Emittance preserving staging optics for PWFA and LWFA

Physics and Applications of High Brightness Beams – *Havana, Cuba*

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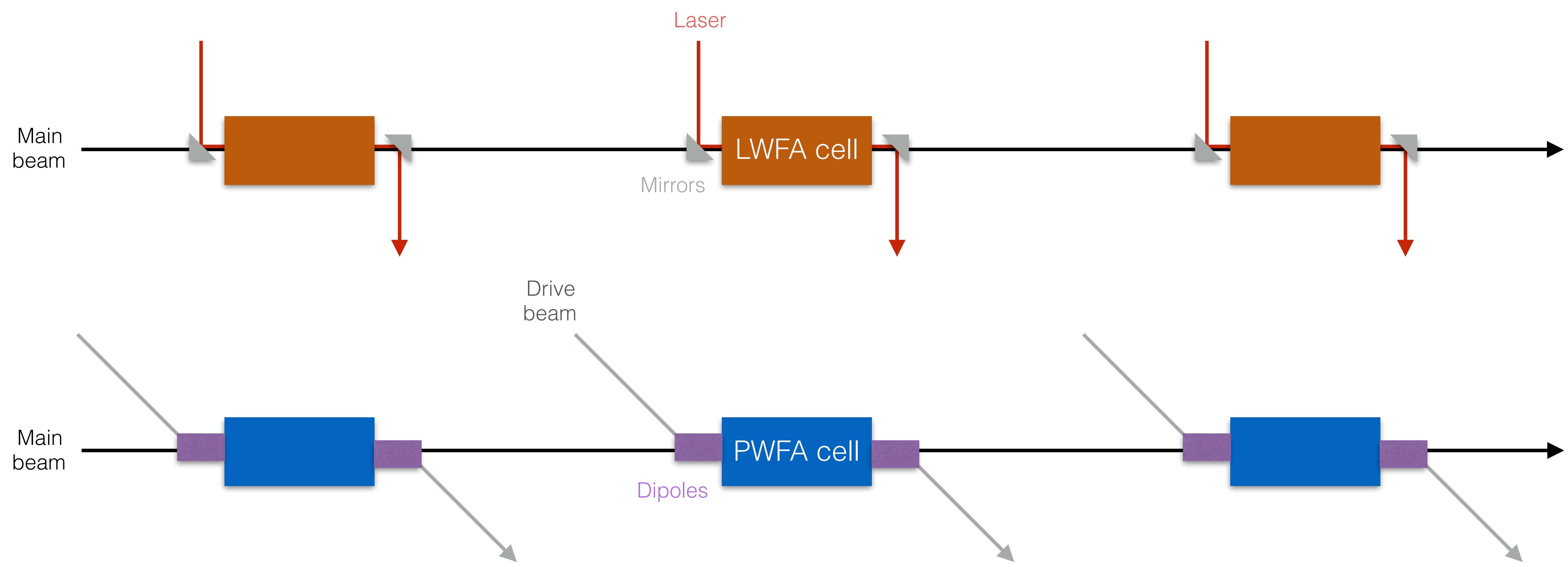
Outline

- Need for staging optics
- Chromatic focusing errors
- Conventional approach to cancellation
- Alternative approach to cancellation
- A few examples (LWFA and PWFA)

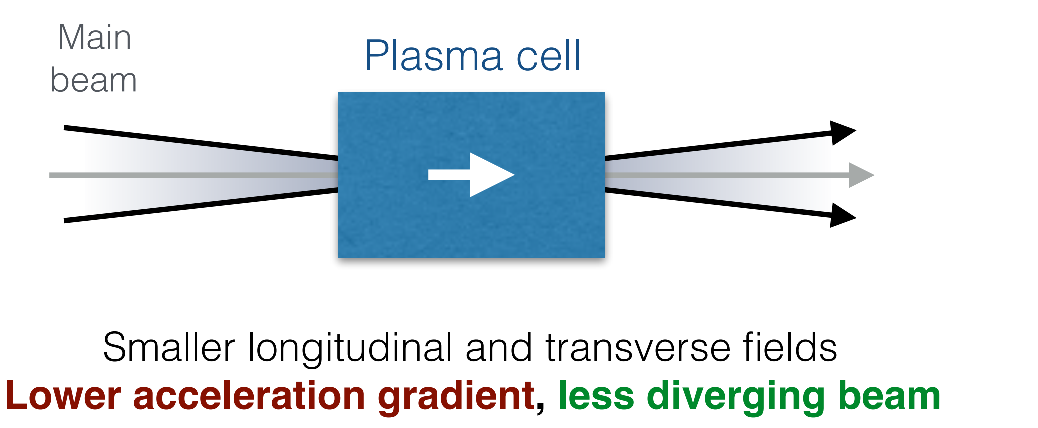
Staging

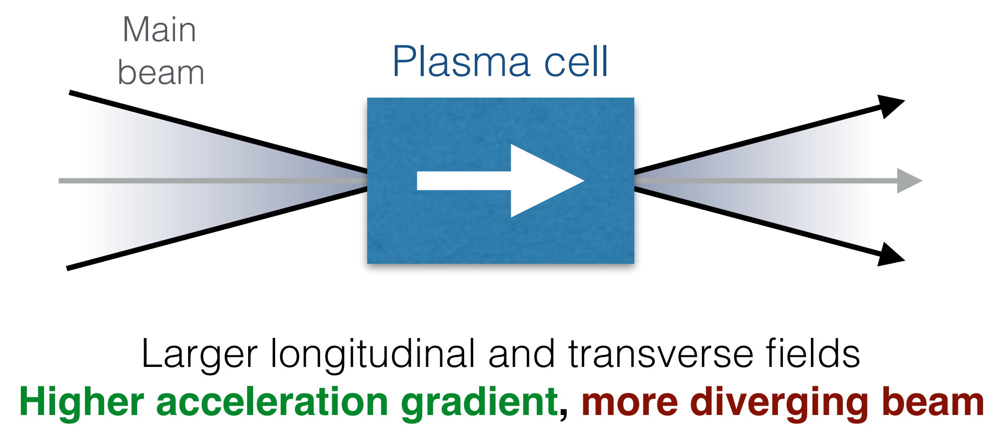
- Assume a working LWFA/PWFA cell (complicated black box).
- The main beam can only gain as much energy as is carried by the drive beam.
- To go higher in energy: **Daisy chain multiple accelerator cells (staging)**



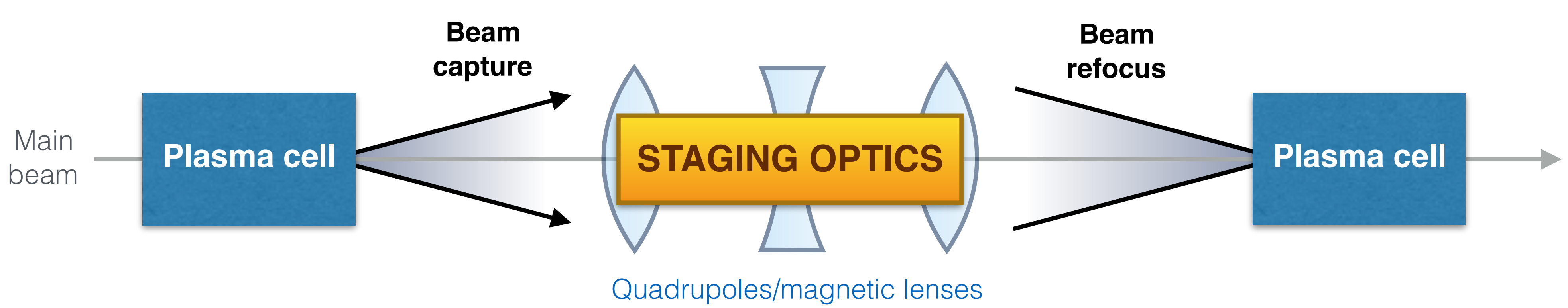
Staging optics

- PWFA/LWFA requires beams matched to a very small beta functions:
  Highly diverging beams exiting the plasma





- Need magnetic beam optics to capture and refocus the main beam: **staging optics**

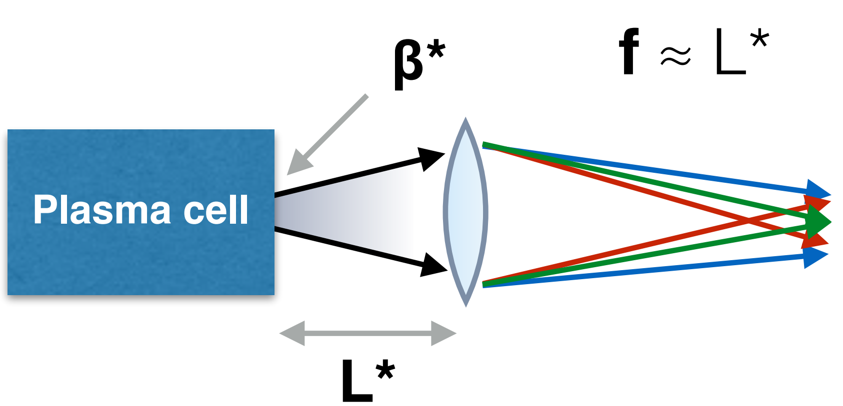


Chromatic focusing errors

- Imperfect focusing of offset energies is a big problem.

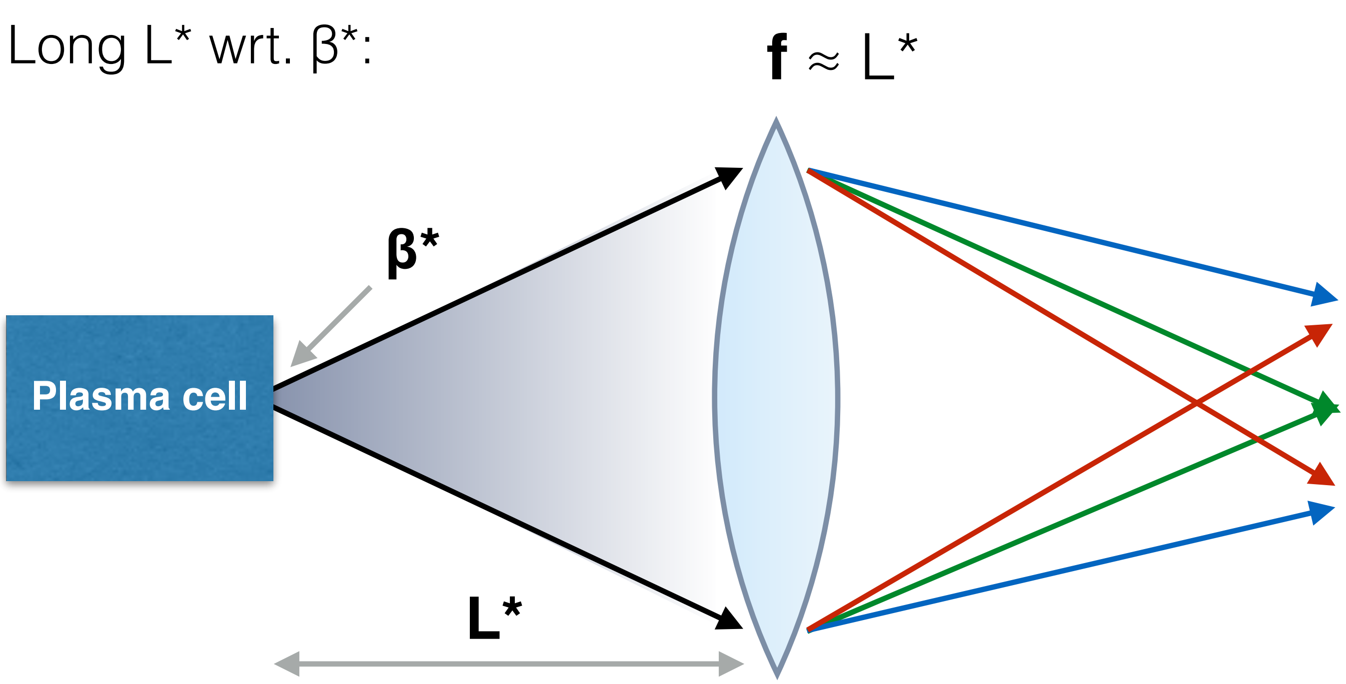
- This arises from:
  - Tightly focused beams (small matched betas)
  - Long drift spaces after the plasma (for injection/extraction)
  - Large energy spreads.

Short L* wrt. β*:



Relative focusing error $\frac{\left(\frac{\Delta\beta}{\Delta\delta}\right)}{\beta}$ is SMALL

Long L* wrt. β*:



Relative focusing error $\frac{\left(\frac{\Delta\beta}{\Delta\delta}\right)}{\beta}$ is LARGE

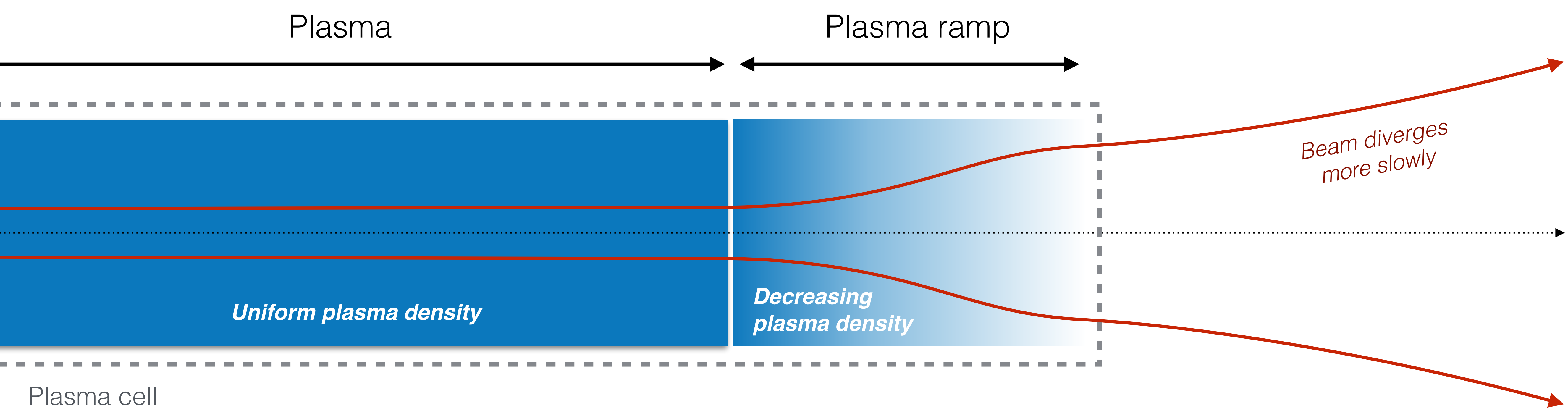
Chromatic focusing errors

- Let's define *W-function (chromatic amplitude)*: (focusing error to 1st order in energy offset δ)
$$W = \sqrt{\left(\frac{\partial\alpha}{\partial\delta} - \frac{\alpha}{\beta}\frac{\partial\beta}{\partial\delta}\right)^2 + \left(\frac{1}{\beta}\frac{\partial\beta}{\partial\delta}\right)^2}$$
- Each lens/quadrupole contributes to **W** by approximately:
$$\Delta W_{quad} \approx \frac{\beta}{f_{quad}}$$
- After a plasma cell, the beam drifts a distance:
$$L^* \gg \beta^* \longrightarrow \beta(L^*) \approx \frac{{L^*}^2}{\beta^*}$$
- The chromatic amplitude added in the first quadrupole:
$$W^* \approx \left(\frac{{L^*}^2}{\beta^*}\right)\frac{1}{L^*} = \frac{L^*}{\beta^*}$$
- The emittance growth of a beam with energy spread:
$$\frac{\Delta\epsilon_{proj}}{\epsilon_0} \approx \frac{1}{2}W^2{\sigma_E}^2$$
- Emittance preserved only for very small energy spreads:
$$\frac{\Delta\epsilon_{proj}}{\epsilon_0} \ll 1 \longrightarrow \boxed{\sigma_E \lll \sqrt{\frac{\Delta\epsilon_{proj}}{\epsilon_0}}\frac{\beta^*}{L^*}}$$

Very small energy acceptance if W is not canceled

Mitigator: Plasma density ramps

- Using specially tapered (adiabatic) plasma density ramps increases the effective β*.
- Lower plasma density ⇒ Larger β* ⇒ Larger acceptable energy spreads
- Problem: Beam-plasma interaction occurring at different densities, which **ruins the beam loading**, and therefore increases energy spread.
- Estimated compromise: about **10 times larger matched betas**. Good, but often insufficient.

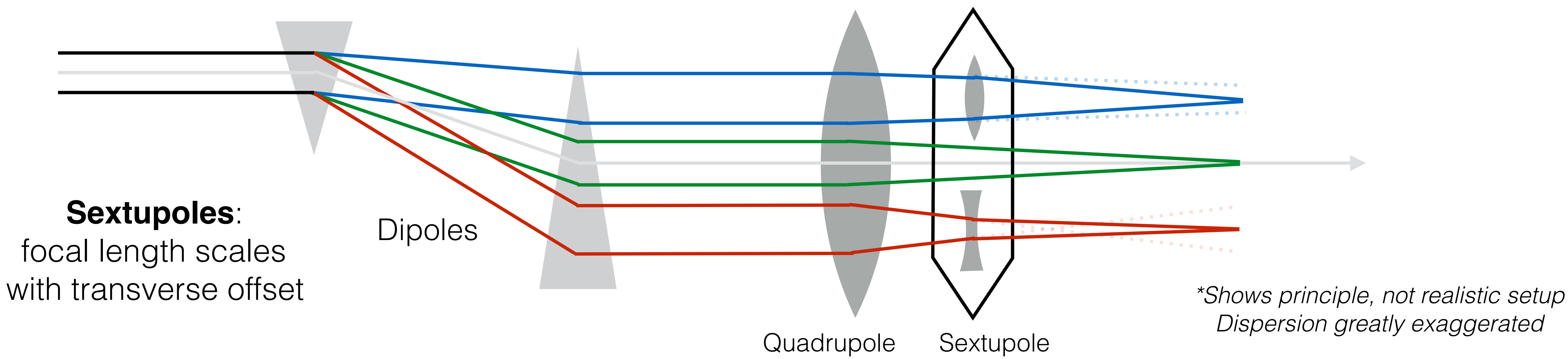


Correcting chromatic errors with sextupoles

- Conventionally, chromatic errors are corrected using sextupoles in regions of large dispersion.

- Cancels the *chromaticity* ξ (energy dependence of the phase advance/tune), which also cancels the W-function.

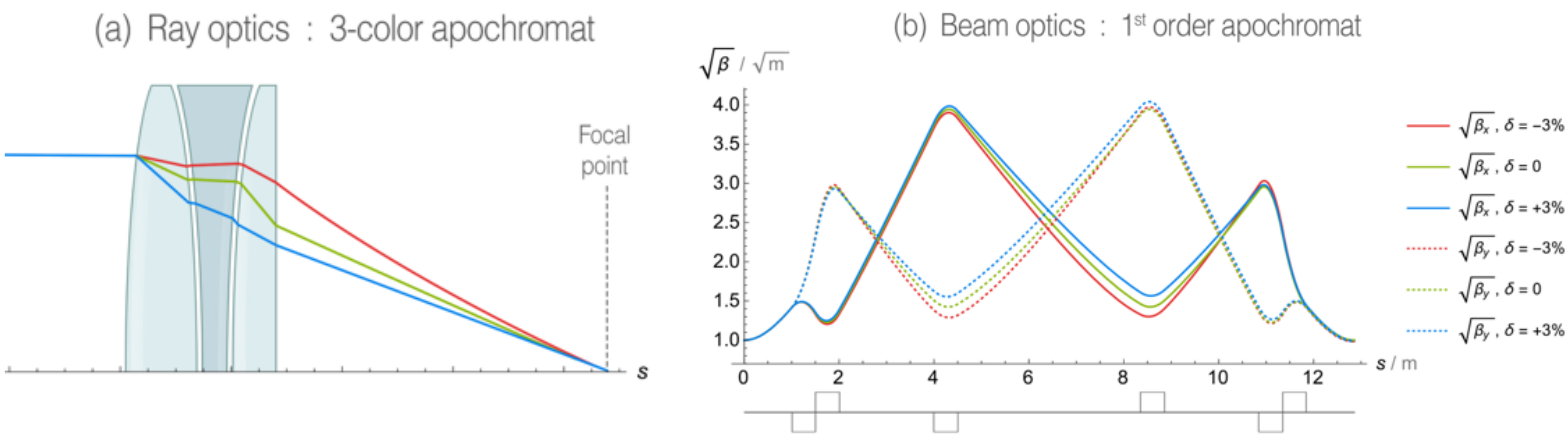
$$\xi = \frac{1}{2\pi}\frac{\partial \mu}{\partial \delta}$$



- Ideal for rings (avoiding tune resonance), but introduces several problems in linear accelerators:
  – **Sextupoles introduce non-linearities**: must be canceled by long, complex lattices.
  – **Strong dispersion is required**: must be canceled.
  – **Bad synchrotron radiation power scaling** with beam energy due to dipoles.

Alternative: Apochromatic focusing

Article submitted to PR-AB: Lindstrøm & Adli,
"Design of general apochromatic drift-quadrupole beamlines"



- *Apochromatic focusing* is how chromatic errors are canceled in light ray optics. Same principle is directly applied to beam optics.

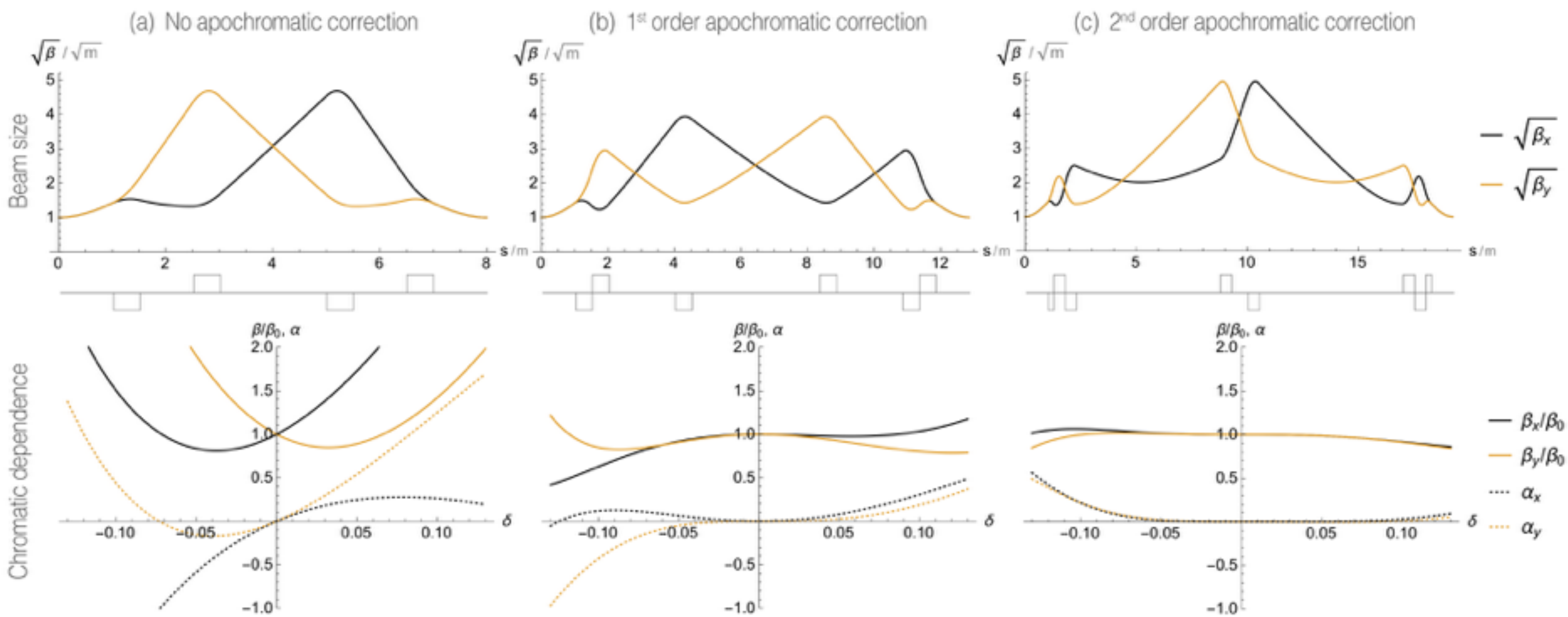
- Mechanism: A range of colors/energies experience **different intermediate focusing**, but **end up focused at the same point**.

- Requires **only quadrupoles**, no sextupoles or dipoles!

Apochromatic focusing: to various orders

Article submitted to PR-AB: Lindstrøm & Adli, "Design of general apochromatic drift-quadrupole beamlines"

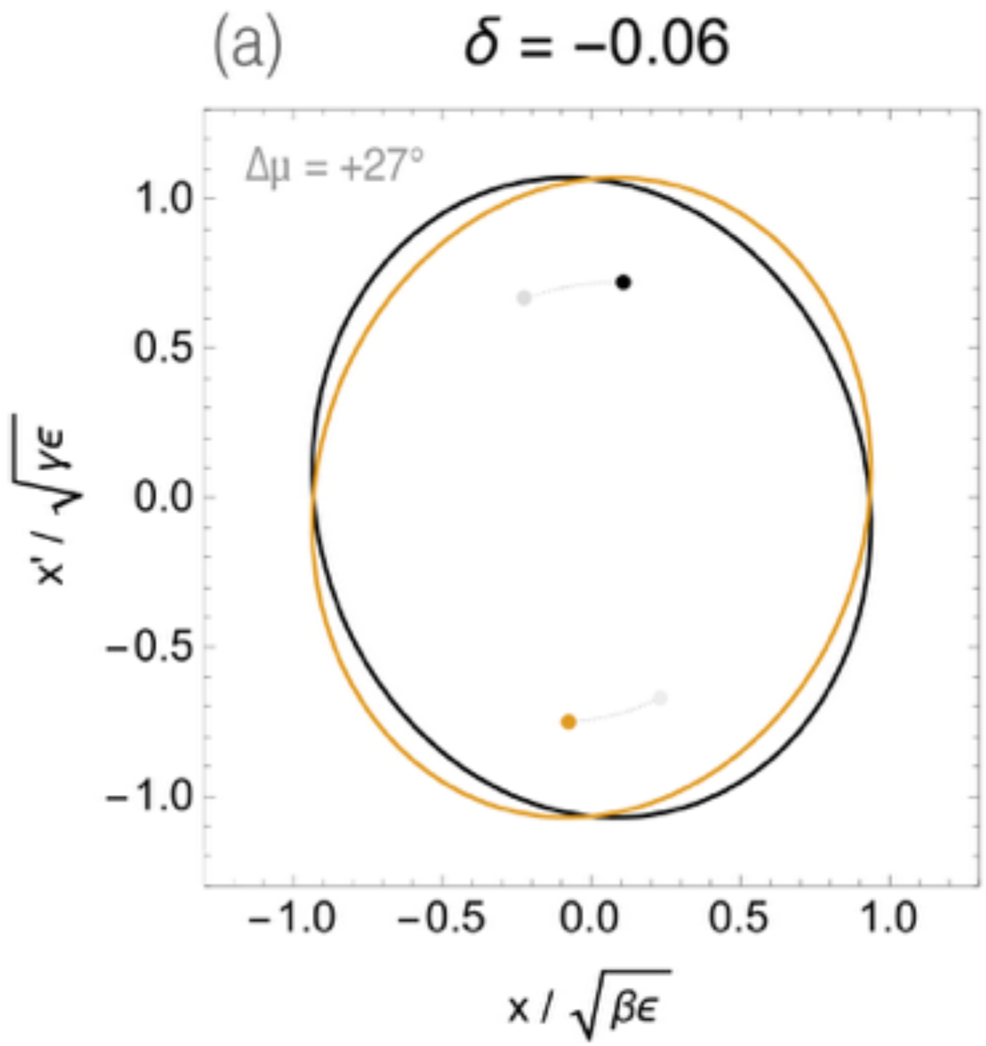
- Apochromatic correction: Flattening the $\alpha(\delta)$ and $\beta(\delta)$ around nominal energy ($\delta = 0$).

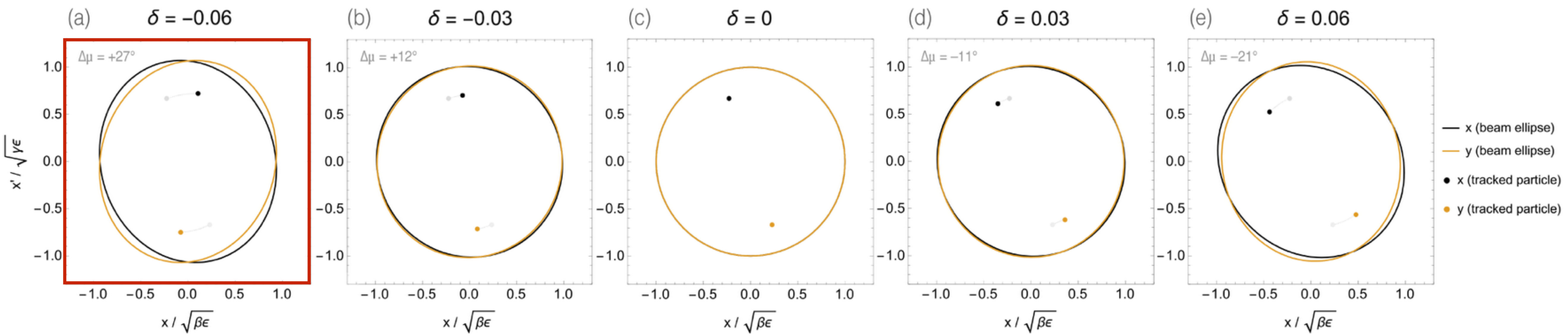


- Higher correction order results in **increasingly energy-independent focusing, at the cost of longer lattices** with more quadrupoles.

Apochromatic focusing: Mechanism

- In a linear accelerator, we **don't care about phase advance**/tunes, as there are no resonances.
- Chromaticity ξ (energy dependent phase advance) is **not canceled**.
- W-function (energy dependent focusing error) is canceled.
- Single particles trajectories: energy dependent (ξ < 0)
  Beam distribution: energy independent (W = 0)





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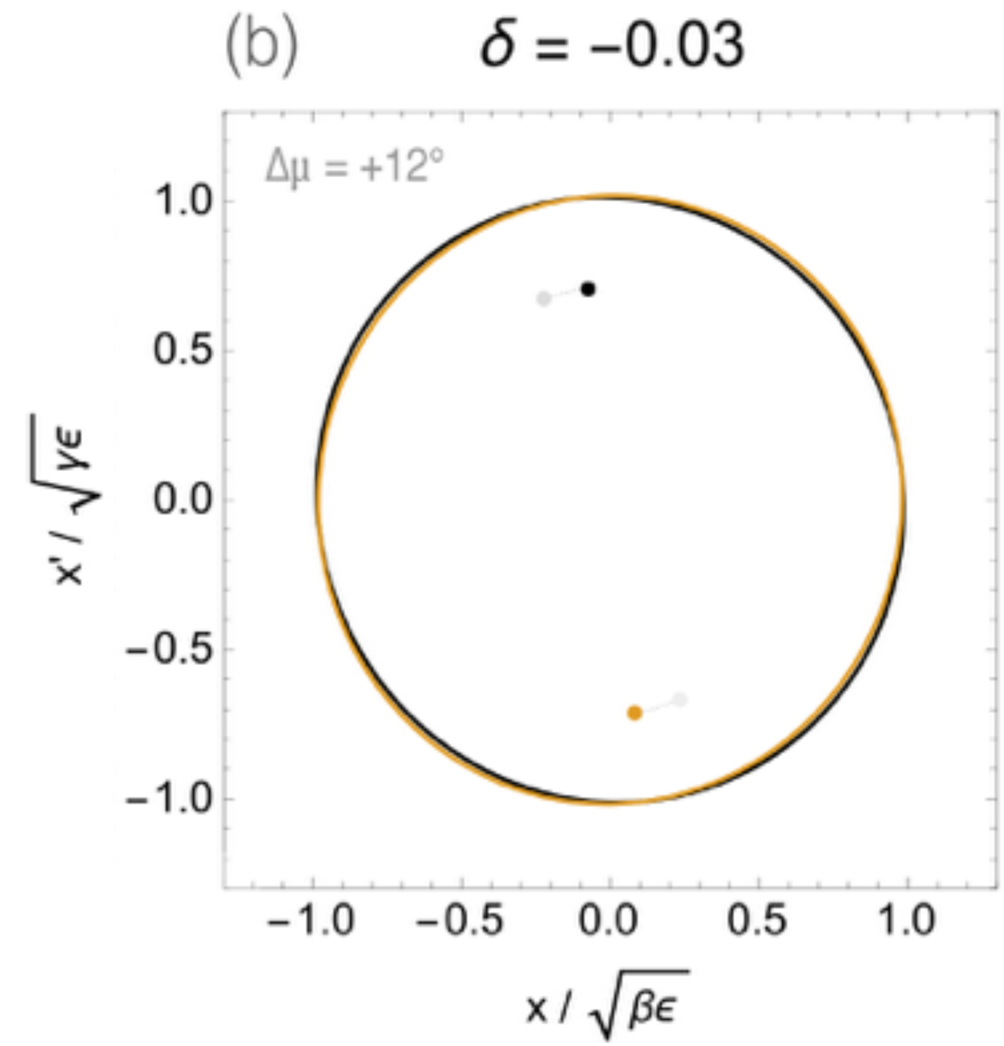

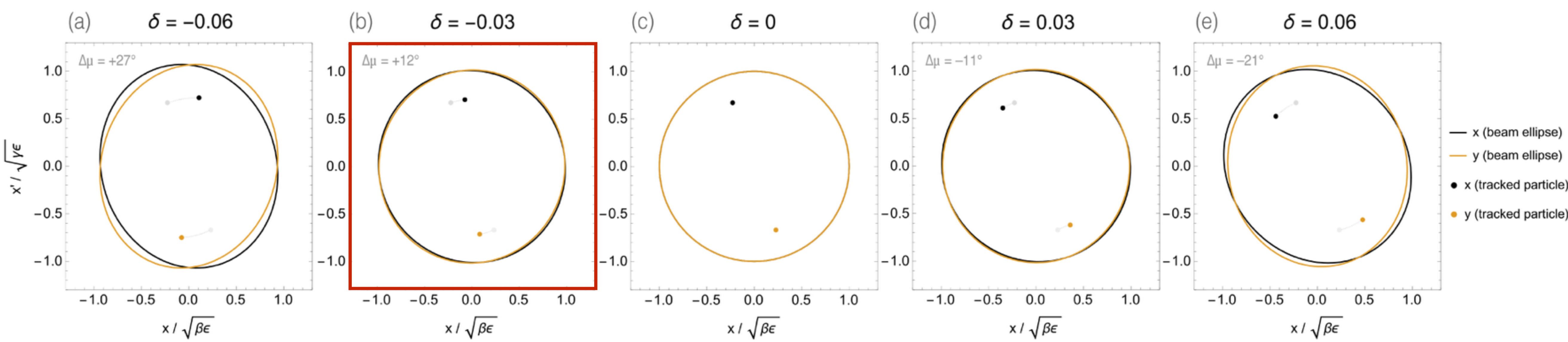

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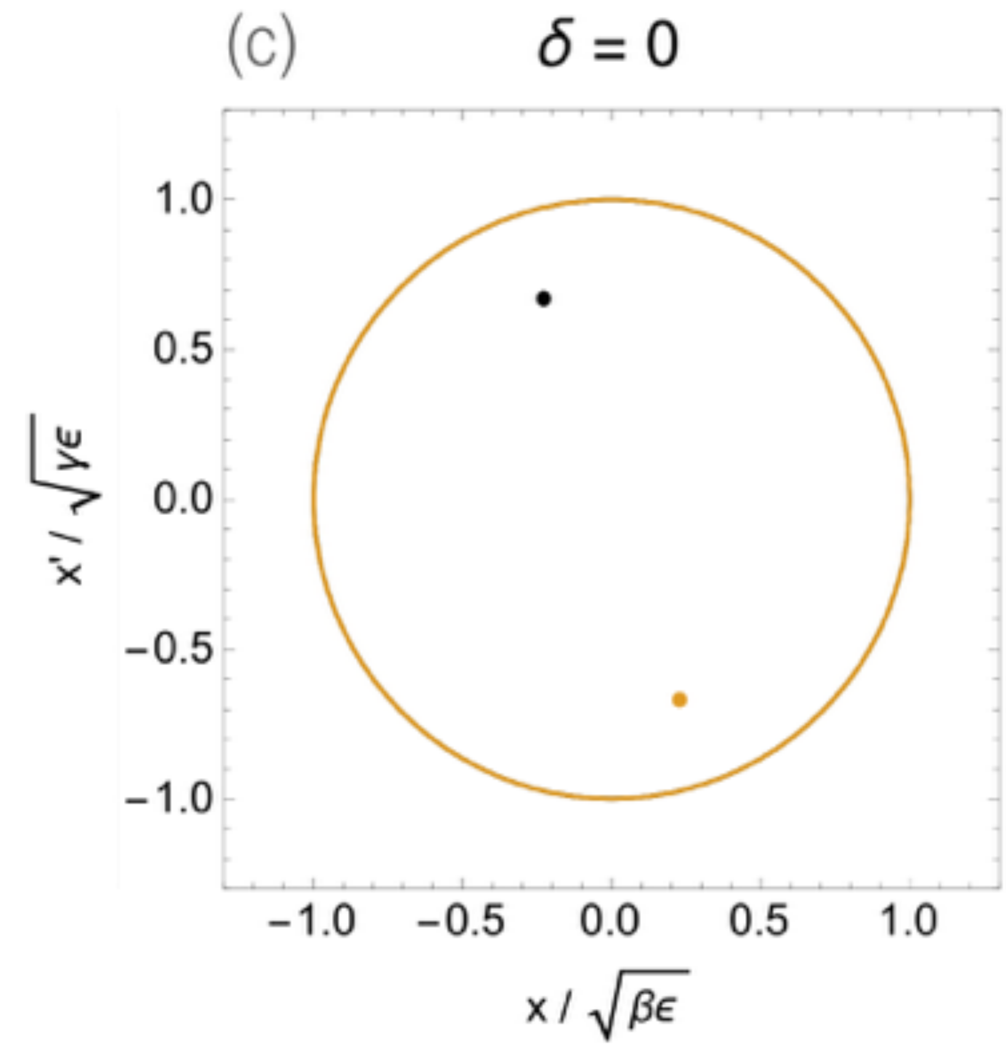

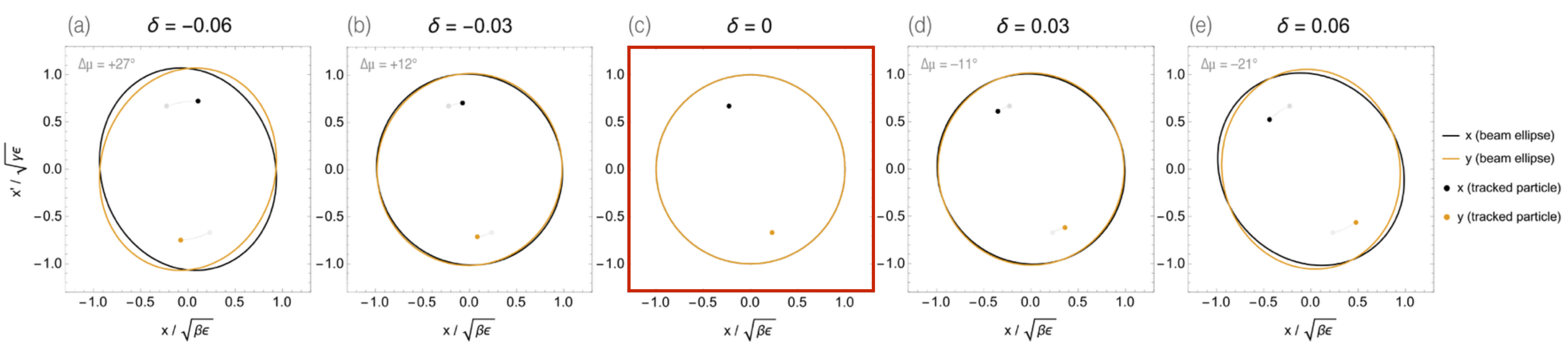

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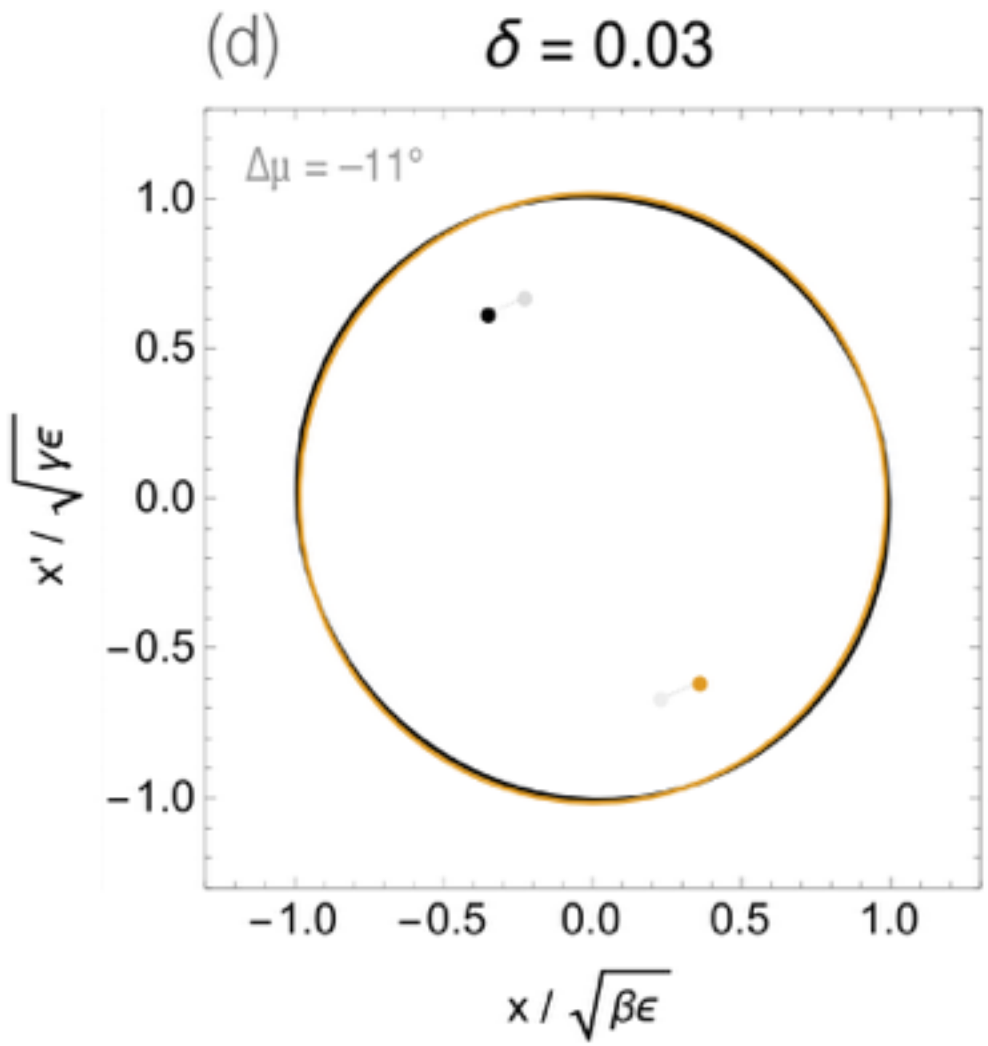

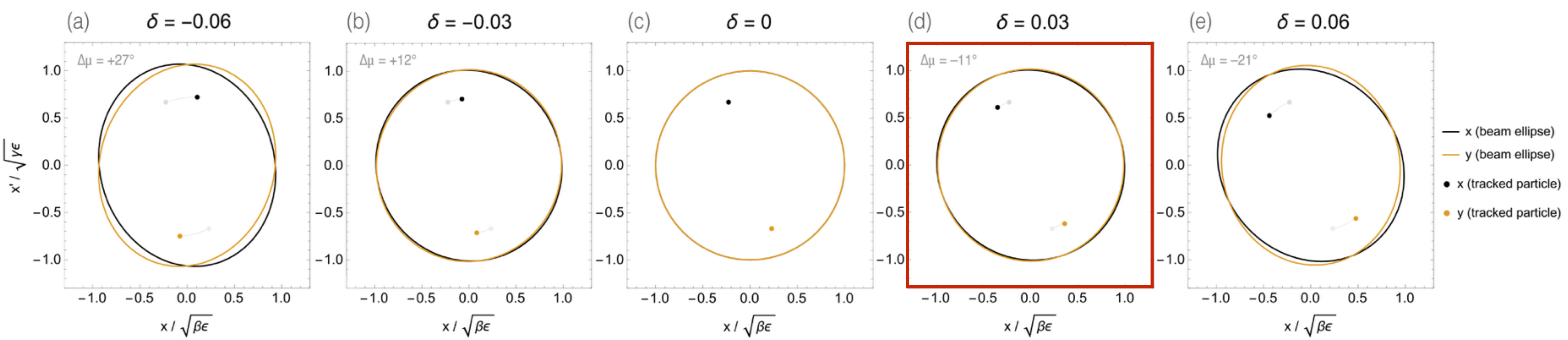

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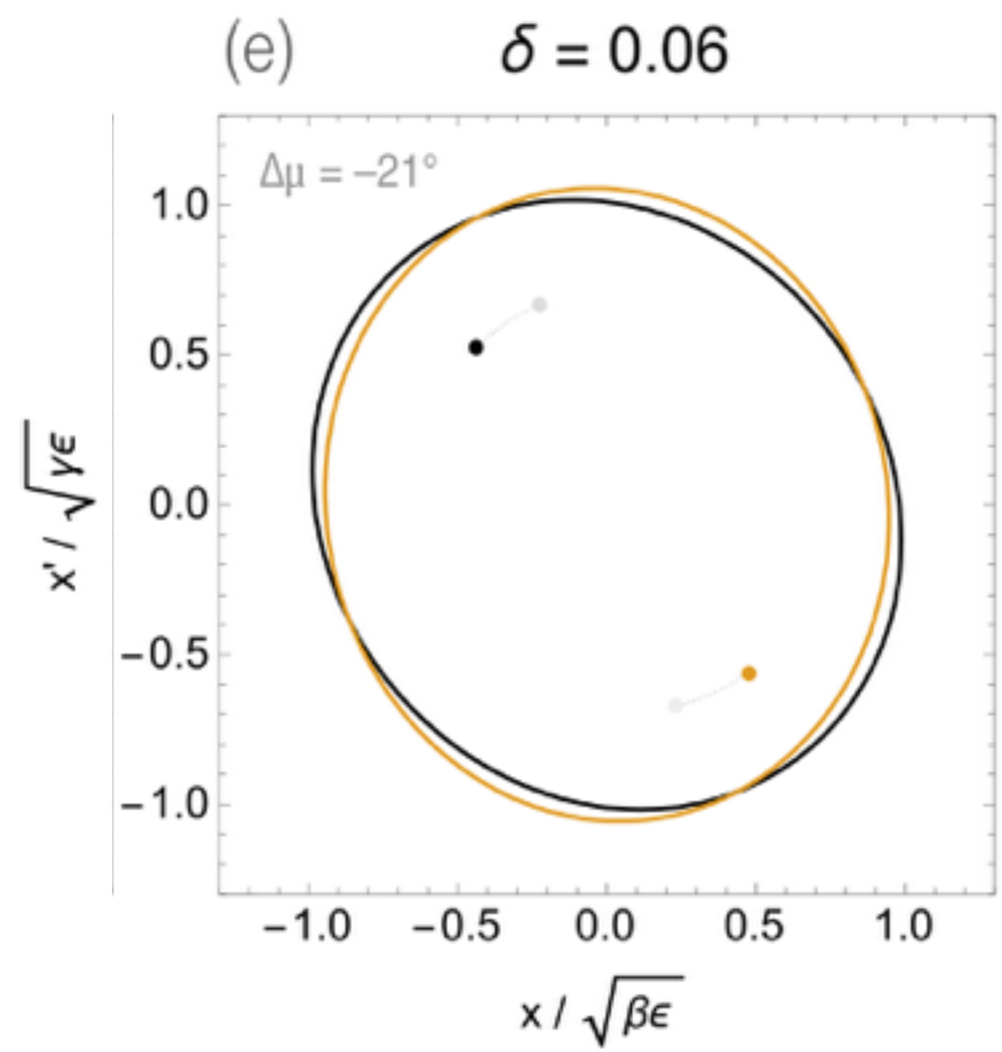
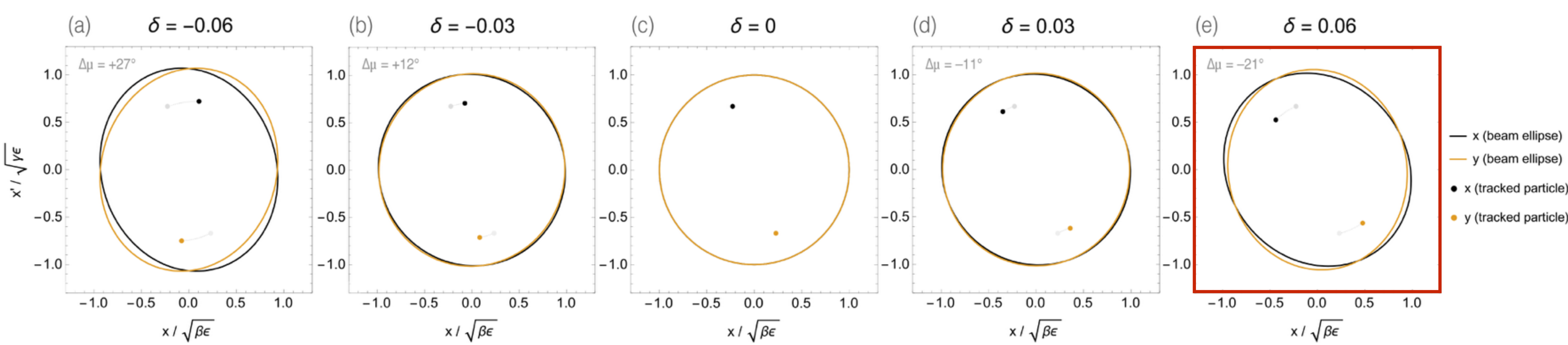
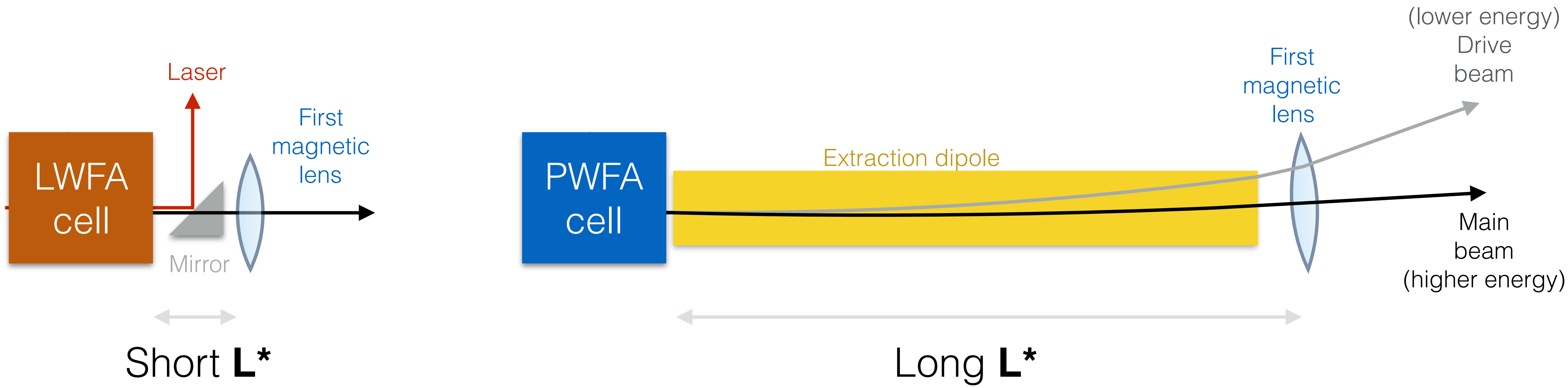




Main differences: PWFA vs. LWFA

- Main LWFA/PWFA difference (wrt. staging optics) is the **injection/extraction length L***:
  - LWFA needs a laser mirror: L^*_{LWFA} = 1-10 cm
  - PWFA needs long separating dipoles: L^*_{PWFA} = 1-10 m

- In addition: **PWFA** injection/extraction dipoles introduce both **transverse dispersion (η)** and **longitudinal dispersion (R_{56})**, which must be canceled (more constraints => more complex solution).



Example 1: LWFA staging (BELLA-like parameters)

- Inspired by S. Steinke *et al.* “Multistage coupling of independent laser-plasma accelerators” (BELLA 2016 Nature paper)

- Single lens, very energy dependent focus. **Prevents matching of most charge into 2nd stage** (charge–coupling efficiency ~3.5%)

- Approx. beam parameters after 1st stage:
  - RMS size ~10 µm, divergence ~ 2-3 mrad
  - Normalized emittance ~ 5 mm mrad
  - Twiss params. : $\alpha^* = 0$, $\beta^* = 5.3$ mm
  - Energy spread ~ 60% FWHM

- Radially symmetric active plasma lens:
  - 3000 T/m
  - 15 mm long, 250 µm radius
  - Approx. 15 mm downstream of exit.

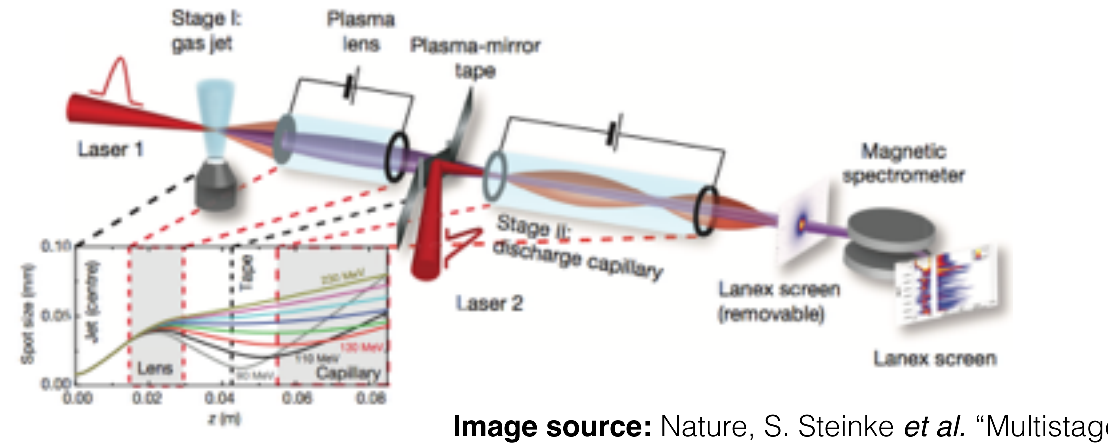
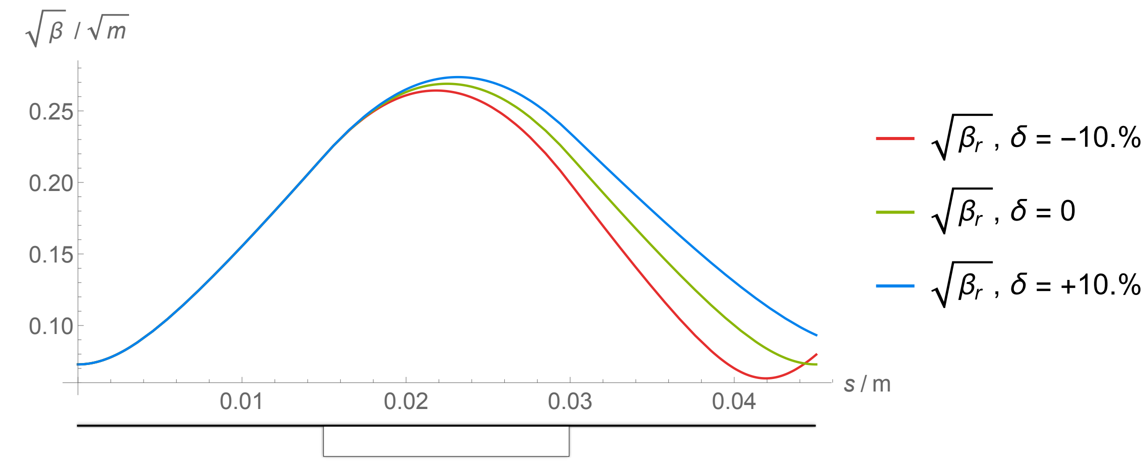
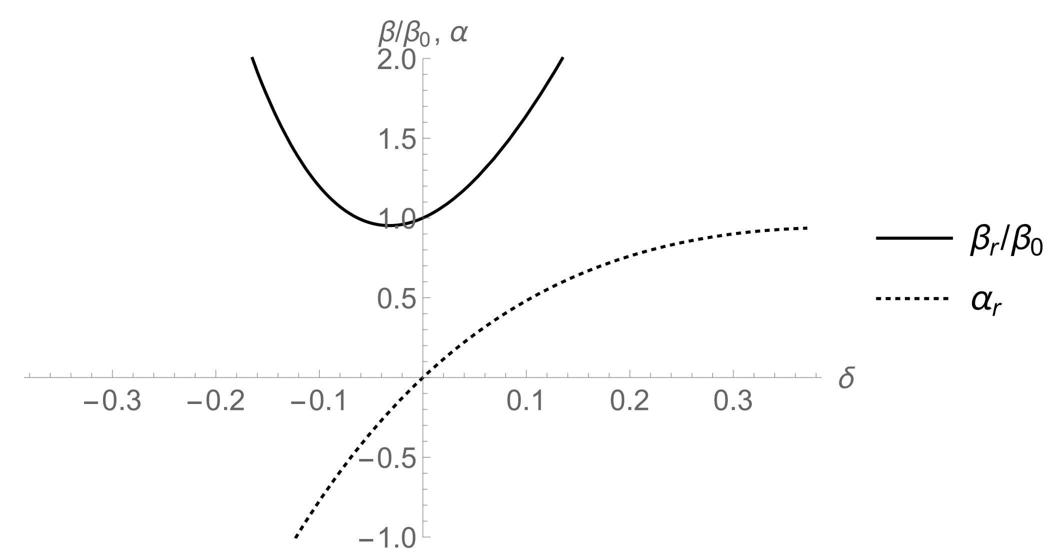


Image source: Nature, S. Steinke *et al.* “Multistage coupling of independent laser-plasma accelerators”

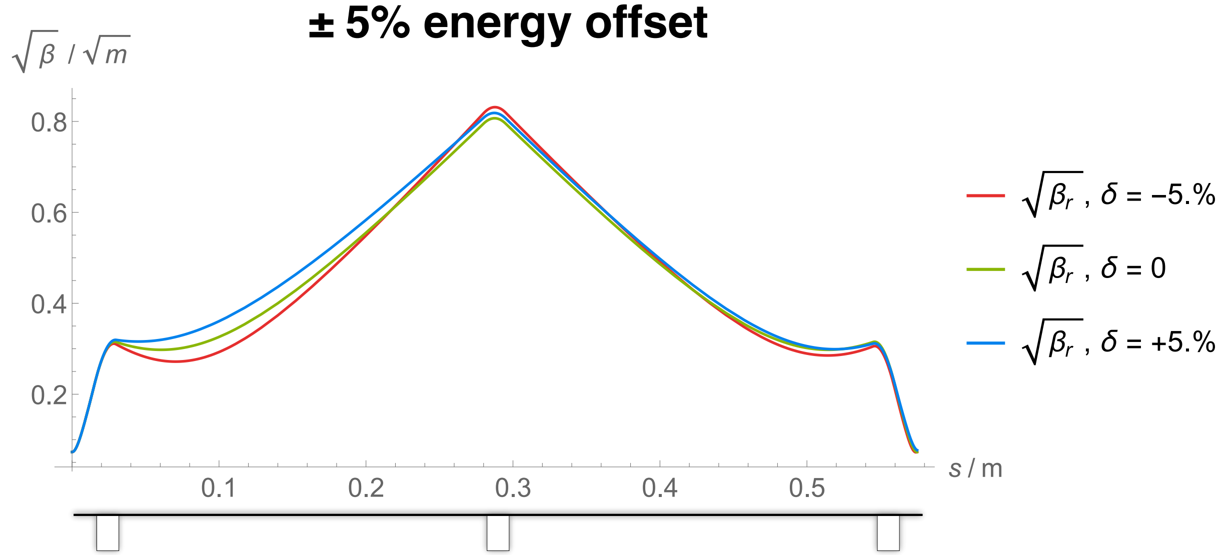




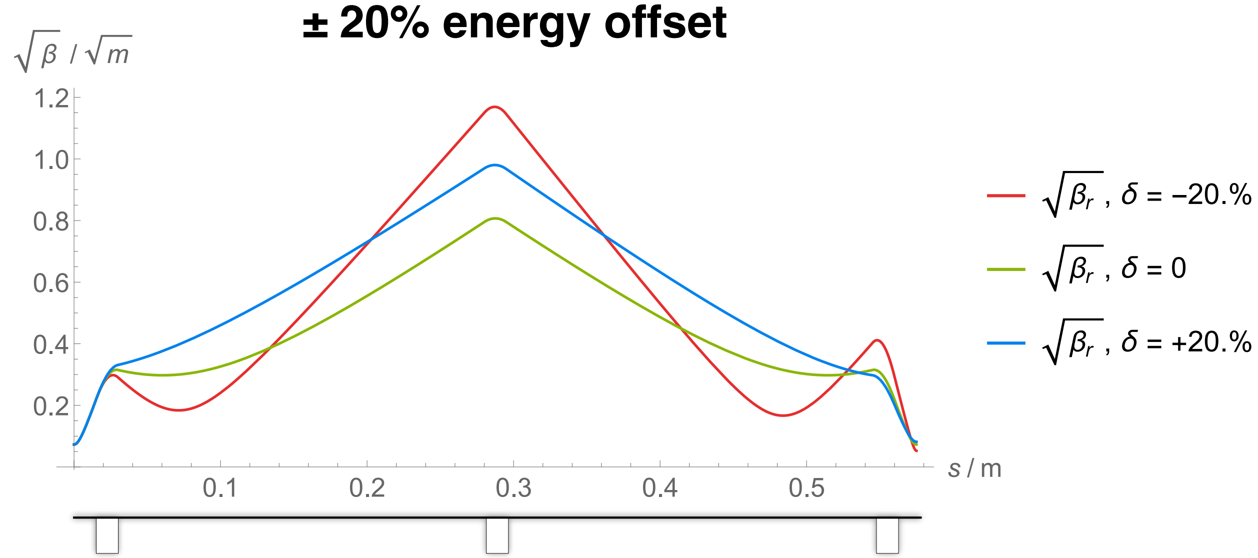
Example 1: LWFA staging (BELLA-like parameters)

- Suggestion #1: **Wide acceptance, not very flat.** *3 lenses, 58 cm long.*

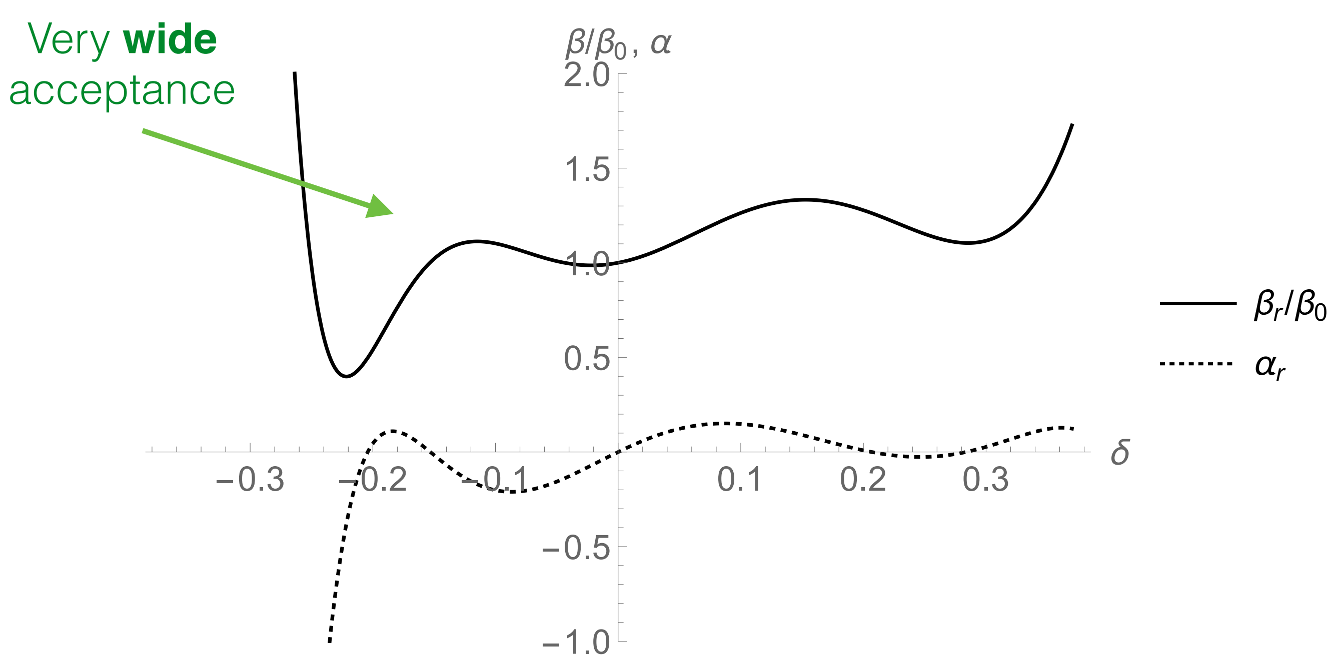
± 5% energy offset



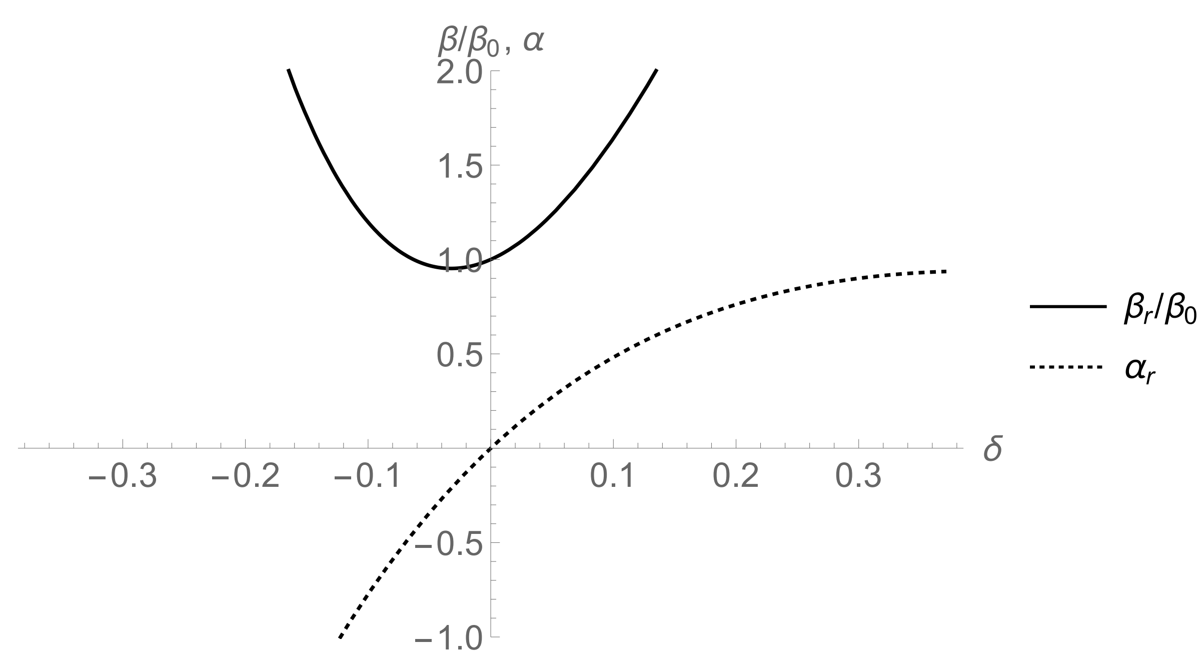
± 20% energy offset



Energy-dependence (suggestion #1)



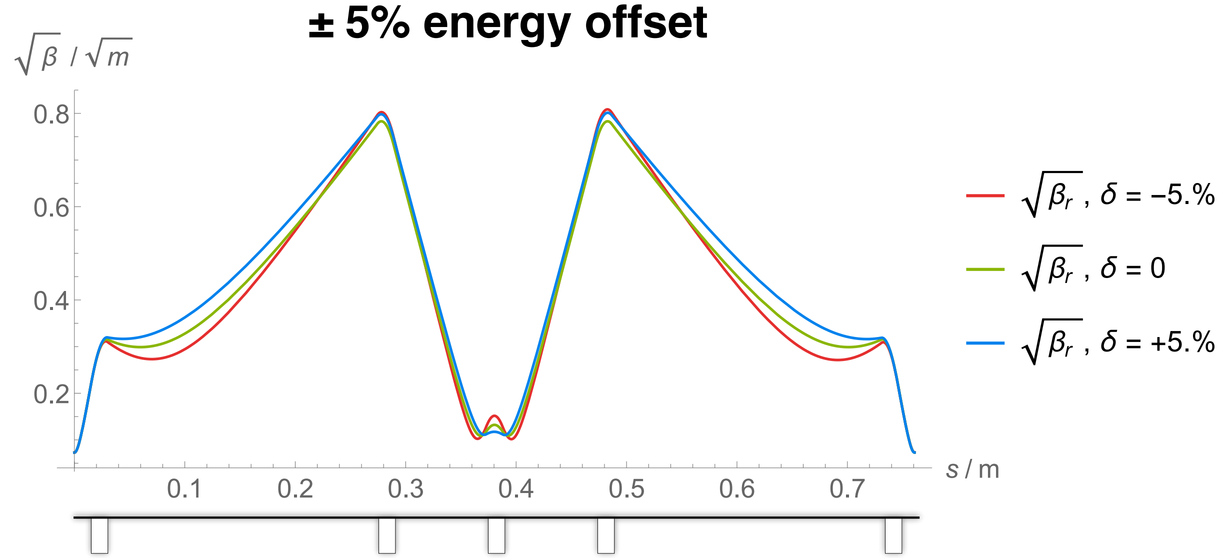
Energy-dependence (no correction)



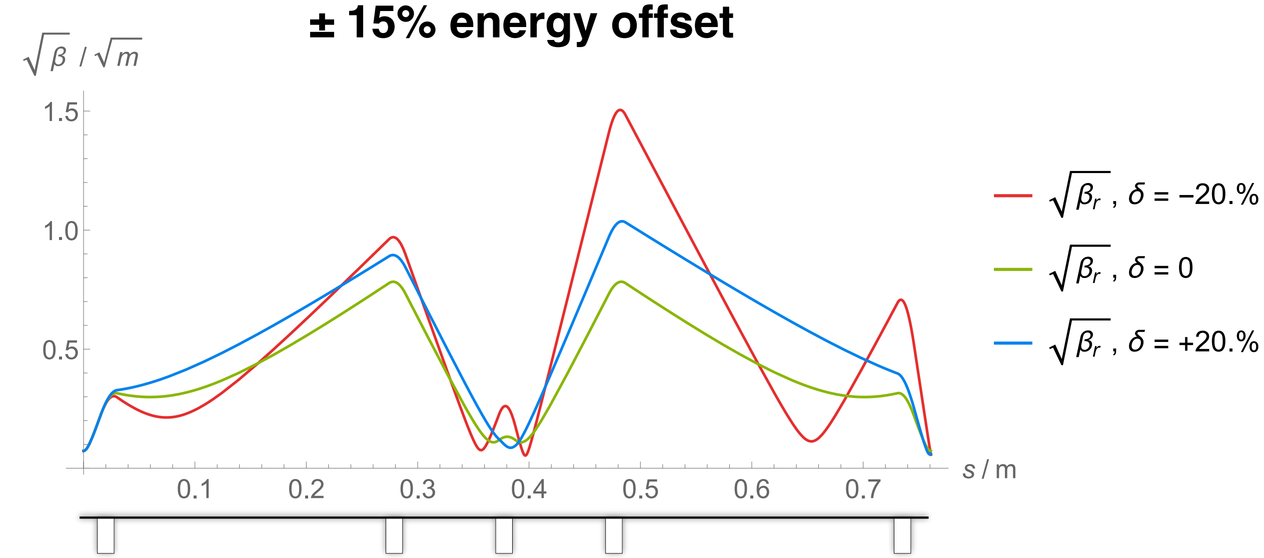
Example 1: LWFA staging (BELLA-like parameters)

- Suggestion #2: **Very flat, not very wide acceptance.** *5 lenses, 76 cm long.*

± 5% energy offset



± 15% energy offset



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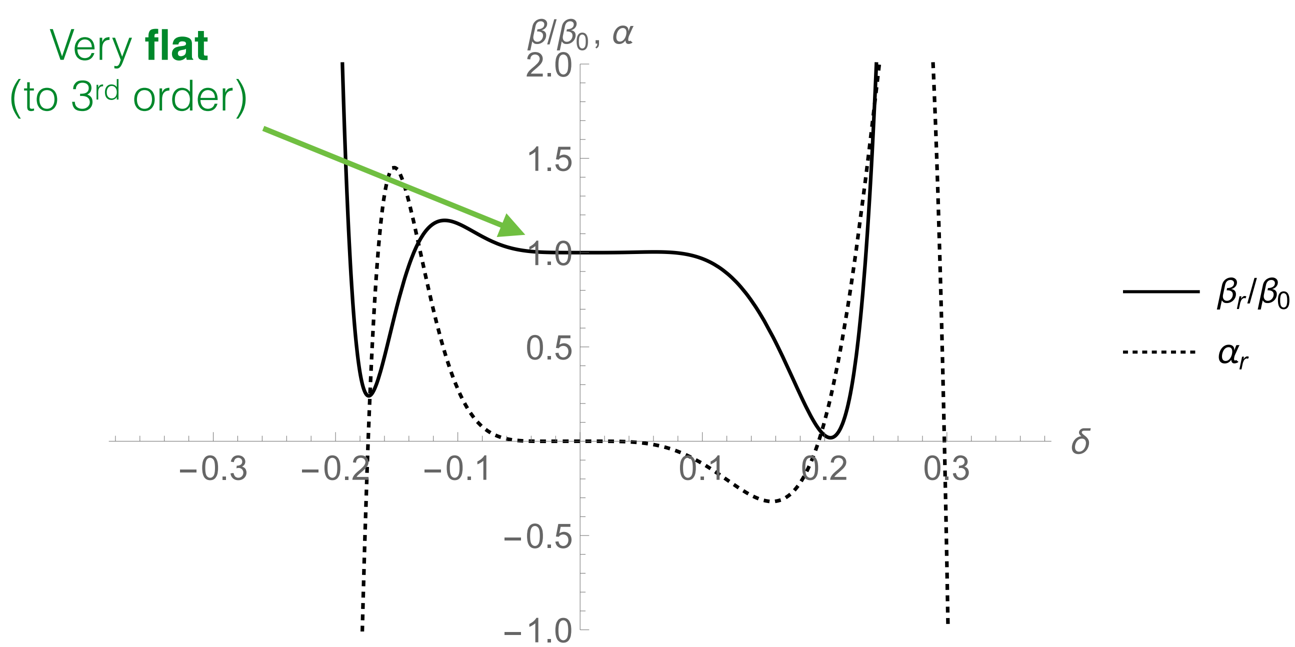
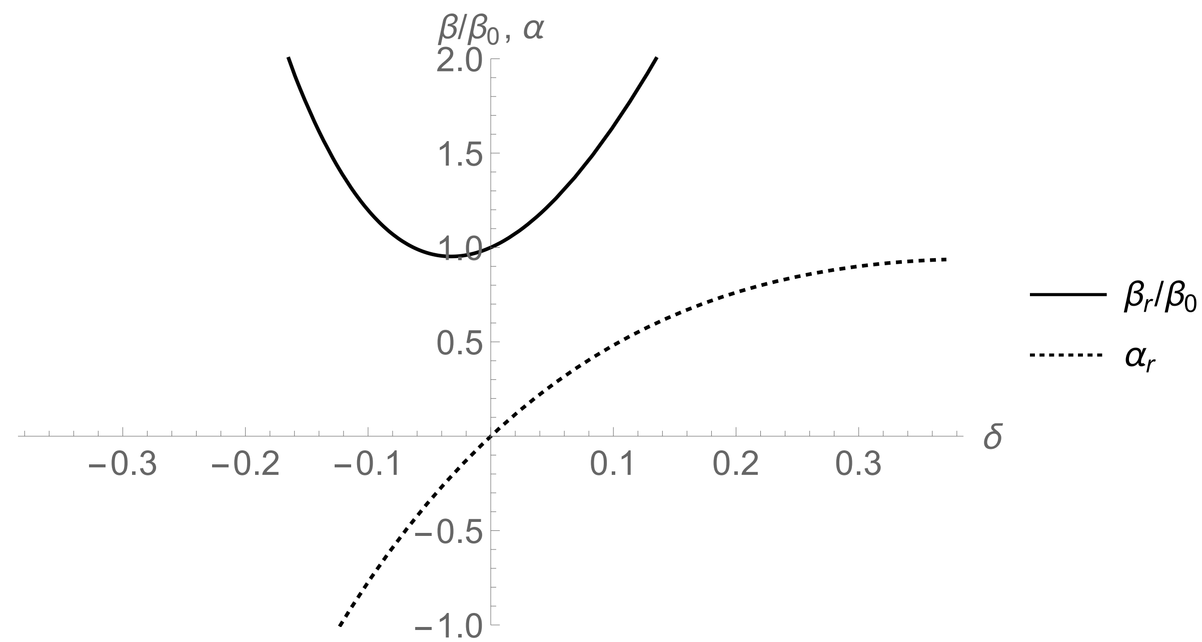


Energy-dependence (no correction)



Example 2: PWFA staging (high energy collider)

- Parameters based on E. Adli *et al.* “A Beam Driven Plasma-Wakefield Linear Collider” (2013)

- Drive beam at 25 GeV:
  ⇒ Reserve **L* ≈ 1 m** for injection/extraction dipoles.

- Main beam (in example):
  - **100 GeV** (between 4th and 5th cell)
  - Effective β* = 30 cm (after a 13x plasma ramp)
  - Assumed **energy spread ~ 1% rms**

- Linear collider requires ultra-low emittances
  ⇒ Acceptable **emittance growth per stage ~ 1%.**

- We will **ignore dispersion and R_{56}-cancellation.** This is a complicated, still unsolved problem.

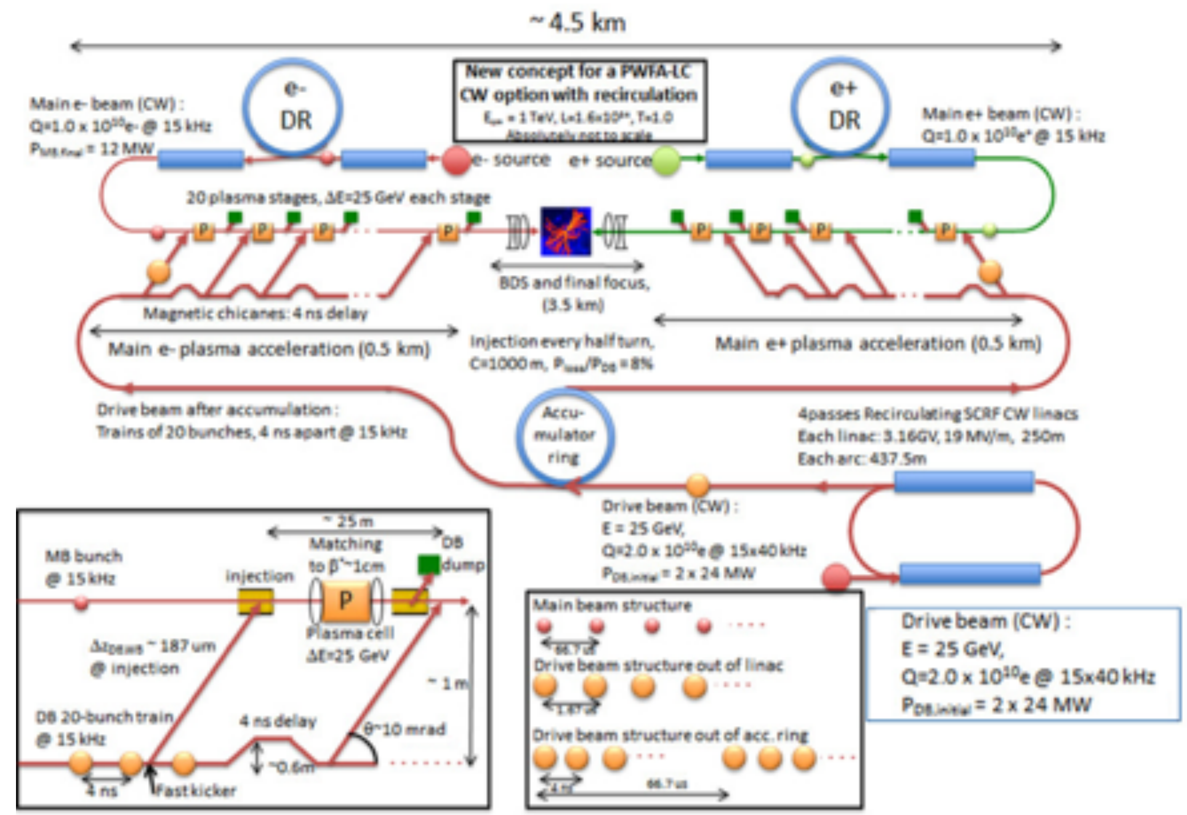


Image source: E. Adli *et al.* “A Beam Driven Plasma-Wakefield Linear Collider”

For more details: → **Article published in NIMA:** Lindstrøm *et al.*, “Staging optics considerations for a PWFA linear collider”

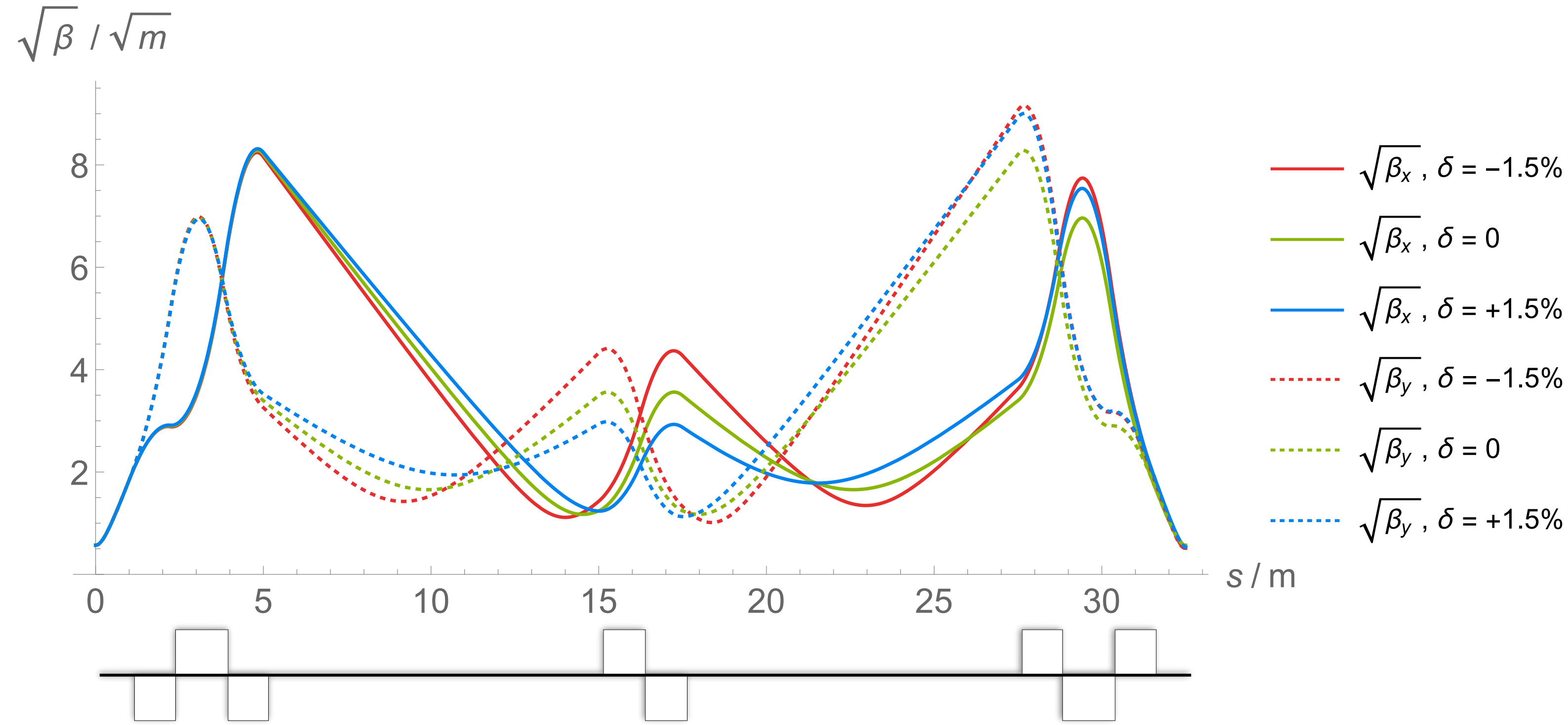
Example 2: PWFA staging (high energy collider)

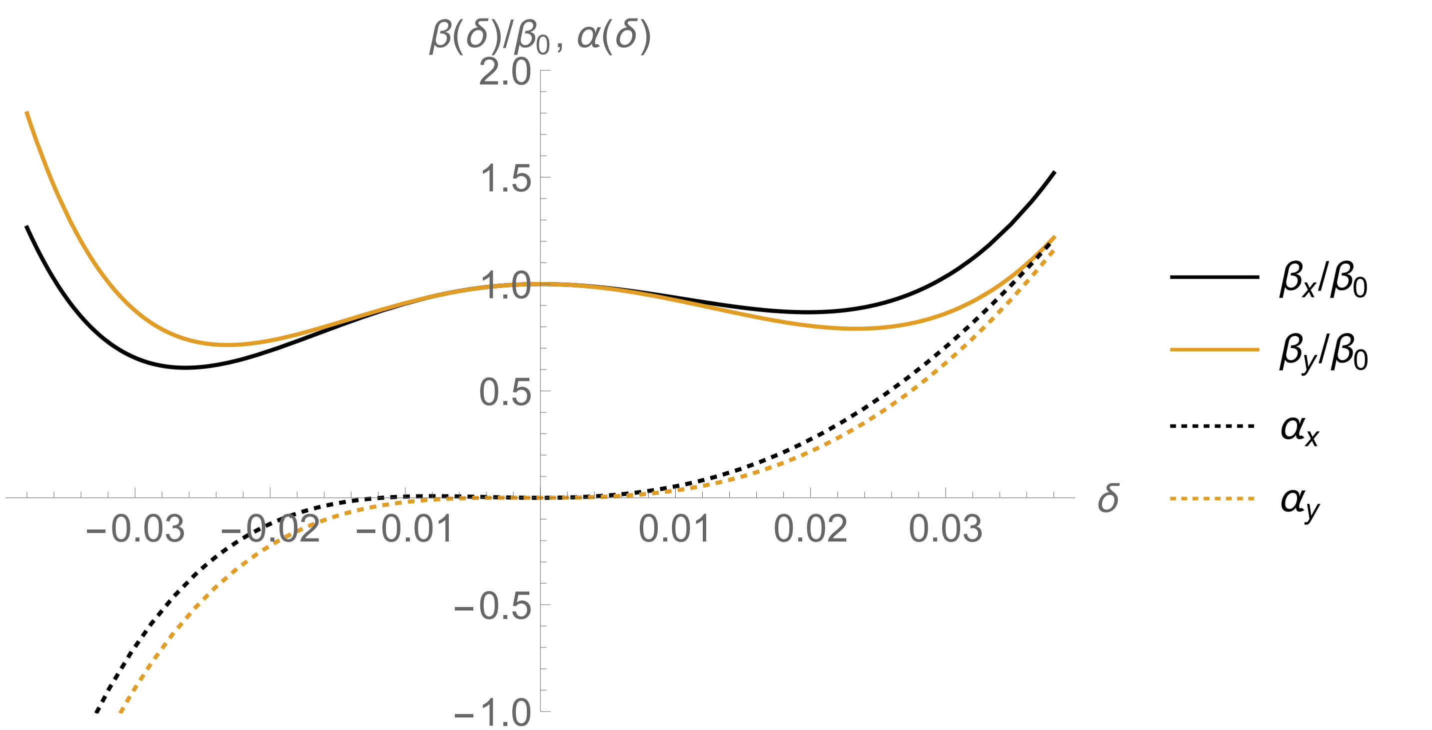
- **Suggestion #1**:
  Strong normal conducting quadrupoles (~160 T/m):

- Length 32 m, 8 quadrupoles.

- **First order apochromat** + normal matching in x/y:
  4 degrees of freedom in a anti-symmetric lattice (8 quads).

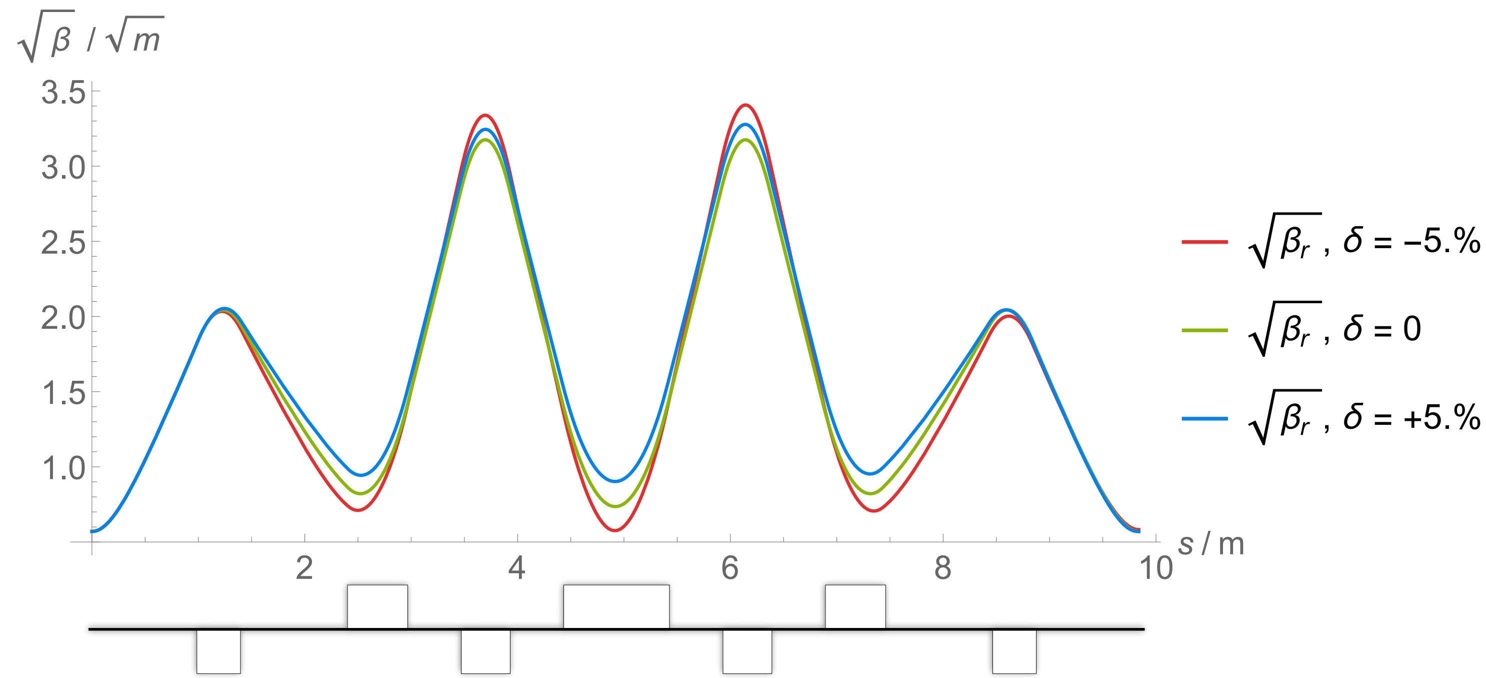
- For a 1% rms energy spread:
  0.96% emittance growth.
  (just within requirements)..

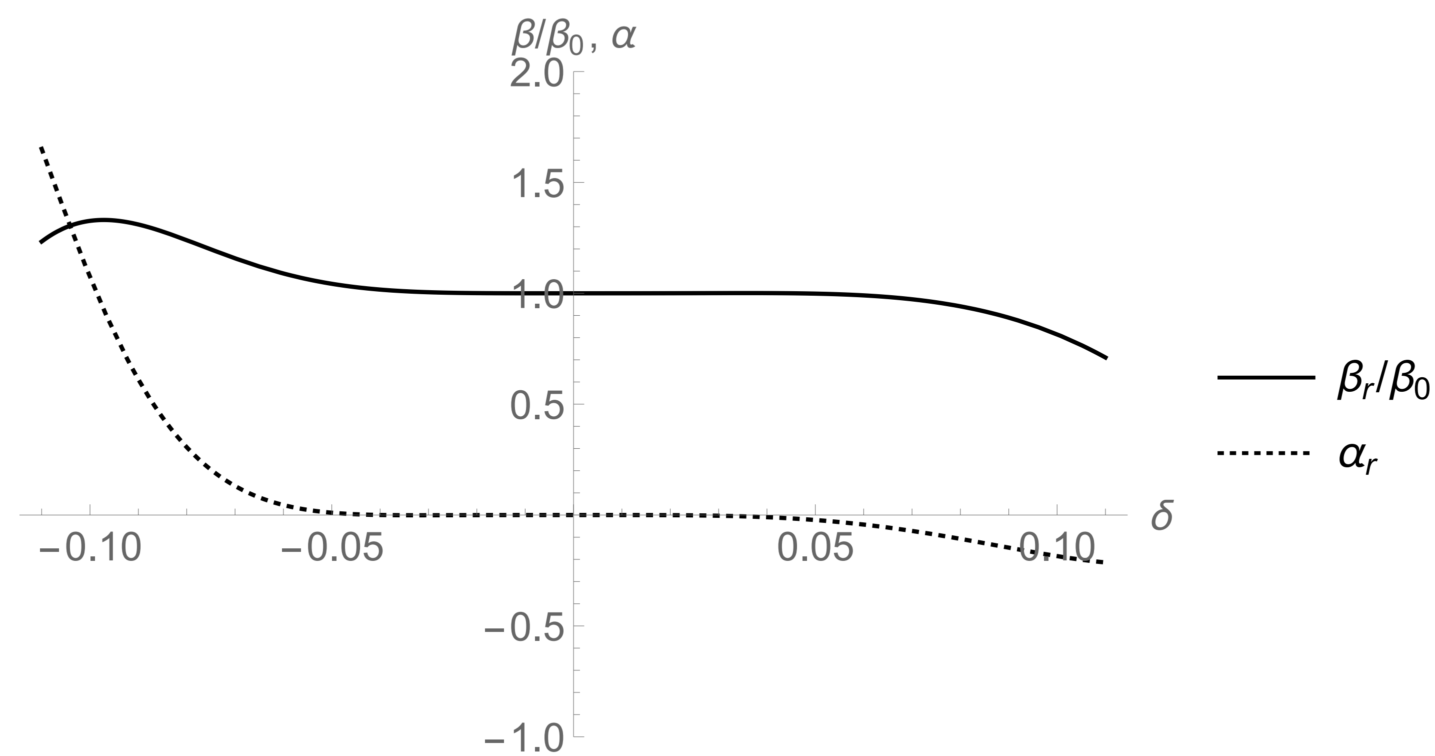




Example 2: PWFA staging (high energy collider)

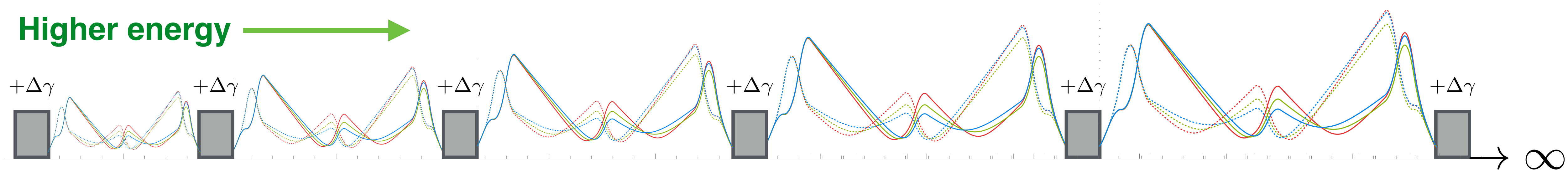
- **Suggestion #2**: Plasma lenses (~3000 T/m) (very long)
- Length 10 m, 7 lenses.
- **Third order apochromat** + normal matching in only r: 4 degrees of freedom in a symmetric lattice (7 lenses).
- For a 1% rms energy spread: **0.000013% emittance growth.** ("achromatic" up to ~3% offset)





Final remarks: Scaling laws with beam energy

- The matched beta-function in a plasma goes as: $\beta^* = \frac{\sqrt{2\gamma}}{k_p} \sim \sqrt{\gamma}$
- Assuming drift-quadrupole beamlines with constant filling factor, constant magnetic field gradient: $l_{quad} \sim \sqrt{\gamma}$ → $\beta(s) \sim \sqrt{\gamma}$
- Staging optics solution can be reused, if scaled by length: $L_{stage} \sim \sqrt{\gamma}$



- ⇒ **One staging optics solution for the entire beamline!**

  ⇒ Emittance growth per stage is constant.

  ⇒ Length of entire beamline scales as: $L_{total} \sim \gamma^{\frac{3}{2}}$

- Always true for LWFA. Increasingly true for PWFA with higher energy, as dipoles bend less.

Summary

- LWFA/PFWA staging requires staging optics with chromatic correction.
- Sextupoles inadequate: **Apochromatic correction** is a good alternative.
- Can be applied both to LWFA (good cancellation) and PWFA (more difficult).

THANK YOU FOR YOUR ATTENTION