Modeling gravitational waves from compact-object binaries

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Introduction
Synergy of numerical relativity and analytical relativity = waveform models crucial for
1. establishing 5-sigma significance of detection [LVC1602.03839]
2. measuring properties of the source [LVC1602.03840, LVC1606.01210, LVC1606.01262]
3. performing tests of general relativity (GR) [PRL116 (2016) 221101]
Template-based online pipeline was needed to observe it

NR+AR waveforms as important for significance, parameter estimation, and tests of GR
Quasicircular binary black holes: numerical relativity
BBH coalescence as predicted by GR

**Intrinsic parameters: BH masses and BH spin vectors**
Numerical relativity catalogs of BBHs

... and many more NR waveforms [SXS, GATech, RIT, Cardiff-UIB, Cactus] (also generated for followup of LVC observations)
Challenging configurations

- **Longterm** BBH simulations at mass ratio 7 [Szilagyi+14, Kumar+15]

- **Almost extremal** BBH simulations: equal-mass, aligned-spins 0.99, 0.994 [Scheel+14]

- New initial data for challenging configurations [Ossokine+15]

<table>
<thead>
<tr>
<th>q</th>
<th>$\vec{\chi}_1$</th>
<th>$\vec{\chi}_2$</th>
<th>$D_0/M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,0,0.9999)</td>
<td>(0,0,0.9999)</td>
<td>14.17</td>
</tr>
<tr>
<td>3</td>
<td>(0,0.49,-0.755)</td>
<td>(0,0,0)</td>
<td>15.48</td>
</tr>
<tr>
<td>10</td>
<td>(0.815,-0.203,0.525)</td>
<td>(-0.087,0.619,0.647)</td>
<td>15.09</td>
</tr>
<tr>
<td>50</td>
<td>(-0.045,0.646,-0.695)</td>
<td>(0,0,0)</td>
<td>16</td>
</tr>
</tbody>
</table>
Direct use of numerical relativity waveforms

- Direct comparison of NR catalogs to observations

\[
\ln \mathcal{L}(\lambda; \theta) = -\frac{1}{2} \sum_k \langle h_k(\lambda, \theta) - d_k | h_k(\lambda, \theta) - d_k \rangle_k - \langle d_k | d_k \rangle_k 
\]

[LVC1602.03843] [LVC1606.01262]
Direct use of numerical relativity waveforms

- **NR followup to observations** [Lovelace+16]

- **Surrogate waveform models** [Blackman+15, (in prep)]
  1. restricted parameter space (high mass, $q \leq 2$, spins $\leq 0.8$, one spin aligned)
  2. many NR simulations to construct basis
  3. interpolation across NR runs
  4. they do not extrapolate to low mass: need models or long NR
Quasicircular binary black holes: models
How good is a model?

\[ \langle h_{\text{NR}}, h_{\text{model}} \rangle = 4 \text{Re} \int_{f_{\text{low}}}^{f_{\text{high}}} \frac{\tilde{h}_{\text{NR}}(f) \tilde{h}_{\text{model}}^*(f)}{S_n(f)} \, df \]

\[ \mathcal{O}(h_{\text{NR}}, h_{\text{model}}) = \frac{\langle h_{\text{NR}}, h_{\text{model}} \rangle}{\sqrt{\langle h_{\text{NR}}, h_{\text{NR}} \rangle \langle h_{\text{model}}, h_{\text{model}} \rangle}} \]

- **Template banks** accept 97% overlaps ~ 10% loss in event rate

- **Parameter estimation**: (sufficient) accuracy requirement
  
  \[ \mathcal{O}(h_{\text{NR}}, h_{\text{model}}) > 1 - \frac{1}{2 \text{SNR}^2} \]
Effective-one-body models of nonprecessing BBHs

- **Nonspinning case:** particle in deformation of Schwarzschild \([\text{Buonanno \\& Damour99}]\). **Spinning case:** *spinning particle in deformation of Kerr* \([\text{Barausse \\& Buonanno10,11; Nagar+14}]\)

- **Inspiral waveforms/radiation reaction** from *resummation post-Newtonian formulas* \([\text{Damour+07,09; Pan+11; Nagar+16}]\)

- **Ringdown from superposition of quasinormal modes** of remnant BH

\[
H_{\text{real}} = Mc^2 \sqrt{1 + 2\nu \left( \frac{H_{\text{eff}}}{\mu c^2} - 1 \right)} - Mc^2
\]

\[
A(R) \left[ 1 + \frac{P^2}{\mu^2 c^2} + \frac{1}{\mu^2 c^2} \left( \frac{A(R)}{D(R)} - 1 \right) \left( \frac{R \cdot P}{R} \right)^2 \right]^{\frac{1}{2}}
\]

\[
A = \sqrt{1 - 2u} + 2\nu u^3 + \left( \frac{94}{3} - \frac{42}{32} \pi^2 \right) \nu u^4 + a_5 u^5 + \cdots \quad (u = GM/Rc^2)
\]

\[
\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}
\]

example of tuning parameter
Effective-one-body models of nonprecessing BBHs

- Tuning to numerical-relativity simulations

![Graph showing gravitational-wave cycles with different calibration and NQC corrections.

- No calibration, no NQC corrections
- Calibration, no NQC corrections
- Calibration + NQC corrections

NR (blue) and EOB (red) simulations.
Effective-one-body model of nonprecessing BBHs for O1

- **SEOBNRv2** calibrated to better than 99% overlap with NR for design aLIGO [AT+14]

- Used in its **reduced-order-model** version [Pürrer14,15] in O1 for filtering and parameter estimation

- Similar set of calibration waveforms used in IHES models [Nagar +15,16]
Effective-one-body model of nonprecessing BBHs for O2

- **SEOBNRv4** [Bohe, Shao, AT+ (in prep)]

\[ \nu = \frac{m_1 m_2}{(m_1 + m_2)^2} \]

\[ \chi_{\text{eff}} = \left( \frac{S_1}{m_1} + \frac{S_2}{m_2} \right) \cdot \hat{L} \]

\[ \chi_A = \left( \frac{S_1}{m_1^2} + \frac{S_2}{m_2^2} \right) \cdot \hat{L} \]

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Design aLIGO

\[ 1 - O(h_{\text{NR}}, h_{\text{EOB}}) \]

\[ M/M_\odot \]

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Phenomenological model of nonprecessing BBHs

- Fit to **hybrids** of uncalibrated EOB and NR [Husa+15, Khan+15]

\[ \phi_{\text{Ins}} = \phi_{\text{TF}2}(M_f; \Xi) \]
\[ + \frac{1}{\eta} \left( \sigma_0 + \sigma_1 f + \frac{3}{4} \sigma_2 f^{4/3} + \frac{3}{5} \sigma_3 f^{5/3} + \frac{1}{2} \sigma_4 f^2 \right) \]
\[ \phi_{\text{Int}} = \frac{1}{\eta} \left( \beta_0 + \beta_1 f + \beta_2 \log(f) - \frac{\beta_3}{3} f^{-3} \right) \]
\[ \phi_{\text{MR}} = \frac{1}{\eta} \left\{ \alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3} \alpha_3 f^{3/4} \right. \]
\[ + \left. \alpha_4 \tan^{-1}\left( \frac{f - \alpha_5 f_{\text{RD}}}{f_{\text{damp}}} \right) \right\} . \]
Comparing nonprecessing BBH models

\[ O(h_1, h_2) \]

\( O(h_1, h_2) \)
maximized over masses and spins (in template bank)

[Bohe, Shao, AT+(in prep)]

(O1 aLIGO)
Comparing nonprecessing BBH models

new, long numerical-relativity simulations are needed

\[ \mathcal{O}(h_1, h_2) \]

maximized over masses and spins (in template bank)

\[ \mathcal{O}(h_1, h_2) \]

[Bohe, Shao, AT+(in prep)]

(O1 aLIGO)
When BH spins are not parallel to angular momentum of the binary, the orbital plane precesses

**Precessing frame** [Buonanno+03, Schmidt+11, O’Shaughnessy+11, Boyle+11]

1. In precessing frame, use calibrated nonprecessing model
2. Inertial-frame modes from rotation of precessing-frame modes according to motion of orbital angular momentum

Both effective-one-body [Pan+13, Babak, AT, Buonanno16] and phenomenological [Hannam+13] models available
Effective-one-body model for precessing BBHs

$1 - \mathcal{O}(h_{\text{NR}}, h_{\text{EOB}})$ averaged over sky location and polarization

[Babak, AT, Buonanno16]
Different models vs GW150914

- Nonprecessing EOB, precessing EOB, and precessing Phenom measure consistent parameters for GW150914
  1. SNR
  2. comparable mass
  3. face off/on
  4. short signal

[LVC1606.01262]
Quasicircular binary black holes: open problems
Open problems for quasicircular BBH models

- Problem of extrapolation outside calibration domains, i.e., high mass ratios, spins
- IMR higher-order modes for spinning binaries are not available

PN+NR with HM vs SEOBNRv1 bank

[Caponi+13]

[Calderon-Bustillo+15]

[LVC(in prep)]
Open problems for quasicircular BBH models

- **Precessional effects** not fully modeled
  1. mode asymmetry in coprecessing frame [O’Shaughnessy+13, Pekowsky+14, Boyle+14]
  2. radiation axis keeps precessing during ringdown [O’Shaughnessy+13]
  3. no calibration to precessing NR

![Graph showing precessional effects](image)

- q=1, chi1=0.5 in plane, chi2=0
  
  ![Graph with two plots](image)
Binary neutron stars
BNS in numerical relativity

- **Longer simulations** with polytropic EOS: SACRA longterm simulations [Hotokezaka et al. 2015], 22 orbits in SpEC [Haas et al. 2016]

- Evolutions with *spin precession* [Tacik et al. 2015, Dietrich et al. 2015], *unequal mass* [Lehner et al. 2015, Dietrich et al. 2016], *more physics* (neutrino cooling, nuclear EOS, magnetic fields) [Foucart et al. 2015, Palenzuela et al. 2015, Endrizzi et al. 2016]

- New schemes that allow *smaller errors* [Bernuzzi et al. 2016]
Why modeling BNS waveforms is important

- Templates that are good for detecting BNSs can create large biases in measurement of tidal parameters [Yagi+14, Favata+14, Wade+14, Hotokezaka+16]

\[
\Lambda = \frac{8}{13} \left[ (1 + 7\eta - 31\eta^2) (\Lambda_1 + \Lambda_2) + \sqrt{1 - 4\eta (1 + 9\eta - 11\eta^2)} (\Lambda_1 - \Lambda_2) \right]
\]
Models of inspiraling BNSs

- Splicing long NR BBH with PN tidal effects [Barkett+15]

- Augment EOB potentials by tidal effects: (i) **gravitational self-force** [Bernuzzi+15], (ii) **dynamic tides** [Hinderer+16]
Models of the post-merger signal

- If $M<2.9\text{M}_\odot$, hypermassive NS forms after BNS merger

- **Peak frequencies** correlate with radius at fiducial mass, compactness, etc., in an EOS-independent way [Bauswein+12, Hotokezaka+13, Takami+14, Bernuzzi+14]
Neutron-star / black-hole binaries
Neutron star / black hole binaries

- **Long NSBHs**: small errors [Foucart+15], with precession [Kawaguchi+15]
- Model for disruption frequency and freq-domain amplitude model [Pannarale+15]
- EOB model with dynamical tides [Hinderer+16]
Conclusions
Conclusions

- **Binary black holes**
  1. many new NR runs (calibration, surrogates, direct use)
  2. challenging configurations becoming feasible
  3. models include info from NR catalogs
  4. towards complete IMR models with eccentricity

- **Binary neutron stars & neutron-star / black-hole binaries**
  1. longterm accurate NR runs
  2. inclusion of tidal effects in accurate point-mass models
  3. universality relations for postmerger
  4. models of disruption frequency

- **Numerical + analytical relativity crucial for best characterization of future GW observations**
Additional slides
Eccentric binaries
Eccentric binaries

- **Dynamical formation** scenarios instead of field binary evolution

- Searches for BNS using quasicircular templates ok for $e<=0.02$ ($M=2.6\text{Msun}$) \cite{Huerta+13}

- Small residual eccentricity can **bias** parameter estimation \cite{Favata14}
  
  residual eccentricity @ ISCO
  
  ($e=0.4$ @ 10Hz)

\[\text{residual eccentricity @ ISCO} \]

\[e=0.4 @ 10\text{Hz}\]
Eccentric binaries

- Frequency/time-domain PN inspiral waveforms [Arun+09, Yunes+09, Huerta+14, Tanay+16]. Small-ecc corrections up to 3PN [Moore,Favata+16]

- IMR waveforms based on geodesic motion in Kerr [East+13]

- IMR waveforms based on PN inspiral + self force + NR-informed ringdown [Huerta+16]

- Ongoing work on eccentric IMR waveforms based on EOB/Phenom