d}z_k = \mathfrak{g} + \frac{\mathrm{i}}{2}\omega, \tag{3.3}$$

where

$$\mathfrak{g} = \frac{1}{2}\sum_k (\mathrm{d}\bar{z}_k \otimes \mathrm{d}z_k + \mathrm{d}z_k \otimes \mathrm{d}\bar{z}_k) = \sum_k \mathrm{d}\bar{z}_k \otimes_s \mathrm{d}z_k, \tag{3.4}$$

$$\omega = -\mathrm{i}\sum_k (\mathrm{d}\bar{z}_k \otimes \mathrm{d}z_k - \mathrm{d}z_k \otimes \mathrm{d}\bar{z}_k) = -\mathrm{i}\sum_k \mathrm{d}\bar{z}_k \wedge \mathrm{d}z_k \tag{3.5}$$

are a Riemannian metric and a symplectic form, respectively. In terms of the real coordinates, they read

$$\mathfrak{g} = \sum_k (\mathrm{d}q_k \otimes \mathrm{d}q_k + \mathrm{d}p_k \otimes \mathrm{d}p_k), \quad \omega = 2\sum_k \mathrm{d}q_k \wedge \mathrm{d}p_k, \tag{3.6}$$

respectively. Let us introduce the dual coordinate vector fields

$$\frac{\partial}{\partial z_k} = \frac{1}{2}\left(\frac{\partial}{\partial q_k} - \mathrm{i}\frac{\partial}{\partial p_k}\right), \quad \mathrm{d}z_j\left(\frac{\partial}{\partial z_k}\right) = \frac{\partial}{\partial z_k} z_j = \delta_j^k, \quad \mathrm{d}\bar{z}_j\left(\frac{\partial}{\partial z_k}\right) = 0 \tag{3.7}$$

and analogously for $\frac{\partial}{\partial \bar{z}_k}$. In terms of them we can write the contravariant form of the Hermitian structure

$$K = \sum_k \frac{\partial}{\partial \bar{z}_k} \otimes \frac{\partial}{\partial z_k} = G + \frac{\mathrm{i}}{2}\Omega, \tag{3.8}$$

where

$$G=\sum_k \frac{\partial}{\partial \bar z_k}\otimes_s \frac{\partial}{\partial z_k}=\frac{1}{4}\sum_k\left(\frac{\partial}{\partial q_k}\otimes\frac{\partial}{\partial q_k}+\frac{\partial}{\partial p_k}\otimes\frac{\partial}{\partial p_k}\right), \tag{3.9}$$

$$\Omega=-\mathrm{i}\sum_k \frac{\partial}{\partial \bar z_k}\wedge\frac{\partial}{\partial z_k}=-\frac{1}{2}\sum_k \frac{\partial}{\partial q_k}\wedge\frac{\partial}{\partial p_k}. \tag{3.10}$$

By using Ω we can construct the Hamiltonian vector field X_f generated by any (smooth) Hamiltonian function f, in the standard way, i.e.

$$X_f=\Omega(\mathrm{d}f,\cdot)=-\mathrm{i}\sum_k\left(\frac{\partial f}{\partial \bar z_k}\frac{\partial}{\partial z_k}-\frac{\partial f}{\partial z_k}\frac{\partial}{\partial \bar z_k}\right)=\frac{1}{2}\sum_k\left(\frac{\partial f}{\partial p_k}\frac{\partial}{\partial q_k}-\frac{\partial f}{\partial q_k}\frac{\partial}{\partial p_k}\right). \tag{3.11}$$

It is easy to verify that it satisfies the equation $\omega(X_f,\cdot)=\mathrm{d}f$. The tensor Ω defines also a Lie-algebra structure on functions, with the Poisson brackets

$$\begin{aligned}\{f,g\}&=\Omega(\mathrm{d}f,\mathrm{d}g)\\ &=-\mathrm{i}\sum_k\left(\frac{\partial f}{\partial \bar z_k}\frac{\partial g}{\partial z_k}-\frac{\partial f}{\partial z_k}\frac{\partial g}{\partial \bar z_k}\right)=\frac{1}{2}\sum_k\left(\frac{\partial f}{\partial p_k}\frac{\partial g}{\partial q_k}-\frac{\partial f}{\partial q_k}\frac{\partial g}{\partial p_k}\right).\end{aligned} \tag{3.12}$$

On the other hand, G defines a Jordan algebra on all quadratic functions, with the Jordan brackets

$$\begin{aligned}\{f,g\}_+&=G(\mathrm{d}f,\mathrm{d}g)\\ &=\frac{1}{2}\sum_k\left(\frac{\partial f}{\partial \bar z_k}\frac{\partial g}{\partial z_k}+\frac{\partial f}{\partial z_k}\frac{\partial g}{\partial \bar z_k}\right)=\frac{1}{4}\sum_k\left(\frac{\partial f}{\partial q_k}\frac{\partial g}{\partial q_k}+\frac{\partial f}{\partial p_k}\frac{\partial g}{\partial p_k}\right).\end{aligned} \tag{3.13}$$

The real quadratic functions are the expectation values of observables

$$f_A(\psi)=\langle\psi|A|\psi\rangle=\sum_{k,l}\bar z_k A_{kl} z_l. \tag{3.14}$$

(Ω,G) defines a Lie–Jordan algebra on these expectation values,

$$\{f_A,f_B\}=f_{\mathrm{i}(AB-BA)},\quad \{f_A,f_B\}_+=f_{\frac{1}{2}(AB+BA)}, \tag{3.15}$$

isomorphic to the Lie–Jordan algebra of the observables.

The Hamilton equations, whose Hamiltonian vector fields (3.11) are generated by quadratic functions $f_H=\langle\psi|H|\psi\rangle$,

$$\dot z_k=X_{f_H}(z_k)=-\mathrm{i}\frac{\partial f}{\partial \bar z_k}=-\mathrm{i}\sum H_{kl}z_l, \tag{3.16}$$

are nothing but the Schrödinger equation with Hamiltonian operator H, $\dot\psi=H\psi$. The evolution of any expectation value (3.14) is given in terms of the Poisson brackets

$$\dot f_A=\{f_H,f_A\}=f_{\mathrm{i}(HA-AH)}, \tag{3.17}$$

which is the Heisenberg equation for the operator A, $\langle\psi|\dot A|\psi\rangle=\langle\psi|\mathrm{i}(HA-AH)|\psi\rangle$.

Projectability onto $\mathbb{P}\mathcal{H}$ requires a conformal factor

$$\tilde{G}(\psi) = \langle\psi|\psi\rangle G(\psi), \quad \tilde{\Omega}(\psi) = \langle\psi|\psi\rangle \Omega(\psi). \tag{3.18}$$

Observe that $\tilde{\Omega}$ does not satisfy the Jacobi identity anymore. However, it does on expectation values. Thus, $\tilde{\Omega}$ projects onto $\mathbb{P}\mathcal{H}$, a Poisson space. Moreover, one should consider homogeneous expectation values

$$\tilde{f}_A = \frac{\langle\psi|A|\psi\rangle}{\langle\psi|\psi\rangle}. \tag{3.19}$$

We are now ready to make the first key observation: the Zeno time (2.4) is the inverse length of the Hamiltonian vector field on the complex projective space, i.e.

$$\tau_{\mathrm{Z}}^{-2} = \tilde{G}(\mathrm{d}\tilde{f}_H, \mathrm{d}\tilde{f}_H). \tag{3.20}$$

Indeed, we have

$$\mathrm{d}\tilde{f}_H = \frac{1}{\|\psi\|^2}(\mathrm{d}f_H - \tilde{f}_H \mathrm{d}\|\psi\|^2), \tag{3.21}$$

and from (3.15) we get

$$G(\mathrm{d}f_H, \mathrm{d}f_H) = \{f_H, f_H\}_+ = f_{H^2} = \langle\psi|H^2|\psi\rangle. \tag{3.22}$$

Therefore,

$$\begin{aligned} \tilde{G}(\mathrm{d}\tilde{f}_H, \mathrm{d}\tilde{f}_H) &= \frac{1}{\|\psi\|^2} G(\mathrm{d}f_H - e_H \mathrm{d}\|\psi\|^2, \mathrm{d}f_H - e_H \mathrm{d}\|\psi\|^2) \\ &= \tilde{f}_{H^2} - (\tilde{f}_H)^2 = (\triangle H)^2. \end{aligned} \tag{3.23}$$

4. An Example: The Qubit

Let $\mathcal{H} \simeq \mathbb{C}^2 \simeq \mathbb{R}^4$ with Hamiltonian

$$H = h_0 \mathbb{I} + \vec{h} \cdot \vec{\sigma}, \tag{4.1}$$

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of Pauli matrices, $h_0 \in \mathbb{R}$, and $\vec{h} = (h_x, h_y, h_z) \in \mathbb{R}^3$. We introduce coordinates z_1, z_2 in the eigenbasis of σ_z, i.e. $\sigma_z|e_j\rangle = (-1)^j|e_j\rangle$, $j = 1, 2$. The expectation value (3.14) of the Hamiltonian H reads

$$f_H = \langle\psi|H|\psi\rangle = h_0 u + h_x x + h_y y + h_z z = h_0 u + \vec{h} \cdot \vec{x}, \tag{4.2}$$

where the quadratic functions u, x, y, z are the expectation values of the Pauli matrices

$$\begin{aligned} u &= f_{\mathbb{I}} = |z_1|^2 + |z_2|^2 = q_1^2 + p_1^2 + q_2^2 + p_2^2, \\ x &= f_{\sigma_x} = \bar{z}_1 z_2 + \bar{z}_2 z_1 = 2(q_1 q_2 + p_1 p_2), \\ y &= f_{\sigma_y} = -\mathrm{i}(\bar{z}_1 z_2 - \bar{z}_2 z_1) = 2(q_1 p_2 - p_1 q_2), \\ z &= f_{\sigma_z} = |z_1|^2 - |z_2|^2 = q_1^2 + p_1^2 - q_2^2 - p_2^2. \end{aligned} \tag{4.3}$$

Notice that u and $\vec{x}$ are not independent, since

$$u^2 = (|z_1|^2 + |z_2|^2)^2 = x^2 + y^2 + z^2 = \vec{x}^2. \tag{4.4}$$

By making use of (4.4) we easily get that the Zeno time (2.4) is

$$\tau_Z^{-2} = \tilde{G}(\mathrm{d}\tilde{f}_H, \mathrm{d}\tilde{f}_H) = \tilde{f}_{H^2} - (\tilde{f}_H)^2 = \vec{h}^2 - \frac{(\vec{h}\cdot\vec{x})^2}{u^2} = \frac{(\vec{h}\wedge\vec{x})^2}{\vec{x}^2}, \tag{4.5}$$

where $\wedge$ is the vector product in $\mathbb{R}^3$.

Let us now consider the Zeno effect with projection

$$P = |e_1\rangle\langle e_1| = \frac{1}{2}(\mathbb{I} + \sigma_3)\,. \tag{4.6}$$

The Zeno dynamics (2.6) becomes

$$U_Z(t) = \mathrm{e}^{-\mathrm{i}H_Z t}P, \quad H_Z = PHP = \frac{h_0 + h_z}{2}(\mathbb{I} + \sigma_3). \tag{4.7}$$

The Zeno Hamiltonian function reads

$$f_{H_Z} = \frac{h_0 + h_z}{2}(u + z) = (h_0 + h_z)|z_1|^2 = (h_0 + h_z)(q_1^2 + p_1^2), \tag{4.8}$$

which should be compared to the "free" Hamiltonian function (4.2) and (4.3). The infinitesimal generator of the Zeno dynamics is

$$X_Z = -\mathrm{i}(h_0 + h_z)\left(z_1\frac{\partial}{\partial z_1} - \bar{z}_1\frac{\partial}{\partial \bar{z}_1}\right) = 2(h_0 + h_z)\left(p_1\frac{\partial}{\partial q_1} - q_1\frac{\partial}{\partial p_1}\right) \tag{4.9}$$

and the dynamics reads

$$\dot{u} = 0, \quad \dot{x} = -2(h_0 + h_z)y, \quad \dot{y} = 2(h_0 + h_z)x, \quad \dot{z} = 0. \tag{4.10}$$

Observe that, due to (4.4), the first equation, $\dot{u} = 0$, entails that the flow is on the sphere $|\vec{x}| = $ constant. This flow is depicted in Fig. 1.

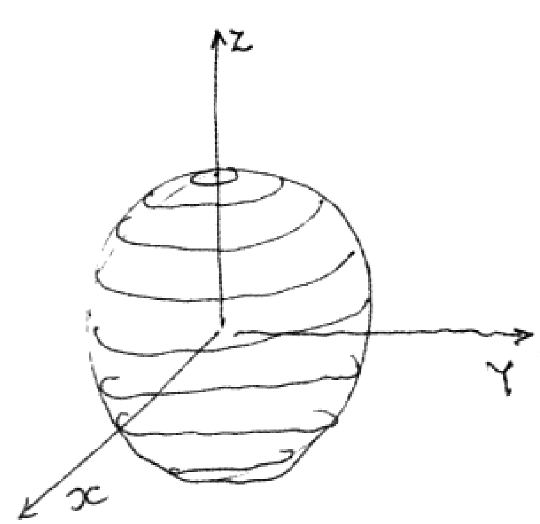

Fig. 1. Zeno flow of a qubit subject to measurement $P = \frac{1}{2}(\mathbb{I} + \sigma_3)$, according to Beppe Marmo, author of the drawing.

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measure → $|0\rangle = |\text{in}\rangle$

system remains in initial state

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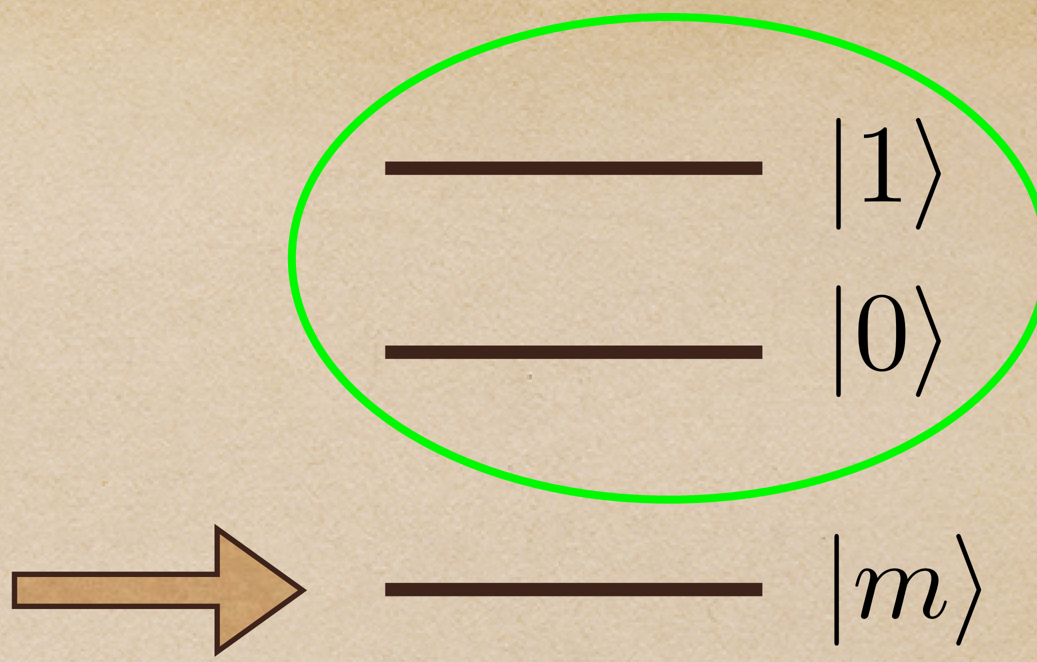

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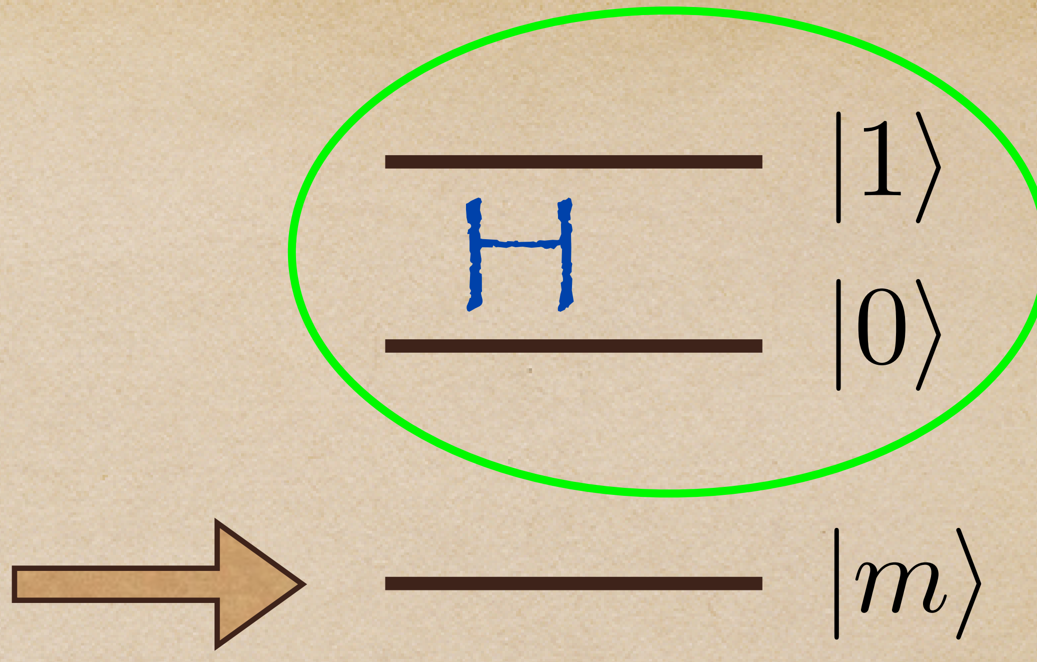

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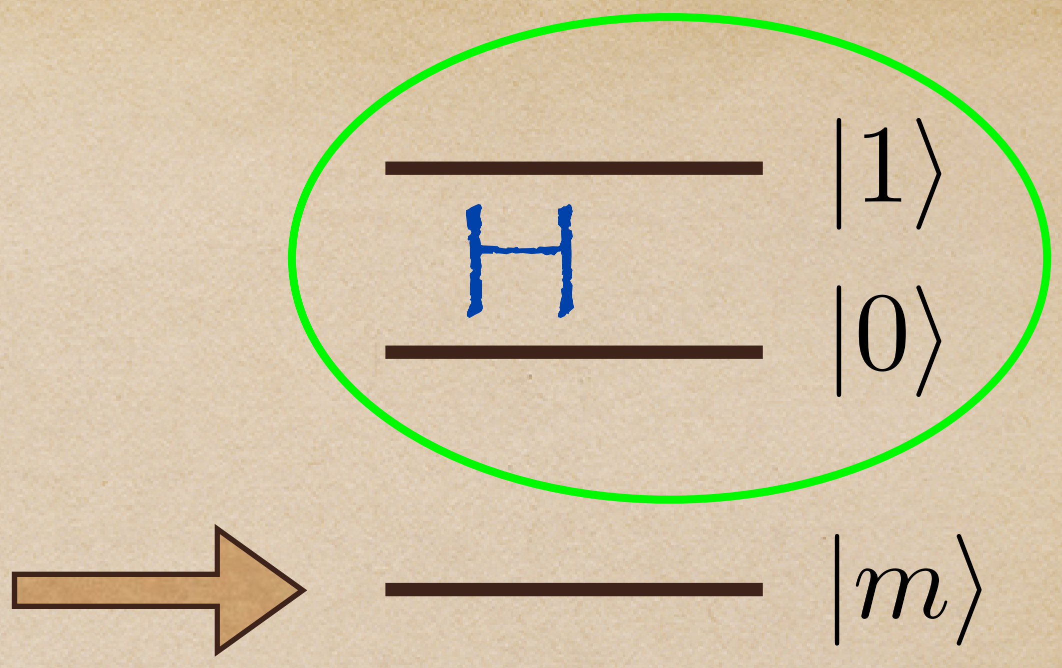

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Quantum Zeno DYNAMICS

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bounded Hamiltonian, $\|H\| < \infty$,

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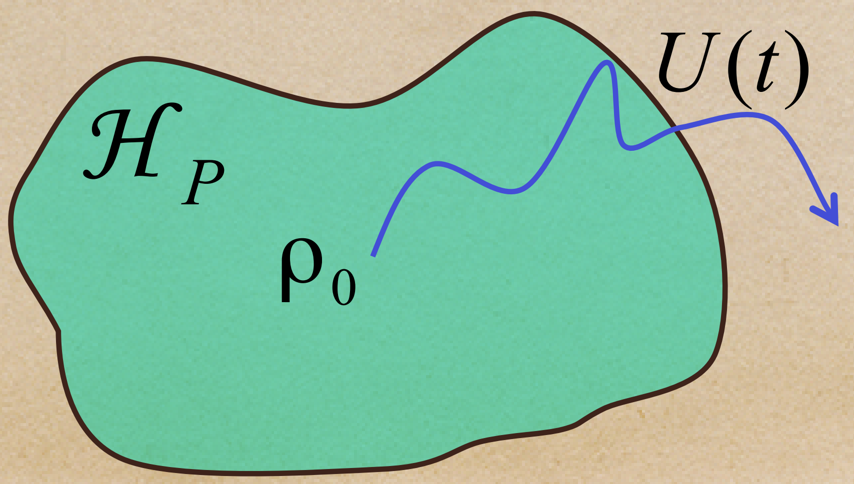
Limiting time evolution

$$V_Z(t) = \lim_{N\to\infty} V_N(t) = \lim_{N\to\infty}[PU(t/N)P]^N = P\exp(-iPHPt)$$

unitary in $\mathcal{H}_P$, i.e. PHP self-adjoint in $\mathcal{H}_P$.



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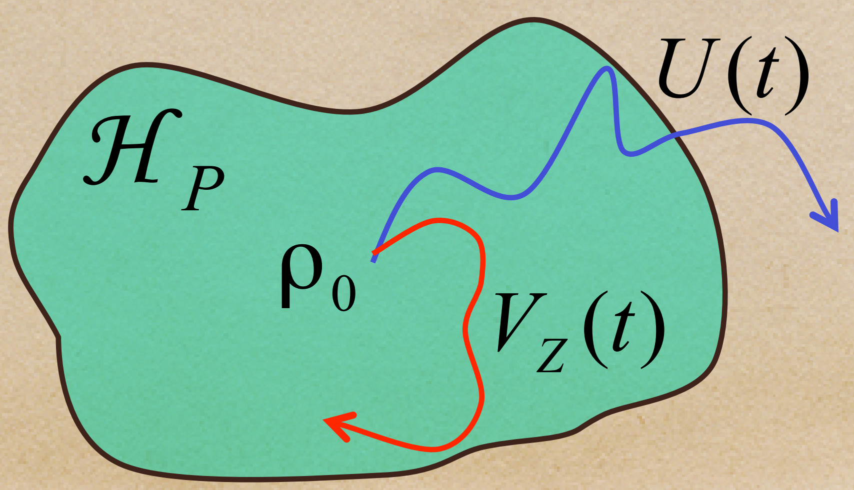

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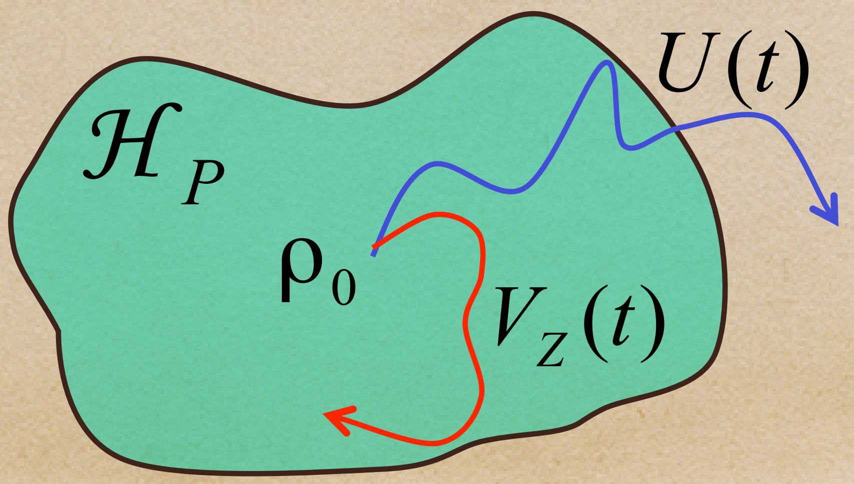
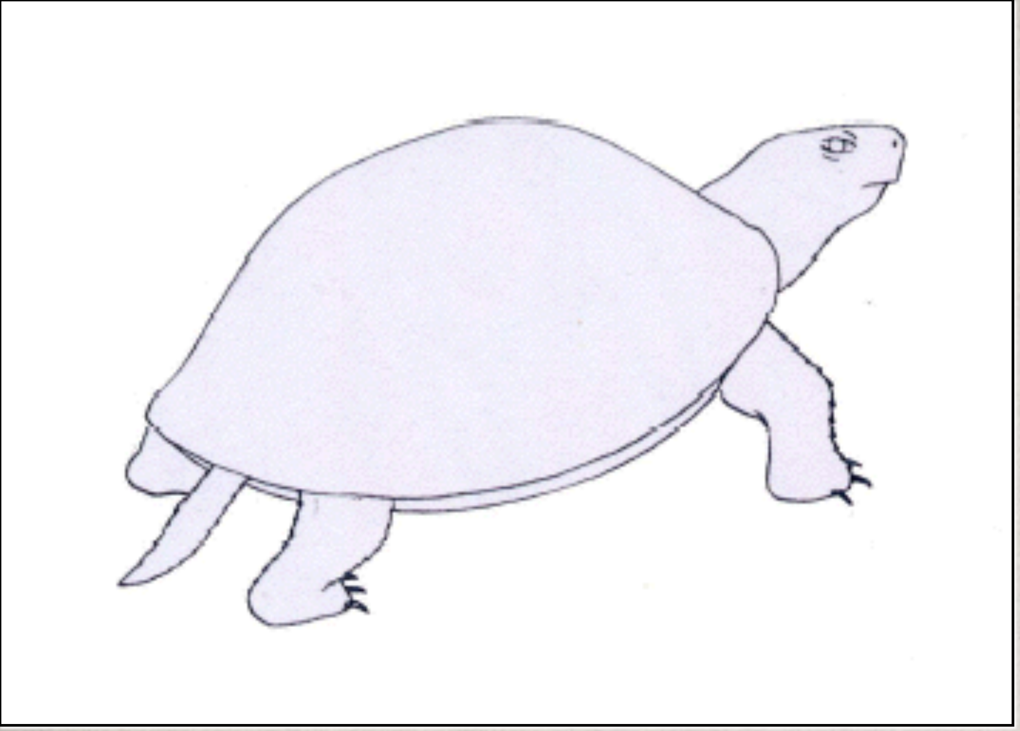
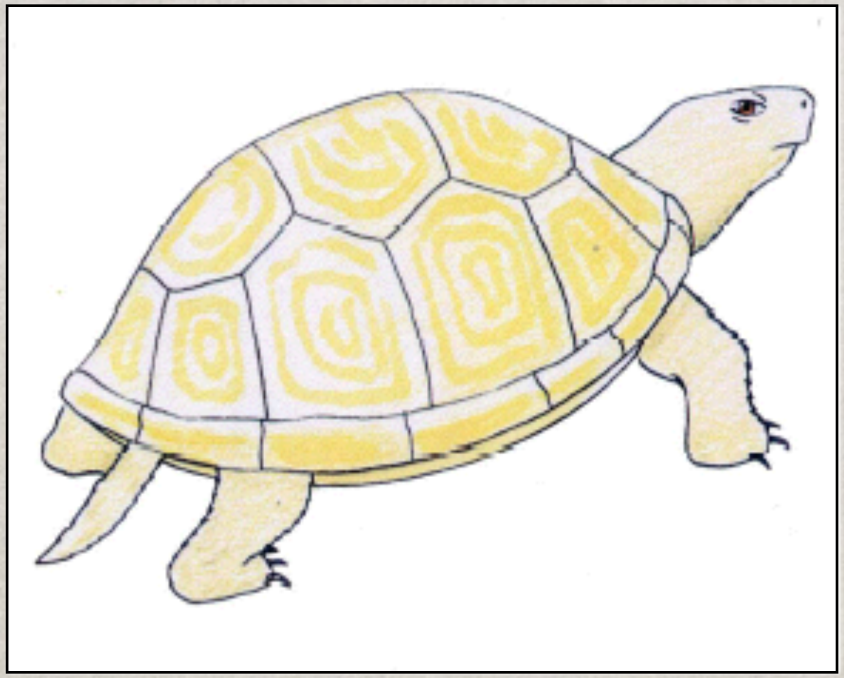


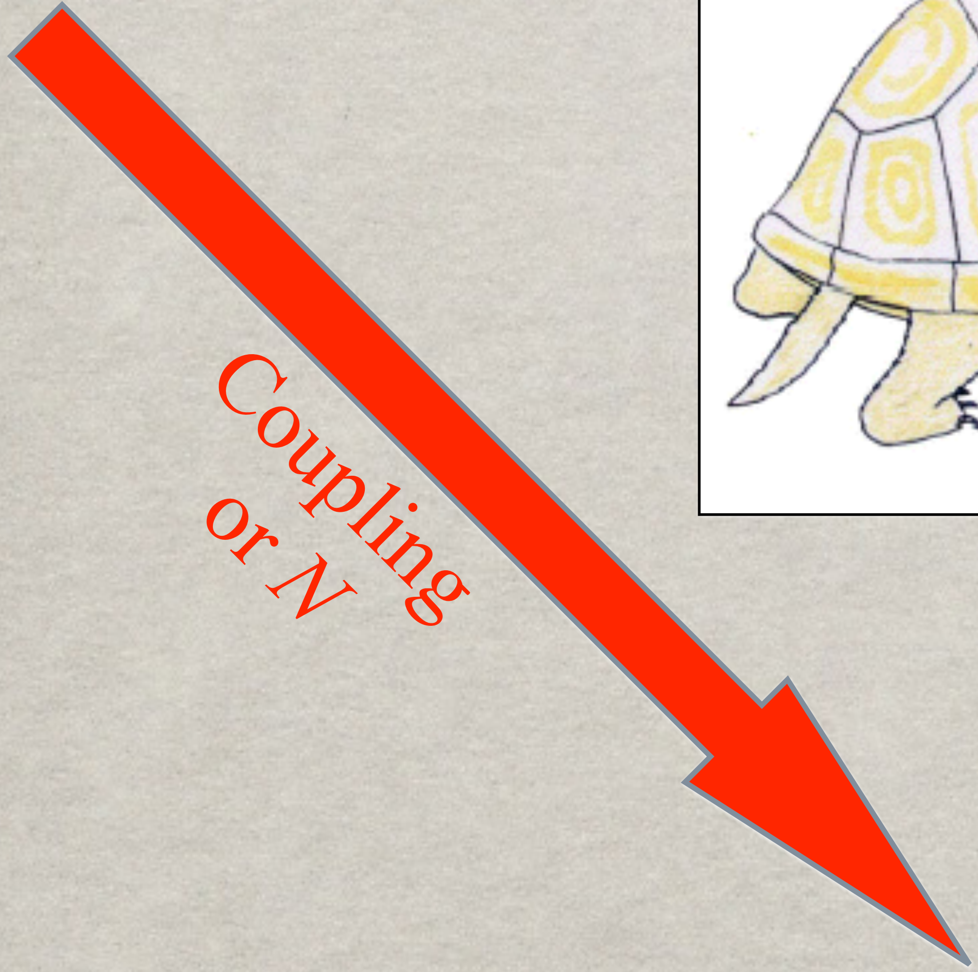
Friedman 1972
Facchi, Gorini, Marmo, Pascazio, Sudarshan 2000
Exner, Ichinose 2005
Facchi et al >2008

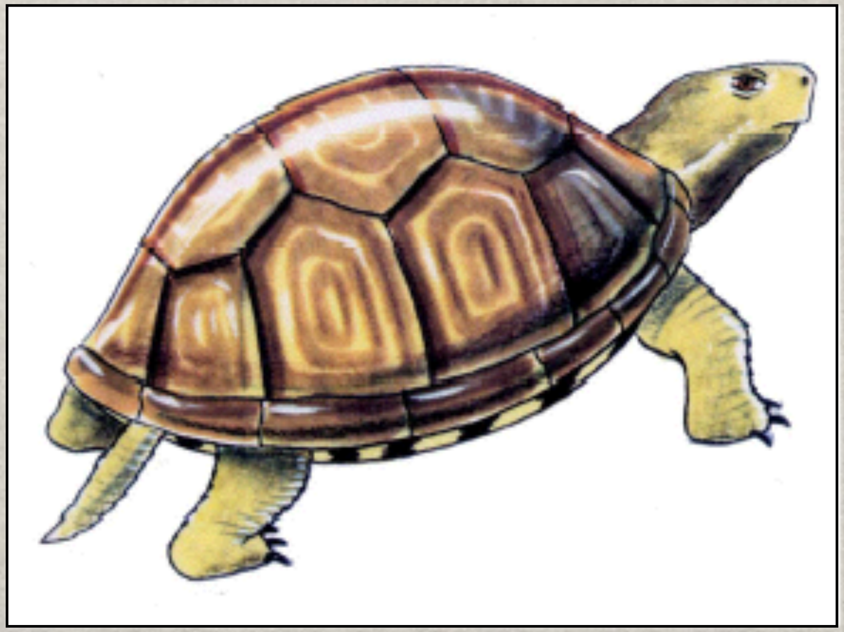
Quantum Zeno subspaces



DYNAMICAL SUPERSELECTION SECTORS







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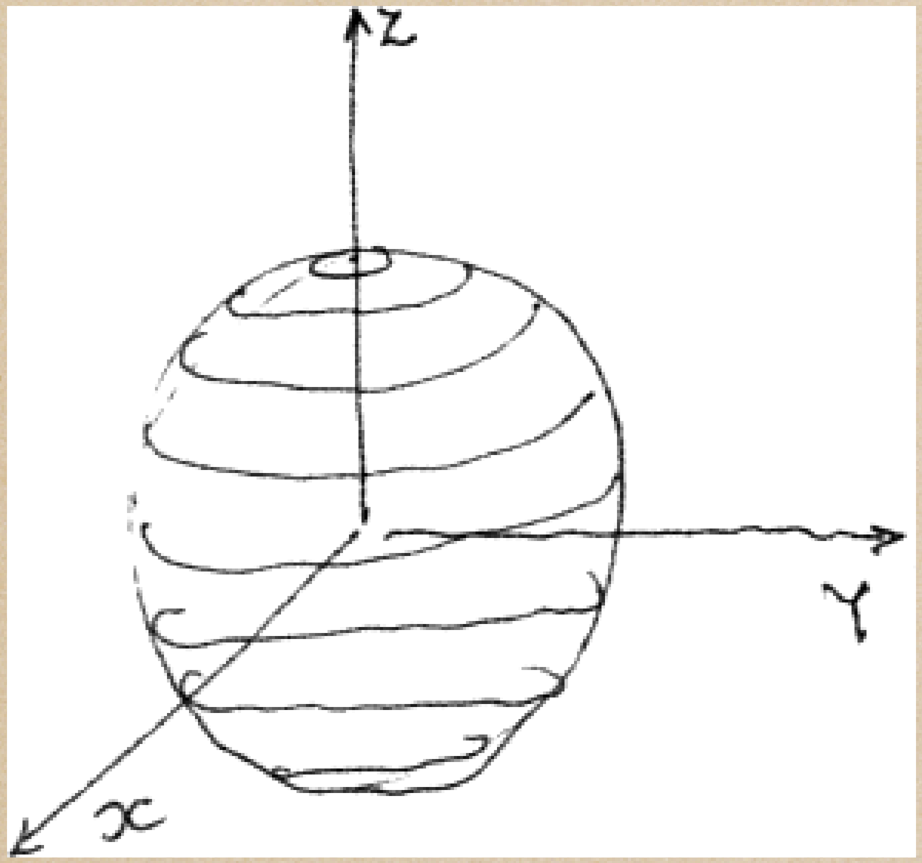

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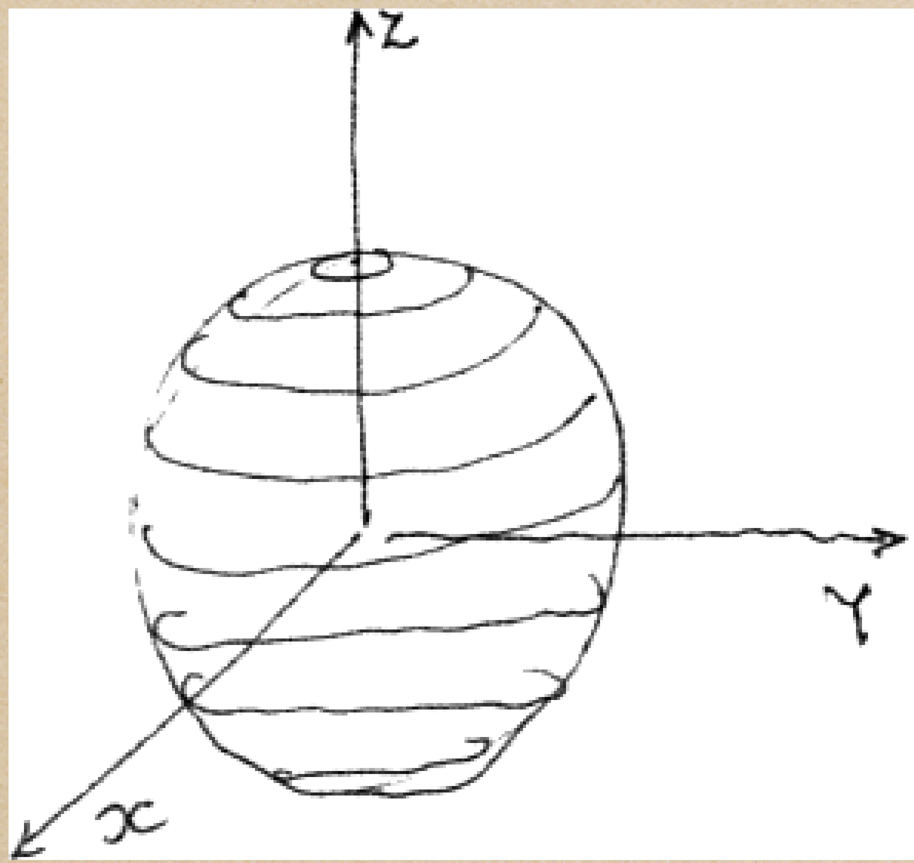

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challenge (for the youngest in the audience): extend to qutrit

H

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→ $|m\rangle$

Folding and unfolding

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Carinena, Ibort, Marmo, Morandi, Geometry from dynamics, classical and quantum (Springer 2015)

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The unmanifest world is simple and linear,
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G. Marmo

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- definitions are better than proofs
- remove unnecessary details
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- back to Parmenides-Zeno:
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