



Modeling gravitational waves from compact-object binaries

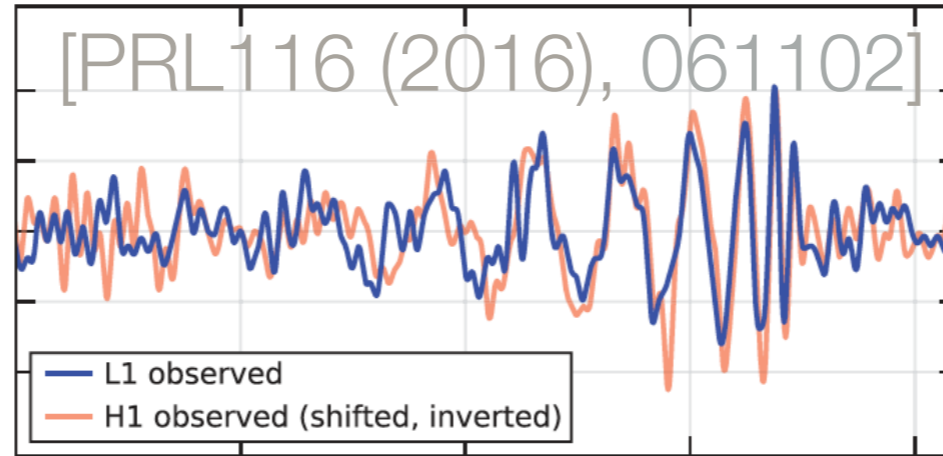
Andrea Taracchini

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Potsdam, Germany)

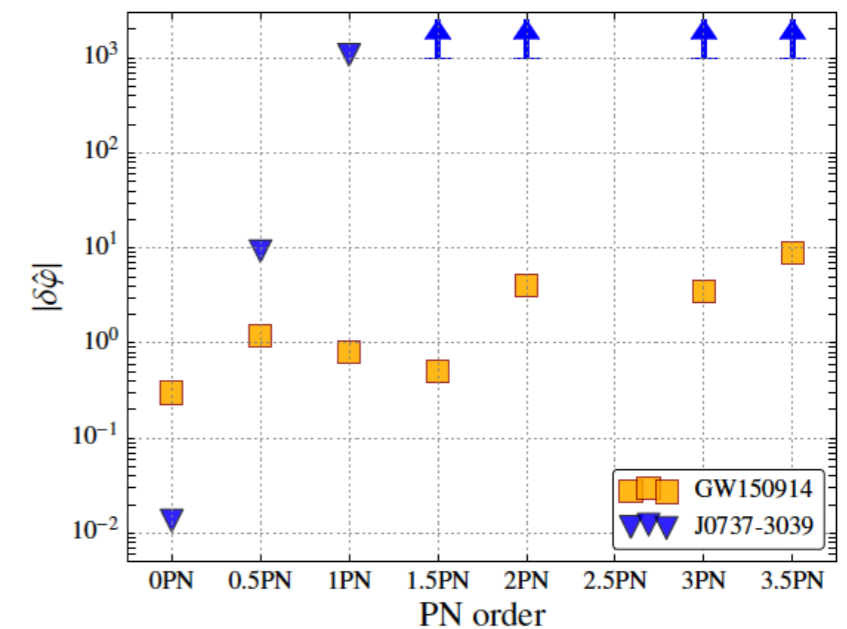
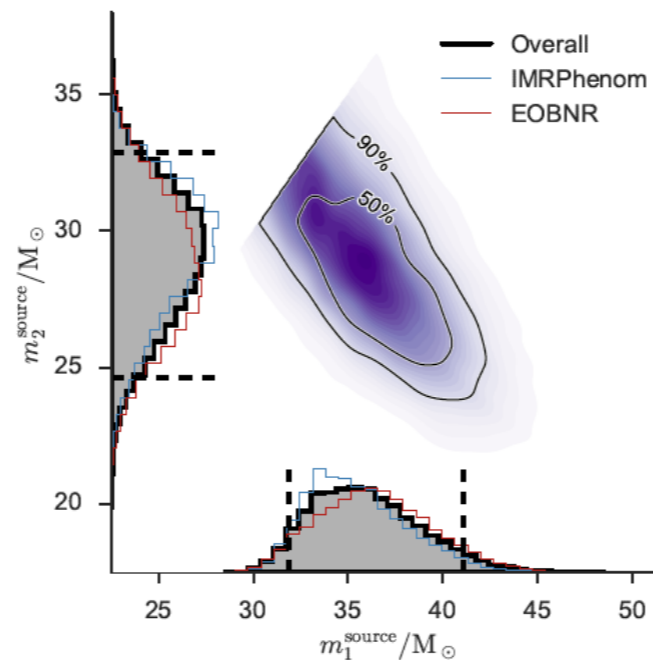
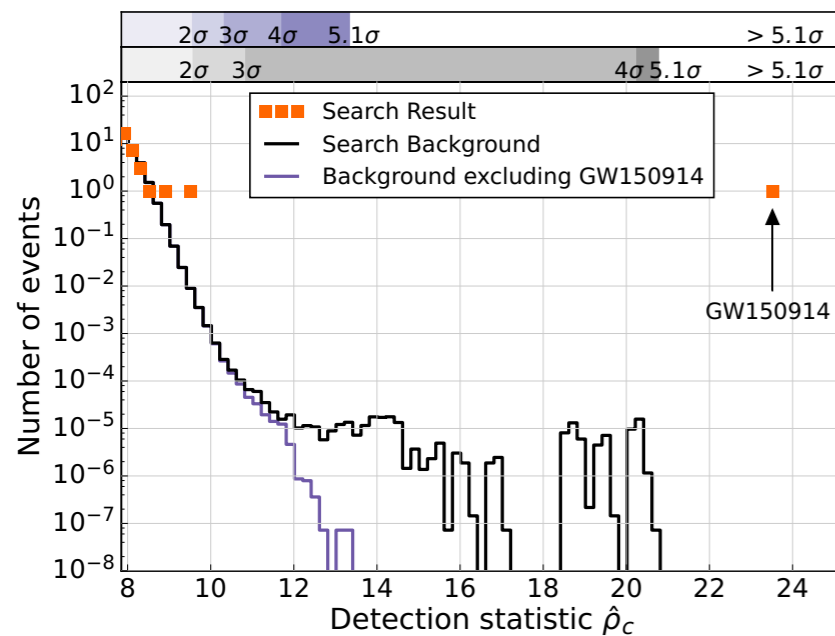
[<https://dcc.ligo.org/G1602133>]

Introduction

Role of numerical/analytical relativity in GW150914

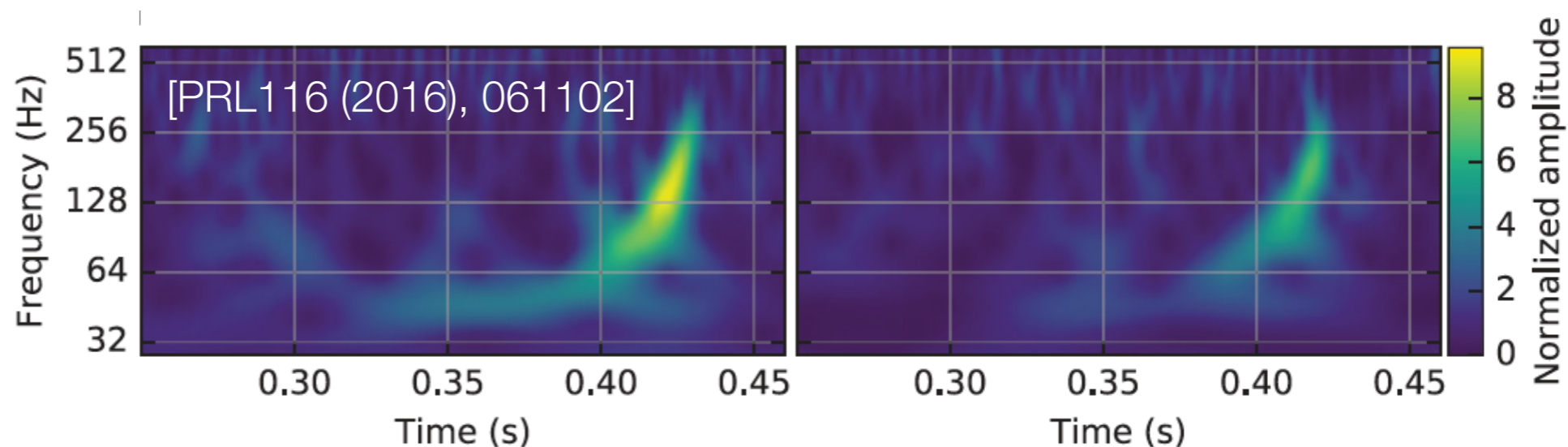


- Synergy of numerical relativity and analytical relativity = waveform models** crucial for
 - establishing 5-sigma **significance** of detection [LVC1602.03839]
 - measuring **properties of the source** [LVC1602.03840, LVC1606.01210, LVC1606.01262]
 - performing **tests of general relativity** (GR) [PRL116 (2016) 221101]

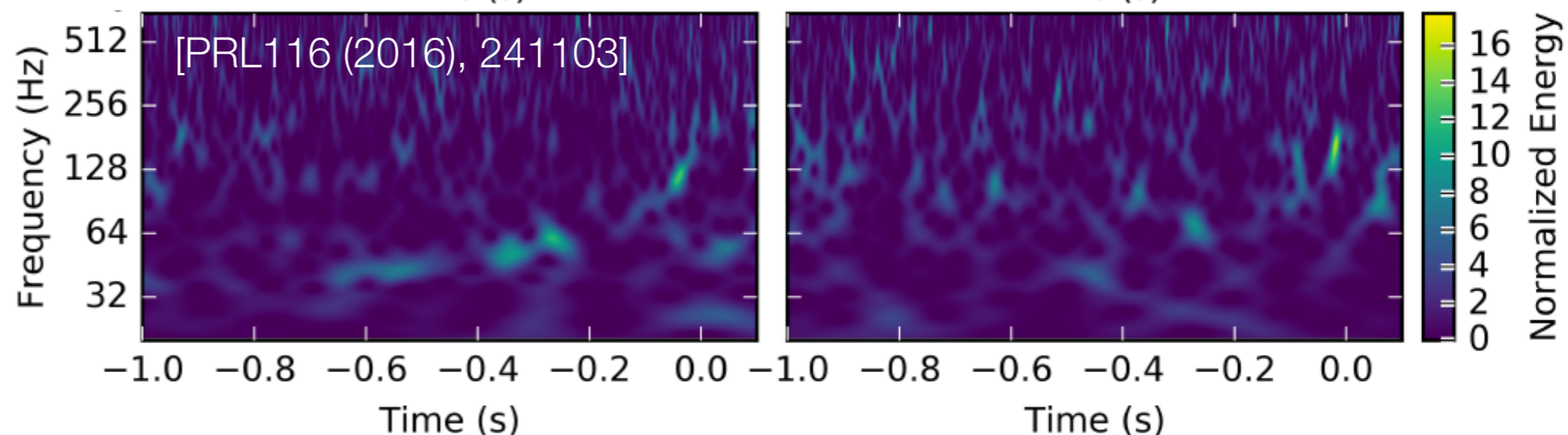


Role of numerical/analytical relativity in GW151226

GW150914



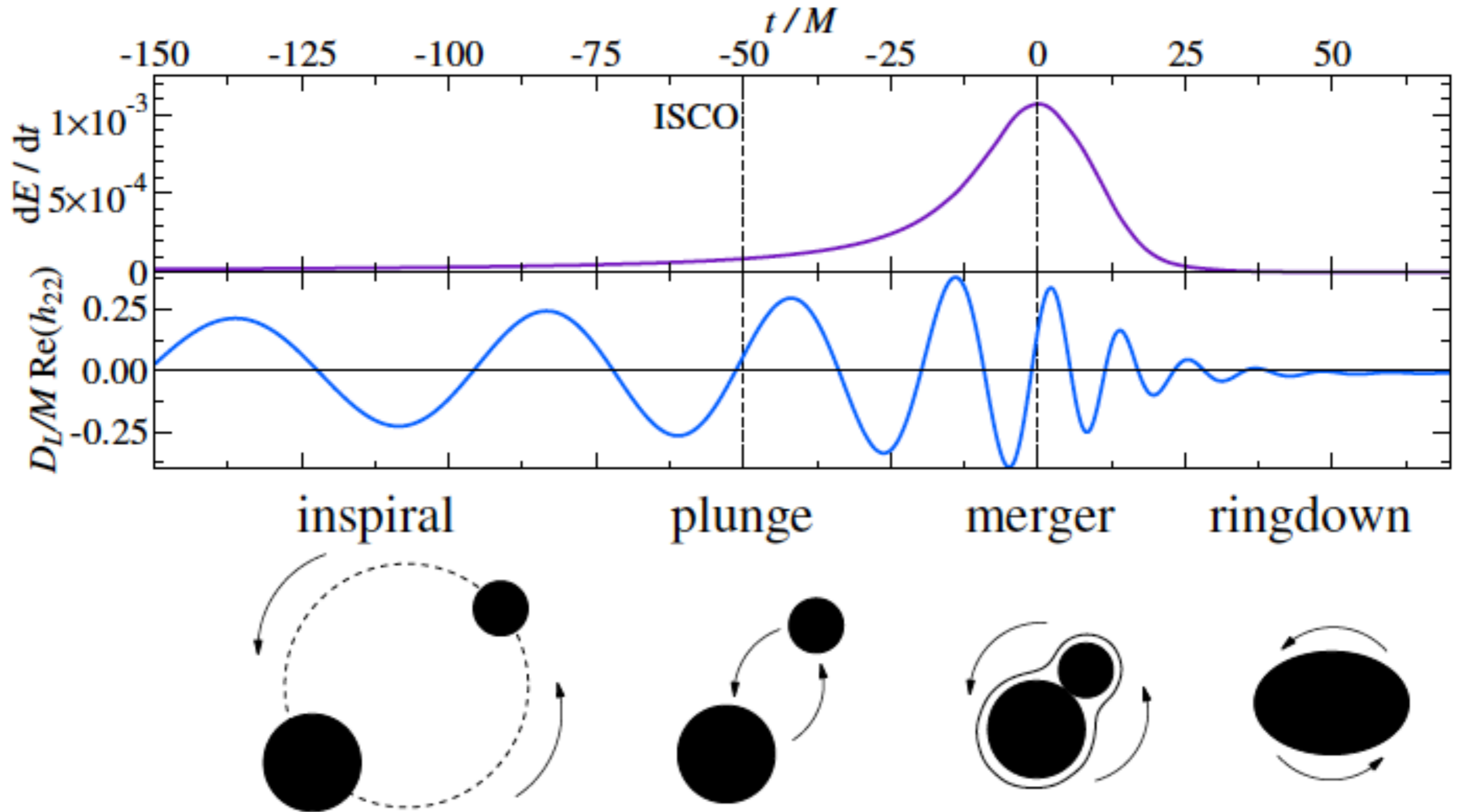
GW151226



- **Template-based** online pipeline was needed to observe it
- NR+AR waveforms as important for significance, parameter estimation, and tests of GR

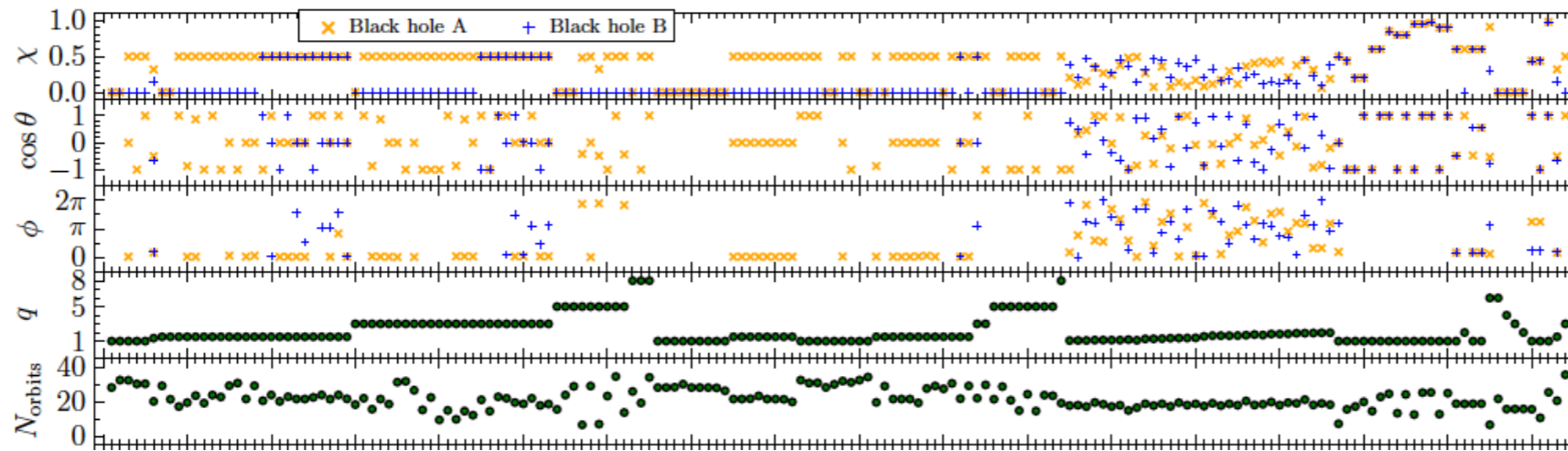
Quasicircular binary black holes:
numerical relativity

BBH coalescence as predicted by GR

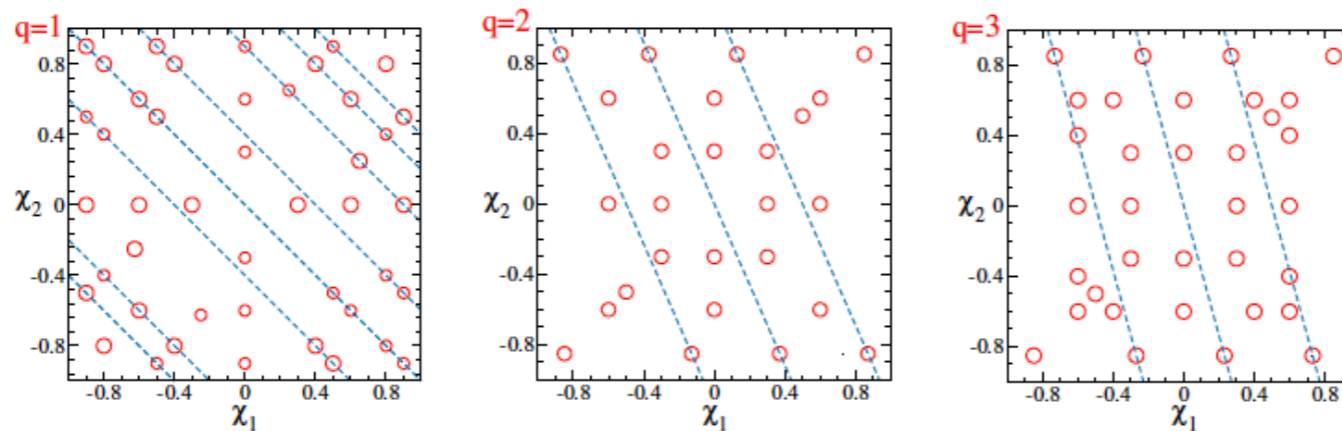


Intrinsic parameters: BH masses and BH spin vectors

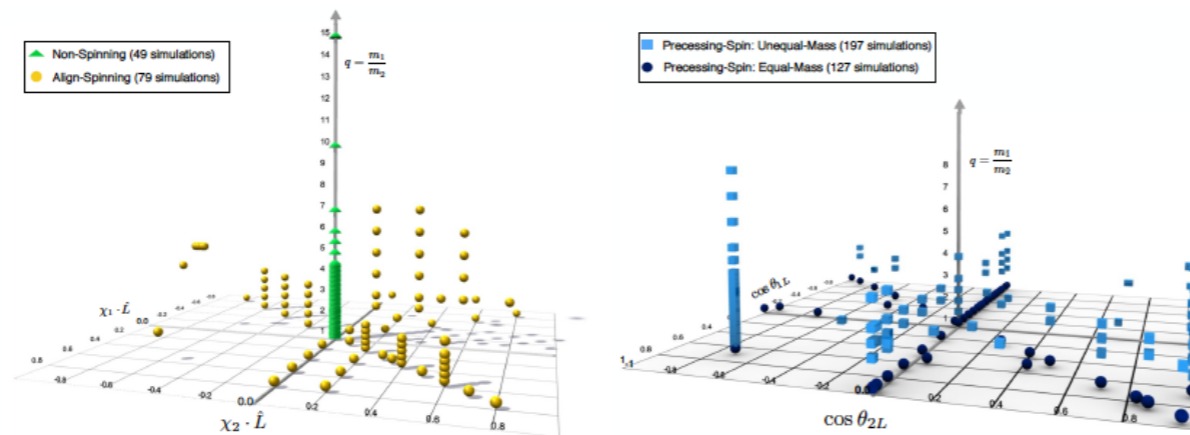
Numerical relativity catalogs of BBHs



[Mroue+13]



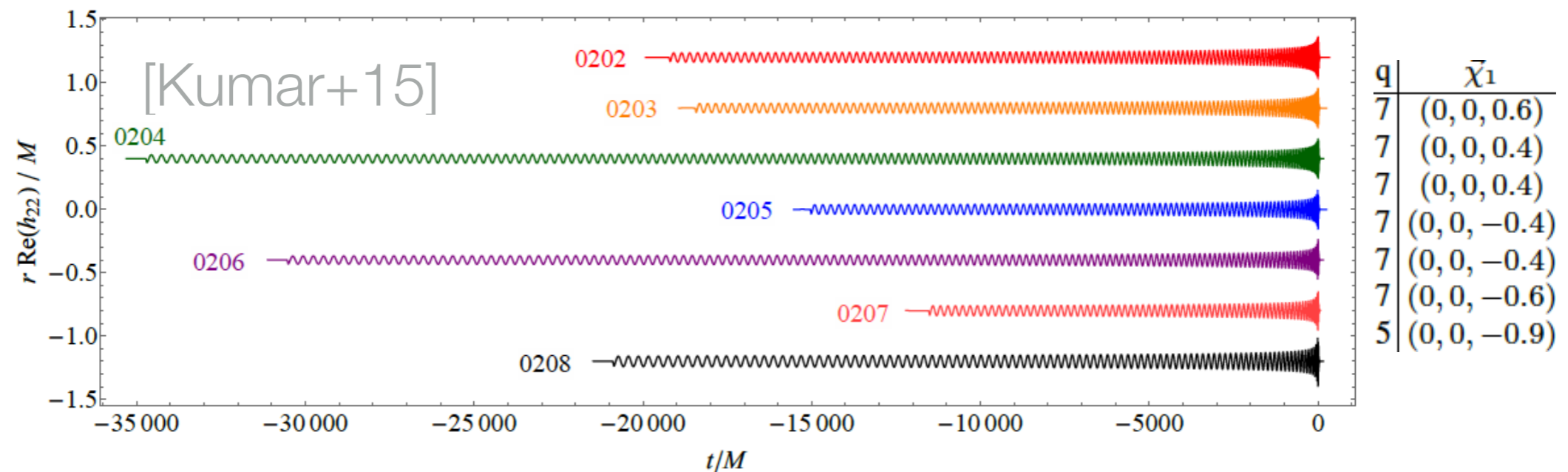
[Chu+15]



[Jani+16]

... and many more NR waveforms [SXS, GATech, RIT, Cardiff-UIB, Cactus]
(also generated for followup of LVC observations)

Challenging configurations



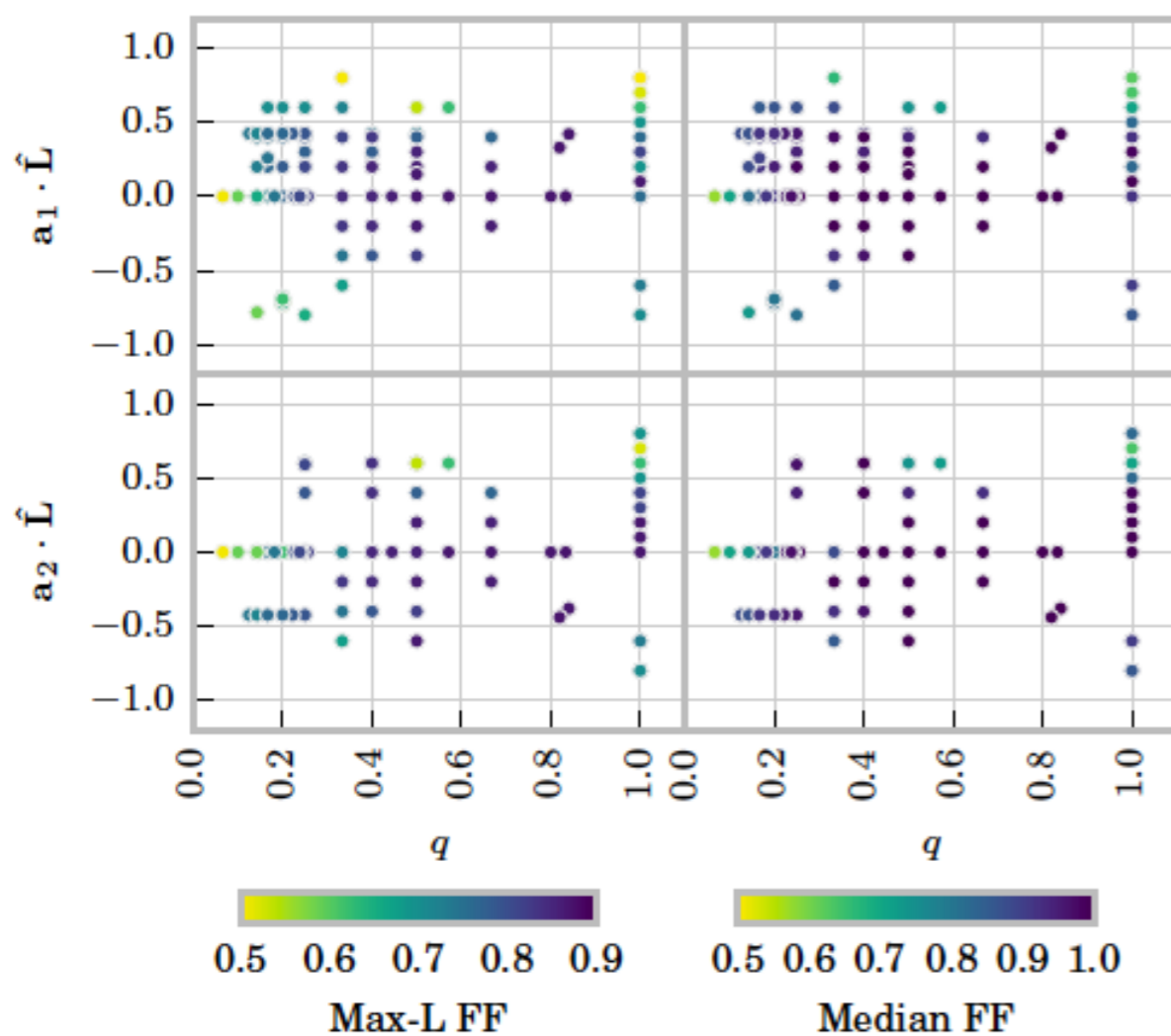
- **Longterm** BBH simulations at mass ratio 7 [Szilagyi+14, Kumar+15]
- **Almost extremal** BBH simulations: equal-mass, aligned-spins 0.99, 0.994 [Scheel+14]
- New initial data for challenging configurations [Ossokine+15]

q	χ_1	χ_2	D_0/M
1	(0, 0, 0.9999)	(0, 0, 0.9999)	14.17
3	(0, 0.49, -0.755)	(0, 0, 0)	15.48
10	(0.815, -0.203, 0.525)	(-0.087, 0.619, 0.647)	15.09
50	(-0.045, 0.646, -0.695)	(0, 0, 0)	16

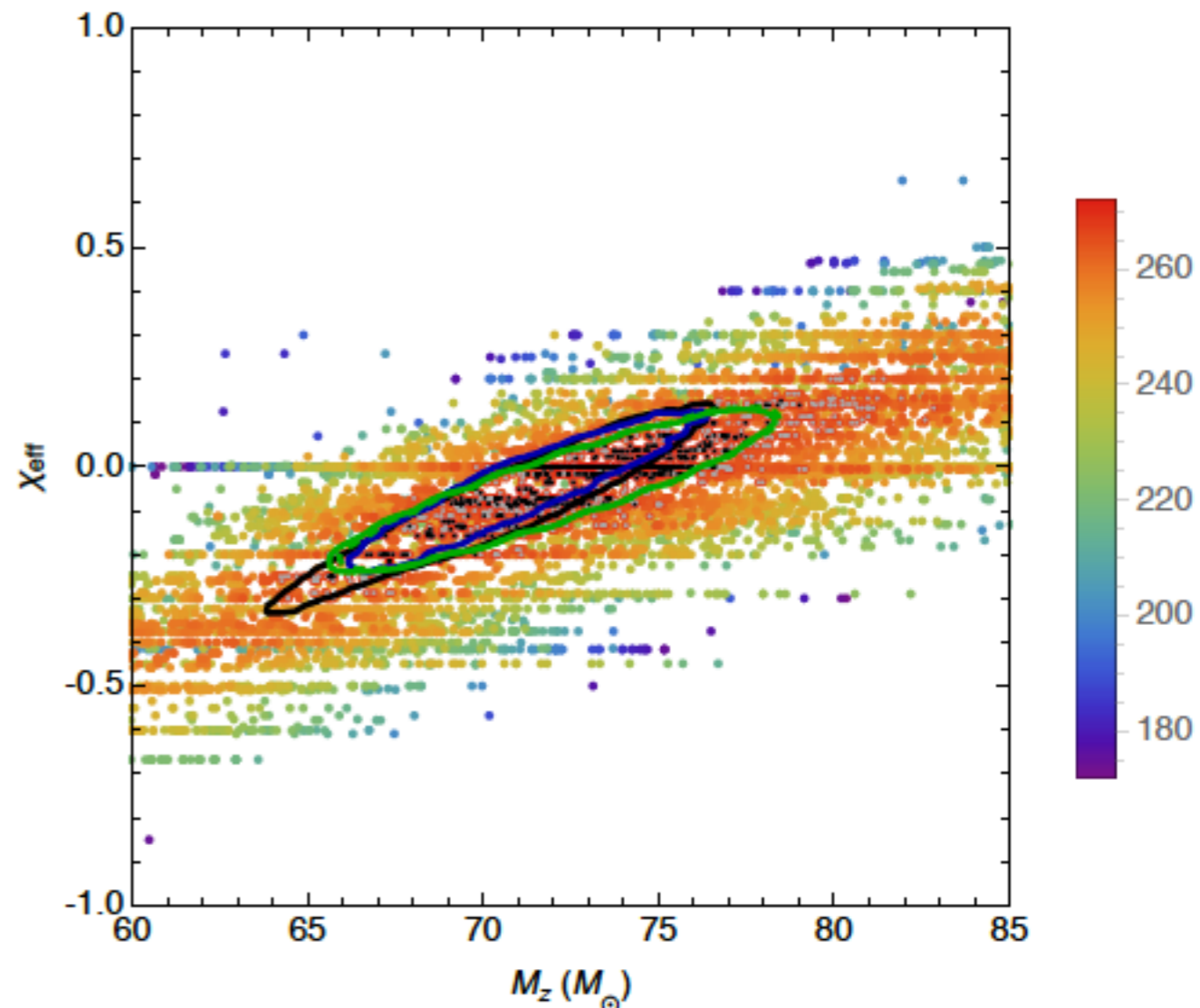
Direct use of numerical relativity waveforms

- Direct comparison of NR catalogs to observations

$$\ln \mathcal{L}(\lambda; \theta) = -\frac{1}{2} \sum_k \langle h_k(\lambda, \theta) - d_k | h_k(\lambda, \theta) - d_k \rangle_k - \langle d_k | d_k \rangle_k$$



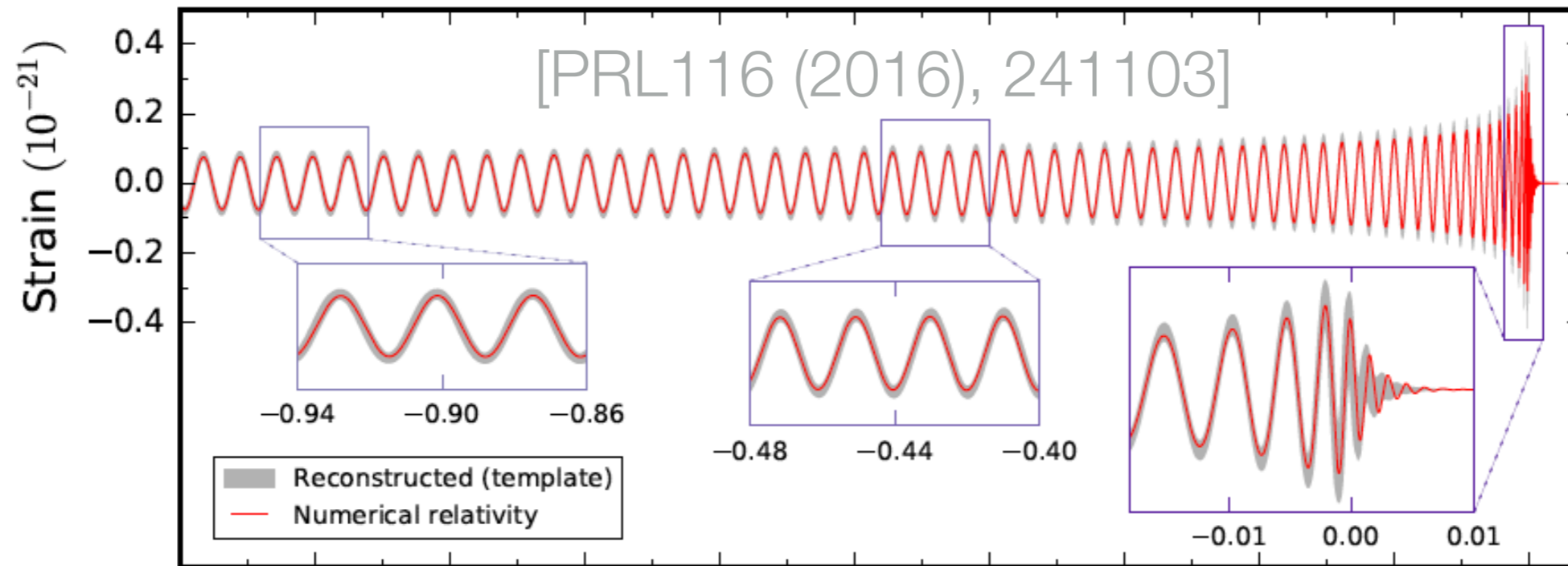
[LVC1602.03843]



[LVC1606.01262]

Direct use of numerical relativity waveforms

- **NR followup to observations** [Lovelace+16]



- **Surrogate waveform models** [Blackman+15,(in prep)]

1. restricted parameter space (high mass, $q \leq 2$, spins ≤ 0.8 , one spin aligned)
2. many NR simulations to construct basis
3. **interpolation** across NR runs
4. they do not extrapolate to low mass: need models or long NR

Quasicircular binary black holes: models

How good is a model?

$$\langle h_{\text{NR}}, h_{\text{model}} \rangle = 4 \operatorname{Re} \int_{f_{\text{low}}}^{f_{\text{high}}} \frac{\tilde{h}_{\text{NR}}(f) \tilde{h}_{\text{model}}^*(f)}{S_n(f)} df$$

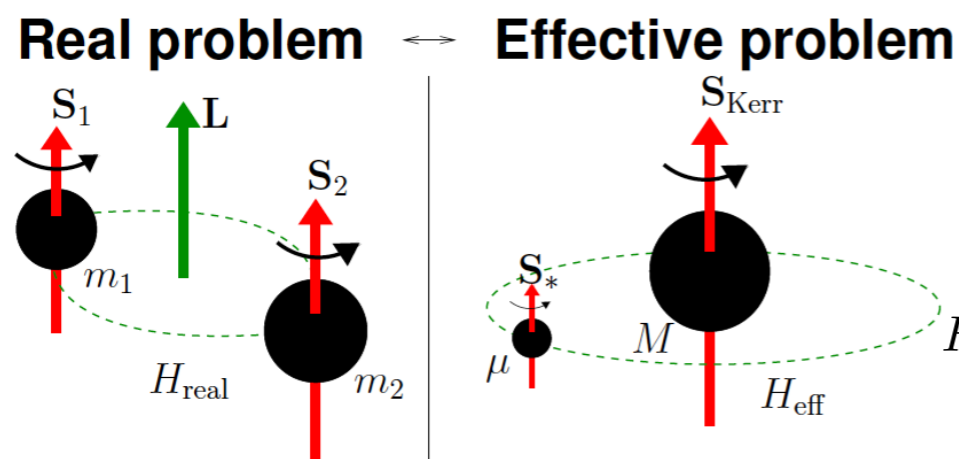
$$\mathcal{O}(h_{\text{NR}}, h_{\text{model}}) = \frac{\langle h_{\text{NR}}, h_{\text{model}} \rangle}{\sqrt{\langle h_{\text{NR}}, h_{\text{NR}} \rangle \langle h_{\text{model}}, h_{\text{model}} \rangle}}$$

- **Template banks** accept 97% overlaps \sim 10% loss in event rate
- **Parameter estimation**: (sufficient) accuracy requirement
[Lindblom+08]

$$\mathcal{O}(h_{\text{NR}}, h_{\text{model}}) > 1 - \frac{1}{2 \text{SNR}^2}$$

Effective-one-body models of nonprecessing BBHs

- Nonspinning case: particle in deformation of Schwarzschild [Buonanno & Damour99]. Spinning case: **spinning particle in deformation of Kerr** [Barausse & Buonanno10,11;Nagar+14]
- Inspiral waveforms/radiation reaction from **resummation post-Newtonian formulas** [Damour+07,09; Pan+11;Nagar+16]
- Ringdown from **superposition of quasinormal modes** of remnant BH



$$H_{\text{real}} = Mc^2 \sqrt{1 + 2\nu \left(\frac{H_{\text{eff}}}{\mu c^2} - 1 \right)} - Mc^2$$

$$H_{\text{eff}} = \mu c^2 \sqrt{A(R) \left[1 + \frac{\mathbf{P}^2}{\mu^2 c^2} + \frac{1}{\mu^2 c^2} \left(\frac{A(R)}{D(R)} - 1 \right) \left(\frac{\mathbf{R} \cdot \mathbf{P}}{R} \right)^2 \right]}$$

Schwarzschild

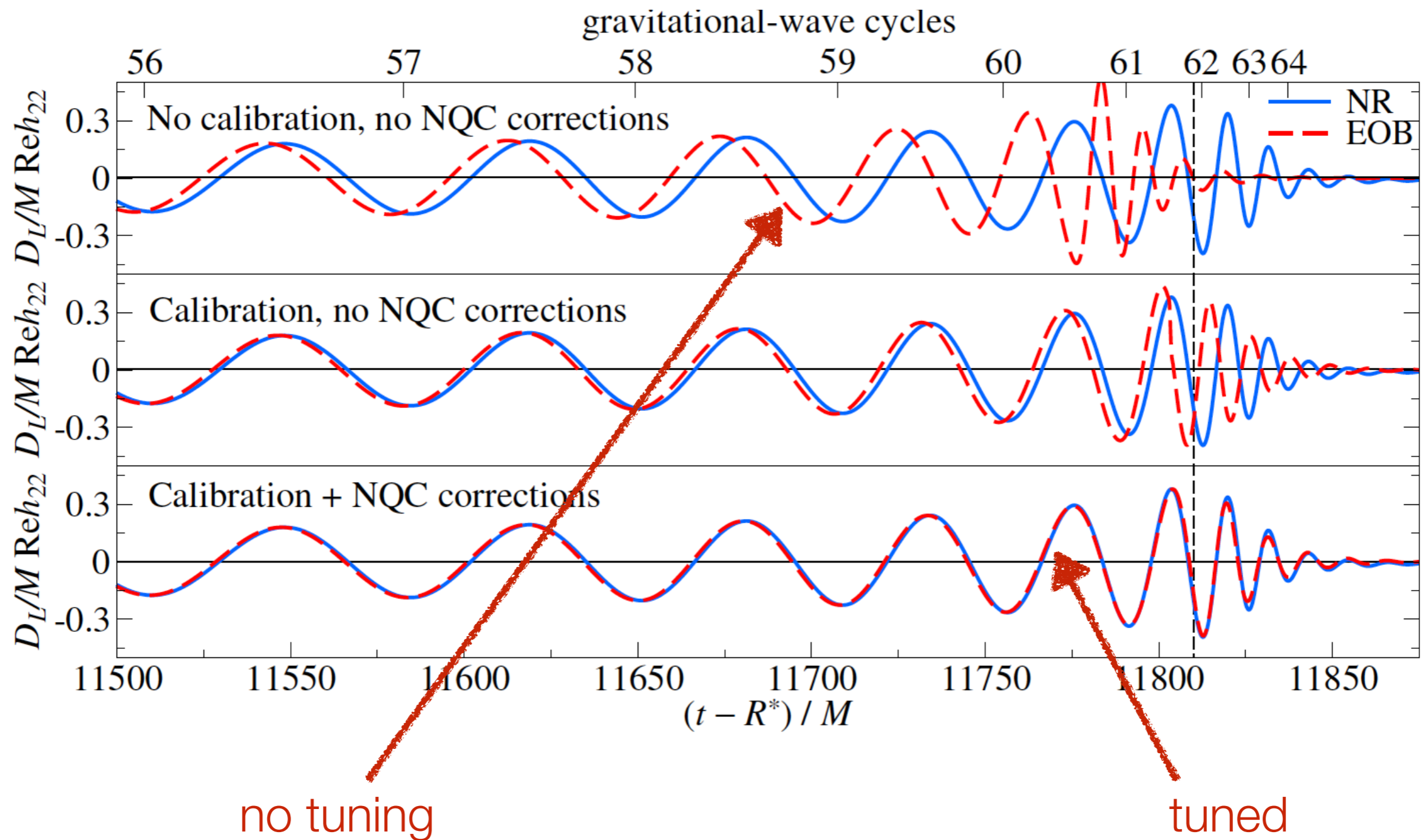
$$A = \overbrace{1 - 2u}^{\text{Schwarzschild}} + 2\nu u^3 + \left(\frac{94}{3} - \frac{42}{32} \pi^2 \right) \nu u^4 + a_5 u^5 + \dots \quad (u = GM/Rc^2)$$

example of tuning parameter →

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

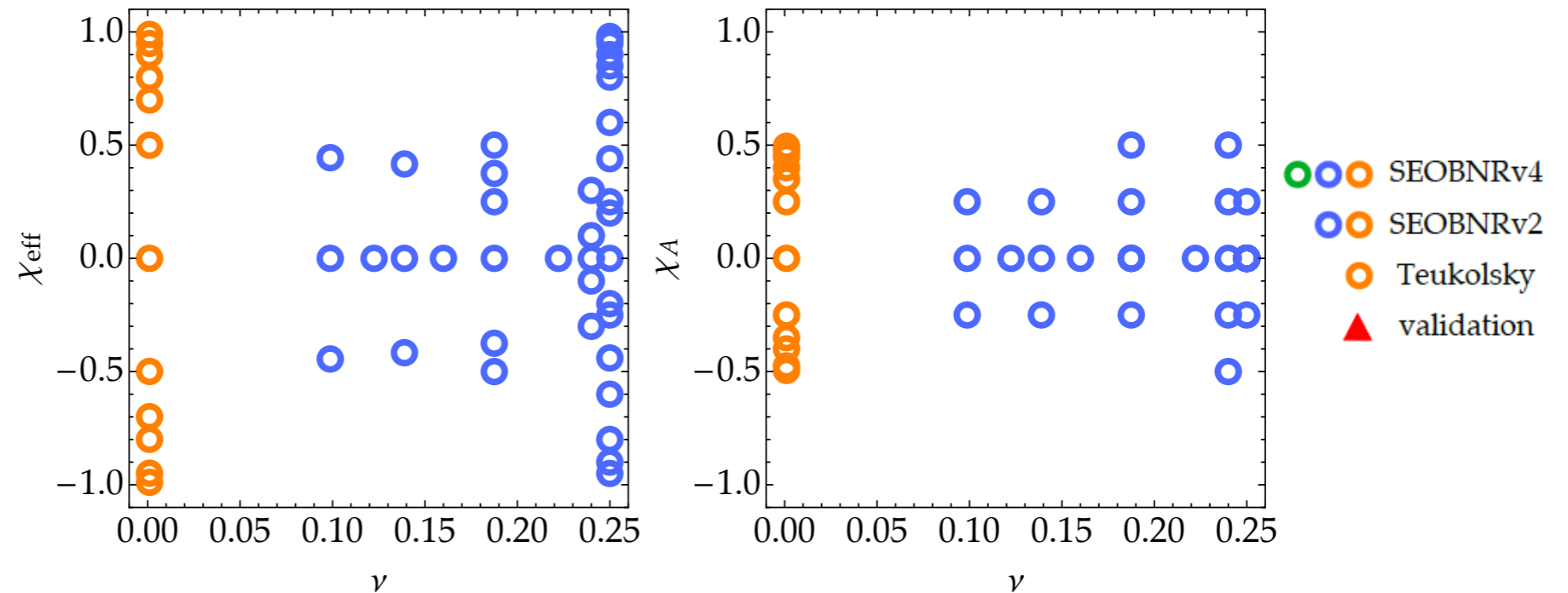
Effective-one-body models of nonprecessing BBHs

- Tuning to numerical-relativity simulations



Effective-one-body model of nonprecessing BBHs for O1

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$
$$\chi_{\text{eff}} = \left(\frac{\mathbf{S}_1}{m_1} + \frac{\mathbf{S}_2}{m_2} \right) \cdot \hat{\mathbf{L}}$$
$$\chi_A = \left(\frac{\mathbf{S}_1}{m_1^2} + \frac{\mathbf{S}_2}{m_2^2} \right) \cdot \hat{\mathbf{L}}$$



- **SEOBNRv2** calibrated to better than 99% overlap with NR for design aLIGO [AT+14]
- Used in its **reduced-order-model** version [Pürrer14,15] in O1 for filtering and parameter estimation
- Similar set of calibration waveforms used in IHES models [Nagar+15,16]

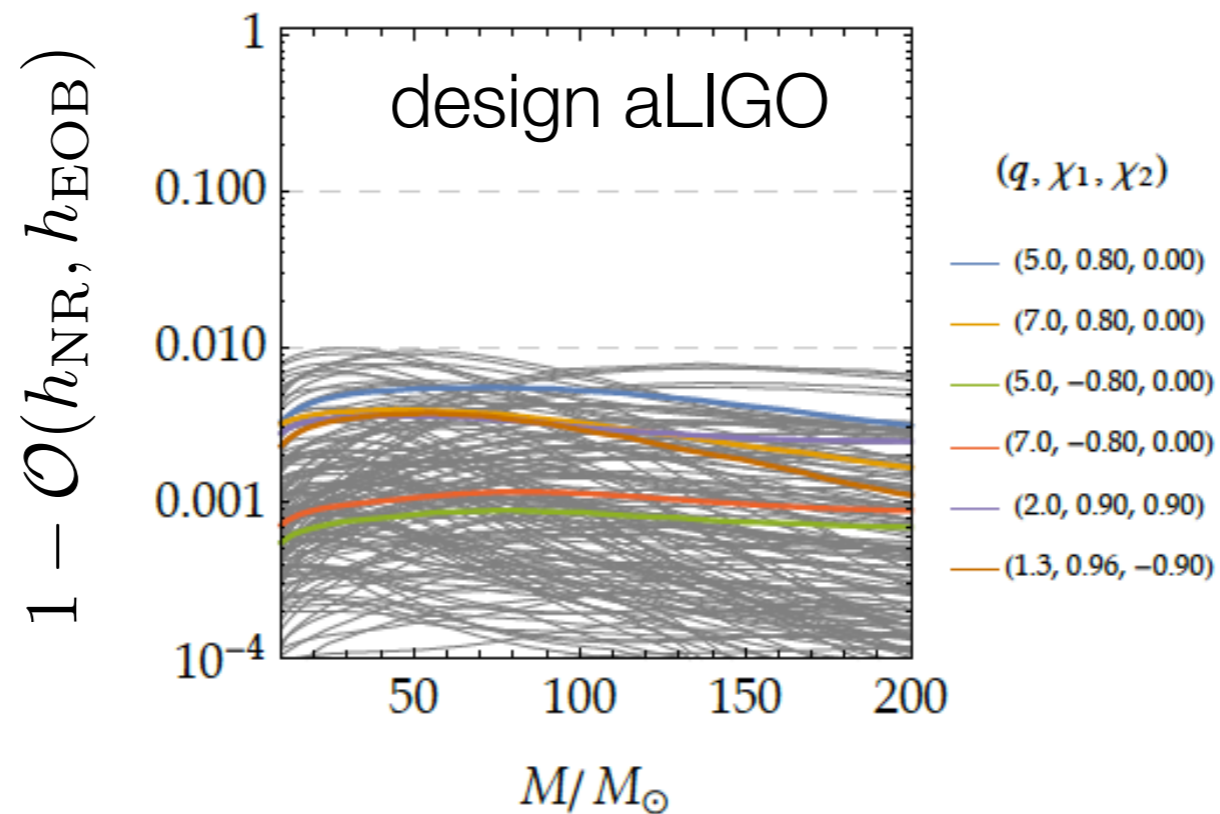
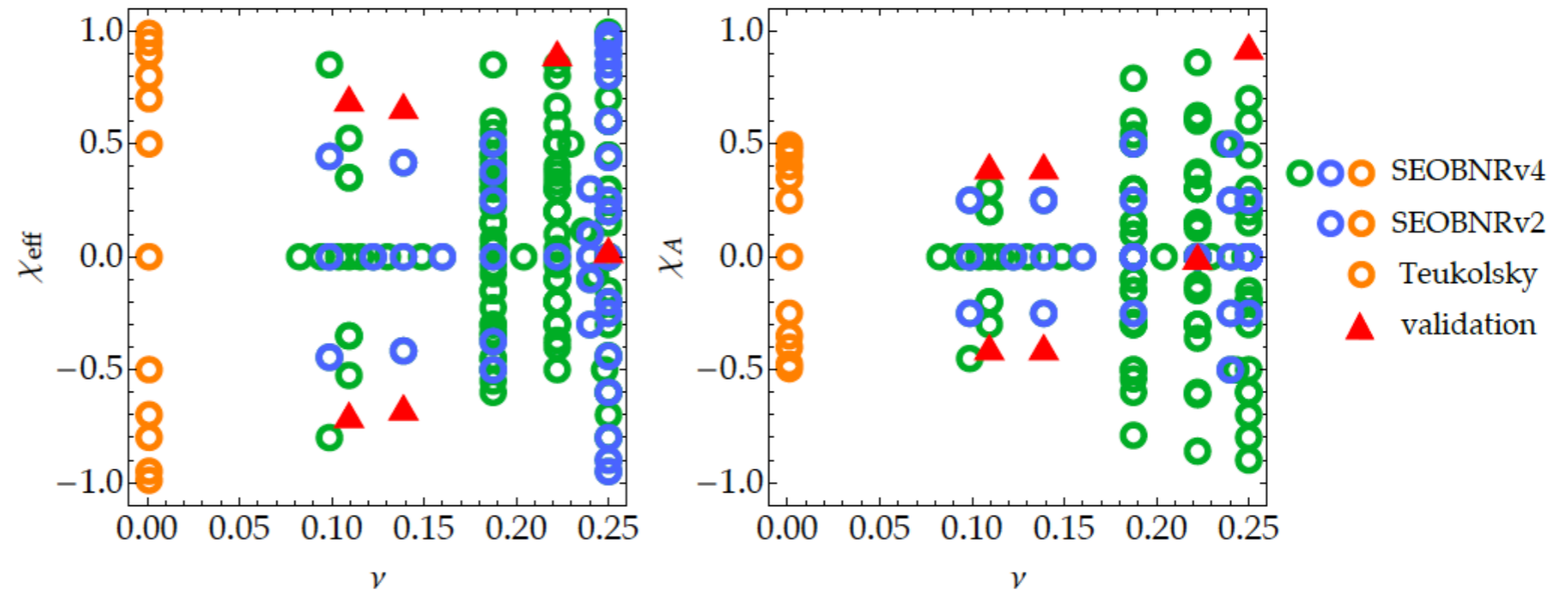
Effective-one-body model of nonprecessing BBHs for O2

- **SEOBNRv4** [Bohe, Shao, AT+(in prep)]

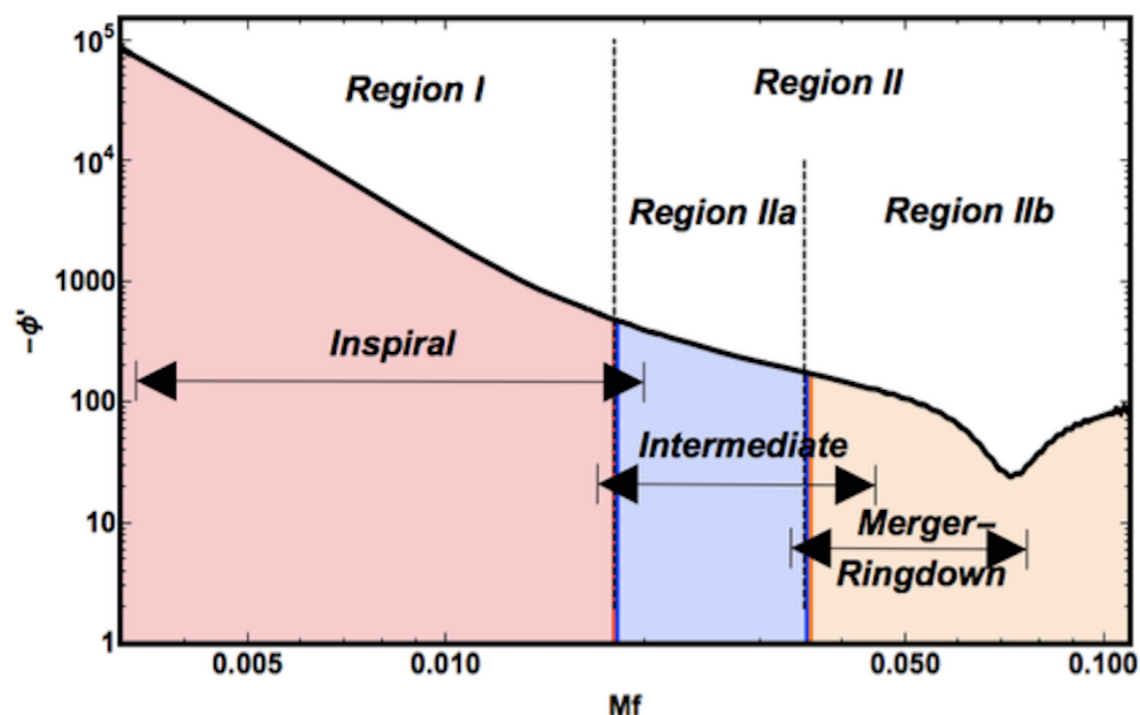
$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$\chi_{\text{eff}} = \left(\frac{\mathbf{S}_1}{m_1} + \frac{\mathbf{S}_2}{m_2} \right) \cdot \hat{\mathbf{L}}$$

$$\chi_A = \left(\frac{\mathbf{S}_1}{m_1^2} + \frac{\mathbf{S}_2}{m_2^2} \right) \cdot \hat{\mathbf{L}}$$

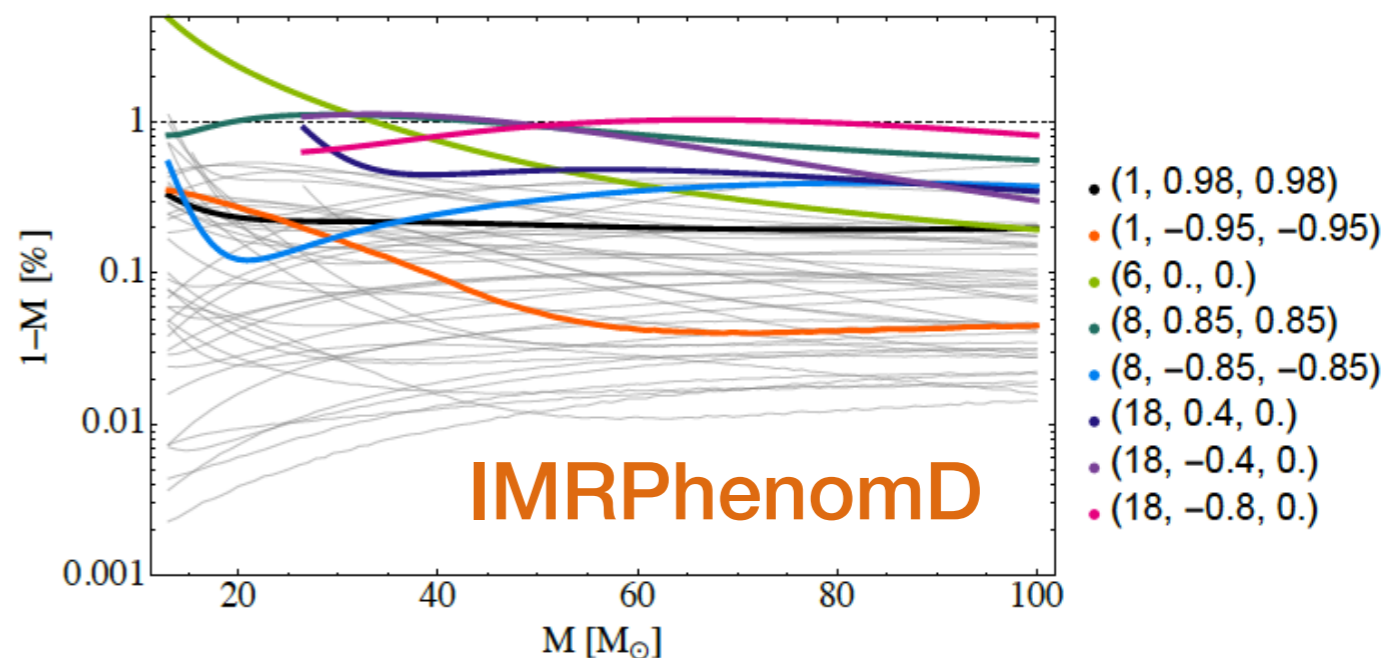
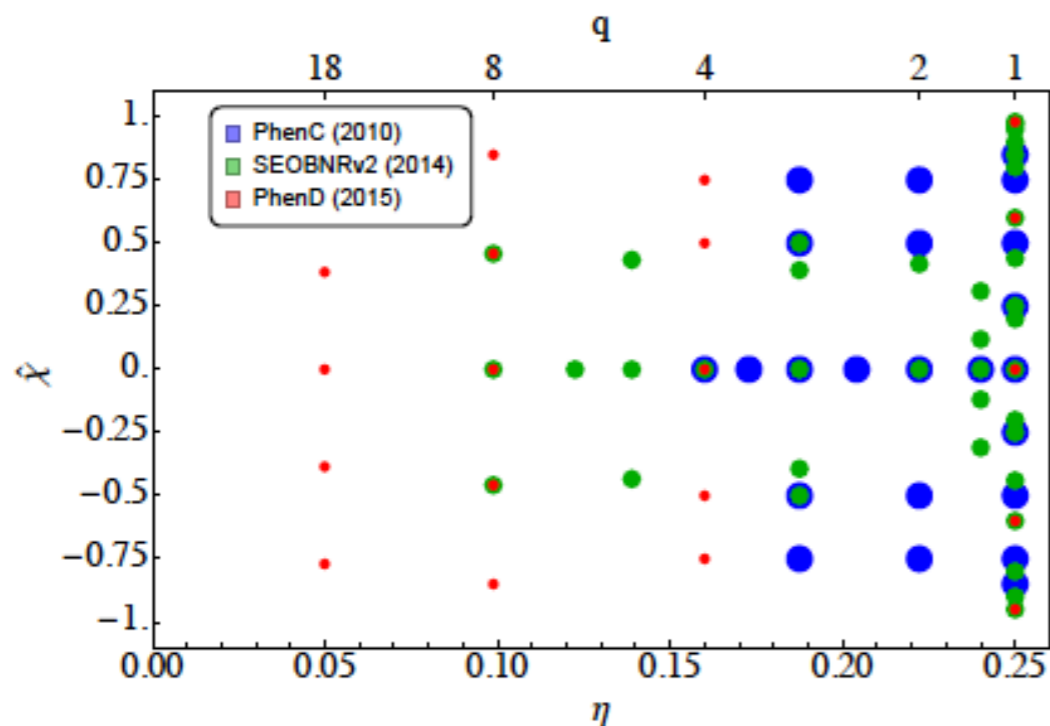


Phenomenological model of nonprecessing BBHs

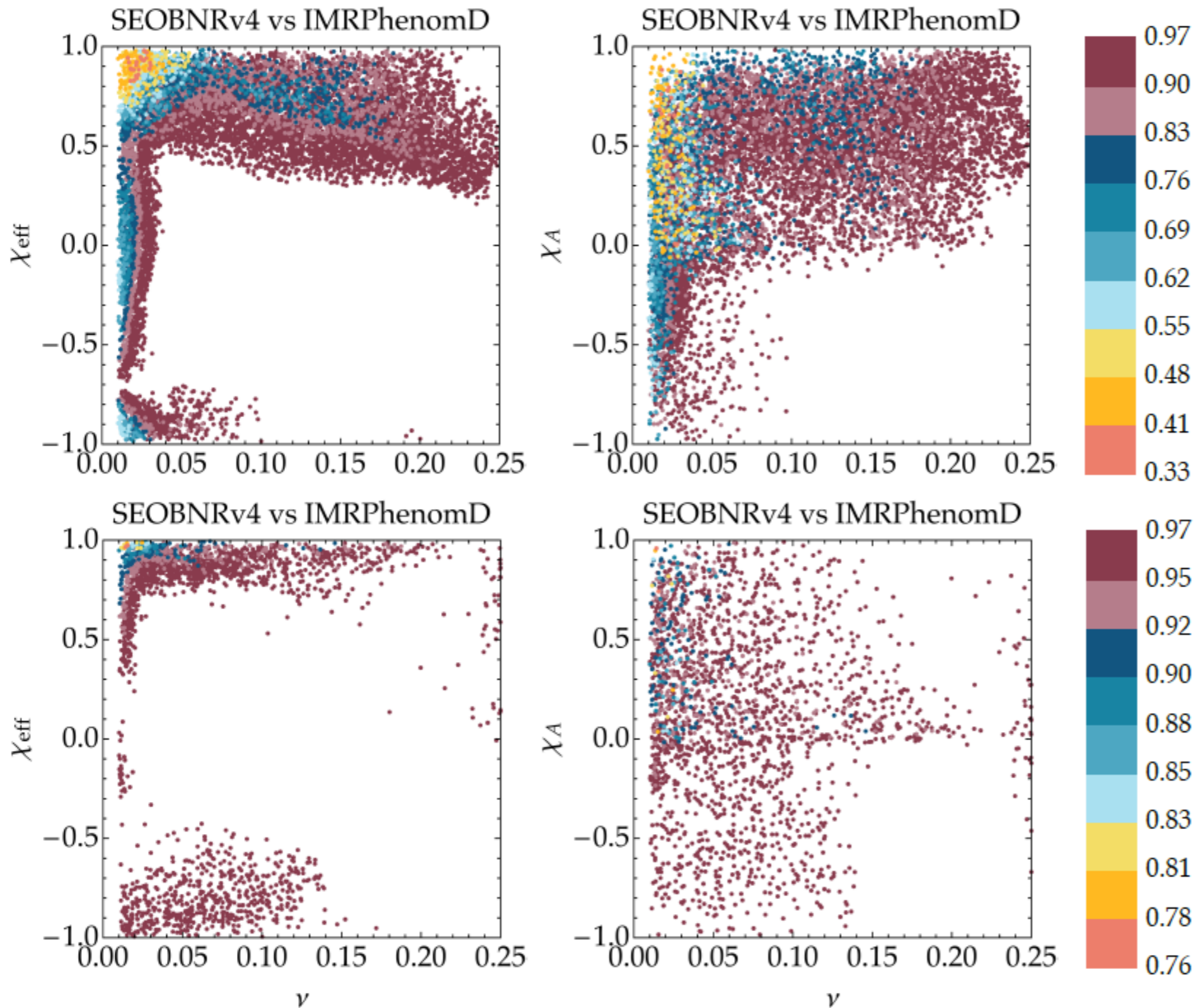


$$\begin{aligned} \phi_{\text{Ins}} &= \phi_{\text{TF2}}(Mf; \Xi) \\ &+ \frac{1}{\eta} \left(\sigma_0 + \sigma_1 f + \frac{3}{4} \sigma_2 f^{4/3} + \frac{3}{5} \sigma_3 f^{5/3} + \frac{1}{2} \sigma_4 f^2 \right) \\ \phi_{\text{Int}} &= \frac{1}{\eta} \left(\beta_0 + \beta_1 f + \beta_2 \text{Log}(f) - \frac{\beta_3}{3} f^{-3} \right) \\ \phi_{\text{MR}} &= \frac{1}{\eta} \left\{ \alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3} \alpha_3 f^{3/4} \right. \\ &\left. + \alpha_4 \tan^{-1} \left(\frac{f - \alpha_5 f_{\text{RD}}}{f_{\text{damp}}} \right) \right\}. \end{aligned}$$

- Fit to **hybrids** of uncalibrated EOB and NR [Husa+15, Khan+15]



Comparing nonprecessing BBH models



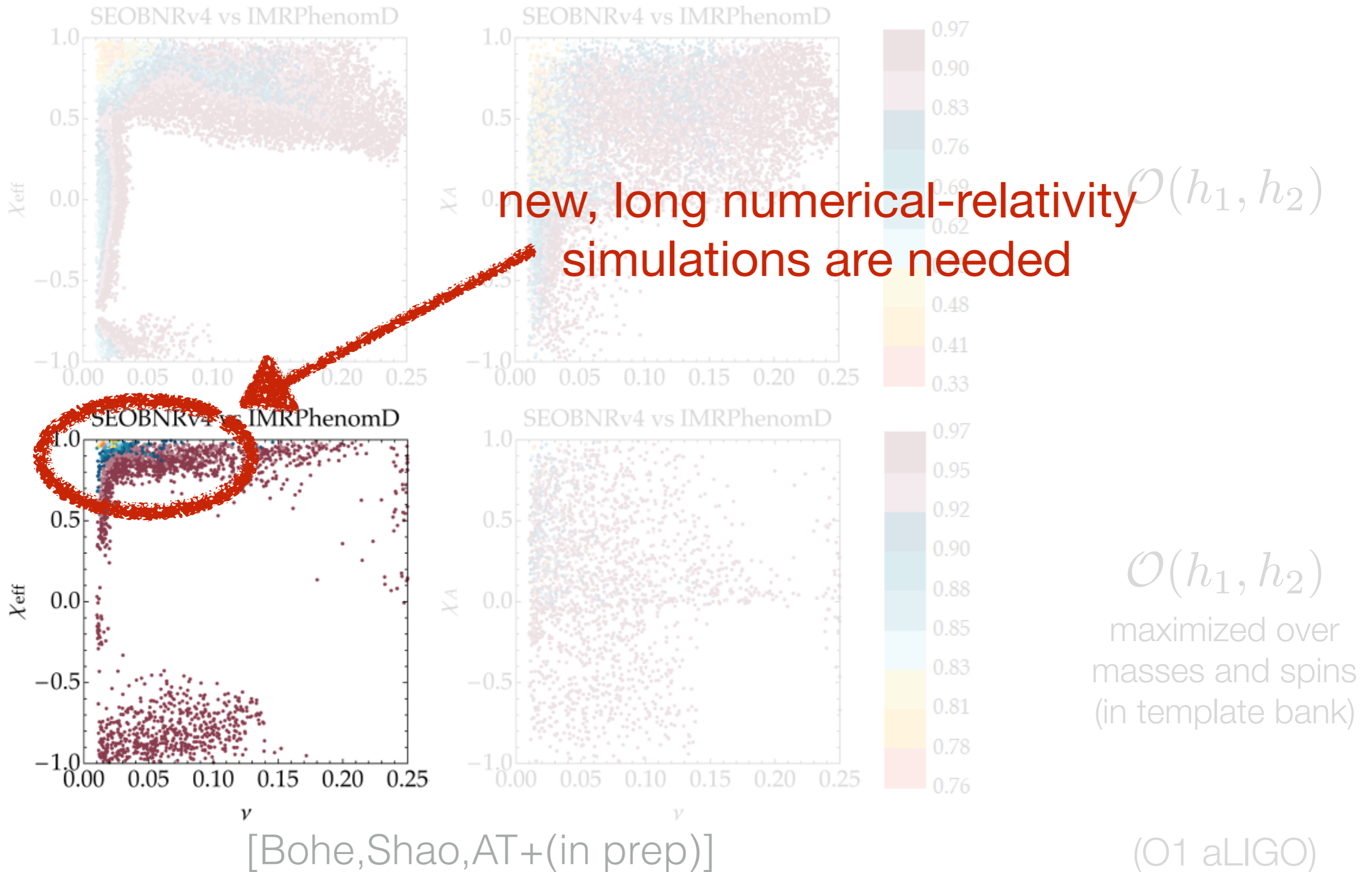
$$\mathcal{O}(h_1, h_2)$$

$\mathcal{O}(h_1, h_2)$
maximized over
masses and spins
(in template bank)

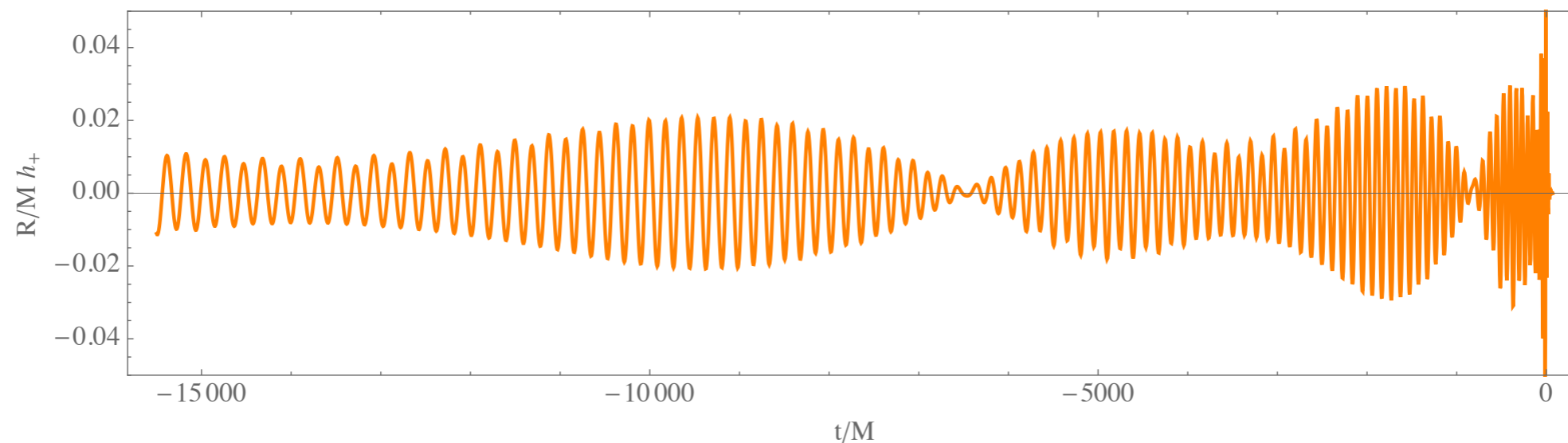
[Bohe, Shao, AT+(in prep)]

(O1 aLIGO)

Comparing nonprecessing BBH models

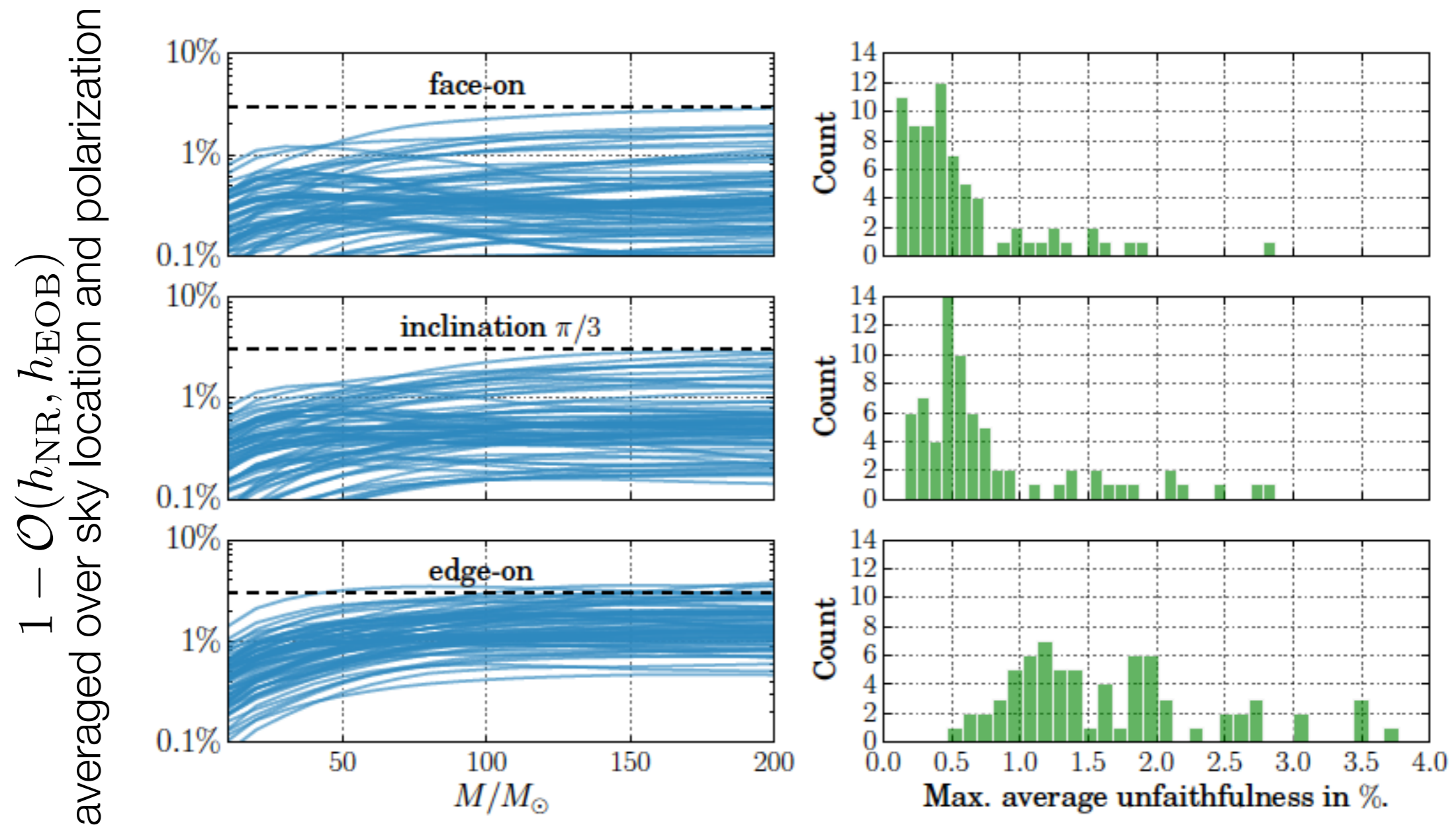


Precessing BBH models



- When BH spins are not parallel to angular momentum of the binary, the **orbital plane precesses**
- **Precessing frame** [Buonanno+03, Schmidt+11, O’Shaughnessy+11, Boyle+11]
 1. In precessing frame, use **calibrated nonprecessing model**
 2. Inertial-frame modes from **rotation of precessing-frame modes according to motion of orbital angular momentum**
- Both effective-one-body [Pan+13, Babak, AT, Buonanno16] and phenomenological [Hannam+13] models available

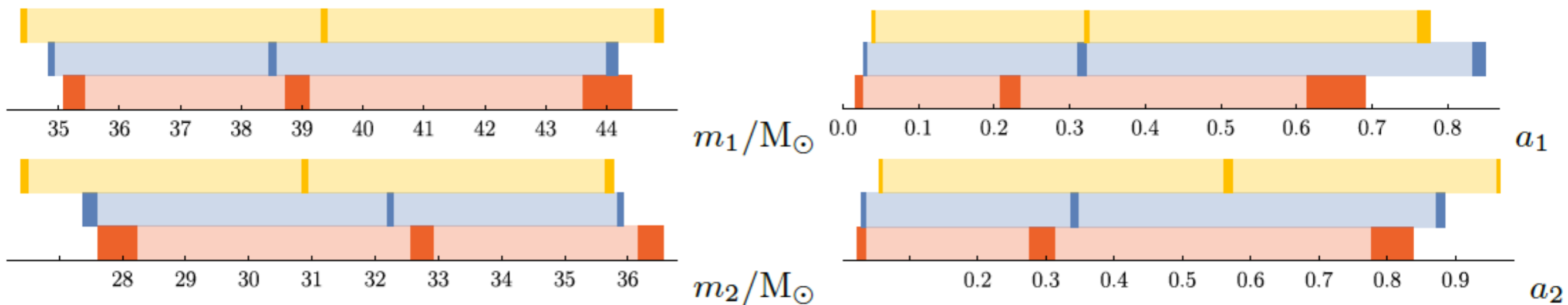
Effective-one-body model for precessing BBHs



[Babak, AT, Buonanno16]

Different models vs GW150914

- **Nonprecessing EOBNR**, **precessing EOBNR**, and **precessing Phenom** measure consistent parameters for GW150914
 1. SNR
 2. comparable mass
 3. face off/on
 4. short signal

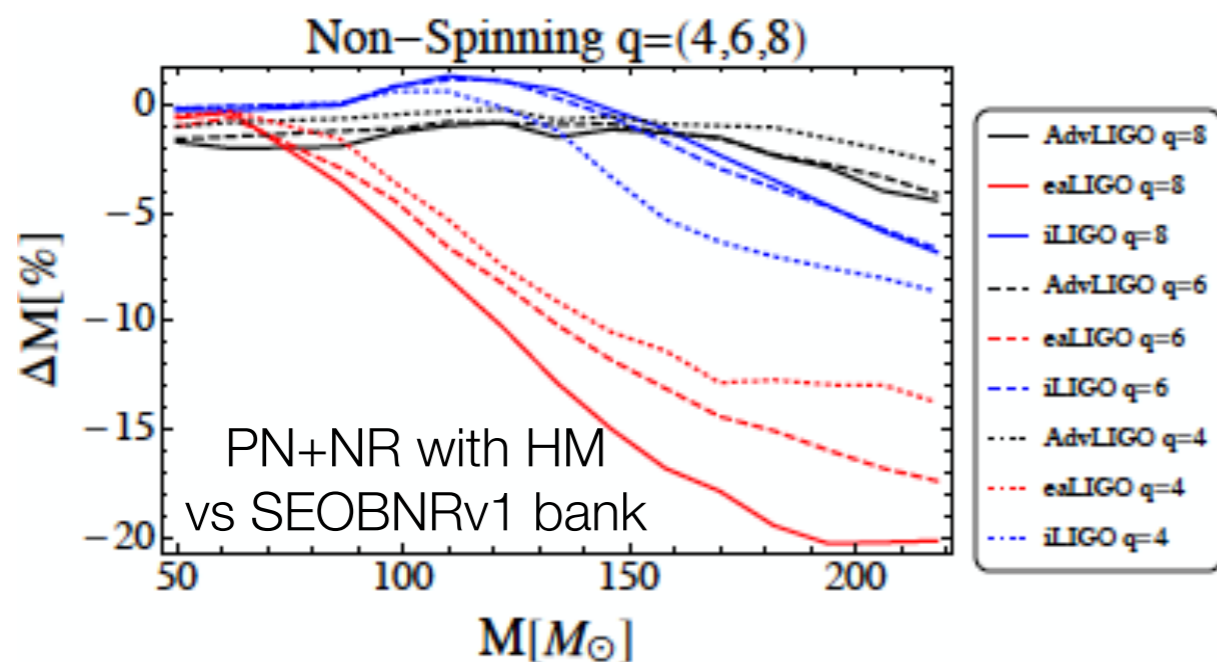
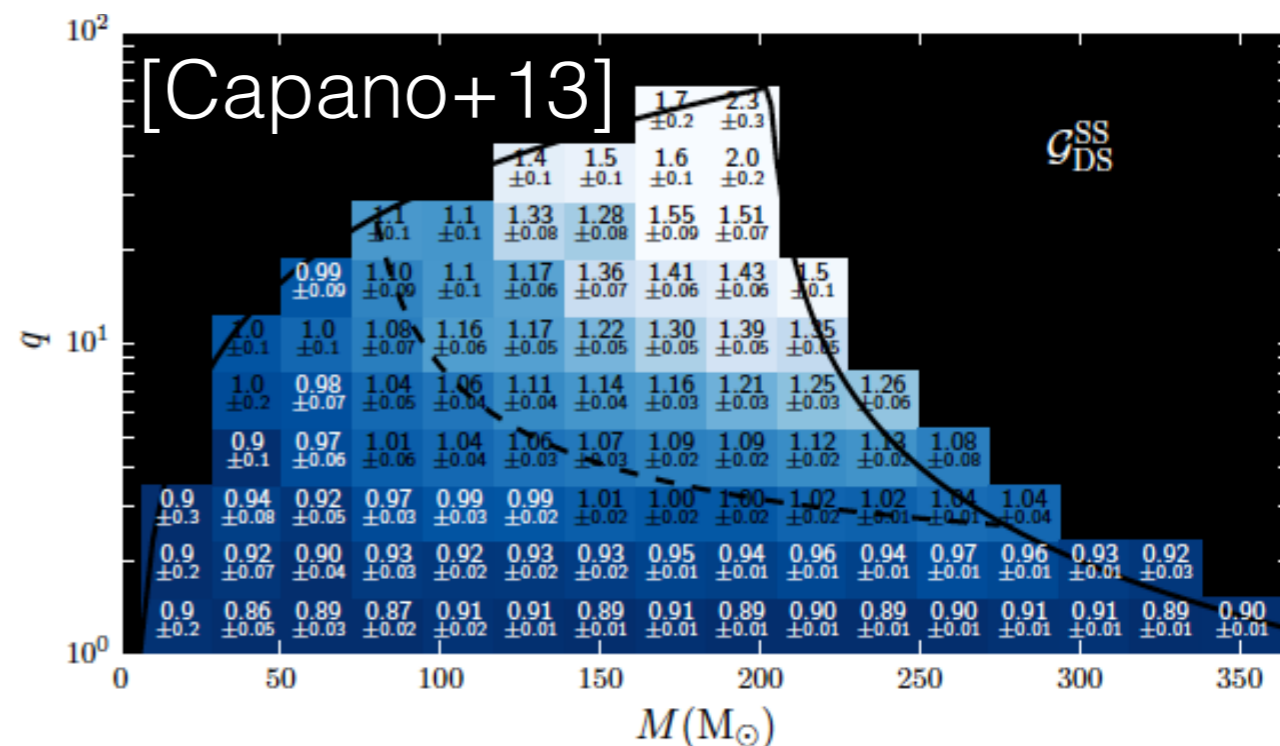


[LVC1606.01262]

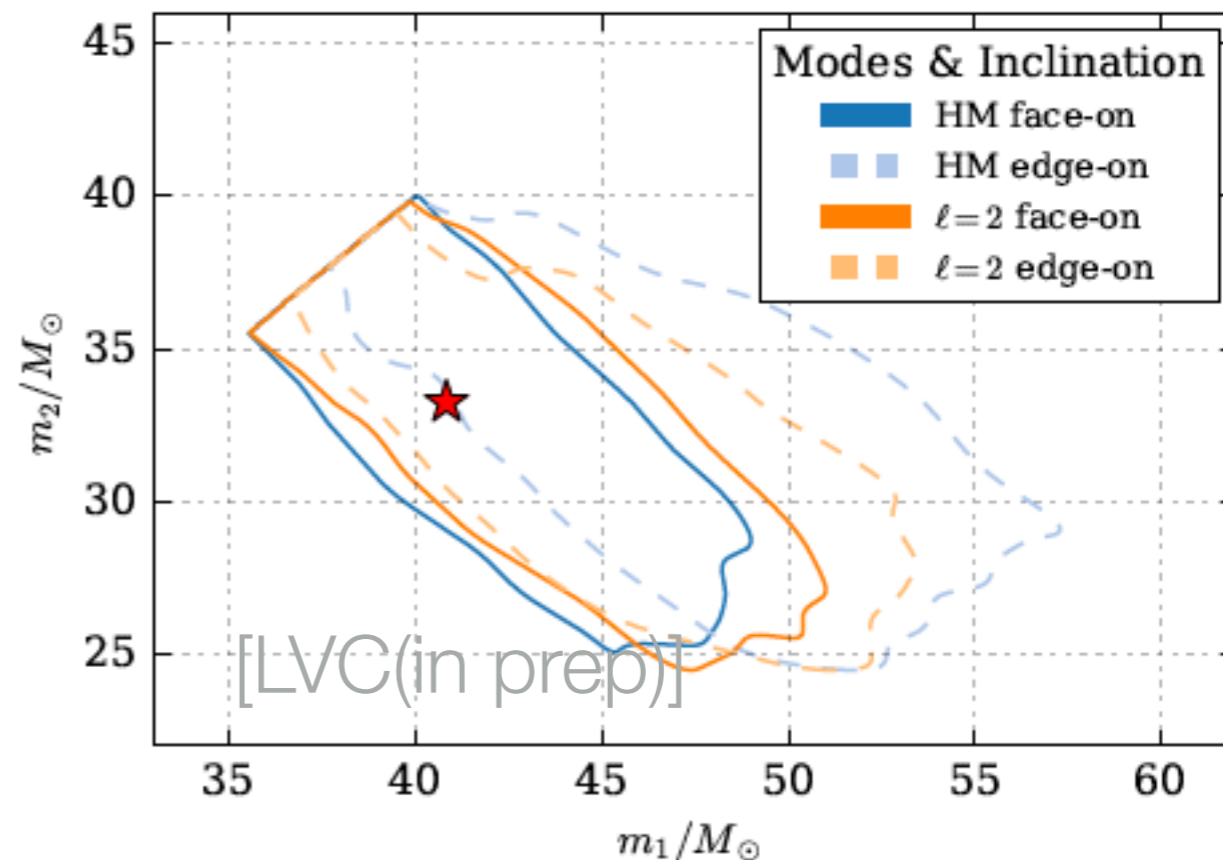
Quasicircular binary black holes:
open problems

Open problems for quasicircular BBH models

- Problem of extrapolation** outside calibration domains, i.e., high mass ratios, spins
- IMR higher-order modes** for spinning binaries are not available

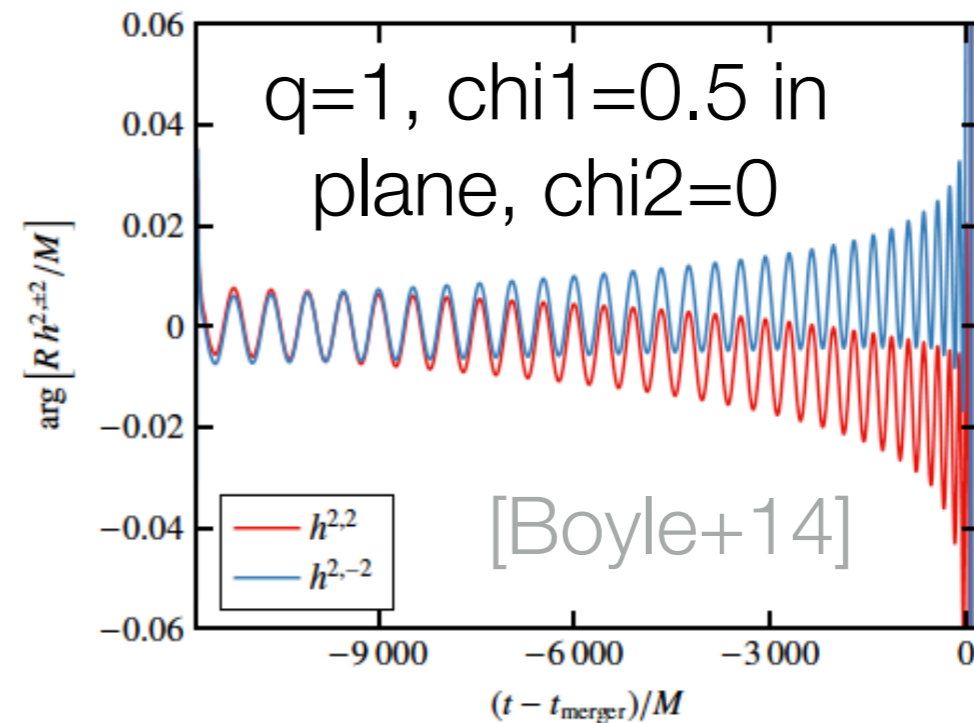
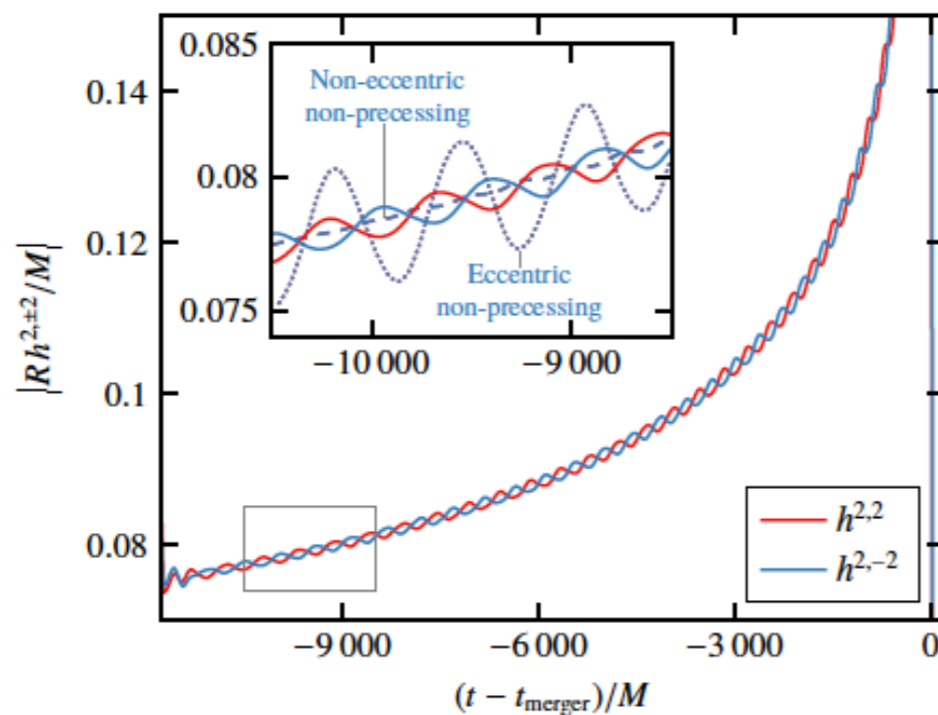


[Calderon-Bustillo+15]



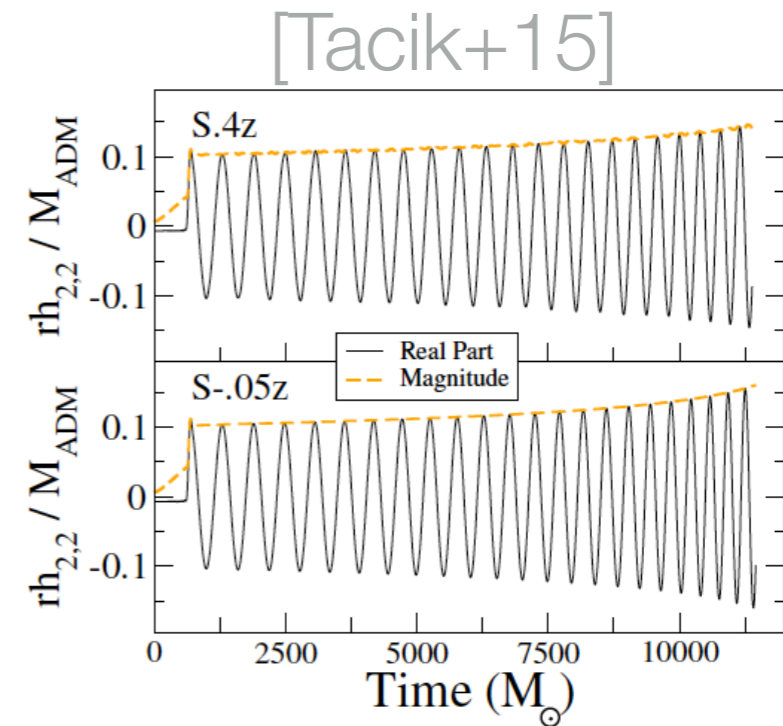
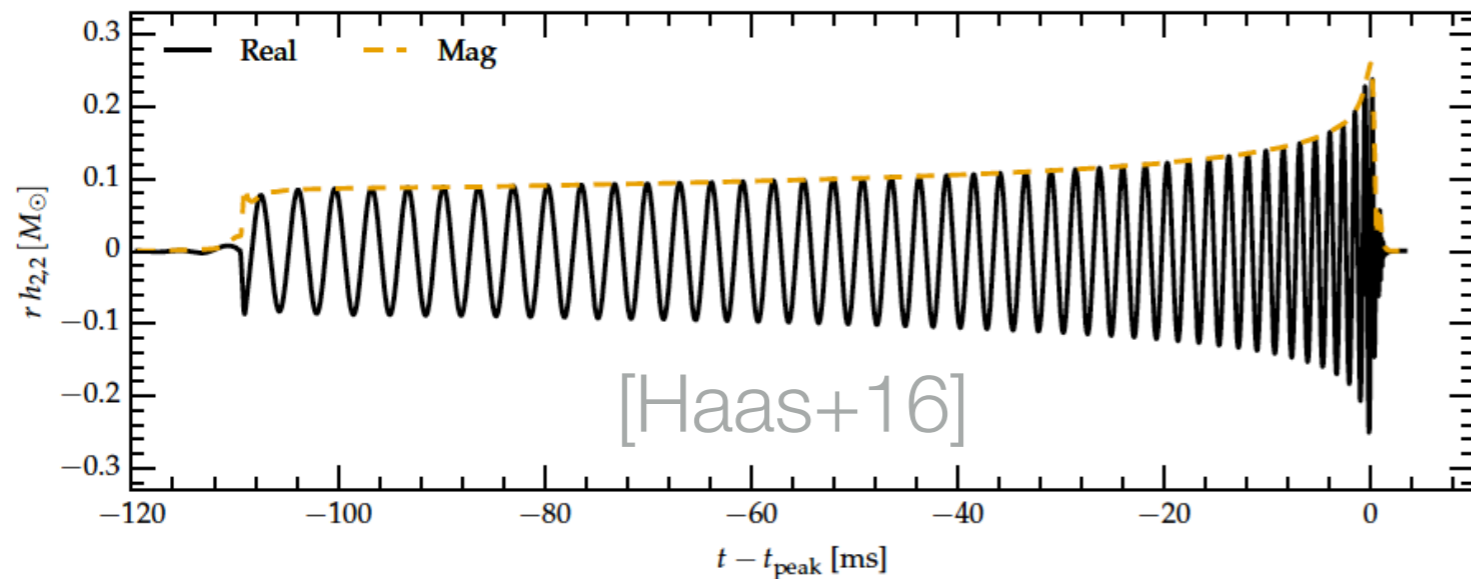
Open problems for quasicircular BBH models

- **Precessional effects** not fully modeled
 1. mode asymmetry in coprecessing frame [O'Shaughnessy+13, Pekowsky+14, Boyle+14]
 2. radiation axis keeps precessing during ringdown [O'Shaughnessy+13]
 3. no calibration to precessing NR



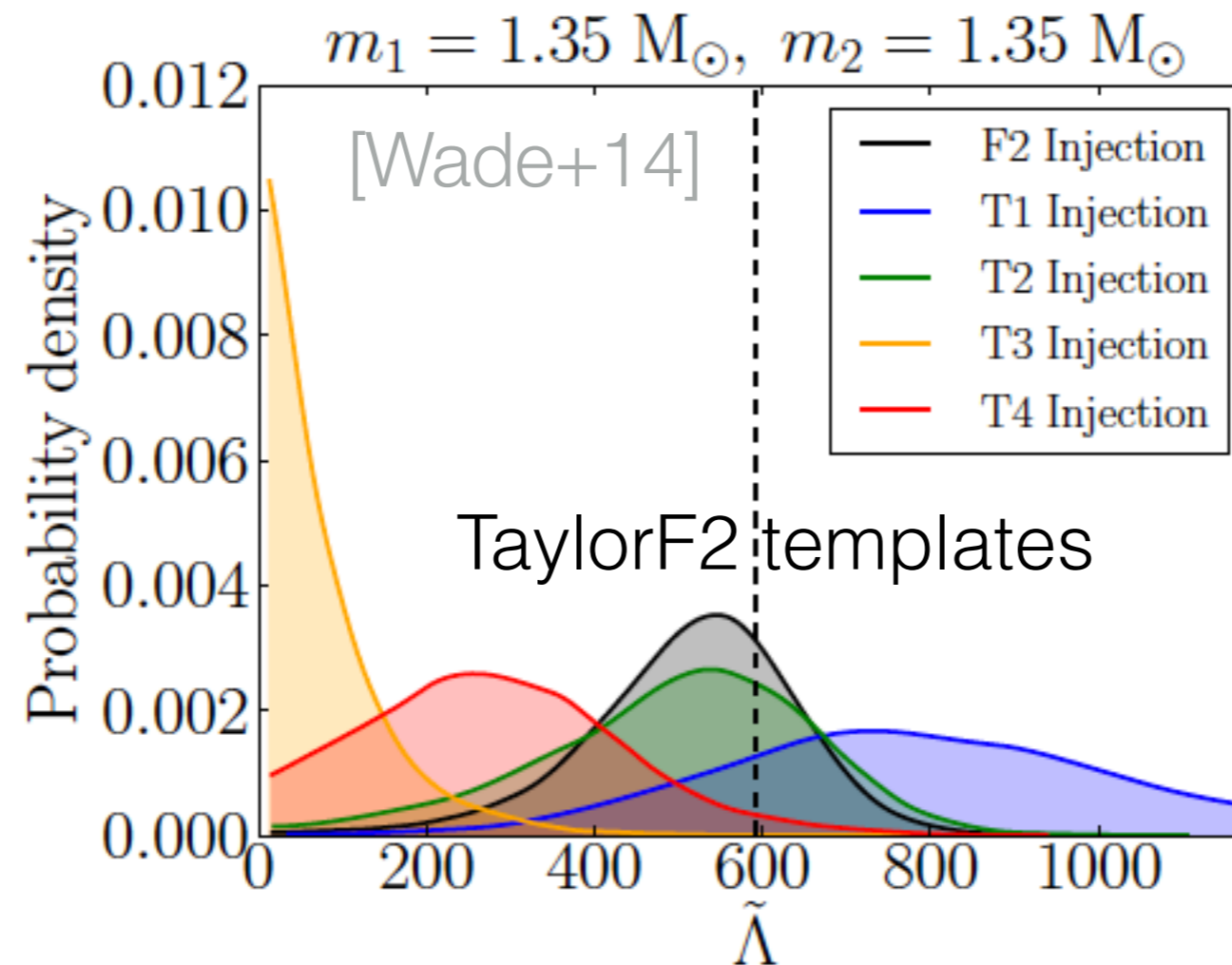
Binary neutron stars

BNS in numerical relativity



- **Longer simulations** with polytropic EOS: SACRA longterm simulations [Hotokezaka+15], 22 orbits in SpEC [Haas+16]
- Evolutions with **spin precession** [Tacik+15, Dietrich+15], **unequal mass** [Lehner+15, Dietrich+16], **more physics** (neutrino cooling, nuclear EOS, magnetic fields) [Foucart+15, Palenzuela+15, Endrizzi+16]
- New schemes that allow **smaller errors** [Bernuzzi+16]

Why modeling BNS waveforms is important

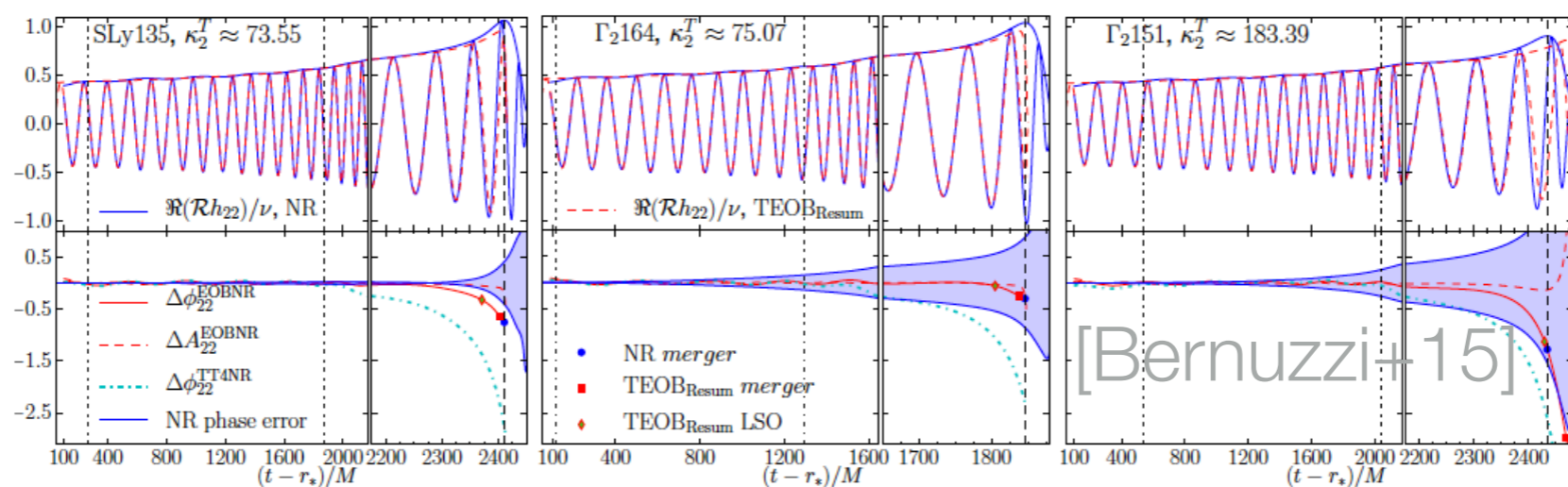


$$\tilde{\Lambda} = \frac{8}{13} \left[(1 + 7\eta - 31\eta^2) (\Lambda_1 + \Lambda_2) + \sqrt{1 - 4\eta} (1 + 9\eta - 11\eta^2) (\Lambda_1 - \Lambda_2) \right]$$

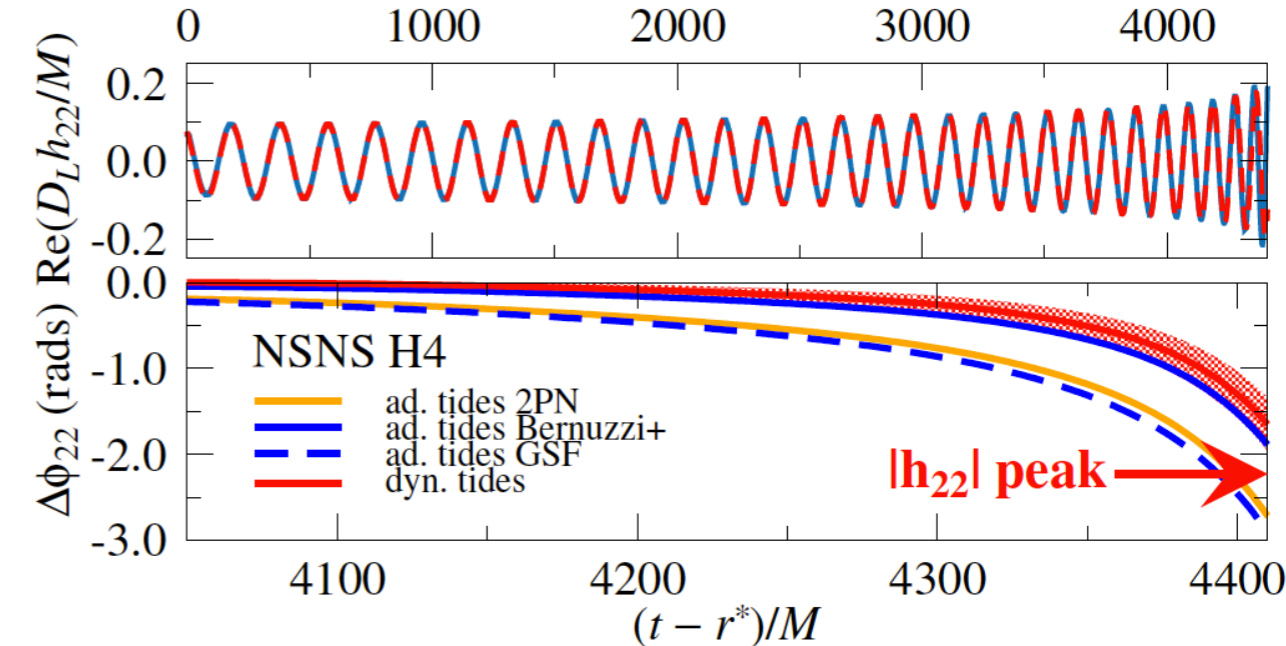
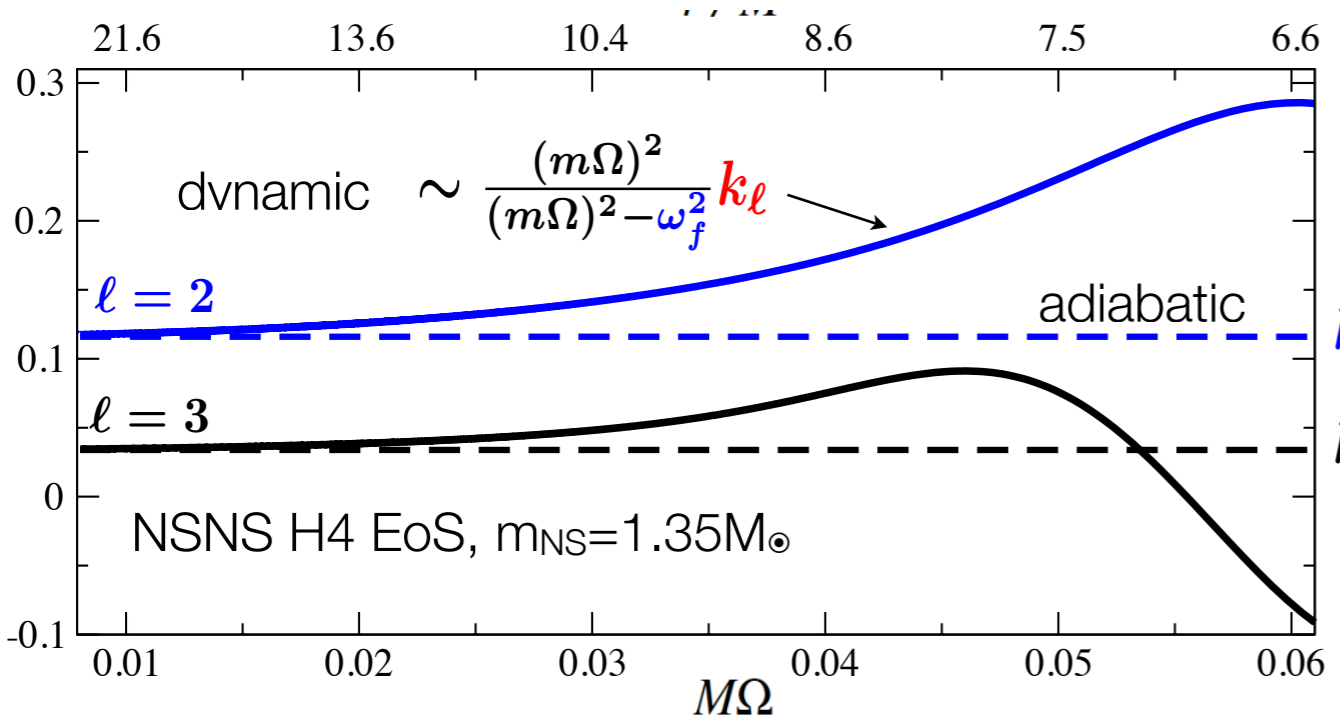
- Templates that are good for detecting BNSs can create **large biases** in measurement of tidal parameters [Yagi+14, Favata+14, Wade+14, Hotokezaka+16]

Models of inspiraling BNSs

- Splicing long NR BBH with PN tidal effects [Barkett+15]
- Augment EOB potentials by tidal effects: (i) **gravitational self-force** [Bernuzzi+15], (ii) **dynamic tides** [Hinderer+16]



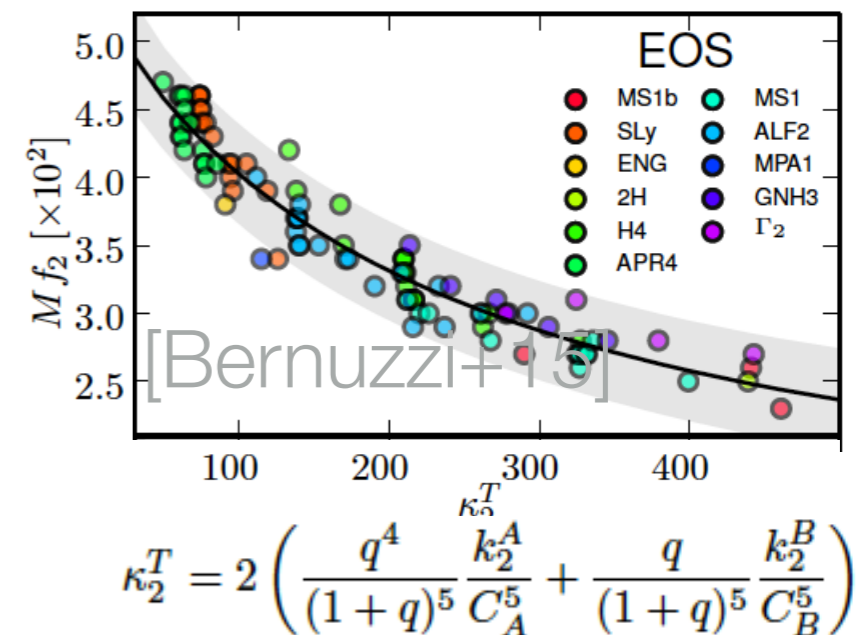
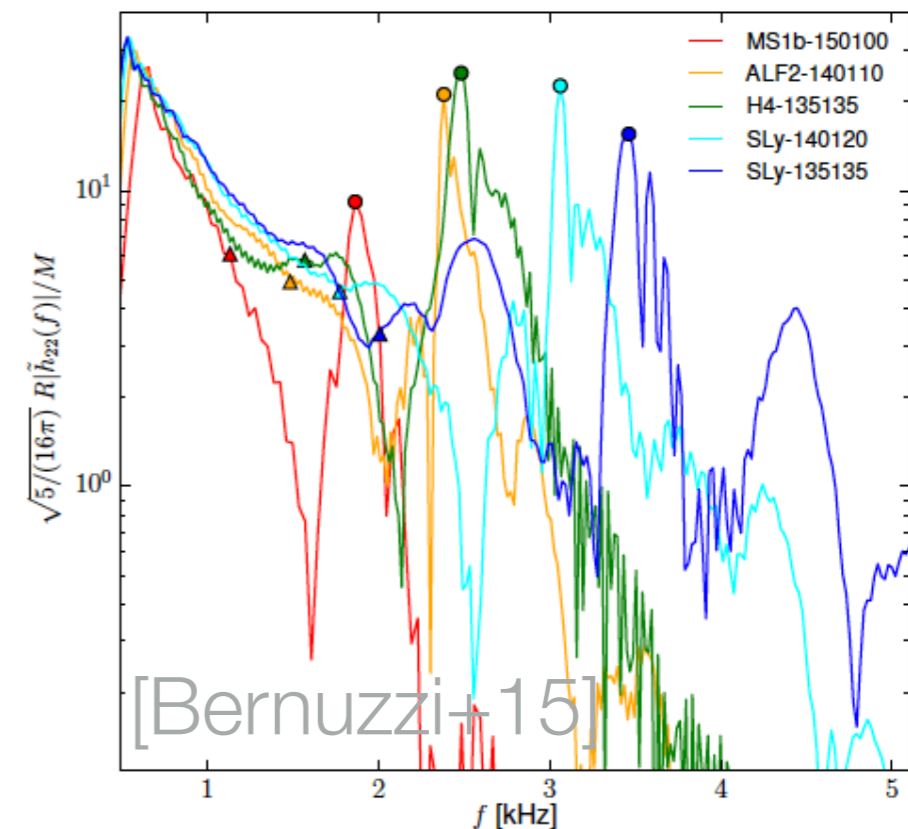
[Bernuzzi+15]



[Hinderer+16]

Models of the post-merger signal

- If $M < 2.9 M_{\text{Sun}}$, hypermassive NS forms after BNS merger
- **Peak frequencies** correlate with radius at fiducial mass, compactness, etc., in an EOS-independent way [Bauswein+12, Hotokezaka+13, Takami+14, Bernuzzi+14]

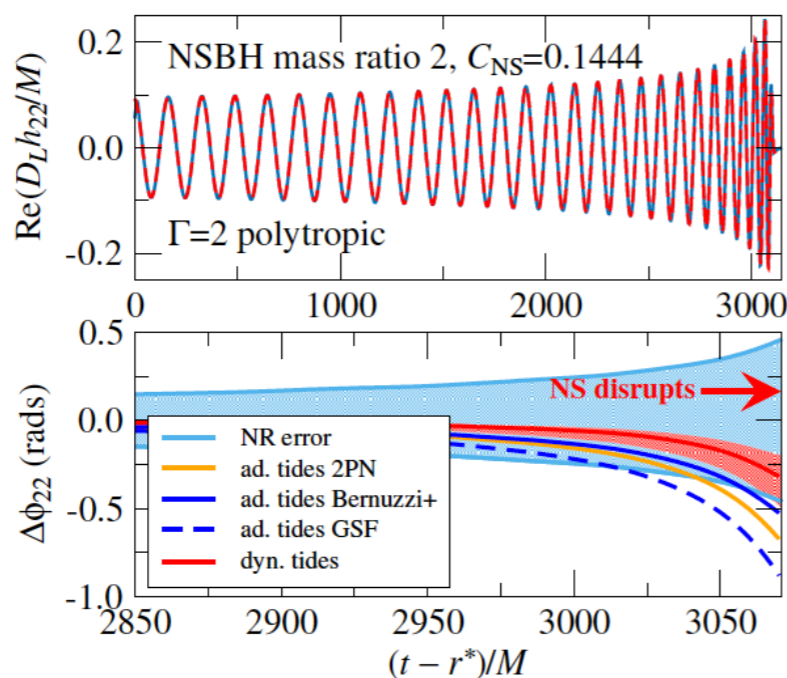
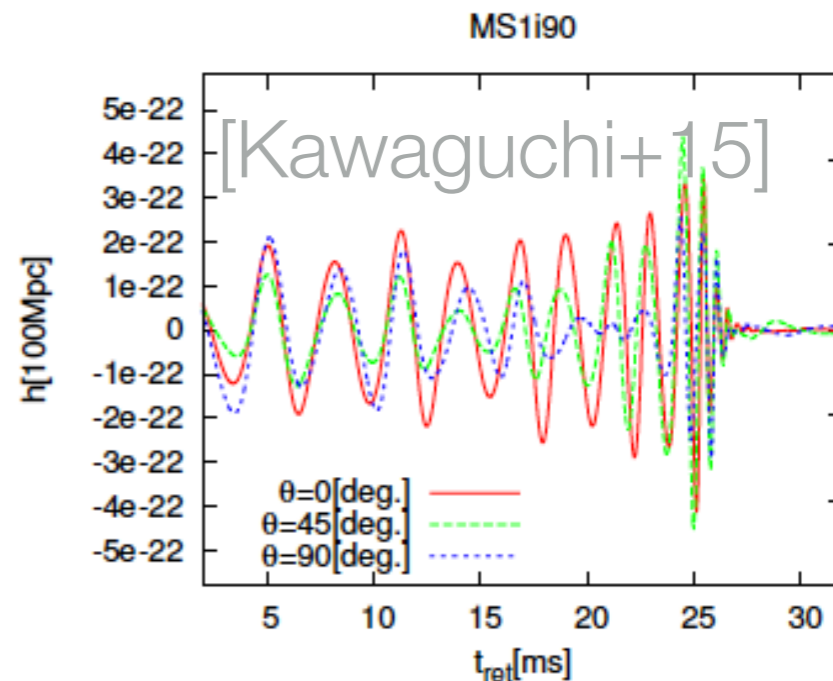


$$\kappa_2^T = 2 \left(\frac{q^4}{(1+q)^5} \frac{\kappa_2^A}{C_A^5} + \frac{q}{(1+q)^5} \frac{\kappa_2^B}{C_B^5} \right)$$

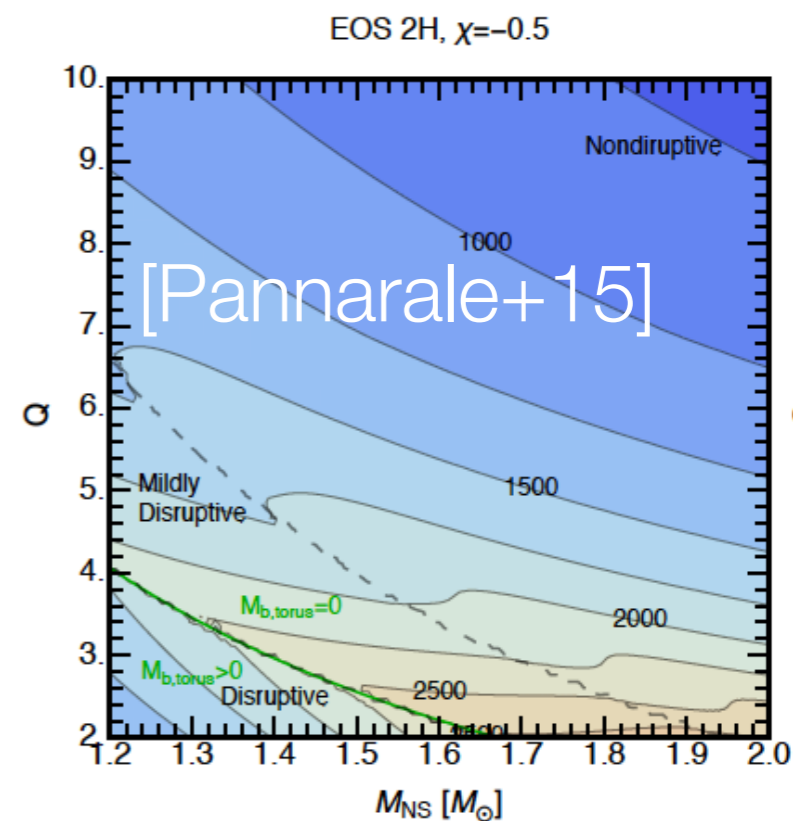
Neutron-star / black-hole binaries

Neutron star / black hole binaries

- **Long NSBHs**: small errors [Foucart+15], with precession [Kawaguchi+15]
- Model for **disruption frequency** and frequency-domain amplitude model [Pannarale+15]
- EOB model with dynamical tides [Hinderer+16]



[Hinderer+16]



Conclusions

Conclusions

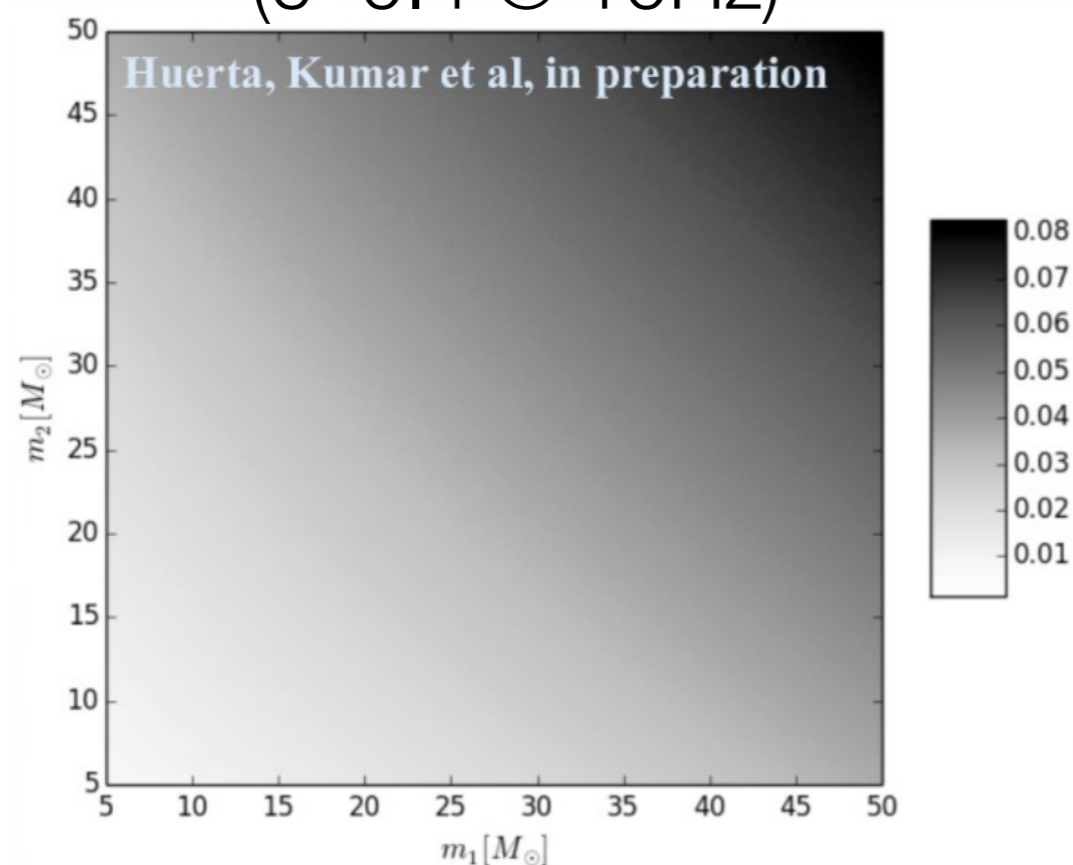
- **Binary black holes**
 1. many new NR runs (calibration, surrogates, direct use)
 2. challenging configurations becoming feasible
 3. models include info from NR catalogs
 4. towards complete IMR models with eccentricity
- **Binary neutron stars & neutron-star / black-hole binaries**
 1. longterm accurate NR runs
 2. inclusion of tidal effects in accurate point-mass models
 3. universality relations for postmerger
 4. models of disruption frequency
- **Numerical + analytical relativity crucial for best characterization of future GW observations**

Additional slides

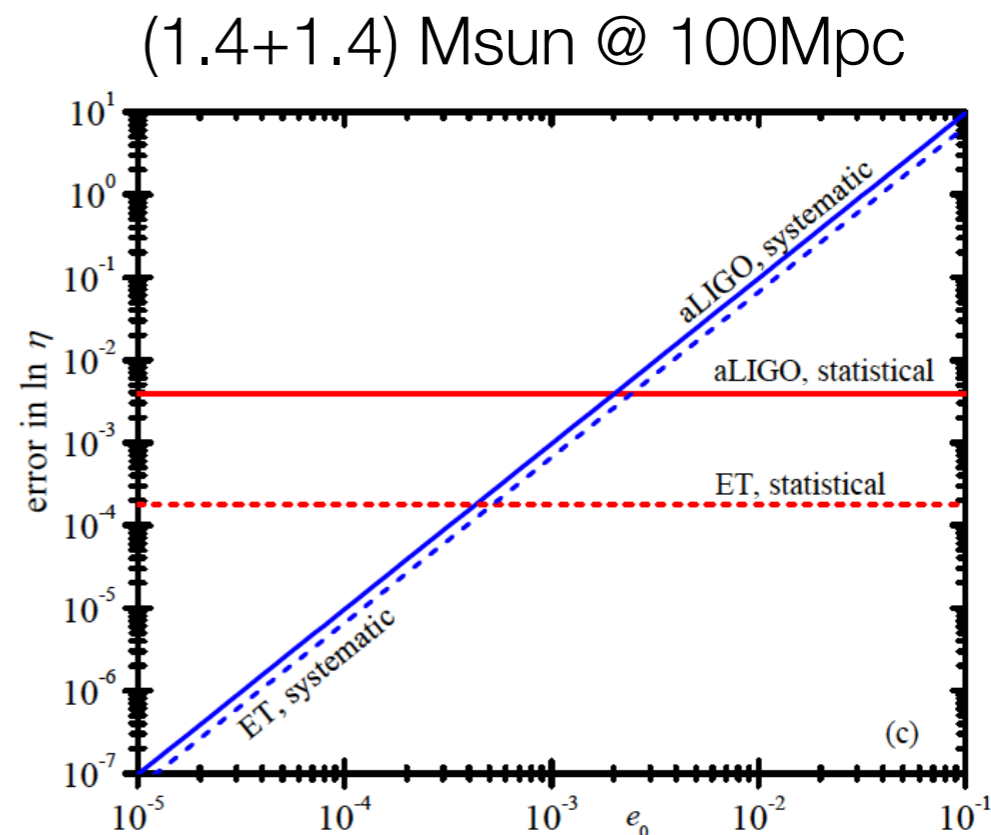
Eccentric binaries

Eccentric binaries

- **Dynamical formation** scenarios instead of field binary evolution
- Searches for BNS using quasicircular templates ok for $e \leq 0.02$ ($M=2.6M_{\text{sun}}$) [Huerta+13]
- Small residual eccentricity can **bias** parameter estimation [Favata14]
residual eccentricity @ ISCO
($e=0.4$ @ 10Hz)



[Huerta+16]



[Favata14]

Eccentric binaries

- Frequency/time-domain **PN inspiral** waveforms [Arun+09, Yunes+09, Huerta+14, Tanay+16]. Small-ecc corrections up to 3PN [Moore, Favata+16]
- IMR** waveforms based on **geodesic** motion in Kerr [East+13]
- IMR** waveforms based on **PN inspiral + self force + NR-informed ringdown** [Huerta+16]
- Ongoing work on eccentric IMR waveforms based on EOB/Phenom

