Flavour physics in a Randall Sundrum model with custodial protection

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Outline

1. **Part 1: Introduction to Warped Extra Dimensions**
2. **Part 2: The Model**
3. **Part 3: $\Delta F = 2$ Processes**
4. **Part 4: Rare decays**

Based on:

Blanke, Buras, Duling, Gori, Weiler [arXiv:0809.1073]

Blanke, Buras, Duling, KG, Gori [arXiv:0812.3803]

Albrecht, Blanke, Buras, Duling, KG [arXiv:0903.2415]

The Randall-Sundrum I Model

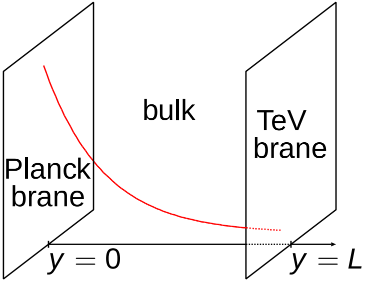
Randall, Sundrum [hep-ph/9905221]

Goal: Solution of the gauge hierarchy problem

- **warped metric** is solution of **5D Einstein's equations**:

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy$$

x^μ - coordinate of 4d space-time
y - coordinate of the **extra dimension** is restricted to $0 \leq y \leq L$
k - curvature scale with $kL \sim 36$



- fundamental mass scale M_{pl}
- Higgs confined to TeV brane

Result: two effective mass scales
Planck brane $\Rightarrow M_{pl}$
TeV brane $\Rightarrow M_{pl}e^{-kL} \sim \mathcal{O}(TeV)$

The Flavour problem

$\Longrightarrow$ gauge hierarchy problem solved

- Hierarchies in masses of quarks and leptons:

$$m_u \approx 5\,MeV, \ldots, m_t \approx 172.5\,GeV$$

$$m_e \approx 0.5\,MeV, \ldots, m_\tau \approx 1800\,MeV$$

- Hierarchies in the CKM mixing:

$$|V_{ud}| \approx 1, \ldots, |V_{ub}| \approx 0.0038$$

Goal: Solution to the flavour problem

- allow the SM fields to propagate in the bulk (except of the Higgs) $\Rightarrow$ 5D fields

Bulk fields

Gherghetta, Pomarol [hep-ph/0003129]
Grossman, Neubert [hep-ph/9912408]

- **Kaluza-Klein decomposition**:

$$\Psi(x^\mu, y) \sim \sum_{n=0}^{\infty} \psi^{(n)}(x^\mu) f^{(n)}(y)$$

$\psi^{(n)}(x^\mu)$	- Kaluza-Klein mode (KK mode)
$f^{(n)}(y)$	- profile of the KK mode in the bulk

- **Solution for bulk profiles:**

$$f^{(n)}(y) = \frac{e^{ky/2}}{N_n}\left[J_\alpha(\frac{m_n}{k}e^{ky}) + b_\alpha(m_n) Y_\alpha(\frac{m_n}{k}e^{ky})\right]$$

- **KK modes have KK mass:**

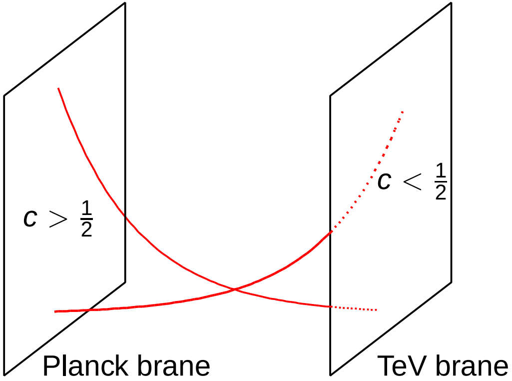
$$m_n \sim n\pi k e^{-kL} \sim n\,\mathcal{O}(TeV)$$

Example: Fermion zero modes

- zero mode profile depends on the bulk mass parameter c:

$$f^{(0)}(y,c) \sim e^{(\frac{1}{2}-c)ky}$$

- **Localization**: $c > \frac{1}{2} \Rightarrow$ Planck brane, $c < \frac{1}{2} \Rightarrow$ TeV brane



Result:
$\Rightarrow$ zero modes exist for only (++) BCs
$\Rightarrow$ zero modes have no KK mass
$\Rightarrow$ **identify SM particle with the zero mode**
$\Rightarrow$ different localization for each quark flavour

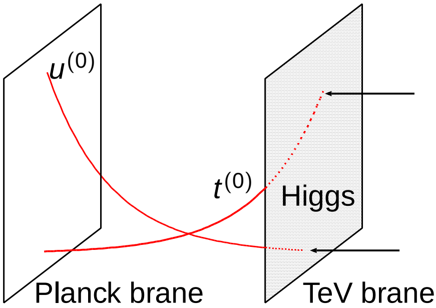
Hierarchical Masses and Mixings

- 4D effective theory is determined by overlap integrals
- **effective Yukawa couplings:**

$$Y_{ij} \sim \int dy\, \lambda_{ij}^{(5)}\, f^{(0)}(y, c_i)\, f^{(0)}(y, c_j)\, \delta(y - L)$$

Geometrical interpretation:

- overlap with the Higgs field determines fermion mass



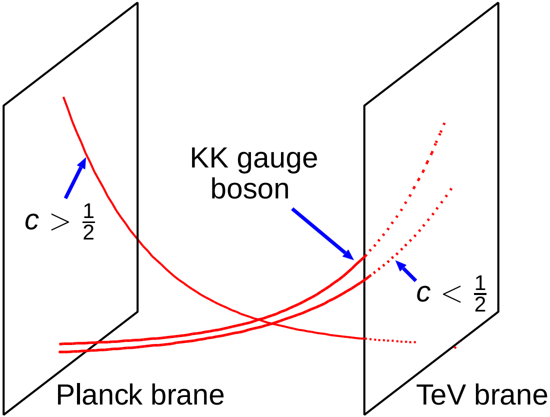
Result:
anarchical $\lambda_{ij}^{(5)} = \mathcal{O}(1)$ and $c = \mathcal{O}(1)$ lead to hierarchical Y_{ij}

Tree level FCNCs

- **effective gauge couplings:**

$$\varepsilon_{L,R}(k) \sim \int_0^L dy\, e^{ky} f_{L,R}^{(n)}(y, c_k) f_{L,R}^{(m)}(y, c_k) f_{\text{gauge}}^{(l)}(y)$$

- KK gauge bosons are localized towards the TeV brane
- Localization of fermions differs depended on c_k



- **Why non-universalities?**
  1. **heavy modes:**
     $\varepsilon_{L,R}(k)$ differs due to the different localization of the fermions
  2. **zero modes:**
     EW gauge bosons of the same charge mix due to EWSB
     $\Rightarrow$ non-universalities e.g. in the Z coupling
- **Coupling matrices after rotation** to fermion mass eigenstates:

$$\Delta_{L,R} = \mathcal{D}^{\dagger}_{L,R}\mathrm{diag}\big(\varepsilon_{L,R}(1), \varepsilon_{L,R}(2), \varepsilon_{L,R}(3)\big)\mathcal{D}_{L,R}$$

Result:
Non-universalities lead after rotation to mass eigenstates to
Flavour Changing Neutral Currents at Tree Level
$\Rightarrow$ new flavour violating parameters and CP-violating phases
$\Rightarrow$ beyond MFV

Constraints from EW precision measurements

Problem:

T parameter constrains the KK scale to $M_{\rm KK} \geq 10$ TeV

Our model should be protected from ...

- **... large T parameter contributions.**

additional bulk $SU(2)_R \Rightarrow$ custodial symmetry in the Higgs sector

Agashe, Delgado, May, Sundrum [hep-ph/0308036]
Csaki, Grojean, Pilo, Terning [hep-ph/0308038]

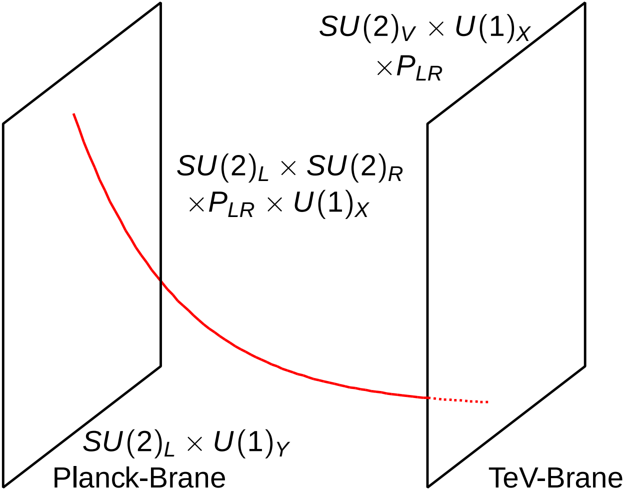
- **... problematic contributions to $Zb_L\bar{b}_L$.**

P_{LR} symmetry

Agashe, Contino, Da Rold, Pomarol [hep-ph/0605341]

$\Longrightarrow$ enlarged bulk symmetry:
$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times P_{LR}$

The symmetry breaking pattern



- **TeV-Brane:** spontaneous symmetry breaking
- **Planck-Brane:** symmetry breaking through BCs

- spontaneous symmetry breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$
  $\Rightarrow$ **mass of zero modes**

Field content

$\Longrightarrow$ **additional heavy gauge bosons and fermions**
due to extended gauge group and heavy KK modes
(restriction to first KK mode)

- three fermion representations $\xi^i = \xi^i_L + \xi^i_R$ containing 5d fermion fields:

$$\xi^i_{1L} = \begin{pmatrix} \chi^{u_i}_L(-+)_{5/3} & q^{u_i}_L(++)_{2/3} \\ \chi^{d_i}_L(-+)_{2/3} & q^{d_i}_L(++)_{-1/3} \end{pmatrix} \qquad \xi^i_{2R} = u^i_R(++)_{2/3}$$

$$\xi^i_{3R} = T^i_{3R} \oplus T^i_{4R} = \begin{pmatrix} \psi'^i_R(-+)_{5/3} \\ U'^i_R(-+)_{2/3} \\ D'^i_R(-+)_{-1/3} \end{pmatrix} \oplus \begin{pmatrix} \psi''^i_R(-+)_{5/3} \\ U''^i_R(-+)_{2/3} \\ D^i_R(++)_{-1/3} \end{pmatrix}$$

- transformation to mass eigenstates induces a **mixing** between fermions of the same charge, similarly in the gauge boson sector

- Gauge boson mass eigenstates:

$$\begin{matrix} G^{(0)A}_\mu & A^{(0)}_\mu & Z & W^\pm \\ G^{(1)A}_\mu & A^{(1)}_\mu & Z_H & W^\pm_H \\ & & Z' & W'^\pm_H \end{matrix}$$

$\Delta F = 2$ Processes - some theoretical aspects

Tree level contributions of gauge bosons:

KK gluons

Csaki, Falkowski, Weiler [arXiv:0804.1954]

Blanke, Buras, Duling, Gori, Weiler [arXiv:0809.1073]

KK photon	small contribution
Z_H and Z'	subdominant in ε_K and ΔM_K, but Z_H can compete with KK gluon in $B_{s,d}$ observables
Z boson	custodially and higher order suppressed

New operators:

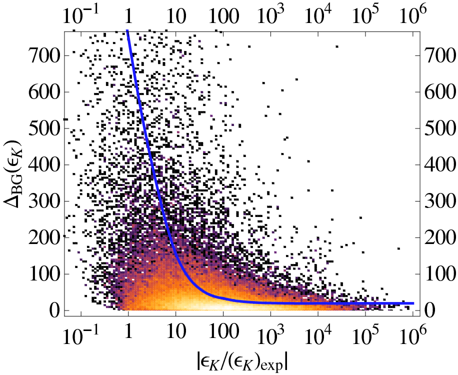
$$\mathcal{Q}_1^{VLL} = (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_L d) \qquad \mathcal{Q}_1^{VRR} = (\bar{s}\gamma_\mu P_R d)(\bar{s}\gamma^\mu P_R d)$$
$$\mathcal{Q}_1^{LR} = (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_R d) \qquad \mathcal{Q}_2^{LR} = (\bar{s}P_L d)(\bar{s}P_R d)$$

$\mathcal{Q}_2^{LR}$	gluons only
$\mathcal{Q}^{LR}$	dominates in $K^0 - \bar{K}^0$ mixing: chirally + RG enhanced
$\mathcal{Q}^{LL}$ and $\mathcal{Q}^{LR}$	dominate in $B^0_{s,d} - \bar{B}^0_{s,d}$ mixing: RG enhanced

Fine-tuning in ε_K

Barbieri-Giudice fine-tuning:
sensitivity of observables to small variation of model parameters

$$\Delta_{BG}(Obs) = \max_i \left| \frac{par_i}{Obs} \frac{\partial Obs}{\partial par_i} \right|$$



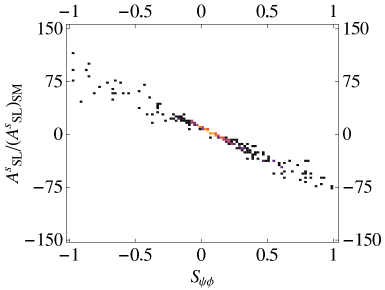
- generically $\varepsilon_K \sim 10^2 (\varepsilon_K)_{exp}$
- $\Delta_{BG}(\varepsilon_K)$ decreases with increasing ε_K
- moderate $\Delta_{BG}(\varepsilon_K)$ with $\varepsilon_K \sim (\varepsilon_K)_{exp}$ possible

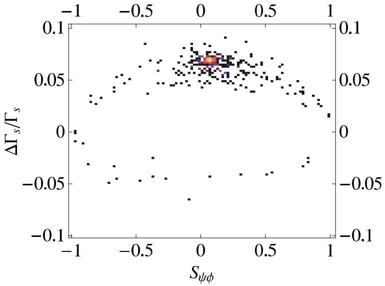
$M_{KK} \simeq 2.45$ TeV

Predictions for observables in the $B_s^0 - \bar{B}_s^0$ system

$\Longrightarrow$

Constraints from $\Delta F = 2$ data: ε_K, ΔM_K, ΔM_s, ΔM_d, $S_{\psi K_S}$
Regions of parameter space with moderate fine tuning





- full range $-1 \leq S_{\psi\phi} \leq 1$ possible
- strong correlation between observables & significant deviations from SM predictions possible

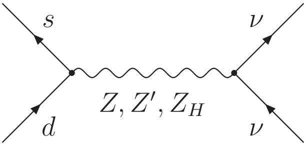
Rare decays - some theoretical aspects

Example: $K^+ \to \pi^+ \nu\bar{\nu}$ **and** $K_L \to \pi^0 \nu\bar{\nu}$ **decays**

- **effective Hamiltonian** for $s \to d\nu\bar{\nu}$ transition:

$$\left[H_{\rm eff}^{\nu\bar{\nu}}\right]^K \sim V_{ts}^* V_{td} \sum_{\ell=e,\mu,\tau} \left[X_{\rm SM} + X_K^{V-A}\right] (\bar{s}d)_{V-A} (\bar{\nu}_\ell \nu_\ell)_{V-A}$$
$$+ V_{ts}^* V_{td} \sum_{\ell=e,\mu,\tau} \left[X_K^{V}\right] (\bar{s}d)_{V} (\bar{\nu}_\ell \nu_\ell)_{V-A} + h.c.$$

- additional **tree level** contributions from electroweak gauge bosons Z, Z_H and Z'
- new operator $(\bar{s}d)_V(\bar{\nu}\nu)_{V-A}$ is present
- **Properties of the master functions** X**:**

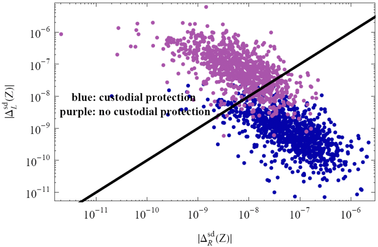


SM	RS
loop function	tree level contribution
flavour universal	non-universal
real	complex

- New contributions to X:

$$X_K^{V-A,V} = f(\Delta_L^{sd}(gauge), \Delta_R^{sd}(gauge), \Delta_L^{\nu\nu}(gauge))$$

- $Z d_R^i \bar{d}_R^j$ yields the dominant contribution $\Rightarrow \Delta_R^{sd}(Z)$ dominant
- phenomenology changes if custodial protection removed

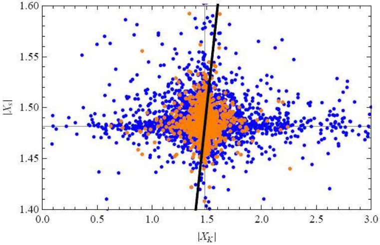


- in the transitions $s \to d\ell^+\ell^-$ and $b \to q\ell^+\ell^-$ ($q = d, s$) also the KK photon $A^{(1)}$ contributes

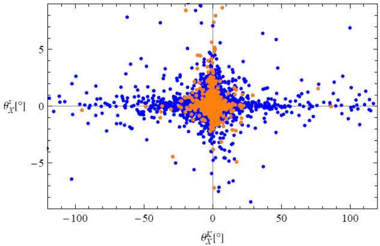
In the following numerical analysis we impose all existing constraints from $\Delta F = 2$ processes.

Breakdown of the universality ...

... between $|X_K|$ and $|X_s|$:



... between θ_X^K and θ_X^s:



e.g. $X_K \equiv X_{\text{SM}} + X_K^{V-A} + X_K^V \equiv |X_K| e^{i\,\theta_X^K}$

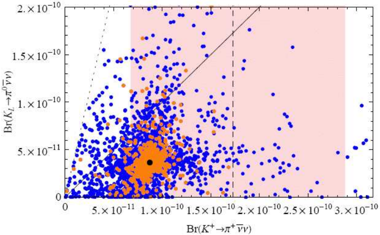
orange points - moderate fine-tuning
blue points - arbitrary fine-tuning $\rightarrow$ larger deviations from SM

Result:
CP-conserving and CP-violating **effects in the** K **system** can be much **larger** than in the B_d and B_s systems

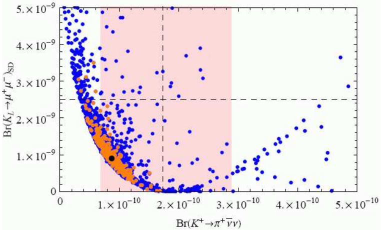
Rare K decays

$Br\left(K_L \to \pi^0 \nu\bar{\nu}\right)$ versus $Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right)$:

- departure from SM expectation:
  $Br\left(K_L \to \pi^0 \nu\bar{\nu}\right) \Rightarrow$ factor 5
  $Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right) \Rightarrow$ factor 2
- simultaneous large effects are possible



Correlation between $Br\left(K_L \to \mu^+\mu^-\right)$ and $Br\left(K^+ \to \pi^+ \nu\bar{\nu}\right)$:

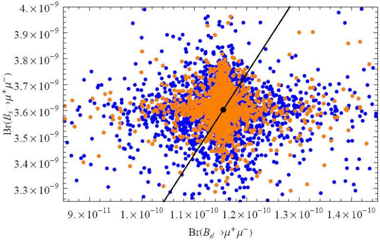


- both CP-conserving decays
- the correlation is an inverse one
- NP enters in both decays with opposite sign

Rare B decays

$Br(B_s \to \mu^+\mu^-)$ versus $Br(B_d \to \mu^+\mu^-)$:

- The branching ratios for $B_{s,d} \to \mu^+\mu^-$ are modified by at most 20%.
- effects are small and challenging to be measured in future experiments



Violation of the golden CMFV relations:

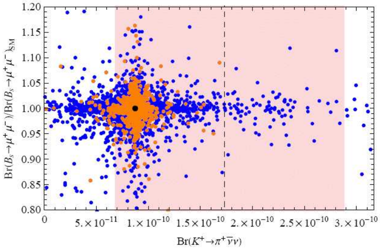
$$\frac{Br(B_s \to \mu^+\mu^-)}{Br(B_d \to \mu^+\mu^-)} = \frac{\hat{B}_{B_d}}{\hat{B}_{B_s}} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d} r\,, \qquad r = \left|\frac{Y_s}{Y_d}\right|^2 \frac{C_{B_d}}{C_{B_s}}\,, \qquad C_{B_{d,s}} = \frac{\Delta M_{d,s}}{(\Delta M_{d,s})_{\text{SM}}}$$

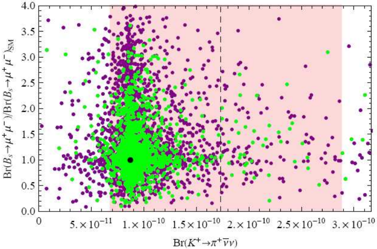
- departure from $r = 1$ measures the violation of the golden CMFV relation between $B_{d,s} \to \mu^+\mu^-$ decays and $\Delta M_{d,s}$

$$0.60 \leq r \leq 1.35$$

Removal of the custodial protection

$\frac{Br(B_s \to \mu^+\mu^-)}{Br(B_s \to \mu^+\mu^-)_{SM}}$ as a function of $Br(K^+ \to \pi^+ \nu\bar{\nu})$:



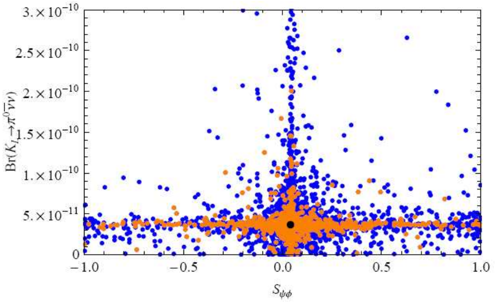


Comparision of the decays with and without custodial protection:

- another factor 2 in $Br(K^+ \to \pi^+ \nu\bar{\nu})$
- significantly larger effects in $Br(B_s \to \mu^+\mu^-)$
- effects in both decays are now of equal size

BUT: agreement with electroweak precision data for KK scale in the reach of LHC is much harder to obtain

Correlations between $\Delta F = 1$ and $\Delta F = 2$ observables:



SM expectation:

- $S_{\psi\phi} \simeq 0.04$

from $\Delta F = 2$ analysis:

- large effects in $S_{\psi\phi}$ possible: $S_{\psi\phi} \simeq 0.4$

$Br\left(K_L \to \pi^0 \nu\bar{\nu}\right)$ versus $S_{\psi\phi}$:

- the CP-asymmetry $S_{\psi\phi}$ can be enhanced by more than an order of magnitude
- effects in both observables are very unlikely

Conclusions

- Model provide a solution to gauge and flavour hierarchies with KK scale in reach of LHC

$\Delta F = 2$ observables:

- agreement with ε_K possible without relevant fine tuning
- large effects in B observables e.g. $S_{\psi\phi}$

$\Delta F = 1$ observables:

- large effects in rare K decays but small effects in rare B decays

Correlation of $\Delta F = 1$ and $\Delta F = 2$ observables:

- a number of branching ratios for rare K decays can differ significantly from SM predictions, but not simultaneously with $S_{\psi\phi}$

WE GIVE GREEN LIGHT FOR EXPERIMENTALISTS!