Hierarchical Squarks and B Physics

Marco Nardecchia
SISSA/ISAS
Trieste

G. F. Giudice, M.N., A. Romanino
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Outline

- FCNC in the MSSM
- MIA and the degenerate scenario
- Hierarchical framework
- Phenomenology and predictions
- Conclusions

FV in the SM and in the MSSM

- In the SM, flavor violation arises from the Yukawa sector:

$$U_R M U_L^\dagger = \text{diagonal} \longrightarrow V = U_L^u U_L^{d\dagger}$$

- In the MSSM, there are new sources of FV:

$$\mathcal{L}_{\text{sdown masses}} = -\left(\tilde{\mathbf{d}}_\mathbf{L}^\dagger \quad \tilde{\mathbf{d}}_\mathbf{R}^\dagger\right) \mathbf{m}_{\tilde{\mathbf{d}}}^\mathbf{2} \begin{pmatrix} \tilde{\mathbf{d}}_\mathbf{L} \\ \tilde{\mathbf{d}}_\mathbf{R} \end{pmatrix} \qquad \tilde{\mathbf{d}}_\mathbf{L} = \begin{pmatrix} \tilde{d}_L \\ \tilde{s}_L \\ \tilde{b}_L \end{pmatrix} = \begin{pmatrix} \tilde{d}_{L1} \\ \tilde{d}_{L2} \\ \tilde{d}_{L3} \end{pmatrix}$$

Soft — Quark masses — EWSB

$$\mathbf{m}_{\tilde{\mathbf{d}}}^\mathbf{2} = \begin{pmatrix} m_Q^2 + m_d^2 + \Delta_{\tilde{d}_L} & v(a_d^* \cos\beta - \mu y_d \sin\beta) \\ v(a_d \cos\beta - \mu^* y_d \sin\beta) & m_{\tilde{d}}^2 + m_d^2 + \Delta_{\tilde{d}_R} \end{pmatrix}$$

Trilinear Soft — Yukawa coupling

How to Suppress FCNC?

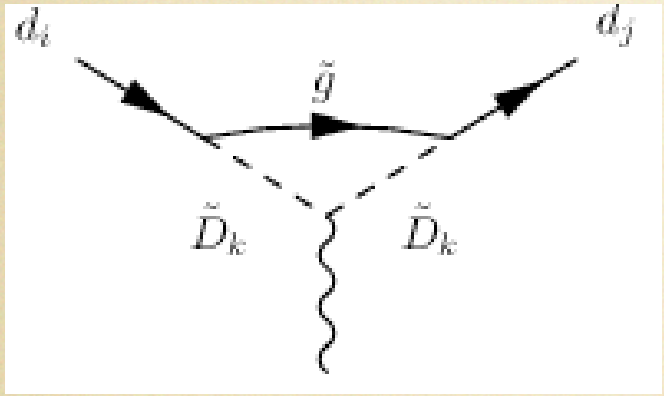
Let us first consider the gluino contribution to a ΔF=1 process of the type: $d_i^A \to d_j^B$

$A, B = \{L, R\}$

$i, j = \{1, 2, 3\}$

$i \neq j$

The Feynman amplitude will be proportional to:



$$\mathcal{W}_{d_i^A k} f(x_k) \mathcal{W}^{\dagger}_{k d_j^B}$$

$$x_k = \frac{m^2_{\tilde{q}_k}}{m^2_{\tilde{g}}} \qquad k = \{1, \ldots, 6\}$$

This amplitude is suppressed if:

- The particles in the loop are very heavy because $\lim_{x\to\infty} f(x) = \lim_{x\to 0} f(x) = 0$
- The squark masses are almost degenerate because $f(x)\mathcal{W}_{d_i^A k}\mathcal{W}^{\dagger}_{k d_j^B} = f(x)\delta_{AB}\delta_{ij} = 0$
- The unitary matrix is close to the identity because $f(x_k)\delta_{A_i k}\delta_{k B_j} = f(x_k)\delta_{AB}\delta_{ij} = 0$

Generically, first solution reintroduces fine tuning.
The last two possibilities imply small off diagonal entries in the squark mass matrix with respect to the diagonal ones

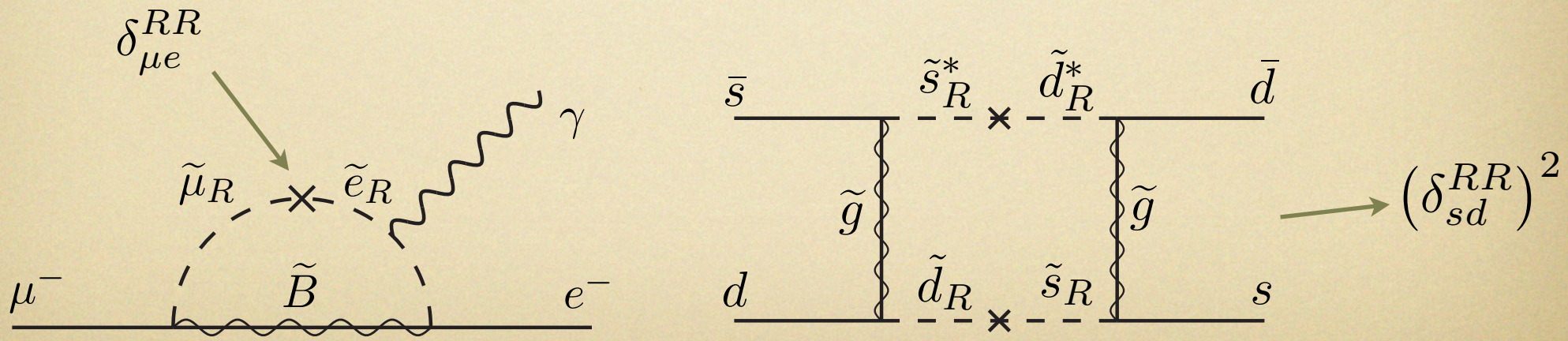
Model independent analysis

- Expand in off-diagonal elements of squark mass $\mathcal{M}^2 = \mathcal{M}_0^2 + \delta\mathcal{M}^2$
- Degeneracy: $\mathcal{M}_0^2 = \tilde{m}^2 I_{6\times 6}$ [Gabbiani Gabrielli Masiero Silvestrini '96]

$$A(\Delta F = 1) = x f^{(1)}(x)\delta_{ij}$$
$$A(\Delta F = 2) = \frac{x^2}{3!} g^{(3)}(x)\delta_{ij}^2$$

$$\delta_{ij} \equiv \frac{(\delta\mathcal{M}^2)_{ij}}{\tilde{m}^2} \qquad x = \frac{\tilde{m}^2}{M^2}$$

- f,g: loop functions (flavor conserving, process dependent)
- δ_{ij} : flavor violating parameters (process independent)



The Hierarchical Case

In the hierarchical scenario, the soft blocks (LL and RR) have the following structure:

$$\tilde{m}^2 = \begin{pmatrix} h_{11} & h_{12} & a_1 \\ h_{21} & h_{22} & a_2 \\ \bar{a}_1 & \bar{a}_2 & l_3 \end{pmatrix}$$

where the "h" block is heavy, and the remaining entries are much lighter

In particular we impose that $l \lesssim \text{TeV}^2$, $h_1, h_2 \gg l$

The LR and RL blocks are suppressed by EW scale. Only for the light family we can have a significant mixing.

The exact diagonalization of the squark mass matrix gives:

$$\mathcal{M}^2 = \mathcal{W}\,\text{Diag}(\tilde{m}^2_{L1}, \tilde{m}^2_{L2}, \tilde{m}^2_{R1}, \tilde{m}^2_{R2}, \tilde{m}^2_1, \tilde{m}^2_2)\mathcal{W}^\dagger$$

The amplitude will be given by:

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Ai,Bj} = \mathcal{W}_{Ai,Lk}\, f\left(\frac{\tilde{m}^2_{Lk}}{m^2_{\tilde{g}}}\right) \mathcal{W}^\dagger_{Lk,Bj} + \mathcal{W}_{Ai,Rk}\, f\left(\frac{\tilde{m}^2_{Rk}}{m^2_{\tilde{g}}}\right) \mathcal{W}^\dagger_{Rk,Bj} + \mathcal{W}_{Ai,p}\, f\left(\frac{\tilde{m}^2_{p}}{m^2_{\tilde{g}}}\right) \mathcal{W}^\dagger_{p,Bj}$$

Heavy LL — Heavy RR — Light third family

Hierarchical Soft Terms

Motivations: [Effective SUSY, Choen Kaplan Lepeintre Nelson '97]

- Complementary to degenerate assumption
- If we start with a degenerate boundary condition at very high energy, we end up to a split situation at low energy because of the Yukawa coupling of the 3rd family
- Welcome to alleviate the SUSY problem

$$A(\Delta F = 1) = f(x)\hat{\delta}_{ij}$$
$$A(\Delta F = 2) = g^{(1)}(x)\hat{\delta}^2_{ij}$$

$$x = \frac{\tilde{m}_3^2}{M^2} \qquad \hat{\delta}_{a3} = \frac{\mathcal{M}^2_{a3}}{\tilde{m}_h^2}$$

There are only 4 flavor violating insertions:

$$\hat{\delta}^{LL}_{bd}, \hat{\delta}^{LL}_{bd}, \hat{\delta}^{RR}_{bd}, \hat{\delta}^{RR}_{bd}$$

$$\hat{\delta}^{LL}_{ij} \equiv \hat{\delta}^{LL}_{i3}\hat{\delta}^{LL*}_{j3}$$

$$\hat{\delta}^{LR}_{ij} \equiv \frac{\mathcal{M}^2_{L3,R3}}{\tilde{m}^2}\hat{\delta}^{LL}_{i3}\hat{\delta}^{RR*}_{j3} \qquad i,j = 1,2$$

$$\hat{\delta}^{LR}_{i3} \equiv \frac{\mathcal{M}^2_{L3,R3}}{\tilde{m}^2}\hat{\delta}^{LL}_{i3}.$$

Correlations in the two frameworks can be very different:

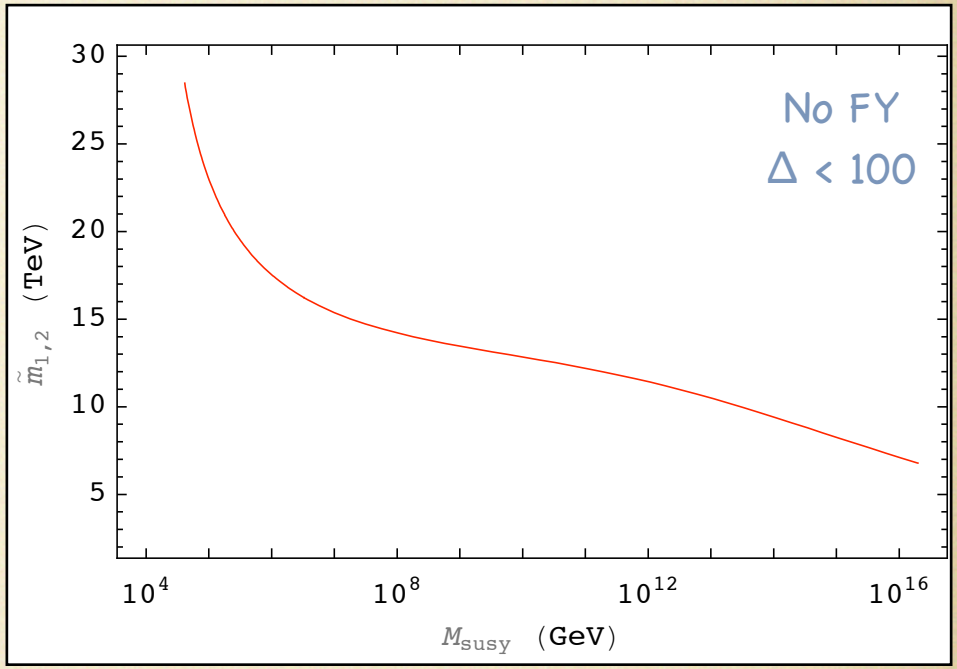
$$\left.\frac{A(\Delta F = 2)}{[A(\Delta F = 1)]^2}\right|_{\text{degenerate}} = \frac{g^{(3)}}{6g^{(1)}}\left(\frac{f}{f^{(1)}}\right)^2 \left.\frac{A(\Delta F = 2)}{[A(\Delta F = 1)]^2}\right|_{\text{hierarchical}}.$$

Naturalness

We can quantify the naturalness criterion by requiring that: [Barbieri Giudice '88]

$$\left| \frac{a_i}{M_Z^2} \frac{\partial M_Z^2(a)}{\partial a_i} \right| < \Delta$$

Where Δ is a measure of the fine tuning, and a_i are the parameters at high scale.



Because of the smallness of the Yukawa coupling the first two generations of sfermions can be naturally heavier with respect to the 3rd one.
[Dimopoulos Giudice '95]

In order to suppress the SUSY contribution to ϵ_K we need:

$$\tilde{m}_h > 800 \text{ TeV}$$

The hypothesis of hierarchical soft terms is not sufficient to solve the flavor problem in the 1-2 sector.

Bounds

x=1

Assuming that the CKM is dominated by the rotation in the down sector, in the hierarchical scenario is plausible to expect:

$$\left|\hat{\delta}_{sb}\right| \approx |V_{ts}^*| \approx 4 \times 10^{-2}$$

$$\left|\hat{\delta}_{db}\right| \approx |V_{td}^*| \approx 8 \times 10^{-3}$$

$$\left|\hat{\delta}_{db}\hat{\delta}_{bs}\right| \approx 3 \times 10^{-4}$$

Hierarchy | Degeneracy

$D_0 - \bar{D}_0$ mixing

$\left\|\hat{\delta}^{LL}_{ut}\hat{\delta}^{LL*}_{ct}\right\| < 8.0 \times 10^{-3} \left(\frac{m_{\tilde{t}}}{350\ \text{GeV}}\right)$	$\left\|\delta^{LL}_{uc}\right\| < 3.4 \times 10^{-2} \left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$

$B \to X_s\gamma$

$\left\|\text{Re}\left(\hat{\delta}^{LL}_{sb}\right)\right\| < 2.2 \times 10^{-2} \left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)^2 \left(\frac{10}{\tan\beta}\right)$	$\left\|\text{Re}\left(\delta^{LL}_{sb}\right)\right\| < 3.8 \times 10^{-2} \left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)^2 \left(\frac{10}{\tan\beta}\right)$
$\left\|\text{Im}\left(\hat{\delta}^{LL}_{sb}\right)\right\| < 6.7 \times 10^{-2} \left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)^2 \left(\frac{10}{\tan\beta}\right)$	$\left\|\text{Im}\left(\delta^{LL}_{sb}\right)\right\| < 1.1 \times 10^{-1} \left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)^2 \left(\frac{10}{\tan\beta}\right)$

Δm_{B_s}

$\left\|\text{Re}\left(\hat{\delta}^{LL}_{sb}\right)\right\| < 9.4 \times 10^{-2} \left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\left\|\text{Re}\left(\delta^{LL}_{sb}\right)\right\| < 4.0 \times 10^{-1} \left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$
$\left\|\text{Im}\left(\hat{\delta}^{LL}_{sb}\right)\right\| < 7.2 \times 10^{-2} \left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\left\|\text{Im}\left(\delta^{LL}_{sb}\right)\right\| < 3.1 \times 10^{-1} \left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$

B^0_d–$\bar{B}^0_d$ mixing

$\left\|\text{Re}\left(\hat{\delta}^{LL}_{db}\right)\right\| < 4.3 \times 10^{-3} \left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\left\|\text{Re}\left(\delta^{LL}_{db}\right)\right\| < 1.8 \times 10^{-2} \left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$
$\left\|\text{Im}\left(\hat{\delta}^{LL}_{db}\right)\right\| < 7.3 \times 10^{-3} \left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\left\|\text{Im}\left(\delta^{LL}_{db}\right)\right\| < 3.1 \times 10^{-2} \left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$

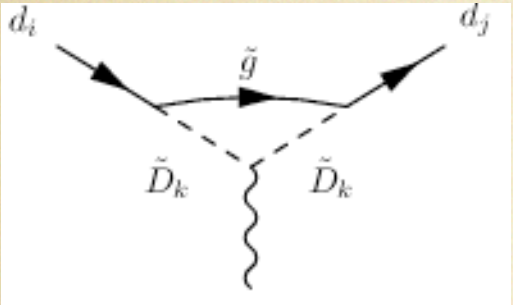
Δm_K

$\sqrt{\left\|\text{Re}\left(\hat{\delta}^{LL}_{db}\hat{\delta}^{LL*}_{sb}\right)^2\right\|} < 1.0 \times 10^{-2} \left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\sqrt{\left\|\text{Re}\left(\delta^{LL}_{ds}\right)^2\right\|} < 4.2 \times 10^{-2} \left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$

ϵ_K

$\sqrt{\left\|\text{Im}\left(\hat{\delta}^{LL}_{db}\hat{\delta}^{LL*}_{sb}\right)^2\right\|} < 4.4 \times 10^{-4} \left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\sqrt{\left\|\text{Im}\left(\delta^{LL}_{ds}\right)^2\right\|} < 1.8 \times 10^{-3} \left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$

Natural size of the insertions



- $\mathcal{W}_{A_i k}$ is the mixing between the fermion "Ai" and the k-th sfermion mass eigenstate
- In a general basis in which the quark mass matrix is not necessary diagonal we have:

$$V = U_L^u U_L^{d\dagger} \qquad \mathcal{W} = \begin{pmatrix} U_L & 0 \\ 0 & U_R \end{pmatrix} \mathcal{W}', \quad U_R M U_L^\dagger = \text{diagonal}, \quad \mathcal{W}'^\dagger \mathcal{M}^2 \mathcal{W}' = \text{diagonal}.$$

- We now focus on LL insertion in the down sector. In the hierarchical scenario we have that: $\hat{\delta}^{LL}_{ij} = \mathcal{W}_{i3}\mathcal{W}^\dagger_{3j}$
- In particular $\hat{\delta}^{LL}_{i3} = \mathcal{W}_{i3}\mathcal{W}^\dagger_{33} \approx \mathcal{W}_{i3} = \left(U_L^d \mathcal{W}'_L\right)_{i3}$
- Now we assume that the CKM matrix is dominated by the rotation in the down sector:

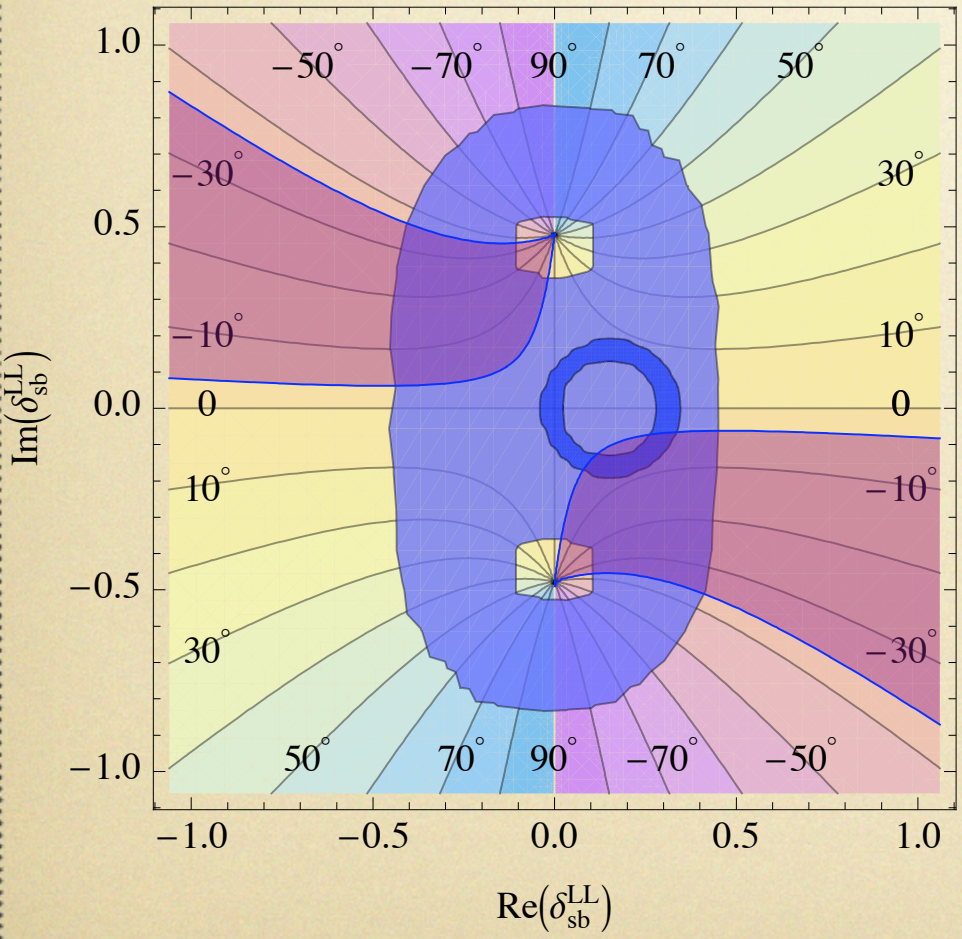
$$\hat{\delta}^{LL}_{i3} \approx (V^\dagger \mathcal{W}'_L)_{i3} = \sum_{k=1}^{3} V^*_{ki} (\mathcal{W}'_L)_{k3}$$

- Barring accidental cancellations:

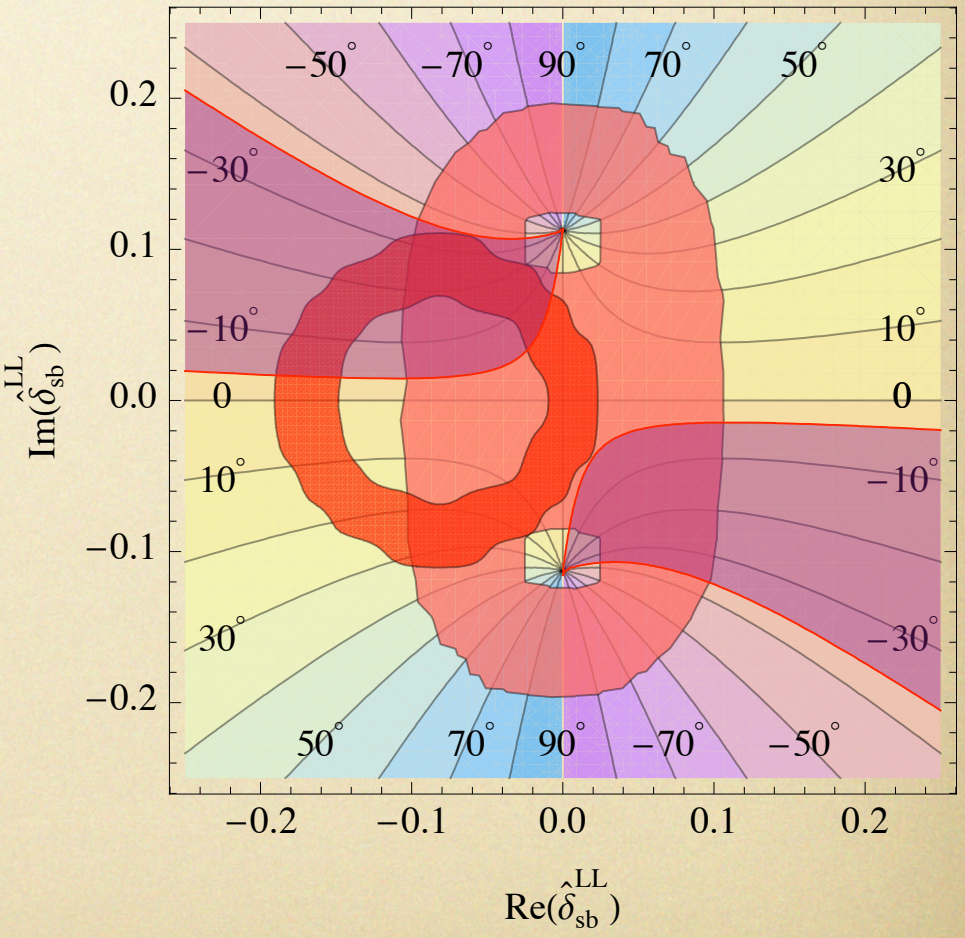
$$\left|\hat{\delta}_{i3}\right| \geq |V^*_{3i}(\mathcal{W}'_L)_{33}| \approx |V^*_{3i}|$$

ΔM_{B_s}, $B \to X_s\gamma$, ϕ_{B_s}

Degenerate Spectrum



Hierarchical Spectrum

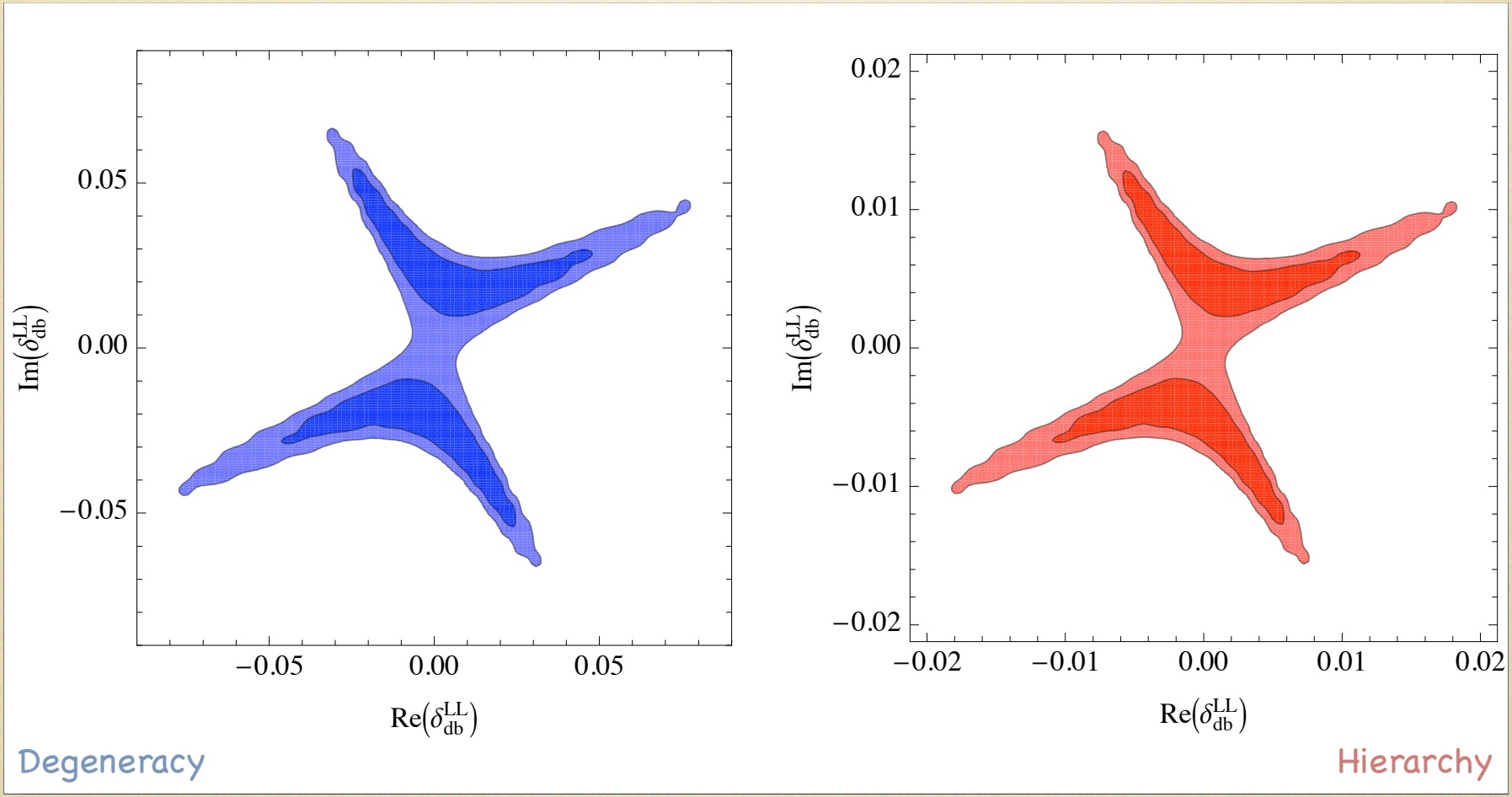


$\tilde{m} = M_{\tilde{g}} = \mu = 350$ GeV, $\tan\beta = 10$, $A = 0$

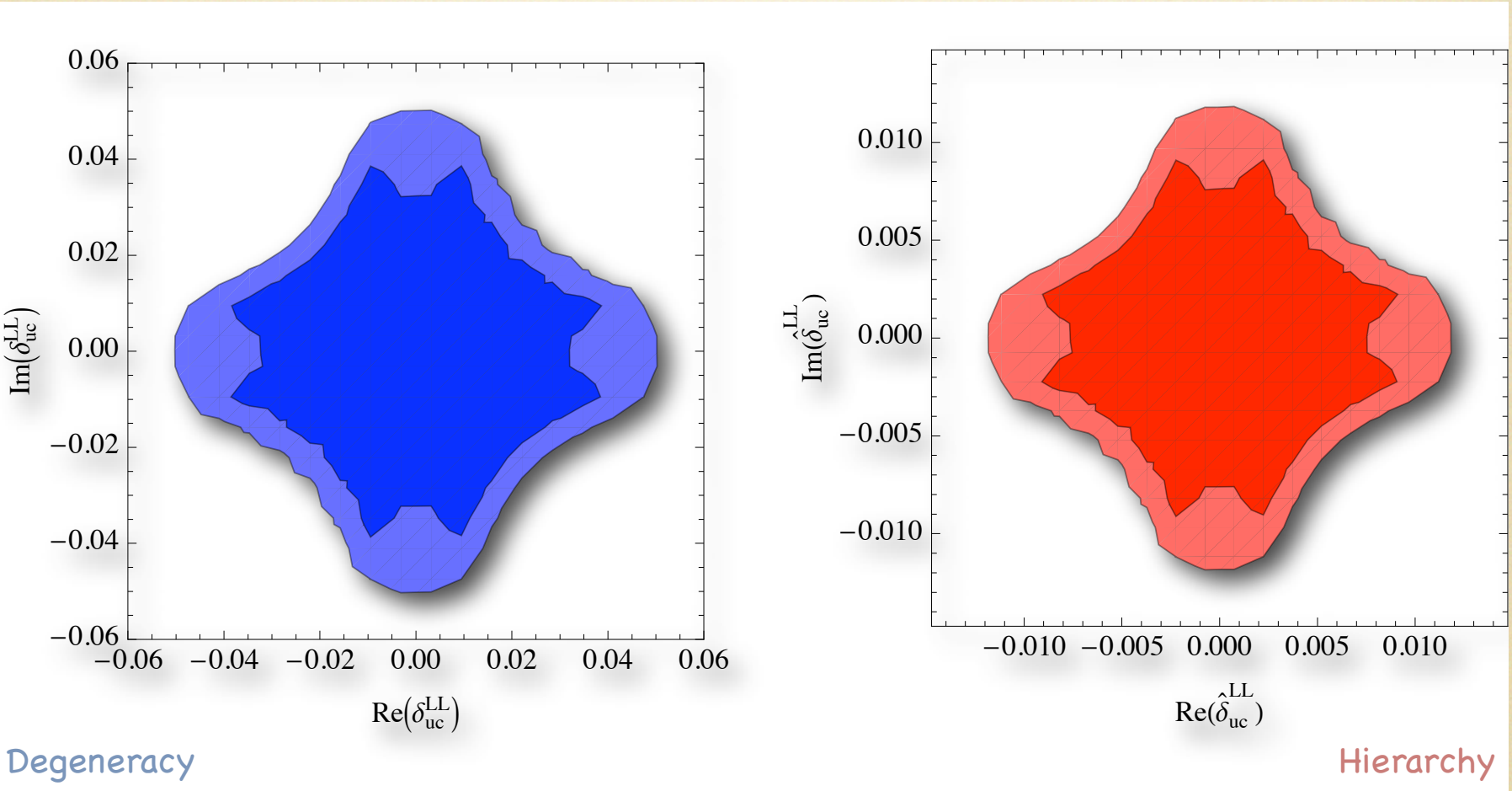
Conclusions

- Heavy first two families are not incompatible with naturalness
- Welcome to alleviate the SUSY flavor problem
- Useful to describe scenarios where the mass separation is moderate but sufficient to make the degenerate assumption a poor starting point.
- Complementary to the degeneracy assumption in a general study of SUSY flavor effects, with predictions different by factor O(1-10)
- Predictions: distinct correlations between different observables.
- Experiments have begun to explore the crucial range of the values for the insertions.

1-3 Transition



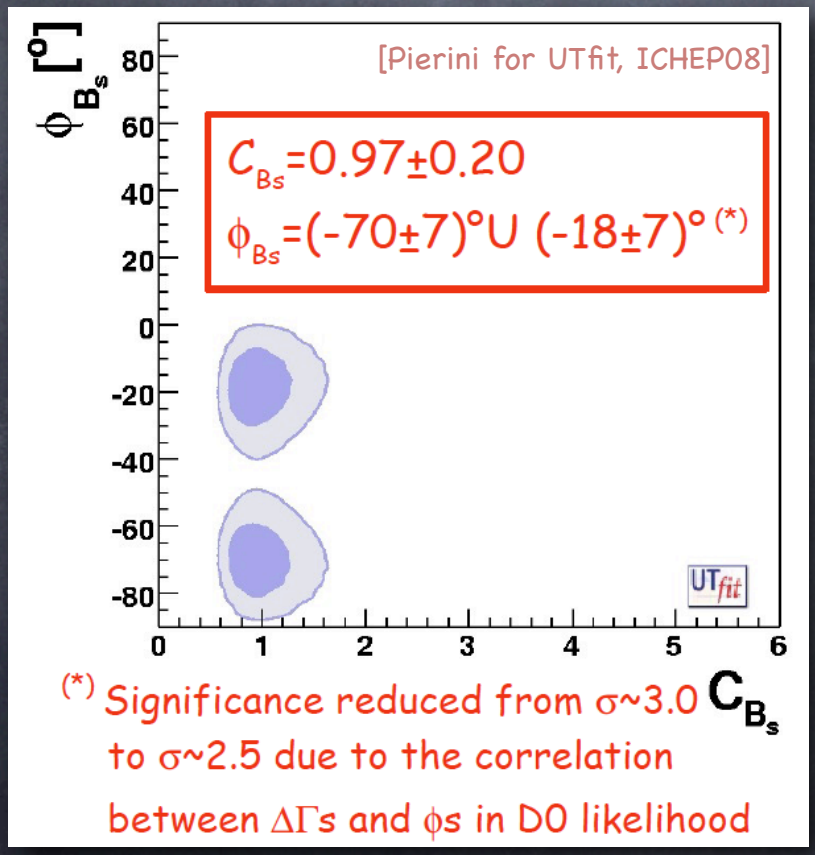
D Mixing

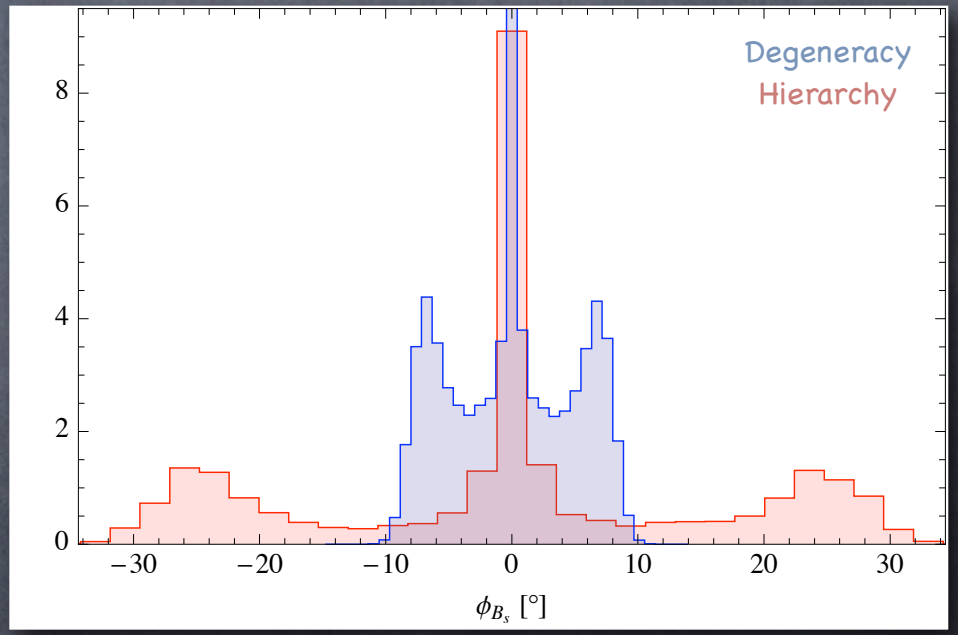

0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.06 −0.04 −0.02 0.00 0.02 0.04 0.06
$\mathrm{Im}(\delta^{LL}_{uc})$
$\mathrm{Re}(\delta^{LL}_{uc})$
0.010
0.005
0.000
−0.005
−0.010
−0.010 −0.005 0.000 0.005 0.010
$\mathrm{Im}(\hat{\delta}^{LL}_{uc})$
$\mathrm{Re}(\hat{\delta}^{LL}_{uc})$
Degeneracy
Hierarchy

ϕ_{Bs}

$$\langle B_s|H_{\rm eff}^{\rm full}|\bar{B}_s\rangle = C_{B_s} e^{2i\phi_{B_s}} \langle B_s|H_{\rm eff}^{\rm SM}|\bar{B}_s\rangle$$

$$\langle B_s|H_{\rm eff}^{\rm SM}|\bar{B}_s\rangle = A_s^{\rm SM} e^{-2i\beta_s} \qquad \beta_s = \arg(-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)) = 0.018 \pm 0.001$$





Inputs

Parameter	Value	Gaussian (σ)	Uniform ($\frac{\Delta}{2}$)	Reference
$\lvert\varepsilon_K\rvert$	2.229×10^{-3}	0.012×10^{-3}	–	[19]
$\Delta m_K\,(\mathrm{ps}^{-1})$	5.292×10^{-3}	0.009×10^{-3}	–	[19]
$\mathrm{BR}(B \to X_s\gamma)$	3.55×10^{-4}	0.26×10^{-4}	–	[20]
$\Delta m_{B_s}\,(\mathrm{ps}^{-1})$	17.77	0.12	–	[19]
$\Delta m_{B_d}\,(\mathrm{ps}^{-1})$	0.507	0.005	–	[19]
$\left\lvert M_{12}^D\right\rvert\,(\mathrm{ps}^{-1})$	7.7×10^{-3}	2.5×10^{-3}	–	[21]
$\bar{\rho}$	0.167	0.051	–	[22]
$\bar{\eta}$	0.386	0.035	–	[22]
λ	0.2255	0.010	–	[19]
$\lvert V_{cb}\rvert$	41.2×10^{-3}	1.1×10^{-3}	–	[19]
$F_K\,(\mathrm{GeV})$	0.160	–	–	[19]
$F_{B_d}\,(\mathrm{MeV})$	189	27	–	[23]
$F_{B_s}\sqrt{B_s}\,(\mathrm{MeV})$	262	35	–	[23]
$F_D\,(\mathrm{MeV})$	201	3	17	[24]
$\hat{B}_K$	0.79	0.04	0.08	[24]
B_1^B	0.88	0.04	0.10	[24]
η_{cc}	0.47	0.04	–	[25]
η_{ct}	0.5765	0.0065	–	[25]
η_{tt}	1.43	0.23	–	[25]
$\overline{m}_t\,(\mathrm{GeV})$	161.2	1.7	–	[24]
$\overline{m}_b\,(\mathrm{GeV})$	4.21	0.08	–	[24]
$\overline{m}_c\,(\mathrm{GeV})$	1.224	0.057	–	[26]

Soft Masses and RGE

soft

$$\mathbf{m}^{\mathbf{2}}_{\tilde{\mathbf{d}}} = \begin{pmatrix} m_Q^2 + m_d^2 + \Delta_{\tilde{d}_L} & v(a_d^* \sin\beta - \mu y_d \cos\beta) \\ v(a_d \sin\beta - \mu^* y_d \cos\beta) & m_{\bar{d}}^2 + m_d^2 + \Delta_{\tilde{d}_R} \end{pmatrix}$$

soft

Roughly speaking, the sfermion mass matrix is dominated by the soft terms, because all the others are suppressed by the EW scale. For the soft terms the RGE are:

First and second family: $$16\pi^2 \frac{d}{dt} m_{\phi_i}^2 = -\sum_{a=1,2,3} 8 C_a(i) g_a^2 |M_a|^2 + \frac{6}{5} Y_i g_1^2 S$$

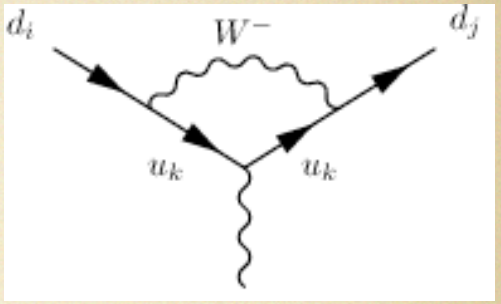
Third family Left: $$16\pi^2 \frac{d}{dt} m_{Q_3}^2 = X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6 g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 + \frac{1}{5} g_1^2 S$$

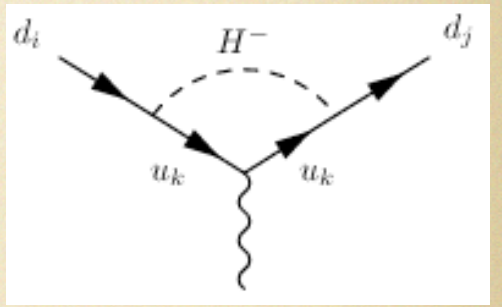
Third family Right: $$16\pi^2 \frac{d}{dt} m_{\bar{d}_3}^2 = 2X_b - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 + \frac{2}{5} g_1^2 S$$

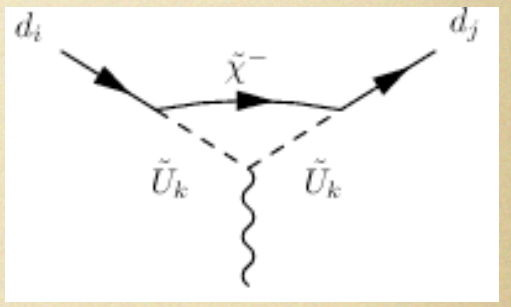
Also if we start with degenerate condition at very high energy, we end up with a split situation because of the Yukawa!!

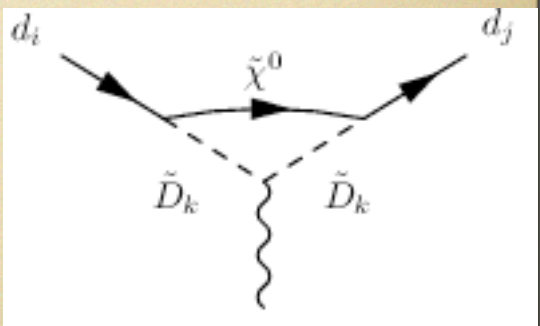
FCNC in the MSSM

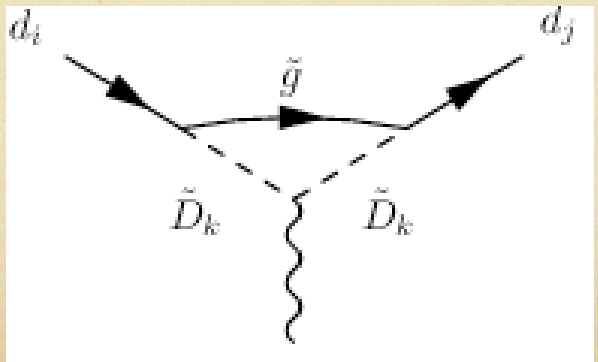
- W and up quarks
- H and up quarks
- Chargino and up squarks
- Neutralino and down squarks
- Gluino and down squarks









Gluino dominance:

- Enhanced by strong coupling
- Gluino does not mix with other particles