A tribute to the memory of Guido Altarelli

Roma, 19 December 2016

Neutrinos Today: an introduction

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Plan

(I) Masses, Mixing and Oscillations:
the data

(II) Implication for the Physics
Beyond the Standard Model

Lecture I
Masses, Mixing and Oscillations: the data

Two-flavour neutrino oscillations in vacuum (ν_e,ν_μ)

here ν_e are produced with average energy E

source

$\longleftarrow$ L $\longrightarrow$

+---+

detector

here we measure

$$P_{ee} \equiv P(\nu_e \rightarrow \nu_e)$$

neutrino interaction eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}}_{U}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{l}_L \gamma^\mu \nu_l$$

as before, but

$$\gamma/2 = \vartheta$$

$$t \approx L$$

$$E_2 - E_1 = \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_2^2} \approx \frac{m_2^2}{2E} - \frac{m_1^2}{2E} \equiv \frac{\Delta m_{21}^2}{2E}$$

$$P_{ee} = \left|\langle \nu_e | \psi(L)\rangle\right|^2 = 1 - \underbrace{4|U_{e1}|^2|U_{e2}|^2}_{\sin^2 2\vartheta} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

no dependence on the phase α more on this later on ….

to see any effect, if Δm² is tiny, we need both θ and L/E large

regimes

$$P_{ee} = |\langle \nu_e | \psi(L) \rangle|^2 = 1 - \underbrace{4|U_{e1}|^2 |U_{e2}|^2}_{\sin^2 2\vartheta} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

$$\frac{\Delta m^2 L}{4E} << 1 \qquad\qquad P_{ee} \approx 1$$

$$\frac{\Delta m^2 L}{4E} >> 1 \qquad \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \approx \frac{1}{2} \qquad P_{ee} \approx 1 - \frac{\sin^2 2\vartheta}{2}$$

by averaging over ν_e energy at the source

$$\frac{\Delta m^2 L}{4E} \approx 1 \qquad\qquad P_{ee} = P_{ee}(E)$$

useful relation $\frac{\Delta m^2 L}{4E} \approx 1.27 \left(\frac{\Delta m^2}{1\, eV^2}\right)\left(\frac{L}{1\, Km}\right)\left(\frac{E}{1\, GeV}\right)^{-1}$

source	L(km)	E(GeV)	$\Delta m^2 (eV^2)$
ν_e, ν_μ (atmosphere)	10^4 (Earth diameter)	1-10	10^{-4} - 10^{-3}
anti- ν_e (reactor)	1	10^{-3}	10^{-3}
anti- ν_e (reactor)	100	10^{-3}	10^{-5}
ν_e (sun)	10^8	10^{-3} - 10^{-2}	10^{-11} - 10^{-10}

neglecting matter effects

Three-flavour neutrino oscillations (ν_e,ν_μ,ν_τ)

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{k>j} \mathrm{Re}(U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j})\sin^2(\frac{\Delta m^2_{kj}L}{4E}) + 2\sum_{k>j} \mathrm{Im}(U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j})\sin(\frac{\Delta m^2_{kj}L}{2E})$$

$P_{\bar\alpha\bar\beta} = P_{\alpha\beta}(U \to U^*)$

CP violation controlled by the Jarlskog invariant

$J = \mathrm{Im}(U^*_{\alpha i}U_{\beta i}U_{\alpha k}U^*_{\beta k})$

$P_{\bar\beta\bar\alpha} = P_{\alpha\beta} \quad (CPT) \quad \Rightarrow \quad P_{\bar\alpha\bar\alpha} = P_{\alpha\alpha}$

no sensitivity to CP violation in disappearance experiments

$P_{\alpha\beta}$ invariant under $U_{\alpha k} \to e^{i\vartheta_\alpha}U_{\alpha k}e^{\varphi_k}$

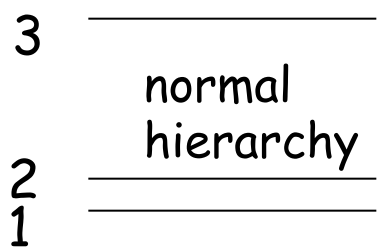
$P_{\alpha\beta}$ only depends on N(N-1)=6 parameters

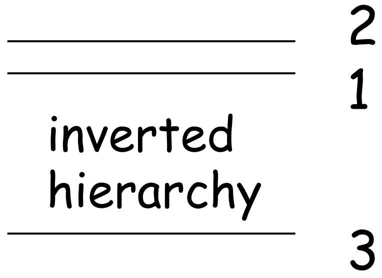
conventions: $[\Delta m^2_{ij} \equiv m_i^2 - m_j^2]$

$m_1 < m_2$

$\Delta m^2_{21} < \left|\Delta m^2_{32}\right|, \left|\Delta m^2_{31}\right|$ i.e. 1 and 2 are, by definition, the closest levels

two possibilities:





Mixing matrix U=U_{PMNS} (Pontecorvo,Maki,Nakagawa,Sakata)

neutrino interaction eigenstates

$$\nu_f = \sum_{i=1}^{3} U_{fi} \nu_i$$

$$(f = e,\mu,\tau)$$

neutrino mass eigenstates

U is a 3 x 3 unitary matrix
standard parametrization

$$U_{PMNS} = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{13}\,s_{23}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{13}\,s_{23}\,e^{-i\delta} & c_{13}\,s_{23} \\ -c_{12}\,s_{13}\,c_{23}\,e^{-i\delta} + s_{12}\,s_{23} & -s_{12}\,s_{13}\,c_{23}\,e^{-i\delta} - c_{12}\,s_{23} & c_{13}\,c_{23} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

$$c_{12} \equiv \cos\vartheta_{12},\ldots$$

three mixing angles $\vartheta_{12},\quad \vartheta_{13},\quad \vartheta_{23}$

three phases (in the most general case) δ $\qquad \underbrace{\alpha,\quad \beta}_{\text{do not enter}}$ $P_{ff'} = P(\nu_f \to \nu_{f'})$

oscillations can only test 6 combinations

$\Delta m^2_{21},\Delta m^2_{32},$ $\vartheta_{12},\quad \vartheta_{13},\quad \vartheta_{23}$ δ

Analysis of Oscillations Data

we anticipate that there are two small parameters

$$|\alpha| \equiv \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right| \approx 0.03$$

$$\Delta m^2_{21} << |\Delta m^2_{32}|, |\Delta m^2_{31}|$$

$$|U_{e3}|^2 \approx \sin^2\vartheta_{13} \approx 0.02$$

we first consider experiments not sensitive to Δm^2_{21} (L not very large, E not very small) and we set $\Delta m^2_{21} = 0$

EXERCISE
derive $P_{ee}, P_{\mu\mu}, P_{\mu e}$ in the limit $\Delta m^2_{21} = 0$ (vacuum osc., no matter effects)

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E} \qquad \Delta \equiv \frac{\Delta m^2_{13} L}{4E} \quad [\Delta m^2_{21} = 0]$$

$$P_{ee} = 1 - 4|U_{e3}|^2 (1 - |U_{e3}|^2) \sin^2\Delta$$

$$P_{\mu\mu} = 1 - 4|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) \sin^2\Delta$$

$$P_{\mu e} = P_{e\mu} = 4|U_{\mu 3}|^2 |U_{e3}|^2 \sin^2\Delta$$

similarly, $P_{\tau\tau}, P_{\tau\mu}, P_{\mu\tau}, P_{\tau e}, P_{e\tau}$ only depend on U_{f3} and Δ for $\Delta m^2_{21} = 0$

we are testing the third column

$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & U_{e3} \\ \cdot & \cdot & U_{\mu 3} \\ \cdot & \cdot & U_{\tau 3} \end{pmatrix}$$

we also consider the limit $\vartheta_{13} = 0$
we are left with one frequency and one mixing angle

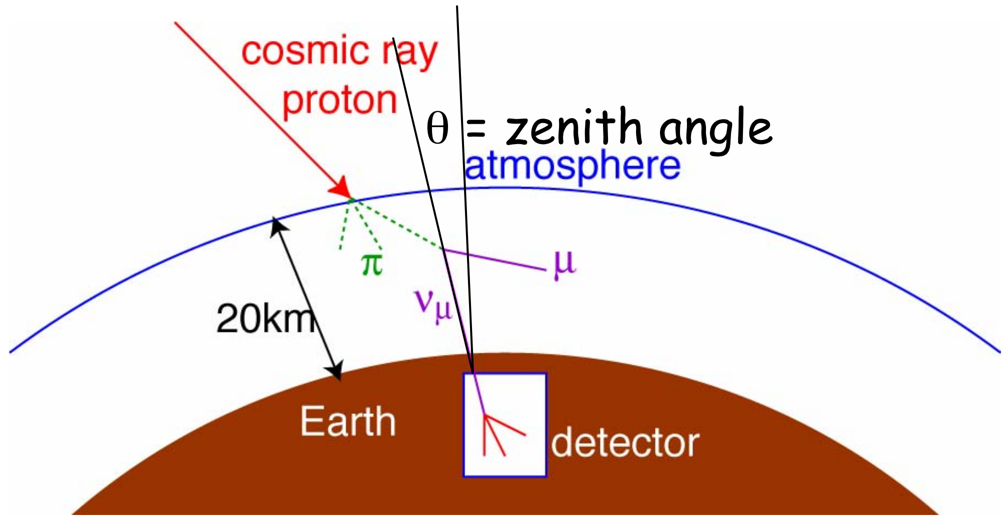
$$|U_{e3}|^2 \approx \sin^2 \vartheta_{13} \approx 0$$

$$P_{ee} = 1$$
$$P_{\mu\mu} = 1 - \sin^2 2\vartheta_{23} \sin^2 \Delta$$
$$P_{\mu e} = P_{e\mu} = 0$$

$$P_{\tau\tau} = P_{\mu\mu}$$
$$P_{\tau\mu} = P_{\mu\tau} = \sin^2 2\vartheta_{23} \sin^2 \Delta$$
$$P_{\tau e} = P_{e\tau} = 0$$

two-flavour oscillations

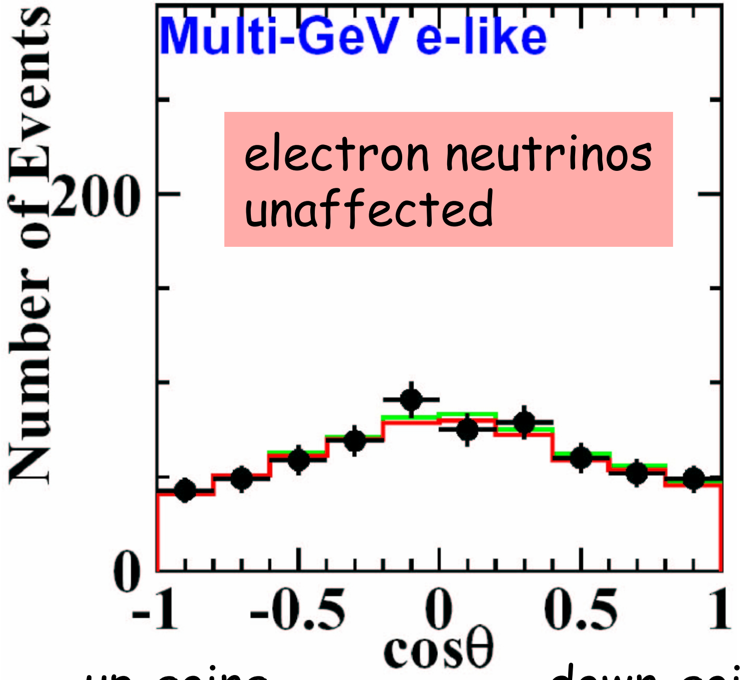
Atmospheric neutrino oscillations



Nobel Prize in Physics 2015
Takaaki Kajita

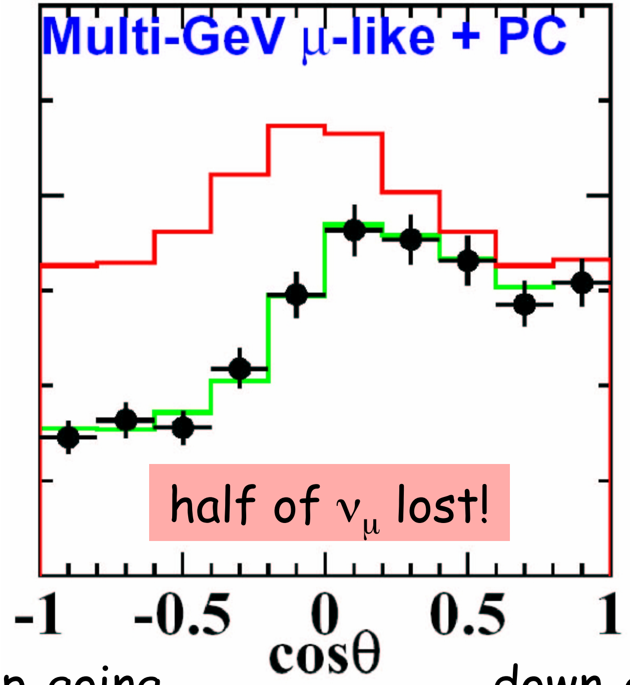
Electron and muon neutrinos (and antineutrinos) produced by the collision of cosmic ray particles on the atmosphere

Experiments:
SuperKamiokande (Japan)
[also IceCube (South Pole)]


Multi-GeV e-like

electron neutrinos unaffected

up-going down-going


Multi-GeV μ-like + PC

half of ν_μ lost!

up-going down-going

electron neutrinos do not oscillate

by working in the approximation $\Delta m^2_{21} = 0$

$$P_{ee} = 1 - \underbrace{4|U_{e3}|^2(1-|U_{e3}|^2}_{\sin^2 2\vartheta_{13}} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \approx 1$$

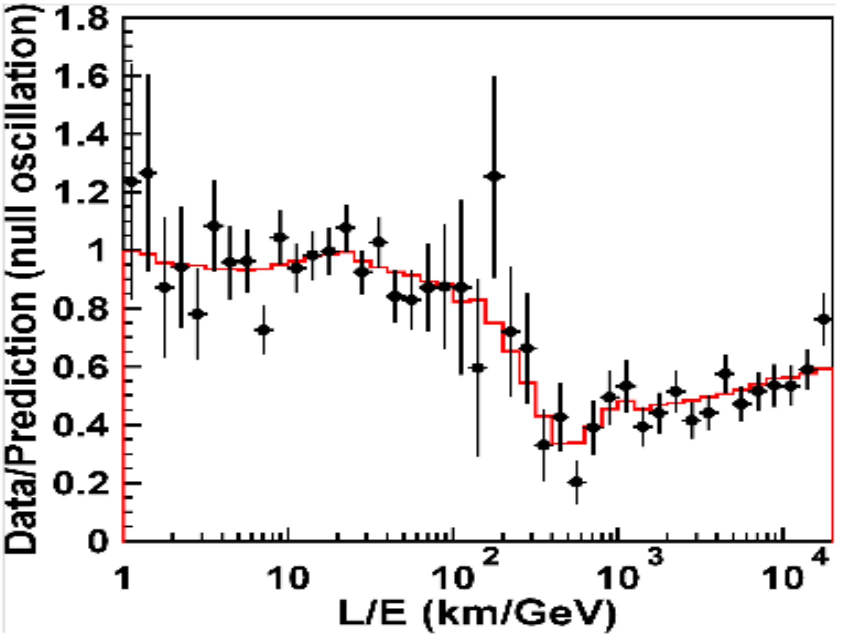
for $U_{e3} = \sin\vartheta_{13} \approx 0$

muon neutrinos oscillate

$$P_{\mu\mu} = 1 - \underbrace{4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2}_{\sin^2 2\vartheta_{23}} \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

$$|\Delta m^2_{32}| \approx 2 \cdot 10^{-3} \quad eV^2$$

$$\sin^2\vartheta_{23} \approx \frac{1}{2}$$



$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & 0 \\ \cdot & \cdot & -\frac{1}{\sqrt{2}} \\ \cdot & \cdot & \frac{1}{\sqrt{2}} \end{pmatrix} + (\text{small corrections})$$

maximal mixing!
not a replica of the quark mixing pattern

other terrestrial experiments measuring $P_{\mu\mu}$

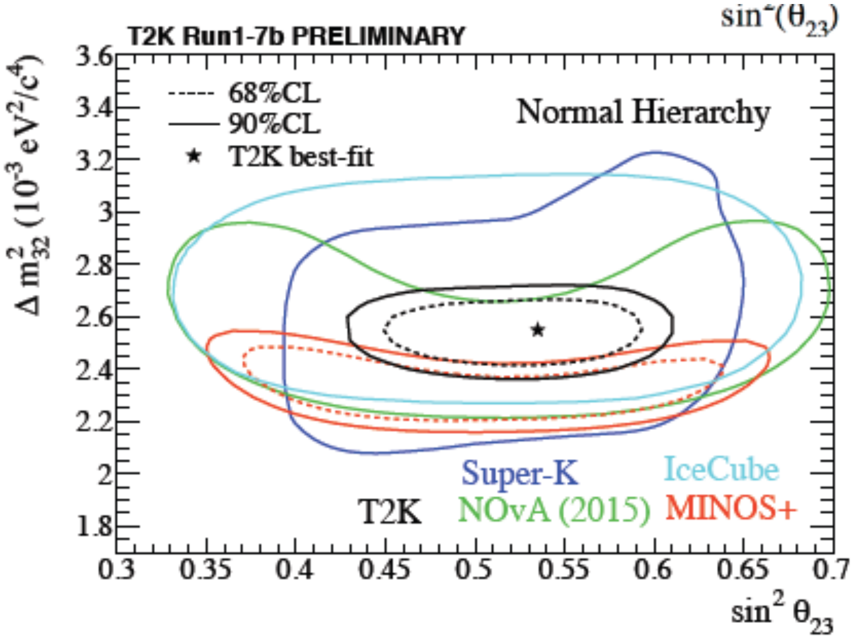
man made neutrino beams

K2K	(Japan, from KEK to Kamioka mine L ≈ 250 Km E ≈ 1.3 GeV)
MINOS	(USA, from Fermilab to Soudan mine L ≈ 735 Km E ≈3 GeV)
T2K	(Japan, from Tokai,J-Park to Kamioka mine L ≈ 295 Km E ≈ 0.6 GeV)
NOνA	(USA, from Fermilab to Ash River L ≈ 810 Km E ≈2 GeV)
OPERA	(CERN-Italy, from CERN to LNGS L ≈ 732 Km E ≈ 17 GeV)

all sensitive to Δm_{32}^2 close to 10^{-3} eV^2

OPERA energy optimized to maximize τ production, via CC events
by the end of 2016 5 τ events have been seen

recent results from T2K [Neutrino 2016]



T2K:

	NH	IH
$\sin^2\theta_{23}$	$0.532^{+0.044}_{-0.060}$	$0.534^{+0.041}_{-0.059}$
$\lvert\Delta m^2_{32}\rvert$ ($/10^{-3}$eV^2)	$2.545^{+0.084}_{-0.082}$	$2.510^{+0.082}_{-0.083}$

$\approx 10\%$

$\approx 3\%$

KamLAND

previous experiments were sensitive to Δm^2 close to 10^{-3} eV^2
to explore smaller Δm^2 we need larger L and/or smaller E

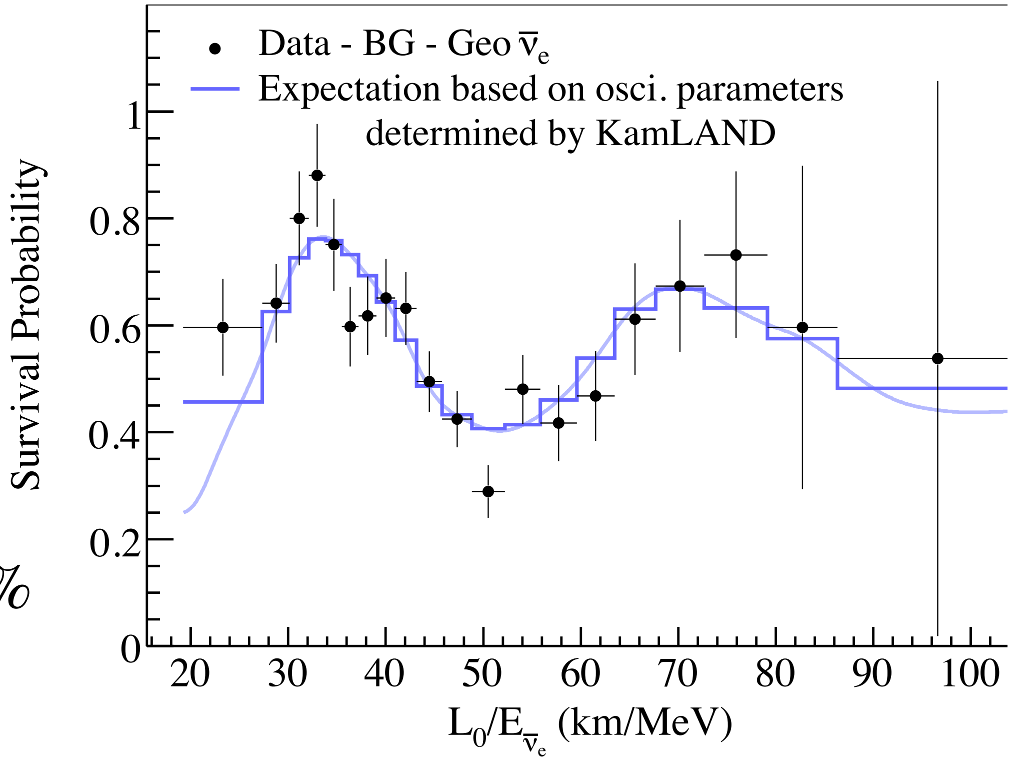
KamLAND experiment exploits the low-energy electron anti-neutrinos ($E\approx 3$ MeV) produced by Japanese and Korean reactors at an average distance of $L\approx 180$ Km from the detector and is potentially sensitive to Δm^2 down to 10^{-5} eV^2

by working in the approximation

$U_{e3} = \sin\vartheta_{13} = 0$ we get

[Exercise]

$$P_{ee} = 1 - \underbrace{4\left|U_{e1}\right|^2\left|U_{e2}\right|^2}_{\sin^2 2\vartheta_{12}} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$



$\Delta m_{21}^2 \approx 8\cdot 10^{-5} \quad eV^2$ 2.5%

$\sin^2\vartheta_{12} \approx 1/3$ 6%

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + (\text{small corrections})$$

this pattern is called tri-bimaximal
completely different from the quark
mixing pattern: two angles are large

by unitarity

historically Δm_{21}^2 and $\sin^2\theta_{12}$ were first determined by solving the solar neutrino problem, i.e. the disappearance of about one third of solar electron neutrino flux, for solar neutrinos above few MeV. The desire of detecting solar neutrinos, to confirm the thermodynamics of the sun, was the driving motivation for the whole field for more than 30 years. Electron solar neutrinos oscillate, but the formalism requires the introduction of matter effects, since the electron density in the sun is not negligible. Experiments: SuperKamiokande, SNO, Borexino

Nobel Prize in Physics 2015: Arthur McDonald

ϑ_{13} from disappearance experiments

These experiments have been realized with reactors. Electron anti-neutrinos are produced by a reactor (E≈3 MeV, L≈1 Km) (by CPT the survival probability in vacuum is the same for neutrinos and antineutrinos and matter effects are negligible).

In this range of (L,E) oscillations driven by Δm^2_{21} are negligible and the survival probability P_{ee} only depends on ($|U_{e3}|$, Δm^2_{31}).

$$P_{ee} = 1 - \underbrace{4|U_{e3}|^2(1-|U_{e3}|^2)}_{\sin^2 2\vartheta_{13}}\sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$E \approx 3\ MeV$$

$$L \approx 1\ Km$$

Experiment	Near Detectors	Far Detectors
CHOOZ (France)	–	(1) 1050m
Double CHOOZ	(1) 400m	(1) 1050m
Reno (Korea)	(1) 290m	(1) 1380m
Daya Bay (China)	(4) (360-530)m	(4) (1600-2000)m

before 2012 there was only an upper bound on $|U_{e3}|$ by CHOOZ

today (end 2016) the value of ϑ_{13} is dominated by the Daya Bay result

$$\sin^2 2\vartheta_{13} = 0.0841 \pm 0.0033 \qquad \Delta m^2_{32} = \begin{cases} (2.45 \pm 0.08) \times 10^{-3}\ eV^2 & [NO] \\ -(2.55 \pm 0.08) \times 10^{-3}\ eV^2 & [IO] \end{cases}$$

$$|U_{e3}|^2 = \sin^2\vartheta_{13} = 0.0215 \pm 0.0009 \quad \vartheta_{13} = (8.4 \pm 0.2)^0$$

4%

ϑ_{13} from appearance experiments

These experiments use a muon-neutrino beam from an accelerator and look for conversion of muon-neutrinos into electron-neutrinos. The (L,E) range is such that they are mainly sensitive to Δm^2_{31}

Experiment	E(GeV)	L(Km)
T2K (Japan)	0.6	295
MINOS (USA)	3	735

at the LO (neglecting Δm^2_{21} and matter effects)

$$P_{\mu e} = 4|U_{\mu 3}|^2 |U_{e3}|^2 \sin^2\Delta = \sin^2\vartheta_{23}\sin^2 2\vartheta_{13}\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

however in this case corrections from Δm^2_{21} and matter effects are non-negligible

EXERCISE

by expanding $P_{\mu e}$ to first order in $\alpha = \Delta m^2_{21}/\Delta m^2_{13}$ show that

$$\begin{aligned} P_{\mu e} \quad = \quad & \sin^2\vartheta_{23}\sin^2 2\vartheta_{13}\sin^2\Delta_{13} \\ & -8\alpha J_{CP}\Delta_{13}\sin^2\Delta_{13} \\ & -8\alpha J_{CP}\frac{\cos\delta}{\sin\delta}\Delta_{13}\cos\Delta_{13}\sin\Delta_{13} \\ & +O(\alpha^2) + matter\ effects \end{aligned}$$

$$\Delta_{13} = \frac{\Delta m^2_{31}L}{4E}$$

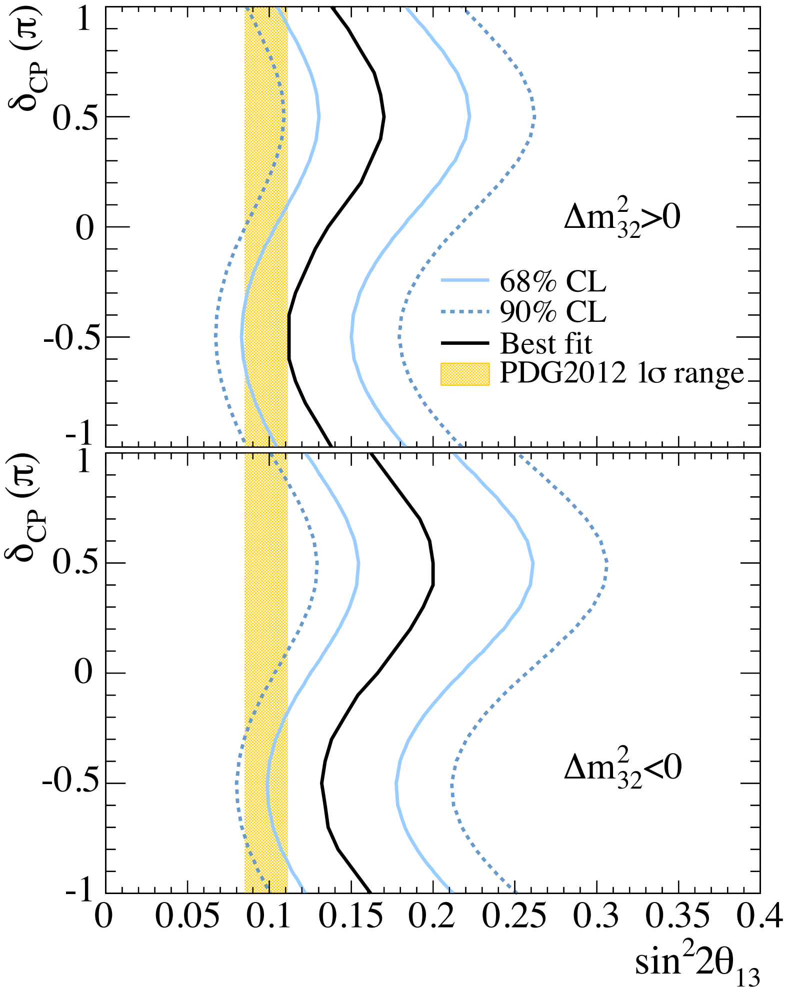
$$\begin{aligned} J_{CP} &= \mathrm{Im}\left(U_{\mu 3}U^*_{e3}U^*_{\mu 2}U_{e2}\right) \\ &= \frac{1}{8}\sin 2\vartheta_{12}\sin 2\vartheta_{23}\sin 2\vartheta_{13}\cos\vartheta_{13}\sin\delta \end{aligned}$$

T2K works near the first oscillation maximum where $|\Delta_{13}|=\pi/2$

$$\begin{aligned} P_{\mu e} &= \sin^2\vartheta_{23}\sin^2 2\vartheta_{13} \\ &\quad -4\pi|\alpha|J_{CP} \\ &\quad +O(\alpha^2)+\textit{matter effects} \end{aligned}$$

the relative subleading corrections are O(20%) and are sensitive to sinδ

[T2K 1311.4750]



At present (end 2016) agreement with the value of ϑ_{13} determined by reactor disappearance experiments requires

$$\sin\delta \approx -1$$

$$\delta \approx \frac{3}{2}\pi$$

i.e. maximal CP violation in the lepton sector

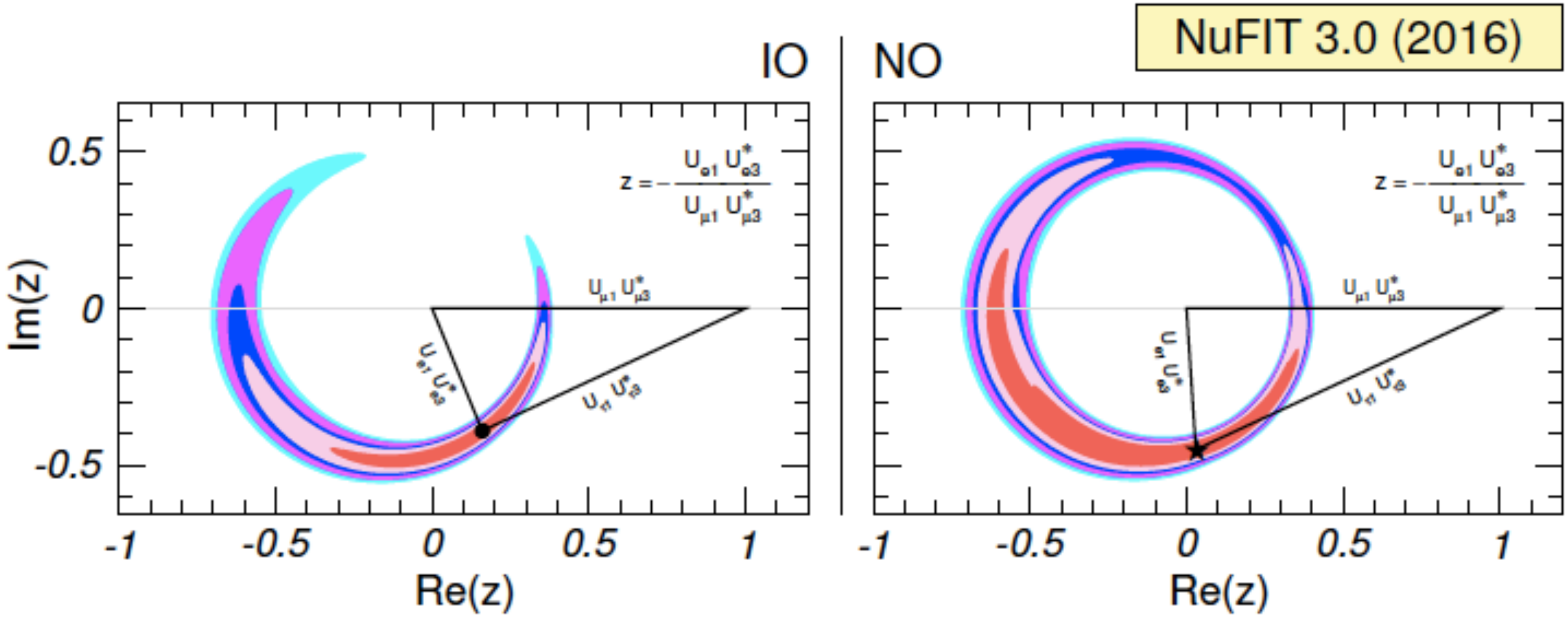


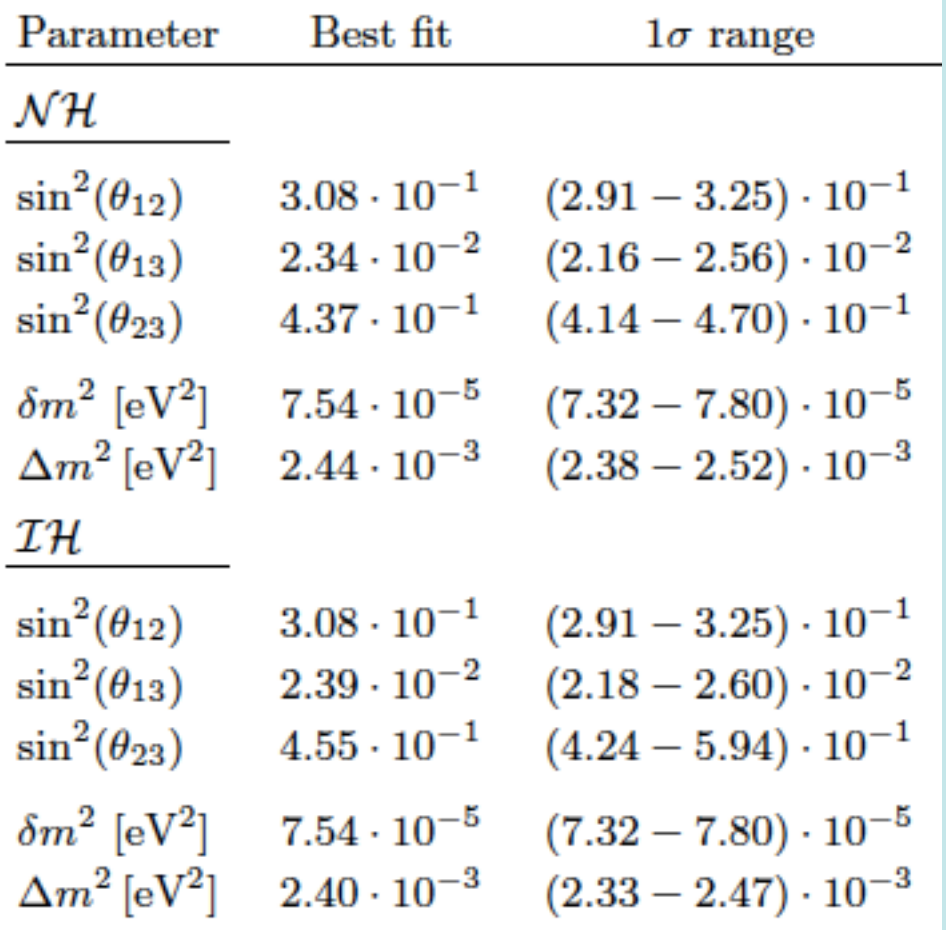
Figure 4. Leptonic unitarity triangle for the first and third columns of the mixing matrix. After scaling and rotating the triangle so that two of its vertices always coincide with $(0, 0)$ and $(1, 0)$ we plot the 1σ, 90%, 2σ, 99%, 3σ CL (2 dof) allowed regions of the third vertex. Note that in the construction of the triangle the unitarity of the U matrix is always explicitly imposed. The regions for both orderings are defined with respect to the common global minimum which is in NO.

I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler and T. Schwetz, arXiv:1611.01514

Summary of data

$m_\nu < 2.2\,eV \quad (95\%\,CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)



Parameter	Best fit	1σ range
$\mathcal{NH}$		
$\sin^2(\theta_{12})$	$3.08 \cdot 10^{-1}$	$(2.91 - 3.25) \cdot 10^{-1}$
$\sin^2(\theta_{13})$	$2.34 \cdot 10^{-2}$	$(2.16 - 2.56) \cdot 10^{-2}$
$\sin^2(\theta_{23})$	$4.37 \cdot 10^{-1}$	$(4.14 - 4.70) \cdot 10^{-1}$
δm^2 [eV^2]	$7.54 \cdot 10^{-5}$	$(7.32 - 7.80) \cdot 10^{-5}$
Δm^2 [eV^2]	$2.44 \cdot 10^{-3}$	$(2.38 - 2.52) \cdot 10^{-3}$
$\mathcal{IH}$		
$\sin^2(\theta_{12})$	$3.08 \cdot 10^{-1}$	$(2.91 - 3.25) \cdot 10^{-1}$
$\sin^2(\theta_{13})$	$2.39 \cdot 10^{-2}$	$(2.18 - 2.60) \cdot 10^{-2}$
$\sin^2(\theta_{23})$	$4.55 \cdot 10^{-1}$	$(4.24 - 5.94) \cdot 10^{-1}$
δm^2 [eV^2]	$7.54 \cdot 10^{-5}$	$(7.32 - 7.80) \cdot 10^{-5}$
Δm^2 [eV^2]	$2.40 \cdot 10^{-3}$	$(2.33 - 2.47) \cdot 10^{-3}$

F. Capozzi, G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo, "Status of three-neutrino oscillation parameters, circa 2013," Phys. Rev. D 89, 093018 (2014).

violation of individual lepton number
implied by neutrino oscillations

Summary of unkowns

absolute neutrino mass scale is unknown [but well-constrained!]

sign $[\Delta m^2_{atm}]$ unknown

[complete ordering (either normal or inverted hierarchy) not known]

δ, α, β unknown

[CP violation in lepton sector not yet established]

violation of total lepton number not yet established

Lecture II
Implication for the Physics Beyond the Standard Model

Beyond the Standard Model

a non-vanishing neutrino mass is evidence of the incompleteness of the Standard Model [SM]

[recall also DM, DE, matter-antimatter asymmetry, strong CP,…]

in the SM neutrinos belong to SU(2) doublets with hypercharge Y=-1/2
they have only two helicities (not four, as the other charged fermions)

$$l = \begin{pmatrix} \nu_e \\ e \end{pmatrix} = (1,2,-1/2)$$

the requirement of invariance under the gauge group G=SU(3)xSU(2)xU(1) forbids pure fermion mass terms in the lagrangian. Charged fermion masses arise, after electroweak symmetry breaking, through gauge-invariant Yukawa interactions

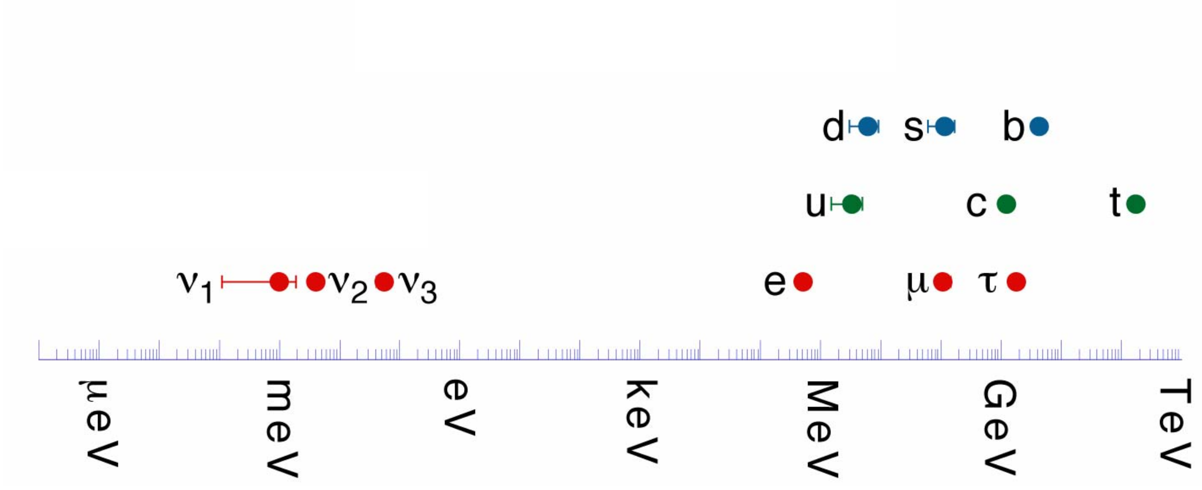
$$\Phi \quad \underbrace{\Psi\Psi'}_{\text{same helicity}}$$

not even this term is allowed for SM neutrinos, by gauge invariance

Questions

how to extend the SM in order to accommodate neutrino masses?

why neutrino masses are so small, compared with the charged fermion masses?



why lepton mixing angles are so different from those of the quark sector?

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{corrections}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^4 \div \lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^4 \div \lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

$\lambda \approx 0.22$

How to modify the SM?

the SM, as a consistent QFT, is completely specified by

0. invariance under local transformations of the gauge group G=SU(3)xSU(2)xU(1) [plus Lorentz invariance]

1. particle content

   three copies of (q, u^c, d^c, l, e^c)

   one Higgs doublet Φ

2. renormalizability (i.e. the requirement that all coupling constants g_i have non-negative dimensions in units of mass: $d(g_i) \geq 0$. This allows to eliminate all the divergencies occurring in the computation of physical quantities, by redefining a finite set of parameters.)

(0.+1.+2.) leads to the SM Lagrangian, L_{SM}, possessing an additional, accidental, global symmetry: (B-L)

0. We cannot give up gauge invariance! It is mandatory for the consistency of the theory. Without gauge invariance we cannot even define the Hilbert space of the theory [remember: we need gauge invariance to eliminate the photon extra degrees of freedom required by Lorentz invariance]!

   We could extend G, but, to allow for neutrino masses, we need to modify 1. (and/or 2.) anyway…

First possibility: modify (1), the particle content

there are several possibilities
one of the simplest one is to mimic the charged fermion sector

Example 1

add (three copies of) right-handed neutrinos $\nu^c \equiv (1,1,0)$ full singlet under G=SU(3)xSU(2)xU(1)

ask for (global) invariance under B-L
(no more automatically conserved as in the SM)

the neutrino has now four helicities, as the other charged fermions, and we can build gauge invariant Yukawa interactions giving rise, after electroweak symmetry breaking, to neutrino masses

$$L_Y = -d^c y_d (\Phi^+ q) - u^c y_u (\tilde{\Phi}^+ q) - e^c y_e (\Phi^+ l) - \nu^c y_\nu (\tilde{\Phi}^+ l) + h.c.$$

$$m_f = \frac{y_f}{\sqrt{2}} v \qquad f = u,d,e,\nu$$

with three generations there is an exact replica of the quark sector and, after diagonalization of the charged lepton and neutrino mass matrices, a mixing matrix U appears in the charged current interactions

$$-\frac{g}{\sqrt{2}} W_\mu \bar{e} \sigma^\mu U_{PMNS} \nu + h.c.$$

U_{PMNS} has three mixing angles and one phase, like V_{CKM}

a generic problem of this approach

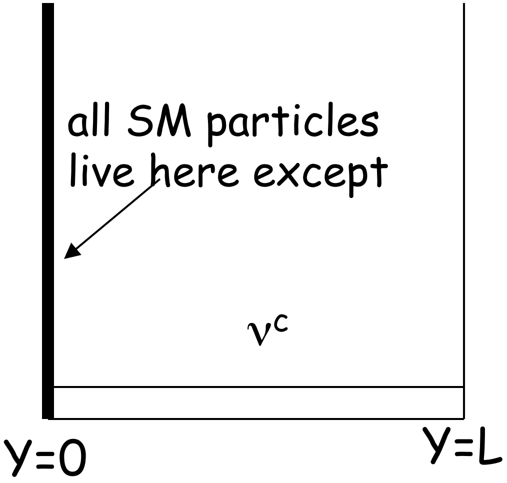
the particle content can be modified in several different ways
in order to account for non-vanishing neutrino masses
(additional right-handed neutrinos, new SU(2) fermion triplets, additional SU(2) scalar triplet(s), SUSY particles,…). Which is the correct one?

a problem of the above example

if neutrinos are so similar to the other fermions, why are so light?

$$\frac{y_\nu}{y_{top}} \le 10^{-12}$$

a speculative answer:
neutrinos are so light, because the right-handed neutrinos have access to an extra (fifth) spatial dimension



neutrino Yukawa coupling

$$\nu^c(y=0)(\tilde{\Phi}^+ l) = \text{Fourier expansion}$$

$$= \frac{1}{\sqrt{L}} \nu_0^c(\tilde{\Phi}^+ l) + \ldots \quad \text{[higher modes]}$$

if L>>1 (in units of the fundamental scale)
then neutrino Yukawa coupling is suppressed

Second possibility: abandon (2) renormalizability

A disaster?

$$L = L^{SM}_{d\leq 4} + \frac{L_5}{\Lambda} + \frac{L_6}{\Lambda^2} + \ldots$$

a new scale Λ enters the theory. The new (gauge invariant!) operators L_5, L_6,… contribute to amplitudes for physical processes with terms of the type

$$\frac{L_5}{\Lambda} \rightarrow \frac{E}{\Lambda} \qquad \frac{L_6}{\Lambda^2} \rightarrow \left(\frac{E}{\Lambda}\right)^2 \qquad \ldots$$

the theory cannot be extrapolated beyond a certain energy scale $E \approx \Lambda$.
[at variance with a renormalizable (asymptotically free) QFT]

If $E \ll \Lambda$ (for example E close to the electroweak scale, 10^2 GeV, and $\Lambda \approx 10^{15}$ GeV not far from the so-called Grand Unified scale), the above effects will be tiny and, the theory will *look like* a renormalizable theory!

$$\frac{E}{\Lambda} \approx \frac{10^2\, GeV}{10^{15}\, GeV} = 10^{-13}$$

an extremely tiny effect, but exactly what needed to suppress m_ν compared to m_{top}!

Worth to explore. The dominant operators (suppressed by a single power of $1/\Lambda$) beyond L_{SM} are those of dimension 5. Here is a list of all d=5 gauge invariant operators

$$\frac{L_5}{\Lambda} = \frac{\left(\tilde{\Phi}^+ l\right)\left(\tilde{\Phi}^+ l\right)}{\Lambda} =$$

$$= \frac{v}{2}\left(\frac{v}{\Lambda}\right)\nu\nu + \ldots$$

a unique operator!
[up to flavour combinations]
it violates (B-L) by two units

it is suppressed by a factor (v/Λ)
with respect to the neutrino mass term
of Example 1:

$$\nu^c(\tilde{\Phi}^+ l) = \frac{v}{\sqrt{2}}\nu^c\nu + \ldots$$

it provides an explanation for the smallness of m_ν:
the neutrino masses are small because the scale Λ, characterizing (B-L) violations, is very large. How large? Up to about 10^{15} GeV

from this point of view neutrinos offer a unique window on physics at very large scales, inaccessible in present (and probably future) man-made experiments.

since this is the dominant operator in the expansion of L in powers of $1/\Lambda$, we could have expected to find the first effect of physics beyond the SM in neutrinos … and indeed this was the case!

L_5 represents the effective, low-energy description of several extensions of the SM

Example 2: see-saw

add (three copies of) $\nu^c \equiv (1,1,0)$

full singlet under G=SU(3)xSU(2)xU(1)

this is like Example 1, but without enforcing (B-L) conservation

$$L(\nu^c, l) = -\nu^c y_\nu (\tilde{\Phi}^+ l) - \frac{1}{2}\nu^c M \nu^c + h.c.$$

mass term for right-handed neutrinos: G invariant, violates (B-L) by two units.

the new mass parameter M is independent from the electroweak breaking scale v. If M>>v, we might be interested in an effective description valid for energies much smaller than M. This is obtained by "integrating out'' the field ν^c

$$L_{eff}(l) = \frac{1}{2}(\tilde{\Phi}^+ l)\left[y_\nu^T M^{-1} y_\nu\right](\tilde{\Phi}^+ l) + h.c. + \underset{\updownarrow}{\ldots}$$

terms suppressed by more powers of M^{-1}

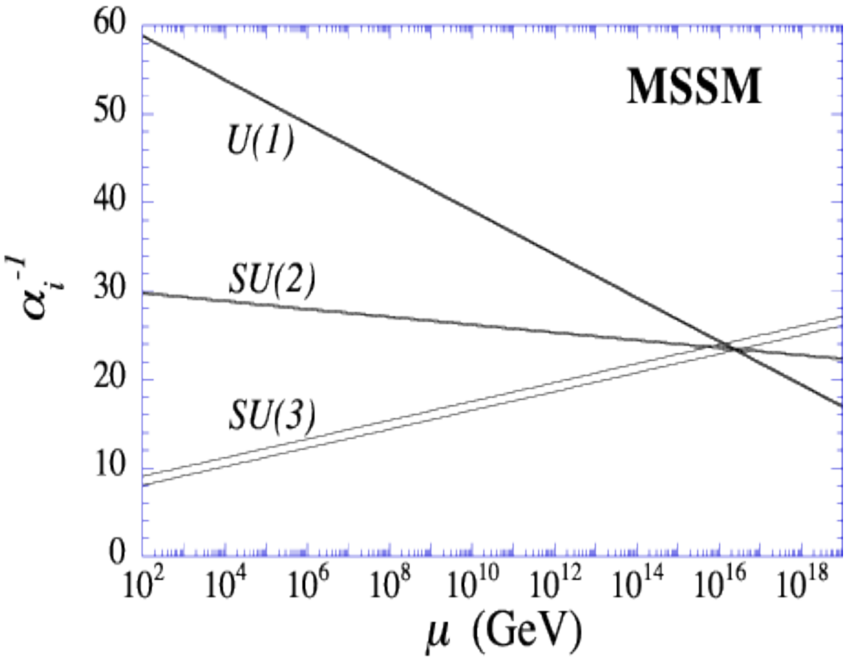
this reproduces L_5, with M playing the role of Λ. This particular mechanism is called (type I) see-saw.

Theoretical motivations for the see-saw

$\Lambda \approx 10^{15}$ GeV is very close to the so-called unification scale M_{GUT}.

an independent evidence for M_{GUT} comes from the unification of the gauge coupling constants in (SUSY extensions of) the SM.

such unification is a generic prediction of Grand Unified Theories (GUTs):
the SM gauge group G is embedded into a simple group such as SU(5), SO(10),…



Particle classification: it is possible to unify all SM fermions (1 generation) into a single irreducible representation of the GUT gauge group. Simplest example: G_{GUT}=SO(10)

$$16 = (q, d^c, u^c, l, e^c, \nu^c)$$

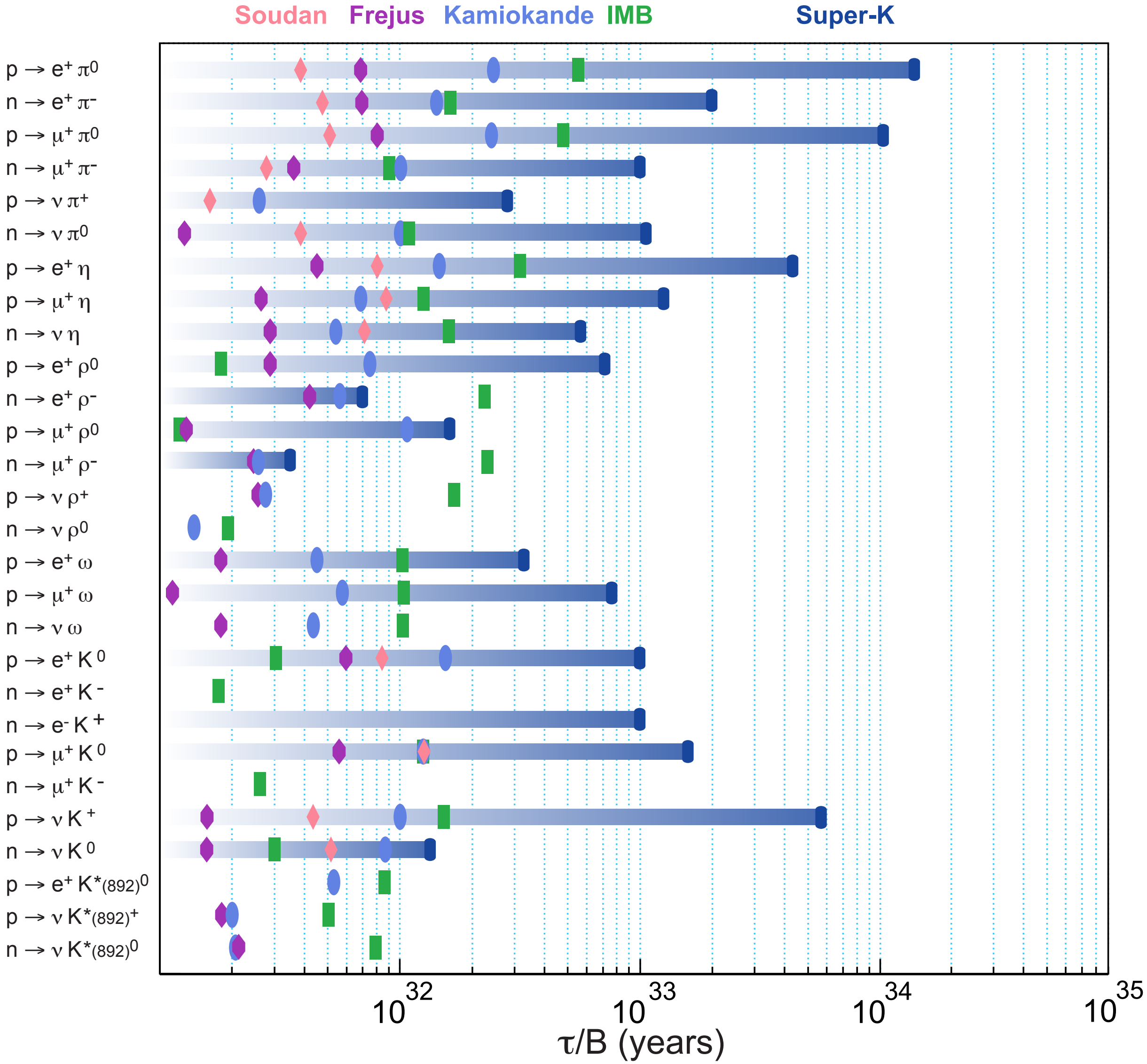
a whole family plus a right-handed neutrino!

quite a fascinating possibility. Unfortunately, it still lacks experimental tests. In GUT new, very heavy, particles can convert quarks into leptons and the proton is no more a stable particle. Proton decay rates and decay channels are however model dependent. Experimentally we have only lower bounds on the proton lifetime.

Unity of All Elementary-Particle Forces
Phys. Rev. Lett. 32, (1974) 438
Howard Georgi and S. L. Glashow

Georgi, H.; Quinn, H.R. and Weinberg, S.
Hierarchy of interactions in unified gauge theories.
Phys. Rev. Lett. 33 (1974) 451

Antilepton + meson two-body modes



2 additional virtues of the see-saw

The see-saw mechanism can enhance small mixing angles into large ones

$$m_\nu = -\left[y_\nu^T M^{-1} y_\nu\right] v^2$$

example

$$y_\nu = \begin{pmatrix} \delta & \delta \\ 0 & 1 \end{pmatrix}$$

δ<<1
small mixing

$$M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

no mixing

$$y_\nu^T M^{-1} y_\nu = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{M_2}$$

$$\approx \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} \qquad \text{for} \quad \frac{M_1}{M_2} << \delta^2$$

The (out-of equilibrium, CP-violating) decay of heavy right-handed neutrinos in the early universe might generate a net asymmetry between leptons and anti-leptons. Subsequent SM interactions can partially convert it into the observed baryon asymmetry

$$\eta = \frac{(n_B - n_{\overline{B}})}{s} \approx 6 \times 10^{-10}$$

Sakharov conditions met by the see-saw theory

1. (B-L) violation at high-temperature and (B+L) violation by pure SM interactions
2. C and CP violation by additional phases in see-saw Lagrangian
3. out-of-equilibrium condition

restrictions imposed by leptogenesis on neutrinos

here: thermal leptogenesis
dominated by lightest ν^c
no flavour effects]

active neutrinos should be light

out-of-equilibrium controlled
by rate of RH neutrino decays

$$\frac{M_1}{8\pi}(y_\nu y_\nu^+)_{11} < \left.\frac{T^2}{M_{Pl}}\right|_{T\approx M_1}$$

$$\frac{(y_\nu y_\nu^+)_{11} v^2}{M_1} \equiv \tilde{m}_1 < 10^{-3} \text{ eV}$$

more accurate estimate $m_i < 0.15$ eV

RH neutrinos should be heavy

[efficiency factor ≤1
washout effects]

$$\eta_B \approx 10^{-2}\,\varepsilon_1\,\eta$$

$$\varepsilon_1 = \frac{\Gamma(\nu_1^c \to l\Phi) - \Gamma(\nu_1^c \to \bar{l}\,\Phi^*)}{\Gamma(\nu_1^c \to l\Phi) + \Gamma(\nu_1^c \to \bar{l}\,\Phi^*)} = -\frac{3}{16\pi}\sum_{j=2,3}\frac{M_1}{M_j}\frac{\mathrm{Im}\{[(yy^+)_{1j}]^2\}}{(yy^+)_{11}} \approx 0.1\times\frac{M_1 m_i}{v^2}$$

[Yukawas y in mass eigenstate basis for ν^c_i]

$M_1 > 6\times 10^8$ GeV

weak point of the see-saw

full high-energy theory is difficult to test

$$L(\nu^c, l) = \nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2} \nu^c M \nu^c + h.c.$$

depends on many physical parameters:
3 (small) masses + 3 (large) masses
3 (L) mixing angles + 3 (R) mixing angles
6 physical phases = 18 parameters

the double of those describing $(L_{SM})+L_5$:
3 masses, 3 mixing angles and 3 phases, as in lecture 1

few observables to pin down the extra parameters: η,...
[additional possibilities exist under special conditions, e.g. Lepton Flavor Violation at observable rates]

easier to test the low-energy remnant L_5

[which however is "universal" and does not implies the specific see-saw mechanism of Example 2]

look for a process where B-L is violated by 2 units. The best candidate is

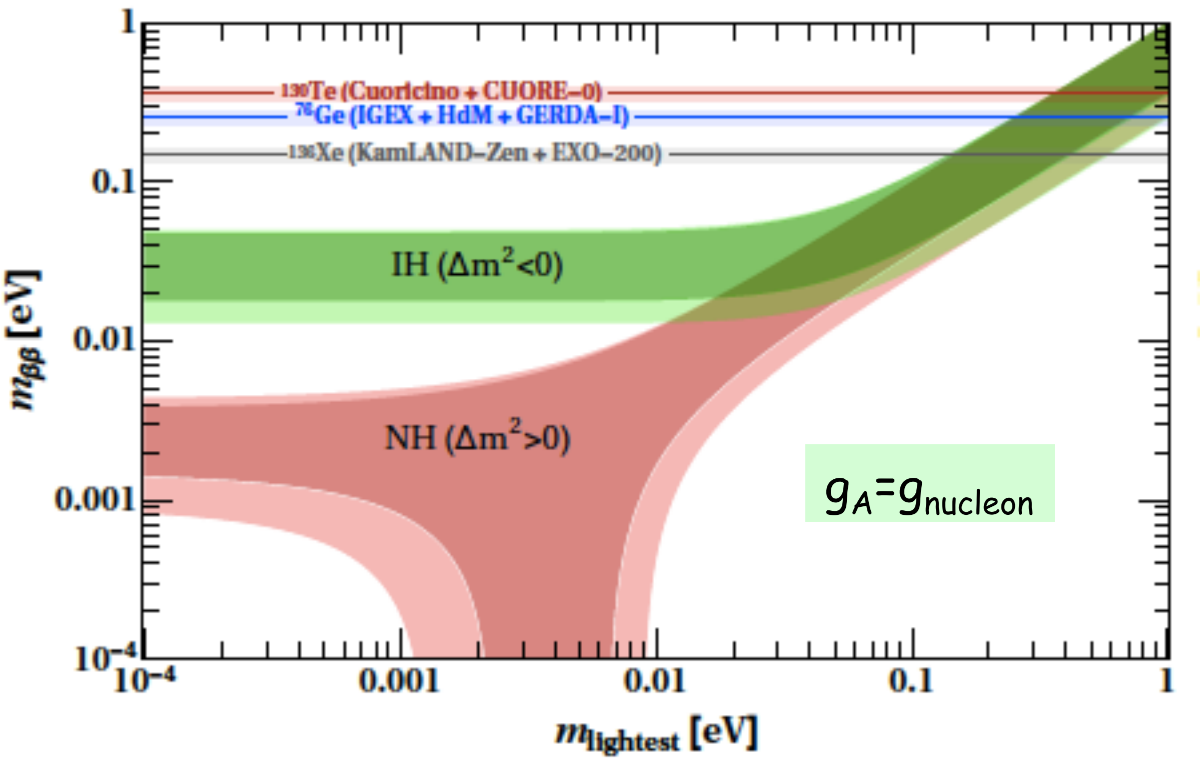
$0\nu\beta\beta$ decay: $(A,Z) \to (A,Z+2)+2e^-$

this would discriminate L_5 from other possibilities, such as Example 1.

The decay in 0νββ rates depend on the combination $$|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$$

$$|m_{ee}| = \left|\cos^2\vartheta_{13}(\cos^2\vartheta_{12}\ m_1 + \sin^2\vartheta_{12}e^{2i\alpha}\ m_2) + \sin^2\vartheta_{13}e^{2i\beta}\ m_3\right|$$

[notice the two phases α and β, not entering neutrino oscillations]



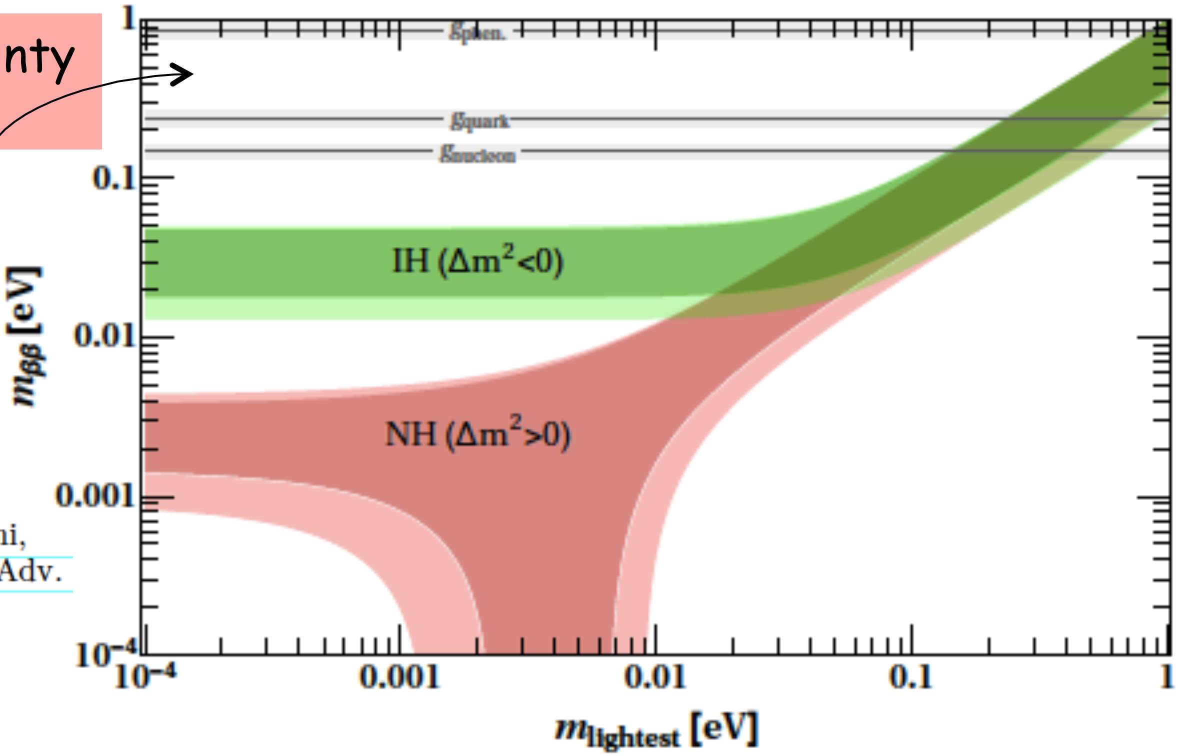
from the current knowledge of $(\Delta m_{ij}^2, \vartheta_{ij})$ we can estimate the expected range of $|m_{ee}|$

Experiment	Isotope	$S^{0\nu}$ (90% C. L.) $[10^{25}$ yr$]$	Lower bound for $m_{\beta\beta}$ [eV]		
			$g_{nucleon}$	g_{quark}	$g_{phen.}$
IGEX + HdM + GERDA-I, [174]	^{76}Ge	3.0	0.25 ± 0.02	0.40 ± 0.04	1.21 ± 0.11
Cuoricino + CUORE-0, [180]	^{130}Te	0.4	0.36 ± 0.03	0.58 ± 0.05	2.07 ± 1.05
EXO-200 + KamLAND-ZEN, [187]	^{136}Xe	3.4	0.15 ± 0.02	0.24 ± 0.03	0.87 ± 0.10

largest theoretical uncertainty is from g_A

limits from ^{136}Xe

S. Dell'Oro, S. Marcocci, M. Viel, and F. Vissani, "*Neutrinoless double beta decay: 2015 review*," Adv. High Energy Phys. **2016** (2016) 2162659, `arXiv:1601.07512`.



Experiment	Isotope	$S^{0\nu}$ (90% C. L.) [10^{25} yr]	Lower bound for $m_{\beta\beta}$ [eV]		
			$g_{nucleon}$	g_{quark}	$g_{phen.}$
CUORE, [189]	^{130}Te	9.5	0.073 ± 0.008	0.14 ± 0.01	0.44 ± 0.04
GERDA-II, [174]	^{76}Ge	15	0.11 ± 0.01	0.18 ± 0.02	0.54 ± 0.05
LUCIFER, [190]	^{82}Se	1.8	0.20 ± 0.02	0.32 ± 0.03	0.97 ± 0.09
MAJORANA D., [191]	^{76}Ge	12	0.13 ± 0.01	0.20 ± 0.02	0.61 ± 0.06
NEXT, [193]	^{136}Xe	5	0.12 ± 0.01	0.20 ± 0.02	0.71 ± 0.08
AMoRE, [194]	^{100}Mo	5	0.084 ± 0.008	0.14 ± 0.01	0.44 ± 0.04
nEXO, [195]	^{136}Xe	660	0.011 ± 0.001	0.017 ± 0.002	0.062 ± 0.007
PandaX-III, [196]	^{136}Xe	11	0.082 ± 0.009	0.13 ± 0.01	0.48 ± 0.05
SNO+, [197]	^{130}Te	9	0.076 ± 0.007	0.12 ± 0.01	0.44 ± 0.04
SuperNEMO, [198]	^{82}Se	10	0.084 ± 0.008	0.14 ± 0.01	0.41 ± 0.04

Conclusion

do we have a theory of neutrino masses ?

No! Neither for neutrinos nor for charged fermions. We lack a unifying principle.

like weak interactions before the electroweak theory

$SU(2)_L \otimes U(1)_Y$

gauge invariance

all fermion-gauge boson interactions in terms of 2 parameters: g and g'

?

Yukawa interactions between fermions and spin 0 particles: many free parameters (up to 22 in the SM!)

many ideas and prejudices but we lack a baseline model

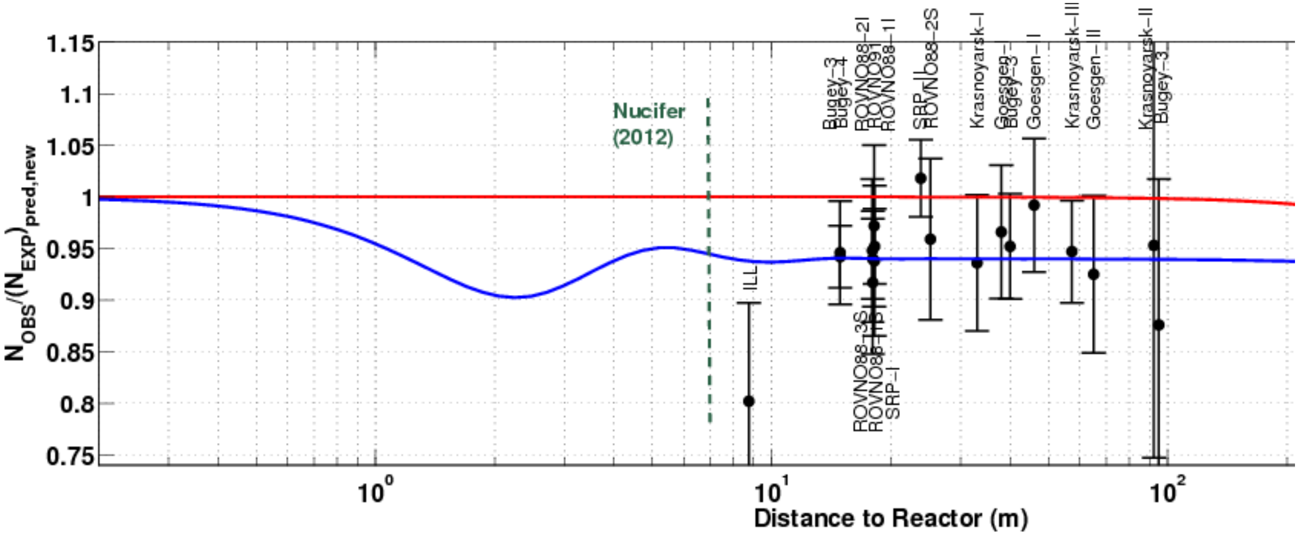
caveat: several prejudices turned out to be wrong in the past!
- $m_\nu \approx 10$ eV because is the cosmologically relevant range
- solution to solar is MSW Small Angle
- atmospheric neutrino problem will go away because it implies a large angle

Back up slides

sterile neutrinos ?

1 reactor anomaly (anti-ν_e disappearance)

re-evaluation of reactor anti-ν_e flux: new estimate 3.5% higher than old one



$$(\Phi_{\exp} - \Phi_{th}) / \Phi_{th} \approx -6\%$$

[th. uncertainty?]

very SBL L ≲ 100 m

$$\vartheta_{es} \approx 0.2$$

$$\Delta m^2 \approx m_s^2 \geq 1\, eV^2$$

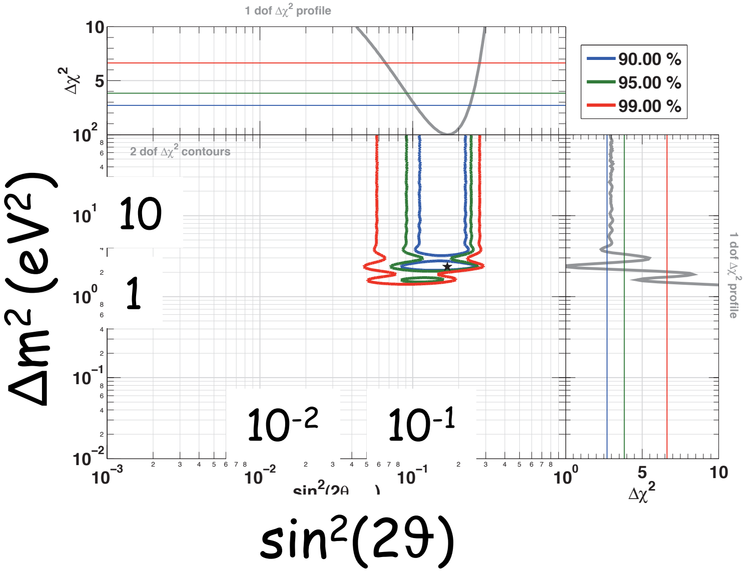
supported by the Gallium anomaly

ν_e flux measured from high intensity radioactive sources in Gallex, Sage exp

$$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$$

[error on σ or on Ge extraction efficiency]

… but disfavoured by cosmological limits



long-standing claim

evidence for $\nu_\mu \to \nu_e$ appearance in accelerator experiments

exp		$E(MeV)$	$L(m)$
LSND	$\bar{\nu}_\mu \to \bar{\nu}_e$	$10 \div 50$	30
MiniBoone	$\nu_\mu \to \nu_e$ $\bar{\nu}_\mu \to \bar{\nu}_e$	$300 \div 3000$	541

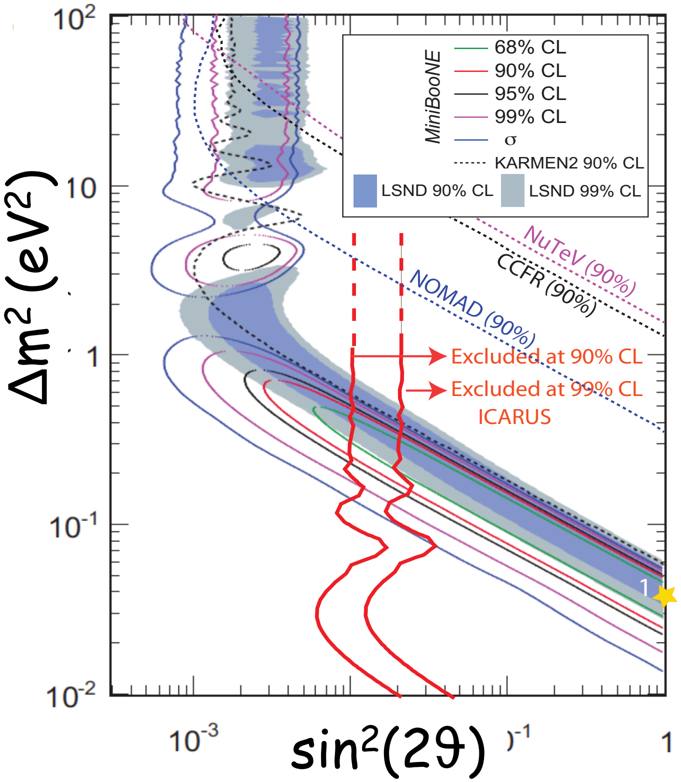
3.8σ

3.8σ [signal from low-energy region]

parameter space limited by negative results from Karmen and ICARUS

$$\vartheta_{e\mu} \approx 0.035$$

$$\Delta m^2 \approx 0.5\, eV^2$$



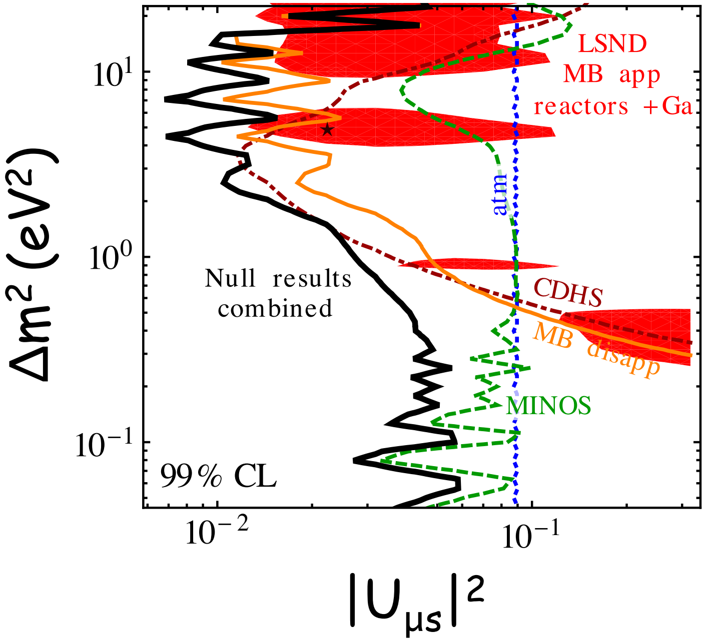
3

interpretation in 3+1 scheme: inconsistent
(more than 1s disfavored by
cosmology)

$$\underbrace{\vartheta_{e\mu}}_{0.035} \approx \underbrace{\vartheta_{es}}_{0.2} \times \vartheta_{\mu s} \Rightarrow \vartheta_{\mu s} \approx 0.2$$

predicted suppression in ν_μ disappearance
experiments: undetected

by ignoring LSND/Miniboone data the
reactor anomaly can be accommodated
by $m_s \geq 1$ eV and $\vartheta_{es} \approx 0.2$
[not suitable for Warm DM]



EXERCISE

estimate Δm^2_{21} from position of second oscillation dip in previous plot

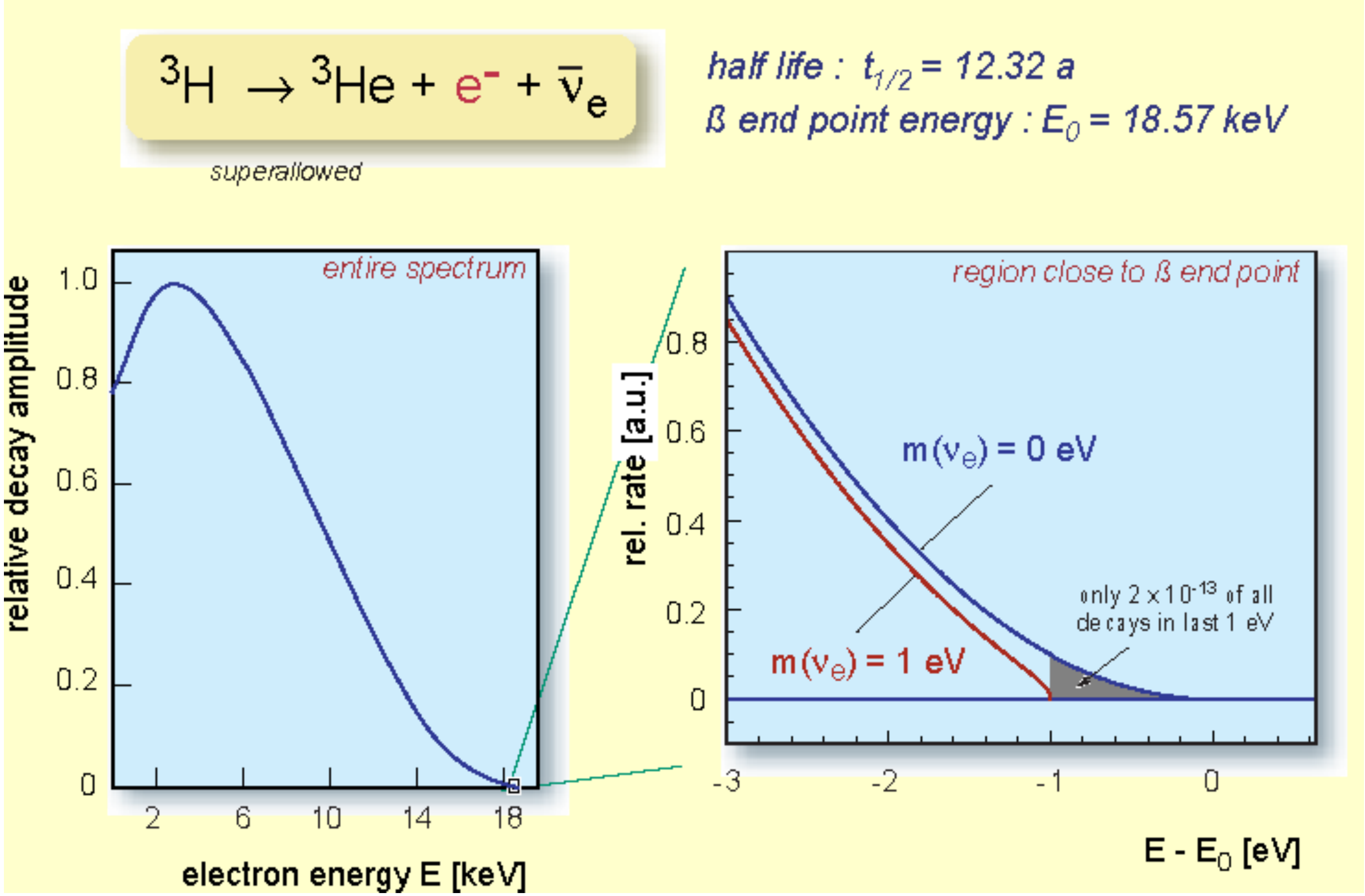
$$\Delta m^2_{21} = 6\pi \left.\frac{E}{L}\right|_{dip} \approx 6\pi \times \frac{1}{50}\, MeV/Km = 7.5\times10^{-5}\ eV^2$$

EXERCISE

work out P_{ee} by keeping U_{e3} non-vanishing

$$P_{ee} \approx \left|U_{e3}\right|^4 + (1-\left|U_{e3}\right|^2)^2\left(1-\sin^2 2\vartheta_{12}\sin^2\Delta_{21}\right)$$

Upper limit on neutrino mass (laboratory)



$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$

superallowed

half life : $t_{1/2} = 12.32$ a

ß end point energy : $E_0 = 18.57$ keV

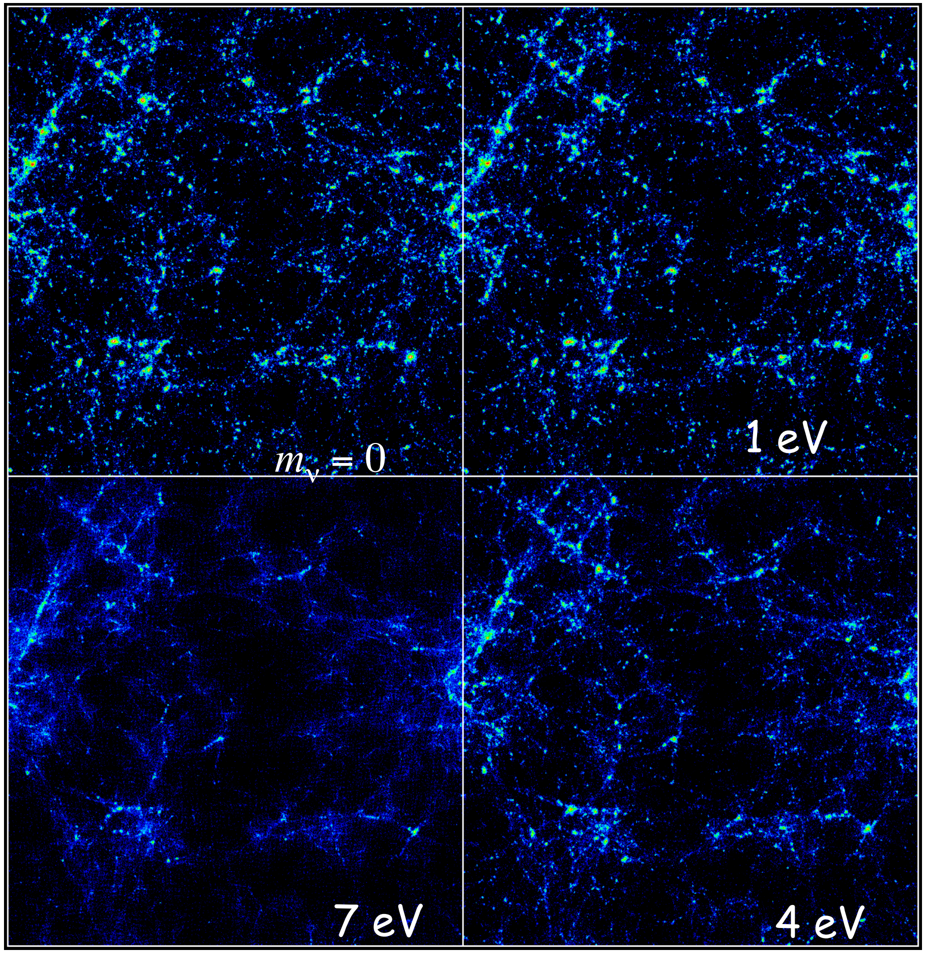
$$m_\nu < 2.2\,eV \quad (95\%\ CL)$$

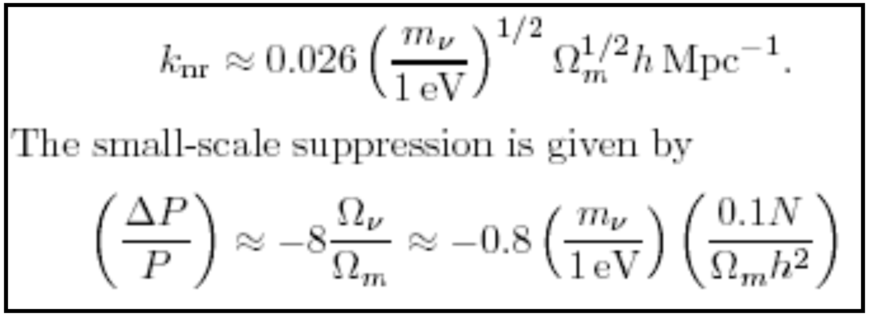
Upper limit on neutrino mass (cosmology)

massive ν suppress the formation of small scale structures

$$\sum_i m_i < 0.2 \div 1 \quad eV$$

depending on
- assumed cosmological model
- set of data included
- how data are analyzed





$$k_{\rm nr} \approx 0.026 \left(\frac{m_\nu}{1\,{\rm eV}}\right)^{1/2} \Omega_m^{1/2} h\,{\rm Mpc}^{-1}.$$

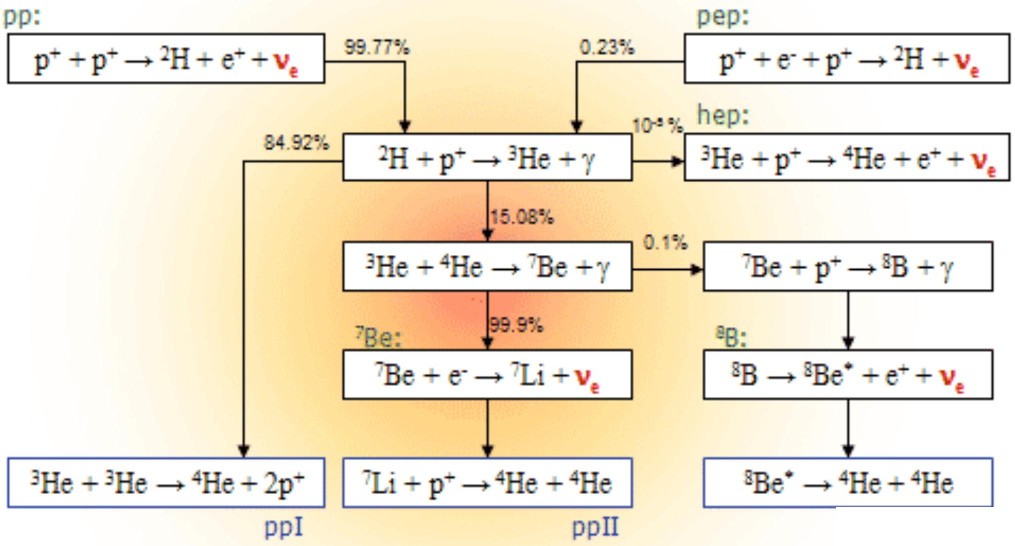
The small-scale suppression is given by

$$\left(\frac{\Delta P}{P}\right) \approx -8\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{m_\nu}{1\,{\rm eV}}\right)\left(\frac{0.1N}{\Omega_m h^2}\right)$$

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)} P(\vec{k})$$

Solar Neutrinos



pp:

$p^+ + p^+ \rightarrow {}^2H + e^+ + \nu_e$

99.77%

0.23%

pep:

$p^+ + e^- + p^+ \rightarrow {}^2H + \nu_e$

84.92%

${}^2H + p^+ \rightarrow {}^3He + \gamma$

10^{-5} %

hep:

${}^3He + p^+ \rightarrow {}^4He + e^+ + \nu_e$

15.08%

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

0.1%

${}^7Be + p^+ \rightarrow {}^8B + \gamma$

99.9%

^{7}Be:

${}^7Be + e^- \rightarrow {}^7Li + \nu_e$

^{8}B:

${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

${}^3He + {}^3He \rightarrow {}^4He + 2p^+$

${}^7Li + p^+ \rightarrow {}^4He + {}^4He$

${}^8Be^* \rightarrow {}^4He + {}^4He$

ppI

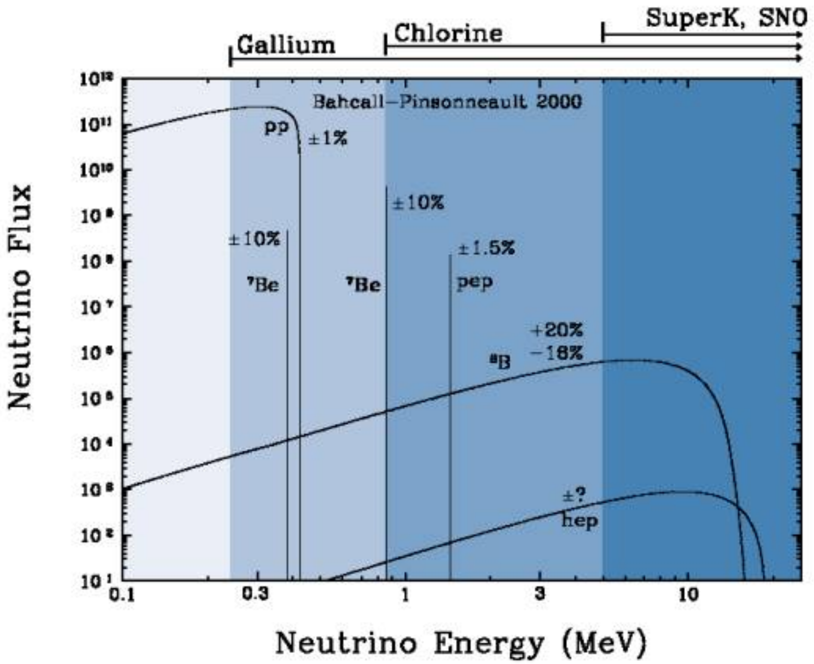
ppII

ν_e produced in the core of the sun through several chains/reactions

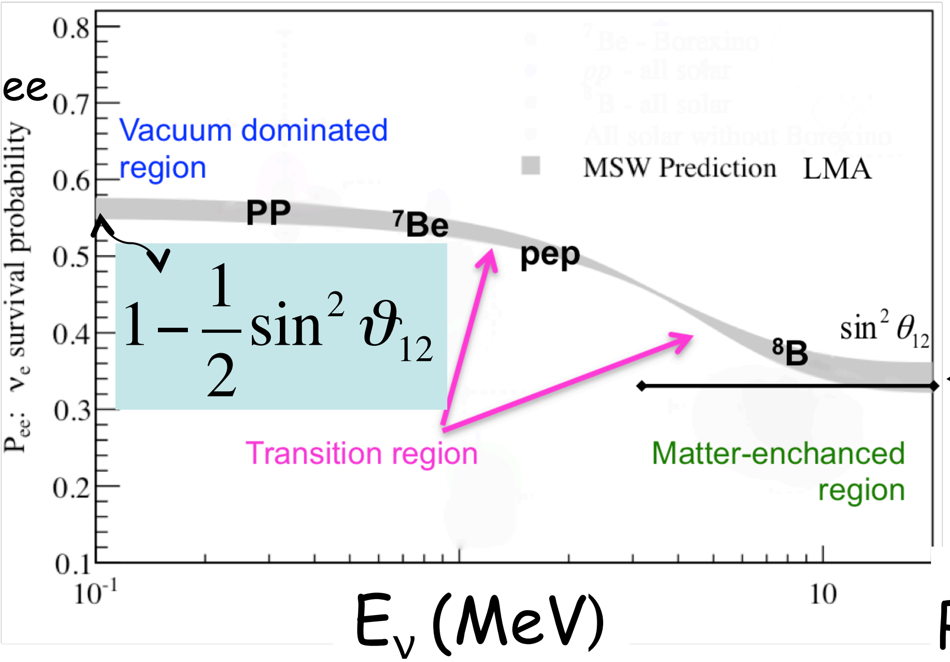
with different energy spectrum

most neutrinos come from pp fusion $E_{max} \approx 0.4$ MeV

most energetic neutrinos come from ^{8}B decay $E_{max} \approx 15$ MeV



Theory prediction for P_{ee}

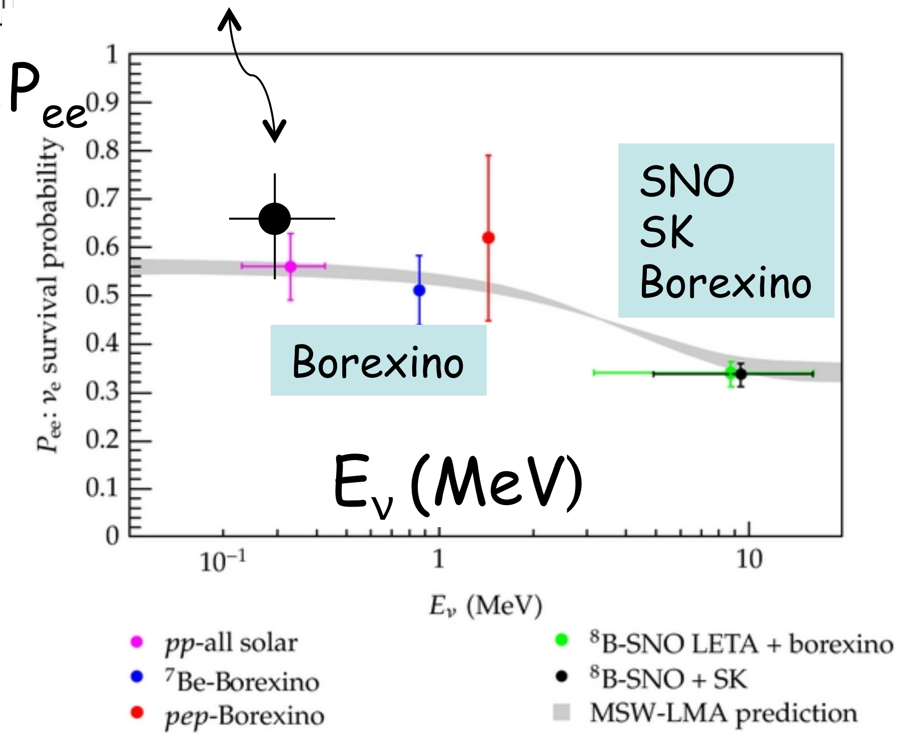


$$1-\frac{1}{2}\sin^2\vartheta_{12}$$

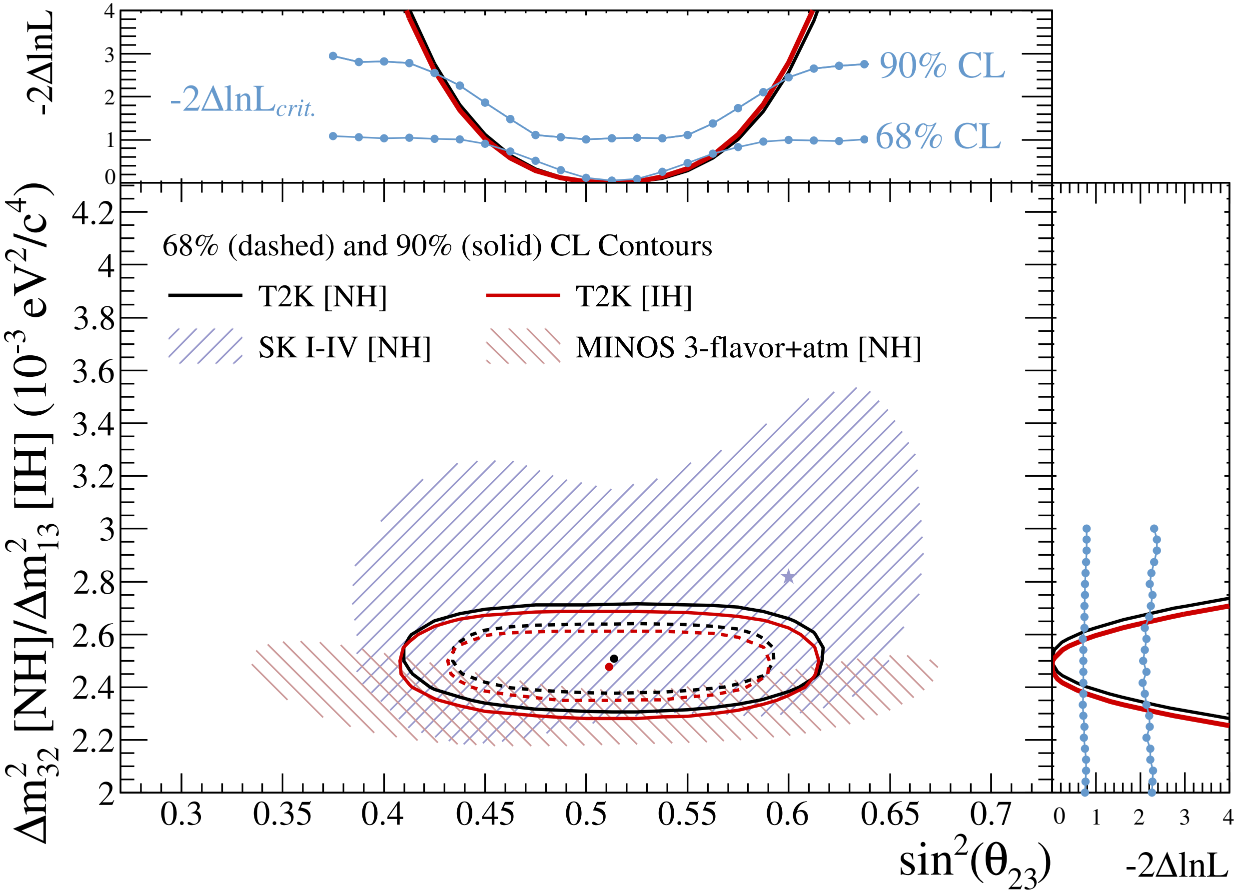
$$\sin^2\vartheta_{12}$$

[Borexino, Nature 512 (2014) 383]

experiments reveal solar neutrinos through different processes and have different energy thresholds



recent results from T2K [hep-ex/1403.1532]



$$\sin^2\vartheta_{23} = \begin{cases} 0.514^{+0.055}_{-0.056} & (NO) \\ 0.511 \pm 0.055 & (IO) \end{cases}$$

$$\Delta m^2_{32} = (2.51 \pm 0.10) \times 10^{-3}\, eV^2 \quad (NO)$$

$$\Delta m^2_{13} = (2.48 \pm 0.10) \times 10^{-3}\, eV^2 \quad (IO)$$

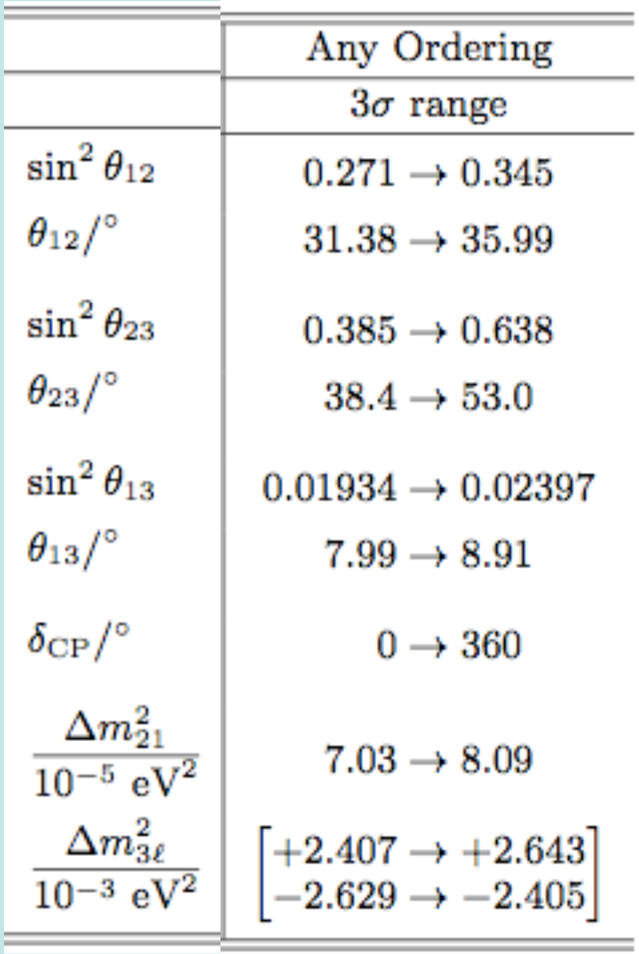
main detection processes

Neutrinos	Experiment	Process
atmospheric ν	SK K2K, MINOS, T2K, Opera	$\nu N \to l X$
solar ν	SK, Borexino SNO	$\nu_X e \to \nu_X e$ $\nu_X D \to \nu_X pn, \nu_e D \to e pp$
reactor ν	KamLand, Chooz, DoubleChooz, Reno, Daya Bay	$\bar{\nu}_e p \to e^+ n \quad (e^+ D\gamma)$

Summary of data

$m_\nu < 2.2\, eV \quad (95\%\ CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)



	Any Ordering
	3σ range
$\sin^2\theta_{12}$	$0.271 \to 0.345$
$\theta_{12}/^\circ$	$31.38 \to 35.99$
$\sin^2\theta_{23}$	$0.385 \to 0.638$
$\theta_{23}/^\circ$	$38.4 \to 53.0$
$\sin^2\theta_{13}$	$0.01934 \to 0.02397$
$\theta_{13}/^\circ$	$7.99 \to 8.91$
$\delta_{CP}/^\circ$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.03 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$\begin{bmatrix} +2.407 \to +2.643 \\ -2.629 \to -2.405 \end{bmatrix}$

[Esteban, G.-Garcia, Maltoni, M-Soler, Schwetz 1611.01514]

violation of individual lepton number implied by neutrino oscillations

Summary of unkowns

absolute neutrino mass scale is unknown [but well-constrained!]

sign $[\Delta m^2_{atm}]$ unknown

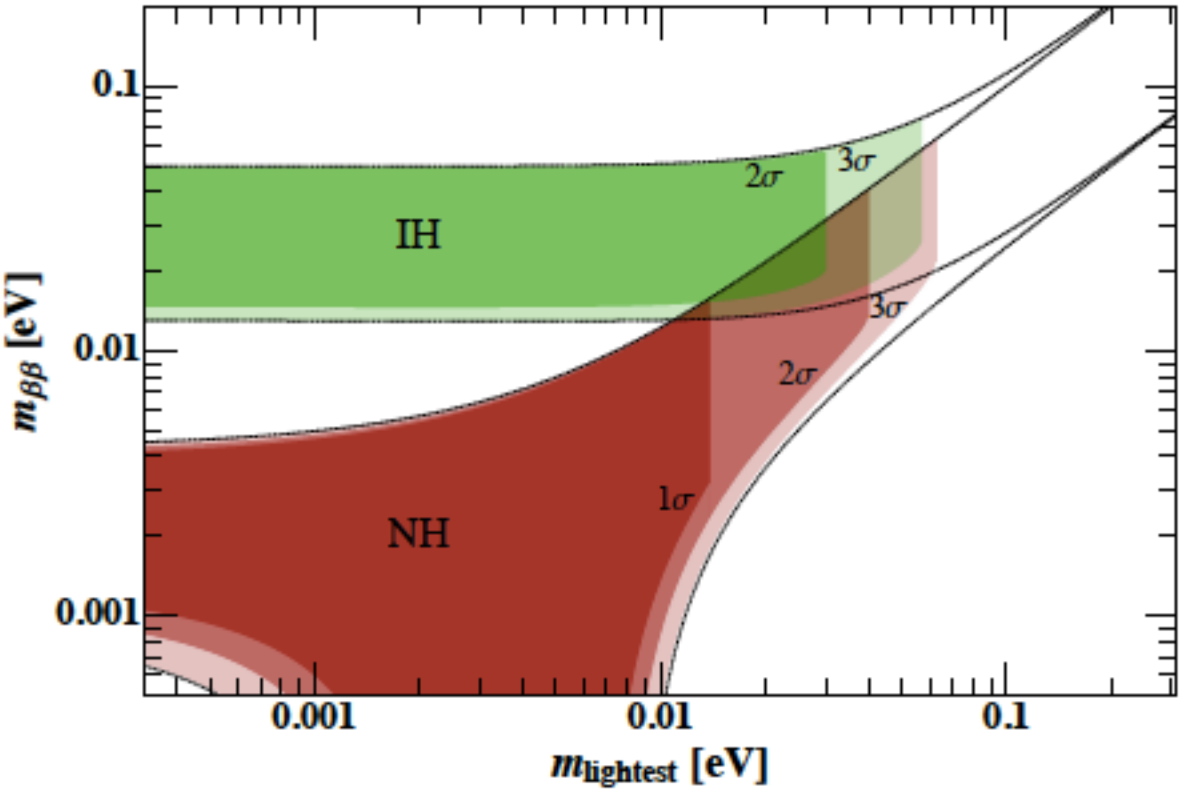
[complete ordering (either normal or inverted hierarchy) not known]

δ, α, β unknown

[CP violation in lepton sector not yet established]

violation of total lepton number not yet established

impact of limits from cosmology



flavor puzzle made simpler in SU(5) ?

Higgs

$\bar{5} = (l, d^c) \qquad 10 = (q, u^c, e^c) \qquad 1 = \nu^c$

$\Phi_5 = (\Phi_D, \Phi_T)$

$$L_Y = -10\, y_u 10\, \Phi_5 - \bar{5}\, y_d 10\, \Phi_5^+ - 1 y_\nu \bar{5}\, \Phi_5 - \frac{1}{2} 1 M 1 + h.c.$$

$y_d = y_e^T$

$m_b = m_\tau$ O.K.

$m_s = m_\mu$ wrong, but not by orders of magnitude

$m_d = m_e$ can be fixed with additional Higgs

$m_s \approx m_\mu / 3$

$m_d \approx 3\, m_e$

suppose that y_u, y_e, y_ν and M/Λ are anarchical matrices [O(1) matrix elements] and that the observed hierarchy is due to the wave function renormalization of matter multiplets (we will see how later on)

$$10 \to F_{10}\, 10 \qquad \bar{5} \to F_{\bar{5}}\, \bar{5} \qquad 1 \to F_1\, 1$$

$$F_X = \begin{pmatrix} \lambda^{Q_{X_1}} & 0 & 0 \\ 0 & \lambda^{Q_{X_2}} & 0 \\ 0 & 0 & \lambda^{Q_{X_3}} \end{pmatrix}$$

$\lambda \approx 0.22$

$Q_{X_1} \geq Q_{X_2} \geq Q_{X_3}$

F_1 dependence cancels in m_ν

$\Upsilon_u = F_{10} y_u F_{10} \qquad \Upsilon_d = F_{\bar{5}} y_d F_{10} \qquad \Upsilon_e = F_{10} y_d^T F_{\bar{5}} \qquad m_\nu \propto F_{\bar{5}} y_\nu^T M^{-1} y_\nu F_{\bar{5}}$

large mixing in lepton sector suggests $F_{\bar{5}} \approx \text{diag}(1,1,1)$

hierarchy mostly due to F_{10} $\quad m_u : m_c : m_t \approx m_d^2 : m_s^2 : m_b^2 \approx m_e^2 : m_\mu^2 : m_\tau^2$

large l mixing corresponds to a large d^c mixing: unobservable in weak int. of quarks

how can a wave function renormalization (effectively) arise?

several possibilities

here (Exercise 5): bulk fermions in a compact extra dimension S^1/Z_2

$$\mathcal{L} = i\bar{\Psi}_1\Gamma^M\partial_M\Psi_1 + i\bar{\Psi}_2\Gamma^M\partial_M\Psi_2 - m_1\varepsilon(y)\bar{\Psi}_1\Psi_1 + m_2\varepsilon(y)\bar{\Psi}_2\Psi_2 - \left[\delta(y)\frac{y}{\Lambda}\bar{f}_1(h+v)f_2 + h.c.\right]$$

$$\Psi_1 = \begin{pmatrix} E_1 \\ \bar{f}_1 \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} f_2 \\ \bar{E}_2 \end{pmatrix}$$

solve the e.o.m. for the fermion zero modes with the b.c.

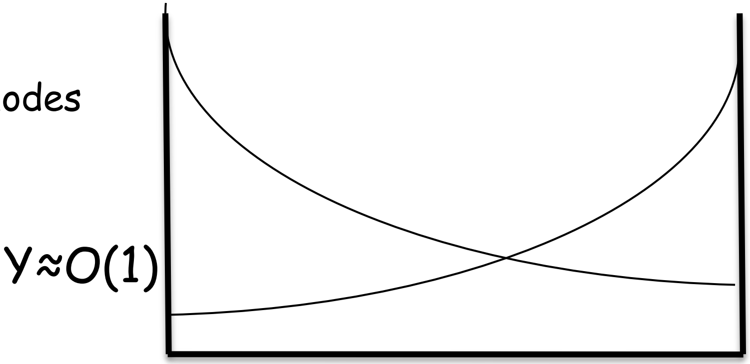
$$-\gamma_5\partial_y\Psi^0_{1,2} \pm m_{1,2}\,\varepsilon(y)\Psi^0_{1,2} = 0$$

$$\Psi_1(-y) = +\gamma_5\Psi_1(y)$$

$$\Psi_2(-y) = -\gamma_5\Psi_2(y)$$

$$f_i^0(y) = \sqrt{\frac{2m_i}{1-e^{-2m_i\pi R}}}e^{-m_i y}$$

vanishing zero-modes for $(E_1, \bar{E}_2)$



$$\mathcal{L}_Y = -\frac{1}{\Lambda\pi R}\bar{f}_1(F_1 y F_2)(h+v)f_2$$

$$F_i = \sqrt{\frac{x_i}{1-e^{-x_i}}} \approx \begin{cases} e^{-x_i/2} & x_i >> 1 \\ 1 & x_i \approx 0 \\ \sqrt{-x_i} & x_i << -1 \end{cases}$$

Flavor symmetries I (the hierarchy puzzle)

hierarchies in fermion spectrum

quarks

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

leptons

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = (0.025 \div 0.049) \approx \lambda^2 << 1 \quad (2\sigma)$$

$$|U_{e3}| < 0.18 \leq \lambda \quad (2\sigma)$$

call ξ_i the generic small parameter. A modern approach to understand why ξ_i<<1 consists in regarding ξ_i as small breaking terms of an approximate flavour symmetry. When ξ_i=0 the theory becomes invariant under a flavour symmetry F

Example: why y_e<<y_{top}? Assume F=U(1)$_F$

F(t)=F(t^c)=F(h)=0 $\quad y_{top}(h+v)t^c t \quad$ allowed

F(e^c)=p>0 F(e)=q>0 $\quad y_e(h+v)e^c e \quad$ breaks U(1)$_F$ by (p+q) units

if ξ=<φ>/Λ<1 breaks U(1) by one negative unit $\quad y_e \approx O(\xi^{p+q}) << y_{top} \approx O(1)$

provides a qualitative picture of the existing hierarchies in the fermion spectrum

Exercise 1: anomalies of B and L_i

the anomaly of the baryonic current and the individual leptonic currents are proportional to tr[Q $\{T^A,T^B\}$] and tr[Q $\{Y,Y\}$] where Q=(B,L_i) and (T^A,Y) are the generators of the electroweak gauge group

compute these traces in the SM with 3 fermion generations

$$\frac{1}{2}\mathrm{tr}[B\{T^A,T^B\}] = 3(gen)\times 3(col)\times\frac{1}{3}(B)\times\left[\frac{1}{4}(up)+\frac{1}{4}(down)\right]\delta^{AB} = \frac{3}{2}\delta^{AB}$$

$$\frac{1}{2}\mathrm{tr}[L_i\{T^A,T^B\}] = 1(L_i)\times\left[\frac{1}{4}(nu)+\frac{1}{4}(e)\right]\delta^{AB} = \frac{1}{2}\delta^{AB}$$

$$\frac{1}{2}\mathrm{tr}[B\{Y,Y\}] = 3(gen)\times 3(col)\times\frac{1}{3}(B)\times\left[\frac{1}{18}(Doubl)-\frac{10}{18}(Singl)\right] = -\frac{3}{2}$$

$$\frac{1}{2}\mathrm{tr}[L_i\{Y,Y\}] = 1(L_i)\times\left[\frac{1}{2}(Doubl)-1(Singl)\right] = -\frac{1}{2}$$

(B+L) is anomalous, (B/3-L_i) [and (B-L)] are anomaly-free

Exercise 2

derive the see-saw relation by integrating out the fields ν^c through their e.o.m. in the heavy M limit. Compute the 1st order corrections in p/M

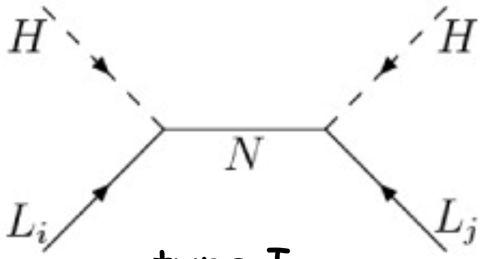
equations of motion of ν^c

$$\begin{pmatrix}\nu^c\\ \bar{\nu}^c\end{pmatrix} = \begin{pmatrix} i\bar{\sigma}^\mu\partial_\mu & -M^+ \\ -M & i\sigma^\mu\partial_\mu\end{pmatrix}^{-1}\begin{pmatrix} y_\nu^*\bar{\omega} \\ y_\nu\omega\end{pmatrix} = \begin{pmatrix}-M^{-1}y_\nu\omega \\ -M^{*-1}y_\nu^*\bar{\omega}\end{pmatrix}+\dots \qquad \omega\equiv(\tilde{\Phi}^+ l)$$

$$L_{eff} = i\bar{l}\,\bar{\sigma}^\mu\partial_\mu l + \frac{1}{2}\Big[\underbrace{\omega(y_\nu^T M^{-1}y_\nu)\omega}_{d-=5} + h.c.\Big] + \underbrace{i\bar{\omega}(y_\nu^+ M^{+-1}M^{-1}y_\nu)\bar{\sigma}^\mu\partial_\mu\omega}_{d-=6} + O(M^{-3})$$

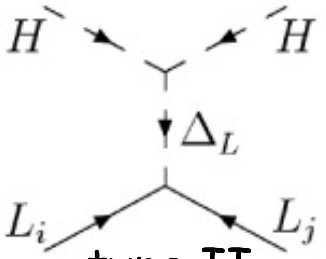
renormalizes the KE of ν by v^2/M^2

there are 3 types of see-saw depending on the particle we integrate out
they all give rise to the same d=5 operator



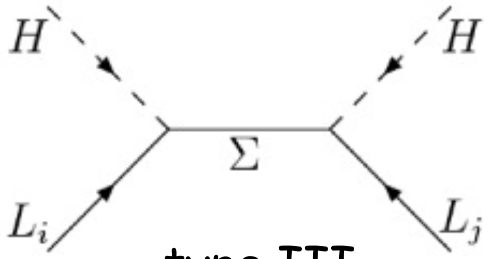
type I

$$y_N^T(M_N)^{-1}y_N$$



type II

$$y_\Delta\frac{\mu}{M_\Delta^2}$$



type III

$$y_\Sigma^T(M_\Sigma)^{-1}y_\Sigma$$

Exercise 3: gauge coupling unification

0th order approximation

justify this $$\sqrt{\frac{5}{3}}g_Y = g_2 = g_3$$ $$\sin^2\vartheta_W = \frac{g_Y^2}{g_Y^2+g_2^2} = \frac{3}{8} \approx 0.375$$

include 1-loop running

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(m_Z)} + \frac{b_i}{2\pi}\log\frac{Q}{m_Z}$$

$$\left(\begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array}\right)_{MSSM} = \left(\begin{array}{c} 33/5 \\ 1 \\ -3 \end{array}\right) \qquad \left(\begin{array}{c} b_1 \\ b_2 \\ b_3 \end{array}\right)_{SM} = \left(\begin{array}{c} 41/10 \\ -19/6 \\ -7 \end{array}\right)$$

knowledge of b.c. M_{GUT} and $\alpha_U=\alpha(M_{GUT})$ would allow to predict $\alpha_i(m_Z)$
in practice, we use as inputs

$$\alpha_{em}^{-1}(m_Z)\Big|_{\overline{MS}} = 127.934 \qquad \sin^2\vartheta(m_Z)\Big|_{\overline{MS}} = 0.231$$

to predict
[MSSM]

$$\alpha_3(m_Z)\Big|_{\overline{MS}} = \frac{7\alpha_{em}(m_Z)}{15\sin^2\vartheta(m_Z)-3} \approx 0.118$$

$$\alpha_U = \frac{28\alpha_{em}(m_Z)}{36\sin^2\vartheta(m_Z)-3} \approx \frac{1}{25}$$

$$\log\left(\frac{M_{GUT}}{m_Z}\right) = \pi\frac{3-8\sin^2\vartheta(m_Z)}{14\alpha_{em}(m_Z)} \quad\Rightarrow\quad M_{GUT} \approx 2\times 10^{16}\,\text{GeV}$$

[corrections from 2-loop RGE,
threshold corrections at M_{SUSY},
threshold corrections at M_{GUT}]

Exercise 4: effective lagrangian for nucleon decay

recognize that, the with the SM particle content, the lowest dimensional operators violating B occur at d=6. Make a list of them

$$\frac{1}{\Lambda_B^2} \times \left\{ \begin{array}{cc} q q u^{c+} e^{c+} & q q q l \\ q l u^{c+} d^{c+} & u^c u^c d^c e^c \end{array} \right.$$

color and SU(2) indices contracted

notice that they respect ΔB=ΔL: nucleon decay into antileptons e.g. p->e^+ π^0, n->e^+ π^- [n->$e^-\pi^+$ suppressed by further powers of Λ_B]

naïve estimate

$$\tau_p \approx \frac{\Lambda_B^4}{m_p^5}$$

assuming

$$\tau_p(p \rightarrow e^+ \pi^0) > 1.4 \times 10^{34}\, ys$$ [SK]

we get

$$\Lambda_B > 2.6 \times 10^{16} \quad \text{GeV}$$

in GUTs Λ_B is related to the scale M_{GUT} at which the grand unified symmetry is broken down to SM gauge group

the observed proton stability is guaranteed by the largeness of M_{GUT}

In SUSY extensions of the SM the lowest dimensional operators violating B occur at d=5: why?

more refined bound [Davidson and Ibarra 0202239]

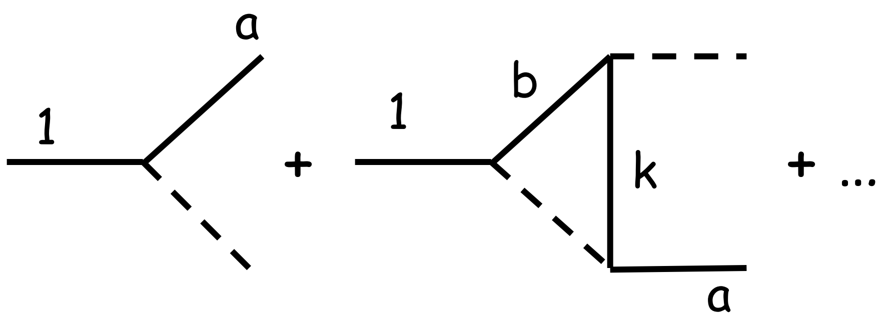
$$\left|\varepsilon_1^{\infty}\right| \le \varepsilon_1^{DI} = \frac{3}{16\pi}\frac{M_1}{v^2}(m_3 - m_1)$$

$$T_R \approx M_1 > (4\times 10^8 \div 2\times 10^9)\ GeV$$

in conflict with the bound on T_R in SUSY models
to avoid overproduction of gravitinos

$$T_R^{SUSY} < 10^{7-9}\ GeV$$

Exercise 7: reconstruct the flavour structure of ε_1



$$\mathcal{A}(\nu_1^c \to l_a \Phi) \propto y^+_{a1} + W\ y_{1b} y^+_{bk} y^+_{ak}$$

$$\mathcal{A}(\nu_1^c \to \bar{l}_a \Phi^*) \propto y_{1a} + W\ y^+_{b1} y_{kb} y_{ka}$$

$$\varepsilon_1 \propto \frac{\left|y^+_{a1} + W\ y_{1b} y^+_{bk} y^+_{ak}\right|^2 - \left|y_{1a} + W\ y^+_{b1} y_{kb} y_{ka}\right|^2}{\left|y^+_{a1} + W\ y_{1b} y^+_{bk} y^+_{ak}\right|^2 + \left|y_{1a} + W\ y^+_{b1} y_{kb} y_{ka}\right|^2} \approx \frac{\mathrm{Im}(W)\,\mathrm{Im}\{[(yy^+)_{1k}]^2\}}{(yy^+)_{11}}$$

[sums understood]

$$\mathrm{Im}(W) \approx \frac{M_1}{M_k}$$

Exercise 8: count the number of physical parameters in the type I see-saw model distinguish between moduli and phases

y_e, y_ν and M depend on (18+18+12)=48 parameters, 24 moduli and 24 phases

we are free to choose any basis leaving the kinetic terms canonical
(and the gauge interactions unchange)

$$e^c \to \Omega_{e^c} e^c \qquad \nu^c \to \Omega_{\nu^c} \nu^c \qquad l \to \Omega_l l \qquad [U(3)^3]$$

these transformations contain 27 parameters (9 angles and 18 phases)
and effectively modify y_e, y_ν and M

$$y_e \to \Omega^T_{e^c} y_e \Omega_l \qquad y_\nu \to \Omega^T_{\nu^c} y_\nu \Omega_l \qquad M \to \Omega^T_{\nu^c} M \Omega_{\nu^c}$$

so that we can remove 27 parameters from y_e, y_ν and M

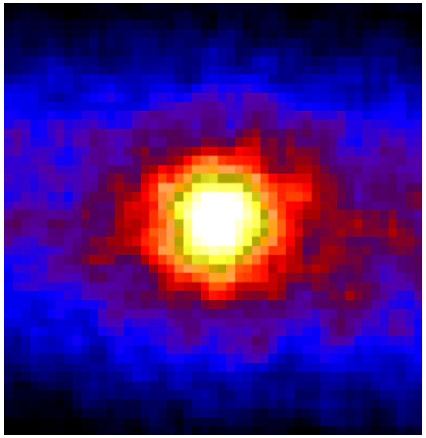
we remain with 21 parameters: 15 moduli and 6 phases
the moduli are 9 physical masses and 6 mixing angles

the same count in the quark sector would give a total of 9 moduli
(6 masses amd 3 mixing angles) and 0 phases <- wrong
how the above argument should be modified, in general?

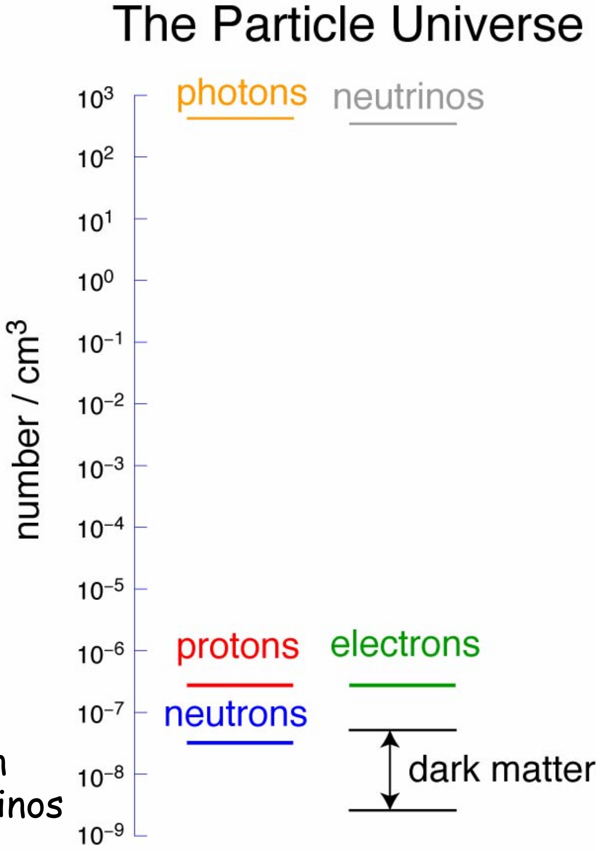
General remarks on neutrinos

the more abundant particles in the universe after the photons: about 300 neutrinos per cm^3

produced by stars: most of the sun energy emitted in neutrinos. As I speak more than 1 000 000 000 000 000 solar neutrinos go through your bodies each second.



this is a picture of the sun reconstructed from neutrinos



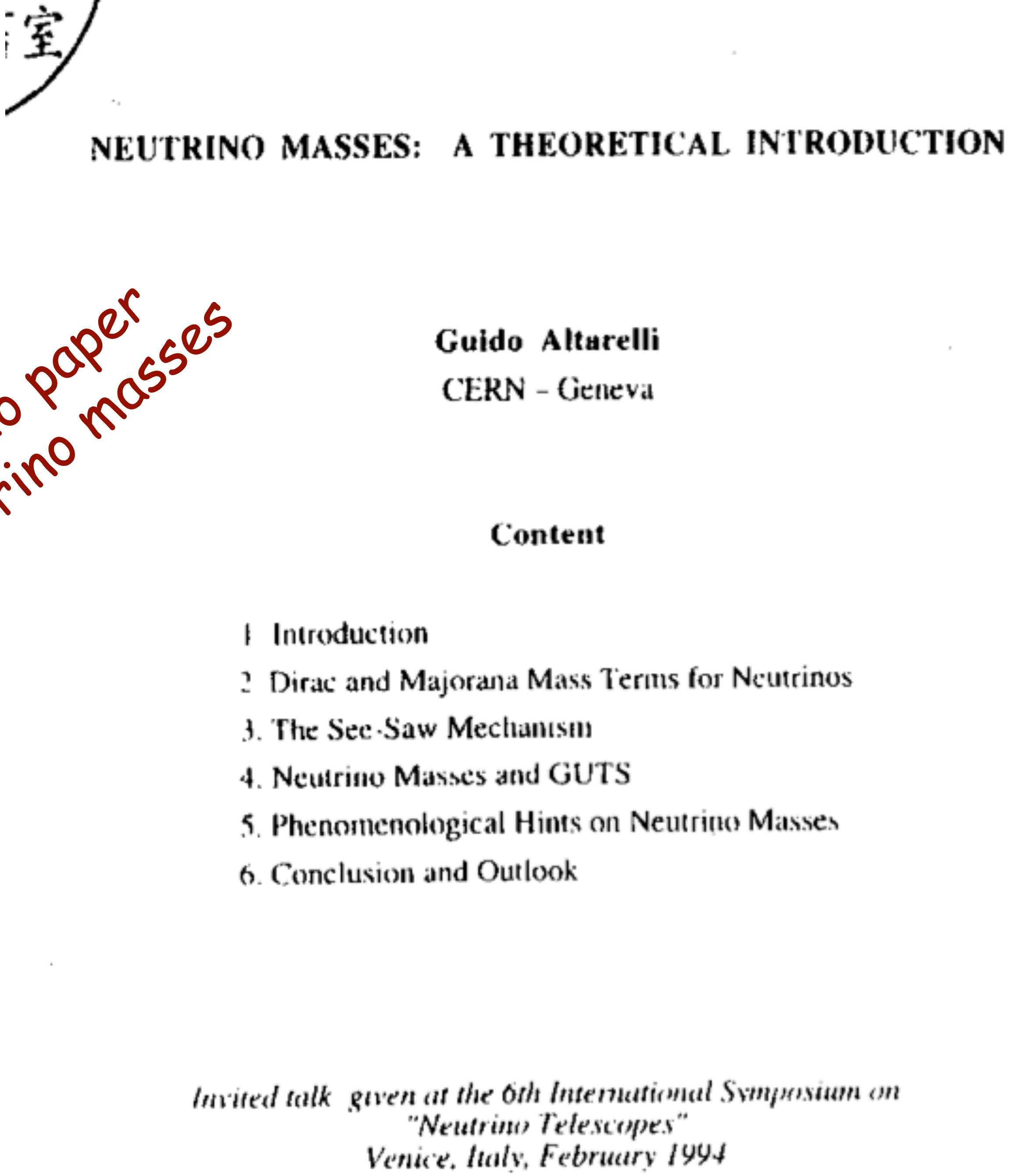
electrically neutral and extremely light:
they can carry information about extremely large length scales e.g. a probe of supernovae dynamics: neutrino events from a supernova explosion first observed almost 30 years ago

in particle physics:
they have a tiny mass (1 000 000 times smaller than the electron's mass) the discovery that they are massive allows us to explore, at least in principle, extremely high energy scales, otherwise inaccessible to present laboratory experiments

structure of the mixing matrix

$$\begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23}-c_{12}\,s_{13}\,s_{23}\,e^{-i\delta} & c_{12}\,c_{23}-s_{12}\,s_{13}\,s_{23}\,e^{-i\delta} & c_{13}\,s_{23} \\ -c_{12}\,s_{13}\,c_{23}\,e^{-i\delta}+s_{12}\,s_{23} & -s_{12}\,s_{13}\,c_{23}\,e^{-i\delta}-c_{12}\,s_{23} & c_{13}\,c_{23} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



NEUTRINO MASSES: A THEORETICAL INTRODUCTION

Guido Altarelli
CERN – Geneva

Content

1. Introduction
2. Dirac and Majorana Mass Terms for Neutrinos
3. The See-Saw Mechanism
4. Neutrino Masses and GUTS
5. Phenomenological Hints on Neutrino Masses
6. Conclusion and Outlook

Invited talk given at the 6th International Symposium on "Neutrino Telescopes"
Venice, Italy, February 1994

1st Guido paper
on neutrino masses