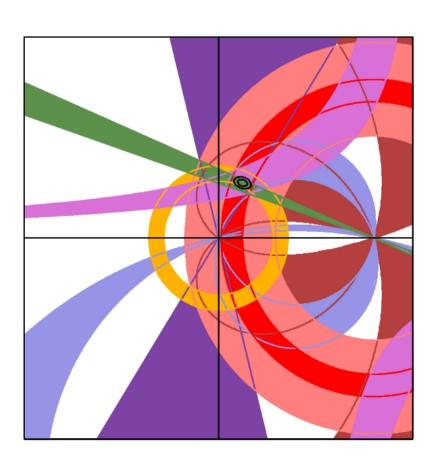
Model-independent new physics analysis with the Unitarity Triangle fit



Marcella Bona



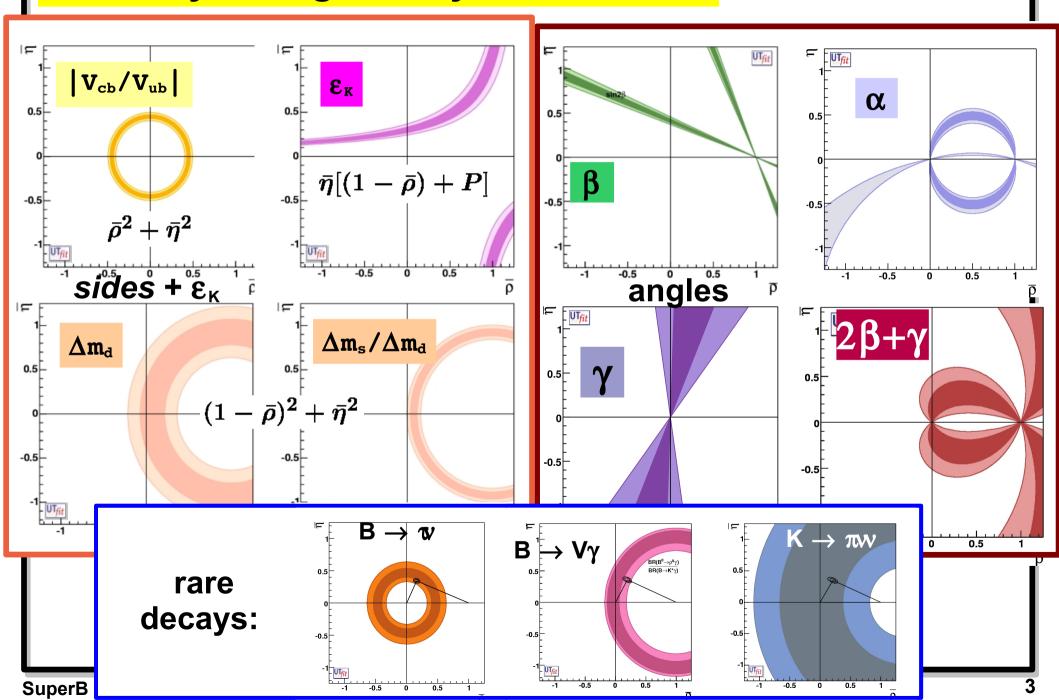
Workshop on New Physics with SuperB, Warwick University, April 15th, 2009



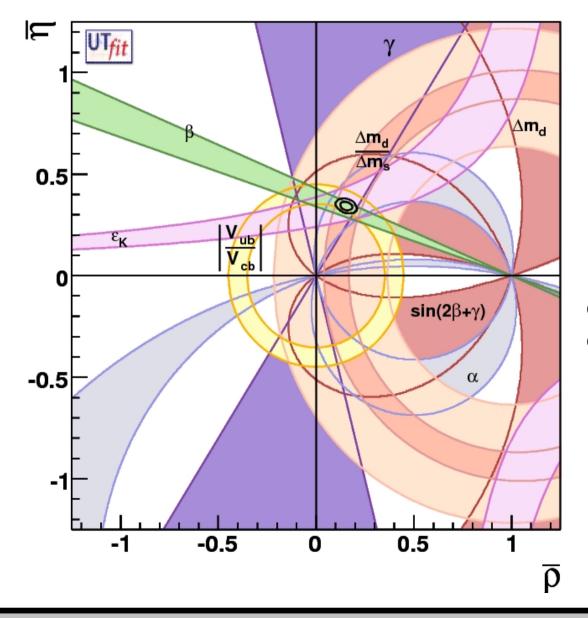
www.utfit.org

M.B., M. Ciuchini, E. Franco, V. Lubicz, G. Martinelli, F. Parodi, M. Pierini, C. Schiavi, L. Silvestrini, A. Stocchi, V. Sordini, C. Tarantino and V. Vagnoni

Unitarity Triangle analysis in the SM



Unitarity Triangle analysis in the SM

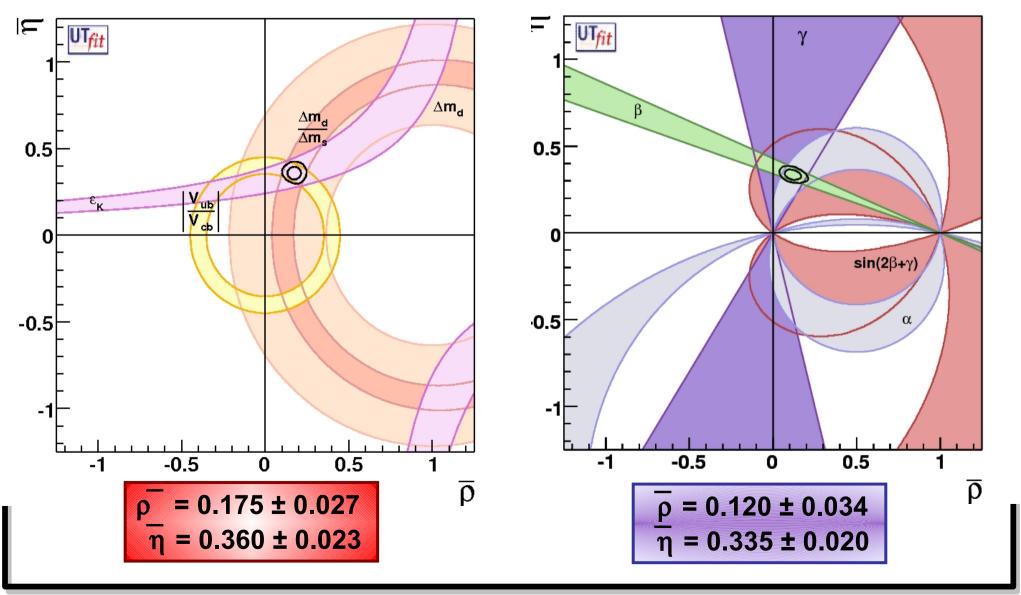


$$\frac{\overline{\rho}}{\eta}$$
 = 0.155 ± 0.022
 $\frac{\overline{\rho}}{\eta}$ = 0.342 ± 0.014

- Data in agreement
- NP, if any, seems not to introduce additional CP or flavour violation in b ↔ d transitions at current experimental precision

the LEP-style analysis vs the angle analysis

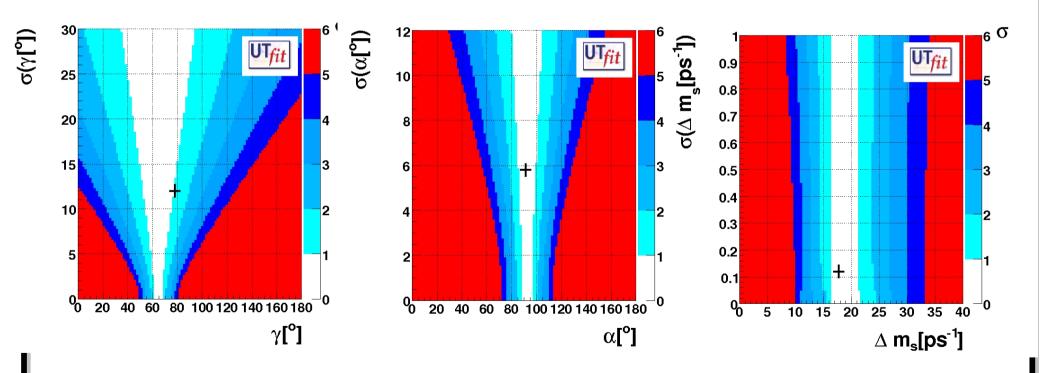
levels @ 95% Prob



compatibility plots

A way to "measure" the agreement of a single measurement with the indirect determination from the fit using all the other inputs: test for the SM description of the flavor physics

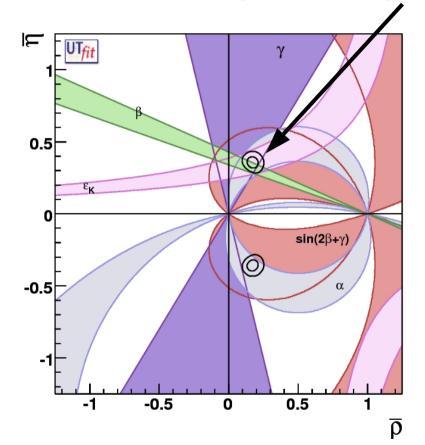
The cross has the coordinates (x,y)=(central value, error) of the direct measurement

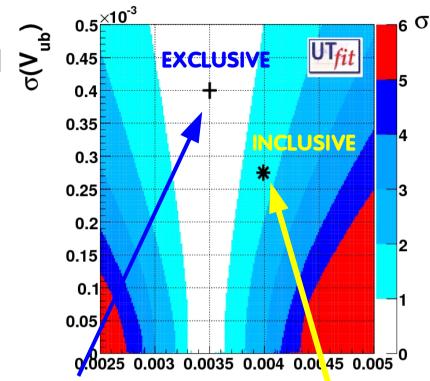


Color code: agreement between the predicted values and the measurements at better than 1, 2, ... $n\sigma$

the current status of the tension

Contours (68% and 95%) for the vertex position determined by Δ ms/ Δ md, $|V_{ub}/V_{cb}|$





Relying on semileptonic form Vub factors determined from Lattice QCD and QCD sum rules

$$Vub_{excl} = (35.0 \pm 4.0) \cdot 10^{-4}$$

 $Vub_{UTfit} = (34.8 \pm 1.6) \cdot 10^{-4}$

Relying on some HQET parameters extracted from experimental fits with some model dependence

 $Vub_{incl} = (39.9 \pm 1.5 \pm 4.0) \cdot 10^{-4}$

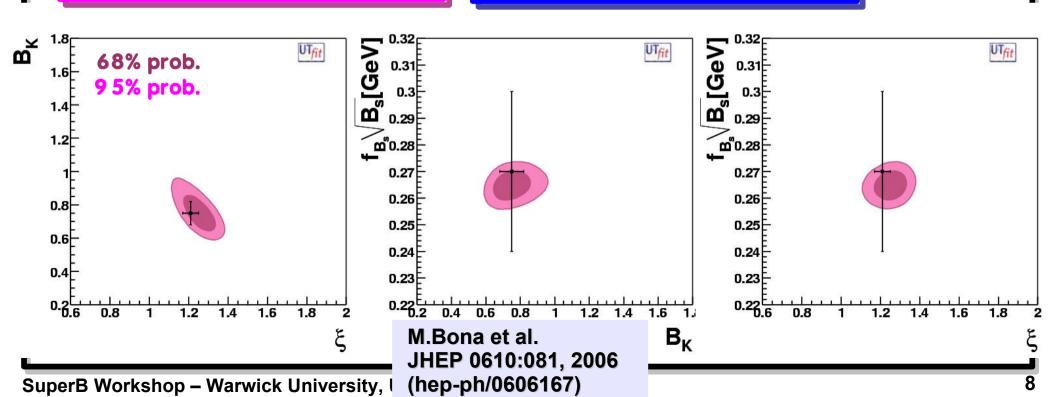
some a-posteriori determinations: lattice QCD

Through the Standard Model Unitarity Triangle analysis, without using the lattice inputs, we also obtain the updated values of the predictions for the lattice parameters

$$B_{K}^{UT} = 0.75 \pm 0.07$$
 $f_{Bs} \sqrt{B_{Bs}}^{UT} = 265 \pm 4 \text{ MeV}$
 $\xi^{UT} = 1.26 \pm 0.05$

$$B_{K}^{lat} = 0.75 \pm 0.07$$
 $f_{Bs} \sqrt{B_{Bs}}^{lat} = 270 \pm 30 \text{ MeV}$
 $\xi^{lat} = 1.21 \pm 0.04$

Averages by UTfit: V. Lubicz, C. Tarantino

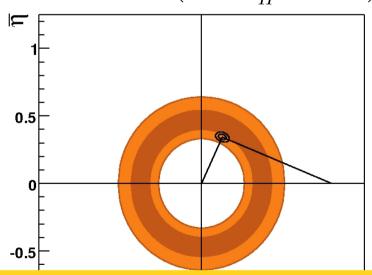


some standard model determinations: B→w

$$\mathcal{B}(B \to \ell \nu) = \frac{G_F^2 m_B m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_B^2} \right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

SM prediction enhanced or reduced by factor r_H :

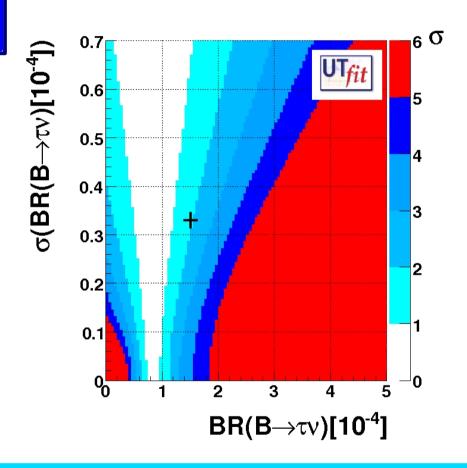
$$r_H = \left(1 - \frac{m_B^2}{m_H^2} \tan^2 \beta\right)^2$$



current HFAG world average

BR(B
$$\to \tau v$$
) = (1.51 ± 0.33) 10⁻⁴

-0.5 0 0.5 1



indirect determination from UT without using lattice QCD and Vub

 $BR(B \rightarrow \tau V) = (0.73 \pm 0.12) 10^{-4}$

the tree level fit:

B factories are constraining the UT with tree-level processes

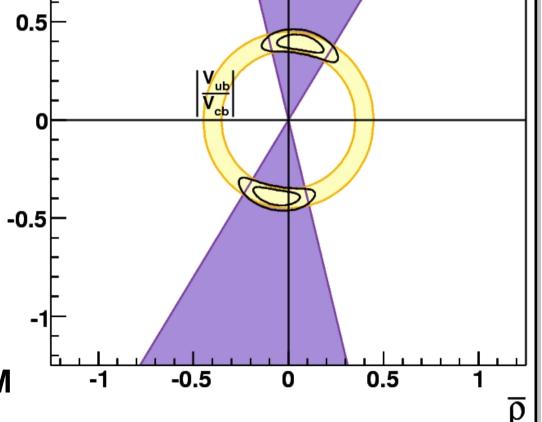
Assuming no NP at tree level (the effect of the \overline{D} - D^0 mixing to γ are small wrt the present error and can be accounted for in the future)

We can determine ρ and η regardless of NP

$$\frac{\overline{\rho}}{\eta} = \pm 0.06 \pm 0.08$$

 $\frac{\overline{\rho}}{\eta} = \pm 0.39 \pm 0.03$

Values in agreement with SM within the errors



UT analysis including new physics (NP)

Consider for example B_s mixing process. Given the SM amplitude, we can define

$$C_{B_s}e^{-2\mathrm{i}\phi_{B_s}} = \frac{\langle \overline{B}_s|H_{eff}^{SM} + H_{eff}^{NP}|B_s\rangle}{\langle \overline{B}_s|H_{eff}^{SM}|B_s\rangle} = 1 + \frac{A_{NP}e^{-2\mathrm{i}\phi_{NP}}}{A_{SM}e^{-2\mathrm{i}\beta_s}}$$

All NP effects can be parameterized in terms of one complex parameter for each meson mixing, to be determined in a simultaneous fit with the CKM parameters (now there are enough experimental constraints to do so).

For kaons we use Re and Im, since the two exp. constraints ϵ_K and Δm_K are directly related to them (with distinct theoretical issues)

$$C_{\varepsilon_{K}} = \frac{\operatorname{Im}\left\langle K^{0} \left| H_{eff}^{full} \right| \overline{K}^{0} \right\rangle}{\operatorname{Im}\left\langle K^{0} \left| H_{eff}^{SM} \right| \overline{K}^{0} \right\rangle}$$

$$C_{\Delta m_{K}} = \frac{\operatorname{Re}\left\langle K^{0} \left| H_{eff}^{full} \right| \overline{K}^{0} \right\rangle}{\operatorname{Re}\left\langle K^{0} \left| H_{eff}^{SM} \right| \overline{K}^{0} \right\rangle}$$

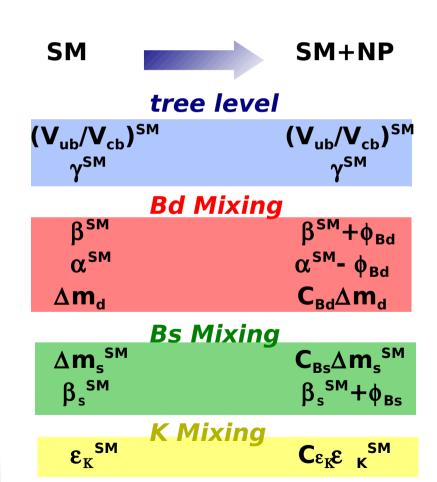
UT analysis including new physics (NP)

M. Bona et al. (UTfit)

Phys.Rev.Lett.97:151803,2006

I Hys.Rev.De				
	ρ, η	C_{Bd} , ϕ_{Bd}	$C_{arepsilonK}$	C_{Bs}, ϕ_{Bs}
V_{ub}/V_{cb}	X			
γ (DK)	X			
ϵ_{K}	X		X	
sin2β	X	X		
Δm_{d}	X	Χ		
α (ρρ,ρπ,ππ)	X	X		
A _{SL} B _d	X	XX		
$\Delta\!\Gamma_{\rm d}/\Gamma_{\rm d}$	X	XX		
$\Delta\Gamma_{\rm s}/\Gamma_{\rm s}$	X			XX
Δm_s				X
A _{CH}	Х	XX		ХХ

model independent assumptions



new-physics-specific constraints

$$A_{\rm SL}^s \equiv \frac{\Gamma(\bar{B}_s \to \ell^+ X) - \Gamma(B_s \to \ell^- X)}{\Gamma(\bar{B}_s \to \ell^+ X) + \Gamma(B_s \to \ell^- X)} = \operatorname{Im}\left(\frac{\Gamma_{12}^s}{A_s^{\rm full}}\right)$$

 Laplace et al. Phys.Rev.D 65: 094040,2002

$$A_{\rm SL}^s \times 10^2 = -0.20 \pm 1.19$$

D0 ICHEP08

• same-side dilepton charge asymmetry A_{CH} : admixture of B_d and B_s dependent on ρ and η and on NP effects

$$A_{\rm SL}^{\mu\mu} \times 10^3 = -4.3 \pm 3.0$$

$$A_{\rm SL}^{\mu\mu} = \frac{f_d \chi_{d0} (A_{\rm SI}^d) + f_s \chi_{s0} (A_{\rm SI}^s)}{f_d \chi_{d0} + f_s \chi_{s0}}$$

• lifetime τ_s in flavour-specific final states: fit for a single exponential for B_s and \overline{B}_s the average lifetime is a function of the width and width difference

$$\tau_{B_s}^{\rm FS} [\rm ps] = 1.461 \pm 0.032$$

$$au_{B_s}^{FS} = rac{1}{\Gamma_s} rac{1 - \left(rac{\Delta \Gamma_s}{2\Gamma_s}
ight)^2}{1 + \left(rac{\Delta \Gamma_s}{2\Gamma_s}
ight)^2}$$

Dunietz et al., hep-ph 0012219

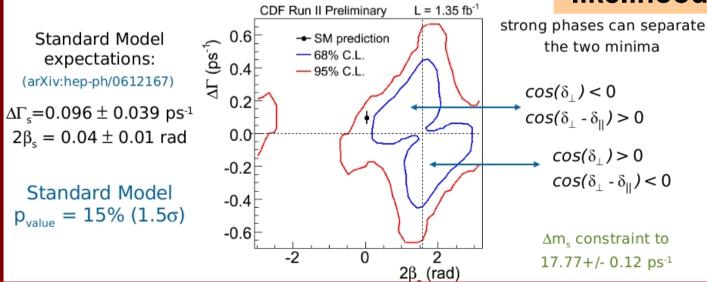
 C_{pen} and ϕ_{pen} are

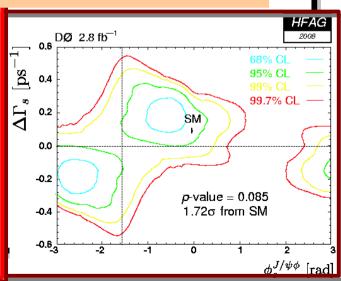
new-physics-specific constraints (II)

B meson mixing matrix element NLO calculation Ciuchini et al. IHFP 0308:031 2003

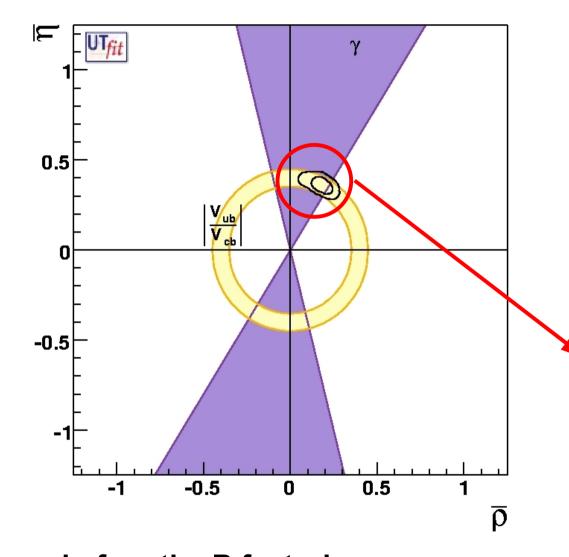
$$\begin{split} \frac{\Gamma_{12}^q}{A_q^{\text{full}}} &= -2\underbrace{\frac{\kappa}{C_{B_q}}} \left\{ \underbrace{\frac{2\phi_{B_q}}{n_1 + \frac{n_6B_2 + n_{11}}{B_1}} - \frac{e^{(\phi_q^{\text{SM}} + 2\phi_{B_q})}}{R_t^q} \left(n_2 + \frac{n_7B_2 + n_{12}}{B_1}\right)}_{R_t^q} \right. \\ &\quad + \underbrace{\frac{e^{2(\phi_q^{\text{SM}} + \phi_{B_q})}}{R_t^q} \left(n_3 + \frac{n_8B_2 + n_{13}}{B_1}\right) + e^{(\phi_q^{\text{Pen}} + 2\phi_{B_q})} C_q^{\text{Pen}} \left(n_4 + n_9 \frac{B_2}{B_1}\right)}_{-e^{(\phi_q^{\text{SM}} + \phi_q^{\text{Pen}} + 2\phi_{B_q})}} \underbrace{\frac{C_q^{\text{Pen}}}{R_t^q} \left(n_5 + n_{10} \frac{B_2}{B_1}\right)}_{R_t^q} \right\} \end{split}$$

 ϕ_s and $\Delta\Gamma_s$: 2D experimental likelihood from CDF and D0





UT analysis including NP



$$\frac{\overline{\rho}}{\eta}$$
 = 0.177 ± 0.044 $\frac{\overline{\rho}}{\eta}$ = 0.360 ± 0.031

Allowing for NP we go back to the SM solution

$$\frac{\overline{\rho}}{\eta}$$
 = 0.155 ± 0.022 $\frac{\overline{\rho}}{\eta}$ = 0.342 ± 0.014

before the B factories: the uncertainty on CKM parameters with NP was the limiting factor.

NP parameters in the K & B_d sectors

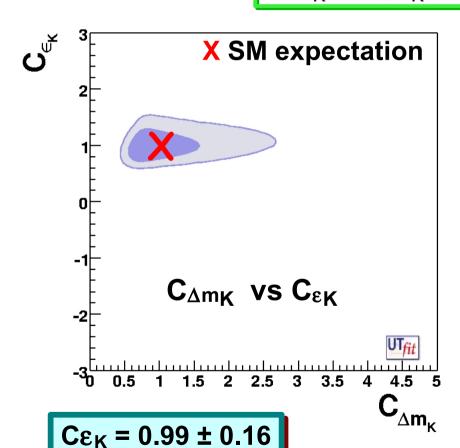
$$\operatorname{Im} A_{K} = C_{\varepsilon} \operatorname{Im} A_{K}^{SM}$$

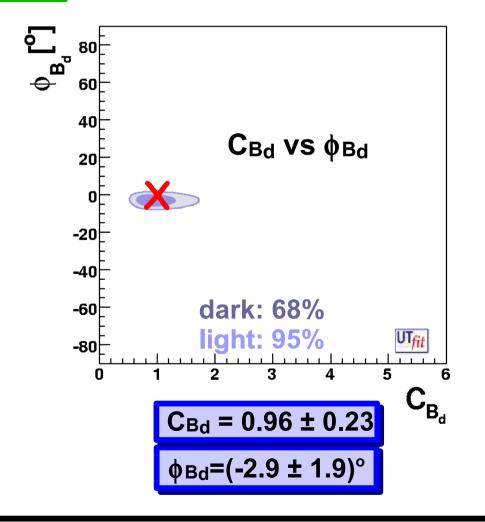
$$\operatorname{Re} A_{K} = C_{\Delta m_{K}} \operatorname{Re} A_{K}^{SM}$$

$$\Delta m_{K} = \mathbf{C}_{\Delta m_{K}} (\Delta m_{K})^{SM}$$

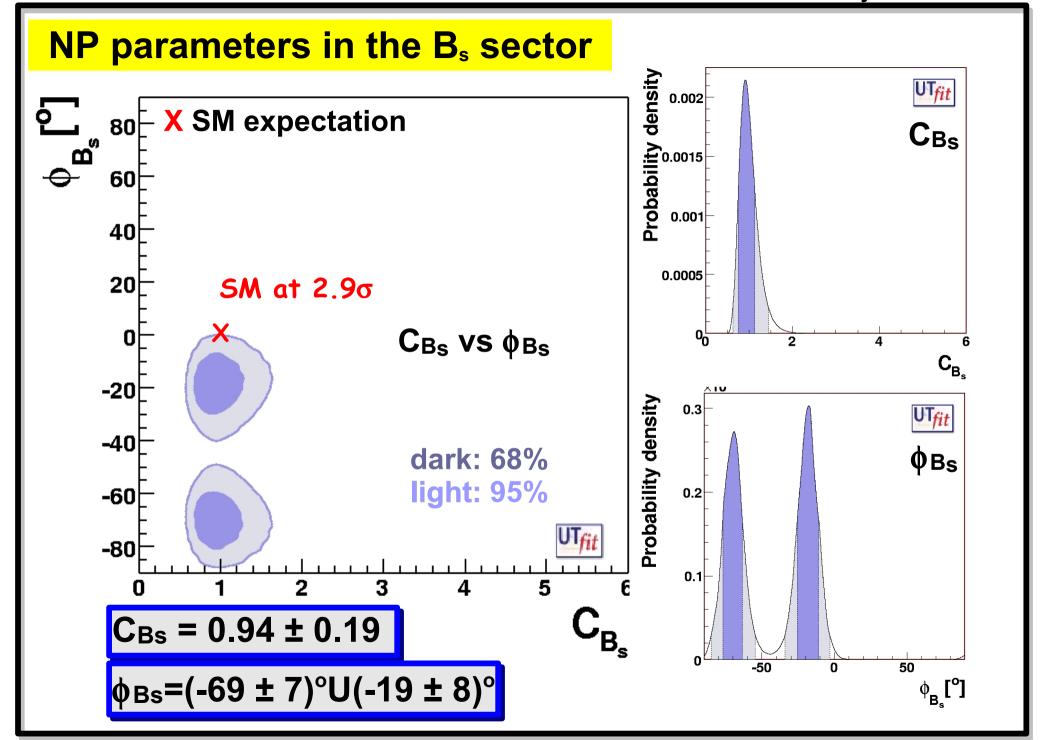
$$\varepsilon_{K} = \mathbf{C}_{\varepsilon} \varepsilon_{K}^{SM}$$

$$C_{B_q}\,e^{2i\phi_{B_q}} = \frac{\langle B_q|H_{\mathrm{eff}}^{\mathrm{full}}|\bar{B}_q\rangle}{\langle B_q|H_{\mathrm{eff}}^{\mathrm{SM}}|\bar{B}_q\rangle}$$





 $C\Delta m_{K} = 0.96 \pm 0.34$



R

Testing the new-physics scale

At the high scale

new physics enters according to its specific features

At the low scale use OPE to write the most general effective Hamiltonian. the operators have different chiralities than the SM NP effects are in the Wilson **Coefficients C**

NP effects are enhanced

- up to a factor 10 by the values of the matrix elements especially for transitions among quarks of different chiralities
- up to a factor 8 by RGE

$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^{5} \left(C_i \right) Q_i^{bq} + \sum_{i=1}^{3} \left(\tilde{C}_i \right) \tilde{Q}_i^{bq}$$

$$Q_1^{q_i q_j} = \bar{q}_{jL}^{\alpha} \gamma_{\mu} q_{iL}^{\alpha} \bar{q}_{jL}^{\beta} \gamma^{\mu} q_{iL}^{\beta} ,$$

$$Q_2^{q_i q_j} = \bar{q}_{jR}^{\alpha} q_{iL}^{\alpha} \bar{q}_{jR}^{\beta} q_{iL}^{\beta} ,$$

$$Q_3^{q_i q_j} = \bar{q}_{jR}^{\alpha} q_{iL}^{\beta} \bar{q}_{jR}^{\beta} q_{iL}^{\alpha} ,$$

$$Q_4^{q_i q_j} = \bar{q}_{jR}^{\alpha} q_{iL}^{\alpha} \bar{q}_{jL}^{\beta} q_{iR}^{\beta} ,$$

$$Q_5^{q_i q_j} = \bar{q}_{jR}^{\alpha} q_{iL}^{\beta} \bar{q}_{jL}^{\beta} q_{iR}^{\alpha} .$$

M. Bona et al. (UTfit) JHEP 0803:049,2008 arXiv:0707.0636

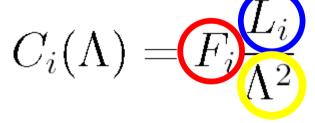
Effective BSM Hamiltonian for $\Delta F=2$ transitions

Most general form of the effective Hamiltonian for $\Delta F=2$ processes

$$\mathcal{H}_{\text{eff}}^{K-\bar{K}} = \sum_{i=1}^{5} C_i \, Q_i^{sd} + \sum_{i=1}^{3} \tilde{C}_i \, \tilde{Q}_i^{sd}$$

$$\mathcal{H}_{\text{eff}}^{B_q - \bar{B}_q} = \sum_{i=1}^{5} C_i \, Q_i^{bq} + \sum_{i=1}^{3} \tilde{C}_i \, \tilde{Q}_i^{bq}$$

The Wilson coefficients C_i have in general the form



Putting bounds on the Wilson coefficients give insights into the NP scale in different NP scenarios that enter through F_i and L_i

F_i: function of the NP flavour couplings

L_i: loop factor (in NP models with no tree-level FCNC)

 Λ : NP scale (typical mass of new particles mediating Δ F=2 transitions)

Contribution to the mixing amplitutes

analytic expression for the contribution to the mixing amplitudes

Lattice QCD

$$\langle \bar{B}_q | \mathcal{H}_{\mathrm{eff}}^{\Delta B=2} | B_q \rangle_i = \sum_{j=1}^5 \sum_{r=1}^5 \left(b_j^{(r,i)} \right) + \eta c_j^{(r,i)} \eta^{aj} C_i(\Lambda) \left(\bar{B}_q | Q_r^{bq} | B_q \right)$$

arXiv:0707.0636: for "magic numbers" a,b and c, $\eta = \alpha_s(\Lambda)/\alpha_s(m_t)$

analogously for the K system

$$\langle ar{K}^0 | \mathcal{H}_{ ext{eff}}^{\Delta S = 2} | K^0
angle_i = \sum_{j=1}^5 \sum_{r=1}^5 \left(b_j^{(r,i)} + \eta \, c_j^{(r,i)}
ight) \eta^{a_j} \, C_i(\Lambda) \, R_r \, \langle ar{K}^0 | Q_1^{sd} | K^0
angle$$

to obtain the p.d.f. for the Wilson coefficients $C_i(\Lambda)$ at the new-physics scale, we switch on one coefficient at a time in each sector and calculate its value from the result of the NP analysis.

Testing the TeV scale

 $C_i(\Lambda) = F_i \frac{L_i}{\Lambda^2}$

The dependence of C on Λ changes on flavor structure. we can consider different flavour scenarios:

• Generic: $C(\Lambda) = \alpha/\Lambda^2$ $F_i \sim 1$, arbitrary phase

• NMFV: $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$ $F_i \sim |F_{SM}|$, arbitrary phase

• MFV: $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2 F_1 \sim |F_{SM}|, F_{i\neq 1} \sim 0$, SM phase

 α (L_i) is the coupling among NP and SM

- \odot α ~ 1 for strongly coupled NP
- α ~ α_w (α_s) in case of loop coupling through weak (strong) interactions

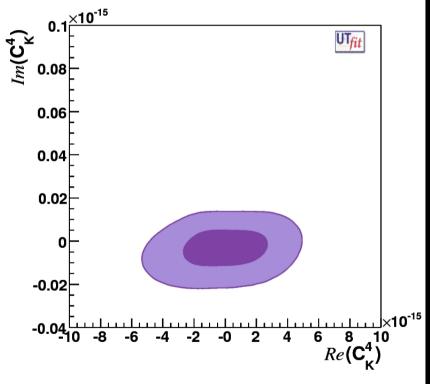
F_{SM} is the combination of CKM factors for the considered process

If no NP effect is seen lower bound on NP scale Λ if NP is seen upper bound on NP scale Λ

Results from the Wilson coefficients

the results obtained for the flavour scenarios: In deriving the lower bounds on the NP scale, we assume Li = 1, corresponding to strongly-interacting and/or tree-level NP.

Parameter	95% allowed range	Lower limit on Λ (TeV)	Lower limit on Λ (TeV)
	(GeV^{-2})	for arbitrary NP	for NMFV
$\mathrm{Re} C_K^1$	$[-9.6, 9.6] \cdot 10^{-13}$	$1.0\cdot 10^3$	0.35
$\mathrm{Re}C_K^2$	$[-1.8, 1.9] \cdot 10^{-14}$	$7.3\cdot 10^3$	2.0
$\mathrm{Re}C_K^3$	$[-6.0, 5.6] \cdot 10^{-14}$	$4.1\cdot 10^3$	1.1
$\mathrm{Re} C_K^4$	$[-3.6, 3.6] \cdot 10^{-15}$	$17 \cdot 10^3$	4.0
$\mathrm{Re} C_K^5$	$[-1.0, 1.0] \cdot 10^{-14}$	10 · 10 ³	2.4
$\mathrm{Im}C_K^1$	$[-4.4, 2.8] \cdot 10^{-15}$	$1.5\cdot 10^4$	5.6
${\rm Im} C_K^2$	$[-5.1, 9.3] \cdot 10^{-17}$	$10 \cdot 10^4$	28
${\rm Im} C_K^3$	$[-3.1, 1.7] \cdot 10^{-16}$	$5.7 \cdot 10^4$	19
${\rm Im} C_K^4$	$[-1.8, 0.9] \cdot 10^{-17}$	$24\cdot 10^4$	62
$\mathrm{Im}C_{K}^{5}$	$[-5.2, 2.8] \cdot 10^{-17}$	$14\cdot 10^4$	37

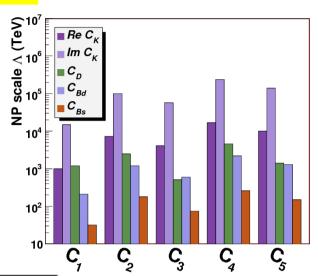


To obtain the lower bound for loop-mediated contributions, one simply multiplies the bounds by $\alpha_s \sim 0.1$ or by $\alpha_w \sim 0.03$.

Upper and lower bound on the scale

Lower bounds on NP scale from K and B_d physics (in TeV at 95% prob.)

Scenario	strong/tree	α_s loop	α_W loop
MFV	5.5	0.5	0.2
NMFV	62	6.2	2
General	24000	2400	800



Upper bounds on NP scale from Bs:

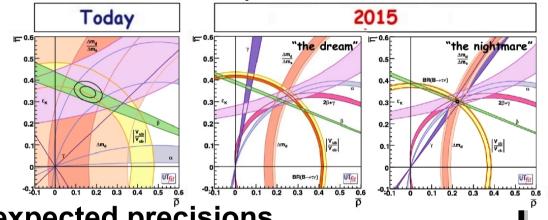
Scenario	strong/tree	α_s loop	α_W loop
NMFV	35	4	2
General	800	80	30

- the general case was already problematic (well known flavour puzzle)
- NMFV has problems with the size of the B_s effect vs the (insufficient) suppression in B_d and (in particular) K mixing

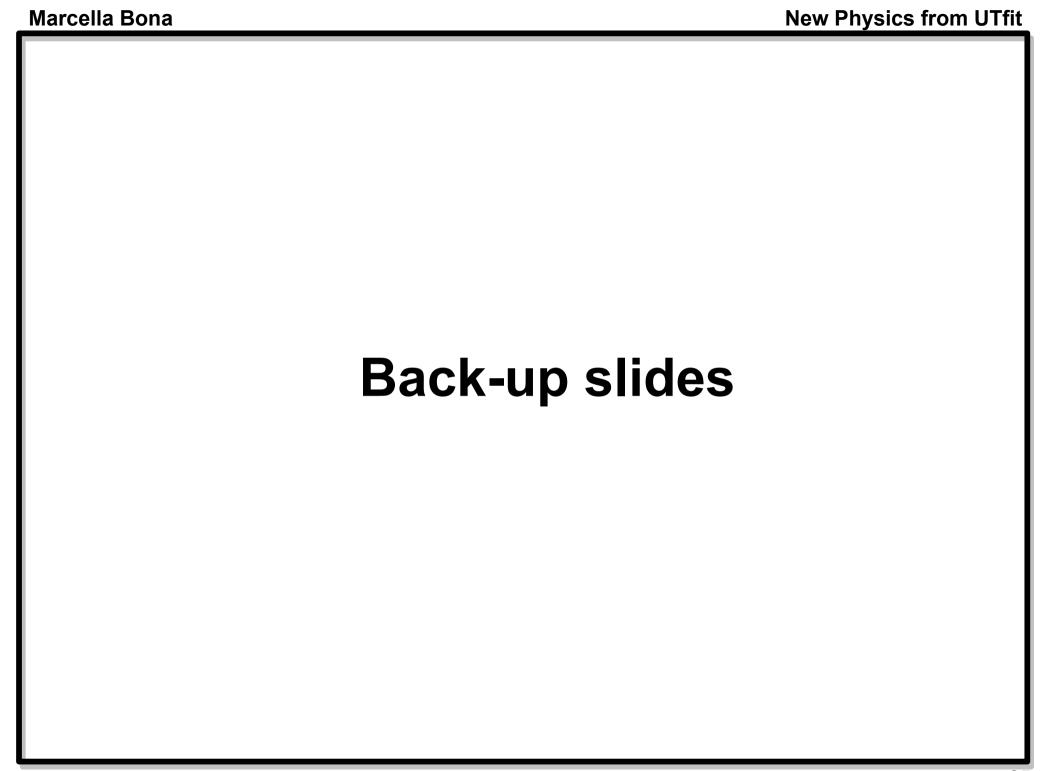
Data suggest some hierarchy in NP mixing which is stronger than the SM one

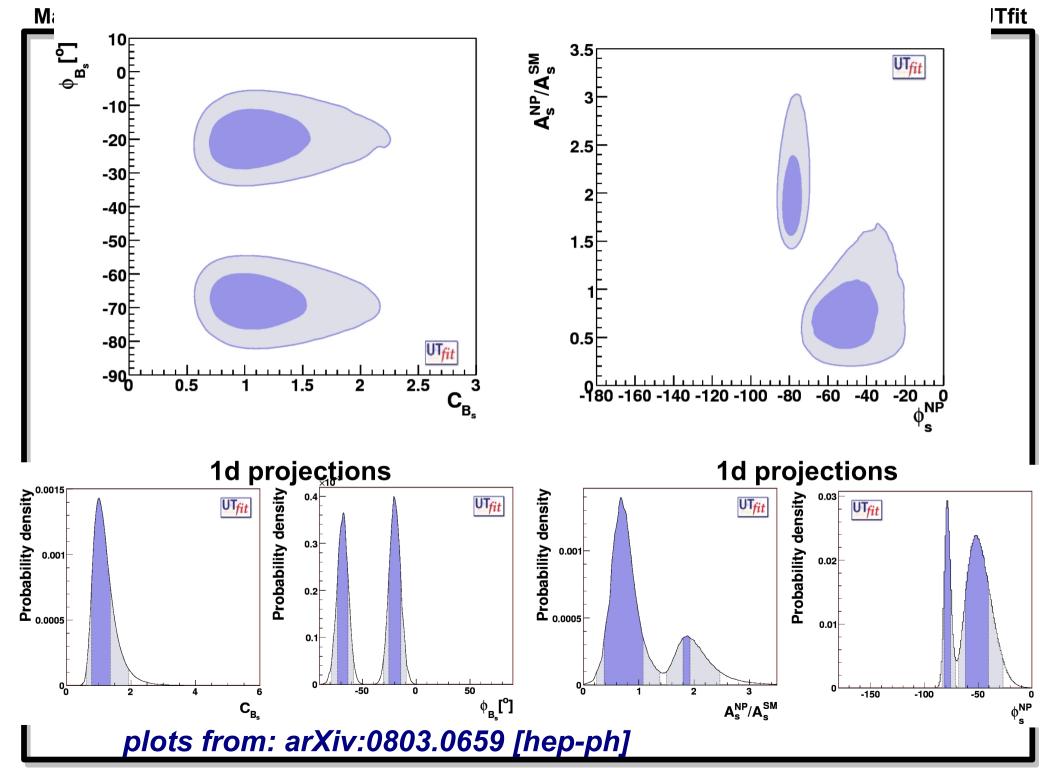
some conclusions

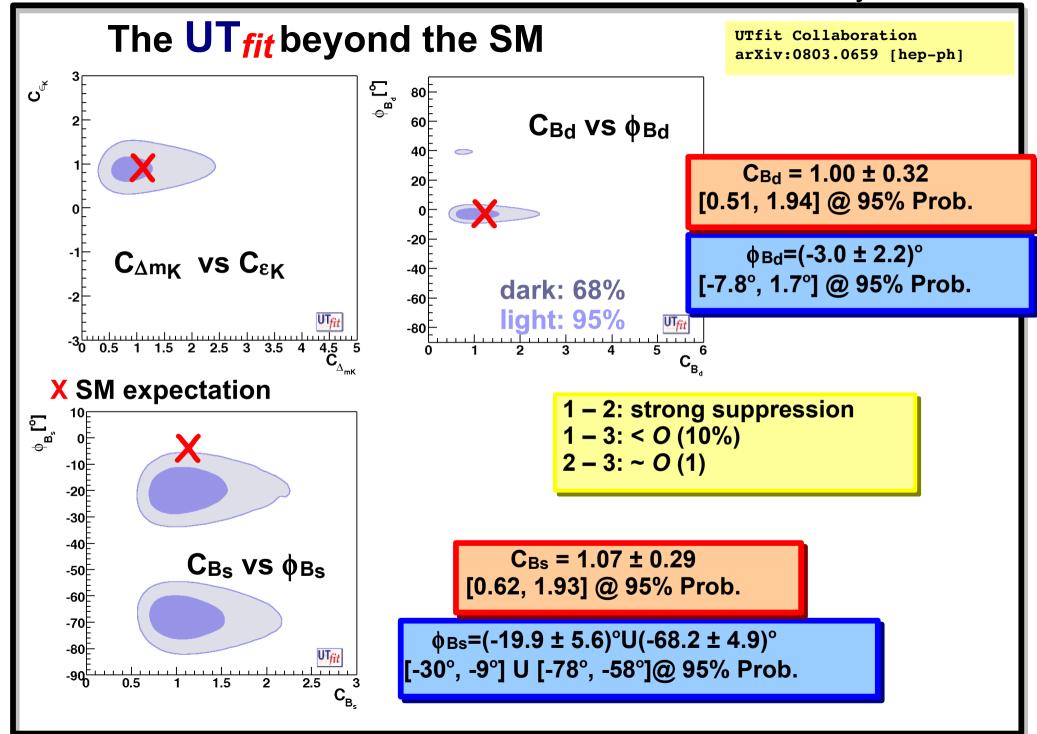
- test of the SM consistency and the CKM mechanism: comparison between inputs and indirect determinations
 - Tevatron data show a hint of discrepancy wrt SM: we are looking forward to be able to use the updated results (latest CDF likelihood still not available)
- LHCb and superB will reach better precision and provide new measurements
 - o an interesting exercise should be to repeat the scale study with the superB expected precisions...



- NP scale bounds extracted from model-independent UT fit
 - ⊚ some models have problems to accommodate the current effects: MFV for B_s, NMFV for B_d vs B_s
- the challenge is for theory
 - \odot flavour hierarchy needs to be stronger than the CKM λ expansion







The future of CKM fits

LHCb reach from: O. Schneider, 1st LHCb Collaboration Upgrade Workshop

LHCb

10/fb (5 years)

0.07%(+0.5%)

 $\phi_s (J/\psi \phi)$

 Δm_s

As SL

0.01+syst

 $sin2\beta (J/\psi K_s)$ 0.010

γ (all methods) 2.4°

 α (all methods) 4.5°

|V_{cb}| (all methods) no

|Vub| (all methods)

SuperB

SuperB reach from: SuperB Conceptua Design Report, arXiv:0709.0451

1/ab (1 month

at Y(55))

0.006

0.14

75/ab (5 years)

0.005

1-2°

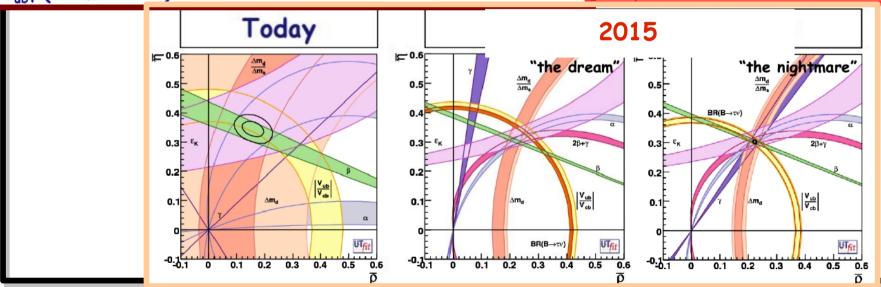
1-2°

< 1%

1-2%

92007 V. Lubicz			
Hadronic	Current	60 TFlop	1-10 PFlop
matrix	lattice	Year	Year
element	error	[2011 LHCb]	[2015 SuperB
$f_{+}^{K\pi}(0)$	0.9%	0.4%	< 0.1%
-+ (*)	(22% on 1-f ₊)	(10% on 1-f ₊)	(2.4% on 1-f ₊)
$\hat{\mathbf{B}}_{\mathtt{K}}$	11%	3%	1%
$f_{\mathtt{B}}$	14%	2.5 - 4.0%	1-1.5%
$f_{\mathtt{B}_\mathtt{S}}B_{\mathtt{B}_\mathtt{S}}^{1/2}$	13%	3 - 4%	1-1.5%
ξ	5%	1.5 - 2 %	0.5 - 0.8 %
2	(26% on ξ-1)	(9-12% on ξ-1)	(3-4% on \(\xi-1\)
$\mathcal{F}_{\mathtt{B} o \mathtt{D}/\mathtt{D}^*\mathtt{I} \mathtt{v}}$	4% (40% on 1-F)	1.2% (13% on 1-F)	0.5% (5% on 1-F)
$f_+^{ B\pi}, \ldots$	11%	4 - 5%	2-3%
T ₁ B → K */ρ	13%		3 – 4%
5 Sharpe @ Lattice QCD: Present and Future, Orsay, 2004			

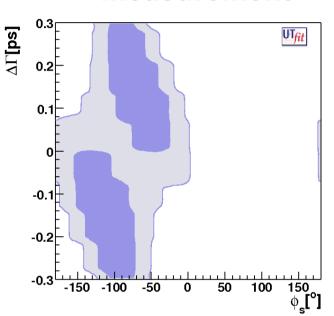
and report of the U.S. Lattice QCD Executive Committee



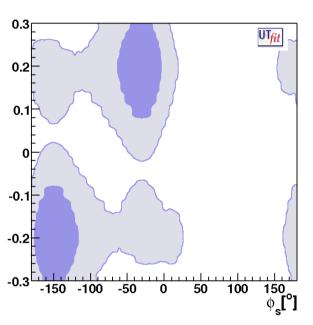
no

More than two measurements (I)

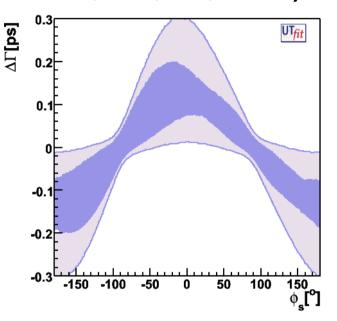
CDF tagged measurement



D0 tagged measurement



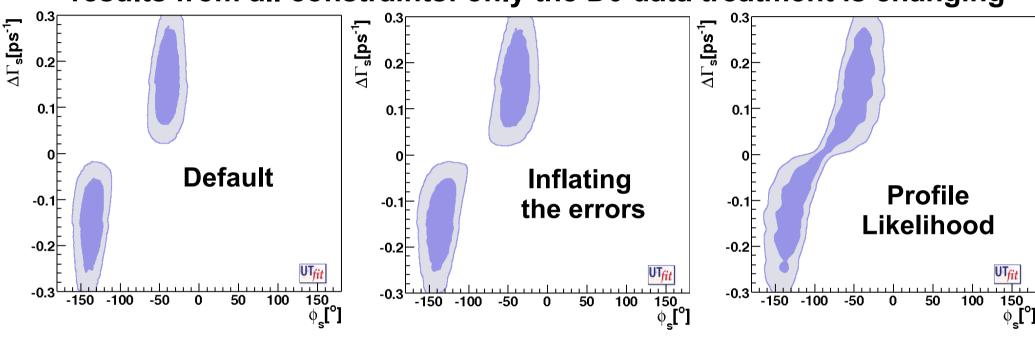
Our analysis (using A_{SL} , A_{CH} , τ_{Bs} , $\Delta\Gamma/\Gamma$)



- $\ \ \, \ \ \, \ \ \, \ \, \ \,$ CDF and D0 measurements consider $\Delta\Gamma$ and β_s as uncorrelated parameters
- In our analysis, we enforce the dependence of △□ from SM and NP parameters
- There is more physics information in our fit than in a simple combination of the two experimental results

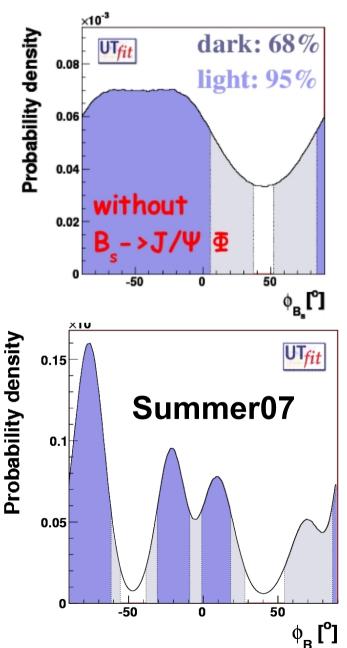
Dependence on the D0 data model

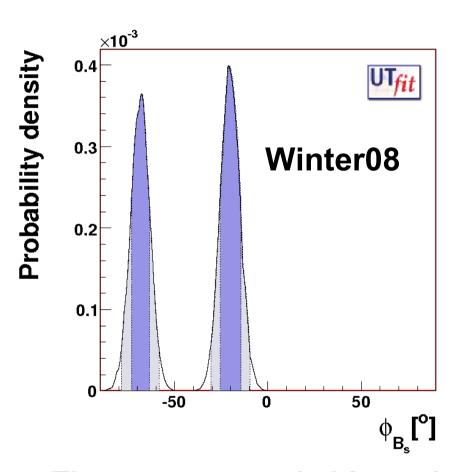
results from all constraints: only the D0 data treatment is changing



- The details on how we model D0 are crucial on the side opposite to the SM prediction
- The distance from the SM value depends on the approach, but not by O(1) effects
- A reduction of the significance is expected when going from the default to the conservative approaches

Did the result move by a lot?

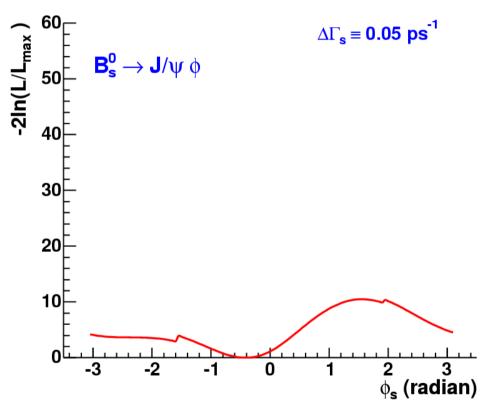


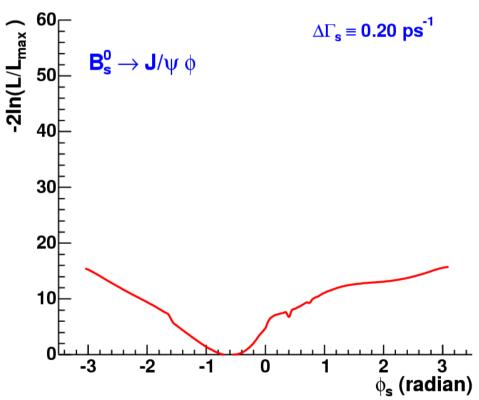


The two most probable peaks last summer are those that survived.

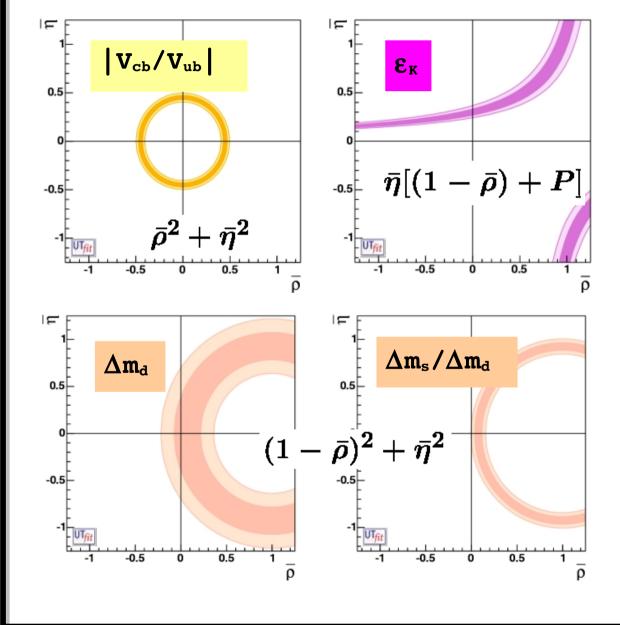
A new 2D likelihood scan from D0

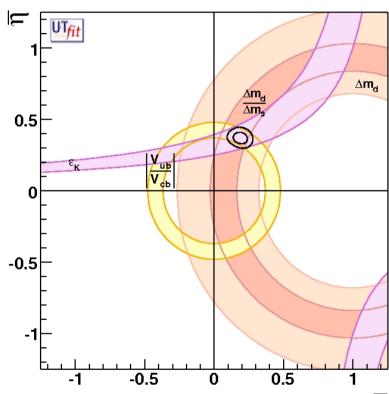
Appeared two weeks ago on the D0 web-site it hasn't the SU(3) assumption but the fit looks preliminary:

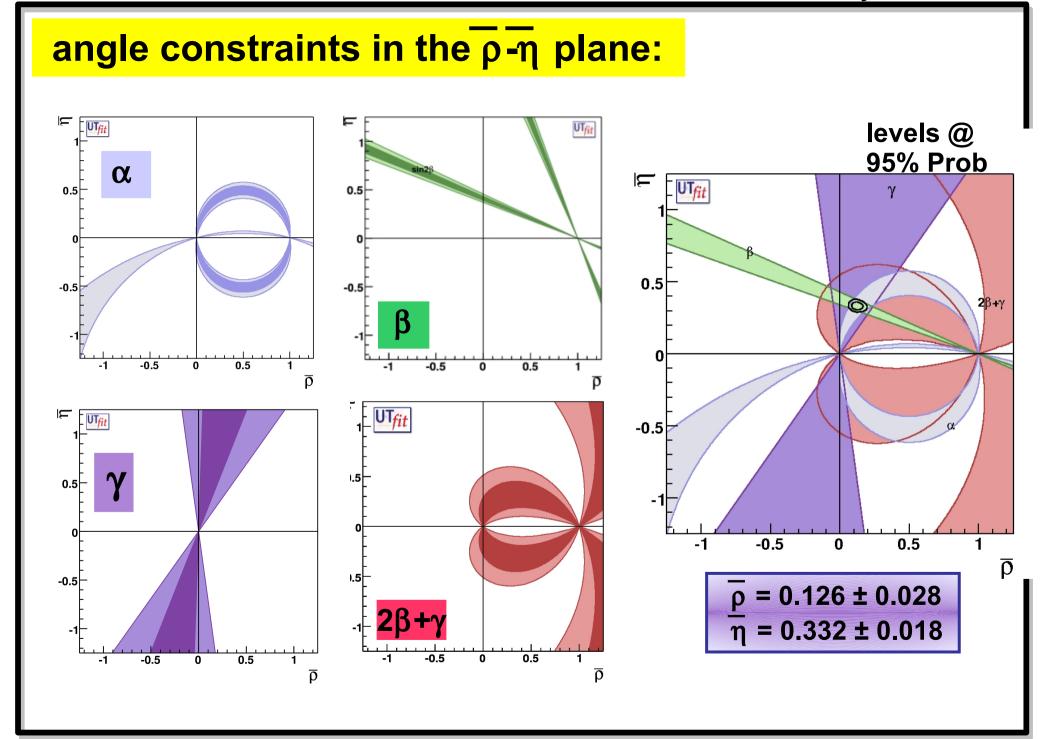




the LEP-style analysis in the ρ-η plane:







Update of the LQCD parameters

Lubicz, Tarantino for UTfit

$$\begin{split} \widehat{B}_K &= 0.75 \pm 0.07 \ , \\ f_{B_s} &= 245 \pm 25 \ \text{MeV} \quad , \quad f_B = 200 \pm 20 \ \text{MeV} \quad , \quad f_{B_s}/f_B = 1.21 \pm 0.04 \ , \\ f_{B_s} \sqrt{\widehat{B}_{B_s}} &= 270 \pm 30 \ \text{MeV} \quad , \quad f_B \sqrt{\widehat{B}_{B_d}} = 225 \pm 25 \ \text{MeV} \quad , \quad \xi = 1.21 \pm 0.04 \ , \\ \widehat{B}_{B_d} &= \widehat{B}_{B_s} = 1.22 \pm 0.12 \quad , \quad \widehat{B}_{B_s}/\widehat{B}_{B_d} = 1.00 \pm 0.03 \ , \\ |V_{cb}| \ (\text{excl.}) &= (39.2 \pm 1.1) \cdot 10^{-3} \quad , \quad |V_{ub}| \ (\text{excl.}) = (35.0 \pm 4.0) \cdot 10^{-4} \end{split}$$

These averages can be compared with the previous ones used by UTfit

$$\begin{split} \widehat{B}_K &= 0.79 \pm 0.04 \pm 0.08 \;, \\ f_{B_s} &= 230 \pm 30 \; \mathrm{MeV} \quad , \quad f_B = 189 \pm 27 \; \mathrm{MeV} \quad , \quad f_{B_s}/f_B = 1.22^{+0.05}_{-0.06} \;, \\ \hline f_{B_s} \sqrt{\widehat{B}_{B_s}} &= 262 \pm 35 \; \mathrm{MeV} \quad , \quad f_B \sqrt{\widehat{B}_{B_d}} = 214 \pm 38 \; \mathrm{MeV} \quad , \quad \xi = 1.23 \pm 0.06 \;, \\ \widehat{B}_{B_d} &= 1.28 \pm 0.05 \pm 0.09 \quad , \quad \widehat{B}_{B_s}/\widehat{B}_{B_d} = 1.02 \pm 0.02^{+0.06}_{-0.02} \;, \\ |V_{cb}| \; (\mathrm{excl.}) = (39.1 \pm 0.6 \pm 1.7) \cdot 10^{-3} \quad , \quad |V_{ub}| \; (\mathrm{excl.}) = (34.0 \pm 4.0) \cdot 10^{-4} \;. \end{split}$$

If this evidence is confirmed...

M.Ciuchini CERN 08

- * MFV models are ruled out, including the simplest realizations of the MSSM
- * the following pattern of flavour violation in NP emerges:

1 <-> 2: strong suppression

 $1 \leftrightarrow 3$: $\leq O(10\%)$

2 <-> 3: O(1)

this pattern is not unexpected in flavour models and SUSY-GUTs

* In progress: (i) update of the ΔF =2 operator analysis, (ii) correlations with ΔF =1 in MSSM

Marco Ciuchini

IFAE - Bologna, 28 March 2008

Page 23

$$A^{\text{NP}}_{\text{d}}/A^{\text{SM}}_{\text{d}}\sim 0.1$$
 and $A^{\text{NP}}_{\text{s}}/A^{\text{SM}}_{\text{s}}\sim 0.7$ correspond to $A^{\text{NP}}_{\text{d}}/A^{\text{NP}}_{\text{s}}\sim \lambda^2$ i.e. to an additional λ suppression.

L.Silvestrini Capri 08

• Lower bounds on NP scale from K and B_d physics: (in TeV at 95% probability)

Scenario	strong/tree	α_s loop	α_W loop
MFV	5.5	0.5	0.2
NMFV	62	6.2	2
General	24000	2400	800

• Upper bounds on NP scale from ϕ_s :

Scenario	strong/tree	α_s loop	α_W loop
NMFV	35	4	2
General	800	80	30

Need a flavour structure, but not NMFV!

Large NP contributions to b

s
transitions are natural in nonabelian flavour
models, given the large breaking of flavour
SU(3) due to the top quark mass

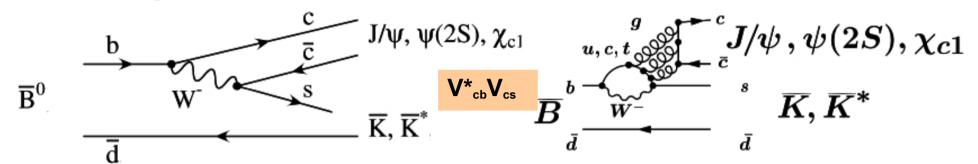
Pomarol, Tommasini; Barbieri, Dvali, Hall; Barbieri, Hall; Barbieri, Hall, Romanino; Berezhiani, Rossi; Masiero et al;

- GUTs can naturally connect the large
 mixing in v oscillations with a large b ↔ s
 mixing

 Back et al.; Moroi; Akama et al.; Chang, Masiero, Murayama; Hisano, Shimizu; Goto et al.; ...
- In a given model expect correlation between b \leftrightarrow s (B_s mixing) and b \rightarrow s (penguin decays) transitions
- This correlation is welcome given the large room for NP in b \rightarrow s hadronic penguins (S_{peng} , $A_{K\pi}$, ...)

 Beneke; Buchalla et al.; Buras et al.; London et al.; Hou et al.; Lunghi & Soni Feldmann et al.; ...
- The correlation is however affected by large hadronic uncertainties

$Sin2\beta$ in golden b \rightarrow ccs modes



branching fraction: O (10⁻³) the colour-suppressed tree dominates and the t penguin has the same weak phase of the tree

$$egin{align} igsplus A_{CP}(t) &= rac{\Gamma(ar{B}^0(t)
ightarrow f_{CP}) - \Gamma(B^0(t)
ightarrow f_{CP})}{\Gamma(ar{B}^0(t)
ightarrow f_{CP}) + \Gamma(B^0(t)
ightarrow f_{CP})} & egin{align} igsplus \sim \sin 2eta \ igsplus \sim 0 \ &= S \sin \Delta mt - C \cos \Delta mt \ \end{bmatrix}$$

theoretical uncertainty:

M.Ciuchini, M.Pierini, L.Silvestrini Phys. Rev. Lett. 95, 221804 (2005)

⇒ model-independent data-driven estimation from J/ψπ ⁰ data:

$$\Delta S_{J/\psi K0} = S_{J/\psi K0} - \sin 2\beta = 0.000 \pm 0.012$$

model-dependent estimates of the u- and c- penguin biases

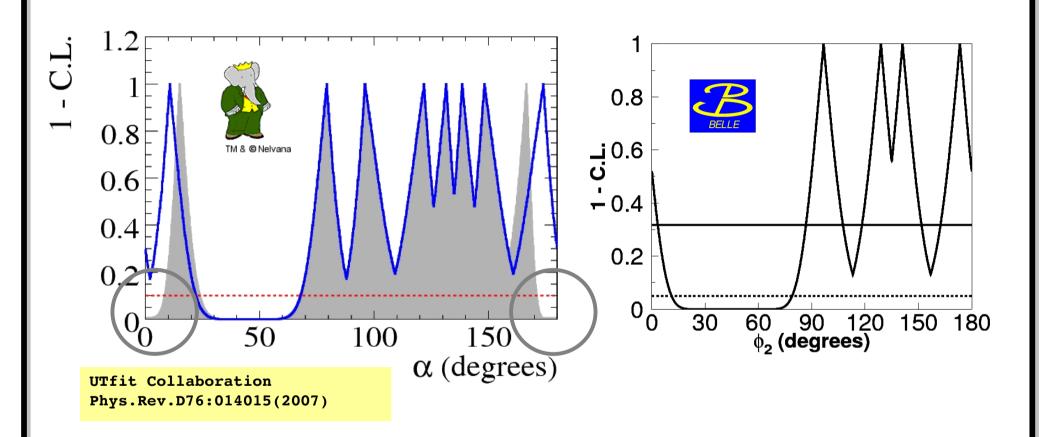
$$\Delta S_{J/\psi K0} = S_{J/\psi K0} - \sin 2\beta \sim O (10^{-3})$$

 $\Delta S_{J/\psi K0} = S_{J/\psi K0} - \sin 2\beta \sim O (10^{-4})$

H.Li, S.Mishima JHEP 0703:009 (2007)

H.Boos et al.
Phys. Rev. D73, 036006 (2006)

At last: α from $\pi\pi$ decays



BaBar: $25^{\circ} < \alpha < 66^{\circ}$ excluded @ 90% CL

Belle: $11^{\circ} < \alpha < 79^{\circ}$ excluded @ 95% CL

But there is more: α from $\rho\rho$ decays

Vector-Vector modes: angular analysis required to determine the CP

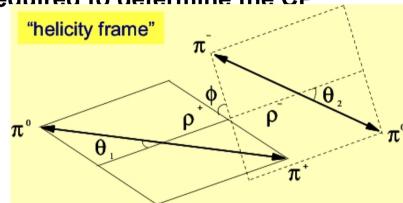
content. L=0,1,2 partial waves:

→ longitudinal: CP-even state

transverse: mixed CP states

 \bullet +-: two π^0 in the final state

wide ρ resonance



but

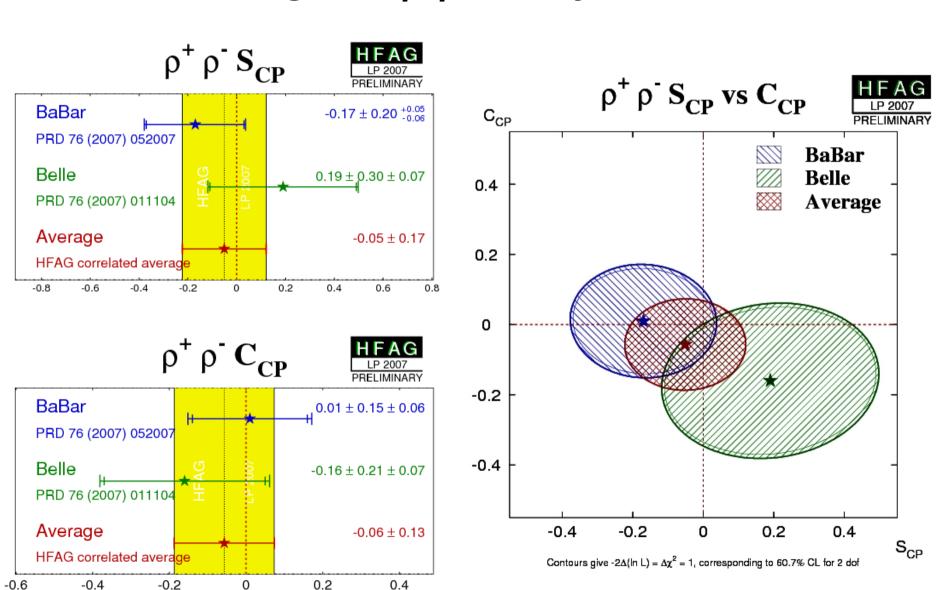
- BR 5 times larger with respect to ππ
- penguin pollution might be smaller than in ππ
- ρ are almost 100% polarized:
 - almost a pure CP-even state
- world average longitudinal fraction:

$$rightarrow f_{long} (\rho^+ \rho^-) = 0.978 \pm 0.025$$

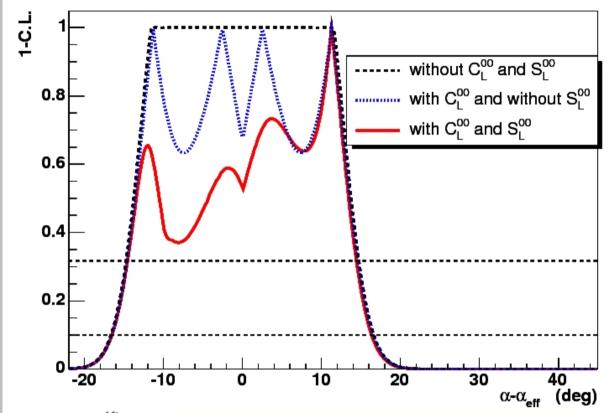
$$- f_{long} (\rho^{\pm} \rho^{\circ}) = 0.912 \pm 0.045$$

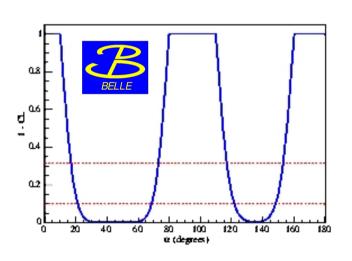
 $- f_{long} (\rho^0 \rho^0)$ still to be measured

World averages in $\rho^+\rho^-$ decays











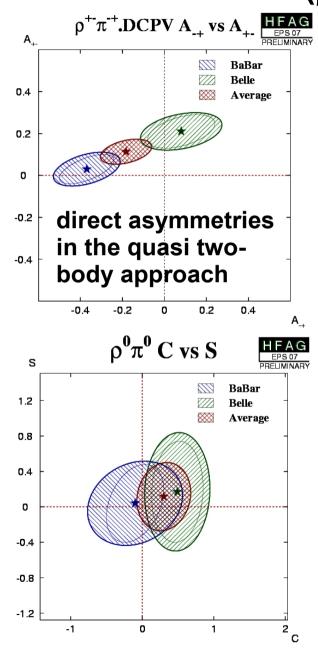
using BaBar ρ⁰ρ⁰:

 $|\alpha - \alpha_{eff}| < 16.5^{\circ}$ @ 90% CL

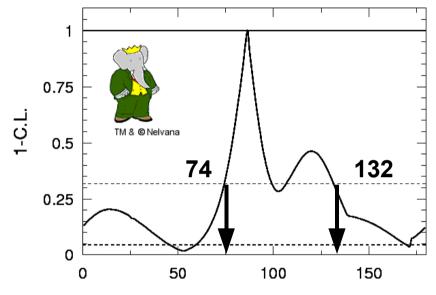
in $\pi\pi$:

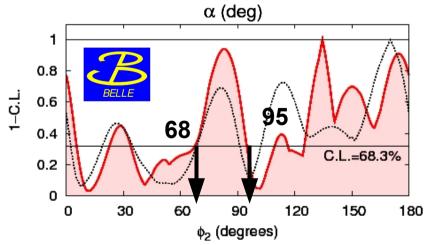
 $|\alpha - \alpha_{\text{eff}}| < 39^{\circ}$ @ 90% CL

Results from $(\rho \pi)^0$



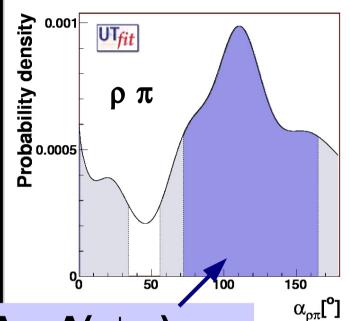
• this analysis allows for a direct determination of α without ambiguities

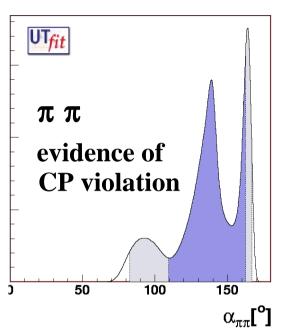


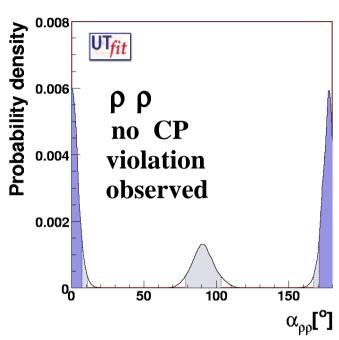


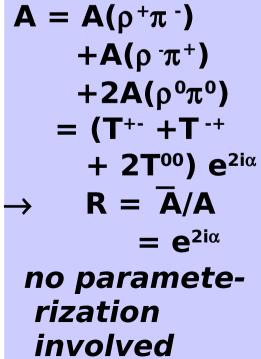
no values excluded, no values selected yet

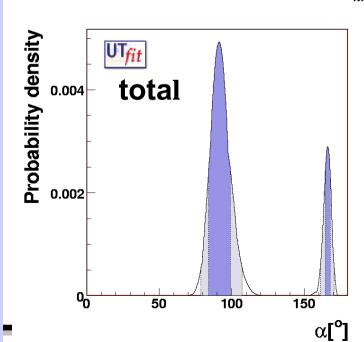
α extraction from the three analyses

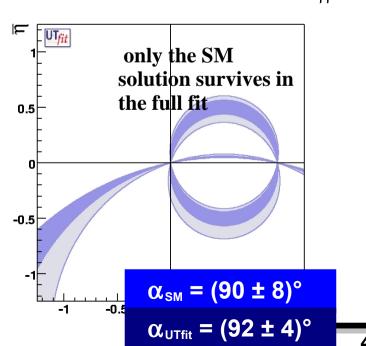








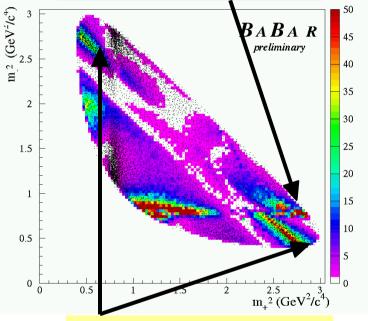




γ measurement: Dalitz method

- neutral D mesons reconstructed in three-body CP-eigenstate final states
 (typically D⁰ → K_sπ⁻π⁺)
- → the complete structure (amplitude and strong phases) of the D⁰ decay in the phase space is obtained on independent data sets and used as input to the analysis
- use of the cartesian coordinate:
 - $\mathbf{a} \mathbf{x}_{\pm} = \mathbf{r}_{\mathrm{B}} \cos (\delta \pm \gamma)$
 - $y_{\pm} = r_{\rm B} \sin (\delta \pm \gamma)$
- \clubsuit γ , r_B and δ_B are obtained from a simultaneous fit of the $K_s\pi^+\pi^-$ Dalitz plot density for B^+ and B^-
- need a model for the Dalitz amplitudes
- 🤟 2-fold ambiguity on γ

Interference of $B^- \to D^0 K^-$, $D^0 \to K^{*+}\pi^-$ (suppressed) with $B^- \to \overline{D}{}^0 K^-$, $\overline{D}{}^0 \to K^{*+}\pi^-$ ~ ADS like



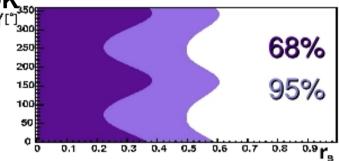
Interference of $B^- \to D^0 K^-, \, D^0 \to K^0{}_s \rho^0$ with $B^- \to \overline{D}{}^0 K^-, \, \overline{D}{}^0 \to K^0{}_s \rho^0$ ~ GLW like

More ways to γ

- with neutral B's in the final states D^0K^{*0} with $D^0 \to K_s\pi^-\pi^+$ and $K^* \to K^-\pi^+$,
 - the charge of the K from the K* tags the flavour of the B⁰ so no time-dependent analysis
 - \Rightarrow first analysis to extract γ from neutral B \rightarrow DK,
 - → BaBar performed it with 371M BB

```
\gamma = (162 \pm 56)^{\circ} \text{ (mod.180}^{\circ})

r_s (D^0 K^{*0}) < 0.55 @ 95\% \text{ Prob.}
```



- again with neutral B's, time-dependent Dalitz plot analysis of the three-body final state $B^0 \to D^-K^0\pi^+$
 - \Rightarrow interference between b \rightarrow u and b \rightarrow c transitions through the mixing: sensitivity to 2β+γ
 - BaBar performed it with 347M BB

$$2\beta + \gamma = (83 \pm 53 \pm 20)^{\circ} \text{ (mod.} 180^{\circ})$$

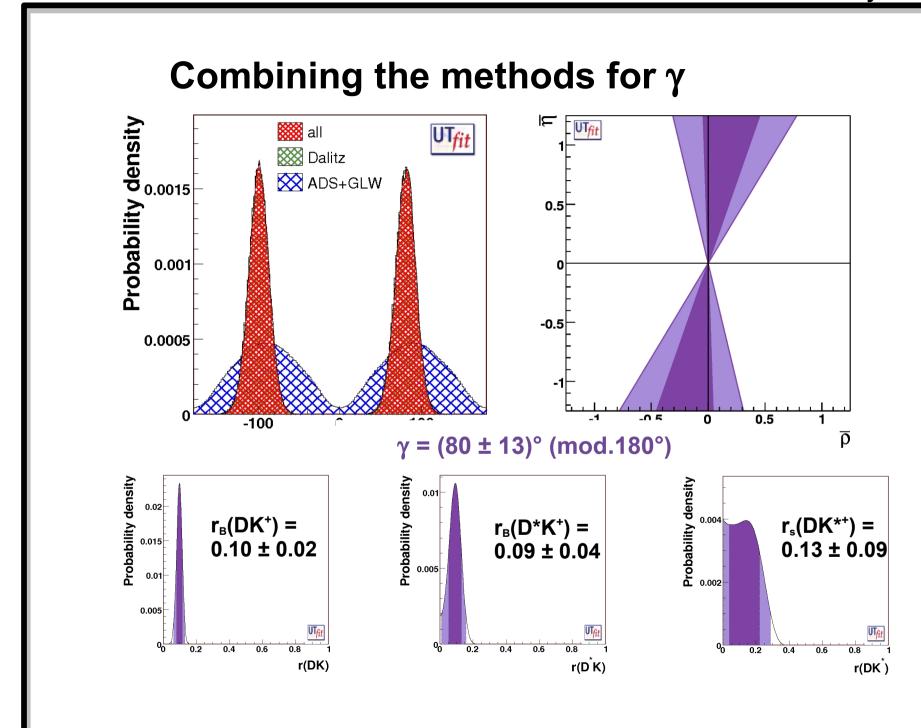
Dalitz method: carthesian coordinates

from previous studies, we know that (γ, δ_B and r_B) are not a good choice from the fit point of view
 → no sensitivity to γ if r_B < 0.10
 (underestimation of the errors)
 → fit bias on r_B for r_B ~ 0.10
 (physical bound + low statistics)
 → fit for cartesian coordinates instead: x_□, y_□
 → x₊ = Re[r_R e^{i(δ ± γ)}], y₊ = Im[r_R e^{i(δ ± γ)}]
 → gaussian errors: no unphysical zones
 → (x+, y+), (x-, y-) uncorrelated
 → unbiased results for all possible r_B

 $\mathbf{x}_{\pm} = [\mathbf{R}_{CP+}(1 \mp \mathbf{A}_{CP+}) - \mathbf{R}_{CP-}(1 \mp \mathbf{A}_{CP-})]/4$

47

also in the GI W:



the method and the inputs:

$$f(ar{
ho},ar{\eta},X|c_1,...,c_m) \sim \prod f_j(\mathcal{C}|ar{
ho},ar{\eta},X) *$$

Bayes Theorem

$$\prod_{j=1,m} f_j(\mathcal{C}|ar{
ho},ar{\eta},X) *$$

$$\prod_{i=1,N} f_i(x_i) f_0(ar
ho,ar\eta)$$

$$X\equiv x_1,...,x_n=m_t,B_K,F_B,...$$

$$\mathcal{C} \equiv c_1,...,c_m = \epsilon, \Delta m_d/\Delta m_s, A_{CP}(J/\psi K_S),...$$

$$egin{array}{c|c} (b
ightarrow u)/(b
ightarrow c) & ar
ho^2+ar\eta^2 \ \hline \epsilon_K & ar\eta[(1-ar
ho)+\ \Delta m_d & (1-ar
ho)^2+\ \hline \Delta m_d/\Delta m_s & (1-ar
ho)^2+ \end{array}$$

$$A_{CP}(J/\psi K_S)$$

 $\sin 2\beta$

 $ar{\Lambda}, \lambda_1, F(1), \ldots$ B_{K} $f_B^2 B_B$

Standard Model + OPE/HQET/ **Lattice QCD** to go from quarks to hadrons

M. Bona et al. (UTfit Collaboration) JHEP 0507:028,2005 hep-ph/0501199

M. Bona et al. (UTfit Collaboration) JHEP 0603:080,2006 hep-ph/0509219

Gronau-London method:



CP:
$$\alpha \rightarrow -\alpha$$

6 parameters:

|T+-|, |T^\infty|, |P|,

 δ^{00} , δ^{P} , α

$$\frac{6 \text{ observables:}}{B_{\pi\pi}^{+-,00} = \frac{1}{2} \left(\left| A^{+-,00} \right|^2 + \left| \overline{A}^{+-,00} \right|^2 \right) , \quad B_{\pi\pi}^{+0} = \frac{\tau_{B^+}}{\tau_{B^0}} \frac{1}{2} \left(\left| A^{+0} \right|^2 + \left| \overline{A}^{+0} \right|^2 \right)}{\left| A^{ij} \right|^2 + \left| \overline{A}^{ij} \right|^2} , \quad S_{\pi\pi}^{ij} = \frac{\operatorname{Im}}{\left| A^{ij} \right|^2 + \left| \overline{A}^{ij} \right|^2}}{\left| A^{ij} \right|^2 + \left| \overline{A}^{ij} \right|^2}$$

solutions

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Marcella Bona

 α extraction with hadronic amplitudes

what kind of "other information"?

The GL method already requires some a priori "minimal assumptions" on strong interactions, namely:

- Flavour blind and CP conserving strong interactions
- Negligible isospin symmetry breaking effects, including e.m. corrections

This is because we believe that:

- QCD is the theory of strong interactions
- QCD is a renormalizable theory with a dimensionless coupling constant and a natural scale $\Lambda_{\rm ocn} \sim 1$ GeV

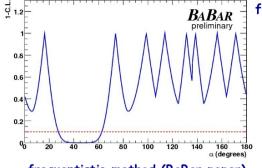
 $\langle M_1 M_2 | O | M \rangle \sim (\Lambda_{ova})^3$ case of a single scale

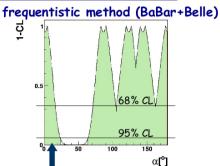
Therefore we do not expect:

 $\langle \pi \pi | O | B \rangle \sim (1 \text{ TeV})^3 \text{ or } (M_{\text{planck}})^3$

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interpretation of the results





frequentistic method (BaBar paper)

"some of the solutions, and the region around a=0 can be disfavoured by other physics information"

The region around $\alpha = 0$ is not excluded, despite the experimental observation of CP violation.

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a extraction with hadronic amplitudes

scales and dimensions

In our case: two scales enter in the process, $\mathbf{M}_{\mathtt{B}}$ and $\Lambda_{\mathtt{OCD}}$

so we expect: $\langle \pi\pi|\hat{O}|B\rangle \sim f_\pi M_B^2 f^+(0) \sim f_\pi M_B^2 \left(\frac{\Lambda_{QCD}}{M_B}\right)^{3/2} \sim M_B^{1/2} \Lambda_{QCD}^{5/2}$

Note: the scaling law has a more general validity than factorization

This gives:

[We use "natural units": the BR \times 106 are simply given by the squared amplitude]

But we can think of other considerations:

several theoretical predictions exist(ed)

[17] Ciuchini et al.'98

 $|T^{+-}|^2 = \frac{G_F^2 \tau_B |V_{ub}V_{ud}^*|^2}{32\pi M_B} |C_1(M_B)\langle \pi^+\pi^-|O_1|B_d^0\rangle + C_2(M_B)\langle \pi^+\pi^-|O_2|B_d^0\rangle|^2 \times 10^6$ $= \frac{G_F^2 \tau_B |V_{ub}V_{ud}^*|^2}{32 \pi M_D} \left| \frac{C_1(M_B)}{3} + C_2(M_B) \right|^2 \times |M_B^2 f_\pi f^+(0)|^2 \times 10^6$

using strict factorization

[18] BBNS'99 [19] Keum et al.'02

 $BR(\pi^+\pi^0)$

ref. [17] ref. [18] ref. [19] $3.7^{+1.3}_{-1.1}$ 3.6 - 5.3 $4.3(1 \pm 0.3)$

This gives:

further considerations

scaling between B and D decays

In the heavy quark limit, the dependence on M_{μ} cancels in the decay rate.

$$R = \frac{|T^{+-}(B_d^0 \to \pi^+\pi^-)|^2}{|T^{+-}(D^0 \to \pi^+\pi^-)|^2} \sim \frac{|V_{ub}V_{ud}^*|^2}{|V_{cd}V_{ud}^*|^2} \qquad |T^{+-}|^2 = BR(D^0 \to \pi^+\pi^-) \times 10^6 \frac{\tau_{B_d^0}}{\tau_{D^0}} R$$

$$|T^{+-}|^2 = BR(D^0 \to \pi^+\pi^-) \times 10^6 \frac{\tau_{B_d^0}}{\tau_{D^0}} R$$

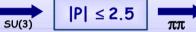
→ This gives: |T+-| = 1.3

• extract P from the $B_s \to K^{\dagger}K^{-}$ decay

Up to DCS terms

$$|P|^2 = BR(B_s \to K^+K^-) \times 10^6 \frac{\tau_{B_d^0}}{\tau_{B_s^0}} \frac{|V_{td}V_{tb}^*|^2}{|V_{ts}V_{tb}^*|^2} : |P_s|^2 \frac{SU(3)}{|P|^2}$$

 $|P_{s}| = 1.1$





assuming that SU(3) breaking effects are not larger than 100%

Marcella Bona

a extraction with hadronic amplitudes

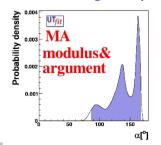
using the available information (priors)

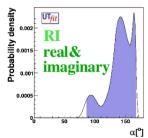
In previous UTfit analyses: $|Ti| \le 10$, $|P| \le 10$

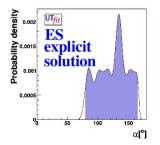
Now: $|T^{ij}| \le 10$, $|P| \le 2.5$, $\le SU(3)$ breaking $\le 100\%$ arbitrary phases

ITil will be automatically limited

1) The information on the matrix elements has the effect of eliminating some of the eight solutions, including the pathological solution at $\alpha \sim 0$



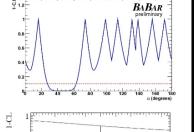


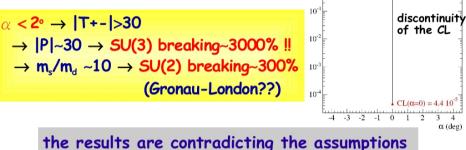


some consequences

• solution for $\alpha \to 0$

in order to reproduce the experimental values of BR($\pi^+\pi^-$) and BR($\pi^0\pi^0$) we may have |T|>>1 and |P|>>1. but |T| ~ |P|





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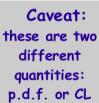
Marcella Bona

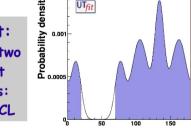
a extraction with hadronic amplitudes

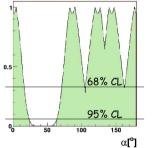
Bayesian vs Frequentistic analysis

Compare the 2 methods using the same assumptions

- In the Bayesian approach: extract BR's and CP parameters with gaussian p.d.f. according with their experimental values and errors
- In the frequentistic analysis: no additional information on the hadronic amplitudes is introduced (besides the GL method)







The two approaches give equivalent results at a meaningful CL/Prob.

 $\alpha_{\pi\pi}[^{\circ}]$

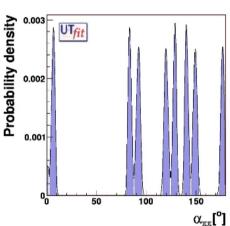
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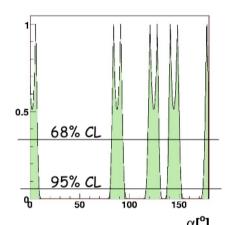
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 α extraction with hadronic amplitudes

further comparison:

Reducing the experimental errors by a factor of 10 at fixed central values





Not yet
really
separated
at a
meaningful
CL

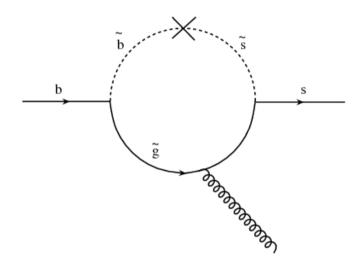
The eight solutions "start" to be separated both in the Bayesian and frequentistic case

Provided the same assumptions are done, the two approaches lead to similar results

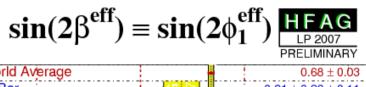
2

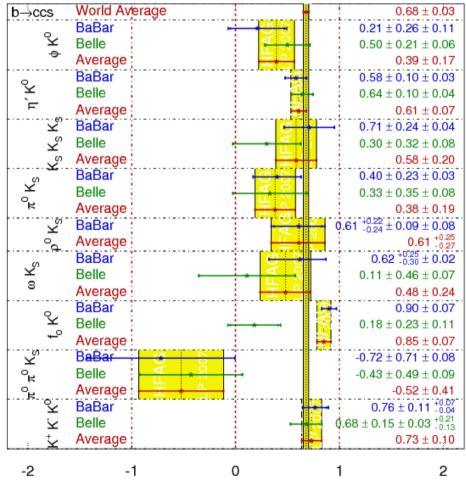
52

b → s penguins



- Extra sources of FCNC: investigation looking at b ↔ s penguin decays
- Some "hints" seen on sin2β in penguin decays
- Difficult interpretation
 due to theoretical issues
 (but SM hadron corrections
 are expected to induce positive shifts)



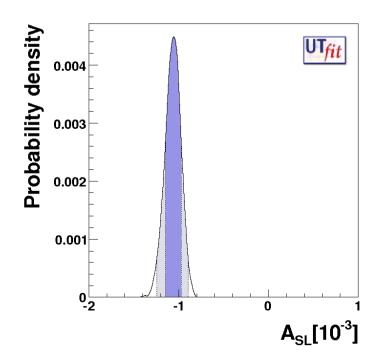


Semileptonic Asymmetry A_{SL}

$$A_{\rm SL} \equiv \frac{\Gamma(\bar{B}^0 \to \ell^+ X) - \Gamma(B^0 \to \ell^- X)}{\Gamma(\bar{B}^0 \to \ell^+ X) + \Gamma(B^0 \to \ell^- X)}$$
$$= -\text{Re} \left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\sin 2\phi_{B_d}}{C_{B_d}} + \text{Im} \left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\cos 2\phi_{B_d}}{C_{B_d}}$$

SM prediction $(-1.06\pm0.09)10^{-3}$ Direct measurement $(-0.3\pm5.0)10^{-3}$

Laplace, Ligeti, Nir and Perez Phys.Rev.D 65:094040,2002



Similar constraint available both Bs decays

$\Delta\Gamma$ for B_d and B_s

$$\frac{\Delta\Gamma_{q}}{\Delta m_{q}} = -2\frac{\kappa}{C_{B_{q}}} \left\{ \cos\left(2\phi_{B_{q}}\right) \left(n_{1} + \frac{n_{6}B_{2} + n_{11}}{B_{1}}\right) - \frac{\cos\left(\phi_{q}^{\mathrm{SM}} + 2\phi_{B_{q}}\right)}{R_{t}^{q}} \left(n_{2} + \frac{n_{7}B_{2} + n_{12}}{B_{1}}\right) + \frac{\cos\left(2(\phi_{q}^{\mathrm{SM}} + \phi_{B_{q}})\right)}{R_{t}^{q^{2}}} \left(n_{3} + \frac{n_{8}B_{2} + n_{13}}{B_{1}}\right) + \cos\left(\phi_{q}^{\mathrm{Pen}} + 2\phi_{B_{q}}\right) C_{q}^{\mathrm{Pen}} \left(n_{4} + n_{9}\frac{B_{2}}{B_{1}}\right) - \cos\left(\phi_{q}^{\mathrm{SM}} + \phi_{q}^{\mathrm{Pen}} + 2\phi_{B_{q}}\right) \frac{C_{q}^{\mathrm{Pen}}}{R_{t}^{q}} \left(n_{5} + n_{10}\frac{B_{2}}{B_{1}}\right) \right\}$$

 The constraint on B_d is not effective (experimental error~ 10 times the precision from the rest of the fit)

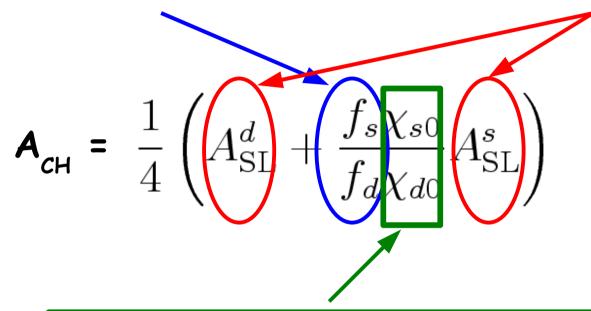
	SM	SM+NP	exp
$10^3 \Delta \Gamma_d / \Gamma_d$	2.8 ± 2.7	2.0 ± 1.8	9 ± 37
$\Delta\Gamma_s/\Gamma_s$	0.10 ± 0.06	0.00 ± 0.08	0.25 ± 0.09

- The experimental measurement of $\Delta\Gamma_s$ actually measures $\Delta\Gamma_s \cos(\beta_s + \phi_{Bs})$ (Dunietz et al., hep-ph/0012219)
- NP can only decrease the experimental result wrt the SM value
- Experimental WA > SM expectation (NP suppressed)

NLO calculation of the matrix element of B meson mixing Ciuchini et al. JHEP 0308:031,2003.

Same Sign dilepton charge asymmetry

Ratio of B_d and B_s production at Tevatron



Semileptonic asymmetries of Bd and Bs mesons

$$\chi_{q}^{(-)} = \frac{\frac{\Delta\Gamma_{q}}{\Gamma_{q}}^{2} + 4\frac{\Delta m_{q}}{\Gamma_{q}}^{2}}{\frac{\Delta\Gamma_{q}}{\Gamma_{q}}^{2}(z_{q}^{(-)} - 1) + 4(2z_{q}^{(-)} + \frac{\Delta m_{q}}{\Gamma_{q}}^{2}(1 + z_{q}^{(-)}))}$$

$$\text{With } \mathbf{z} = |q/p|^{2} \text{ and } \mathbf{z} = |p/q|^{2}$$

From NLO calculation of the B meson mixing

τ_{Bs} in Flavor Specific final states

- \odot B_s and \overline{B}_s lifetime difference induced by $\Delta\Gamma_s$
- Experimental fit done with a single exponential rather than two exponentials
- The "average" lifetime is a function of the width and width difference

$$au_{
m Bs}$$
 in Flavor Specific final states $au_{Bs}^{FS} = rac{1}{\Gamma_s} rac{1-\left(rac{\Delta\Gamma_s}{2\Gamma_s}
ight)^2}{1+\left(rac{\Delta\Gamma_s}{2\Gamma_s}
ight)^2}$

Time-dependent angular analysis

TAGGED

UNTAGGED

2-fold ambiguity 4-fold ambiguity $(\pi-\phi_s, -\Delta\Gamma_s, \pi-\delta_{1,2})$ $(\pi+\phi_s, -\Delta\Gamma_s, \pm\delta_{1,2})$

 $(-\phi_s, \Delta\Gamma_s, \pm(\pi-\delta_{1.2}))$

$$(\pi - \phi_s, -\Delta \Gamma_s, \pm (\pi - \delta_{1,2}))$$

$$\frac{d^4\Gamma}{dt d\cos\theta d\varphi d\cos\psi} \propto$$

Dunietz, Fleischer, Nierste hep-ph/0012219

$$2\cos^2\psi(1-\sin^2\theta\cos^2\varphi)|A_0(t)|^2$$

$$+\sin^2\psi(1-\sin^2\theta\sin^2\varphi)|A_{\parallel}(t)|^2$$

$$+\sin^2\psi\sin^2\theta|A_{\perp}(t)|^2$$

$$+(1/\sqrt{2})\sin 2\psi \sin^2\theta \sin 2\varphi \operatorname{Re}(A_0^*(t)A_{\parallel}(t))$$

+
$$(1/\sqrt{2})\sin 2\psi \sin 2\theta \cos \varphi \operatorname{Im}(A_0^*(t)A_{\perp}(t))$$

$$-\sin^2\psi\sin 2\theta\sin\varphi \operatorname{Im}(A_{\parallel}^*(t)A_{\perp}(t)).$$

$$|A_0(t)|^2 = |A_0(0)|^2 e^{-\Gamma t} \left[\cosh \frac{\Delta \Gamma t}{2} - \cos \phi | \sinh \frac{\Delta \Gamma t}{2} + \sin \phi \sin(\Delta m t) \right]$$

$$|\overline{A}_0(t)|^2 = |A_0(0)|^2 e^{-\Gamma t} \left[\cosh \frac{\Delta \Gamma t}{2} - |\cos \phi| \sinh \frac{|\Delta \Gamma| t}{2} - \sin \phi \sin(\Delta m t) \right]$$

$$\operatorname{Im} \left\{ A_0^*(t) A_{\perp}(t) \right\} = |A_0(0)| |A_{\perp}(0)| e^{-\Gamma t}$$

$$\times \left[\sin \delta_2 \, \cos(\Delta m \, t) \, - \, \cos \delta_2 \, \cos \phi \, \sin(\Delta m \, t) \, - \, \cos \delta_2 \, \sin \phi \, \sinh \frac{\Delta \Gamma \, t}{2} \right]$$

$$\operatorname{Im} \left\{ \overline{A}_{0}^{*}(t) \overline{A}_{\perp}(t) \right\} = |A_{0}(0)| |A_{\perp}(0)| e^{-\Gamma t}$$

$$\times \left[-\sin\delta_2 \cos(\Delta m t) + \cos\delta_2 \cos\phi \sin(\Delta m t) - \cos\delta_2 \sin\phi \sinh\frac{\Delta\Gamma t}{2} \right]$$

