Digital Processing with Focus onto Neutron Detection

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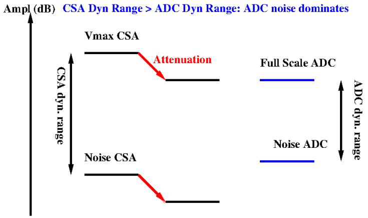
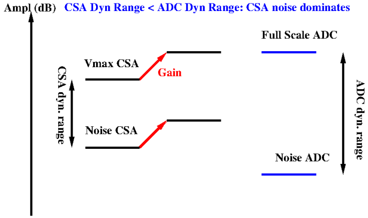
Second Lesson (2016-10-26)

Timing as a study case

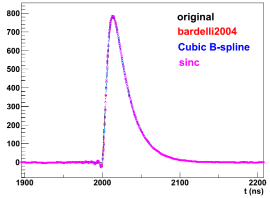
Remind from first lesson

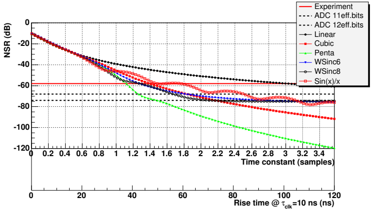
- ADCs can add noise to your signal ($\propto 2^{N-\mathrm{ENOB}}$)





- Signal reconstruction can add artifacts and "noise" for fast transients ($\leq \mathrm{Kernel\,Lenght} \times T_s$)





Timing with LED

- Timing: extracting a "time mark" from a signal, e.g. with a leading edge discriminator (LED);

Timing with LED

- Timing: extracting a “time mark” from a signal, e.g. with a leading edge discriminator (LED);
- LED: device emitting a logic “true” signal when input voltage crosses a fixed threshold (e.g. oscilloscope trigger)

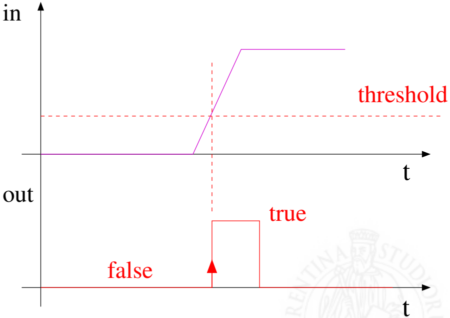


Figure 4: Leading edge discriminator

LED and *amplitude walk*

In a LED, threshold crossing depends on amplitude for a fixed risetime. Reason: threshold is fixed.

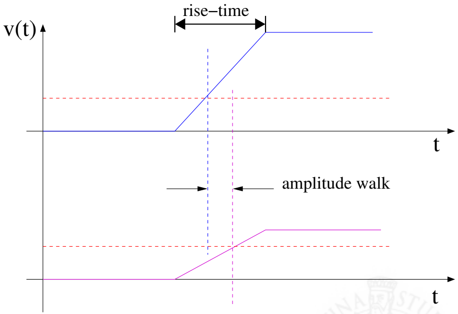
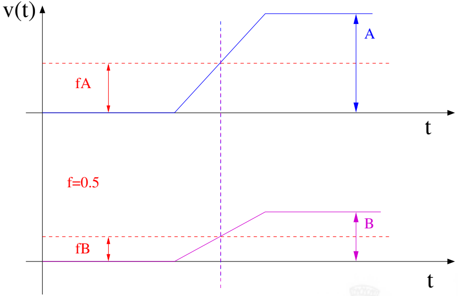


Figure 5: Amplitude walk of a LED.

Constant Fraction Discrimination

- a Constant Fraction Discriminator acts as if its threshold could move dynamically: threshold is a fixed fraction *f* of full amplitude;



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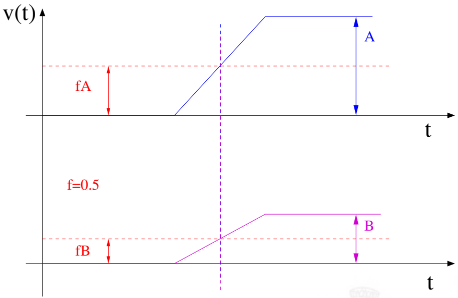
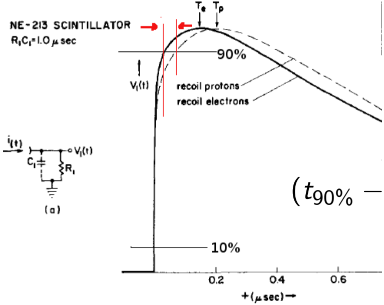


Figure 6: Constant Fraction Discriminator principle.

- amplitude walk reduced (eliminated exactly for a linear rising edge)

CFD and PSD

CFD useful also in Pulse Shape Discrimination: NE-213 anode current signal integrated on RC parallel $\Longrightarrow$ the slower component of a proton signal (i.e. neutron detected) is associated to a longer risetime with respect to electron signal (i.e. gamma detected)

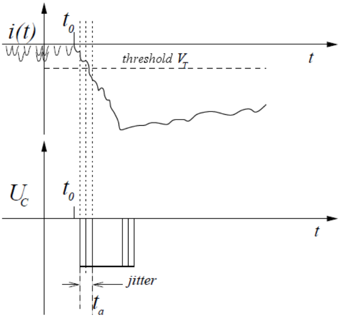


$$(t_{90\%} - t_{10\%})_{\mathrm{p}} > (t_{90\%} - t_{10\%})_{\mathrm{e}}$$

Figure 7: PSD from risetime (adapted from [Roush1964]).

Timing and noise: jitter

noise fluctuations affect signal $\Longrightarrow$ time mark fluctuates around average



$\Longrightarrow$ *jitter*: statistical time-mark fluctuations

Jitter: a simple model

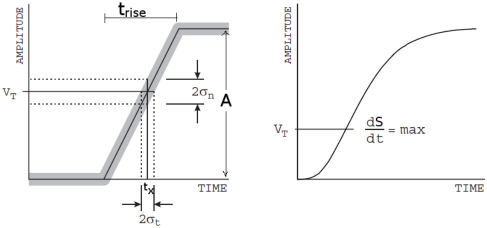


Figure 8: Noise and jitter (adapted from [Spieler2005])

- σ_n std. dev. **amplitude** fluctuations → "noise band" $2\sigma_n$ wide

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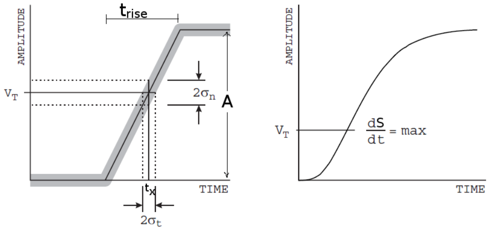

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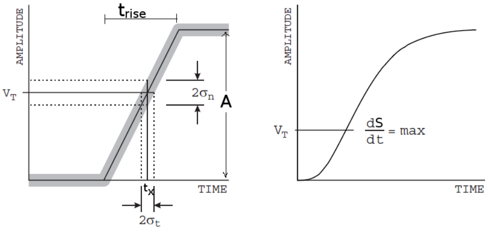

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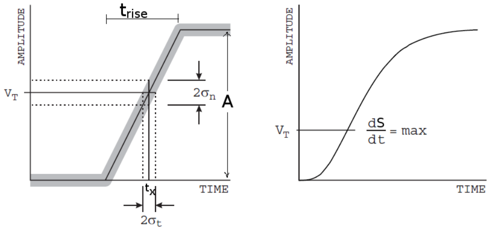

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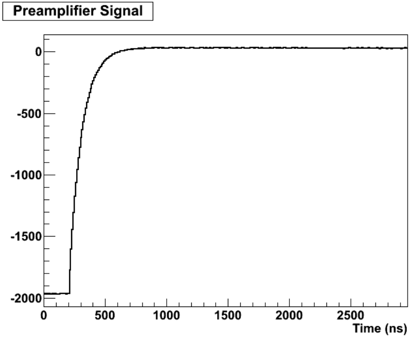
- we put threshold where $|dS/dt|$ is max $\Longrightarrow \sigma_t$ minimum
- linear signal front: $\left|\dfrac{dS}{dt}\right| = \dfrac{A}{t_{rise}} \Longrightarrow$

$$\sigma_t = \frac{\sigma_n t_{rise}}{A} \propto \frac{t_{rise}}{\mathrm{SNR}} \tag{3}$$

A digital-CFD (dCFD)

CFD procedure for a "tail" signal (e.g. from charge preamp):

1) apply pole-zero cancellation + integration
to get rid of tail



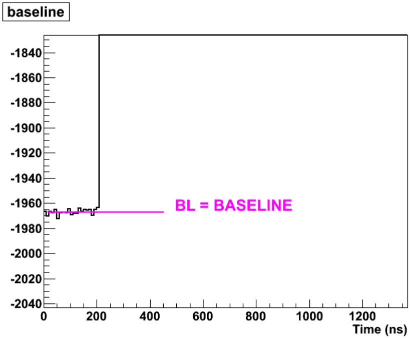
Please note:

1. the time axis unit is ns;
2. original (not interpolated) signal has $T_s = 10\,\mathrm{ns}$.

A digital-CFD (dCFD)

CFD procedure for a "tail" signal (e.g. from charge preamp):

2) calculate the baseline BL (e.g. averaging flat part: also consider noise autocorrelation, e.g. when calculating rise-time)



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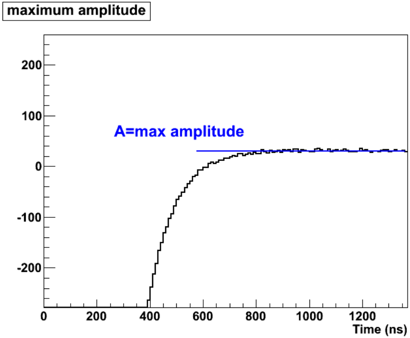
CFD procedure for a “tail” signal (e.g. from charge preamp):

3) calculate max amplitude A (samples average or amplitude of unit gain shaper); step amplitude $= A - BL$



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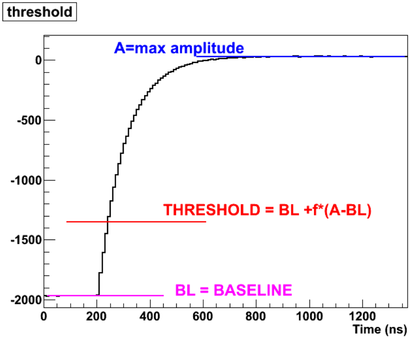
CFD procedure for a "tail" signal (e.g. from charge preamp):

4) calculate dynamic threshold as

$T = BL + f\,(A - BL)$



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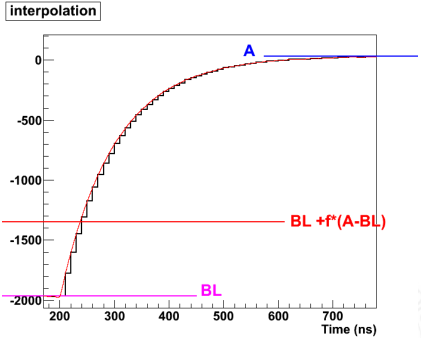
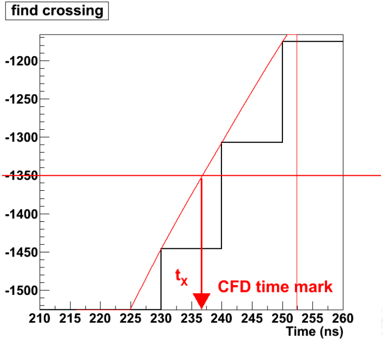
CFD procedure for a “tail” signal (e.g. from charge preamp):
5) apply interpolation (whole signal shown...
in real-life region around threshold is enough)



A digital-CFD (dCFD)

CFD procedure for a "tail" signal (e.g. from charge preamp):

6) time mark = intersection interpolation-threshold
(find it iteratively in complex cases)



Intersection time t_X is in units of T_s (fraction of the sampling period). Time in seconds from first sample $= t_X \cdot T_s$. If $x[n]$ last sample before t_X then $0 < t_x - n < 1$ (in this example, $n = 23$ $t_X \sim 23.68$).

t-measurement: Sampling ADC vs Analog

Effects affecting resolution of digital timing:

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- detector signals have wide frequency bandwidth (*wideband* signals) $\Longrightarrow$ signal reconstruction from samples affected by interpolation errors $\Longrightarrow$ timing affected by interpolation "noise" (an effect not present in analog chains)

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- NB: analog CFD similar formula except: equal sign, no ADC noise, a factor $\sqrt{1+f^2}$

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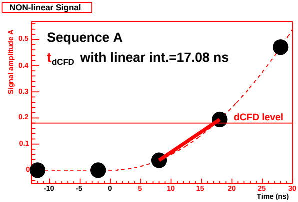

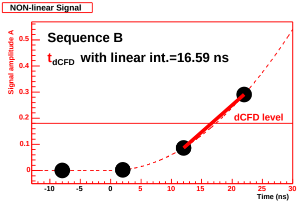

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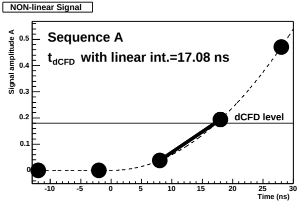

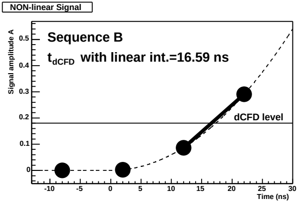

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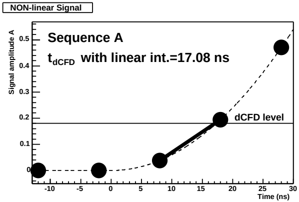

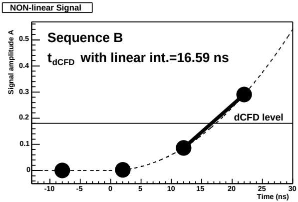

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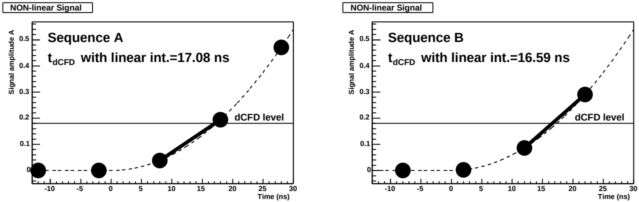

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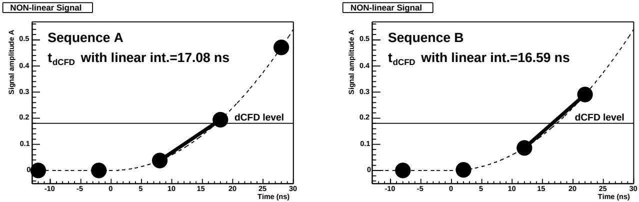
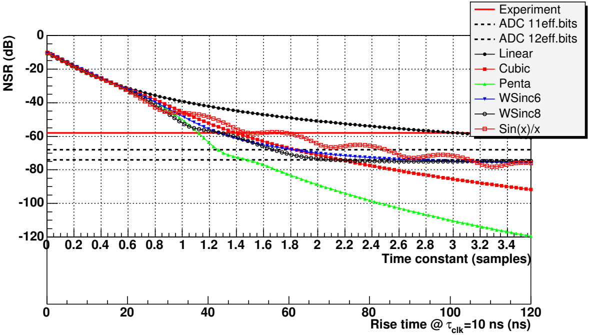


- non linear front $\Longrightarrow$ reconstruction not perfect
- for a fixed signal shape, t_x depends on **where** samples are taken
- i.e. on phase of sampling clock w/ respect to signal front
- will happen anyway w/ other kernels (not BW limited signal)

Questions about interpolation "noise"

- effect of interpolation different for linear and cubic;
- we know there are many kernels available...
- which kernel is the "best" one?
- for a given T_s what is the minimum risetime safe from interpolation noise?
- from the previous lesson:



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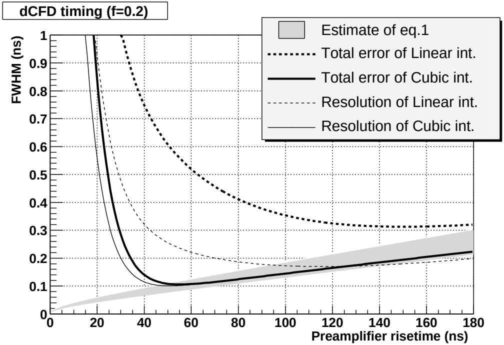
- same procedure employed for simulation of interpolation noise;
- random noise added to each signal (noise standard deviation constant for all signals);
- noise variance and spectrum depends on two contributions: the simulated front-end electronics bandwidth (σ_e in eq. (4)) and the simulated ADC noise, derived from ENOB ($\sigma_q = \frac{1}{\sqrt{12} \cdot 2^{\mathrm{ENOB}}}$ in eq. (4)).

dCFD simulation: 12 bit, 10.8 ENOB, 100 MHz ADC

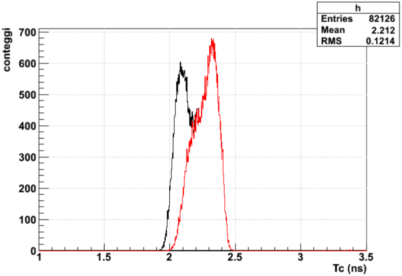
- FWHM of t_x spectrum vs signal risetime [Bardelli2004]:



- cubic interpolation much better than linear: min{FWHM}=100 ps!
- $t = 0$ known $\Longrightarrow$ fluctuations due to t_x determination only
- $t_{rise} > 60\,\text{ns} \Longrightarrow \text{FWHM} \propto t_r$ (SNR constant!) (cfr. eq. (3));
- FWHM increases rapidly as risetime decreases under 60 ns.

Interpolation artifacts: double coincidence peak

- When interpolation dominates resolution strange artifacts appear;
- Example: experimental data (time coincidence between two Si detectors exposed to diffused UV pulsed laser) [Pastore2013]:



- rise-time less than $4T_s$; cubic interp. (4 consecutive samples)
- coincidence peak not gaussian; left peak: signals for which first (out of 4) interpolation node (sample) lies on baseline; right peak: signals for which first node already above baseline.

Questions about ADC's

- fast signals (characteristic times $\leq 3 \div 4\ T_s$): interpolation affects FWHM;
- the faster the ADC the better? buy the ADCs with highest F_s?
- remember ENOB? lower ENOB $\Longrightarrow$ more time jitter;
- in real ADCS, high ENOB and high F_s are conflicting requirements.

Timing measurement: simulation of different ADC's

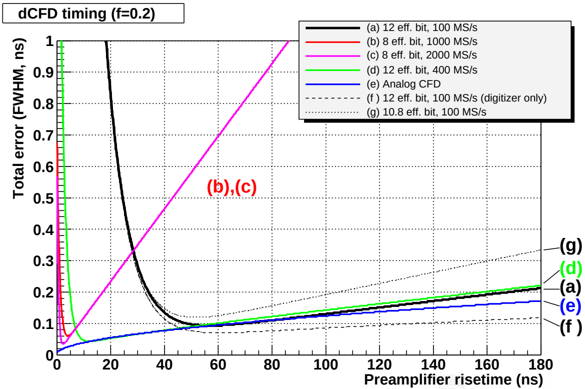


Figure 9: Time resolution (FWHM) for different ENOB/F_s combinations vs charge preamp risetime [Bardelli2004]. Cubic interpolation used.

high sampling rate and high ENOB: conflicting requirements

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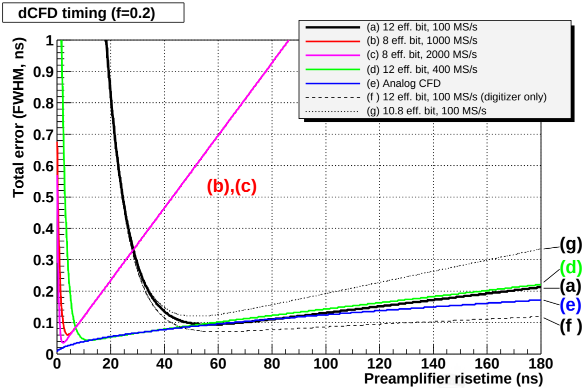

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we use the analog CFD curve (curve e), in blue) as reference

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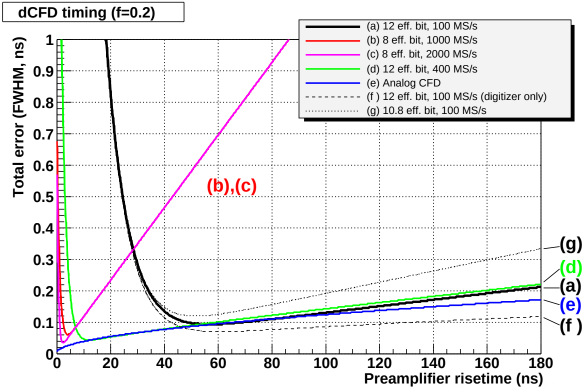

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risetime >60ns: ENOB= 12 (a, d, f) ≈ analog CFD at 400 **and** 100 MS/s

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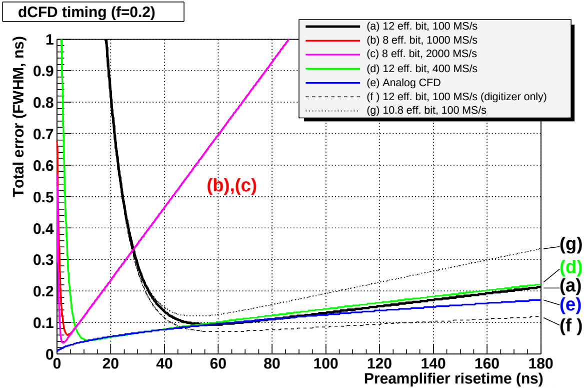

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ENOB$=8$ at 1 GS/s too noisy! far from analog (except for rise-time $\sim 2 \div 3$ ns); worse than 12 ENOB at 100 MS/s for rise-time > 30 ns.

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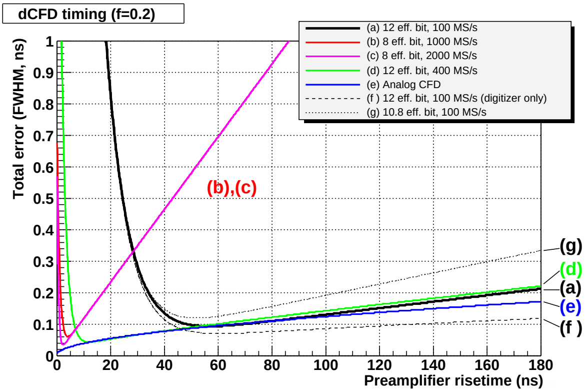

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risetime $\approx 60\,\mathrm{ns}$: even ENOB=10.8 100 MS/s comes close to analog ENOB= 12, $T_s = 10\,\mathrm{ns} \Longrightarrow \min\{\mathrm{FWHM}\} = 100 \div 200\,\mathrm{ps}!$

Timing measurement: simulation of different ADC's

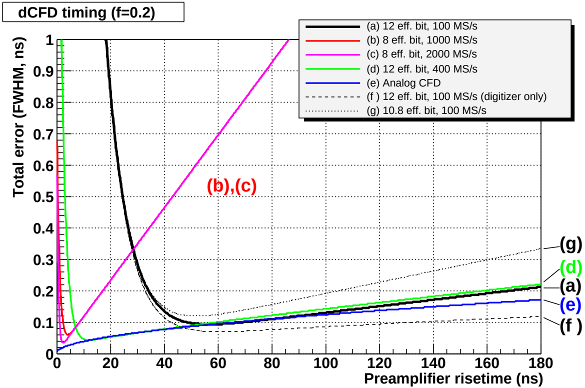


Figure 9: Time resolution (FWHM) for different ENOB/F_s combinations vs charge preamp risetime [Bardelli2004]. Cubic interpolation used.

risetime <60 ns: at 100 MS/s interpolation dominates! $\Longrightarrow$ $F_s = 100\,\mathrm{MS/s}$ not enough

Timing measurement: simulation of different ADC's

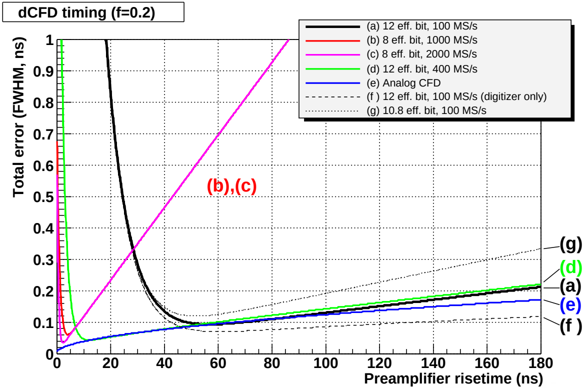
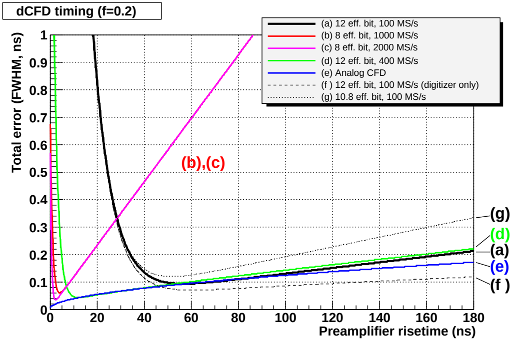


Figure 9: Time resolution (FWHM) for different ENOB/F_s combinations vs charge preamp risetime [Bardelli2004]. Cubic interpolation used.

N.B. (ENOB$=$ 12 F_s $=400\,\mathrm{MS/s}$) better than (ENOB$=$ 8 F_s $= 1 \div 2\,\mathrm{GHz}$) down to risetime$=$ 7 ns!

Final message



enough samples on front (about 4÷5)

$\Longrightarrow$ better high ENOB than high F_s

Time resolution and PSD in Si detectors

- FAZIA (Four π A Z Identification Array) collaboration;
- charge (Z) id of nuclei stopped in 300 μm thick Si;

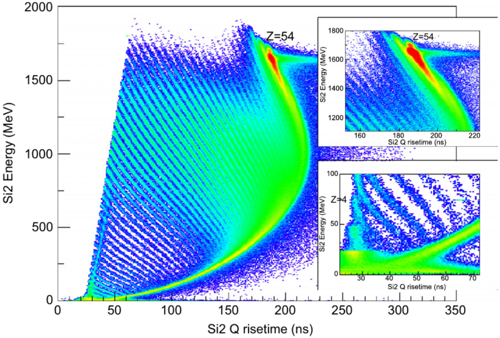
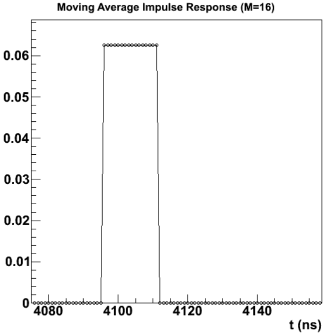


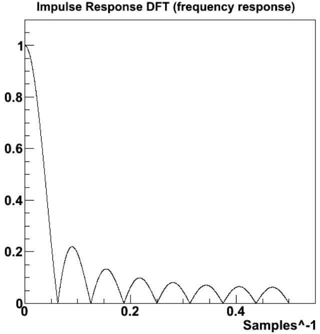
Figure 10: "Si-Energy vs Charge rise-time" (from [Carboni2012]).

- elements from Z=2 to Z=54 are resolved;
- risetimes from 20 to 220 ns $\Longrightarrow$ Z id possible thanks to $\approx$100 ps resolution. (ADC is 14 bit, 100 MS/s, digitizer ENOB=11.2);

Moving average, a simple Low Pass filter

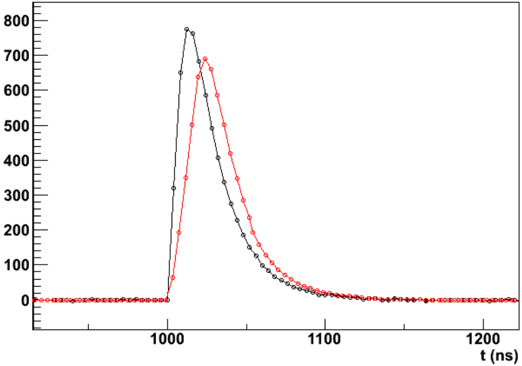
Causal mov. average of M samples from $x[n-M+1]$ to $x[n]$
Convolution: $y[n] = \frac{1}{M}\sum_{i=0}^{M-1} x[n-i]$
Also recursion works: $y[n] = y[n-1] + \frac{1}{M}(x[n] - x[n-M])$
Frequency response: Low Pass Filter





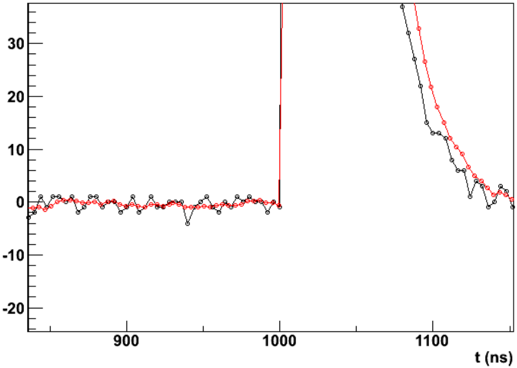
Moving average, a simple Low Pass filter

Effect of moving average on a detector pulse. The processed signal is in red. Transients are slowed down (low-pass!).



Moving average, a simple Low Pass filter

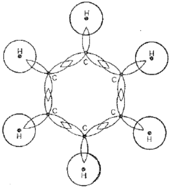
The same picture expanded to show how the noise on the baseline is reduced by the moving average.



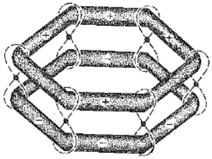
Application to n/γ PSD

n/γ PSD: introduction

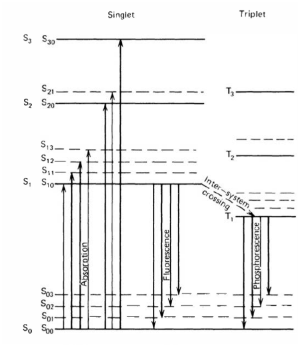
- liquid organic scintill. (e.g. BC501), cyclic aromatic compounds



The σ-hybrid orbitals of the carbon atoms of benzene (Coulson, 1952).



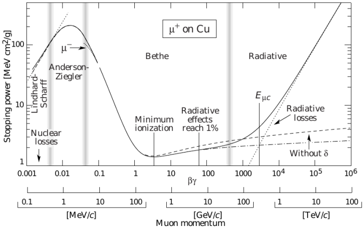
The π-molecular orbitals in benzene (Coulson, 1952).



- scintillation emitted by excited molecules featuring π level structure
- emission involving only singlet states $\Longrightarrow$ shorter emission time
- emission through triplet states $\Longrightarrow$ longer emission time

n/γ PSD: introduction

- density of triplet states along particle track affects overall emission time
- remind: γ must transfer energy to an electron, neutron to a proton
- density of triplet states greater where greater specific energy loss:



- take 1 MeV kinetic energy: then $(\beta\gamma)_{electron} = 2.8$ and $(\beta\gamma)_{proton} = 4.5\ 10^{-2}$
- much higher density for p $\Longrightarrow$ longer emission time ("tail" in signal).

PSD with charge comparison 1

- two integrations, usually *slow* (a.k.a. *tail*) and *total*

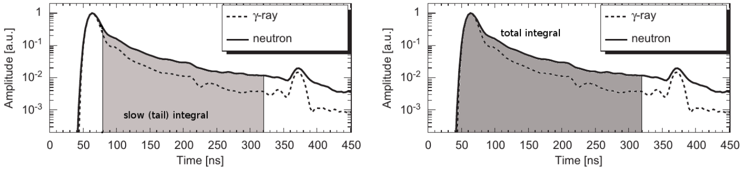


Figure 11: Slow and total integral (adapted from [Söderstrom2008]).

PSD with charge comparison 1

- two integrations, usually *slow* (a.k.a. *tail*) and *total*

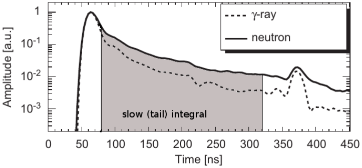
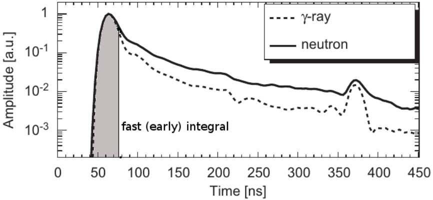


Figure 11: Slow and total integral (adapted from [Söderstrom2008]).

- sometimes *fast* (a.k.a. *early*) and *total*



PSD with charge comparison 2

- most used PSD method (see ref. table at the end);

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  1. to determine the time mark reference for integral start (either with a LED or CFD algorithm or using interpolation to find “real” maximum);

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  2. to evaluate integrals starting/ending “in between samples” (most often previous point will take you in between);

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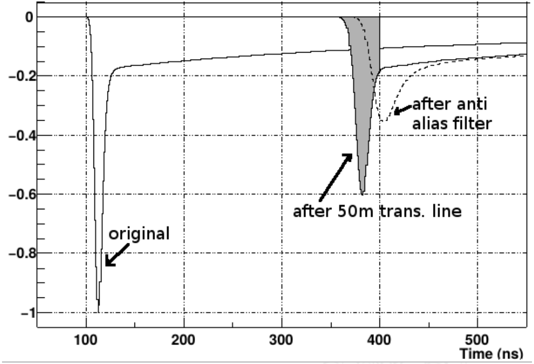
- most used PSD method (see ref. table at the end);
- baseline statistical uncertainty: keep it below other causes of uncertainty (use enough samples for average), see [Bardelli2006].
- interpolation: from what we have learnt, we can exploit it:
  1. to determine the time mark reference for integral start (either with a LED or CFD algorithm or using interpolation to find "real" maximum);
  2. to evaluate integrals starting/ending "in between samples" (most often previous point will take you in between);
- really consider 2) if $\Delta t \approx T_s$ ($\Delta t \gg T_s$: it is OK to just sum samples);

PSD with charge comparison 3

- antialiasing filter could slow down first part $\Longrightarrow$ increase Δt of *fast* with respect to analog FEE (part of fig.1 in [Bardelli2002]);



PSD with charge comparison 4

- to minimize ADC noise fluctuations, *fast* (shorter) could be better than *slow* (longer). If $s[i]$ is signal and $n[i]$ is noise

$$\mathrm{V}\left\{\sum_{i=1}^{M}(s[i] + n[i])\right\} = \mathrm{V}\left\{\sum_{i=1}^{M} s[i]\right\} + \mathrm{V}\left\{\sum_{i=1}^{M} n[i]\right\} =$$

$$= \mathrm{V}\left\{\sum_{i=1}^{M} s[i]\right\} + M\,\mathrm{V}\left\{n[i]\right\}$$

where $\mathrm{V}\left\{\cdot\right\}$ is variance operator and we assume same noise variance on all samples $\Longrightarrow$ noise contribution $\propto M$;

PSD with charge comparison 4

- more complex weigthing function $w(t)$ than "rectangular gated" integral can be used [Gatti1962, Söderstrom2008]
- the optimal is very close to rectangular anyway:

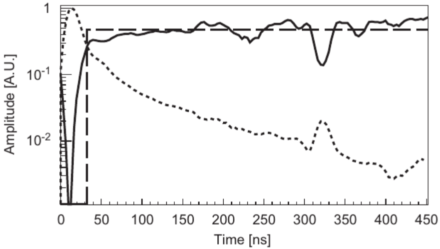


Figure 12: Optimal weigthing function (solid) and rectangular slow integral (dashed). An average neutron signal shape is also shown, from [Söderstrom2008].

PSD: zero crossing and risetime

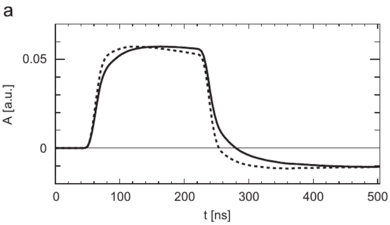
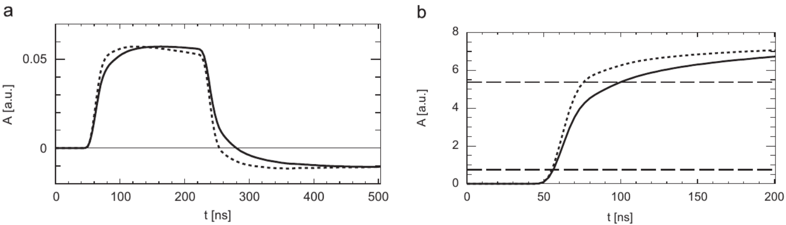


Figure 13: Zero crossing and risetime methods, from [Södestrom2008].

- zero-crossing signal obtained differentiating charge signal (e.g. bipolar DL shaping, usually need also low-pass: mov. average);

PSD: zero crossing and risetime



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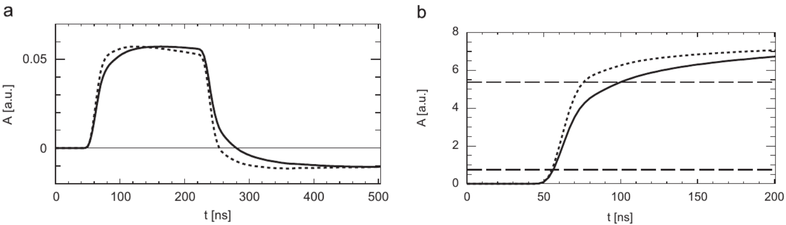
PSD: zero crossing and risetime



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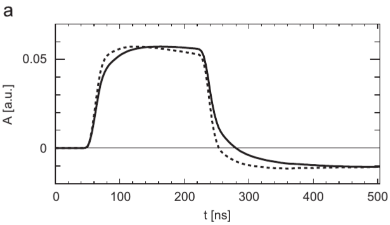
- risetime: time for amplitude to go from, e.g., 10% to 90% of maximum;

PSD: zero crossing and risetime



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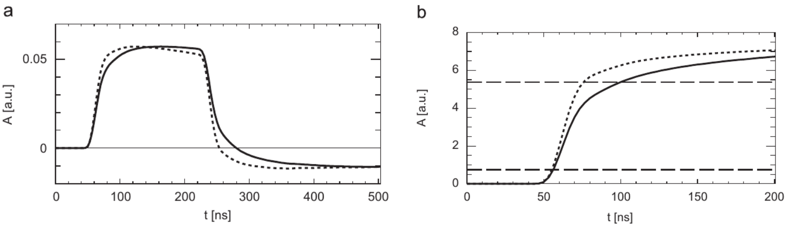
PSD: zero crossing and risetime



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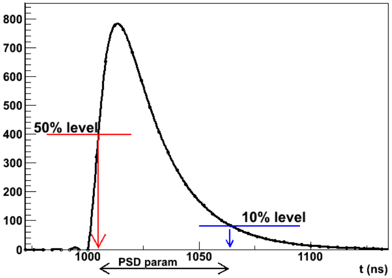
- strictly related: zc-time $\leftrightarrow$ time of zero derivative $\leftrightarrow$ time of max;
- most used, together with charge comparison;

PSD: Time over Threshold and Q-Risetime

- relevance of interpolation for precise time mark evaluation (both $t = 0$ mark and zero crossing);
- risetime: digital integration+interpolation based dCFD algorithm;
- risetime equivalent: “time over threshold”;

Basic principle of Time over Threshold



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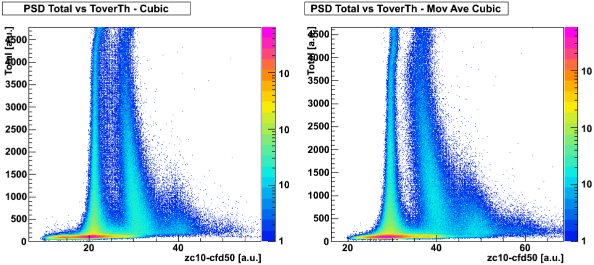
BC501 sampled with 12 bit, 250 MSPS, 10.5 ENOB, Am-Be source, Time Over Threshold



Cubic interpolation: moving average helps getting better separation.

PSD: Time over Threshold and Q-Risetime

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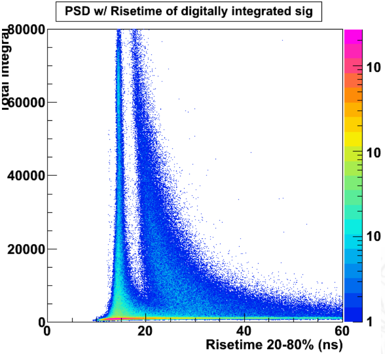
BC501 12 bit etc., Am-Be source, Charge Risetime



Cubic dCFD at 20 and 80% to get t_{rise} of integrated PMT signal.

PSD with reference shapes (a.k.a. NGMA)

- exploits “reference” shapes;

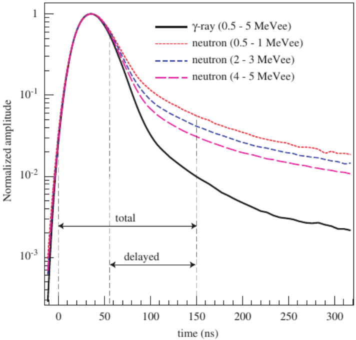
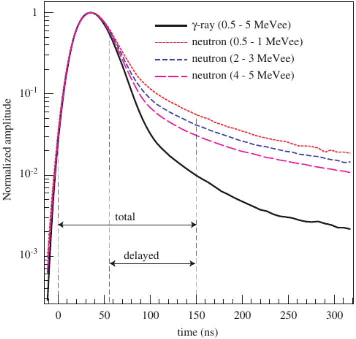


Figure 14: Reference signal shapes, from [Guerrero2008]. Note the energy dependence.

PSD with reference shapes (a.k.a. NGMA)

- exploits "reference" shapes;
- compares digitized signal to reference, a "similarity" parameter is extracted (e.g. $\sum_i (S[i] - \mathcal{S}_{ref}[i])^2$, etc.);



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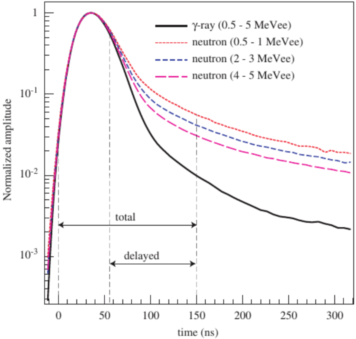
- "most similar" type is assigned;



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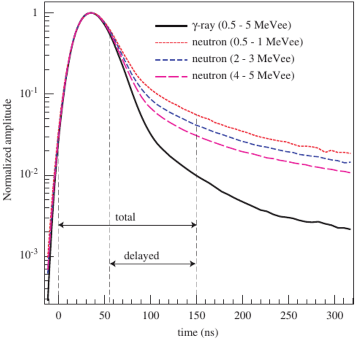
- reference shapes: averages over thousands of digitized signals



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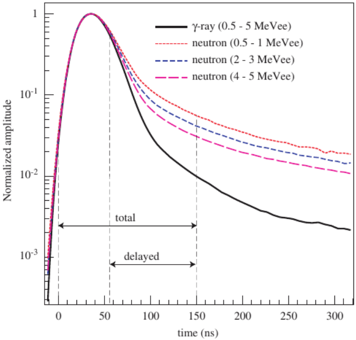
- asynchronous sampling clock $\Longrightarrow$ carefully align shapes before averaging



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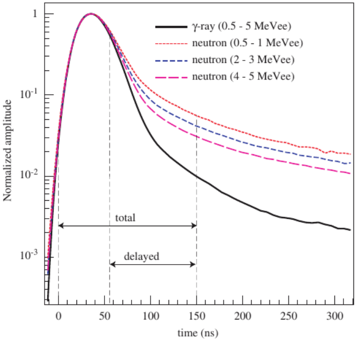
- interpolation can help:



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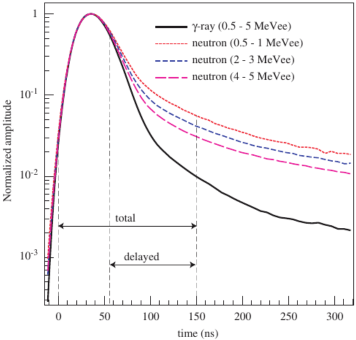
  1. evaluating real start of the signal (dCFD);



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- "most similar" type is assigned;
- reference shapes: averages over thousands of digitized signals
- asynchronous sampling clock $\Longrightarrow$ carefully align shapes before averaging
- interpolation can help:
  1. evaluating real start of the signal (dCFD);
  2. calculating samples "in between" $\Longrightarrow$ "oversampled" shapes can be aligned with better precision;

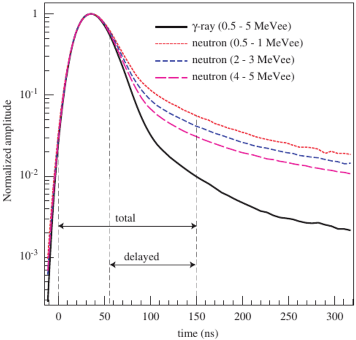
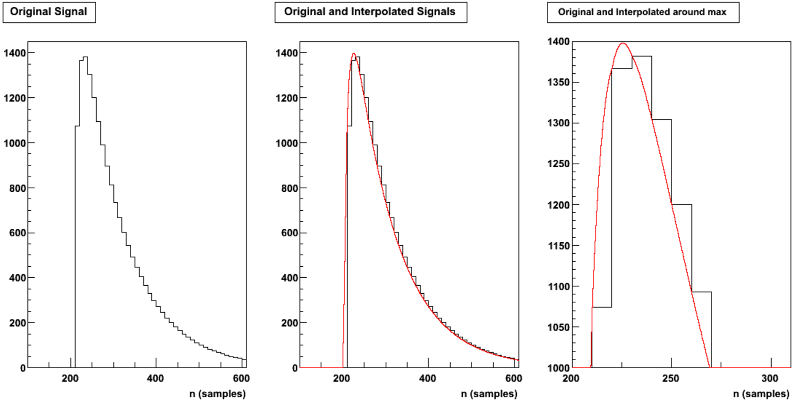


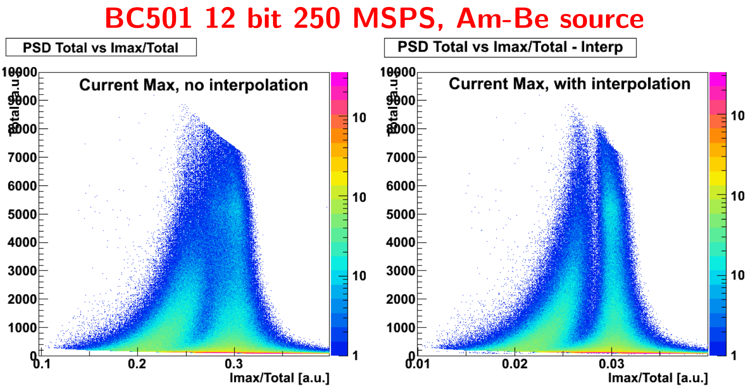
Figure 14: Reference signal shapes, from [Guerrero2008]. Note the energy dependence.

PSD: current maximum

Maximum of current signal (at a given energy) depends on signal duration. In [Cavallaro2013] it is implemented with analog electronics. Digital signals: interpolation critical to get real maximum!



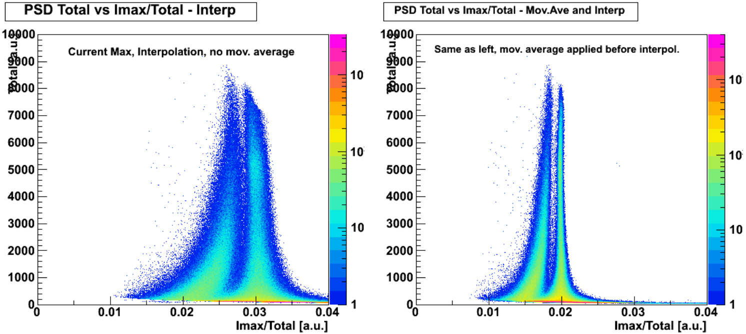
PSD w/ BC501: Current Maximum



Comparing left to right: beneficial effect of interpolation (Imax).

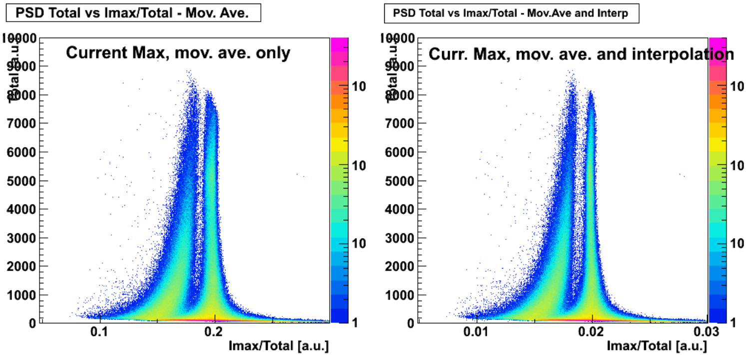
PSD w/ BC501: Current Maximum

BC501 12 bit 250 MSPS, Am-Be source



Comparing left to right: beneficial effect of moving average w/ interp.

PSD w/ BC501: Current Maximum



Comparing left to right: with mov. ave. you get "almost" there...

PSD: Pulse Gradient Analysis [D'Mellow2007]

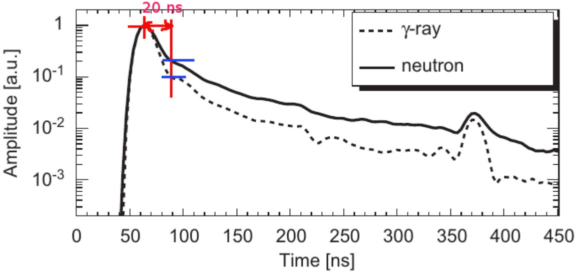


Figure 15: Principle of PGA according to [D'Mellow2007] (picture taken from [Söderstrom2008]).

- normalized shape; PSD param.=amplitude at Δt after max;
- interpolation: both peak determination and amplitude after Δt;
- "smoothing" needed to reduce noise/fluctuations (method relies on a single amplitude, there is no intrinsic averaging).

Selected n-γ PSD literature (1)

	Scint.	Analog	Digital	ADC	Ref.
Adams1978	NE213	CC			NIM 156(1978)459
Alexander1961	NE213, UGLLT	ZC			NIM 13(1961)244
Ambers2011	EJ-309		CC+NGMA	12bit/250MHz	NIM A638(2011)116
Barnabà1998	BC501A	ZC			NIM A410(1998)220
Bell1981	NE213	CC			NIM 188(1981)105
Cao1988	NE213	ZC			NIM A416(1988)32
Cavallaro2013	NE213	IMAX			NIM A700(2013)65
Ĉerny2004	BC501	CC			NIM A527(2004)512
Cester2013	EJ-309		CC	10bit/1GHz	NIM A719(2013)81
Cester2014	EJ-299-33		CC	12bit/250MHz	NIM A735(2014)202
D'Mellow2007	EJ301		CC, PGA	10bit/250MHz	NIM A578(2007)191
Esposito2004	stil, NE213		CC	12bit/200MHz	NIM A518(2004)626
Flaska2007	BC-501A		CC	8bit/5GHz	NIM A577(2007)654
Flaska2009	BC-253A		CC	12bit/250MHz	NIM A599(2009)221
Flaska2013	EJ-309		CC	10÷14bit/0.25÷2GHz	NIM A729(2013)456
Gamage2011	BC501A		PGA,CC,NGMA,SD	12bit/500MHz	NIM A642(2011)78
Guerrero2008	BC501A		NGMA	8bit/1GHz	NIM A597(2008)212
Hawkes2013	cust. plast.		shape study	8bit/2.5GHz	NIM A729(2013)522
Hellesen2013	BC400, NE213		CC	12bit/2GHz	NIM A720(2013)135
Heltsley1988	NE213	CC			NIM A263(1988)441
Kaplan2013	EJ309		CC	12bit/250MHz	NIM A729(2013)463
Kaschuck2005	ant,stil,NE213		CC	12bit/200MHz	NIM A551(2005)420

Selected n-γ PSD literature (2)

	Scint.	Analog	Digital	ADC	Ref.
Kalyna1970	NE213	ZC			NIM 88(1970)277
Jastaniah2002	BC523A		RT,ToT	8bit/500MHz	NIM A517(2004)202
Jhingan2008	BC501	CC			NIM A585(2008)165
Pai1989	NE213	ZC			NIM A278(1989)749
Pawelzak2013	EJ309		CC	12bit/200MHz	NIM A711(2013)21
Savran2010	BC501A		CC, NGMA	12bit/500MHz	NIM A624(2010)675
Söderstrom2008	BC501	ZC	WCC, ZC, CC	14bit/100MHz	NIM A594(2008)79
Wolski1995	BC501A	ZC, CC			NIM A360(1995)584
Nakhostin2010	NE213		ZC	8bit/1GHz	NIM A621(2010)498
Roush1964	NE213	ZC			NIM 31(1964)112
Söderstrom2008	BC501	ZC	ZC, CC	14bit/100MHz	NIM A594(2008)79
Stevanato2012	LaBr(Ce)	CC	CC	12bit/250MHz	NIM A678(2012)83
Yousefi2009	phoswich for β/γ disc.		wavelets	12bit/100MHz	NIM A599(2009)66
Zaitseva2012	cust. plast.		CC	14bit/200MHz	NIM A668(2012)88

CC=charge comparison
WCC=weigthed charge comparison (see Gatti1962)
ZC=zero crossing
ToT=Time over threshold
NGMA=neutron gamma model analysis (a.k.a true shape)
PGA=pulse gradient analysis
SD=simplified digital charge collection
RT=rise time
IMAX=maximum of current (anode) signal

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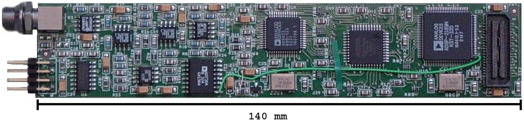
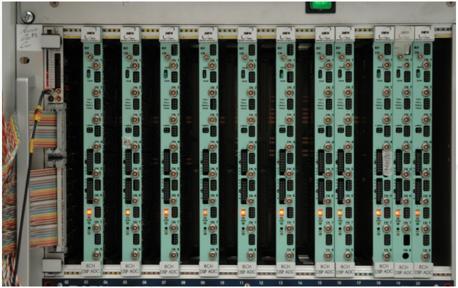
- we can learn from > years experience in imaging and telecommunication (cubic convolution, splines, smoothing splines, wavelets,...)... literature is rich on this topic (some papers in the references of these lessons).
- Sometimes better use your human/technical resources (if available!) to design your own digitizer

GARFIELD+RCo at LNL: digitizers [Pasquali2007]



- 1 channel/board
- 12 bit; 125 MSPS
- 9.5 ENOB
- sel. polarity



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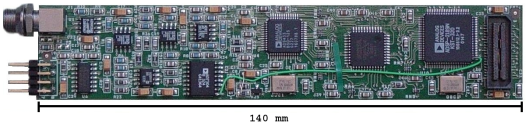

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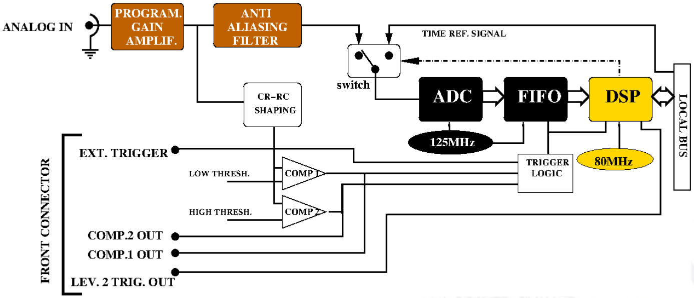
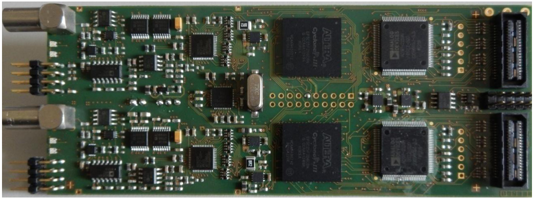
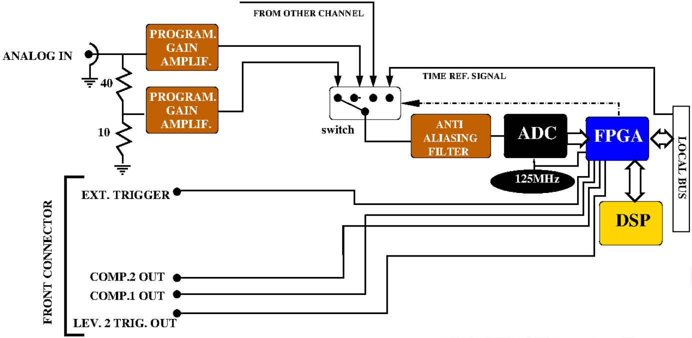


GARFIELD+RCo at LNL: new digitizers (start 2011)



- 2 channel/board
- 14 bit; 125 MSPS
- 11.5 ENOB
- adj. DC offset



New digitizer

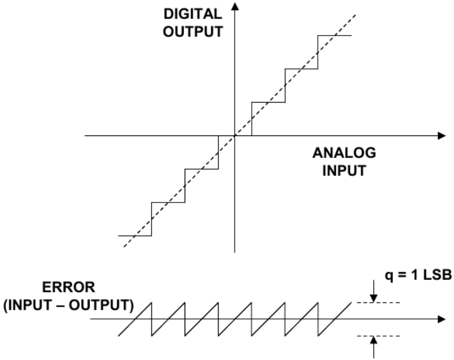
- design: Stefano Meneghini (INFN-Bo), Luigi Bardelli, Maurizio Bini, G.P.
- 14 bit; 125 MSPS;
- two coarse dynamic ranges (better SNR)+ fine gain (12 bit DAC); adjustable range from 100 mV to 10 V
- DC coupled
- adjustable DC offset (polarity selection)
- two channels per board (sampling clocks have opposite phase)
- FPGA centric
- cost: about 300 euros/channel
- DSP: ADSP2189N; FPGA: Altera Cyclone III; Clock gen: AD9572
- VCA: AD8337; ADC: AD9255

Thank you!

Backup slides

Quantization noise: a picture

- comes from <u>second</u> step of A/D conversion (quantization)
- subtract quantized and not-yet-quantized signals:



- the difference is usually correlated to the input for simple signals, e.g. sine (cfr. exercise with pClasses test_quant_noise() in test.C)

Quantization noise: some math

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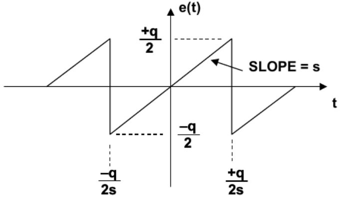

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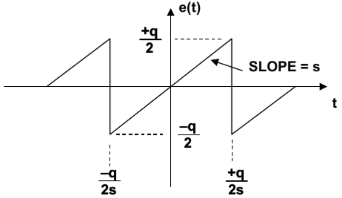

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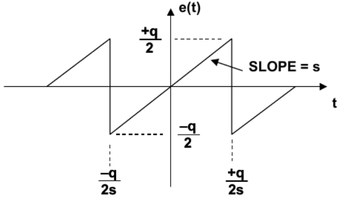

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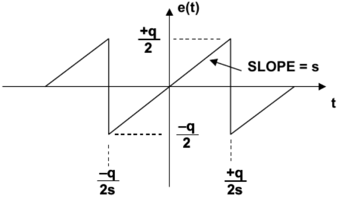

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- rms value $\dfrac{q}{\sqrt{12}}$ same as uniform distribution in $(-q/2, q/2)$

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- $\Longrightarrow$ "white" noise of spectral density $w = 2\frac{\sigma_Q^2}{F_s} = \frac{1}{6\,F_s}\left(\frac{R}{2^{\mathrm{N}}}\right)^2$ in $(0, \frac{F_s}{2})$

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$$SNR(dB) = 20\, log_{10}\left(\frac{FS}{q}\right) + 20\, log_{10}\left(\sqrt{\frac{3}{2}}\right) =$$

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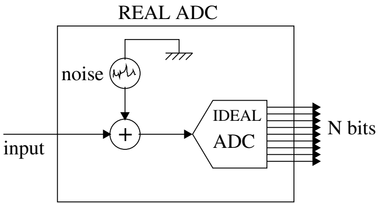
- Some values:

N (bits)	SNR (dB)
10	61.96
12	74.00
14	86.04

as a rule of thumb: 6 dB per bit!

Real ADC and noise...

- we know that a real ADC can be modelled as an "ideal" ADC plus a noise generator adding noise to the input (see figure);



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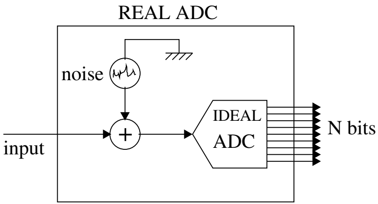

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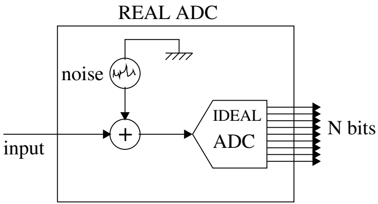

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- real ADC noise has variance $\sigma^2_{eff} > \sigma^2_Q$

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  2. $\sigma_{eff} \propto \frac{R}{2^{\mathrm{ENOB}}}$
  3. in bits ($R = 2^{\mathrm{N}}$) we get $\sigma_{eff} \propto 2^{\mathrm{N-ENOB}} \Longrightarrow \mathrm{N} - \mathrm{ENOB}$ controls how much noise we get;

ENOB: equivalence of the two definitions

$$\sigma^2_{eff} = \frac{1}{12}\left(\frac{R}{2^{\mathrm{ENOB}}}\right)^2 \Longrightarrow \mathrm{ENOB} = \tfrac{1}{2}\, log_2\left(\tfrac{R^2}{12\,\sigma^2_{eff}}\right) \qquad (*)$$

Sine wave, amplitude $R \Longrightarrow V_{rms} = \dfrac{R}{2\sqrt{2}}$

If σ_{eff} only contribution to SNR: $SNR = \dfrac{V_{rms}}{\sigma_{eff}} = \dfrac{R}{2\sqrt{2}\sigma_{eff}}$.

Invert and obtain: $\dfrac{R^2}{\sigma^2_{eff}} = (2\sqrt{2}\,SNR)^2$ and substitute in (*) to find

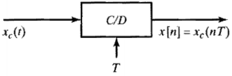
$$\mathrm{ENOB} = \tfrac{1}{2}\, log_2 \tfrac{(2\sqrt{2}\,SNR)^2}{12} = log_2\, SNR - log_2 \tfrac{\sqrt{12}}{2\sqrt{2}} = log_2\, SNR - log_2 \sqrt{\tfrac{3}{2}}$$

1. multiply and divide by $log_{10}\, 2 = 0.301$, then use log rules to change log base to 10;
2. multiply and divide by 20, so that $20\, log_{10} SNR = SNR(db)$.

$$\mathrm{ENOB} = \tfrac{SNR(dB) - 20 log_{10}\sqrt{1.5}}{20\, log_{10} 2} = \tfrac{SNR(dB) - 1.76}{6.02}$$ Q.E.D.

Shannon: Ideal continuous-discrete time converter

- Ideal C/D converter: $x_c(t) \Longrightarrow x[n] = x_c(n\,T_s)$ (no quantization)



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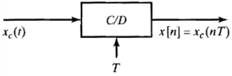

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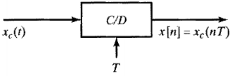

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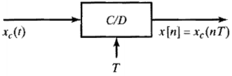

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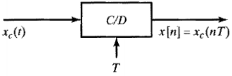

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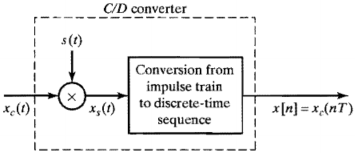

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- the $\mathcal{FT}\{x_s(t)\}$ is made of f-shifted images of $\mathcal{FT}\{x_c(t)\}$ (exploiting linearity of convolution and exploiting the result $X_c(j\Omega) * \delta(\Omega - k\Omega_s) = X_c(\Omega - k\Omega_s)$)

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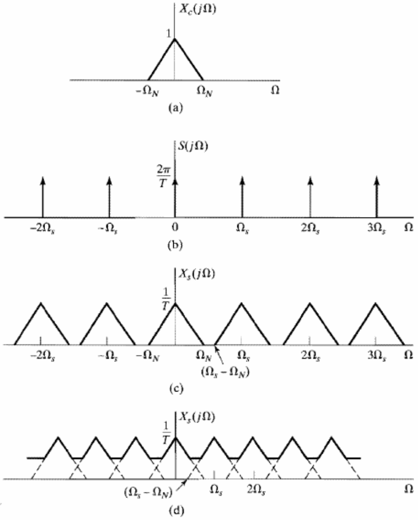

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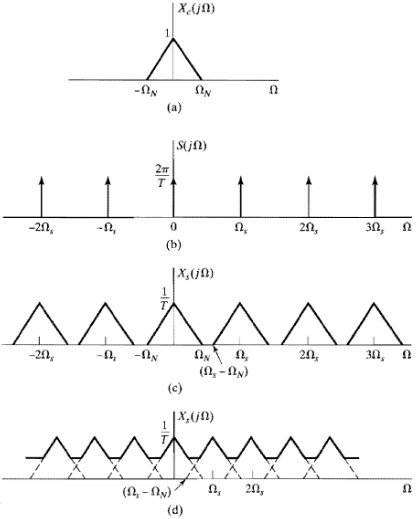

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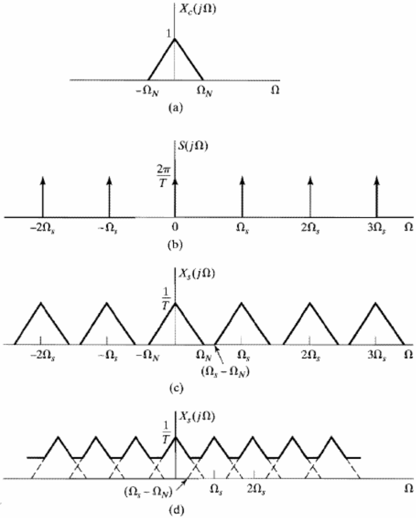

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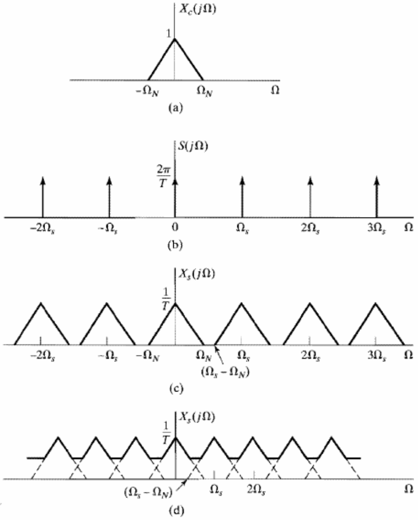
- copies must NOT overlap $\Longrightarrow$ if Ω_N is maximum frequency in $x_c(t)$ then we want

$$\Omega_s - \Omega_N \geq \Omega_N \implies \Omega_s \geq 2\Omega_N$$



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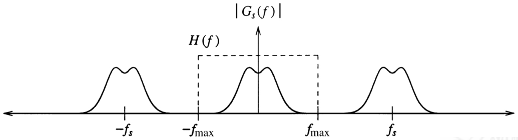

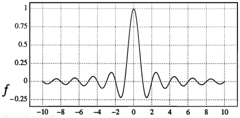

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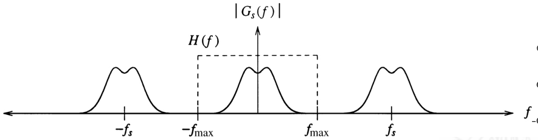

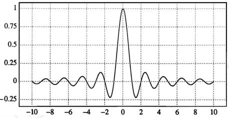

- multiplication in f-domain $\Longrightarrow$ convolution with filter's impulse response (right picture) in t-domain (N.B: $T_s = 1$ in right panel)

Sinc interpolation

- in t-domain, convolution of $x_s(t)$ with $\mathcal{FT}^{-1}$ of brick-wall f-response: $sinc(t) = \frac{sin(\pi t/T_s)}{\pi t/T_s}$ (*normalized sinc*). We assumed a cut-off at $f_{max} = \frac{1}{2}\frac{1}{T_s}$

$$x_r(t) = x_s(t) * sinc(t) = \sum_n x[n] \int_{-\infty}^{+\infty} sinc(x)\delta(t - nT_s - x)dx =$$

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- interpolation: for $t = mT_s$, $sinc\left(mT_s - nT_s\right) = 0\ \forall m \in \mathbb{Z}$ except $m = n$ where $sinc(0) = 1 \Longrightarrow x(nT_s) = x[n] \Longrightarrow x_r(t)$ goes through known samples; in between we get "interpolated" values

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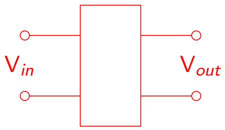

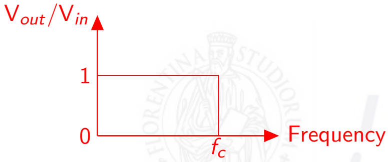

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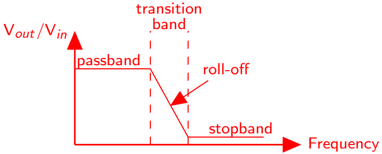

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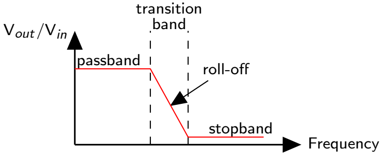

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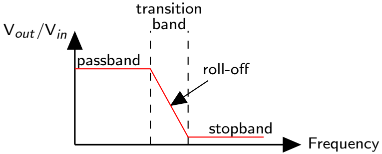

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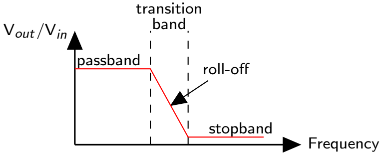

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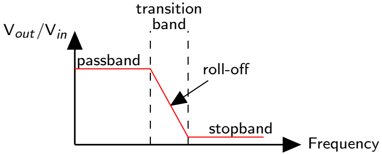

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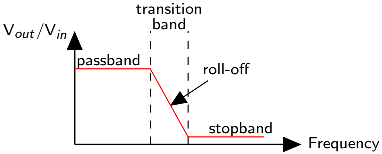

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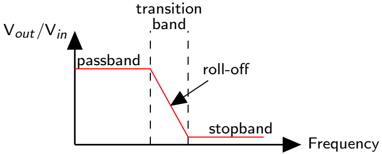

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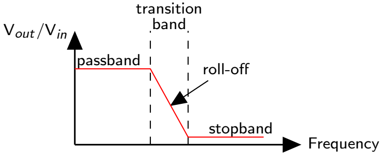

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- N.B. attenuation usually not constant in stopband: stopband begins when a certain minimum attenuation is reached

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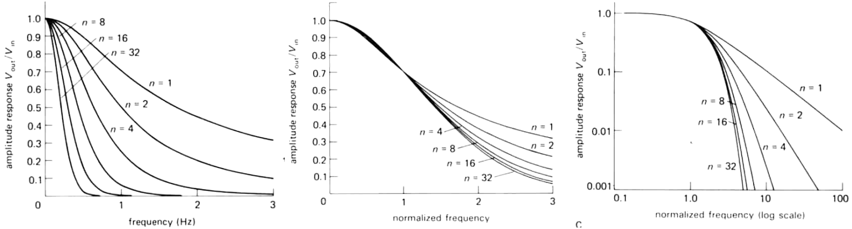
- step response $\Longleftrightarrow$ filter action on t-domain info
- time domain coded info: any sample contains some info

Filter impulse, step and frequency response

- filter has: **impulse**, **step** and **frequency response**
- impulse response: filter output when input is a pulse (Dirac's δ)
- step response: filter output when input is a perfect step
- each one contains complete info about the filter
- frequency response $\Longleftrightarrow$ filter action on f-domain info
- step response $\Longleftrightarrow$ filter action on t-domain info
- time domain coded info: any sample contains some info
- frequency domain coded info: relationship between many samples (no info in single sample)

Anti-aliasing stage: RC low pass stage [Horowitz1989]



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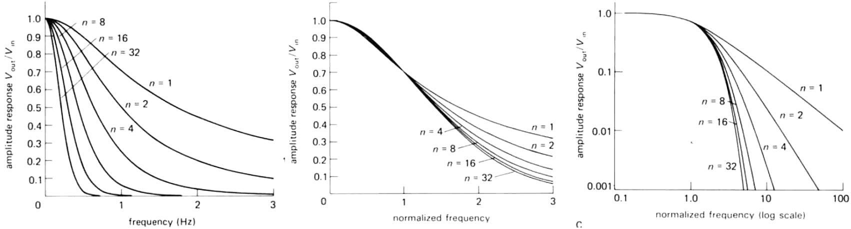

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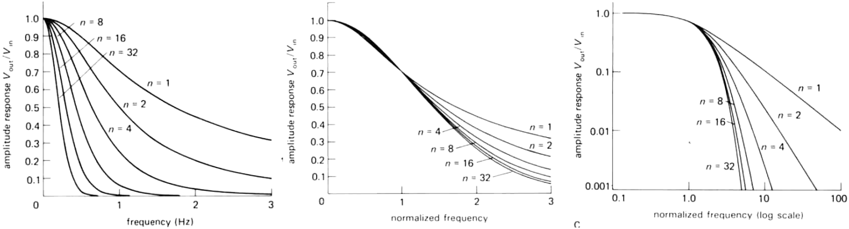

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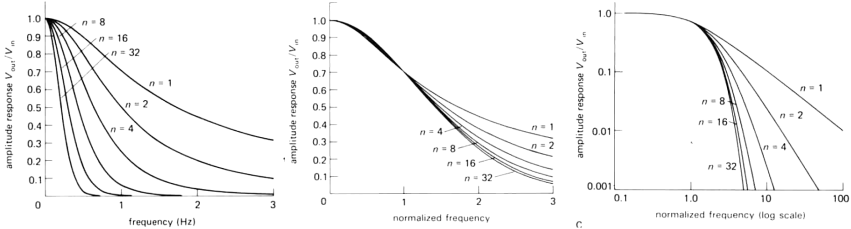

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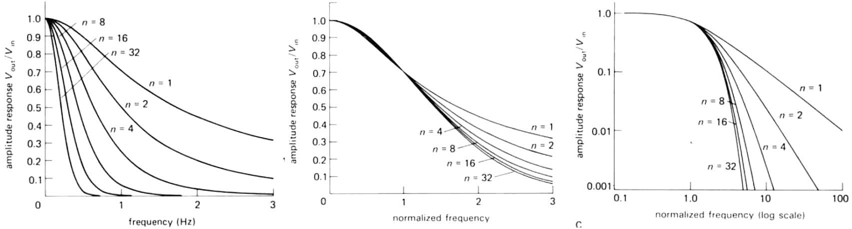

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- n is a.k.a. the number of "poles" (zeros at denominator in the transfer function, cfr. Laplace or Fourier transform)
- however we don't get a sharper knee at -3dB cutoff: "many soft knees do not a hard knee make" (cit. Horowitz-Hill); this clearly appears when plotting response vs f/f_c (normalized frequency)

Anti-aliasing stage: Butterworth, Chebyshev, Bessel

- solution: active filters using amplifiers and feedback

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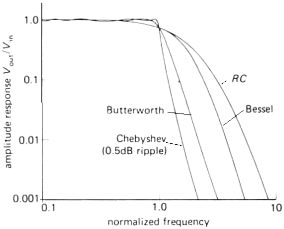
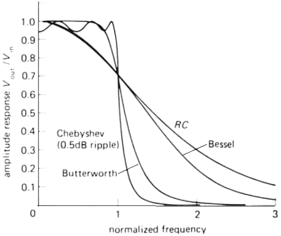
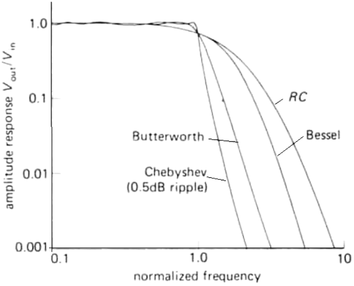
- frequency response for 6-poles active filters [Horowitz1989]

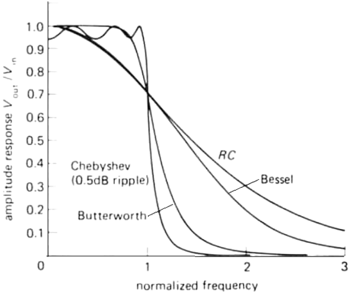




Anti-aliasing stage: f-response

- frequency response for 6-poles active filters





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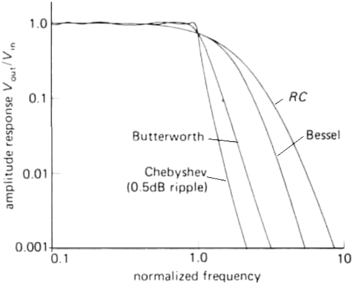

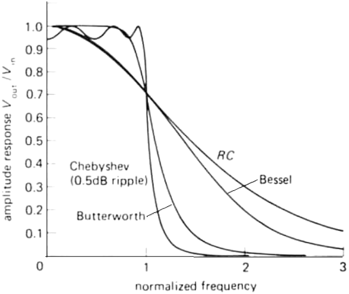

- Butterworth: maximally flat passband response

Anti-aliasing stage: f-response

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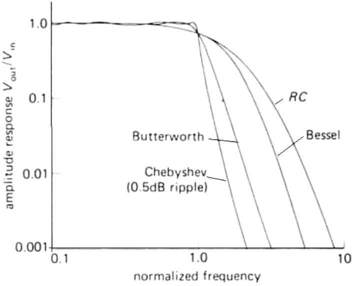

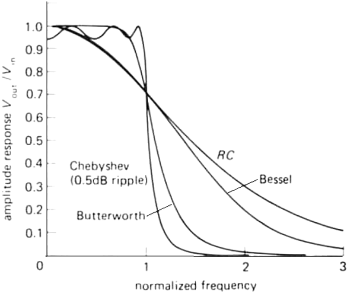

- Butterworth: maximally flat passband response
- Chebyshev: accept some passband ripple to get steeper roll-off

Anti-aliasing stage: t-response

- step response for 6-poles active filters

Anti-aliasing stage: t-response

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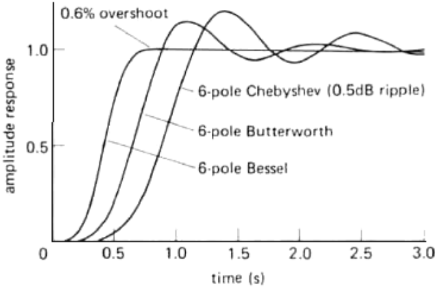
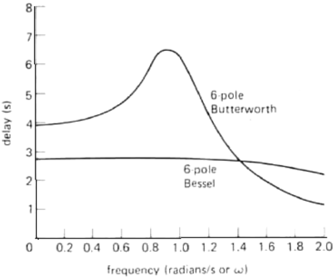
- Butterworth and Chebyshev: bad step-response (left) due to not constant delay ($\equiv$ non linear phase resp.) (right)





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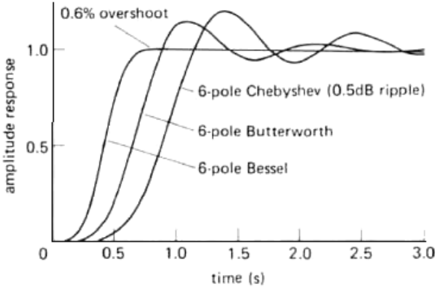
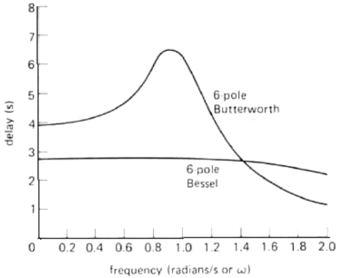


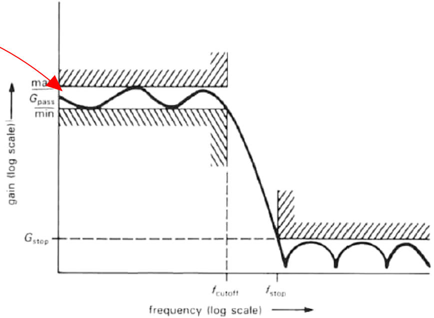


- Bessel: trades roll-off slope for step-response

Designing an anti-aliasing filter

To design a LPF:

- choose allowed range of gain in passband (ripple)



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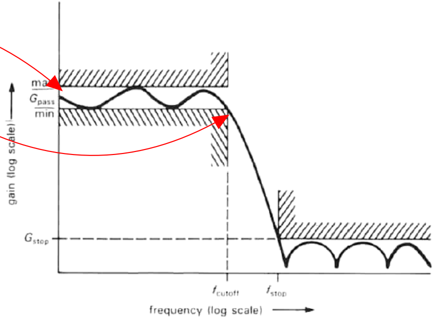

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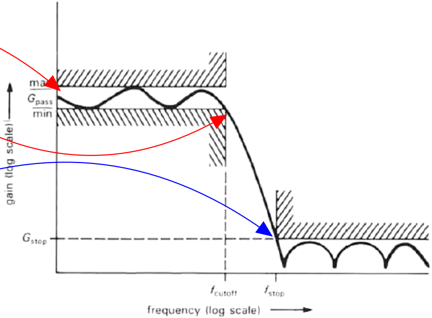

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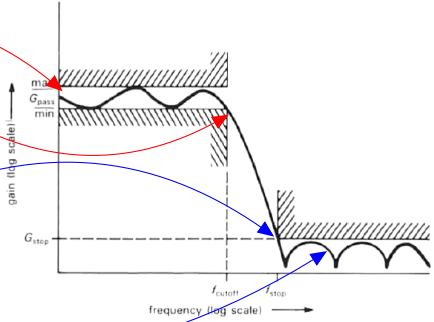

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To design a LPF:

- choose allowed range of gain in passband (ripple)
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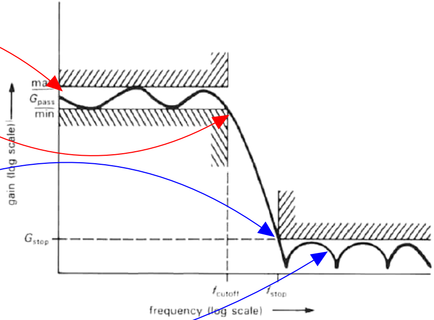
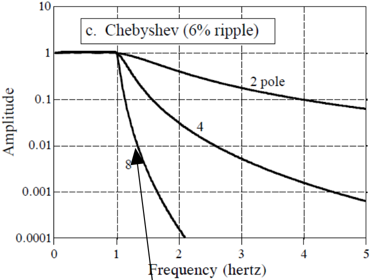
- choose minimum attenuation in stopband
- not necessarily in this order...



Exercise: design an anti-aliasing filter!

DATA: ADC has $F_s = 100$ MHz, 12 bit; allow for 6 % ripple in p-b and require at least 10^{-2} attenuation (-40 dB) at Nyquist frequency ($F_s/2 = 50$ MHz)



- 8-pole Cheb (6 % ripple): -40 dB at $1.35 \times f_c \Longrightarrow 1.35 \times f_c = 50$ MHz $\Longrightarrow f_c = 37$ MHz $\Longrightarrow$ choose 8-pole filter

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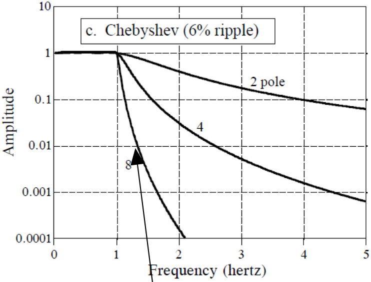

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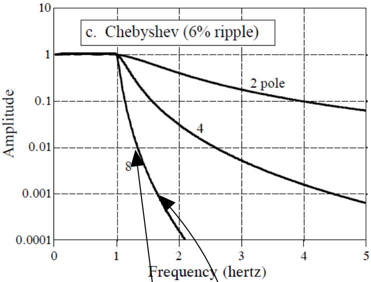

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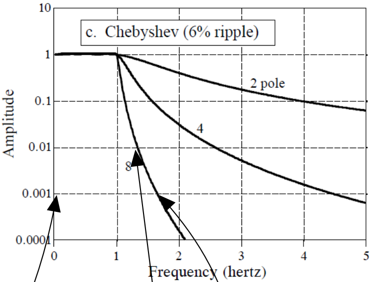

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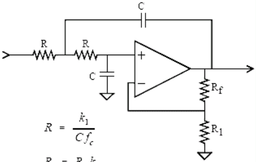
- passband stops at 37 MHz $\implies$ alias in passband for $f > 50 + (50 - 37) = 100 - 37 = 63$ MHz $(= 1.7 f_c)$
- at $1.7 f_c$ attenuation is ≈ 0.001 (60 dB), compatible with effective dynamic range

Sallen-Key circuit

How is the antialias implemented in electronics? Most used electronic scheme to get Bessel/Chebyshev/Butterworth response: Sallen-Key architecture. Same circuit gives all responses by suitable choice of ratios k_1 and k_2

FIGURE 3-8
The modified Sallen-Key circuit, a building block for active filter design. The circuit shown implements a 2 pole low-pass filter. Higher order filters (more poles) can be formed by cascading stages. Find k_1 and k_2 from Table 3-1, arbitrarily select R_1 and C (try 10K and 0.01µF), and then calculate R and R_f from the equations in the figure. The parameter, f_c, is the cutoff frequency of the filter, in hertz.

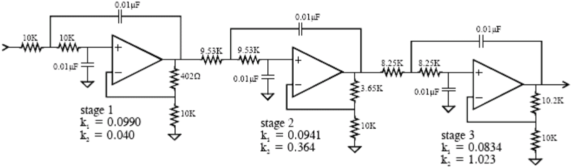


$$R = \frac{k_1}{C f_c}$$

$$R_f = R_1 k_2$$

TABLE 3-1
Parameters for designing Bessel, Butterworth, and Chebyshev (6% ripple) filters.

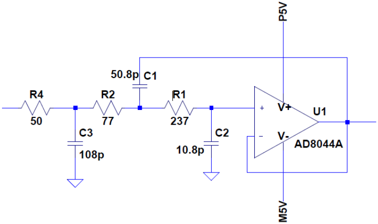
# poles		Bessel k_1	Bessel k_2	Butterworth k_1	Butterworth k_2	Chebyshev k_1	Chebyshev k_2
2	stage 1	0.1251	0.268	0.1592	0.586	0.1293	0.842
4	stage 1	0.1111	0.084	0.1592	0.152	0.2666	0.582
	stage 2	0.0991	0.759	0.1592	1.235	0.1544	1.660
6	stage 1	0.0990	0.040	0.1592	0.068	0.4019	0.537
	stage 2	0.0941	0.364	0.1592	0.586	0.2072	1.448
	stage 3	0.0834	1.023	0.1592	1.483	0.1574	1.846
8	stage 1	0.0894	0.024	0.1592	0.038	0.5359	0.522
	stage 2	0.0867	0.213	0.1592	0.337	0.2657	1.379
	stage 3	0.0814	0.593	0.1592	0.889	0.1848	1.711
	stage 4	0.0726	1.184	0.1592	1.610	0.1582	1.913



Sallen-Key circuit

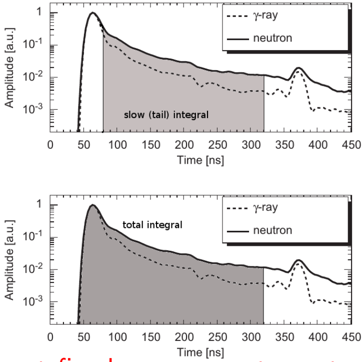
Many stages: increase complexity, noise, power dissipation...can we use just one?

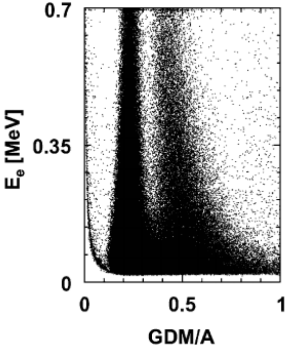
Example of 1-stage 3-pole Bessel Sallen-Key as implemented in Luigi Bardelli's “year 2000” board



One usually studies actual response using circuit simulators (spice, pspice, ltspice...). In this case, we get ≈ 20 dB attenuation of aliased frequencies in passband...is it acceptable?

PSD: n/γ discrim. in liquid organic scintillators





- at fixed energy, protons stopping power $\gg$ than electrons

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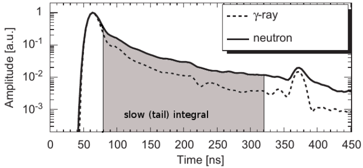

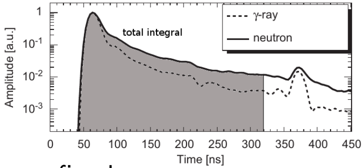

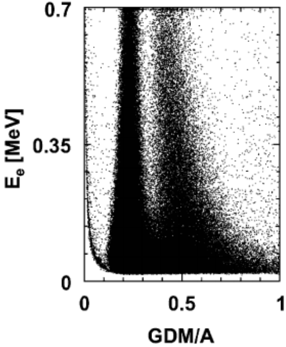

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- higher density of triplet states along track $\Longrightarrow$ signal has longer tail

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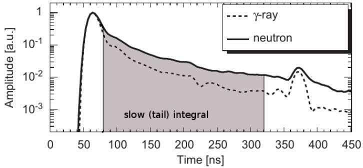

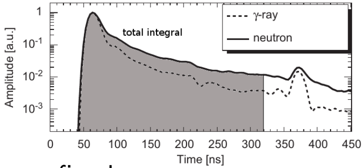

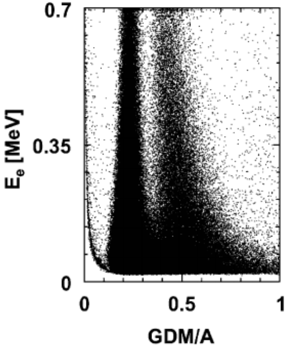

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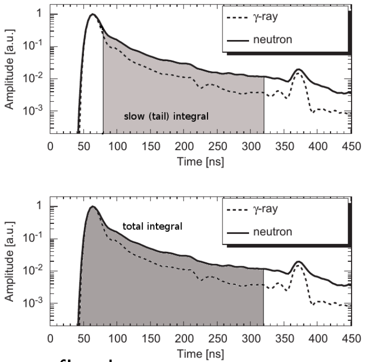

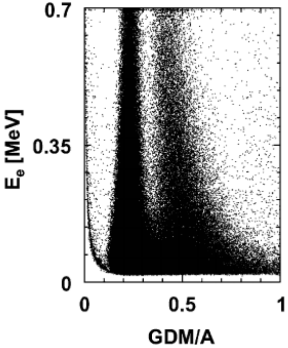

- at fixed energy, protons stopping power $\gg$ than electrons
- higher density of triplet states along track $\Longrightarrow$ signal has longer tail
- two integrations, usually *slow* (top left) and *total* (bottom left)
- Right picture: "total (E) vs slow (GDM)". GDM normalized to pulse amplitude. Gammas (i.e. electrons) on the left, neutrons (i.e. protons) on the right.

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