τ— LEPTON DIPOLE MOMENTS @ SUPER B FACTORIES

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April 2009

OUTLINE

1. τ - ELECTRIC DIPOLE MOMENT (EDM)
   - 1.1 Definition, measurements
   - 1.2 Observables

2. τ - MAGNETIC DIPOLE MOMENT (MDM)
   - 2.1 Definition, measurements
   - 2.2 Observables

3. CONCLUSIONS

1. EDM 1.1 Definition, measurements

BOUNDS:

PDG '08 95% CL

EDM BELLE '02

$$\Re e(d_\gamma^\tau) : (-2.2 \text{ to } 4.5) \times 10^{-17} \text{ e cm}$$

$$\Im m(d_\gamma^\tau) : (-2.5 \text{ to } 0.8) \times 10^{-17} \text{ e cm}$$

1. EDM 1.1 Definition, measurements

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$$\Re e(d^{\tau}_{\gamma}) : (-2.2 \text{ to } 4.5) \times 10^{-17} \text{ e cm}$$

$$\Im m(d^{\tau}_{\gamma}) : (-2.5 \text{ to } 0.8) \times 10^{-17} \text{ e cm}$$

$$d^{e}_{\gamma} = (0.07 \pm 0.07) \times 10^{-26} \text{ e cm}$$

$$d^{\mu}_{\gamma} = (3.7 \pm 3.4) \times 10^{-19} \text{ e cm}$$

1. EDM   1.1 Definition, measurements

P and T- odd first order interaction of a fermion with gauge fields (Landau 1957)

Besides, chirality flipping (some insight into the mass origin)

Classical electromagnetism
Ordinary quantum mechanics

$$H_{EDM} = -\vec{d}\cdot\vec{E}\,, \qquad \vec{d} = d\,\vec{s}$$

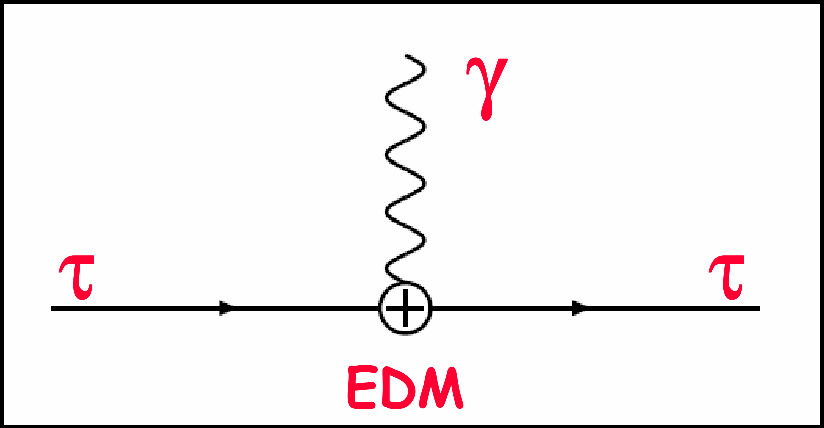
Dirac's equation

$$H = \bar{\Psi}\left(i(\not{\partial} + e\not{A}) - m\right)\Psi + \frac{i}{2} d\,\bar{\Psi}\gamma^5\sigma^{\mu\upsilon}\Psi F_{\mu\upsilon}$$

Interaction term for non-relativistic limit of Dirac's equation gives H_{EDM}

CPT: CP and T are equivalent

1. EDM   1.1 Definition, measurements

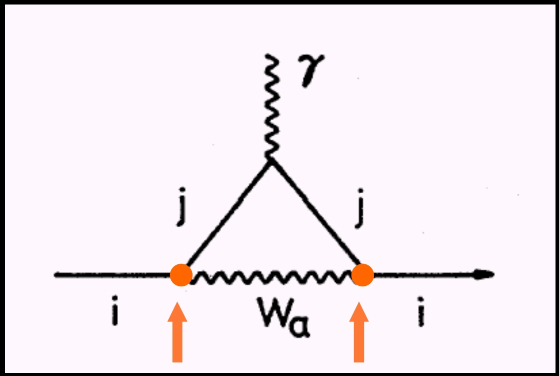


SM :
- vertex correction coming from CKM
- at least 4-loops for leptons

Beyond SM:
- one loop effect (SUSY, 2HDM,...)
- dimension six effective operator

1. EDM 1.1 Definition, measurements

SM :

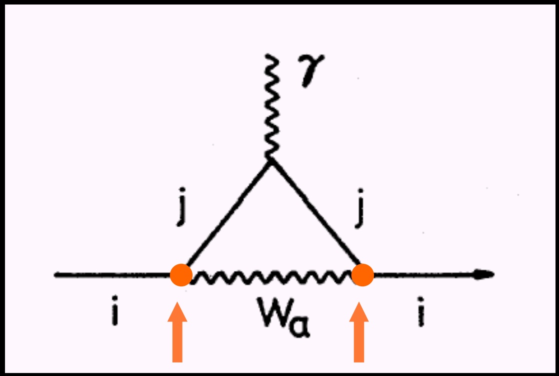


= 0

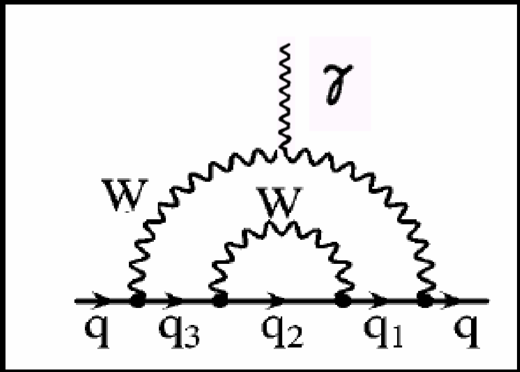
V_{CKM} V^*_{CKM}

1. EDM 1.1 Definition, measurements

SM :



V_{CKM} V^*_{CKM}



= 0 !!!

E.P.Shabalin '78

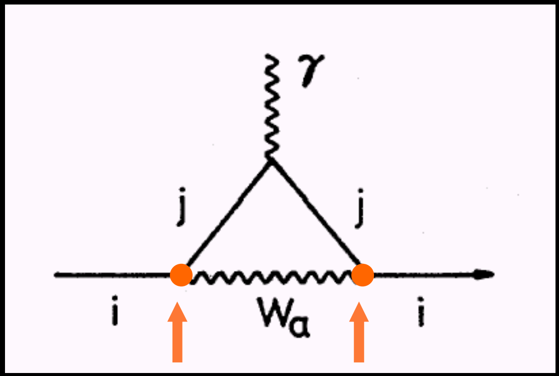
1-loop

2-loops

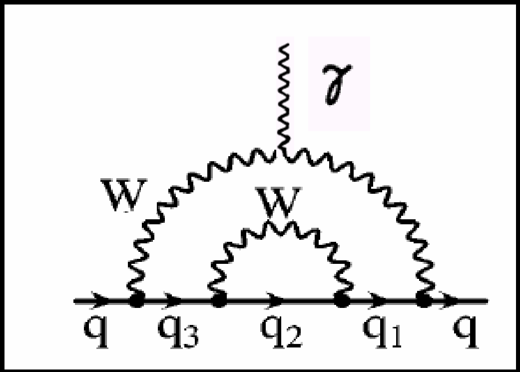
We need 3-loops for a quark EDM

1. EDM 1.1 Definition, measurements

SM :



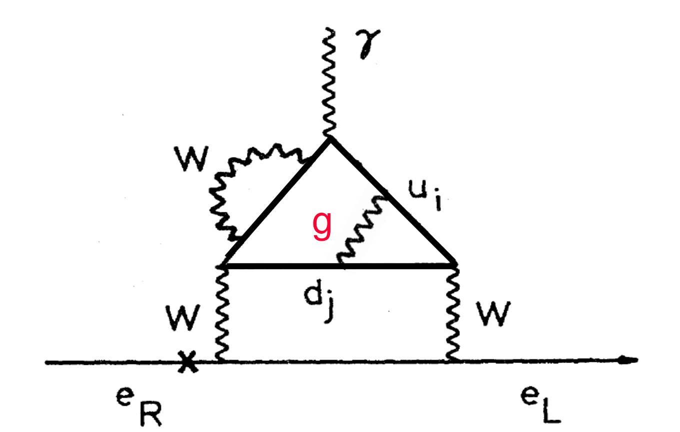
V_{CKM} V^*_{CKM}



= 0 !!!

E.P.Shabalin '78

We need 3-loops for a quark-EDM, and 4 loops for a lepton...



1. EDM   1.1 Definition, measurements

SM - EDM:

$$d_e \approx e\, G_F m_e \alpha^3 \alpha_S J/(4\pi)^5 \approx 10^{-38}\, e\,cm$$

CKM 3-loops

$$d_\gamma^q \approx 10^{-32} - 10^{-34}\, e\,cm$$

CKM+gluon 4-loops

$$d_\gamma^e \approx 10^{-38}\, e\,cm$$

Well below present experimental limits!

Hopefully

$$d_\gamma^\tau \approx \frac{m_\tau}{m_e} d_\gamma^e \approx 10^{-33} - 10^{-34}\, e\,cm$$

in the SM

…still 16 orders of magnitude below experiments…

1. EDM 1.1 Definition, measurements

Non-vanishing signal for a τ-EDM

⇕

NEW PHYSICS

Beyond SM physics predictions for the τ-EDM can go up to $10^{-20/21}$ e cm

1. EDM   1.1 Definition, measurements

Scaling the EDM as $m_{l,}$, then,limits on the e-EDM imply:

⇨ $d_\gamma^\tau < 2.4\times10^{-24}\, e\,cm$

(still many orders of magnitude above SM prediction)

However the EDM do not necessarily scale as m_l

- extended Higgs sector
- Leptoquarks
- heavy Majorana neutrino

1. EDM 1.1 Definition, measurements

Scalar Leptoquarks models

$$d_e : d_\mu : d_\tau = m_u^2 \; m_e : m_c^2 \, m_\mu : m_t^2 \, m_\tau$$

$$(m_\tau / m_e)^2 = 10^7 \quad , \quad (m_\tau / m_e)^3 = 10^{11}$$

Then, stronger than present τ - EDM bounds can be more competitive than e - EDM bounds

This will probably be the case for Super B Factories

1. EDM 1.1 Definition, measurements

Light Fermions :

- Stables or with enough large lifetime
- EDM : spin dynamics in electric fields

Heavy Fermions:

- Short living particles
- Spin matrix and angular distribution of decay products

  in τ -pair production may depend on the EDM

1. EDM 1.1 Definition, measurements

HOW DO WE MEASURE τ **ELECTRIC DIPOLE MOMENT**?

CP-even observables…

Total cross sections	$e^+e^- \longrightarrow \gamma \longrightarrow \tau^+\tau^-$
	$e^+e^- \longrightarrow e^+e^- \tau^+\tau^-$
Partial widths	$Z \longrightarrow \tau^+\tau^- \gamma$

Triple products $T_{ij} = (q_+ - q_-)_i \, (q_+ \times q_-)_j + (i \leftrightarrow j)$

Triple products, correlations and asymmetries CP-odd observables select EDM by symmetry properties

1. EDM 1.1 Definition, measurements

HOW DO WE MEASURE τ **ELECTRIC DIPOLE MOMENT**?

CP-even observables…

Total cross sections	$e^+e^- \longrightarrow \gamma \longrightarrow \tau^+\tau^-$
	$e^+e^- \longrightarrow e^+e^- \tau^+\tau^-$
Partial widths	$Z \longrightarrow \tau^+\tau^- \gamma$

Spin correlations	$s_i^+ \cdot s_j^-$
Linear polarizations	$s_i^\pm$

Triple products, correlations and asymmetries CP-odd observables select EDM by symmetry properties

1. EDM 1.2 Observables

Tau pair production

EDM: T-odd P-odd

Vs.

NORMAL - TRANSVERSE / LONGITUDINAL CORRELATIONS:
T - odd P - odd
no need of additional P-odd source: unpolarized beam

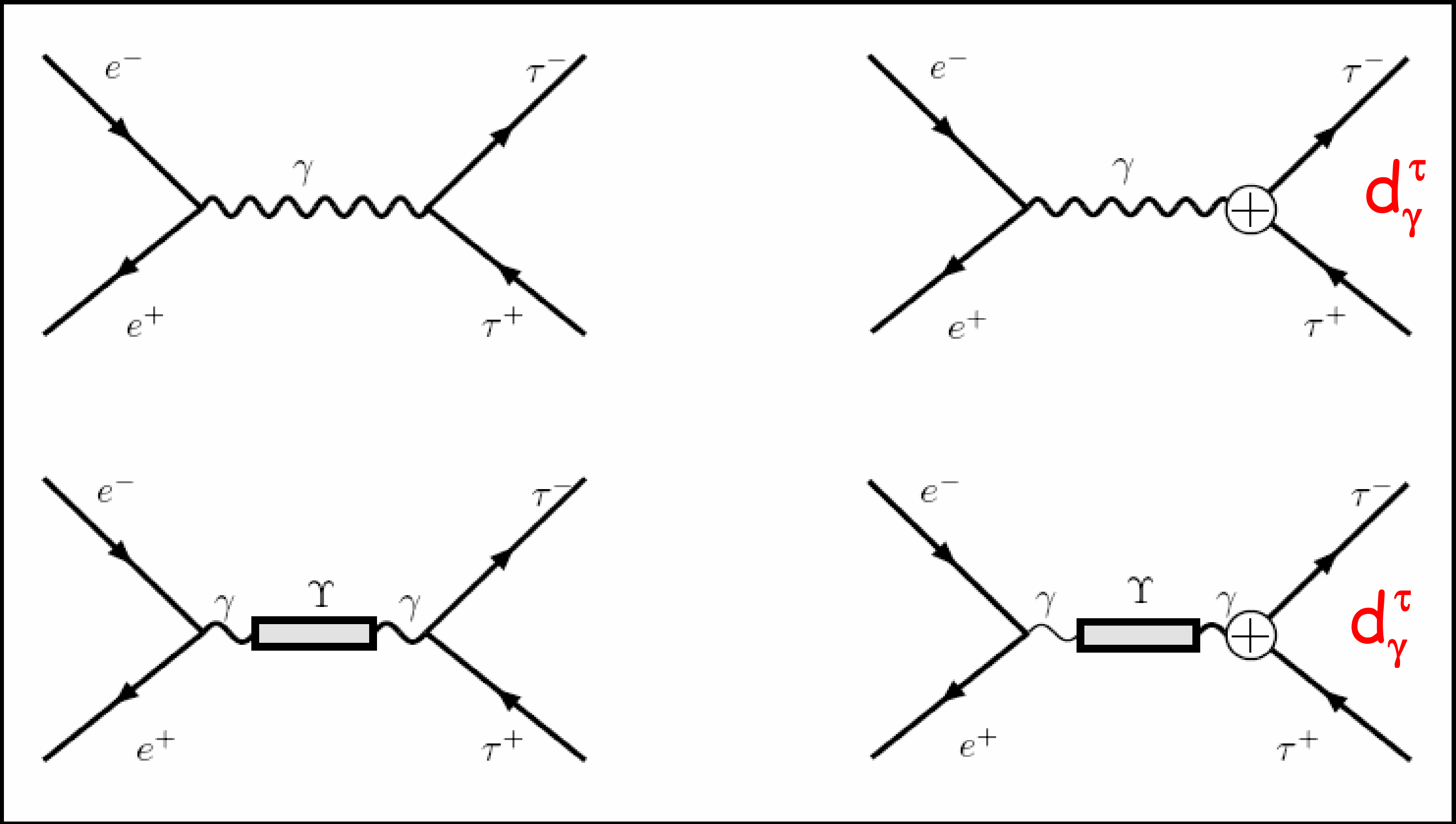
And

NORMAL POLARIZATION:
T - odd but P - even
needs additional P-odd source: beam polarization

1. EDM 1.2 Observables

$$e^+e^- \to \gamma, \Upsilon \to \tau^+(s_+)\,\tau^-(s_-)$$

Diagrams



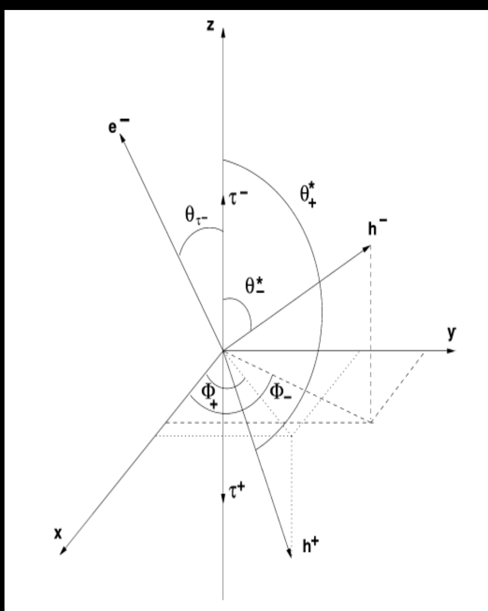
1. EDM 1.2 Observables

Tau pair production

NORMAL - TRANSVERSE / CORRELATIONS:

UNPOLARIZED e⁻ BEAM

x: Transverse   y: Normal   z: Longitudinal



$$\frac{d\sigma^{corr}}{d\Omega_{\tau^-}} = \frac{\alpha^2}{16s}\beta\Big(s_+^x s_-^x C_{xx} + s_+^y s_-^y C_{yy} + s_+^z s_-^z C_{zz} + (s_+^x s_-^y + s_+^y s_-^x)C_{xy}^+ + (s_+^x s_-^z + s_+^z s_-^x)C_{xz}^+ + (s_+^y s_-^z + s_+^z s_-^y)C_{yz}^+ + (\boldsymbol{s}_+ \times \boldsymbol{s}_-)_x C_{yz}^- + (\boldsymbol{s}_+ \times \boldsymbol{s}_-)_y C_{xz}^- + (\boldsymbol{s}_+ \times \boldsymbol{s}_-)_z C_{xy}^-\Big)$$

$$C_{xx} = (2-\beta^2)\sin^2\theta \qquad C_{xz}^+ = \frac{1}{\gamma} sin 2\theta$$

$$C_{yy} = -\sin^2\theta \qquad C_{xy}^- = 2\beta\sin^2\theta \quad \mathrm{Re}(d_\nu^\tau)$$

$$C_{zz} = \beta^2 + (2-\beta^2)\cos^2\theta \qquad C_{yz}^- = \gamma\beta sin^2\theta \quad \mathrm{Re}(d_\gamma^\tau)$$

1. EDM 1.2 Observables

Tau pair production

NORMAL - TRANSVERSE / CORRELATIONS:

UNPOLARIZED e⁻ BEAM

$$\frac{d^2\sigma}{d\phi_-^* d\phi_+^*} = \frac{\alpha^2\beta^2}{192 s^2} Br_- Br_+ \alpha_- \alpha_+ \sin(\phi_-^* - \phi_+^*)\, \mathrm{Re}(d_\gamma^\tau)$$

$$A_{NT} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

$$\sigma^\pm = \int_{w \gtrless 0} \frac{d^2\sigma}{d\phi_-^* d\phi_+^*} d\phi_-^* \, d\phi_+^*$$

$$w = \sin(\phi_-^* - \phi_+^*)$$

$$A_{NT} = \frac{4\beta}{\pi} \frac{\alpha_- \alpha_+}{3 - \beta^2} \mathrm{Re}(d_\gamma^\tau)$$

(NL correlation: similar alnalysis)

1. EDM 1.2 Observables

Tau pair production

Normal polarization: $P_N^{\tau} \leftrightarrow T-odd,\ P-even$

and needs helicity-flip so for the Tau it is mass enhanced

Genuine CPV if $P_N^{\tau^+} \leftrightarrow P_N^{\tau^-}$

NORMAL POLARIZATION: T-odd P-even
Vs.
EDM: T-odd P-odd

POLARIZED e^- BEAM provide another P-odd source

Tau pair production

Normal polarization:

EDM and polarized beams produce a P - even observable

$$\left.\frac{d\sigma^S}{d\Omega_{\tau^-}}\right|_\lambda = \frac{\alpha^2}{16s}\beta\Big\{\lambda\big[(s_- + s_+)_x X_+ + (s_- + s_+)_z Z_+ + (s_- - s_+)_y Y_-\big]$$
$$+ (s_- - s_+)_x X_- + (s_- - s_+)_z Z_-\Big\},$$

$$X_+ = \frac{1}{\gamma}\sin\theta_{\tau^-}, \qquad X_- = -\frac{1}{2}\sin(2\theta)\frac{2m_\tau}{e}\,\mathrm{Im}\{d_\tau^\gamma\},$$

$$Z_+ = +\cos\theta_{\tau^-}, \qquad Z_- = -\frac{1}{\gamma}\sin^2\theta\,\frac{2m_\tau}{e}\,\mathrm{Im}\{d_\tau^\gamma\},$$

$$Y_- = \gamma\beta\sin\theta_{\tau^-}\frac{2m_\tau}{e}\,\mathrm{Re}\{d_\tau^\gamma\}$$

1. EDM 1.2 Observables

For polarized beams

$$P_N^\tau \propto \lambda\gamma\beta^2 \cos\theta_\tau \sin\theta_\tau \frac{m_\tau}{e} \mathrm{Re}(d_\tau^\gamma)$$

Angular asymmetries (P_N^τ) are proportional to EDM

$$A_N^\mp = \frac{\sigma_L^\mp - \sigma_R^\mp}{\sigma_L^\mp + \sigma_R^\mp} = \alpha_\mp \frac{3\pi\gamma\beta}{8(3-\beta^2)} \frac{2m_\tau}{e} \mathrm{Re}(d_\tau^\gamma)$$

One can also measure A for τ^+ and/or τ^-

~~CP~~ :

$$A_N^{CP} \equiv \frac{1}{2}(A_N^+ + A_N^-)$$

1. EDM 1.2 Observables

Bounds:

Polarized beams – Normal polarization observable

$$\left|\Re e(d_{\gamma}^{\tau})\right| \leq 1.6\times10^{-19}\ e\,cm \quad \text{Super B, 1yr running, } 15\,ab^{-1}$$

$$\left|\Re e(d_{\gamma}^{\tau})\right| \leq 7.2\times10^{-20}\ e\,cm \quad \text{Super B, 15yr running, } 75\,ab^{-1}$$

-Some other observables can also be defined in order to measure the EDM imaginary part

-Correlation obs. with unpolarized beams have less sensitivity

2. MDM 2.1 Definition, measurements

Classical physics/quantum mechanics

$$H_{MDM} = -\vec{\mu} \cdot \vec{B}, \qquad \vec{\mu} = \mu \, \vec{s}$$

Dirac's equation **anomalous MDM**

$$H = \bar{\Psi}\left(i(\not{\partial} + e\not{A}) - m\right)\Psi + \frac{i}{2} a \bar{\Psi} \sigma^{\mu\upsilon} \Psi F_{\mu\upsilon}$$

Interaction term for non-relativistic limit of Dirac's equation gives H_{MDM}

$$H_{MDM} = -\underbrace{2(1+a)\frac{e\hbar}{2mc}}_{\mu}\vec{s} \cdot \vec{B}, \qquad \mu = 2(1+a)\mu_B$$

Schwinger 1948 QED: $a_\gamma^e = \dfrac{\alpha}{2\pi} + \cdots$

2. MDM 2.1 Definition, measurements

$$a_e^{exp} = (1159.652\,181\,1 \pm 0.000\,000\,7) \times 10^{-6}$$

$$a_\mu^{exp} = (1165.9208 \pm 0.0006) \times 10^{-6}$$

$$\frac{\alpha}{2\pi} = 0.00116141\ldots$$

2. MDM 2.1 Definition, measurements

$$a_e^{exp} = (1159.652\,181\,1 \pm 0.000\,000\,7) \times 10^{-6}$$

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$$-0.052 < a_\tau^{exp} < 0.013 \quad (95\%CL)$$

$$\frac{\alpha}{2\pi} = 0.00116141\ldots$$

2. MDM 2.1 Definition, measurements

$$a_e^{exp} = (1159.652\,181\,1 \pm 0.000\,000\,7) \times 10^{-6}$$

$$a_\mu^{exp} = (1165.92\underline{08} \pm 0.0006) \times 10^{-6}$$

$$-0.052 < a_\tau^{exp} < 0.013 \quad (95\%CL)$$

$$a_\tau^{SM} = (1\,177.21 \pm 5) \times 10^{-6}$$

HAD ~ 350

WI ~ 48

$$\frac{\alpha}{2\pi} = 0.00116141\ldots$$

2. MDM 2.1 Definition, measurements

Citation: C. Amsler *et al.* (Particle Data Group), PL **B667**, 1 (2008) (URL: http://pdg.lbl.gov)

τ MAGNETIC MOMENT ANOMALY

The q^2 dependence is expected to be small providing no thresholds are nearby.

$\mu_\tau/(e\hbar/2m_\tau)-1 = (g_\tau-2)/2$

For a theoretical calculation $[(g_\tau-2)/2 = 117\,721(5)\times 10^{-8}]$, see EIDELMAN 07.

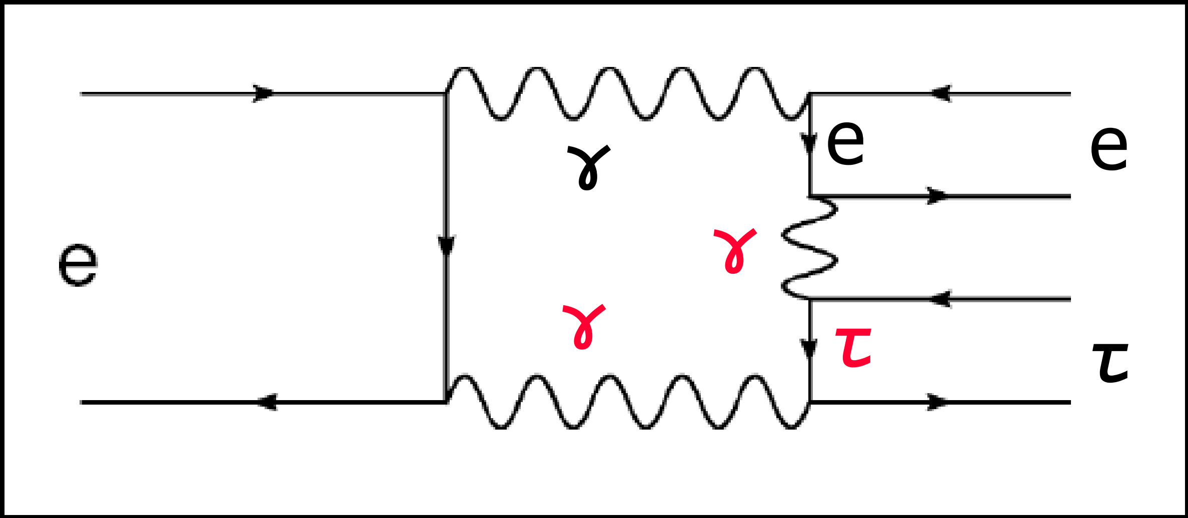
VALUE	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
> -0.052 and < 0.013 (CL = 95%) OUR LIMIT					
> -0.052 and < 0.013	95	[1] ABDALLAH	04K	DLPH	$e^+e^- \to e^+e^-\tau^+\tau^-$ at LEP2
• • • We do not use the following data for averages, fits, limits, etc. • • •					
<0.107	95	[2] ACHARD	04G	L3	$e^+e^- \to e^+e^-\tau^+\tau^-$ at LEP2
> -0.007 and < 0.005	95	[3] GONZALEZ-S...	00	RVUE	$e^+e^- \to \tau^+\tau^-$ and $W \to \tau\nu_\tau$

Model independent /Effective lagragians

2. MDM 2.1 Definition, measurements

Delphi, 2004 $e^+ e^- \rightarrow e^+ e^- \tau^+ \tau^-$

Diagram



Photons and τ OFF-SHELL

2. MDM 2.1 Definition, measurements

The anomaluos magnetic moment is defined by

$$a_\tau = F_2\,(\,q^2 = 0\,).$$

This quantity is gauge independent

AND

for QED, $F_2\,(\,q^2 \neq 0\,)$ is also gauge independent

Besides, weak interactions gauge dependence

vanishes for $q^2 \longrightarrow 0$

1-loop QED:

$$F_2(s) = \left(\frac{\alpha}{2\pi}\right)\frac{2m_\tau^2}{s}\frac{1}{\beta}\left(\log\frac{1+\beta}{1-\beta} - i\pi\right), \quad \text{for } q^2 = s > 4m_\tau^2,$$

2. MDM 2.1 Definition, measurements

BONUS

Schwinger

real and imaginary parts

Flavor dependence

$$F_2(s) = \left(\frac{\alpha}{2\pi}\right)\frac{2m_\tau^2}{s}\frac{1}{\beta}\left(\log\frac{1+\beta}{1-\beta} - i\pi\right), \quad \text{for } q^2 = s > 4m_\tau^2,$$

2. MDM 2.1 Definition, measurements

At B-factories:

$$F_2(M_{\Upsilon}^2) = (\ 265 - 245\ i\ )\ x\ 10^{-6}$$

Real and imaginary parts are same order of magnitude

THIS FORM FACTOR CAN BE MEASURED

AT SUPER B FACTORIES

2. MDM 2.2 Observables

Diagrams

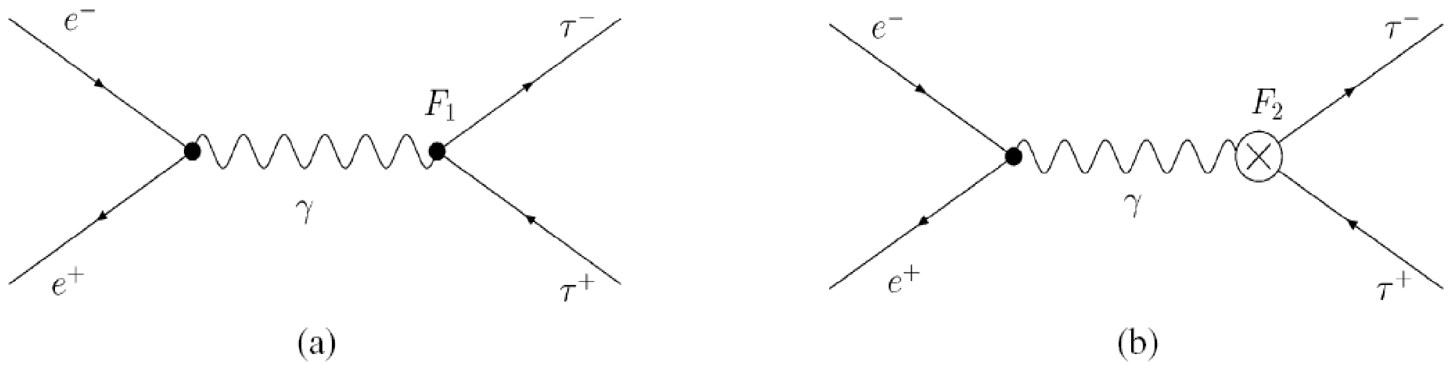


Fig. 1. Diagrams: (a) direct γ exchange, (b) F_2 in γ exchange.

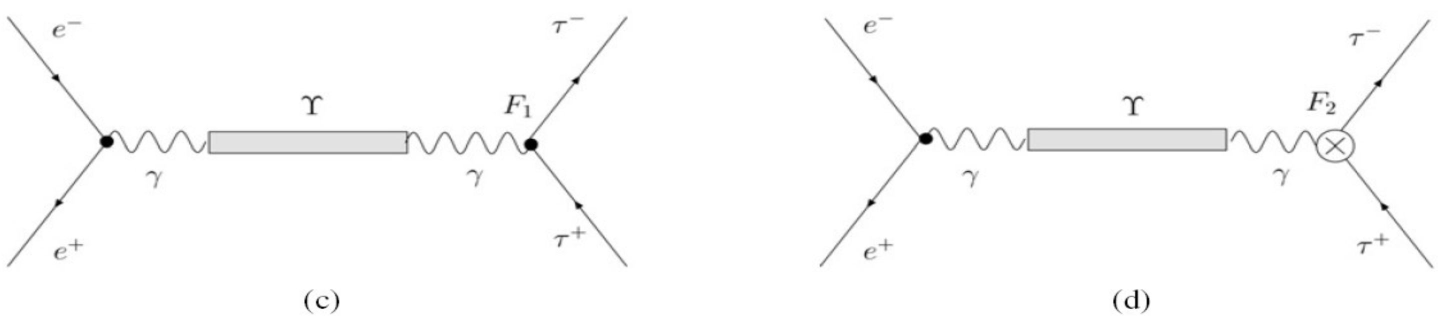
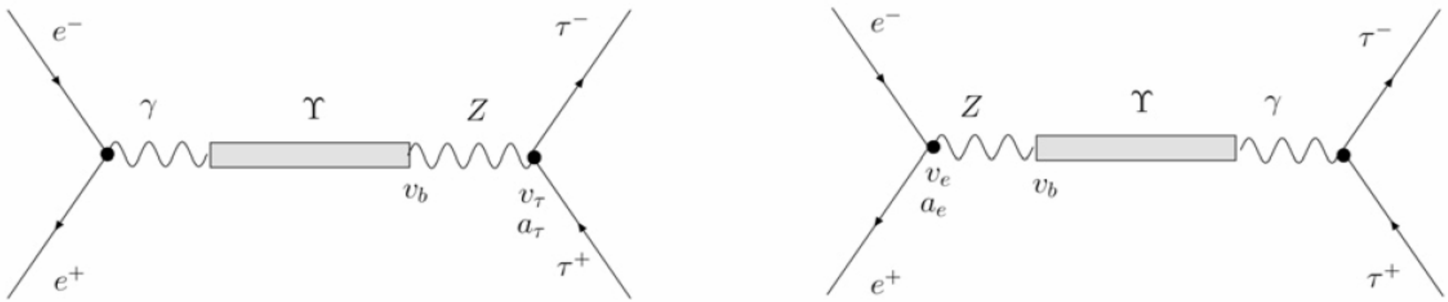


Fig. 2. Diagrams: (c) Υ production, (d) F_2 in Υ production.

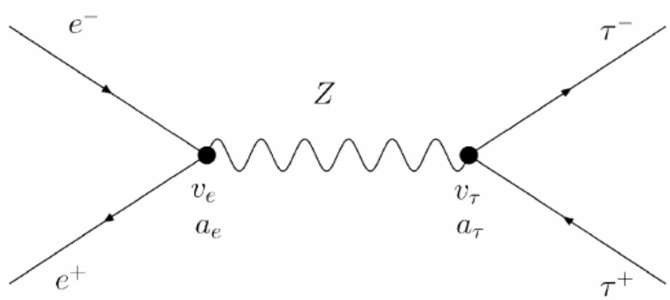
2. MDM 2.2 Observables

Diagrams



(e) (f)

Fig. 3. γ and Z interchange on Υ production.



(g)

Fig. 4. Non-resonant Z interchange.

2. MDM 2.2 Observables

UNPOLARIZED BEAM

Normal polarization asymmetries: Imaginary part

$$\frac{d\sigma^S}{d\cos_{\tau^-}} = \frac{\pi\alpha^2}{4s}\beta(s_- + s_+)_y Y_+,$$

$$Y_+ = \gamma\beta^2(\cos\theta_{\tau^-}\ \sin\theta_{\tau^-})\,\mathrm{Im}\{F_2(s)\}$$

$\gamma = \sqrt{s}/2m_\tau$ is the dilation factor.

Angular distribution:

$$\frac{d\sigma_{FB}}{d\phi_\pm} = \mp\frac{\pi\alpha^2}{12s}\mathrm{Br}(\tau^+ \to h^+\bar{\nu}_\tau)\,\mathrm{Br}(\tau^- \to h^-\nu_\tau)(\alpha_\pm)\beta^3\gamma\,\mathrm{Im}\{F_2(s)\}\sin\phi_\pm.$$

$$A_N^\pm = \frac{\sigma_L^\pm - \sigma_R^\pm}{\sigma} = \pm\alpha_\pm\frac{1}{2(3-\beta^2)}\beta^2\gamma\,\mathrm{Im}\{F_2(s)\}$$

2. MDM 2.2 Observables

POLARIZED BEAM

Longitudinal and Transverse polarization asymmetries:

Real part

L / T pol. are P-odd $\Longrightarrow$ beam polarization needed

$$\left.\frac{d\sigma^S}{d\cos_{\tau^-}}\right|_{\lambda} = \frac{\pi\alpha^2}{8\,s}\beta\Big\{(s_- + s_+)_y Y_+ + \lambda\big[(s_- + s_+)_x X_+ + (s_- + s_+)_z Z_+\big]\Big\},$$

$$X_+ = \sin\theta_{\tau^-}\big[|F_1|^2 + \big(2-\beta^2\big)\gamma^2\,\mathrm{Re}\{F_2\}\big]\frac{1}{\gamma},$$

$$Z_+ = \cos\theta_{\tau^-}\big[|F_1|^2 + 2\,\mathrm{Re}\{F_2\}\big],$$

2. MDM 2.2 Observables

POLARIZED BEAM

Angular distribution L/T polarization asymmetries:

$$A_T^{\pm} = \frac{\sigma_R^{\pm}|_{\mathrm{Pol}} - \sigma_L^{\pm}|_{\mathrm{Pol}}}{\sigma} = \mp\alpha_{\pm}\frac{3\pi}{8(3-\beta^2)\gamma}\left[|F_1|^2 + (2-\beta^2)\gamma^2\,\mathrm{Re}\{F_2\}\right],$$

$$A_L^{\pm} = \frac{\sigma_{\mathrm{FB}}^{\pm}(+)|_{\mathrm{Pol}} - \sigma_{\mathrm{FB}}^{\pm}(-)|_{\mathrm{Pol}}}{\sigma} = \mp\alpha_{\pm}\frac{3}{4(3-\beta^2)}\left[|F_1|^2 + 2\,\mathrm{Re}\{F_2\}\right],$$

Combining both observables

$$\mathrm{Re}\{F_2(s)\} = \mp\frac{8(3-\beta^2)}{3\pi\gamma\beta^2}\frac{1}{\alpha_{\pm}}\left(A_T^{\pm} - \frac{\pi}{2\gamma}A_L^{\pm}\right).$$

2. MDM 2.2 Observables

Z interference is eliminated in this last observable !

$$\mathrm{Re}\{F_2(s)\} = \mp\frac{8(3-\beta^2)}{3\pi\gamma\beta^2}\frac{1}{\alpha_\pm}\left(A_T^\pm - \frac{\pi}{2\gamma}A_L^\pm\right).$$

2. MDM 2.2 Observables

		Babar+Belle	SuperB	
		2 ab^{-1}	1 yr running 15 ab^{-1}	5 yr running 75 ab^{-1}
$Im\{F_2\}$	Normal Single τ Asym.	2.1×10^{-5}	7.8×10^{-6}	3.5×10^{-6}

$$F_2(M_{\Upsilon}^2) = (265 - 245\, i) \times 10^{-6}$$

2. MDM 2.2 Observables

		Babar+Belle	SuperB	
		2 ab^{-1}	1 yr running 15 ab^{-1}	5 yr running 75 ab^{-1}
$Im\{F_2\}$	Normal Single τ Asym.	$2.1x10^{-5}$	$7.8x10^{-6}$	$3.5x10^{-6}$
$Re\{F_2\}$	Transv. – Long. Asym. Combined	$1.0x10^{-5}$	$3.7x10^{-6}$	$1.7x10^{-6}$
	TT - LT	$7.6x10^{-5}$	$2.8x10^{-5}$	$1.2x10^{-5}$
	LL – LT	$5.2x10^{-5}$	$1.9x10^{-5}$	$8.5x10^{-6}$
	NN - LT	$5.1x10^{-5}$	$1.8x10^{-5}$	$8.3x10^{-6}$

$$F_2(M_\Upsilon^2) = (265 - 245\, i) \times 10^{-6}$$

2. MDM   2.2 Observables

Polarized beams Unpolarized beams			SuperB: 1 yr running $15\ ab^{-1}$	SuperB: 5 yr running $75\ ab^{-1}$
$Im\{F_2\}$	Normal Single τ Asym.		7.8×10^{-6}	3.5×10^{-6}
$Re\{F_2\}$	Transv. – Long. Asym. Combined		3.7×10^{-6}	1.7×10^{-6}
	TT - LT		2.8×10^{-5}	1.2×10^{-5}
	LL – LT		1.9×10^{-5}	8.5×10^{-6}
	NN - LT		1.8×10^{-5}	8.3×10^{-6}

$$F_2(M_\Upsilon^2) = (\,265 - 245\, i\,) \times 10^{-6}$$

3. Conclusions

EDM

- Linear polarization and correlation CP-odd observables at super B factories can lower the Tau-EDM bound by 2-3 orders of magnitude.
- Tau-EDM bounds may become competitive with other EDM bounds for “mass dependent” models.
- Polarized beam observables have the best sensitivity and are independent from other low and high energy observables already investigated.

3. Conclusions

MDM

- The Tau-MDM form factor can be *measured* for the first time in asymmetries at Super B factories.
- At least two SM/QED figures can be obtained.
- Both unpolarized and polarized beam observables allow to do so.
- Linear observables for MDM:

  real part polarization beam needed

  imaginary part unpolarized beam