

Guido Altarelli and the Rome School of Weak Interactions

-more than 40 years from the QCD improved effective weak Hamiltonian to the Unitary Triangle Fit -

Roma Tre December 19th 2016

Guido Martinelli

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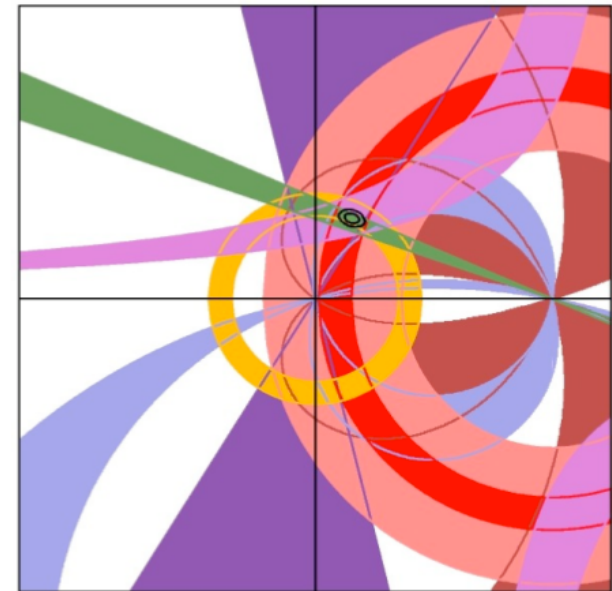
*I am very honoured
for this invitation
to speak at*

*A tribute to the
memory of Guido
Altarelli*



PLAN OF THE TALK

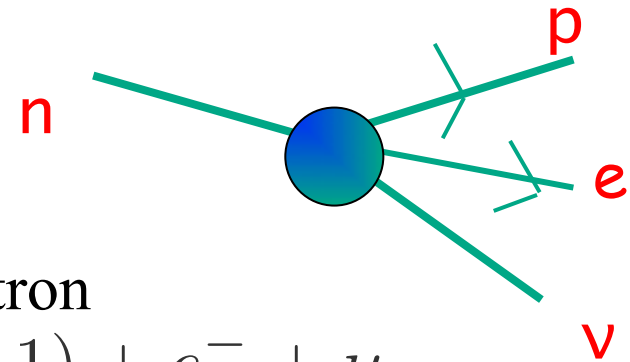
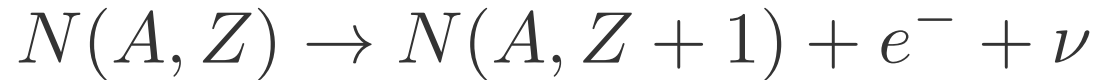
- *The origins: from Fermi to Cabibbo, and then to Altarelli & Maiani;*
- *QCD and Weak Interactions, the first important steps;*
- *My collaboration with Guido: The (first) calculation of the NLO corrections to the Effective Weak Hamiltonian;*
- *The game becomes more complex where we stand now;*
- *Back to Guido A.*
- *Final remarks.*



Some of the slides have been taken from a talk in honour of Guido by
Luigi Maiani @ CERN

The Fermi Theory

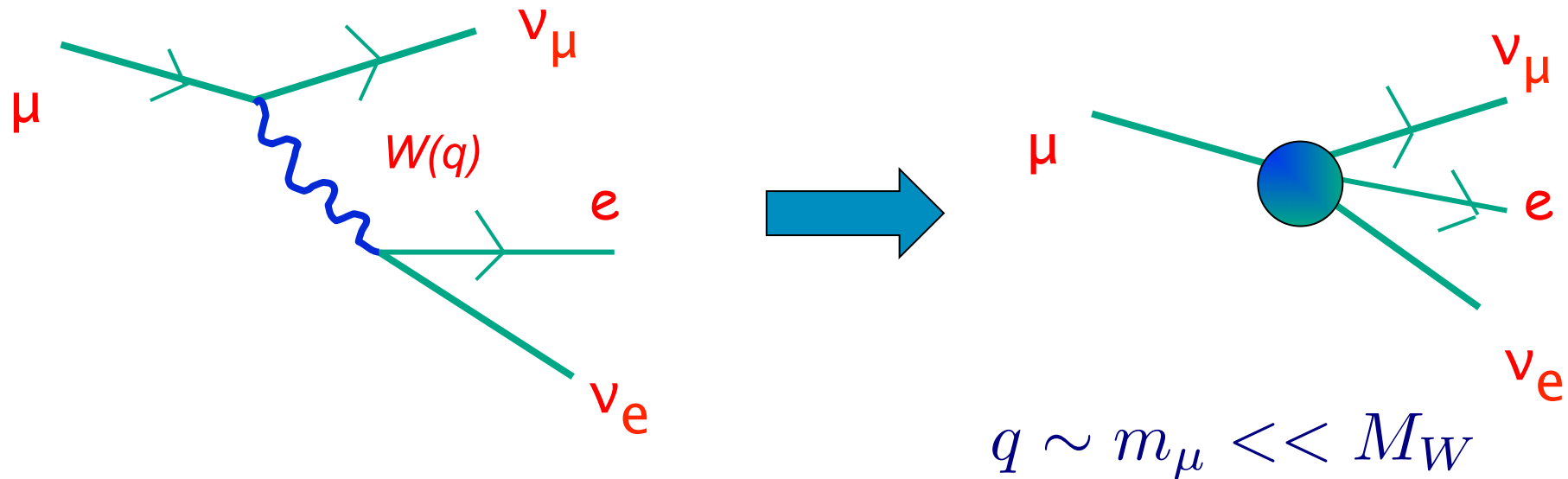
The first quantitative theory of β decays was formulated by Fermi who, following Pauli, assumed that a neutral, unobserved particle, the neutrino, is emitted together with the electron in the process



The interaction is expressed in term of a Hamiltonian given by the product of two terms: the first which induces a transition between the initial and final nucleus and a second which creates the electron-neutrino pair in analogy with the electromagnetic transitions $A^* \rightarrow A + \gamma$ where a photon is created from the vacuum

$$H_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}} (\bar{\psi}_p \Gamma \psi_n) (\bar{\psi}_e \Gamma \psi_\nu)$$

The Fermi Hamiltonian



$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} (\bar{\nu}_\mu \gamma^\rho (1 - \gamma_5) \mu) (\bar{e} \gamma_\rho (1 - \gamma_5) \nu_e)$$

In order to explain the close equality of the muon's and neutron's β -decay Fermi constants, R. Feynman and M. Gell-Mann's proposed the "universality" of weak interactions, mediated by vector currents, closely similar to the universality of the electric charge: a tantalising hint of a common origin of the two interactions

Overture

Twenty Years (* 2) After

Nicola Cabibbo

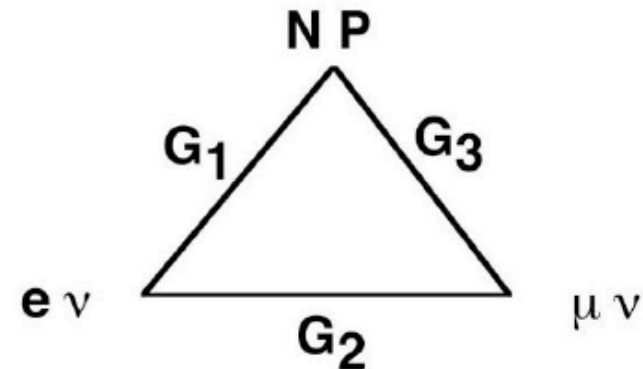
*Original trasparencies by Cabibbo (2003)
translated by G.M.*

Universality of Weak Interactions

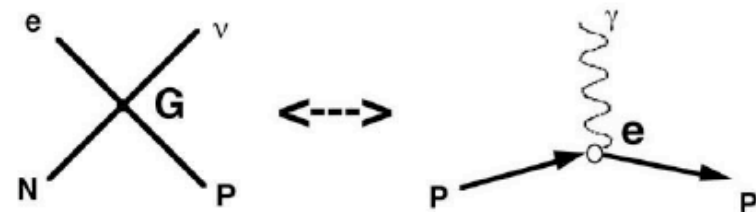
- 1 - $N \rightarrow P + e + \bar{\nu}$ G_1
- 2 - $\mu \rightarrow e + \nu + \bar{\nu}$ G_2
- 3 - $\mu^- + P \rightarrow N + \nu$ G_3

In ~ 1950 $G_1 \approx G_2 \approx G_3$

The Puppi Triangle



Suggestive in view of Fermi's idea:



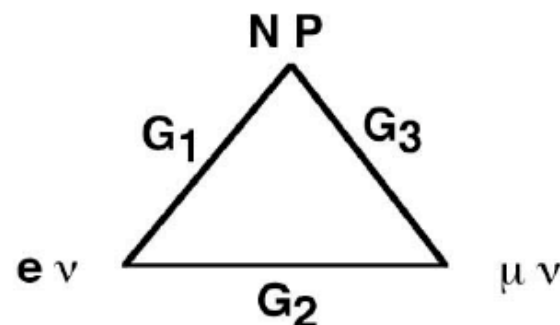
Universality of Weak Interactions 1962-63

$$G_1 \approx G_2 \approx G_3$$

Suggestive, but true?

$$G_{\text{beta decay}} \approx 0.96 G_{\mu \text{ decay}}$$

(Significant Difference)



$$4 - \Lambda \rightarrow P + e + \bar{\nu} \quad G_4$$

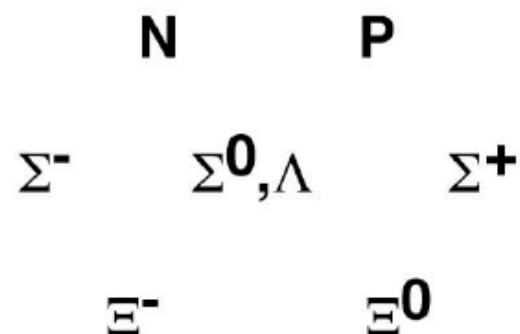
And for strange particle decays....

$$G_4 \approx 0.2 G_{\text{m decay}}$$

Universality of Weak Interactions 1962-63

Towards a solution:

- 1) Gell-Mann's SU(3) symmetry
and its application to weak
transitions.
(N.C. + R. Gatto 1962)
- 2) High statistics (for that time)
bubble chamber experiments.
(V. Soergel, Filthut, P. Franzini,
G. Snow, etc.)



Universality and weak mixing

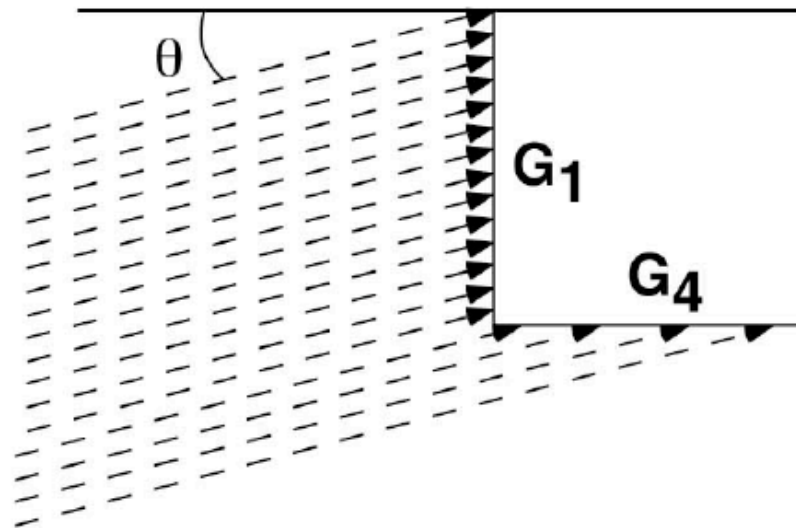
$$N \rightarrow P + e^- + \nu \quad G_1 \approx 0.96 G_{\mu\text{-decay}}$$

$$\Lambda \rightarrow P + e^- + \nu \quad G_4 \approx 0.2 G_{\mu\text{-decay}}$$

Broken Universality?
no, shared intensity

$$G_1 = \cos\theta G_{\mu\text{-decay}}$$

$$G_4 = \sin\theta G_{\mu\text{-decay}}$$



$$\theta \approx 0.2 \text{ (today 0.221)}$$

Nicola found the solution to the puzzle of strange particle weak decays while in CERN, Geneva. He formulated what came to be known as “Cabibbo universality”, in terms of the partially conserved currents associated to the Unitary Symmetry, SU3, recently discovered by Gell-Mann and by Yuval Ne’eman, and of the axial currents associated with the chiral extension, SU3xSU3. He assumed that strangeness changing and non-changing beta decays had to be described by a single hadron weak current, the orthogonal combination of the corresponding SU3xSU3 currents, determined by a single unknown parameter, the Cabibbo angle *L. Maiani Nature 2010*

The Weak Current

According to the proposal of the 1963 (by Cabibbo) the weak current belongs to an octet of currents, J_α^i

$$J_\alpha = \cos\theta_c(J_\alpha^1 + iJ_\alpha^2) = \sin\theta_c(J_\alpha^4 + iJ_\alpha^5)$$

which in terms of the quarks, proposed in 1964 by Gell-Mann and Zweig, are written as

$$J_\alpha = \cos\theta_c(\bar{u}\gamma_\alpha(1 - \gamma_5)d) = \sin\theta_c(\bar{u}\gamma_\alpha(1 - \gamma_5)s)$$

It is then possible to obtain relations between strangeness conserving and strangeness violating processes.

The vectorial part of the weak current belongs to the same octet of the electromagnetic current. Its matrix elements between mesons and baryons are uniquely determined. This, obviously, if we neglect the mass difference between the strange and down quark, in the limit of exact $SU(3)$.

With a value of $\sin\theta\approx 0.22$ and the use of unitary symmetry, Cabibbo could describe the beta decays of strange mesons and baryons as well as explain the small discrepancy of the neutron and muon Fermi constants, the former being about 2.5% smaller than the latter.

The discrepancy had been noticed already by Feynman and was being just confirmed by an accurate experiment performed by Valentino Telegdi in Chicago

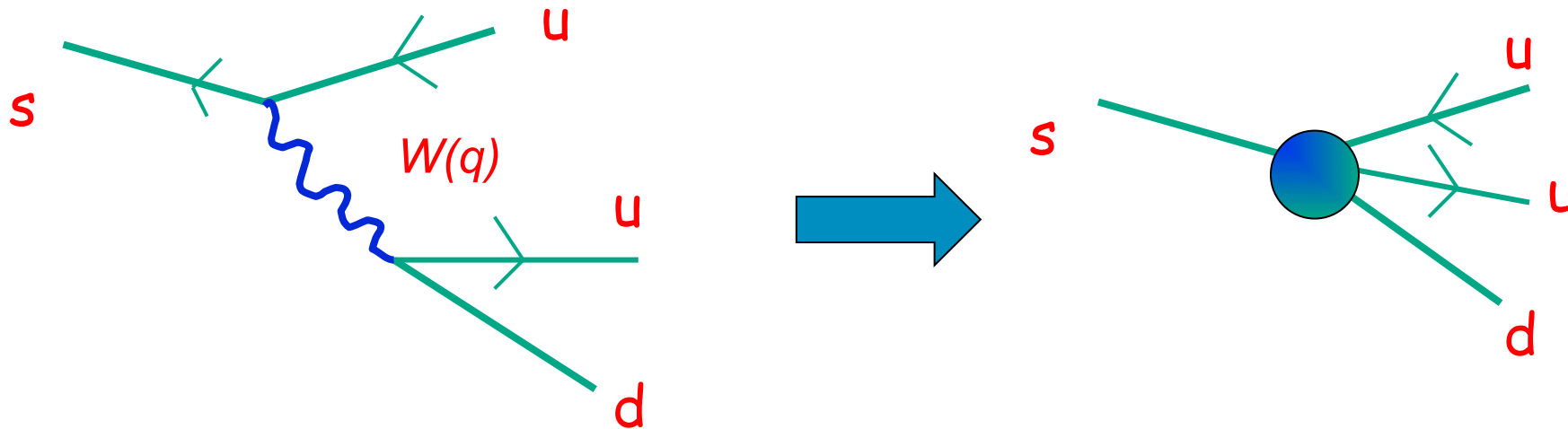
Later, Cabibbo reformulated the same concept in the quark model, as the fact that the weak interaction couples the “up” quark to an orthogonal combination of the “down” and “strange” quarks determined by the angle θ previously introduced

The Development of the Standard Model

- *1954 Yang and Mills Non abelian gauge theories*
- *1961 Glashow $SU(2) \times U(1)$*
- *1964 Brout & Englert + Higgs*
- *1967 Weinberg + 1968 Salam Standard Model with all the ingredients*
- *1971 t'Hooft and Veltman Renormalizability of Weinberg-Salam*
- *1972 Bouchiat, Iliopoulos and Meyer Adler anomalies cured by leptons and fractionally charged colored quarks*
- *1973 Gross, Wilczek and Politzer Asymptotic Freedom and $SU(3)$ -QCD as the theory of strong interactions*

QCD EFFECTS IN WEAK INTERACTIONS

The Effective Hamiltonian



$$q \sim m_K \ll M_W$$

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (\bar{s} \gamma_\mu (1 - \gamma_5) u) (\bar{u} \gamma^\mu (1 - \gamma_5) d)$$

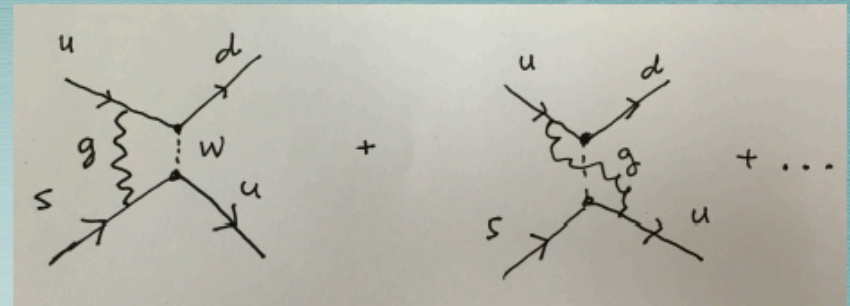
+ strong interactions

QCD Renormalization of 4-fermion operators, 1974

- The octet (or $\Delta I=1/2$) enhancement is a prominent feature of the non leptonic decays
 - the product of the Cabibbo currents for $d \rightarrow u$ ($I=1$) and $s \rightarrow u$ ($I=1/2$) should lead to a balanced mixture of $1/2$ and $3/2$, while the lifetimes of K_S ($\Delta I=1/2$) is much shorter than the lifetime of K^+ ($\Delta I=3/2$)
- Ken Wilson (1969) had noted that the strong interactions, which respect Isospin conservation, could renormalise differently the two components, however, without a theory of the strong interactions he could not test the idea
- But what about QCD?
- Gluons could be exchanged up to momenta of the order of M_W , and perturbation theory would give predictable renormalization effects of order $[\alpha_s \gamma \log(M_W/\mu)]^n$, which would add up to factors of $(M_W/\mu)^d$, with some anomalous dimension d ;
- with the scale of K decays $\mu \ll M_W$, the enhancement could be sizeable for $d > 0$

QCD Renormalization of 4-fermion operators, 1974

- How can flavor-blind QCD tell isospin 1/2 from isospin 3/2?
- answer came from an old Feynman observation: if quarks were bosons, the Fermi interaction of non leptonic would be pure $\Delta I=1/2$
- proof:
 - Fierz rearrangement exchanges $u \leftrightarrow d$
 - the Fierz of Dirac matrices gives -1
 - field exchange gives +1(boson) or -1(fermion)
 - with bosons we get -1, i.e the pair ud is in $I=0$, the operator has $I=1/2$



four fermion operator ($\Delta S = -1$) =
 $\bar{s}\gamma_\mu(1 - \gamma_5)u \times \bar{u}\gamma^\mu(1 - \gamma_5)d$
 with color :=
 $\bar{s}^\alpha\gamma_\mu(1 - \gamma_5)u_\alpha \times \bar{u}^\beta\gamma^\mu(1 - \gamma_5)d_\beta$

- with coloured quarks we have to exchange also: $\alpha \leftrightarrow \beta$
 - QCD renormalizes differently color symmetric and color antisymmetric
 - color antisymmetric gets an additional -1 \Rightarrow ud pair has $I=0$
 - we found that the anomalous dimensions in QCD *enhance the color antisymmetric and suppress the symmetric combination !!!!*

**OCTET ENHANCEMENT OF NON-LEPTONIC WEAK INTERACTIONS
IN ASYMPTOTICALLY FREE GAUGE THEORIES**

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Received 22 June 1974

Octet enhancement of weak non leptonic amplitudes is found to occur in asymptotically free gauge theories of strong interactions, combined with unified weak and e.m. interactions. The order of magnitude of the enhancement factor for different models is discussed.

$$\mathcal{A}^{\Delta S=1}_{FI} (2\pi^4) \delta^4(p_F - p_I) = \text{tadpoles} + (\text{Higgs scalar exchange}) + \int d^4x d^4y D_{\mu\nu}(x, M_W) \langle F | T[J_\mu(y+x/2) J^\dagger_\nu(y-x/2)] | I \rangle$$

1) *Tadpoles cannot give any contribution;*

2) *Higgs contribution suppressed as m^2/M^2_W*

$$\langle F | \mathcal{H}^{\Delta S=1} | I \rangle = G_F/\sqrt{2} V_{ud} V_{us}^* \sum_i C_i(\mu) \langle F | Q_i(\mu) | I \rangle$$

WILSON OPE

$(M_W)^{di-6}$

$\Delta I = \frac{1}{2}$ Rule for Nonleptonic Decays in Asymptotically Free Field Theories

M. K. Gaillard* and Benjamin W. Lee†

National Accelerator Laboratory, Batavia, Illinois 60510

(Received 10 April 1974)

The effective nonleptonic weak interaction is examined assuming the Weinberg-Salam theory of weak interactions and an exactly-conserved-color gauge symmetry for strong interactions. It is shown that the octet part of the nonleptonic weak interaction is more singular at short distances than the 27 part. The resulting enhancement of the octet term in the effective local weak Lagrangian, together with suggested mechanisms for the suppression of matrix elements of the 27 operator, may be sufficient to account for the observed $|\Delta I| = \frac{1}{2}$ rule.

Wilson OPE

$$\mathcal{A}_W \approx \alpha M_W^{-2} \sum_k C_k [\ln(M_W^2/m^2)]^{dk} \langle F | Q_k(0) | I \rangle + \dots$$



Anomalous dimension
of the operator Q_k

“The OPE shows that the amplitude is dominated by the matrix elements of those operators with $dk > 0$ thus giving rise to a possible mechanism to enhance contributions with definite quantum numbers, e.g. $\Delta I=1/2$ vs $\Delta I=3/2$ as first suggested by Wilson”

$$O_L^1 = \bar{\psi} \gamma_\mu L^+ (1 + \gamma_5) \psi \bar{\psi} \gamma^\mu L^- (1 + \gamma_5) \psi$$

$$O_L^2 = \bar{\psi} \gamma_\mu L^+ (1 + \gamma_5) t^A \psi \bar{\psi} \gamma^\mu L^- (1 + \gamma_5) t^A \psi$$

$$O_R^1 = \bar{\psi} \gamma_\mu R^+ (1 - \gamma_5) \psi \bar{\psi} \gamma^\mu R^- (1 - \gamma_5) \psi$$

$$O_R^2 = \bar{\psi} \gamma_\mu R^+ (1 - \gamma_5) t^A \psi \bar{\psi} \gamma^\mu R^- (1 - \gamma_5) t^A \psi$$

$$O_{LR}^1 = \bar{\psi} \gamma_\mu L^+ (1 + \gamma_5) \psi \bar{\psi} \gamma^\mu R^- (1 - \gamma_5) \psi$$

$$O_{LR}^2 = \bar{\psi} \gamma_\mu L^+ (1 + \gamma_5) t^A \psi \bar{\psi} \gamma^\mu R^- (1 - \gamma_5) t^A \psi$$

(4) Definition of the Operators
 Note the perversion:
 (5) $(1 + \gamma_5)$ is left-handed

$$O_L^\pm = \frac{N \pm 1}{N} O_L^1 \pm \frac{1}{2} O_L^2; \quad d_L^\pm = \frac{1}{2b} \left(\frac{3}{8\pi^2} \right) \left(\mp \frac{N \mp 1}{N} \right) \quad (7)$$

same for O_R^\pm , $d_R^\pm = d_L^\pm$, and

$$\delta_{LR}^1 = -\frac{N^2 - 1}{N} O_{LR}^1 + \frac{1}{2} O_{LR}^2;$$

$$\delta_{LR}^2 = \frac{1}{N} O_{LR}^1 + \frac{1}{2} O_{LR}^2;$$

$$d_{LR}^1 = \frac{1}{2b} \left(\frac{3}{8\pi^2} \right) \left(-\frac{1}{N} \right);$$

$$d_{LR}^2 = \frac{1}{2b} \left(\frac{3}{8\pi^2} \right) \left(\frac{N^2 - 1}{N} \right),$$

(8)

First calculation of the LO anomalous dims:
 $\Delta I = 1/2$ dynamically enhanced
 although only qualitatively successful

WEAK INTERACTIONS PHENOMENOLOGY WAS IMPROVING AT A FAST PACE

1. Better and better data on charm production and semileptonic non-leptonic decays (1)
2. The bottom quark was discovered in 1977 and its properties & decays started to be intensively studied
3. The beginning of the Heavy Quark (Effective) Theory (2)

**ENHANCEMENT OF NON-LEPTONIC DECAYS
OF CHARMED PARTICLES**

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Received 14 October 1974

The enhancement of non-leptonic rate due to QCD corrections improved agreement of the prediction of the semileptonic branching ratio with data

Calculations of semileptonic branching ratios were done in the “parton model” i.e. using the free particle

Search for charm

Mary K. Gaillard* and Benjamin W. Lee

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Jonathan L. Rosner

University of Minnesota, Minneapolis, Minnesota 55455

A systematic discussion of the phenomenology of charmed particles is presented with an eye to experimental searches for these states. We begin with an attempt to clarify the theoretical framework for charm. We then discuss the $SU(4)$ spectroscopy of the lowest lying baryon and meson states, their masses, decay modes, lifetimes, and various production mechanisms. We also present a brief discussion of searches for short-lived tracks. Our discussion is largely based on intuition gained from the familiar—but not necessarily understood—phenomenology of known hadrons, and predictions must be interpreted only as guidelines for experimenters.

- [7] B.W. Lee, M.K. Gaillard and G. Rosner, *Rev. Mod. Phys.* 47 (1975) 277;
G. Altarelli, N. Cabibbo and L. Maiani, *Nucl. Phys.* B88 (1975) 285; *Phys. Lett.* 57B (1975) 277
S.R. Kingsley, S. Treiman, F. Wilczek and A. Zee, *Phys. Rev.* D11 (1975) 1914;
J. Ellis, M.K. Gaillard and D. Nanopoulos, *Nucl. Phys.* B100 (1975) 313

THE LIFETIME OF CHARMED PARTICLES

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Received 10 July 1978

We present a computation of the semileptonic decay rate of charmed particles, including the first order gluon corrections and the final quark mass corrections. Taking into account these corrections, the lifetime of charmed particles is estimated to be: $\tau \approx 0.7 \times 10^{-12}$ s.

Ph
-F

The infancy of the Heavy Quark Effective Theory

*just after I came back
from CERN in 1982*

LEPTONIC DECAY OF HEAVY FLAVORS: A theoretical update

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Received 29 June 1982

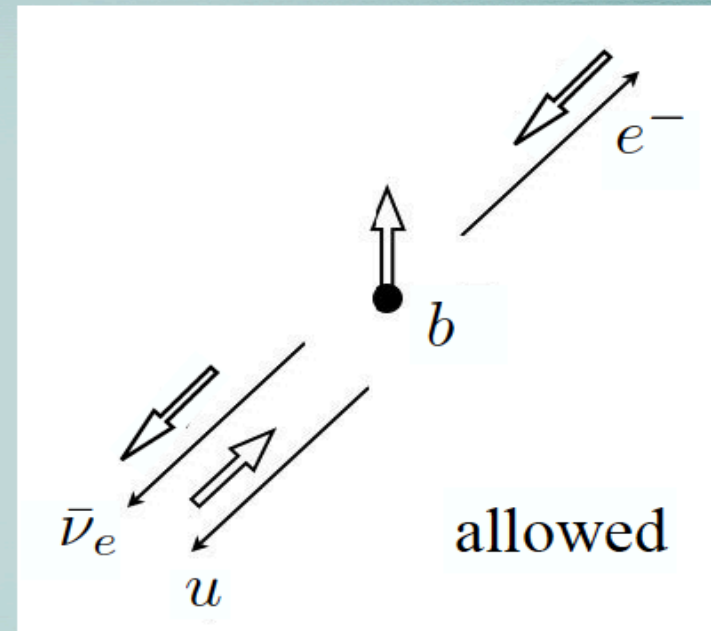
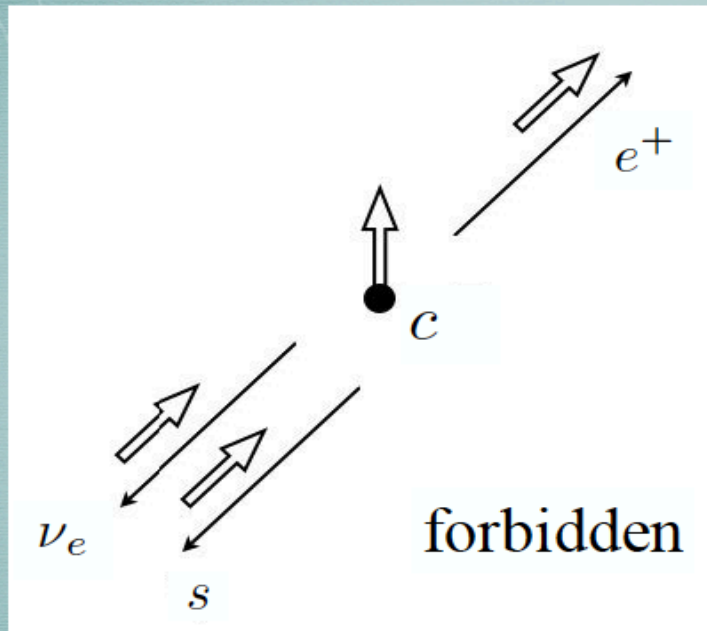
The ``naive'' ancestor of
of the HQET shape
function for semileptonic
and radiative decays

It contains, however, up
to a redefinition of the
non perturbative
parameters, the main
features of the modern
theory

Semileptonic decays of c vs. b quarks

Maiani

Charged lepton energy end point configurations in c and b decay



- However Paolo Franzini (then still in Cornell with CLEO) observed that the lepton end point in b decay corresponds to small hadron masses and therefore non perturbative corrections come in.

- The two Guidos, Altarelli and Martinelli, came in, with the crucial resummation of the perturbative terms and the result provided a valuable tool in the estimate of V_{ub} from inclusive rates

G. Altarelli, N. Cabibbo, G. Corbo, L. Maiani and G. Martinelli, *Leptonic Decay of Heavy Flavors: A Theoretical Update*, Nucl. Phys. B 208 (1982) 365.

Fit of the parameters from The lepton spectrum of D decays

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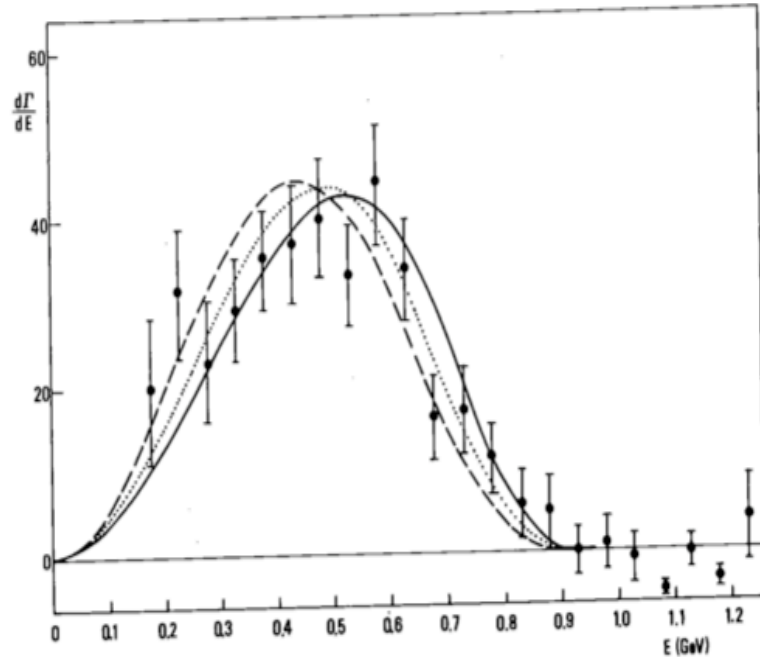
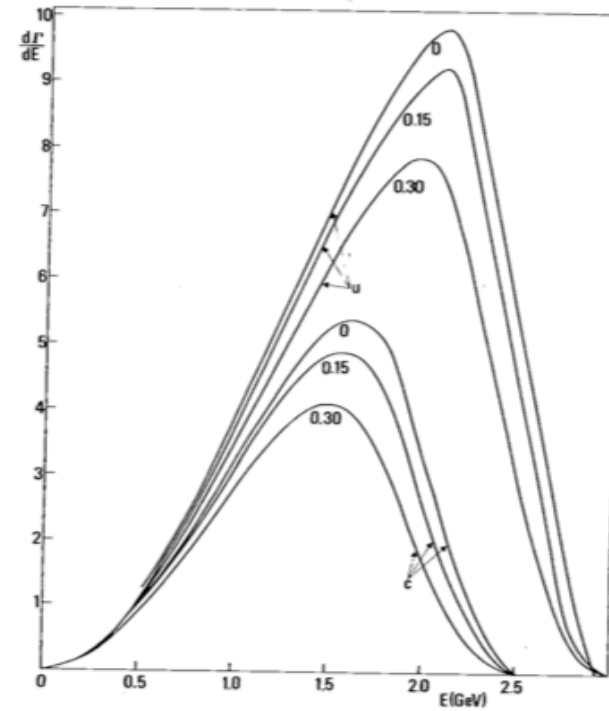


Fig. 3. Charged lepton spectrum in D decay for $M_D = 1.866$ GeV, $m = 0.3$ GeV, $m_{sp} = 0.15$ GeV, $\alpha_s = 0.38$, $P_B = 0.26$ GeV and $P_F = 0$ (solid), $P_F = 0.15$ GeV (dotted), $P_F = 0.3$ GeV (dashed). The normalization is fixed to the number of events.



6. $d\tilde{\Gamma}_{em}/dE$ for B meson decay for $M_B = 5.218$ GeV, $m_u = 0.15$ GeV, $m_c = 1.7$ GeV, $m_{sp} = 0.24$ GeV, various values of P_F as indicated (in GeV) and $P_B = 0.76$ GeV. The absolute scale is arbitrary, but the relative normalizations are correct.

comparing our predictions with the spectra obtained in $e^+e^- \rightarrow Y^m \rightarrow B\bar{B}$, the largest uncertainty, at present, seems to arise from the poor determination of the B mass, i.e. of the B momentum at a given value of the beam energy. The present bounds on the B-meson mass are [14]

$$5.162 \text{ GeV} \leq M_B \leq 5.275 \text{ GeV}, \quad (42)$$

Prediction of the spectrum in B decays for $b \rightarrow c$ and $b \rightarrow u$

5279.17 ± 0.29 PDG FIT

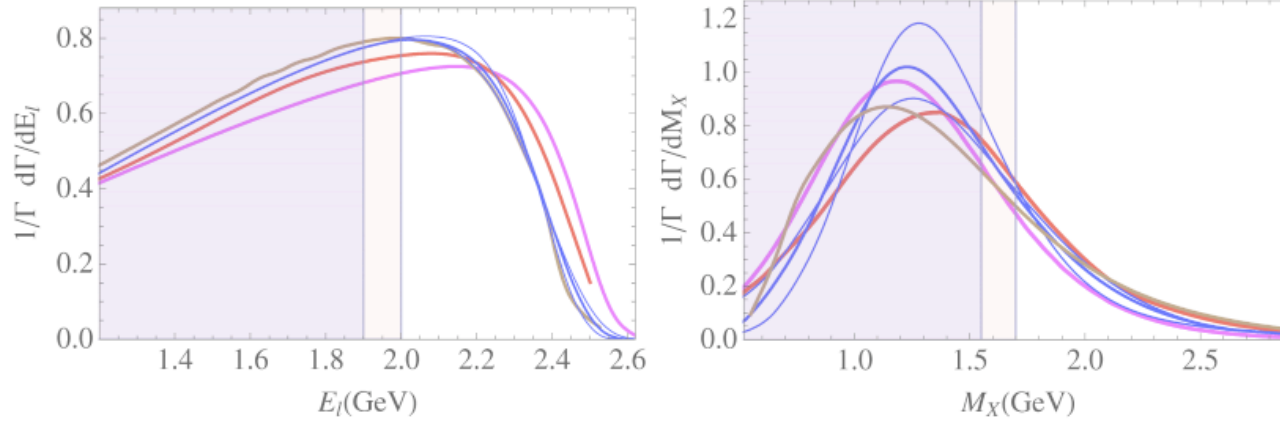


Fig. 38. Comparison of different theoretical treatments of inclusive $b \rightarrow u$ transitions: (a) E_l spectrum; (b) M_X spectrum. Red, magenta, brown and blue lines refer, respectively, to DGE, ADFR, BLNP, GGOU with a sample of three different functional forms. The actual experimental cuts at $E_l = 1.9, 2.0$ GeV and $M_X = 1.55, 1.7$ GeV are also indicated.

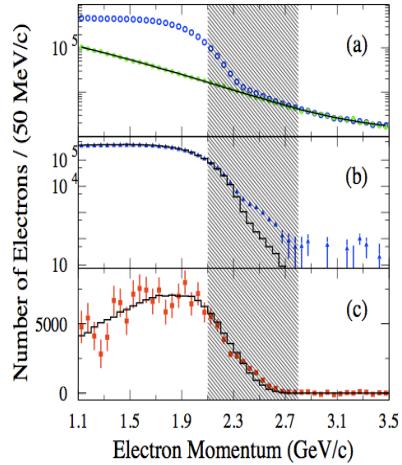


Fig. 40. The inclusive electron energy spectrum [594] from BaBar is shown for (a) on-peak data and q^2 continuum (histogram); (b) data subtracted for non- $B\bar{B}$ contributions (points) and the simulated contribution from B decays other than $b \rightarrow ul\nu$ (histogram); and (c) background-subtracted data (points) with a model of the $b \rightarrow ul\nu$ spectrum (histogram).

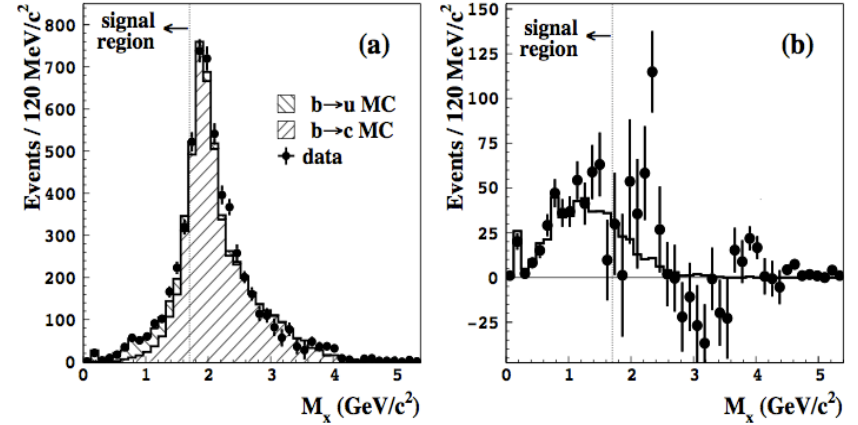


Fig. 41. The hadronic invariant mass spectrum [595] in Belle data (points) is shown in (a) with histograms corresponding to the fitted contributions from $b \rightarrow cl\nu$ and $b \rightarrow ul\nu$. After subtracting the expected contribution from $b \rightarrow cl\nu$, the data (points) are compared to a model $b \rightarrow ul\nu$ spectrum (histogram) in (b).

How Altarelli remembered that period ...

After the Gross-Wilczek and Politzer papers we immediately turned to study the potentiality of QCD for improving the parton model. Myself and Maiani we decided to study the QCD corrections to the effective weak non-leptonic Hamiltonian, written as a Wilson expansion in terms of 4-quark operators of the $(V-A)_x(V-A)$ type obtained by integrating away the W^\pm exchange [18]. The logarithmically enhanced terms of the QCD corrections are fixed by the anomalous dimensions of these operators, much in the same way as the moments of structure functions get logarithmic corrections as computed by Gross et al [2, 3] from the anomalous dimensions of the leading-twist operators in the light-cone expansion. Our hope was to find that the QCD corrections act in the direction of enhancing the $\Delta T = 1/2$ operators with respect to those with $\Delta T = 3/2$, thus explaining, at least in part, the empirical $\Delta T = 1/2$ rule (where T is the isotopic spin). The explicit calculation turned out to lead to precisely this result, as also obtained in a simultaneous work by M. K. Gaillard and B. W. Lee [19] (actually these authors had pointed out to us the crucial role of charm in this problem). These important papers were the first calculations of the QCD corrections to the coefficients of the Wilson expansion in the product of two weak currents, an approach that, suitably generalised (by considering other weak processes) and improved (for example, by computing the anomalous dimensions beyond the leading order), still represents a basic tool in this field. In the following months we applied the method to charm decays [20], before the discovery of charm, and to weak neutral current processes [21]. To this last paper also contributed Keith Ellis, a scottish PhD student of Cabibbo, who was to stay with us in Rome for a few years, eventually speaking a very good italian and fully understanding the roman way of living. Later, in '81 myself with Curci (who, unfortunately, is no more with us), Martinelli and Petrarca [22] we computed the two-loop anomalous dimensions for the operators of the effective weak non-leptonic Hamiltonian.

The (first) calculation of the NLO corrections to the Effective Weak Hamiltonian

The physical motivations for a NLO calculation

For heavy quark decay (especially for charm) a substantial increase in the non-leptonic width is obtained, which leads to a prediction [7] for the (quark) semileptonic branching ratio B^{SL} , which is considerably smaller than the free field value. For charm, the prediction in the LLA is typically $B^{\text{SL}} \approx 13\text{--}16\%$ as compared with the free field value of $\sim 20\%$. Until recently, the results for a charm (c) quark

with real gluon emission [9]. However, the c quark decay prediction should remain essentially valid for D^+ (provided the spectator is really inert [10]) because, in D^+ , the annihilation process can only occur at the Cabibbo suppressed level. Since a value of B^{SL} for D^+ close to 20% is being currently reported [8] it is important to verify whether or not the LLA is supported by a study of the next to leading corrections.

In order to investigate these matters we computed the first non-leading QCD corrections to the effective weak non-leptonic hamiltonian (a summary of our results has already been published elsewhere [11]). The main ingredients for this calculation

Further Motivations:

$$\mathcal{A}_{\text{FI}} (2\pi^4) \delta^4 (p_{\text{F}} - p_{\text{I}}) =$$

$$\int d^4x d^4y D_{\mu\nu}(x, M_{\text{W}}) \langle F | T[J_{\mu}(y+x/2) J_{\nu}^{\dagger}(y-x/2)] | I \rangle$$


$$\langle F | \mathcal{H}^{\Delta S=1} | I \rangle = G_{\text{F}}/\sqrt{2} V_{\text{ud}} V_{\text{us}}^* \sum_i C_i(\mu) \frac{\langle F | Q_i(\mu) | I \rangle}{(M_{\text{W}})^{\text{di}-6}}$$

di= dimension of the operator $Q_i(\mu)$

$C_i(\mu)$ Wilson coefficient: it depends on M_{W}/μ and $\alpha_{\text{W}}(\mu)$

$Q_i(\mu)$ local operator renormalized at the scale μ

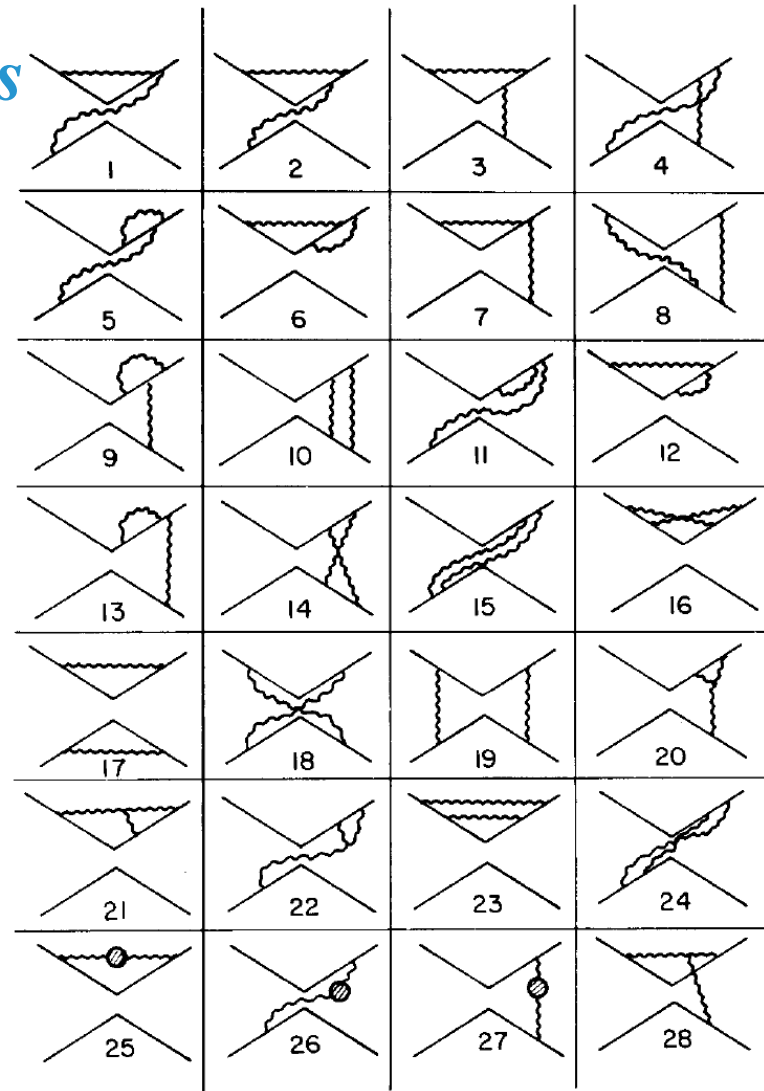
Without the next-to-leading corrections it is impossible to fix the renormalization scale and to match consistently the Wilson coefficients to the matrix elements of the (lattice) operators (see also citation from Buras *)

No penguin diagrams necessary for the charm calculation

466

G. Altarelli et al. / Corrections to weak decays

*Letters exchanges
between the
CERN team
(G. Curci & GM)
And the Rome
Team
(G. Altarelli and
S. Petrarca)*



*Occasionally
some mistake
was found*

Fig. 2. The 28 independent two-loop diagrams for the anomalous dimension of the four-fermion operators of dimension six. Replicas differing by up-down, left-right reflections of diagrams are not shown. "Penguin" like diagrams are absent in the massless theory. They are irrelevant for transition involving four different flavours as in $c \rightarrow s\bar{d}u$.

We were scared of using Naive Dimensional Regularization (NDR) in the presence of chiral currents (γ_5) and decided to use Dimensional Reduction (we were really naïve!!)

Volume 148B, number 1,2,3

PHYSICS LETTERS

22 November 1984

**CONSISTENCY BETWEEN DIFFERENT DIMENSIONAL REGULARIZATIONS
IN TWO-LOOP CALCULATIONS FOR SUPERSYMMETRIC GAUGE THEORIES**

G. CURCI and G. PAFFUTI

*Istituto di Fisica, Università di Pisa, Pisa, Italy
and INFN, Sezione di Pisa, Pisa, Italy*

Received 6 August 1984

We show that dimensional regularization and dimensional reduction are consistent up to two-loop in susy gauge theories. No anomalies are found for supersymmetry at two-loop level.

Recently Van Damme and 't Hooft [1] have raised the problem of compatibility between standard dimensional regularization (DR) [2] and the dimensional reduction scheme (SDR) [3] in supersymmetric gauge theories.

A convenient device to perform calculations for the $N = 1, 2, 4$ models at once is offered by the formalism of ref. [4] used for similar computations in ref. [5].

Let us consider the Yang–Mills theory in D dimensions with fermions in the adjoint representation

Climbing NLO and NNLO Summits of Weak Decays

Andrzej J. Buras *arXiv:1102.5650v4*

In 1981 Guido (M.) took part in the pioneering calculation of the two loop anomalous dimensions of the current-current operators. This calculation done in collaboration with Guido Altarelli, Giuseppe Curci and Silvano Petrarca has been unfortunately performed in the dimensional reduction scheme (DRED) that was not familiar to most phenomenologists and its complicated structure discussed in detail by these authors most probably scared many from checking their results. Moreover it was known that the treatment of γ_5 in the DRED scheme, similarly to the dimensional regularization scheme with anticommuting γ_5 (known presently as the NDR scheme), may lead to mathematically inconsistent results.

Consequently it was not clear in 1988 whether the result of Altarelli et al. was really correct.

The calculation by Buras & Weiz, in NDR and DRED, of the NLO corrections to KK bar mixing confirmed our results and demonstrated that the calculation could have been done in NDR as well.

Further Motivations & Recent Developments

$$\mathcal{A}_{\text{FI}} (2\pi^4) \delta^4 (p_{\text{F}} - p_{\text{I}}) =$$

$$\int d^4x d^4y D_{\mu\nu}(x, M_{\text{W}}) \langle F | T[J_{\mu}(y+x/2) J_{\nu}^{\dagger}(y-x/2)] | I \rangle$$


$$\langle F | \mathcal{H}^{\Delta S=1} | I \rangle = G_{\text{F}}/\sqrt{2} V_{\text{ud}} V_{\text{us}}^* \sum_i C_i(\mu) \frac{\langle F | Q_i(\mu) | I \rangle}{(M_{\text{W}})^{\text{di}-6}}$$

di= dimension of the operator $Q_i(\mu)$

$C_i(\mu)$ Wilson coefficient: it depends on M_{W}/μ and $\alpha_{\text{W}}(\mu)$ @NLO

$Q_i(\mu)$ local operator renormalized at the scale μ FROM LATTICE

Without the next-to-leading corrections it is impossible to fix the renormalization scale and to match consistently the Wilson coefficients to the matrix elements of the (lattice) operators (see also citation from Buras *)

*Numerical Estimates of Hadronic Masses in a Pure
SU(3) Gauge Theory*

H. Hamber & G. Parisi

Phys.Rev.Lett. 47 (1981) 1792

- Weak Hamiltonian on the Lattice Cabibbo et al.
+ Gavela et al. + Bernard & Soni
- Construction and renormalization of the Weak
Hamiltonian on the Lattice Bochicchio et. al.
- Renormalization of composite operators GM et al.
- $K\pi\pi$ amplitudes on a finite volume Lellouch &
Luscher

*Leptonic, Semileptonic, $K\pi\pi$, B and K Mixing,
Radiative, ...*

Andrzej J. Buras Gospel *arXiv:1102.5650v4*

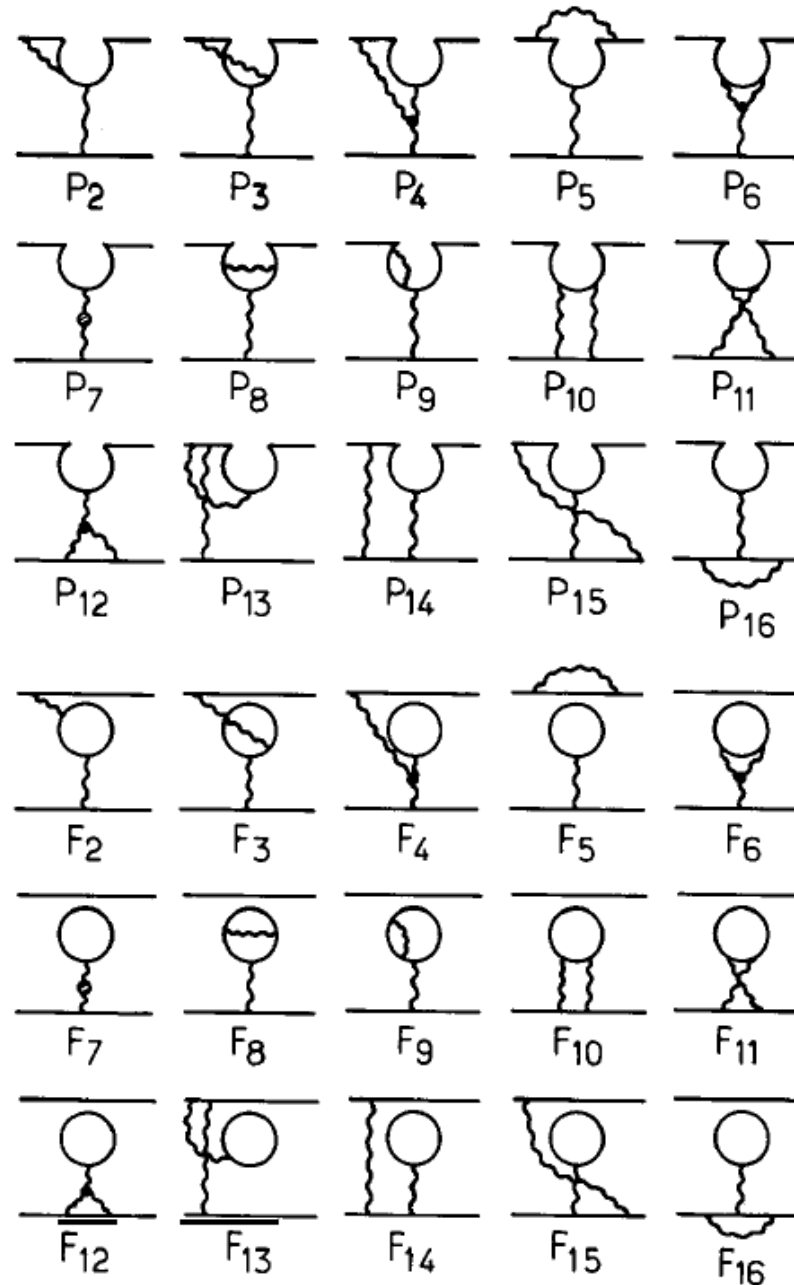
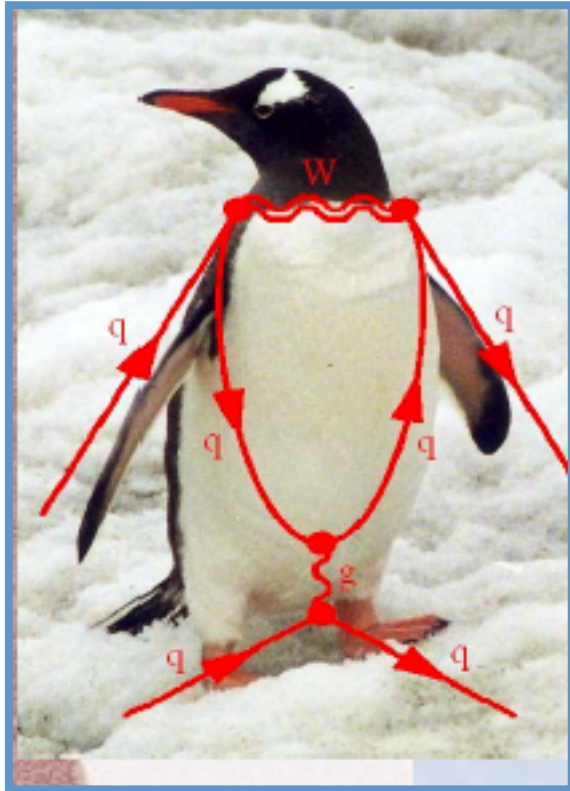
During the last supper of the Ringberg workshop ('88) Guido Martinelli and me realized that it would be important to calculate NLO QCD corrections to the Wilson coefficients of penguin operators relevant for $K \rightarrow \pi\pi$ decays

.. NLO QCD corrections to $\Delta S = 1$ and $\Delta B = 1$ non-leptonic decays... $\Delta S = 2$ & $\Delta B = 2$ transitions, rare K and B decays, in particular $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $B_{s,d} \rightarrow \mu^+ \mu^-$... the inclusive decay $B \rightarrow X_s \gamma$, $B \rightarrow X_s$ gluon, ... $K_L \rightarrow \pi^0 \ell^+ \ell^-$, $B \rightarrow X_s \ell^+ \ell^-$... $B \rightarrow K^(\rho) \ell^+ \ell^-$*

several thousands citations

still the road has been opened by Guido Altarelli

The Penguin Era Begins (J. Ellis)



M. Shifman, A.I. Vainshtein,
V. I. Zakharov
J. Flynn and L. Randall

Fig. 11. Penguin diagrams at two loops.

A concrete (most difficult) example:

$K \rightarrow \pi\pi$ decays

$$\mathcal{H}^{\Delta S=1} = G_F/\sqrt{2} V_{ud} V_{us}^* \left[(1-\tau) \sum_{i=1,2} z_i (Q_i - Q_i^c) + \tau \sum_{i=1,10} (z_i + y_i) Q_i \right]$$

Where y_i and z_i are short distance coefficients, which are known
In perturbation theory at the NLO (Buras et al. + Ciuchini et al.)

$$\tau = -V_{ts}^* V_{td} / V_{us}^* V_{ud}$$

We must compute $\mathcal{A}^{I=0,2}_i = \langle (\pi\pi)_{I=0,2} | Q_i | K \rangle$
with a non perturbative technique (lattice LL,
QCD sum rules, 1/N expansion etc.)

$$\begin{aligned} \mathcal{A}^{I=0,2}_i(\mu) &= \langle (\pi \pi)_{I=0,2} | Q_i(\mu) | K \rangle \\ &= Z_{ik}(\mu a) \langle (\pi \pi)_{I=0,2} | Q_k(a) | K \rangle \end{aligned}$$

Where $Q_i(a)$ is the bare lattice operator
And a the lattice spacing.

The effective Hamiltonian can then be read as:

$$\langle F | H^{\Delta S=1} | I \rangle = G_F / \sqrt{2} V_{ud} V_{us}^* \sum_i C_i(1/a) \langle F | Q_i(a) | I \rangle$$

In practice the renormalization scale (or $1/a$) are the scales which separate short and long distance dynamics

GENERAL FRAMEWORK

$$\langle \mathcal{H}^{\Delta S=1} \rangle = G_F / \sqrt{2} V_{ud} V_{us}^* \dots \sum_i C_i(\mathbf{a}) \langle Q_i(\mathbf{a}) \rangle$$

$$M_W = 100 \text{ GeV}$$

Effective Theory - quark & gluons

$$a^{-1} = 2\text{-}5 \text{ GeV}$$

Hadronic non-perturbative region

$$\Lambda_{\text{QCD}}, M_K = 0.2\text{-}0.5 \text{ GeV}$$

perturbative regime

Chiral regime

100 GeV

perturbative region

Large mass scale: heavy degrees of freedom (m_t, M_W, M_S) are removed and their effect included in the Wilson coefficients

1-2 GeV

non-perturbative region

renormalization scale μ (inverse lattice spacing $1/a$); this is the scale where the quark theory is matched to the effective hadronic theory

Scale of the low energy process
 $\Lambda \sim M_W$

THE SCALE PROBLEM:

Effective theories prefer low scales,
Perturbation Theory prefers large scales

Where we are now?

- *non-perturbative renormalization of the relevant operators*
- *$K \rightarrow \pi\pi$ computed at the physical point using Lellouch-Lüscher (see also Lin, Sachrajda, gm, Testa)*
- *Unquenched and at (almost) physical quark masses*
- *Enormous progresses made by RBC-UKQCD*



RBC-UK QCD

$$\varepsilon'/\varepsilon = (1.4 \pm 7.0) \cdot 10^{-4}$$

$$\left(\frac{\text{Re } A_0}{\text{Re } A_2} \right) = 31.0 \pm 6.6$$

$$\left(\varepsilon'/\varepsilon \right)_{\text{exp}} = (16.6 \pm 2.3) \cdot 10^{-4}$$

$$\left(\frac{\text{Re } A_0}{\text{Re } A_2} \right)_{\text{exp}} = 22.4$$

Courtesy by A. Buras

Four dominant contributions to ε'/ε in the SM

AJB, Jamin, Lautenbacher (1993); AJB, Gorbahn, Jäger, Jamin (2015)

$$\text{Re}(\varepsilon'/\varepsilon) = \left[\frac{\text{Im}(V_{td} V_{ts}^*)}{1.4 \cdot 10^{-4}} \right] 10^{-4} \left[\overset{\text{From Re}A_0}{\downarrow} -3.7 + 21.2 \cdot B_6^{(1/2)} + \overset{\text{From Re}A_2}{\downarrow} 1.1 - 9.6 \cdot B_8^{(3/2)} \right]$$

(Q₄)

(V-A) ⊗ (V-A)
QCD Penguins

(V-A) ⊗ (V+A)
QCD Penguins

(V-A) ⊗ (V-A)
EW Penguins

(V-A) ⊗ (V+A)
EW Penguins

Assumes that $\text{Re}A_0$ and $\text{Re}A_2$ ($\Delta I=1/2$ Rule) fully described by SM (includes isospin breaking corrections)

ε'/ε from RBC-UKQCD

Calculate all contributions directly (no isospin breaking corrections)

$$\left[-(6.5 \pm 3.2) + 25.3 \cdot B_6^{(1/2)} + (1.2 \pm 0.8) - 10.2 \cdot B_8^{(3/2)} \right]$$

ε'/ε from RBC-UKQCD

Anatomy: AJB, Gorbahn, Jäger, Jamin (2015)

Calculate all contributions directly

$$\text{Re}(\varepsilon'/\varepsilon) = \left[\frac{\text{Im}(V_{td} V_{ts}^*)}{1.4 \cdot 10^{-4}} \right] 10^{-4} \left[-6.5 + 25.3 \cdot B_6^{(1/2)} + 1.2 - 10.2 \cdot B_8^{(3/2)} \right]$$

(Q₄)

(V-A) ⊗ (V-A)
QCD Penguins

(V-A) ⊗ (V+A)
QCD Penguins

(V-A) ⊗ (V-A)
EW Penguins

(V-A) ⊗ (V+A)
EW Penguins

Extracted from

RBC-UKQCD

$B_6^{(1/2)} = B_8^{(3/2)} = 1$ in the large N limit

$B_6^{(1/2)} = 0.57 \pm 0.15$

$B_8^{(3/2)} = 0.76 \pm 0.05$

EW penguins in full agreement with BGJJ but

+ third term very similar to BGJJ
 $(\text{Re}A_2)_{\text{Lattice}} \approx (\text{Re}A_2)_{\text{exp}}$

$$\left[\frac{(\text{Re}A_0)}{(\text{Re}A_0)_{\text{exp}}} \approx 1.4 \right]$$

The negative contribution of Q₄ overestimated

$$\left(\frac{\varepsilon'}{\varepsilon} \right)_{\text{Lattice}} = (1.4 \pm 7.0) \cdot 10^{-4}$$

Anatomy of ε'/ε – A new flavour anomaly?

AJB, Gorbahn, Jäger, Jamin,, 1507.xxxx

RBC-UKQCD

$$\varepsilon'/\varepsilon = (1.4 \pm 7.0) \cdot 10^{-4}$$

(3.2 σ) $\varepsilon'/\varepsilon = (2.2 \pm 3.8) \cdot 10^{-4}$

$$\varepsilon'/\varepsilon = (6.3 \pm 2.5) \cdot 10^{-4}$$

$$\varepsilon'/\varepsilon = (9.1 \pm 3.3) \cdot 10^{-4}$$

exp: $\varepsilon'/\varepsilon = (16.6 \pm 3.3) \cdot 10^{-4}$

RBC-QCD values

$$B_6^{(1/2)} = 0.57 \pm 0.15$$

$$B_8^{(3/2)} = 0.76 \pm 0.05$$

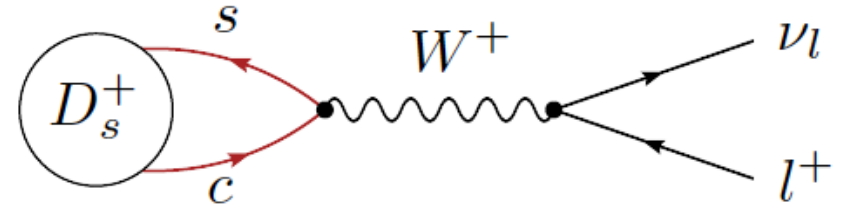
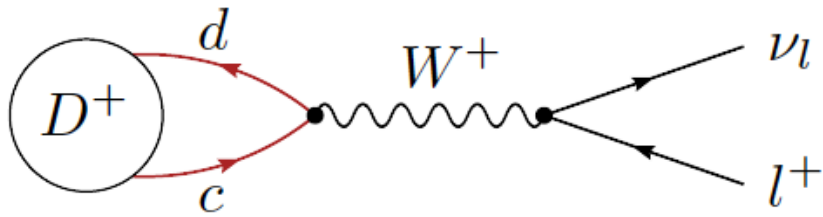
large N bounds (AJB, Gérard)

$$B_6^{(1/2)} = B_8^{(3/2)} = 0.76$$

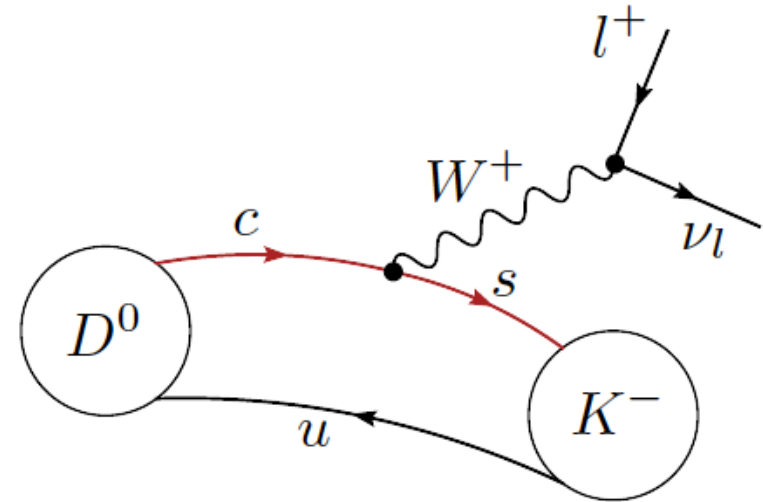
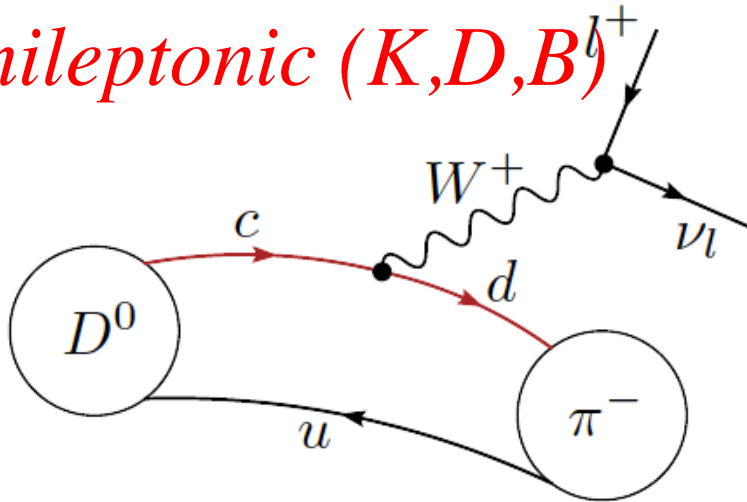
large N bounds (AJB, Gérard)

$$B_6^{(1/2)} = B_8^{(3/2)} = 1.0$$

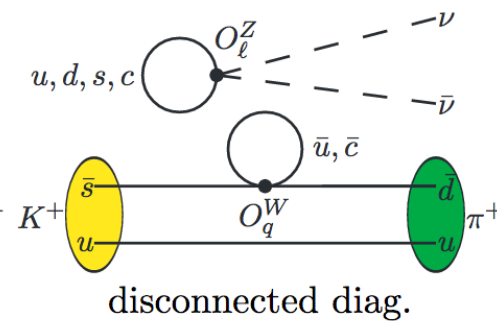
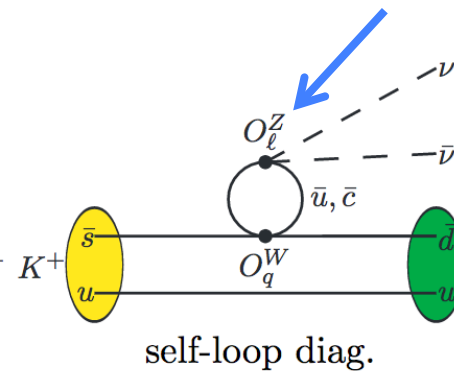
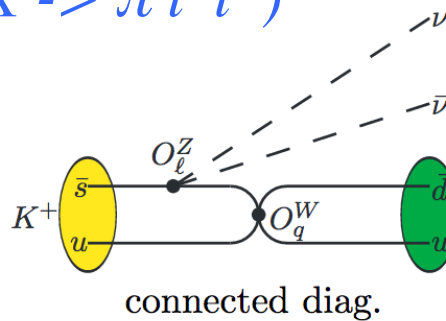
Leptonic (π, K, D, B)



Semileptonic (K, D, B)



(some) Radiative and Rare long distance effects
(also $K \rightarrow \pi l^+ l^-$)

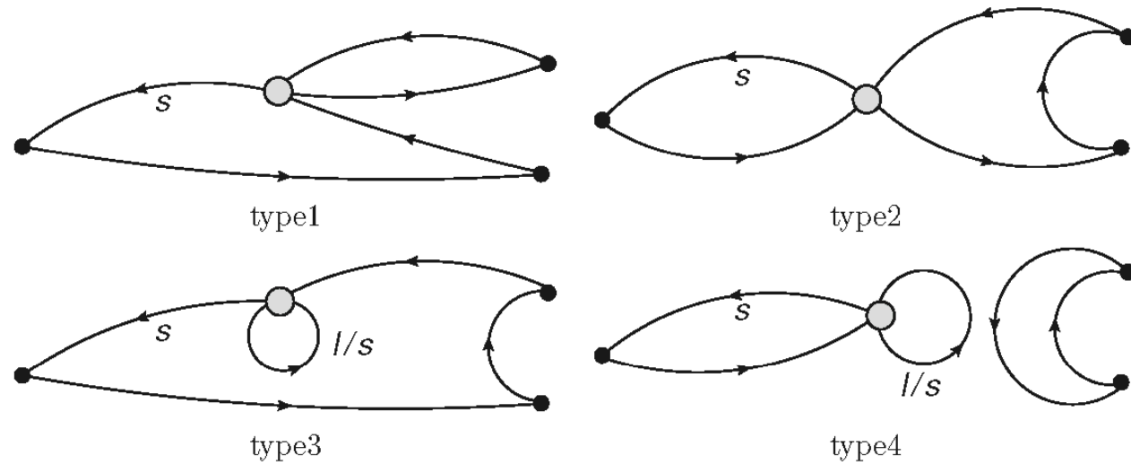


Non-leptonic

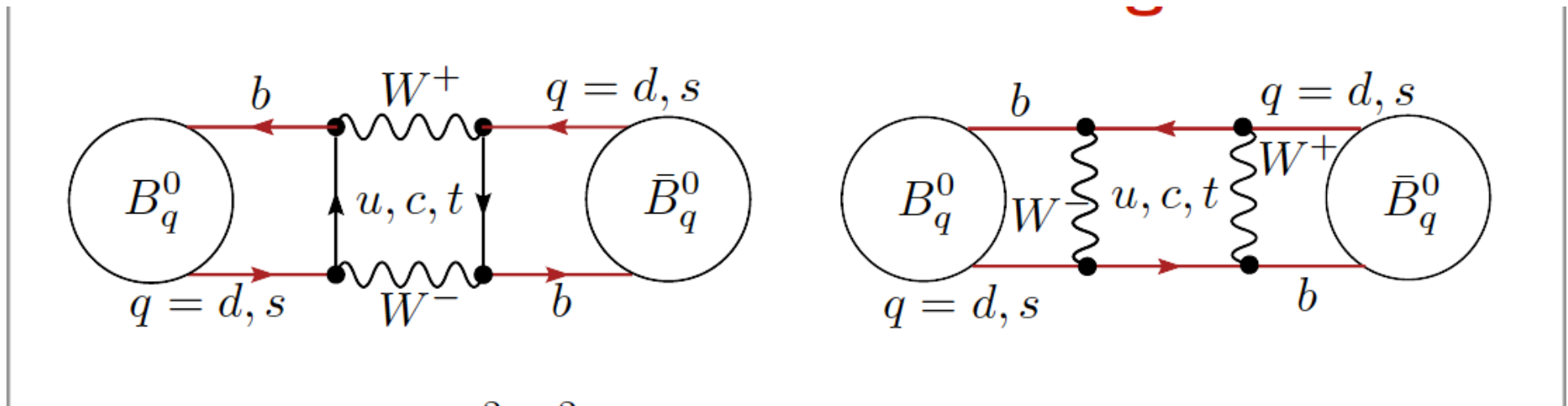
but only below the inelastic threshold

(may be also 3 body decays)

$B \rightarrow \pi\pi, K\pi, \text{ etc. No !}$



Neutral meson mixing (local)



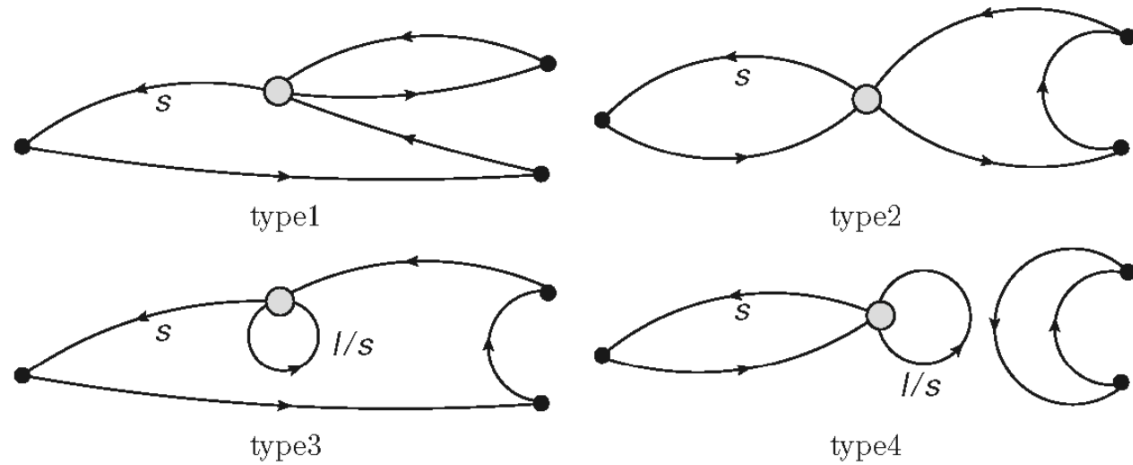
+ some long distance contributions to K and D neutral meson mixing + short distance contributions to $B \rightarrow K^* l^+ l^-$

Non-leptonic

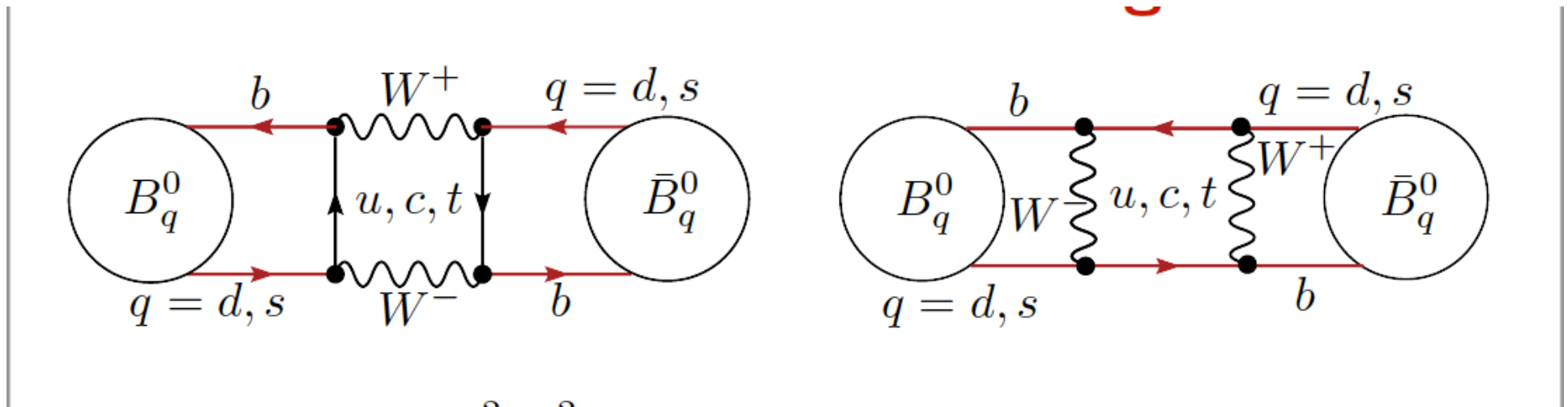
but only below the inelastic threshold

(may be also 3 body decays)

$B \rightarrow \pi\pi, K\pi, \text{ etc. No !}$



Neutral meson mixing (local)



+ some long distance contributions to K and D neutral meson mixing + short distance contributions to $B \rightarrow K^* l^+ l^-$

Radiative corrections to weak amplitudes

important for hadron masses, leptonic and semileptonic decays, $|V_{us}|$, but also for D and B decays

13

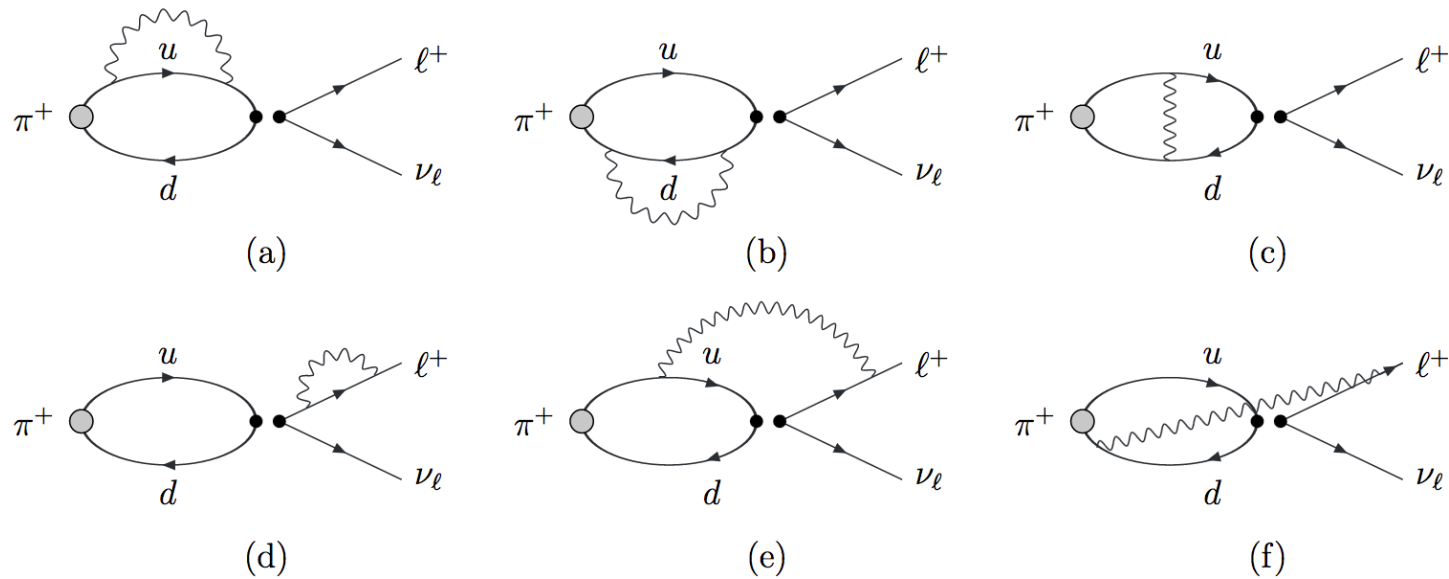
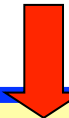


FIG. 5: Connected diagrams contributing at $O(\alpha)$ contribution to the amplitude for the decay $\pi^+ \rightarrow \ell^+ \nu_\ell$.

CP Violation in the Standard Model

Flavor physics in the Standard Model

In the SM, the quark mass matrix, from which the CKM matrix and CP violation originate, is determined by the coupling of the Higgs boson to fermions.



$$\mathcal{L}_{\text{quarks}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Yukawa}}$$

CP invariant

CP and symmetry breaking are strictly correlated

$$\mathcal{L}(\Lambda_{\text{Fermi}}) = \mathcal{L}(\Lambda, H, H^\dagger) + \mathcal{L}^{\text{kin}} + \mathcal{L}_{\text{SM}}^{\text{gauge}} + \mathcal{L}_{\text{SM}}^{\text{Yukawa}} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \dots$$

EWSB

has many accidental symmetries

may violate accidental symmetries

Absence of FCNC at tree level (& GIM suppression of FCNC @loop level)

Almost no CP violation at tree level

Flavour Physics is extremely sensitive to New Physics (NP)

In competition with Electroweak Precision Measurements

WHY RARE DECAYS ?

Rare decays are a manifestation of broken (accidental) symmetries e.g. of physics beyond the Standard Model

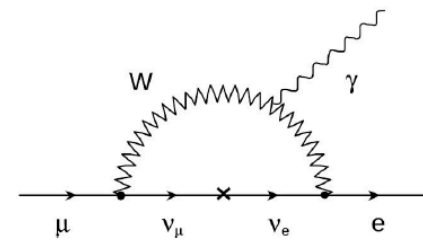
Proton decay

baryon and lepton number conservation

$\mu \rightarrow e + \gamma$

lepton flavor number

$\nu_i \rightarrow \nu_k$ **found !**



$$\mathcal{B}(\mu \rightarrow e\gamma) \sim \alpha \frac{m_\nu^4}{m_W^4} \sim 10^{-52}$$

RARE DECAYS WHICH ARE ALLOWED IN THE STANDARD MODEL

FCNC:

$$q_i \rightarrow q_k + \nu \bar{\nu}$$

$$q_i \rightarrow q_k + l^+ l^-$$

$$q_i \rightarrow q_k + \gamma$$

these decays occur
only via loops because
of *GIM* and are
suppressed by *CKM*

THUS THEY ARE SENSITIVE TO
NEW PHYSICS

Flavour and New Physics

Flavour phenomenology plays a fundamental role in indirect searches of New Physics:

- *looks for deviation from the SM whatever the origin*
- *needs good theoretical control of the SM contribution only*
- *in general cannot provide precise information on the NP scale, but a positive result would be a strong evidence that NP is not too far (i.e. in the multi-TeV region)*

the path leading to TeV NP is narrower after the results of the LHC

this will be further explored in the present run



- 1) A fundamental issue is **to find signatures of new physics** and to unravel the underlying theoretical structure;
- 2) Precision Flavor physics is a key tool, complementary to the large energy searches at the LHC, in this endeavour;
- 3) If the LHC discovers new elementary particles BSM, then precision flavor physics will be necessary to understand the underlying framework;
- 4) **The discovery potential of precision flavor physics should also not be underestimate;**
- 5) Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.

$$Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle$$

$$Q^{EXP} = \sum_i C_{SM}^i(M_W, m_t, \alpha_s) \langle F | \hat{O}_i | I \rangle + \sum_{i'} C_{Beyond}^{i'}(\tilde{m}_\beta, \alpha_s) \langle F | \hat{O}_{i'} | I \rangle$$

In the Standard Model the quark mass matrix, from which the CKM Matrix and \mathcal{CP} originate, is determined by the Yukawa Lagrangian which couples fermions and Higgs

$$\mathcal{L}_{\text{quarks}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{weak int}} + \mathcal{L}_{\text{yukawa}}$$

\mathcal{CP} invariant

\mathcal{CP} and symmetry breaking are closely related !

QUARK MASSES ARE GENERATED BY DYNAMICAL SYMMETRY BREAKING

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad H^C = i\tau_2 H^*$$

$$\phi^+ \rightarrow 0 \quad \phi^0 \rightarrow \frac{V}{\sqrt{2}}$$

Charge +2/3

Elementary Particles

Quarks	u	c	t	γ
	d	s	b	
Leptons	ν_e	ν_μ	ν_τ	Z
	e	μ	τ	

Force Carriers

Three Generations of Matter

$$\mathcal{L}_{\text{yukawa}} \equiv \sum_{i,k=1,N} \left[Y_{i,k} (q_L^i H^C) U_R^k + X_{i,k} (q_L^i H) D_R^k + \text{h.c.} \right]$$

Charge -1/3

$$\sum_{i,k=1,N} \left[m_{i,k}^u (\bar{u}_L^i u_R^k) + m_{i,k}^d (\bar{d}_L^i d_R^k) + \text{h.c.} \right]$$

Diagonalization of the Mass Matrix

Up to singular cases, the mass matrix can always be diagonalized by 2 unitary transformations

$$u_L^i \rightarrow U_{L}^{ik} u_L^k \quad u_R^i \rightarrow U_{R}^{ik} u_R^k$$

$$M' = U_L^\dagger M U_R \quad (M')^\dagger = U_R^\dagger (M)^\dagger U_L$$

$$\mathcal{L}^{\text{mass}} \equiv m_{\text{up}} (\bar{u}_L u_R + \bar{u}_R u_L) + m_{\text{ch}} (\bar{c}_L c_R + \bar{c}_R c_L) \\ + m_{\text{top}} (\bar{t}_L t_R + \bar{t}_R t_L)$$

$$L_{CC}^{\text{weak int}} = \frac{g_W}{\sqrt{2}} (J_\mu^- W_\mu^+ + h.c.)$$

$$\rightarrow \frac{g_W}{\sqrt{2}} (\bar{u}_L V^{CKM} \gamma_\mu d_L W_\mu^+ + \dots)$$

$N(N-1)/2$ angles and $(N-1)(N-2)/2$ phases

$N=3$ 3 angles + 1 phase KM
 the phase generates complex couplings i.e. CP
violation;

6 masses +3 angles +1 phase = 10 parameters

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{td}	V_{ts}	V_{tb}

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

**NO Flavour Changing Neutral Currents (FCNC)
at Tree Level
(FCNC processes are good candidates for observing
NEW PHYSICS)**

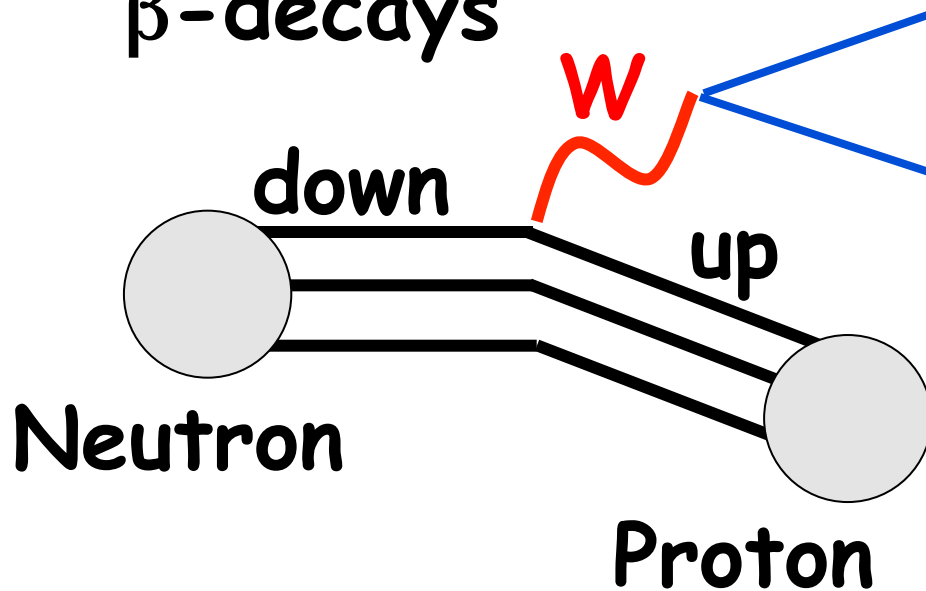
**CP Violation is natural with three quark
generations (Kobayashi-Maskawa)**

**With three generations all CP
phenomena are related to the same
unique parameter (δ)**

Quark masses & Generation Mixing

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{td}	V_{ts}	V_{tb}

β -decays



$|V_{ud}|$

updated values later (0.999)

- $|V_{ud}| = 0.9735(8)$
- $|V_{us}| = 0.2196(23)$
- $|V_{cd}| = 0.224(16)$
- $|V_{cs}| = 0.970(9)(70)$
- $|V_{cb}| = 0.0406(8)$
- $|V_{ub}| = 0.00409(25)$
- $|V_{tb}| = 0.99(29)$

Textures

There is a clear correlation between mixings and masses

$$m_u \sim 4 \text{ MeV} \quad m_c \sim 1200 \text{ MeV} \quad m_t \sim 170 \text{ GeV}$$

$$m_d \sim 8 \text{ MeV} \quad m_s \sim 110 \text{ MeV} \quad m_b \sim 4.3 \text{ GeV}$$

Horizontal $U(2)$: $\psi_L \quad \psi_L^c$

$$\mathcal{L}_{\text{higgs}} = Y H \left[(\psi_L^a)(\psi_L^b)^c S^{ab} + (\psi_L^a)(\psi_L^b)^c A^{ab} \right]$$

Symmetric
tensor

Antisymmetric
tensor

$$M^d = M \begin{pmatrix} 0 & -\sqrt{x} \\ \sqrt{x} & 1+x \end{pmatrix}$$

$$\sin \theta_c \sim \sqrt{m_d / m_s}$$

R. Gatto '70

$$\text{diag}(M) = M \begin{pmatrix} x & \\ & 1 \end{pmatrix} \quad x = m_d / m_s$$

$$V_1 = \begin{pmatrix} 1 \\ \sqrt{x} \end{pmatrix} \quad \lambda_1 = M x$$

$$V_2 = \begin{pmatrix} -\sqrt{x} \\ 1 \end{pmatrix} \quad \lambda_2 = M$$

Masses & Mixings
(including the CP phases)
are related !!

The Wolfenstein Parametrization

$1 - 1/2 \lambda^2$	λ	$A \lambda^3(\rho - i \eta)$
$-\lambda$	$1 - 1/2 \lambda^2$	$A \lambda^2$
$A \lambda^3 \times$ $(1 - \rho - i \eta)$	$-A \lambda^2$	1

V_{ub}

$+ O(\lambda^4)$

V_{td}

$$\lambda \sim 0.2 \quad A \sim 0.8$$

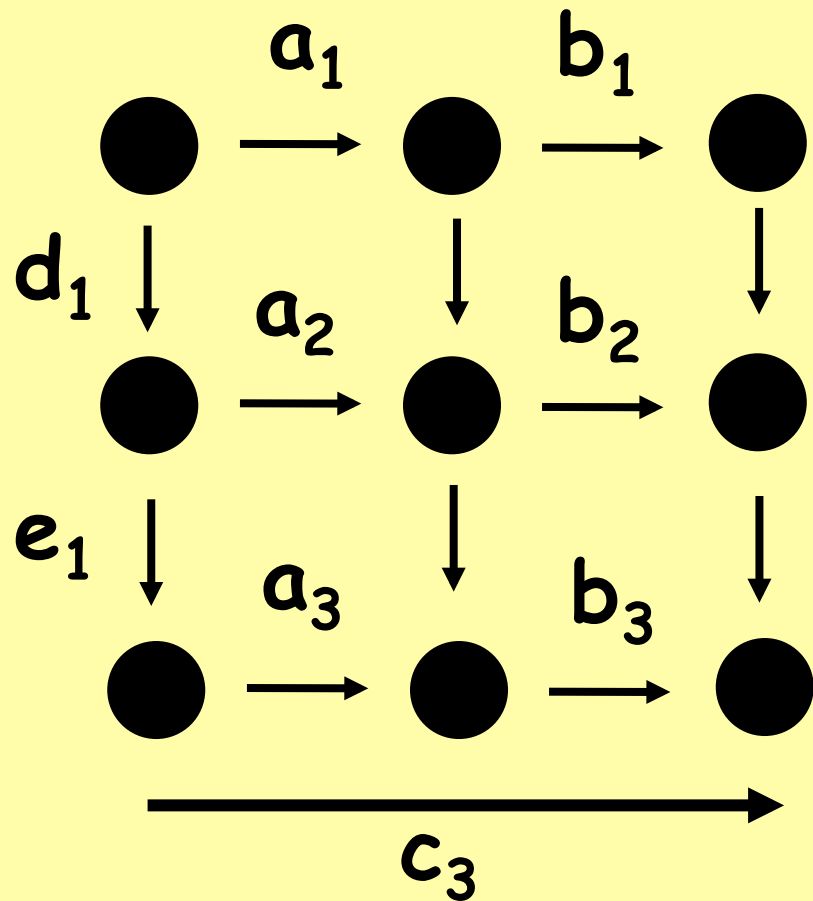
$$\eta \sim 0.2 \quad \rho \sim 0.3$$

$$\sin \theta_{12} = \lambda$$

$$\sin \theta_{23} = A \lambda^2$$

$$\sin \theta_{13} = A \lambda^3(\rho - i \eta)$$

The Bjorken-Jarlskog Unitarity Triangle



$|V_{ij}|$ is invariant under phase rotations

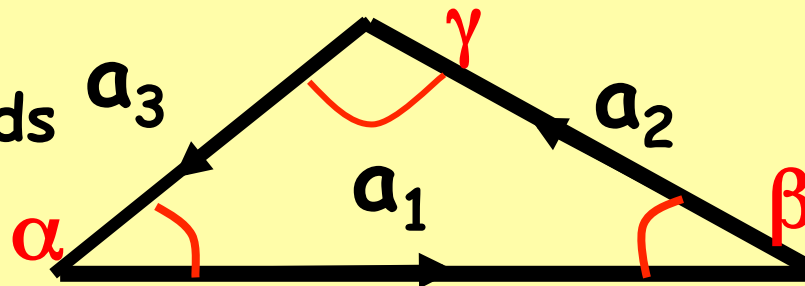
$$a_1 = V_{11} V_{12}^* = V_{ud} V_{us}^*$$

$$a_2 = V_{21} V_{22}^* \quad a_3 = V_{31} V_{32}^*$$

$$a_1 + a_2 + a_3 = 0$$

$$(b_1 + b_2 + b_3 = 0 \text{ etc.})$$

Only the orientation depends on the phase convention



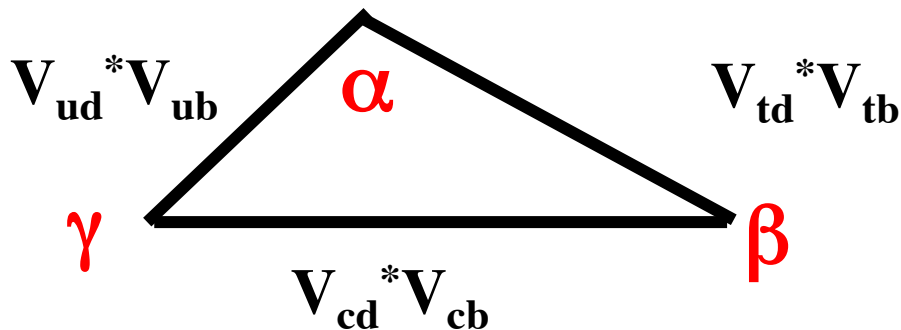
Physical quantities correspond to invariants under phase reparametrization i.e. $|a_1|, |a_2|, \dots, |e_3|$ and the area of the Unitary Triangles

$$J = \text{Im} (a_1 a_2^*) = |a_1 a_2| \sin \beta$$

a precise knowledge of the moduli (angles) would fix J

$$\phi \propto J$$

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$$



$$\gamma = \delta_{CKM}$$

Gluons and quarks

The QCD Lagrangian :

$$L_{STRONG} = -1/4 G^A_{\mu\nu} G^{\mu\nu}_A \leftarrow GLUONS$$

$$+ \sum_{f=\text{flavour}} \bar{q}_f (i \gamma_\mu D_\mu - m_f) q_f$$

QUARKS (& GLUONS)

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_0 f^{ABC} G^B_\mu G^C_\nu$$

$$q_f \equiv q_f^a_\alpha(x) \quad \gamma_\mu \equiv (\gamma_\mu)^{\alpha\beta} \quad D_\mu \equiv \partial_\mu I + i g_0 t^A_{ab} G^A_\mu$$

STRONG CP VIOLATION

$$\mathcal{L}_\theta = \theta \tilde{G}^{\mu\nu a} G_{\mu\nu}^a \quad \tilde{G}_{\mu\nu}^a = \epsilon_{\mu\nu\rho\sigma} G_{\rho\sigma}^a$$

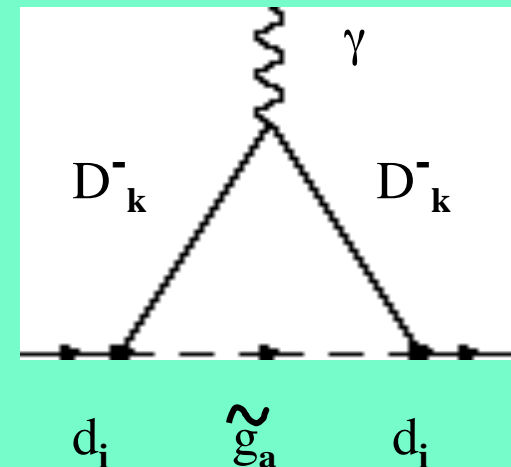
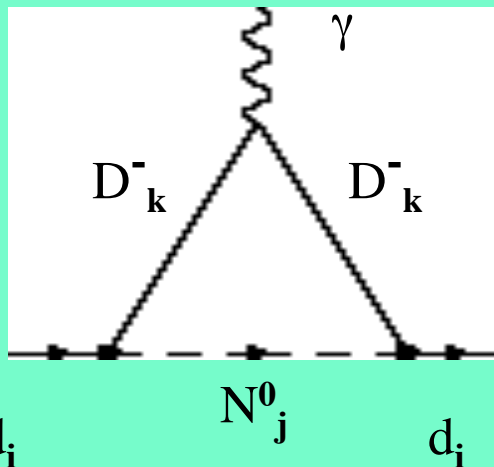
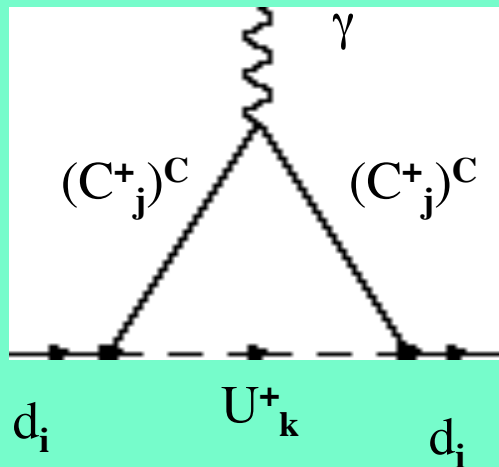
$$\mathcal{L}_\theta \sim \theta \vec{E}^a \cdot \vec{B}^a$$

This term violates CP and gives a contribution to the electric dipole moment of the neutron

$$e_n < 3 \cdot 10^{-26} \text{ e cm}$$

$\theta < 10^{-10}$ which is quite unnatural !!

Neutron electric dipole moment in SuperSymmetry



$$\begin{aligned} \mathcal{L}_{\Delta F=0} = & -i/2 C_e \psi \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu} \\ & -i/2 C_C \psi \sigma_{\mu\nu} \gamma_5 t^a \psi G^{\mu\nu a} \\ & -1/6 C_g f_{abc} G_{\mu\rho}^a G_{\nu\lambda}^b G_{\sigma\tau}^c \epsilon^{\mu\nu\lambda\sigma} \end{aligned}$$

C_e, C_g can be computed perturbatively

(Some) Resolutions of the Strong CP Problem

- Just declare CP to be good in the strong sector
 - Weak sector can reintroduce the problem

- $m_u = 0$ $\bar{q} \left(i \not{D} - m e^{i\theta' \gamma_5} \right) q$

't Hooft PRL **37** 8 (1976)
 Jackiw & Rebbi, PRL **37** 127 (1976)
 Callan, Dashen & Gross PLB **63** 335 (1976)
 Kaplan & Manohar PRL **56** 2004 (1986)

- $m_u \neq 0$
 Gasser & Leutwyler PhysRept **87** 77-169 (1982)

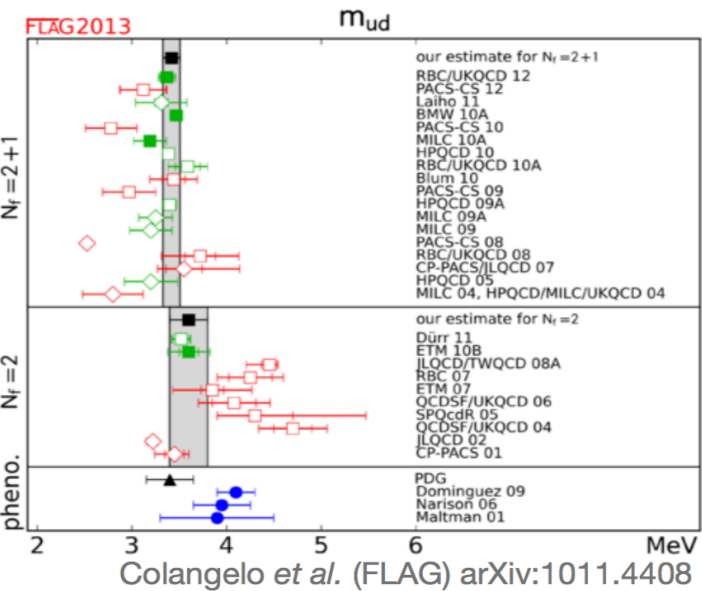
- **Additional Peccei-Quinn symmetry & axions**

Peccei & Quinn: PRL **38** (1977) 1440, PR **D16** (1977) 1791

$$m_u^{\overline{MS}}(2 \text{ GeV}) = 2.40 (15)(17) \text{ MeV}$$

$$m_d^{\overline{MS}}(2 \text{ GeV}) = 4.80 (15)(17) \text{ MeV}$$

$$\frac{m_u^{\overline{MS}}}{m_d^{\overline{MS}}} = 0.50 (2)(3) \quad m_u^{\overline{MS}}(2 \text{ GeV}) = 2.16 (9)(7) \text{ MeV}$$



RM123

Flag

Axions

Peccei & Quinn: PRL **38** (1977) 1440, PR **D16** (1977) 1791

- Couple to topological charge

$$\mathcal{L}_{\text{axions}} = \frac{1}{2} (\partial_\mu a)^2 + \left(\frac{a}{f_a} + \theta \right) \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

- Otherwise have shift symmetry.

$$a \rightarrow a + \alpha$$

- Amenable to effective theory treatment

$$V_{\text{eff}} \sim \cos(\theta + c\langle a \rangle)$$

- PQ symmetry can break before or after inflation.

$$m_a^2 f_a^2 = \chi_{\text{QCD}}$$

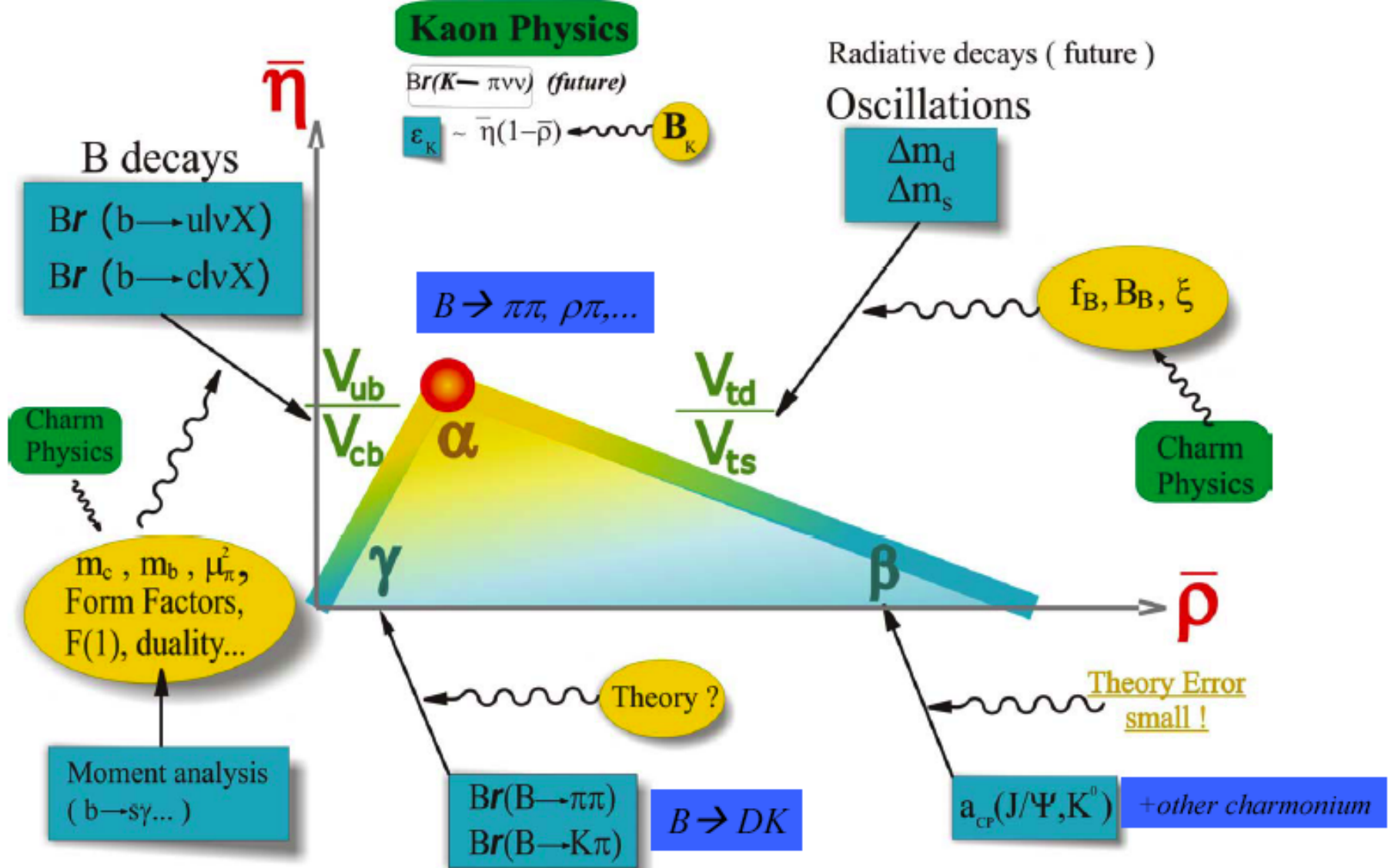
Axion mass Topological Susceptibility

Average over initial θ

Visualization of the unitarity of the CKM matrix

Unitarity Triangle in the $(\bar{\rho}-\bar{\eta})$ plane

From
A. Stocchi
ICHEP 2002



Measure	V_{CKM}	Other NP parameters
$\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$	$\bar{\rho}^2 + \bar{\eta}^2$	$\bar{\Lambda}, \lambda_1, F(1), \dots$
ε_K	$\eta [(1 - \bar{\rho}) + \dots]$	B_K
Δm_d	$(1 - \bar{\rho})^2 + \bar{\eta}^2$	$f_{B_d}^2 B_{B_d}$
$\Delta m_d/\Delta m_1$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$	ξ
$A_{CP}(B_d \rightarrow J/\psi K_s)$	$\sin 2\beta$	—

$$Q^{EXP} = V_{CKM} \times \langle H_F | \hat{O} | H_I \rangle$$

For details see:
 UTfit Collaboration
<http://www.utfit.org>

classical UT analysis

$\sin 2\beta$ is measured directly from $B \rightarrow J/\psi K_s$ decays at Babar & Belle & LHC

$$\mathcal{A}_{J/\psi K_s} = \frac{\Gamma(B_d^0 \rightarrow J/\psi K_s, t) - \Gamma(\bar{B}_d^0 \rightarrow J/\psi K_s, t)}{\Gamma(B_d^0 \rightarrow J/\psi K_s, t) + \Gamma(\bar{B}_d^0 \rightarrow J/\psi K_s, t)}$$

$$\mathcal{A}_{J/\psi K_s} = \sin 2\beta \sin(\Delta m_d t)$$

DIFFERENT LEVELS OF THEORETICAL UNCERTAINTIES (STRONG INTERACTIONS)

- 1) First class quantities, with reduced or negligible theor. uncertainties

$$A_{CP}(B \rightarrow J/\psi K_s) \quad \gamma \text{ from } B \rightarrow DK$$

$$K^0 \rightarrow \pi^0 \nu \bar{\nu}$$

- 2) Second class quantities, with theoretical errors of O(10%) or less that can be reliably estimated

$$\Gamma(B \rightarrow c, u), \quad \varepsilon_K, \quad \Delta M_{d,s}$$

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

- 3) Third class quantities, for which theoretical predictions are model dependent (BBNS, charming, etc.)

In case of discrepancies we cannot tell whether is new physics or we must blame the model

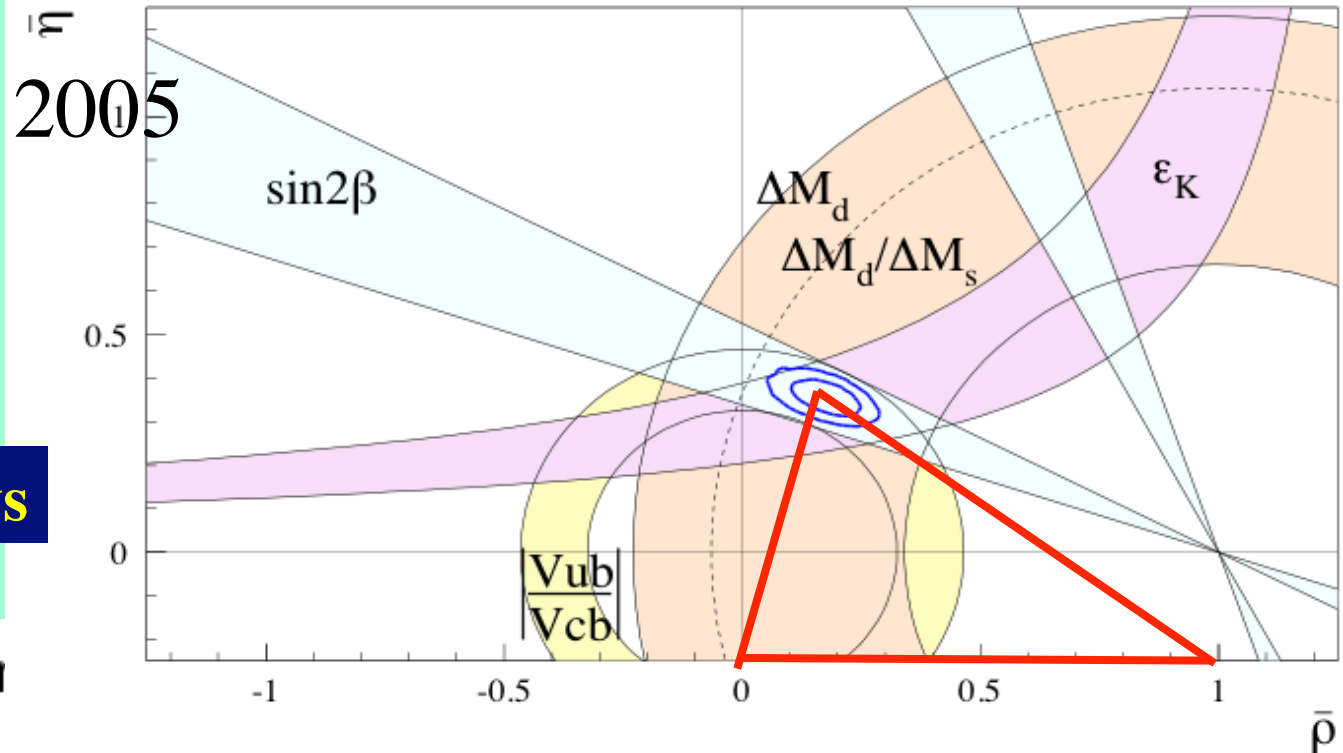
$$B \rightarrow K \pi \quad B \rightarrow \pi^0 \pi^0$$

$$B \rightarrow \phi K_s$$

Unitary Triangle SM

semileptonic decays

Experimental constraints

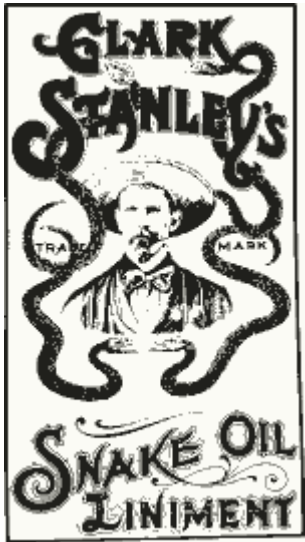


Meas.	$V_{CKM} \times \text{other}$	$(\bar{\rho}, \bar{\eta})$
$\frac{b \rightarrow u}{b \rightarrow c}$	$ V_{ub}/V_{cb} ^2$	$\bar{\rho}^2 + \bar{\eta}^2$
Δm_d	$ V_{td} ^2 f_{B_d}^2 B_{B_d}$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\left \frac{V_{td}}{V_{ts}} \right ^2 \xi^2$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$
ϵ_K	$f(A, \bar{\eta}, \bar{\rho}, B_K)$	$\propto \bar{\eta}(1 - \bar{\rho})$
$A(J/\psi K^0)$	$\sin 2\beta$	$\frac{2\bar{\eta}(1 - \bar{\rho})}{\sqrt{\bar{\eta}^2 + (1 - \bar{\rho})^2}}$

$B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing

$K^0 - \bar{K}^0$ mixing

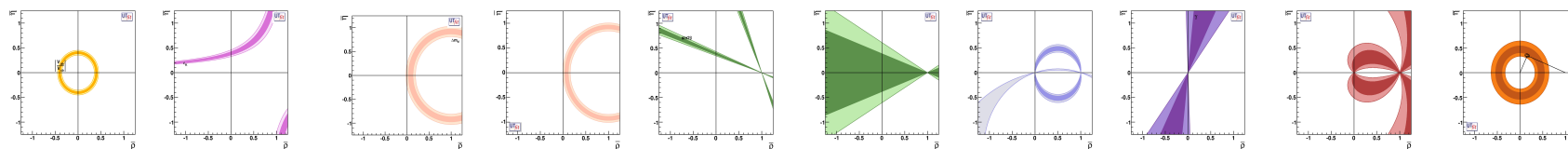
B_d Asymmetry



M.Bona *et al.*, UTfit
JHEP0507:028, 2005

www.utfit.org

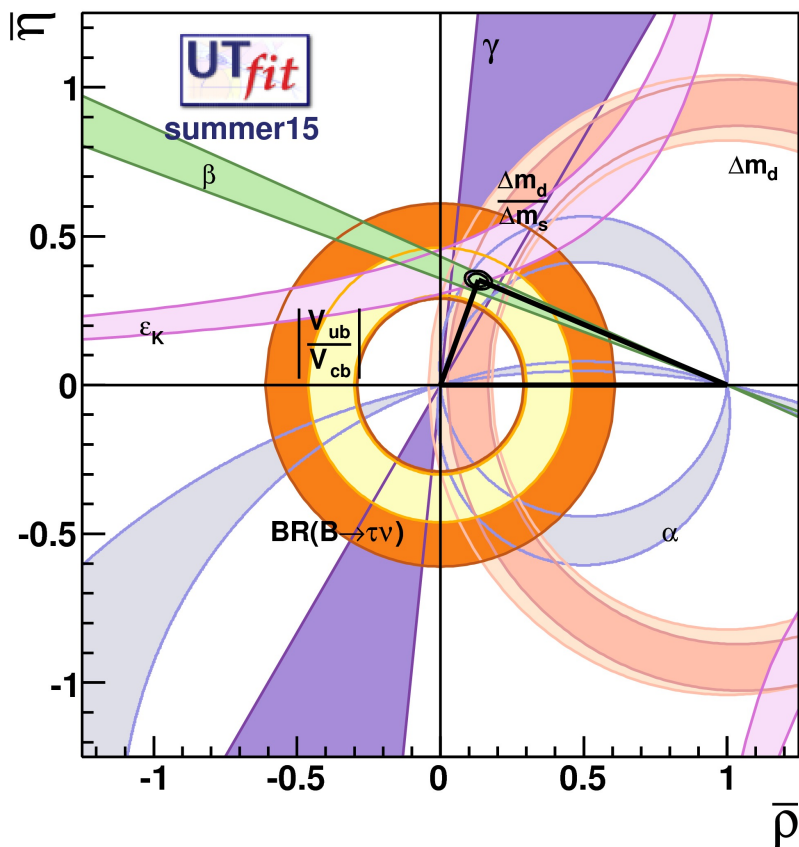
A. Bevan, M. Bona, M. Ciuchini,
D. Derkach, E. Franco, V. Lubicz,
G. Martinelli, F. Parodi, M. Pierini,
C. Schiavi, L. Silvestrini, A. Stocchi,
V. Sordini, C. Tarantino and V. Vagnoni



2015 results

$$\bar{\rho} = 0.142 \pm 0.019 \quad \bar{\eta} = 0.348 \pm 0.013$$

In the hadronic sector, the SM CKM pattern represents the principal part of the flavor structure and of CP violation



$$\alpha = (90.5 \pm 2.6)^\circ$$

$$\sin 2\beta = 0.691 \pm 0.018$$

$$\beta = (21.82 \pm 0.72)^\circ$$

$$\gamma = (67.4 \pm 2.7)^\circ$$

$$A = 0.828 \pm 0.012$$

$$\lambda = 0.22549 \pm 0.00066$$

Consistence on an over constrained fit of the CKM parameters

CKM matrix is the dominant source of flavour mixing and CP violation

Uppsala, July 1, 1987

Schubert

SUMMARY & PERSPECTIVE

G. ALTARELLI



$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$

$$= \begin{pmatrix} c_\theta c_\beta & s_\theta c_\beta & s_\beta e^{-i\delta} \\ -s_\theta s_\gamma - c_\theta s_\beta s_\gamma e^{i\delta} & c_\theta s_\gamma - s_\theta s_\beta s_\gamma e^{i\delta} & c_\beta s_\gamma \\ s_\theta s_\gamma - c_\theta s_\beta c_\gamma e^{i\delta} & -c_\theta s_\gamma - s_\theta s_\beta c_\gamma e^{i\delta} & c_\beta c_\gamma \end{pmatrix}$$

$$V = \begin{pmatrix} .9754 \pm .0004 & .2206 \pm .0018 & 0.000 \pm .0076 \\ -.2203 \pm .0019 & .9743 \pm .0005 & .0474 \pm .0066 \\ .0104 \pm .0075 & -.0462 \pm .0067 & .9989 \pm .0003 \end{pmatrix}$$

$$+i \begin{pmatrix} 0 & 0 & 0 \pm .0076 \\ 0 \pm .0004 & 0 \pm .0001 & 0 \\ 0 \pm .0075 & 0 \pm .0017 & 0 \end{pmatrix}$$

$$\theta = (12.74 \pm 0.11)^\circ$$

$$\beta = (0 \pm 0.43)^\circ$$

$$\gamma = (2.72 \pm 0.38)^\circ$$

$$\left| \frac{V_{ub}}{V_{cb}} \right| < 0.20 \quad 90\% \quad \text{CLEO.}$$

CKM Matrix in the SM

The fit results for all the nine CKM elements are

$$V_{CKM} = \begin{pmatrix} (0.9743 \pm 0.00014) & (0.22509 \pm 0.00061) & (0.00366 \pm 0.00012)e^{i(-67.8 \pm 2.8)^\circ} \\ (-0.22498 \pm 0.00066)e^{i(0.0353 \pm 0.00095)^\circ} & (0.97343 \pm 0.00015)e^{i(-0.00188333 \pm 5 \times 10^{-5})^\circ} & (0.04206 \pm 0.00053) \\ (0.00876 \pm 0.00015)e^{i(-22.03 \pm 0.83)^\circ} & (-0.04129 \pm 0.00054)e^{i(1.054 \pm 0.039)^\circ} & (0.999107 \pm 2.235 \times 10^{-5}) \end{pmatrix}$$

Standard Parametrization (PDG)

$$\sin \theta_{12} = 0.22504 \pm 0.00065$$

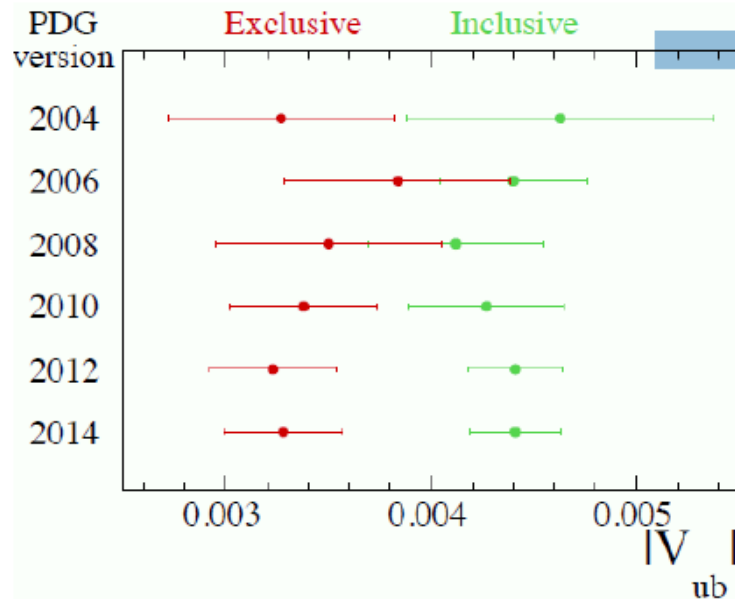
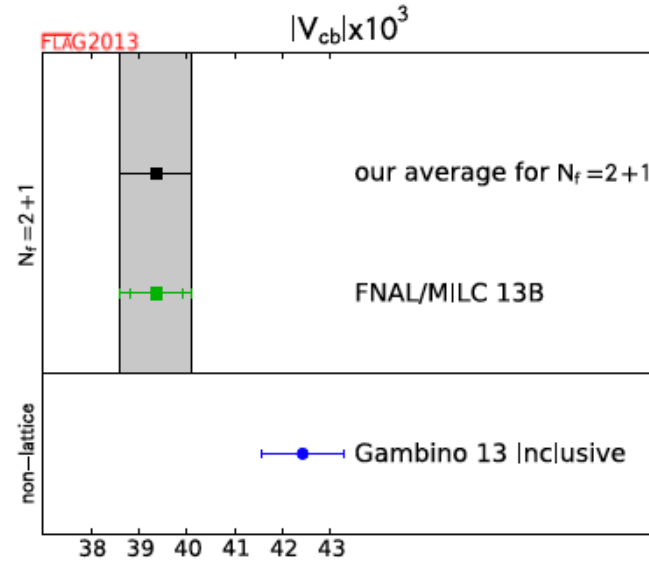
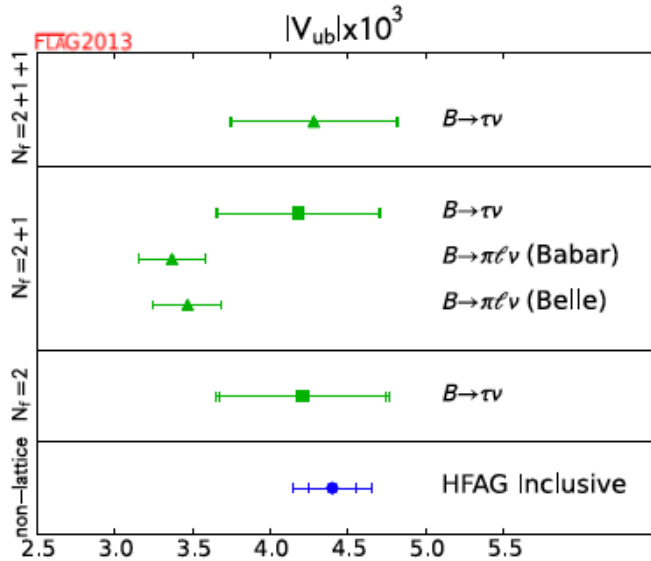
$$\sin \theta_{23} = 0.04206 \pm 0.00054$$

$$\sin \theta_{13} = 0.00366 \pm 0.00012 \quad \delta = 67.8 \pm 2.8$$

Wolfenstein Parametrization (PDG)

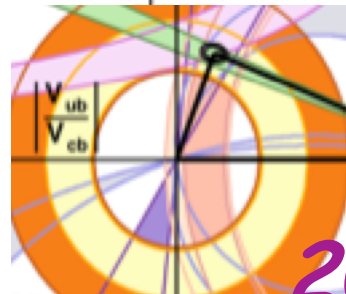
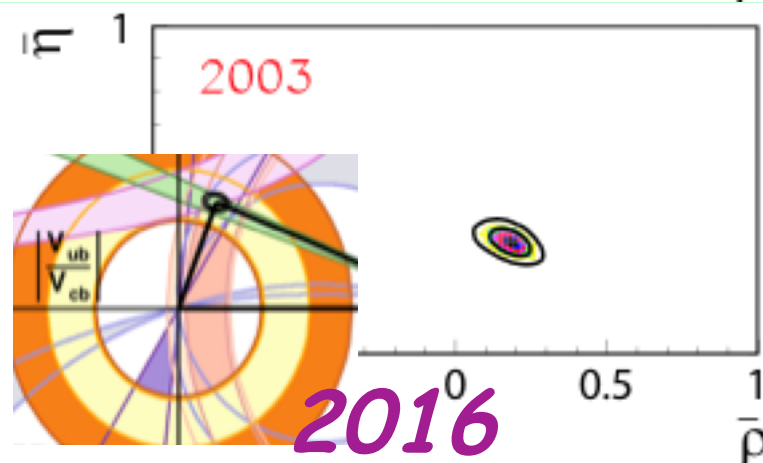
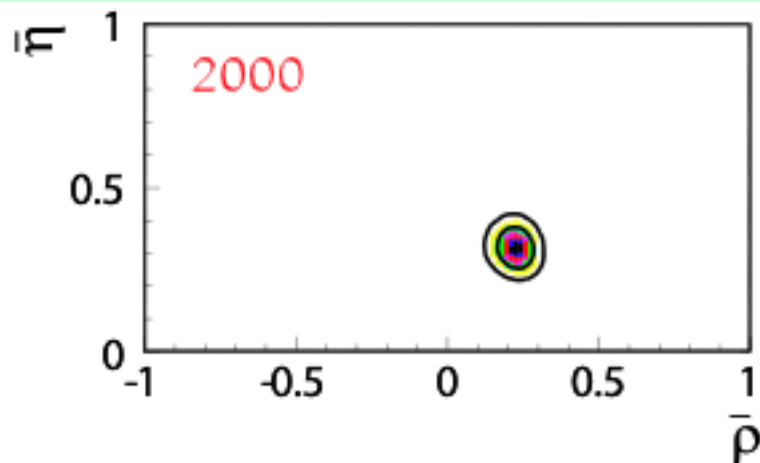
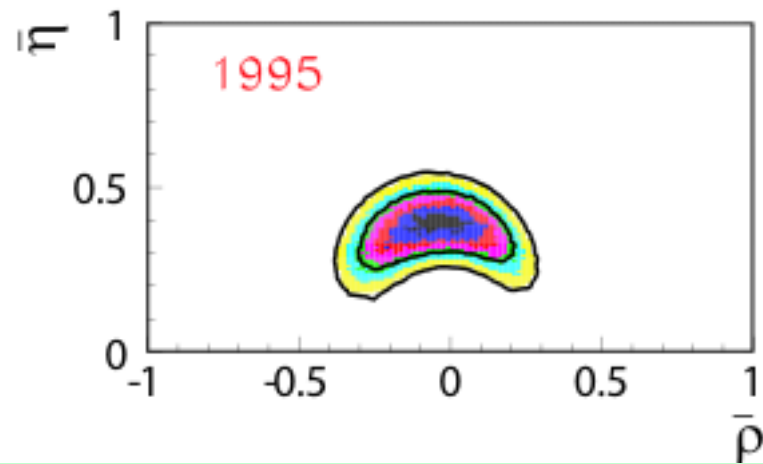
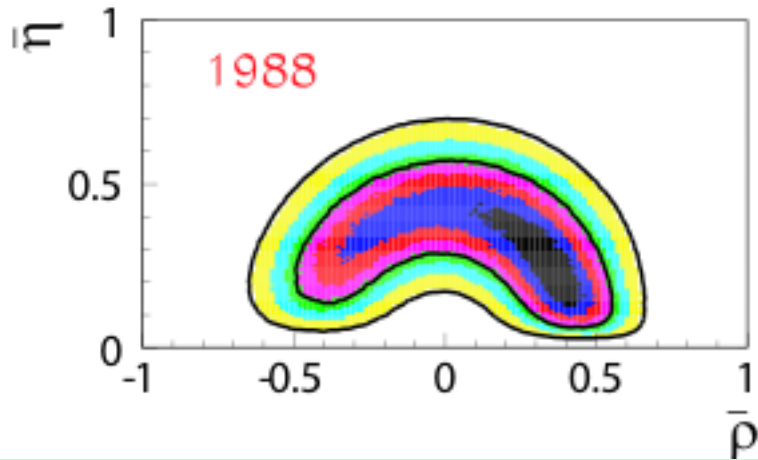
$$\lambda = 0.22514 \pm 0.00066 \quad A = 0.828 \pm 0.012$$

Still some problem persists $|V_{ub}|$, $|V_{cb}|$



$V_{ub} \text{ Exclusive} = 0.00369 \pm 0.00015$
 $V_{cb} \text{ Exclusive} = 0.0392 \pm 0.0007$
 $V_{ub}/V_{cb} \text{ Exclusive} = 0.083 \pm 0.006$
 $V_{ub} \text{ Inclusive} = 0.00441 \pm 0.00022$
 $V_{cb} \text{ Inclusive} = 0.0422 \pm 0.0007$
 $Belle = 0.04247 \pm 0.00100$

PROGRESS SINCE 1988



LATTICE PARAMETERS

	Lattice	Prediction	Pull
\hat{B}_K	0.766 ± 0.010 1.3 %	0.84 ± 0.07 8.3 %	0.9
f_{B_s}	0.226 ± 0.005 2.2 %	0.2256 ± 0.0039 2.7 %	0.0
f_{B_s}/f_{B_d}	1.204 ± 0.016 1.3 %	1.197 ± 0.056 0.4 %	0.0
B_s	0.875 ± 0.040 1.3 %	0.875 ± 0.030 0.4 %	0.0
B_s/B_d	1.03 ± 0.08 7.8 %	1.096 ± 0.062 5.7 %	0.7

Do we still care? Tensions and Unknowns

- 1) A “classical” example $B \rightarrow \tau \nu$
- 2) $|V_{ub}|$ and $|V_{cb}|$ inclusive vs exclusive
- 3) $|V_{cb}|$, B mixing and ϵ_K
- 4) D-mixing
- 5) $R(D)$ and $R(D^*)$
- 6) $B \rightarrow K^* \ell \ell$
- 7) Physics BSM ?

CKM-TRIANGLE ANALYSIS

State of The Art 2015

	Measurement	Fit	Prediction	Pull
α	$(92.7 \pm 6.2)^{\circ}$ 6.7 %	$(90.1 \pm 2.7)^{\circ}$ 2.9 %	$(88.3 \pm 3.4)^{\circ}$ 3.8 %	0.6
$\sin 2\beta$	0.680 ± 0.024 3.5 %	0.696 ± 0.022 2.6 %	0.747 ± 0.039 5.2 %	1.8
γ	$(71.4 \pm 6.5)^{\circ}$ 9.1 %	$(67.4 \pm 2.8)^{\circ}$ 4.2 %	$(66.7 \pm 3.0)^{\circ}$ 4.5 %	0.7
$ V_{ub} \times 10^3$	3.81 ± 0.40 10 %	3.66 ± 0.12 3.3 %	3.64 ± 0.12 3.3 %	0.5
$ V_{cb} \times 10^2$	4.09 ± 0.11 2.6 %	4.206 ± 0.053 1.2 %	4.240 ± 0.062 1.4 %	0.9
$\varepsilon_K \times 10^3$	2.228 ± 0.011 0.5 %	2.227 ± 0.011 0.5 %	2.08 ± 0.18 8.7 %	0.8
Δm_s (ps ⁻¹)	17.761 ± 0.022 0.1 %	17.755 ± 0.022 0.1 %	17.3 ± 1.0 5.7 %	0.2
$BR(B \rightarrow \tau \nu) \times 10^4$	1.06 ± 0.20 18.9 %	0.83 ± 0.07 7.9 %	0.81 ± 0.7 8.2 %	1.3
$BR(B_s \rightarrow \mu\mu) \times 10^9$	2.9 ± 0.7 24.1 %	3.99 ± 0.15 3.8 %	3.94 ± 0.16 4.0 %	1.5 ew corrections not included
$BR(B_d \rightarrow \mu\mu) \times 10^9$	0.39 ± 0.15 38.5 %	0.1098 ± 0.0057 5.2 %	0.1103 ± 0.0058 5.2 %	1.9 ew corrections not included
β_s	$(0.97 \pm 0.95)^{\circ}$ 98 %	$(1.056 \pm 0.039)^{\circ}$ 4.4 %	$(1.056 \pm 0.039)^{\circ}$ 4.1 %	0.1 not included in the fit

$$B(B \rightarrow \tau \nu)_{\text{Old}} = (1.67 \pm 0.30) 10^{-4}$$

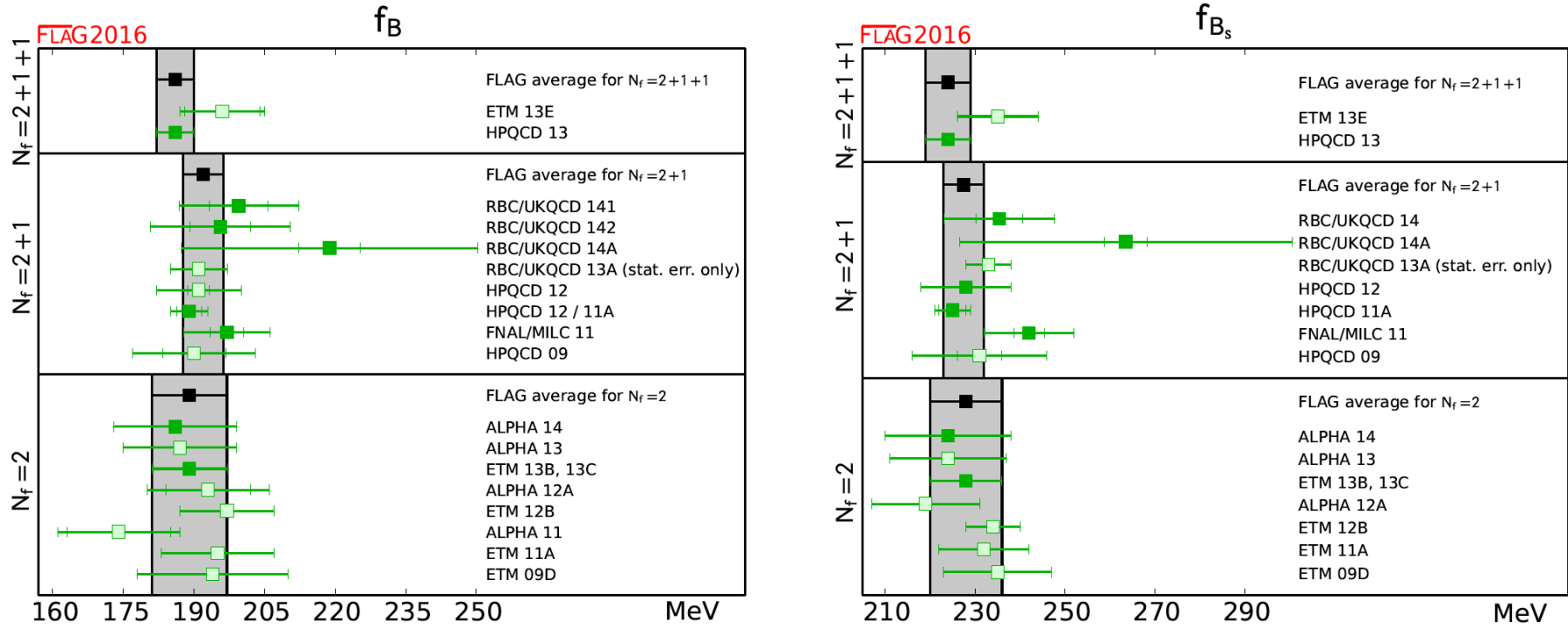


Figure 20: Decay constants of the B and B_s mesons. The values are taken from Tab. 32 (the

$$\begin{aligned}
 & f_B = 192.0(4.3) \text{ MeV} \quad (186) \text{ Refs. [48, 53–56],} \\
 N_f = 2 + 1 : & f_{B_s} = 228.4(3.7) \text{ MeV} \quad (224) \text{ Refs. [48, 53–56],} \\
 N_f = 2+1+1 & f_{B_s}/f_B = 1.201(16) \quad (1.205) \text{ Refs. [48, 53–56].}
 \end{aligned}$$

LATTICE PARAMETERS (2016)

It does not make sense to improve the precision on B_K if we do not control long distance effects; Similarly for f_π or f_K without radiative corrections

obtained excluding the given constraint from the fit

			Pull ($\# \sigma$)
B_K	0.740 ± 0.029	0.81 ± 0.07	< 1
f_{B_s}	0.226 ± 0.005	0.220 ± 0.007	< 1
f_{B_s}/f_{B_d}	1.203 ± 0.013	1.210 ± 0.030	< 1
B_{B_s}/B_{B_d}	1.032 ± 0.036	1.07 ± 0.05	< 1
B_{B_s}	1.35 ± 0.08	1.30 ± 0.07	< 1

in general: average the Nf=2+1+1 and Nf=2+1 FLAG averages, through eq.(28) in arXiv:1403.4504

for B_K , f_{B_s} , f_{B_s}/f_{B_d} :

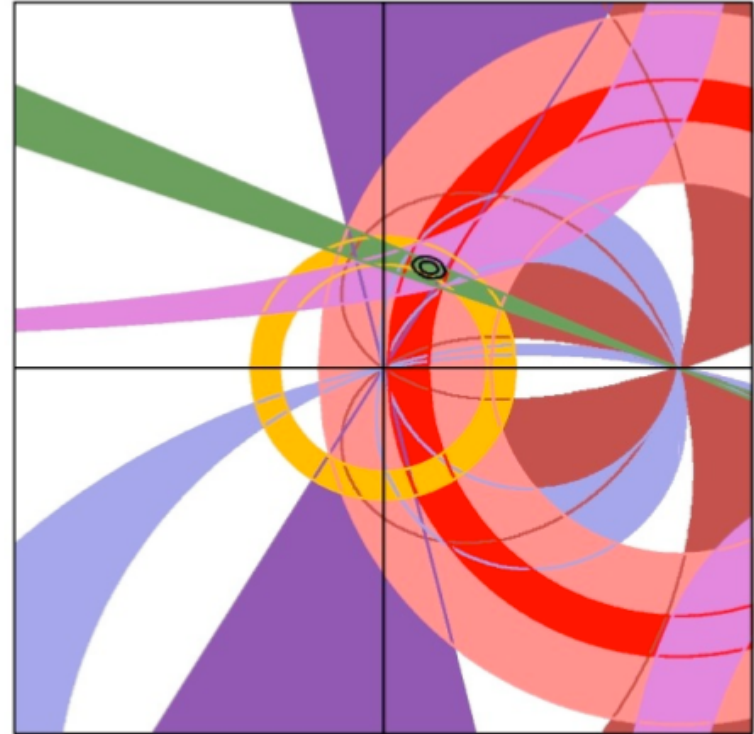
FLAG Nf=2+1+1 (single result) and Nf=2+1 average

for B_{B_s} , B_{B_s}/B_{B_d} :

update w.r.t. the Nf=2+1 FLAG average (no Nf=2+1+1 results yet)

updating the FNAL/MILC result to FNAL/MILC 2016 (1602.13560)

- *Future directions*



Long Distance Effects in Neutral Meson Mixing

N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1212.5931

Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1406.0916

Z.Bai (RBC-UKQCD), arXiv:1411.3210

exp

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 3.483(6) \times 10^{-12} \text{ MeV.}$$

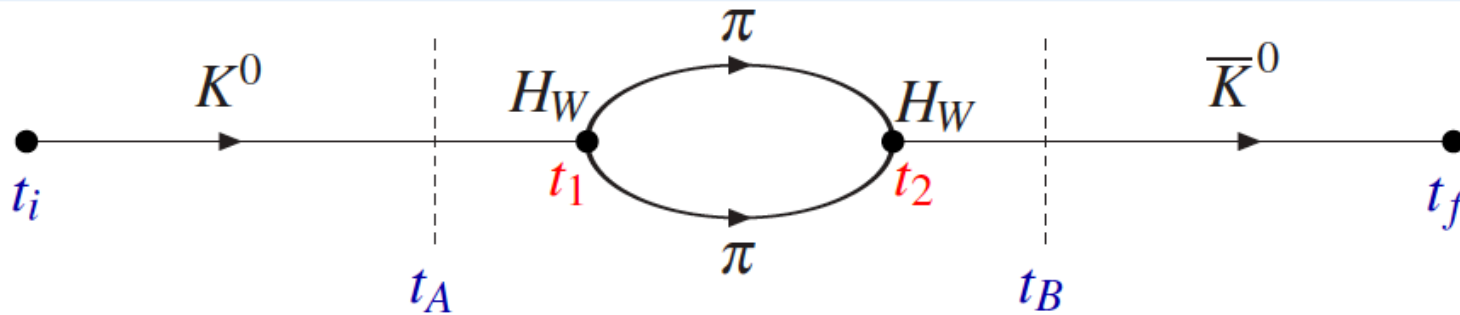
3.19(41)(96)
lattice unphysical
masses

- Historically led to the prediction of the energy scale of the charm quark.
Mohapatra, Rao & Marshak (1968); GIM (1970); Gaillard & Lee (1974)
- Tiny quantity \Rightarrow places strong constraints on BSM Physics.
- Within the standard model, Δm_K arises from $K^0 - \bar{K}^0$ mixing at second order in the weak interactions:

$$\Delta M_K = 2\mathcal{P} \sum_{\alpha} \frac{\langle \bar{K}^0 | H_W | \alpha \rangle \langle \alpha | H_W | K^0 \rangle}{m_K - E_{\alpha}},$$

where the sum over $|\alpha\rangle$ includes an energy-momentum integral.

Long Distance Effects in Neutral Meson Mixing



- Δm_K is given by

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2\mathcal{P} \sum_{\alpha} \frac{\langle \bar{K}^0 | \mathcal{H}_W | \alpha \rangle \langle \alpha | \mathcal{H}_W | K^0 \rangle}{m_K - E_{\alpha}} = 3.483(6) \times 10^{-12} \text{ MeV}.$$

$$\text{LQDC} = 3.19(41)(96) \cdot 10^{-12} \text{ MeV}$$

- The above correlation function gives ($T = t_B - t_A + 1$)

$$C_4(t_A, t_B; t_i, t_f) = |Z_K|^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | \mathcal{H}_W | n \rangle \langle n | \mathcal{H}_W | K^0 \rangle}{(m_K - E_n)^2} \times \left\{ e^{(M_K - E_n)T} - (m_K - E_n)T - 1 \right\}.$$

- From the coefficient of T we can therefore obtain

$$\Delta m_K^{\text{FV}} \equiv 2 \sum_n \frac{\langle \bar{K}^0 | \mathcal{H}_W | n \rangle \langle n | \mathcal{H}_W | K^0 \rangle}{(m_K - E_n)}.$$

Long Distance Effects in Neutral Meson Mixing

- The general formula can be written:

N.H.Christ, G.Martinelli & CTS, arXiv:1401.1362

N.H.Christ, X.Feng, G.Martinelli & CTS, arXiv:1504.01170

$$\Delta m_K = \Delta m_K^{\text{FV}} - 2\pi v \langle \bar{K}^0 | H | n_0 \rangle_V v \langle n_0 | H | K^0 \rangle_V \left[\cot \pi h \frac{dh}{dE} \right]_{m_K},$$

where $h(E, L)\pi \equiv \phi(q) + \delta(k)$.

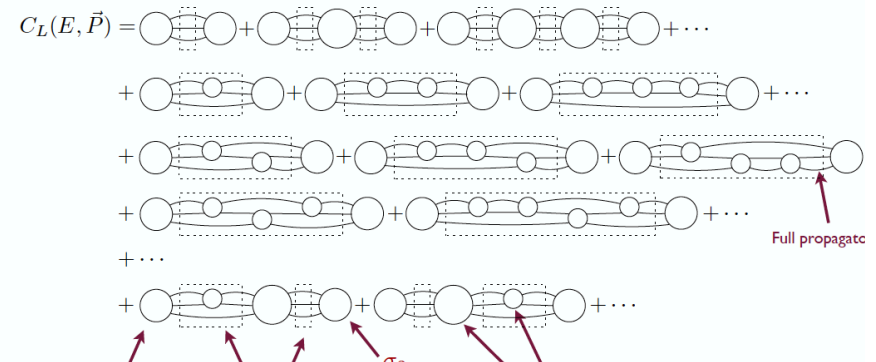
- This formula reproduces the result for the special case when the volume is such that there is a two-pion state with energy $= m_K$. N.H.Christ, arXiv:1012.6034
- Increasing the volumes keeping $h = n/2$ and thus avoiding the power corrections is an intriguing possibility.

***Within reasonable approximations
can be extended to D meson mixing***

***M. Ciuchini, V. Lubicz, L. Silvestrini, S. Simula
(progresses made by M. T. Hansen & S.
Sharpe, 1204.0826v4, 1409.7012v, 1504.04248v1)***

Also CPV in D $\rightarrow \pi\pi$ or KK

3-particle correlator



D MIXING

- D mixing is described by:
 - Dispersive $D \rightarrow \bar{D}$ amplitude M_{12}
 - SM: long-distance dominated, not calculable
 - NP: short distance, calculable w. lattice
 - Absorptive $D \rightarrow \bar{D}$ amplitude Γ_{12}
 - SM: long-distance, not calculable
 - NP: negligible
 - Observables: $|M_{12}|$, $|\Gamma_{12}|$, $\Phi_{12} = \arg(\Gamma_{12}/M_{12})$

Let us assume that the Standard Model contributions to M_{12} and Γ_{12} are real

"REAL SM" APPROXIMATION II

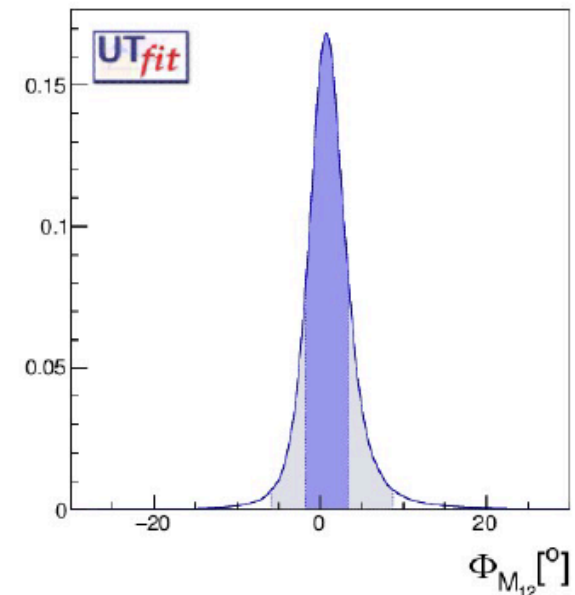
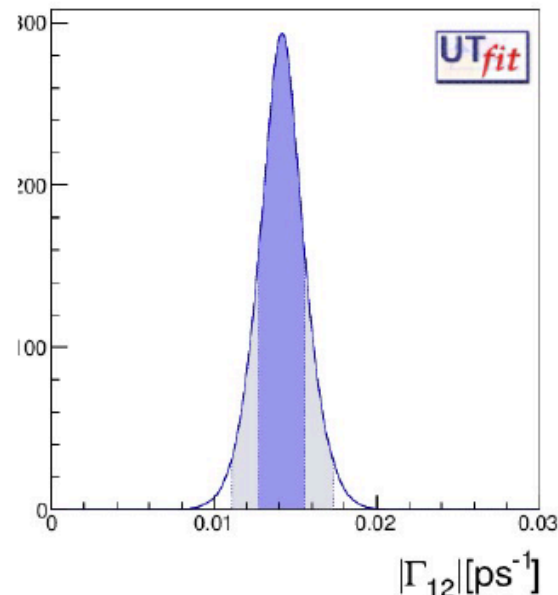
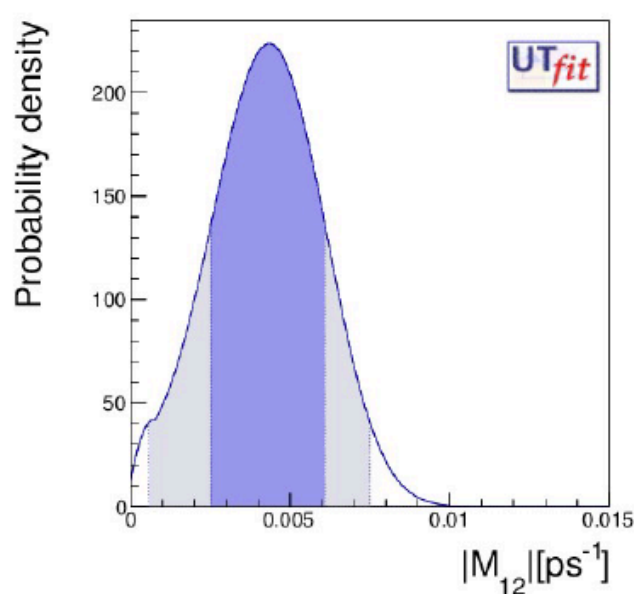
- Define $|D_{S,L}| = p|D^0| \pm q|D^0|$ and $\delta = (1 - |q/p|^2) / (1 + |q/p|^2)$. All observables can be written in terms of $x = \Delta m / \Gamma$, $y = \Delta \Gamma / 2\Gamma$ and δ , with

$$\begin{aligned} \sqrt{2} \Delta m &= \text{sign}(\cos \Phi_{12}) \sqrt{4|M_{12}|^2 - |\Gamma_{12}|^2 + \sqrt{(4|M_{12}|^2 + |\Gamma_{12}|^2)^2 - 16|M_{12}|^2|\Gamma_{12}|^2 \sin^2 \Phi_{12}}}, \\ \sqrt{2} \Delta \Gamma &= 2\sqrt{|\Gamma_{12}|^2 - 4|M_{12}|^2 + \sqrt{(4|M_{12}|^2 + |\Gamma_{12}|^2)^2 - 16|M_{12}|^2|\Gamma_{12}|^2 \sin^2 \Phi_{12}}}, \\ \delta &= \frac{2|M_{12}||\Gamma_{12}| \sin \Phi_{12}}{(\Delta m)^2 + |\Gamma_{12}|^2}, \end{aligned} \quad (7)$$

- Notice that $\phi = \arg(q/p) = \arg(y + i\delta x) - \arg \Gamma_{12}$
- $|q/p| \neq 1 \Leftrightarrow \phi \neq 0$ clear signals of NP

CPV IN CHARM MIXING

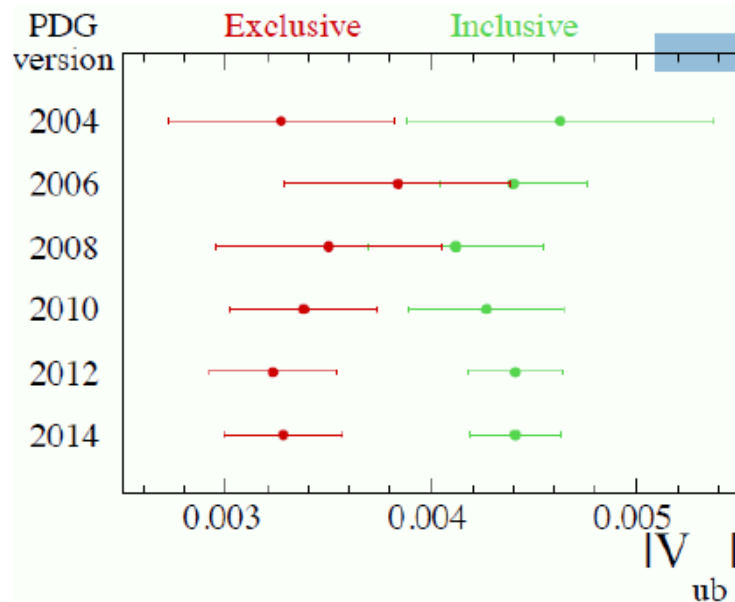
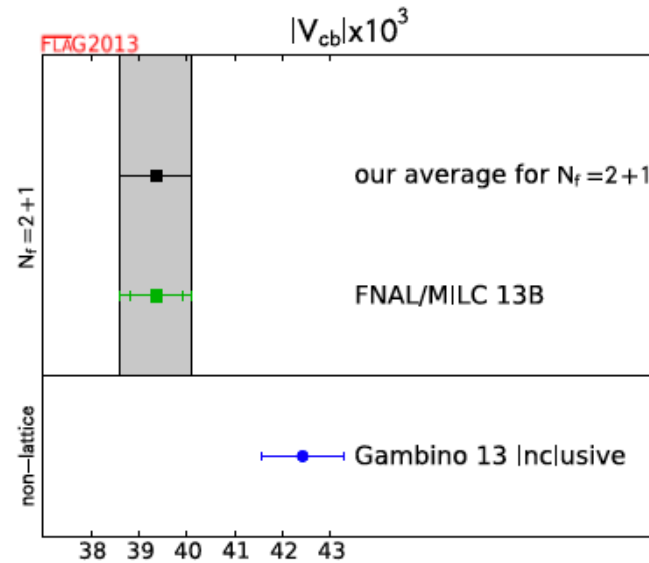
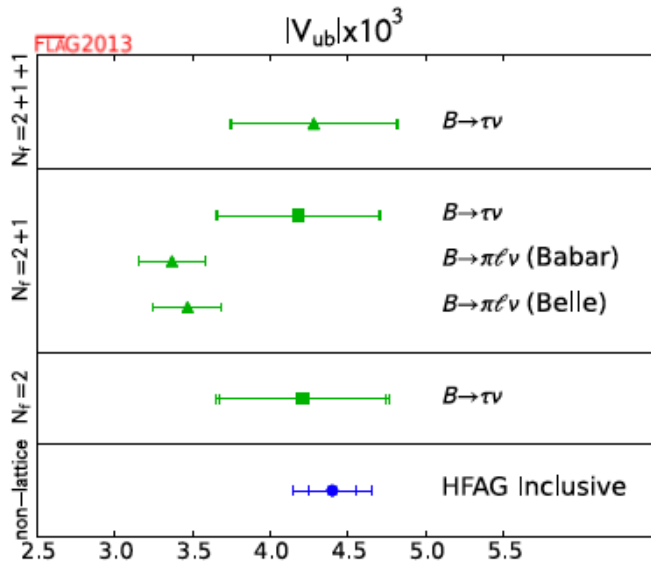
- Latest UTfit average (HFAG very similar):
 $x = (3.5 \pm 1.5) 10^{-3}$, $y = (5.8 \pm 0.6) 10^{-3}$,
 $|q/p|-1 = (0.7 \pm 1.8) 10^{-2}$, $\phi = \arg(q/p) = (0.20 \pm 0.56)^\circ$
 $|M_{12}| = (4 \pm 2)/fs$, $|\Gamma_{12}| = (14 \pm 1)/fs$, $\Phi_{12} = (0 \pm 3)^\circ$



Do we still care? Tensions and Unknowns

- 1) A “classical” example $B \rightarrow \tau \nu$
- 2) $|V_{ub}|$ and $|V_{cb}|$ inclusive vs exclusive
- 3) $|V_{cb}|$, B mixing and ϵ_K
- 4) D-mixing
- 5) $R(D)$ and $R(D^*)$
- 6) $B \rightarrow K^* \ell \ell$
- 7) Physics BSM ?

$|V_{ub}|$, $|V_{cb}|$



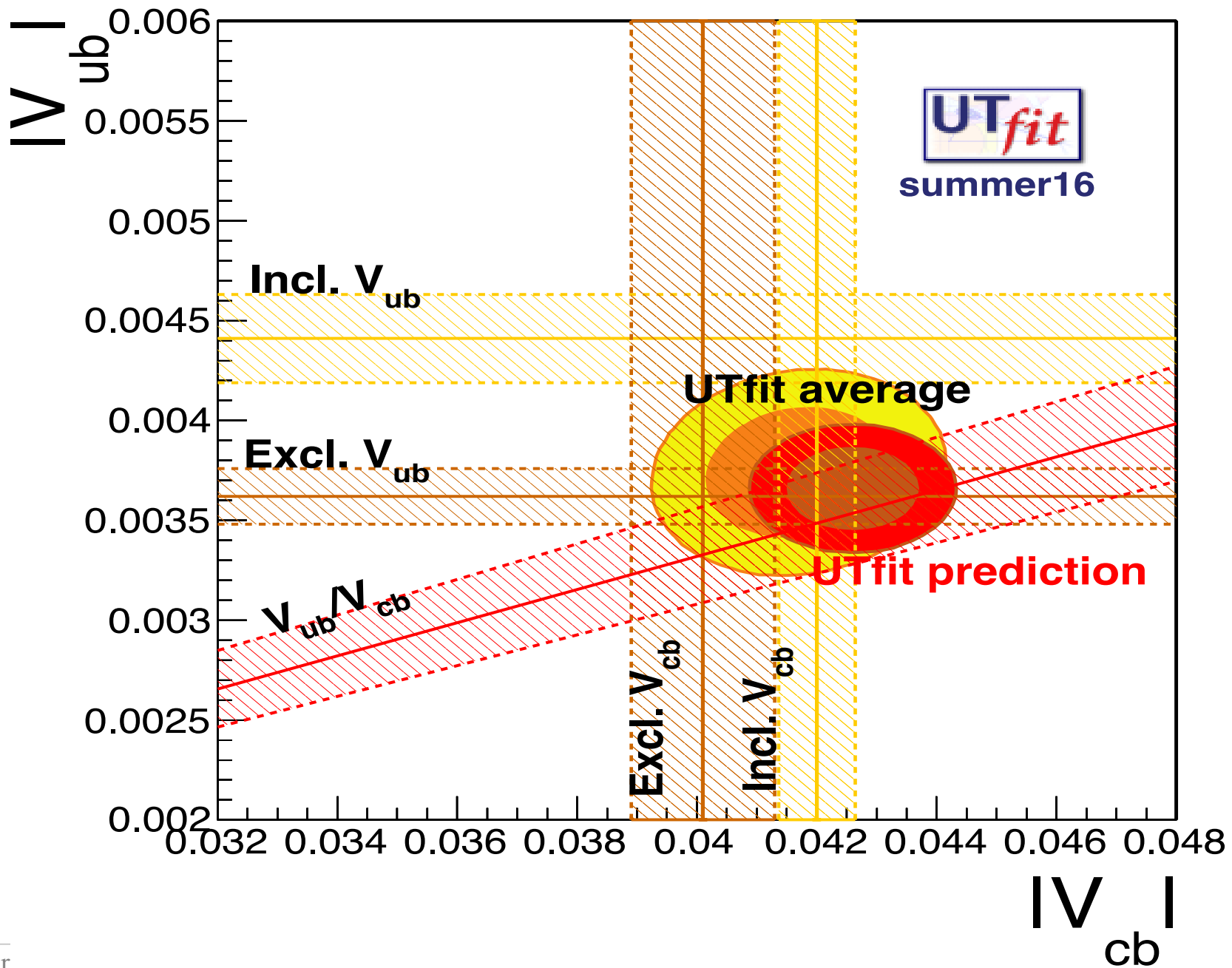
$V_{ub} \text{ Exclusive} = 0.00361 \pm 0.00013$

$V_{cb} \text{ Exclusive} = 0.0400 \pm 0.0011$

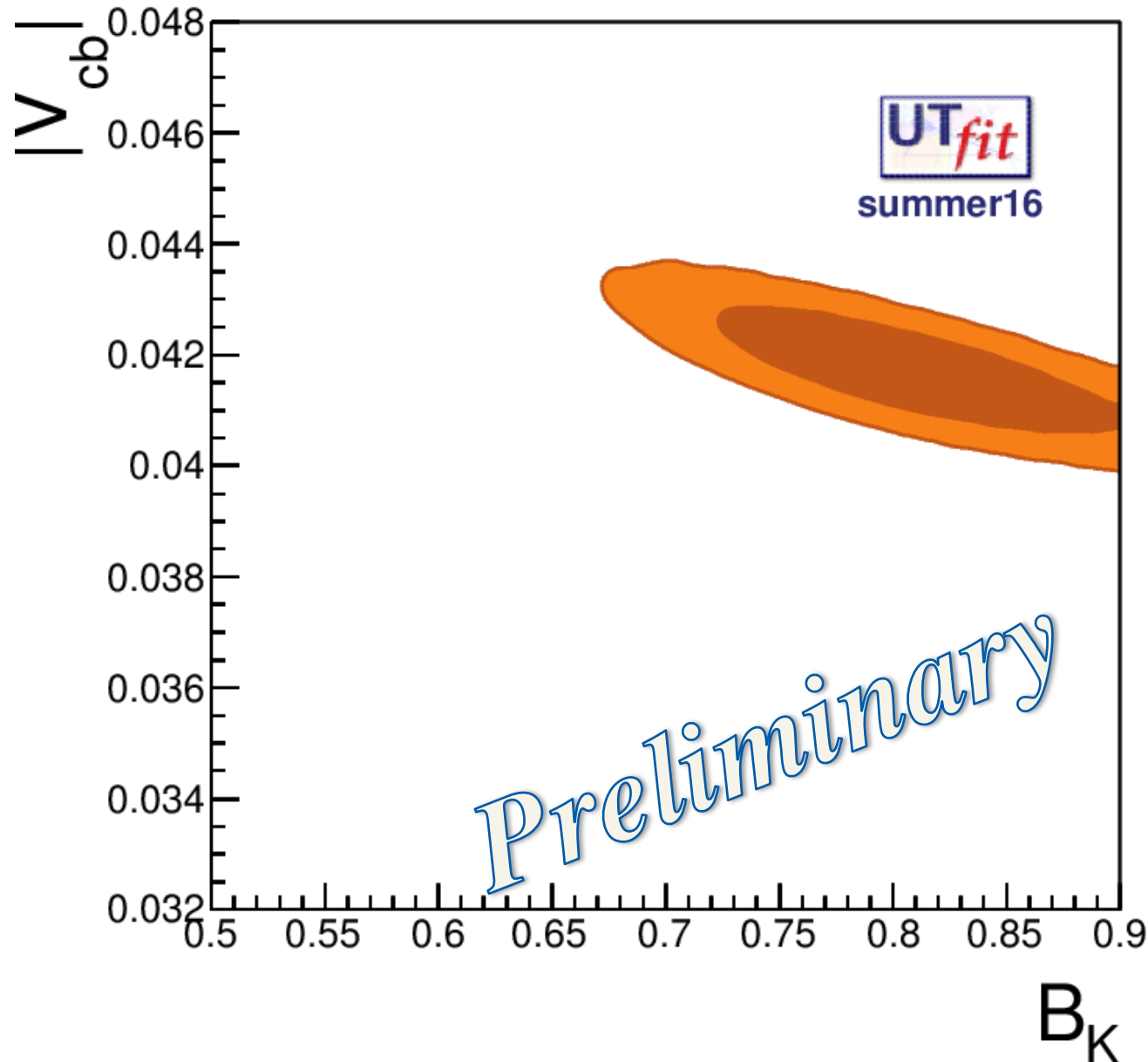
$V_{ub}/V_{cb} \text{ Exclusive} = 0.083 \pm 0.006$

$V_{ub} \text{ Inclusive} = 0.00440 \pm 0.00022$

$V_{cb} \text{ Inclusive} = 0.0420 \pm 0.0006$



UT-fit 2016 Correlation B_k vs V_{cb} in quest for theoretical improvement



- ϵ_K large V_{cb}
- B mixing with large lattice matrix elements small V_{cb}

2015

inclusives

vs

exclusives

$$V_{ub} \quad (4.40 \pm 0.22) \times 10^{-3}$$

$$(3.61 \pm 0.13) \times 10^{-3}$$

$$V_{cb} \quad (4.20 \pm 0.06) \times 10^{-2}$$

$$(4.00 \pm 0.11) \times 10^{-2}$$

$$V_{ub} \quad (3.73 \pm 0.21) \times 10^{-3}$$

$$V_{cb} \quad (4.17 \pm 0.10) \times 10^{-2}$$

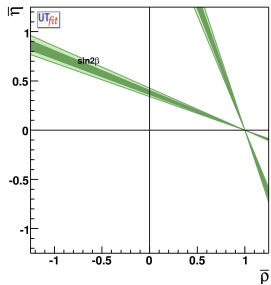
$$\sin 2\beta_{\text{exp}} =$$
$$0.680 \pm 0.023$$

$$\sin 2\beta_{\text{UTfit}} =$$
$$0.740 \pm 0.037$$
$$B_K = 0.81 \pm 0.07$$

$$\sin 2\beta_{\text{incl}} =$$
$$0.784 \pm 0.027$$
$$B_K = 0.74 \pm 0.05$$
$$(2015)$$

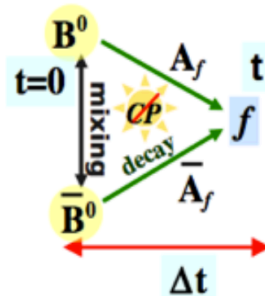
$$\sin 2\beta_{\text{excl}} =$$
$$0.703 \pm 0.021$$
$$B_K = 0.93 \pm 0.07$$
$$(2015)$$

Beta results



$B^0 \rightarrow J/\psi K^0$

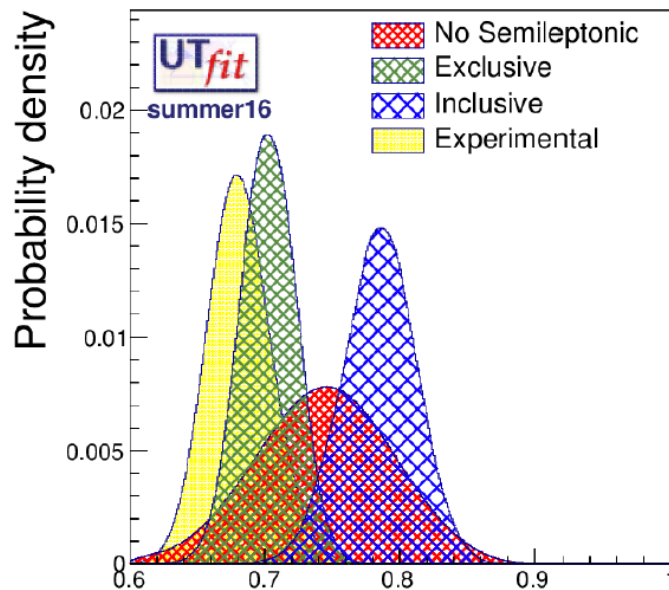
sin2β from time-dependent A_{CP} in $B \rightarrow J/\psi K$



$$a_{f_{CP}}(t) = \frac{\text{Prob}(B^0(t) \rightarrow f_{CP}) - \text{Prob}(\bar{B}^0(t) \rightarrow f_{CP})}{\text{Prob}(B^0(t) \rightarrow f_{CP}) + \text{Prob}(\bar{B}^0(t) \rightarrow f_{CP})} = C_f \cos \Delta m_d t + S_f \sin \Delta m_d t$$

The decay is dominated by a single (tree level) amplitude, thus a can be simplified:

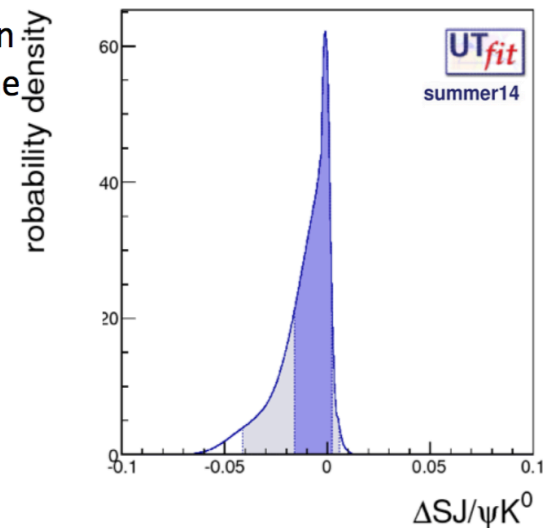
$$a_{f_{CP}}(t) = -\eta_{CP} \sin(\Delta m_d t) \sin 2\beta$$



We also analyse $\bar{B}^0 \rightarrow J/\psi \pi^0$ to obtain the theoretical uncertainty related to the penguin pollution in data-driven way. This gives us an additional correction:

data-driven theoretical uncertainty

$\Delta S \in [-0.02, 0.00]$ at 68% prob.



$$\sin(2\beta) = (0.680 \pm 0.023)$$

CKM Uncertainties

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (8.39 \pm 0.30) \cdot 10^{-11} \left[\frac{|V_{cb}|}{0.0407} \right]^{2.8} \left[\frac{\gamma}{73.2^\circ} \right]^{0.71}$$
$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.36 \pm 0.09) \cdot 10^{-11} \left[\frac{|V_{ub}|}{3.88 \cdot 10^{-3}} \right]^2 \left[\frac{|V_{cb}|}{0.0407} \right]^2 \left[\frac{\sin \gamma}{\sin(73.2)} \right]^2$$

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (65.3 \pm 3.1) \left[\bar{\text{Br}}(B_s \rightarrow \mu^+ \mu^-) \right]^{1.4} \left[\frac{\gamma}{70^\circ} \right]^{0.71} \left[\frac{227 \text{ MeV}}{F_{B_s}} \right]^{2.8}$$

A. Buras

AJB, Buttazzo,
Girrbach-Noe,
Knegjens
1503.02693

For $B_s \rightarrow \mu^+ \mu^-$ we use the formula from [56], slightly modified in [2]

$$\overline{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.65 \pm 0.06) \cdot 10^{-9} \left[\frac{m_t(m_t)}{163.5 \text{ GeV}} \right]^{3.02} \left[\frac{\alpha_s(M_Z)}{0.1184} \right]^{0.032} R_s$$

where

$$R_s = \left[\frac{F_{B_s}}{227.7 \text{ MeV}} \right]^2 \left[\frac{\tau_{B_s}}{1.516 \text{ ps}} \right] \left[\frac{0.938}{r(y_s)} \right] \left[\frac{|V_{ts}|}{41.5 \cdot 10^{-3}} \right]^2.$$

Now,

$$|V_{td}| = |V_{us}| |V_{cb}| R_t, \quad |V_{ts}| = \eta_R |V_{cb}|$$

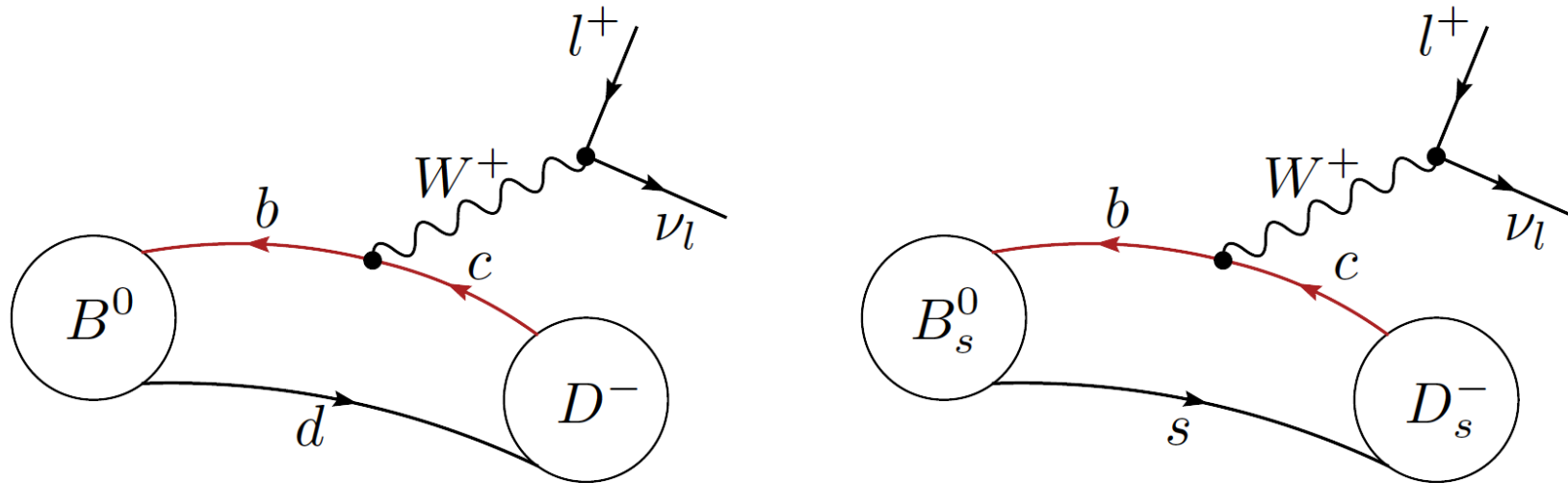
with R_t being one of the sides of the unitarity triangle (see Fig. 1) and

$$\eta_R = 1 - |V_{us}| \xi \sqrt{\frac{\Delta M_d}{\Delta M_s}} \sqrt{\frac{m_{B_s}}{m_{B_d}}} \cos \beta + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4) = 0.9825,$$

Do we still care? Tensions and Unknowns

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- 3) $|V_{cb}|$, B mixing and ϵ_K
- 4) D-mixing (already discussed)
- 5) $R(D)$ and $R(D^*)$ (and V_{cb} of course)
- 6) $B \rightarrow K^* \ell \ell$
- 7) Physics BSM ?

B semileptonic decay: $|V_{cb}|$

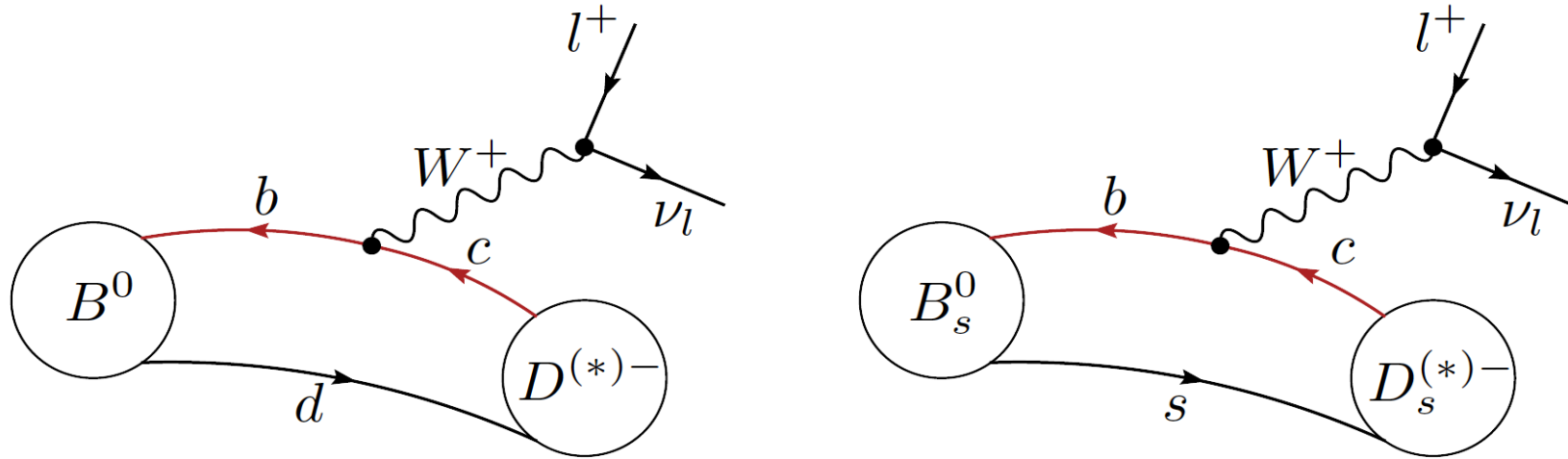


$$\frac{d\Gamma(B_{(s)} \rightarrow Pl\nu)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 \right. \\ \left. + \frac{3m_l^2}{8q^2} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

e, μ suppressed ←

uncertainties from kinematical factors / neglected h.o. OPE at the permille level

B semileptonic decay: $|V_{cb}|$



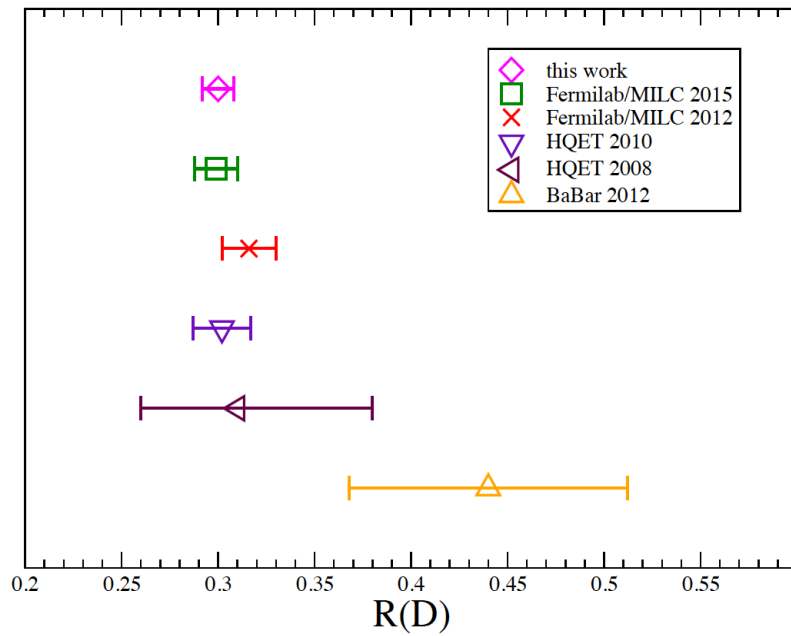
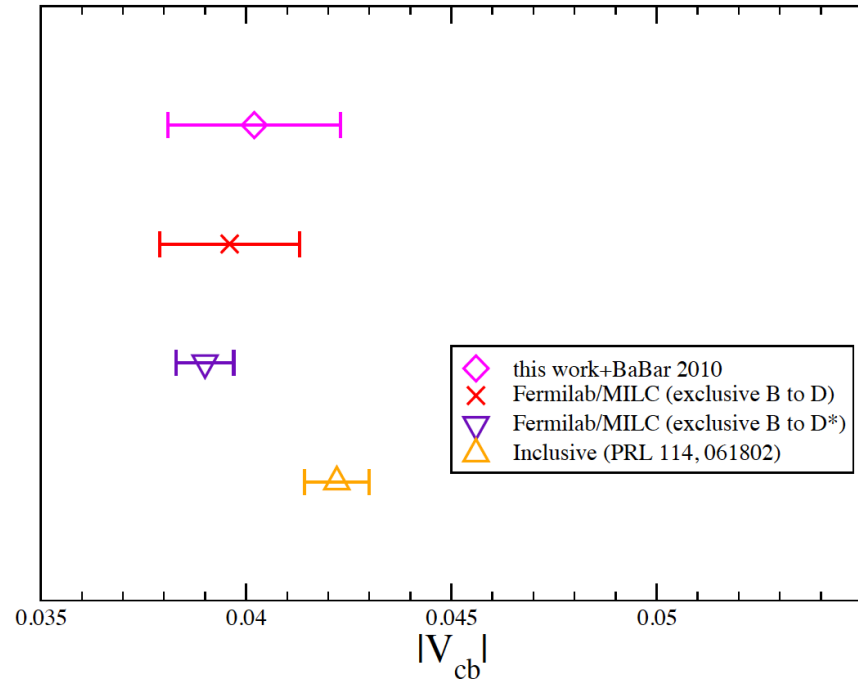
$$\frac{d\Gamma(B \rightarrow D l \nu_l)}{dw} = \frac{G_F^2}{48\pi^3} (m_B + m_D)^2 (w^2 - 1)^{3/2} |\eta_{EW}|^2 |V_{cb}|^2 |\mathcal{G}(w)|^2 + \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$\frac{d\Gamma(B \rightarrow D^* l \nu_l)}{dw} = \frac{G_F^2}{4\pi^3} (m_B - m_{D^*})^2 (w^2 - 1)^{1/2} |\eta_{EW}|^2 \chi(w) |V_{cb}|^2 |\mathcal{F}(w)|^2 + \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$w = \frac{p_B \cdot p_{D^*}}{m_B m_{D^*}} \qquad \mathcal{G}(w) = \frac{4 \frac{m_D}{m_B}}{1 + \frac{m_D}{m_B}} f_+(q^2) \quad \text{etc}$$

Low recoil region ($w=1$) accessible to lattice calculations

HPQCD June 13 2016

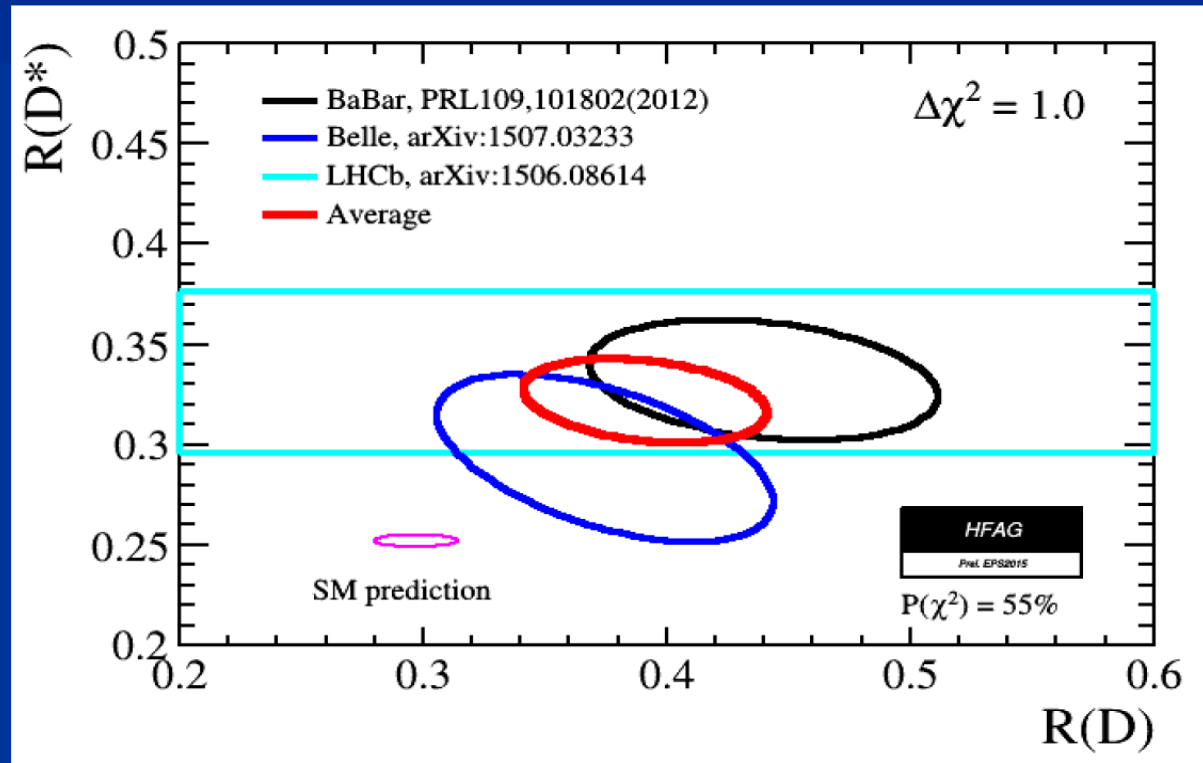


Tauonic B decays

Crivellin 2016

- Tree-level decays in the SM via W-boson

$$R(D^{(*)}) = B \rightarrow D^{(*)} \tau \nu / B \rightarrow D^{(*)} \ell \nu$$



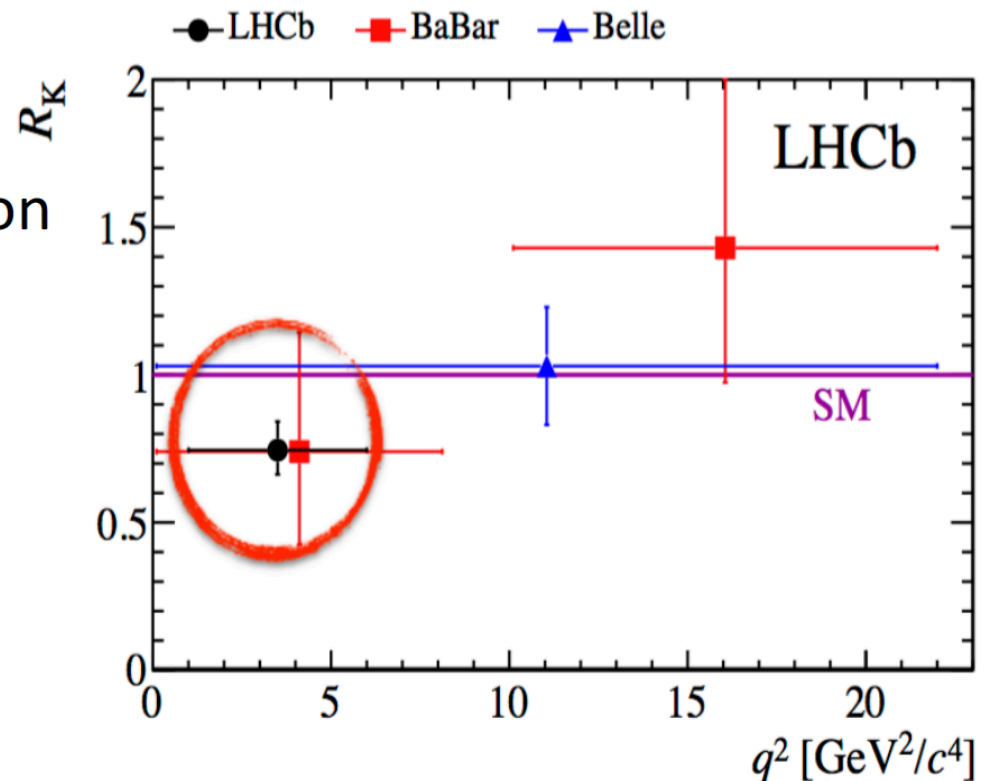
➔ Combined $\approx 4 \sigma$ deviation

More LFU tests

- Ratio (R_K) of branching fractions of $B^+ \rightarrow K^+ \mu^+ \mu^-$ to $B^+ \rightarrow K^+ e^+ e^-$ expected to be unity in the SM with excellent precision

$$R_K = \frac{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\Gamma[B^+ \rightarrow K^+ \mu^+ \mu^-]}{dq^2} dq^2}{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\Gamma[B^+ \rightarrow K^+ e^+ e^-]}{dq^2} dq^2}$$

- Observation of LFU violation would be a clear sign of New Physics
- LHCb observed a 2.6σ deviation from SM in the low q^2 region
- New measurements expected soon, e.g. R_{K^*}



Phys. Rev. Lett. 113 (2014) 151601

Breaking of Lepton Flavor Universality in B decays ?

Greljo, Isidori, Marzocca

Crivellin

etc.

$|V_{ub}|$ & $|V_{cb}|$ inclusive vs exclusive and all that

- 1) On the long run exclusive decays based on non-perturbative (lattice) determination of the relevant form factors will win;
- 2) The precision of the theoretical predictions for inclusive decays cannot be improved (are the present quoted errors reliable?);
- 3) Still (much) more work is needed, and different approaches to the physical B should be used and compared;
- 4) R(D) and R(D*) is an open problem; more lattice collaborations should work on these calculations;
- 5) Theoretical calculations and experimental analyses should not be biased by the HQFT - after all $\Lambda_{\text{QCD}}/m_c \approx O(1)$;
- 6) I hope to be wrong, but the possibility of new physics in tree level $b \rightarrow c$ decays looks to me quite remote.

Do we still care? Tensions and Unknowns

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- 3) $|V_{cb}|$, B mixing and ϵ_K
- 4) D-mixing (already discussed)
- 5) $R(D)$ and $R(D^*)$ (and V_{cb} of course)
- 6) $B \rightarrow K^* \ell \ell$
- 7) Physics BSM ?

The differential decay rate of the process $\bar{B}_d \rightarrow \bar{K}^*(\rightarrow K\pi)\ell^+\ell^-$ can be written as:

$$\begin{aligned} \frac{d^4\Gamma(\bar{B}_d)}{dq^2 d\cos\theta_K d\cos\theta_l d\phi} = & \frac{9}{32\pi} \left[J_{1s} \sin^2\theta_K + J_{1c} \cos^2\theta_K + (J_{2s} \sin^2\theta_K + J_{2c} \cos^2\theta_K) \cos 2\theta_l \right. \\ & + J_3 \sin^2\theta_K \sin^2\theta_l \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_K \sin \theta_l \cos \phi \\ & + (J_{6s} \sin^2\theta_K + J_{6c} \cos^2\theta_K) \cos \theta_l + J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi \\ & \left. + J_9 \sin^2\theta_K \sin^2\theta_l \sin 2\phi \right], \end{aligned} \quad (3)$$

where the kinematical variables ϕ , θ_ℓ , θ_K , q^2 are defined as in Refs. [17, 22, 24] : θ_ℓ and θ_K describe the angles of emission between \bar{K}^{*0} and ℓ^- (in the di-meson rest frame) and between \bar{K}^{*0} and K^- (in the di-hadron rest frame) respectively, whereas ϕ corresponds to the angle between the di-lepton and di-meson planes and q^2 to the di-lepton invariant mass. The decay rate $\bar{\Gamma}$ of the CP-conjugated process $B_d \rightarrow K^*(\rightarrow K\pi)\ell^+\ell^-$ is obtained from Eq. (3) by replacing $J_{1,2,3,4,7} \rightarrow \bar{J}_{1,2,3,4,7}$ and $J_{5,6,8,9} \rightarrow -\bar{J}_{5,6,8,9}$, where \bar{J} is equal to J with all weak phases conjugated. This convention corresponds to taking the same lepton ℓ^- for the definition of θ_ℓ for both B and \bar{B} decays (see for example Ref. [27]). The usual

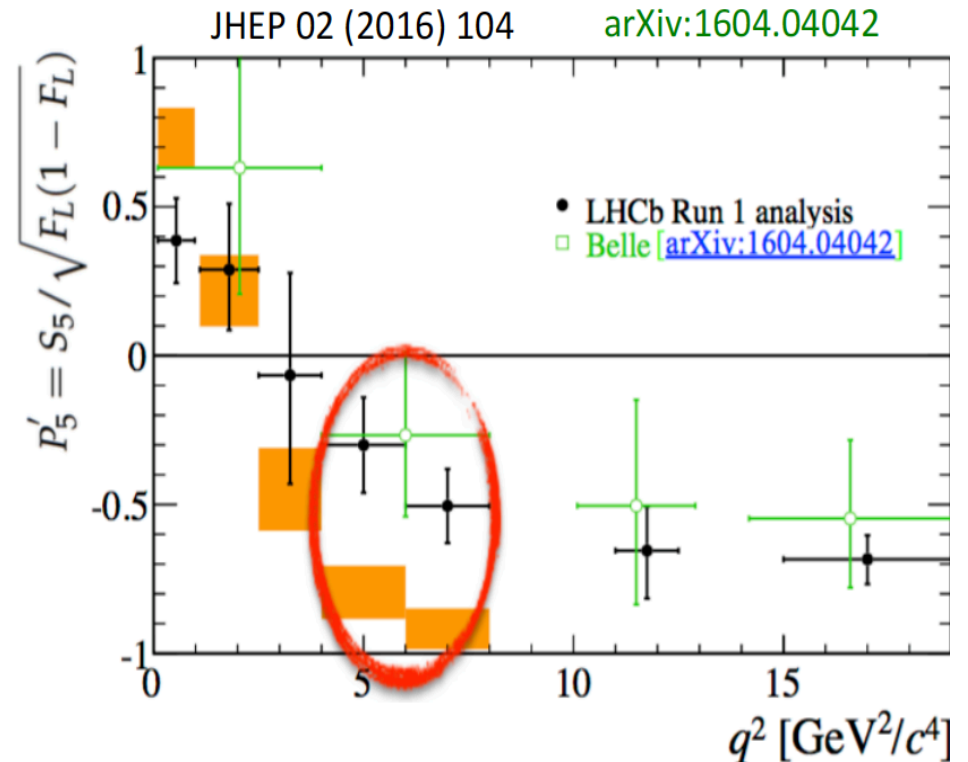
$$\langle P'_5 \rangle_{\text{bin}} = \frac{1}{2\mathcal{N}'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_5 + \bar{J}_5], \quad \langle P'_5{}^{\text{CP}} \rangle_{\text{bin}} = \frac{1}{2\mathcal{N}'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_5 - \bar{J}_5], \quad (26)$$

$$\langle P'_6 \rangle_{\text{bin}} = \frac{-1}{2\mathcal{N}'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_7 + \bar{J}_7], \quad \langle P'_6{}^{\text{CP}} \rangle_{\text{bin}} = \frac{-1}{2\mathcal{N}'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_7 - \bar{J}_7], \quad (27)$$

Angular analysis of $B^0 \rightarrow K^* \mu^+ \mu^-$

- Well established “anomaly”

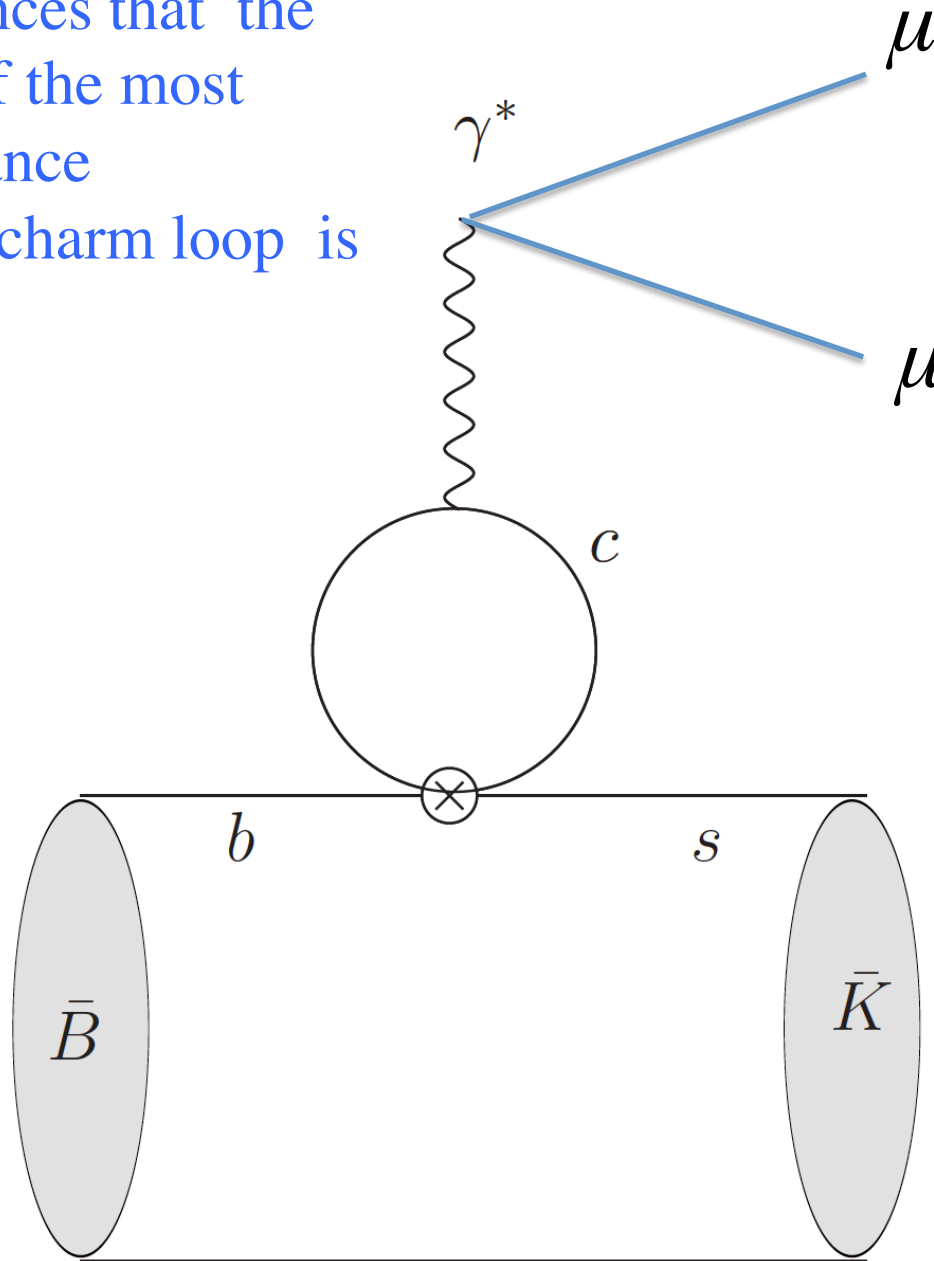
- Observables are q^2 (dimuon mass squared) and 3 angles
- Angular distributions provide many observables sensitive to different sources of New Physics see e.g. [JHEP 05 \(2013\) 137](#)



- Some global theoretical fits require **non-SM contributions** to accommodate the data see e.g. [JHEP 06 \(2016\) 092](#)
- However, genuine QCD effects can also be an explanation
→ **more efforts needed to clarify the picture** see e.g. [JHEP 06 \(2016\) 116](#)

There are good chances that the lattice calculation of the most important long distance contributions via a charm loop is possible

M. Ciuchini,
V.Lubicz, G.M.,
L. Silvestrini,
S. Simula



RADIATIVE/RARE KAON DECAYS

*G. Isidori, G. M., and P. Turchetti, Phys.Lett. B633, 75 (2006),
arXiv:hep-lat/0506026*

N.H. Christ X. Feng A. Portelli and C.T. Sachrajda *Phys.Rev. D92*
(2015) no.9, 094512 [10.1103/PhysRevD.92.094512](https://doi.org/10.1103/PhysRevD.92.094512) *

$$K \rightarrow \pi l^+ l^- \qquad K \rightarrow \pi \nu \bar{\nu}$$

Conserved currents and GIM important

2.1 $K \rightarrow \pi \ell^+ \ell^-$

G. Isidori, G. M., and P. Turchetti

The main non-perturbative correlators relevant for these decays are those with the electromagnetic current. In particular, the relevant T -product in Minkowski space is [7, 8]

$$(\mathcal{T}_i^j)_{\text{em}}^\mu(q^2) = -i \int d^4x e^{-iq \cdot x} \langle \pi^j(p) | T \{ J_{\text{em}}^\mu(x) [Q_i^u(0) - Q_i^c(0)] \} | K^j(k) \rangle , \quad (11)$$

$$J_{\text{em}}^\mu = \frac{2}{3} \sum_{q=u,c} \bar{q} \gamma^\mu q - \frac{1}{3} \sum_{q=d,s} \bar{q} \gamma^\mu q \quad (12)$$

for $i = 1, 2$ and $j = +, 0$. Thanks to gauge invariance we can write

$$(\mathcal{T}_i^j)_{\text{em}}^\mu(q^2) = \frac{w_i^j(q^2)}{(4\pi)^2} [q^2(k+p)^\mu - (m_k^2 - m_\pi^2)q^\mu] . \quad (13)$$

The normalization of (13) is such that the $O(1)$ scale-independent low-energy couplings $a_{+,0}$ defined in [8] can be expressed as

$$a_j = \frac{1}{\sqrt{2}} V_{us}^* V_{ud} \left[C_1 w_1^j(0) + C_2 w_2^j(0) + \frac{2N_j}{\sin^2 \theta_W} f_+(0) C_{7V} \right] . \quad (14)$$

A detailed analysis of the extraction of the amplitude from lattice correlators
by N.H. Christ X. Feng A. Portelli and C.T. Sachrajda

**Is the present picture showing a
Model Standardissimo ?**

An evidence, an evidence, my kingdom for an evidence

From Shakespeare's *Richard III*
and A. Stocchi

- 1) Fit of NP- $\Delta F=2$ parameters in a Model
“independent” way***
- 2) “Scale” analysis in $\Delta F=2$ ***



.... beyond the Standard Model

UT Analysis:

- **Model independent analysis**
- **Limits on the deviations**
- **NP scale update**

Results from a fit to the Wilson Coefficients

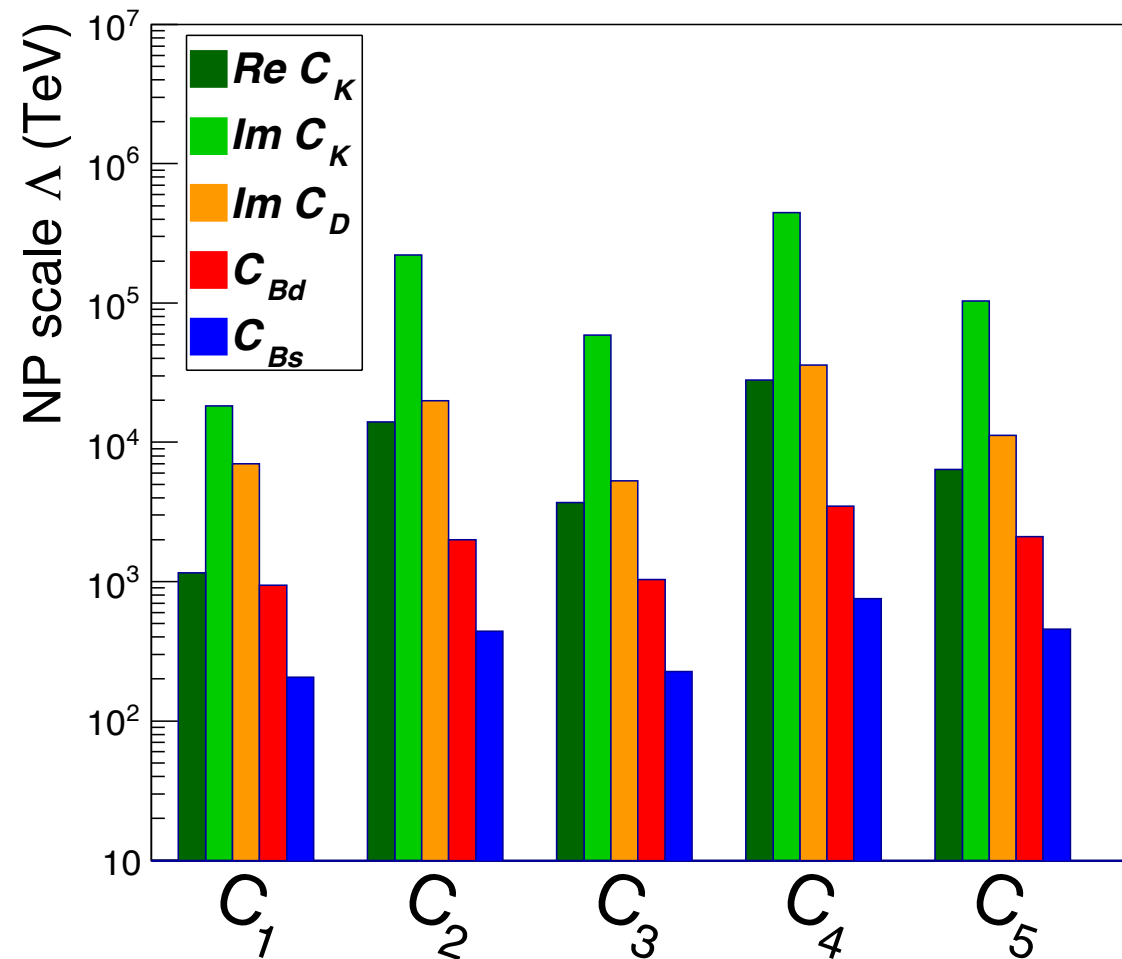
Results obtained with $L=1$ corresponding to tree level NP effects and an arbitrary flavor structure

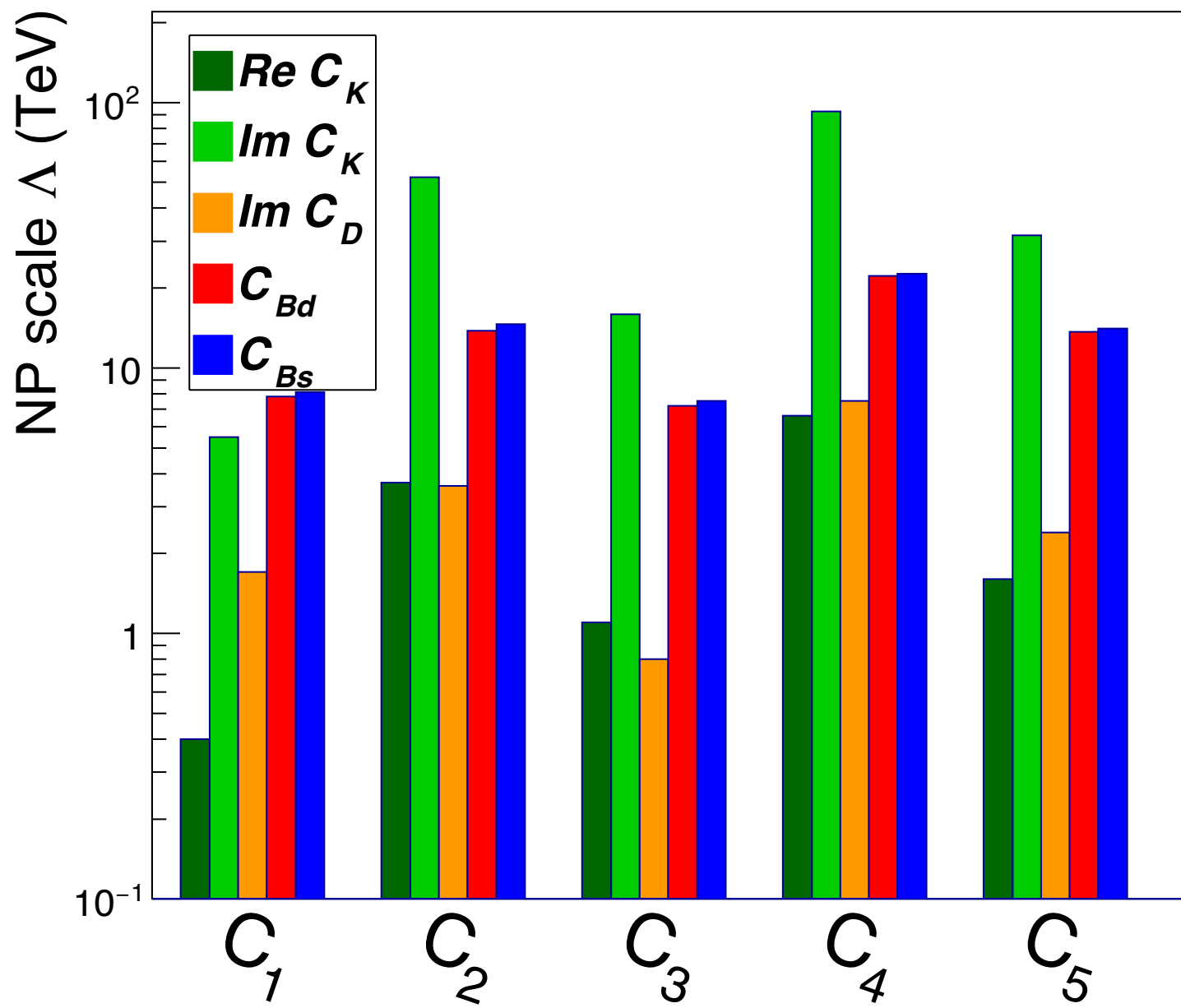
$$\varepsilon_K \quad \Lambda = 5 \cdot 10^5 \text{ TeV}$$

$$D \quad \Lambda = 10^4 \text{ TeV}$$

$$B_d \quad \Lambda = 3 \cdot 10^3 \text{ TeV}$$

$$B_s \quad \Lambda = 8 \cdot 10^2 \text{ TeV}$$





NMFV

This is my last paper with Guido

1. Failure of local duality in inclusive nonleptonic heavy flavor decays

Guido Altarelli (CERN & Rome III U.), G. Martinelli, S. Petrarca, F. Rapuano (Rome U. & INFN, Rome). Mar 1996. 9 pp.

Published in **Phys.Lett. B382 (1996) 409-414**

CERN-TH-96-77, ROME1-1143-96

DOI: [10.1016/0370-2693\(96\)00637-5](https://doi.org/10.1016/0370-2693(96)00637-5)

e-Print: [hep-ph/9604202](https://arxiv.org/abs/hep-ph/9604202) | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

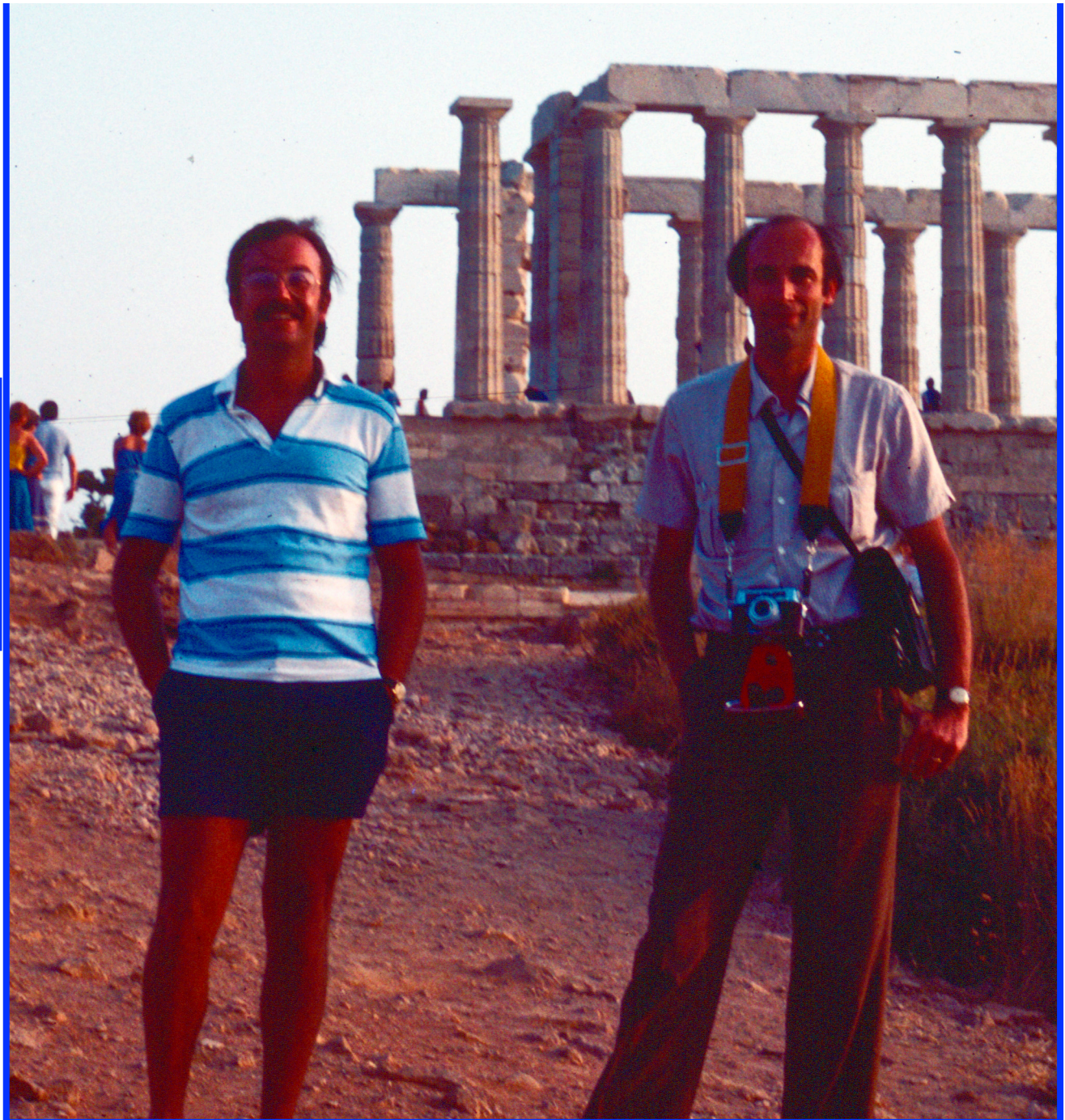
[CERN Document Server](#); [ADS Abstract Service](#)

[Detailed record](#) - [Cited by 62 records](#) 50+

but our friendship continued untouched.

Some personal souvenir....

WE WERE A
LITTLE
YOUNGER
THOUGH !!



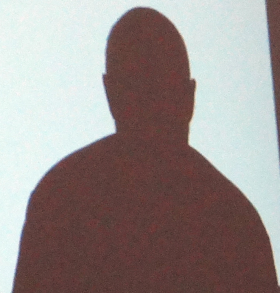
A NICE GROUP AT WORK: Manuel Greco,
myself, GUIDO, Keith Ellis, Mario Greco
Guess who is the non-Italian !





Roma 2012

Happy Birthday Guido!!



Singapore 2014



Per i giovani in generale, e per gli studenti di Dottorato in Fisica in particolare, Guido Altarelli è un esempio a cui ispirarsi:

un grandissimo scienziato, una persona di caratura morale eccezionale, pieno di calore umano, simpatia, gentilezza e integrità.

Chi ha avuto il privilegio di collaborarci o semplicemente di conoscerlo continuerà a ricordarlo con ammirazione e rispetto.

Noi, che di Guido siamo stati amici e gli abbiamo voluto bene, non lo dimenticheremo.



THANKS FOR YOUR ATTENTION



International School for Advanced Studies

