Digital Processing with Focus onto Neutron Detection

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Outline

Digital Processing with Focus onto Neutron Detection

Focus: Key issues behind Pulse Shape Discrimination with digitized signals
Aim: understanding few basic facts (not a comprehensive treatment)

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- First lesson:

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  - Application to n/γ PSD

Introduction: sensor, read-out, digitizer

Detection cell: sensor and read-out circuit

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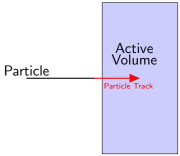

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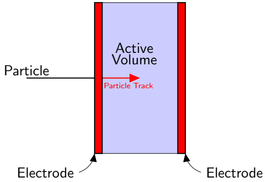

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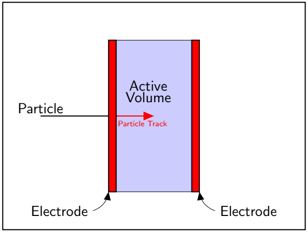
Sensor (gas, semiconductor)



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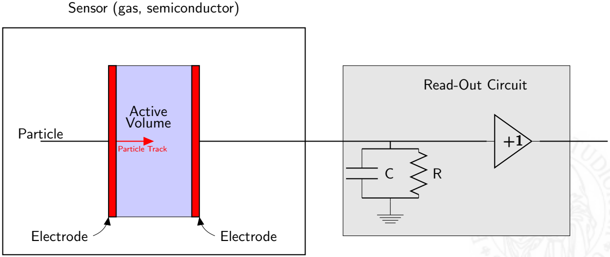

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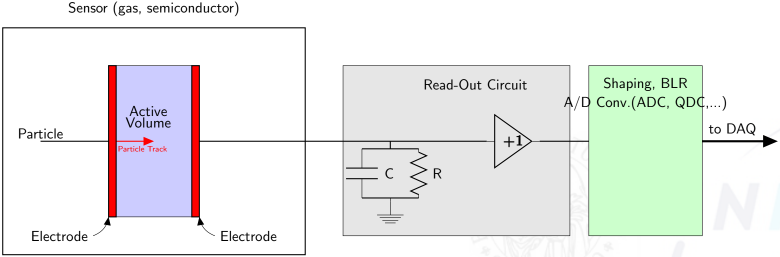

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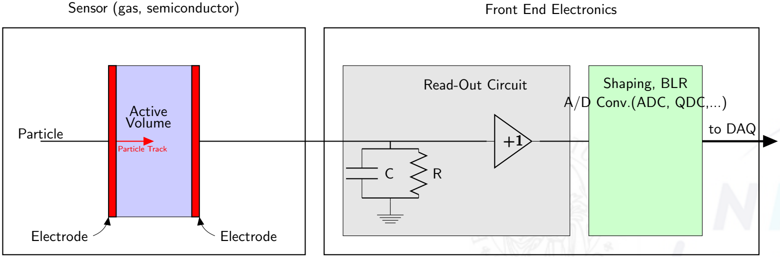
- read-out + signal processing = Front End Electronics (FEE)



Carrier collection and Read-Out characteristic times

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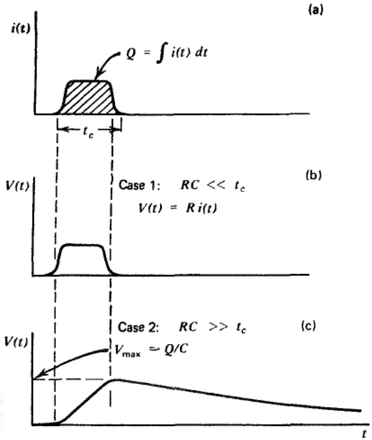

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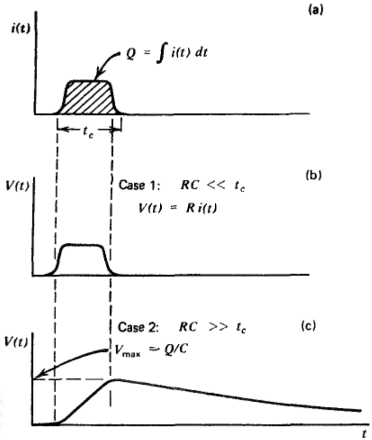

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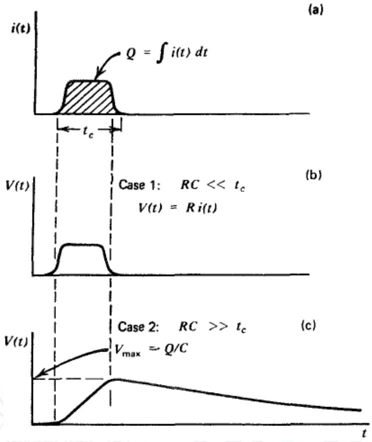

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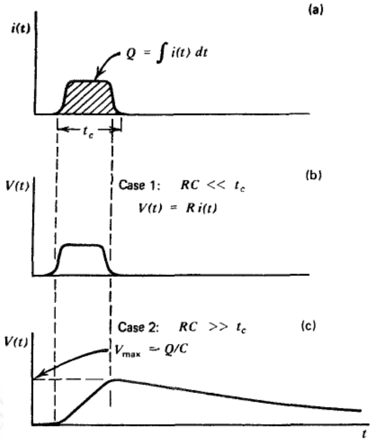

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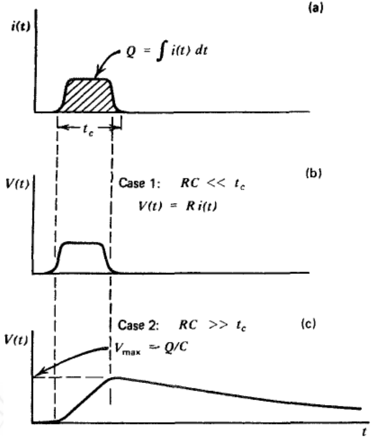
- t_c: few ns (fast scintillators, microchannel plates) $\longrightarrow$ 10-100 ns (semiconductors) $\longrightarrow$ μs (ionization chambers)



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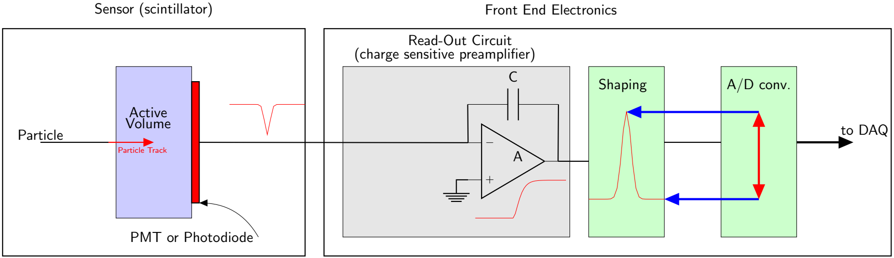
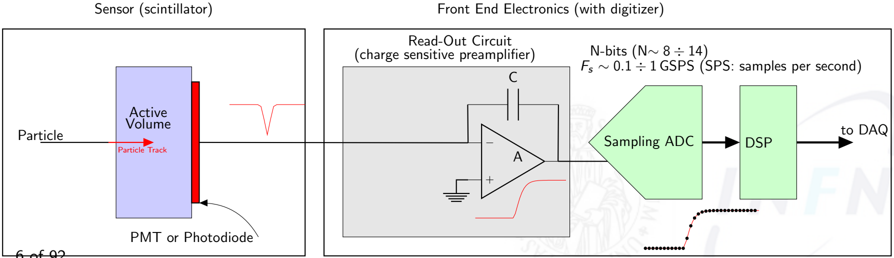
Analog and digitizing chains compared





Advantages of digitizers

- better stability with respect to analog circuits
- flexibility (processing just a matter of calculation)
- easy pulse shape analysis implementation
- predictable and reduced dead time
- easy implementation of pile-up rejection
- processing not possible with analog can be implemented

How's information coded?

Deposited Energy:

- number of generated information carriers (ion pairs, e-h pairs, scintillation photons) $\Longrightarrow$ total produced charge Q;
- retrieved as $\int i(t)\mathrm{d}t$ ($RC \ll t_c \rightarrow$ current signal) or...
- maximum amplitude of $V(t)$ ($RC \gg t_c \rightarrow$ charge signal);

Time of interaction:

- time at which some signal feature (e.g. threshold crossing) occurs (*time mark*);
- unavoidable delay between interaction time and time mark;

How's information coded?

Point of gamma interaction (Hyperpure Ge detectors, e.g. AGATA):

- needed to reconstruct full gamma energy when multiple interaction occur before absorption;
- need segmented electrodes to identify a subvolume [Akkoyun12];
- within subvolume, interaction point obtained from all signal shapes (comparing with waveform database) with 1 mm resolution.

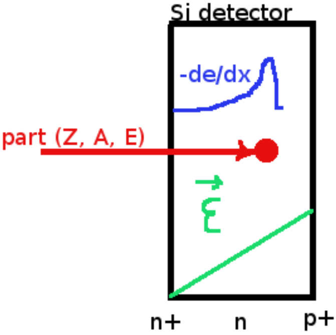
Radiation type (neutron? gamma? charged fragment?):

- usually coded into time evolution of signals;
- technique goes under PSA or PSD names (Pulse Shape Analysis, Pulse Shape Discrimination);
- often obtained from correlations (e.g. PSD param vs energy);
- collection time, relative amplitude fast/slow components.

From signal shape to radiation type: Si detector

Pulse Shape Discrimination example: nuclear fragment stopped in Si detector (FAZIA collaboration).

- collection time and maximum value of current depend on (Z, A) and E



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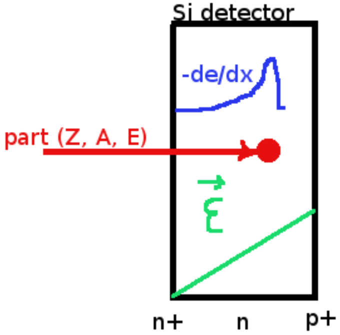

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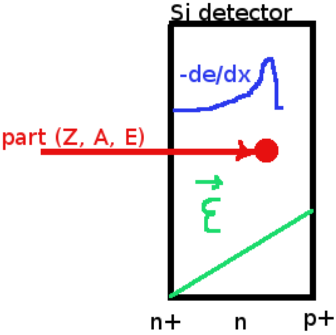

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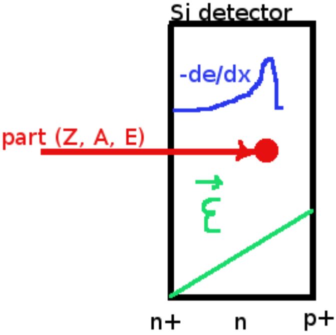

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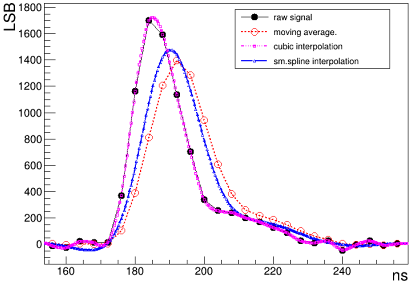

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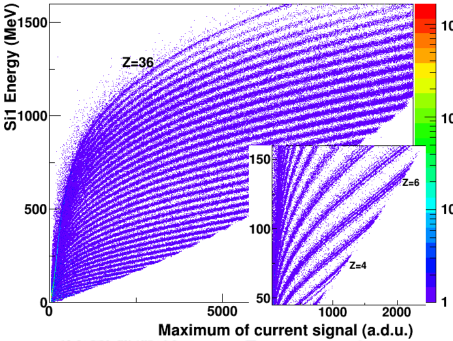

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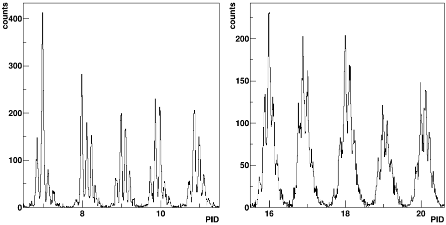
- resulting isotopic identification (PID spectrum)



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Noise from the ADC

First Message: Sampling ADC as noise source

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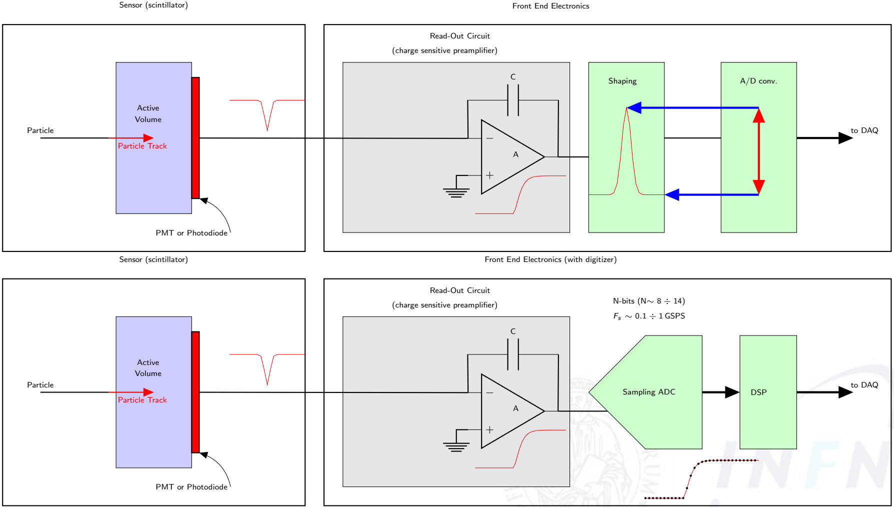

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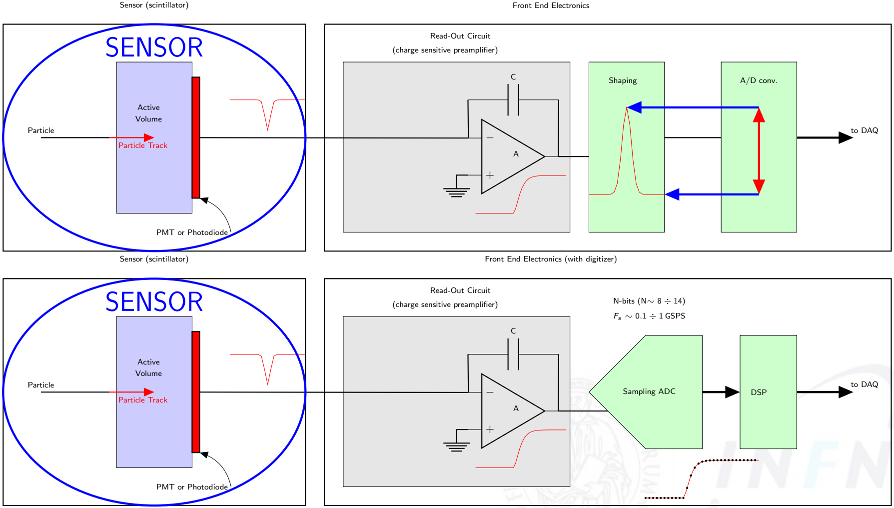

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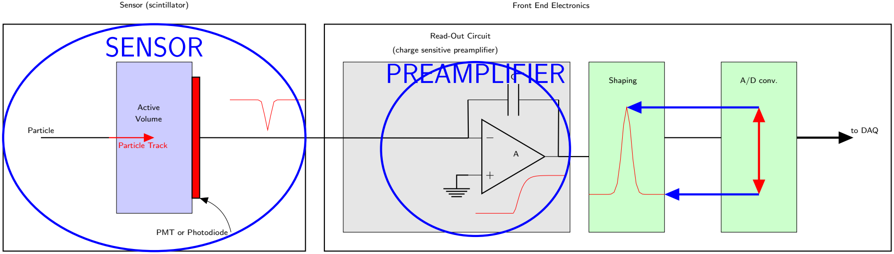

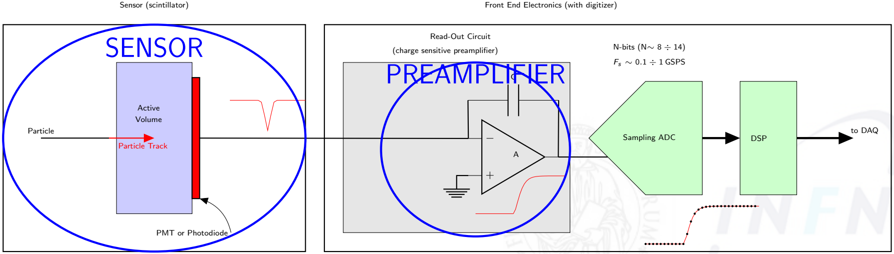

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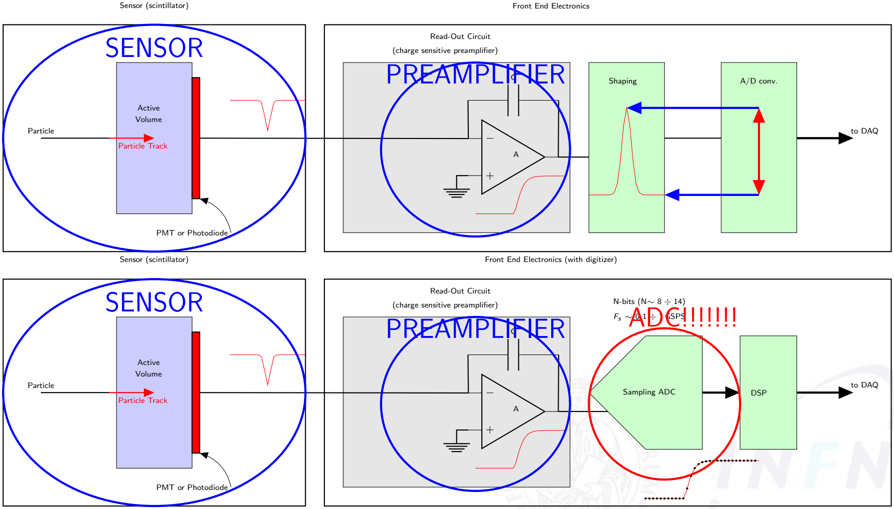

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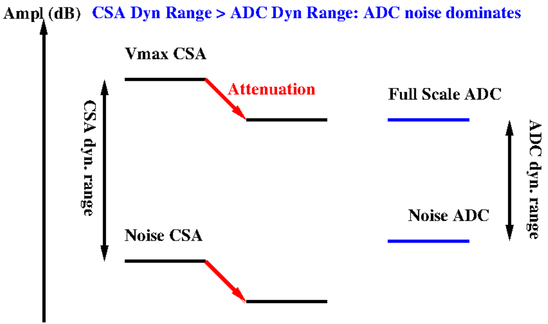

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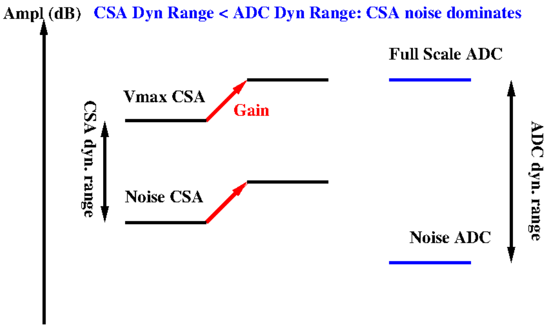

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ADC: math model of sampling

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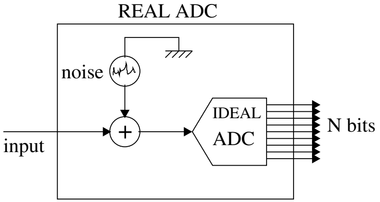

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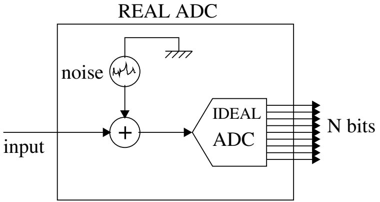

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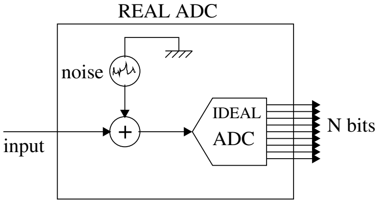

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- The "textbook" ENOB definition is actually $\mathrm{ENOB} = \frac{SNR(dB) - 1.76}{6.02}$ where *SNR* is the signal-to-noise ratio including effects of harmonic distortion, clock jitter etc. My definition is equivalent to this one provided that σ_{eff} is the dominant contribution to *SNR*, which is usually the case in nuclear physics.

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- **in practice** input ref. noise accounts for most SINAD, i.e. determines most of $N - ENOB$
- actual effective resolution and *ENOB* usually differ by a few 0.1 LBS

Shannon, signal reconstruction, interpolation

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- no final answer (ask questions each time);
- useful to grasp the general aspect of the problem.

Discrete-Time and “proper” sampling

1. sampling: discretization of *independent* variable;
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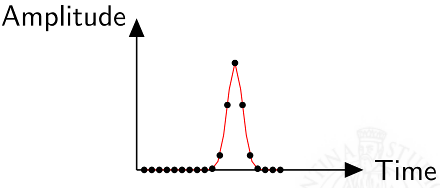
Let’s come to point 1), time-discretization: $x[n] = x_c(n\,T_s)$
Intuition $\Longrightarrow$ maximum acceptable sampling period must exists:
Starting from “close” samples...



...we can still recognize signal shape from samples...

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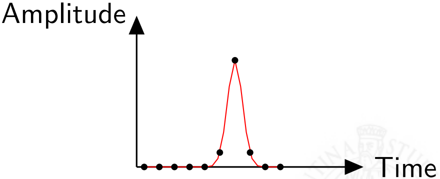
...we increase the sampling period T_s...



...now just a few samples taken...

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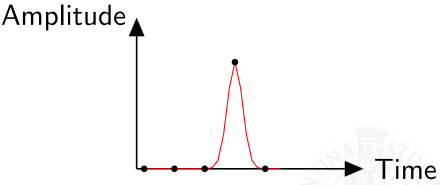

...so that we take just one sample (by chance) during the signal...

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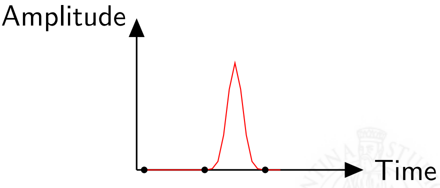

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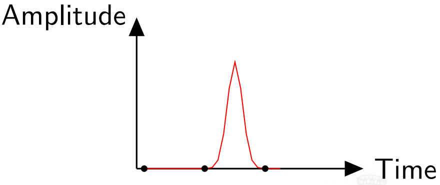

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Need for recipe: what is maximum T_s for which I can get back the original (continuous) signal? How to get it back from samples?

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- Timing: extracting a "time mark" from a signal, e.g. with a leading edge discriminator (LED);
- LED: device emitting a logic "true" signal when input voltage crosses a fixed threshold (e.g. oscilloscope trigger)

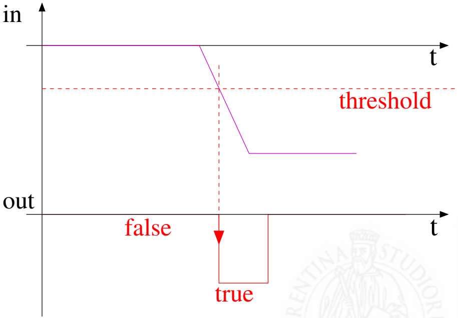

Figure 1: Leading edge discriminator

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- knowing the signal "in between samples" we could get the exact crossing time;

A possible recipe: Shannon Theorem

Shannon Theorem

- in order to obtain "proper" sampling (i.e. being able to reconstruct signal $x_c(t)$ from equally spaced samples $x[n]$ exactly)

1. continuous signal must be bandlimited $\Longrightarrow$ no components in f-domain (Fourier transform) with $f > F_{max}$ (Nyquist freq.)
2. samples must be spaced by $T_s \leq \frac{1}{2} T_{max}$ with $T_{max} = \frac{1}{F_{max}}$, i.e. sampling frequency $F_s \geq 2\, F_{max}$

- recipe to get back $x_c(t)$:

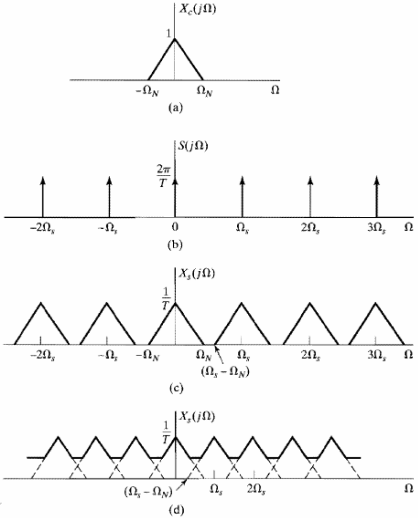
$$x_c(t) = \sum_{n=-\infty}^{+\infty} x[n] \cdot \mathrm{sinc}\left(\frac{t}{T_s} - n\right) \tag{1}$$

where $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$ (sinc: *cardinal sine* function).

Shannon: periodic frequency spectrum

$$X_s(j\Omega) = \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} X_c(\Omega - k\Omega_s)$$

original $X_c(j\Omega)$ plus ∞ copies shifted by $k\Omega_s$



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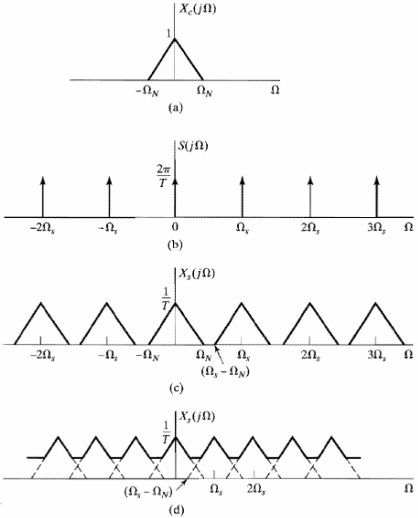

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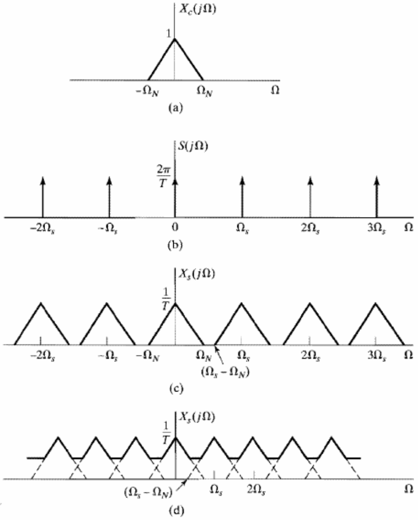

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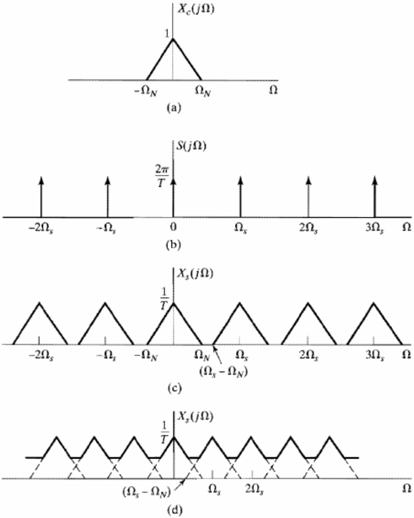
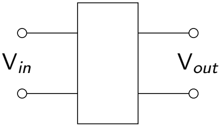
- use inverse $\mathcal{FT}$ to obtain $x_c(t)$
- copies must NOT overlap $\Longrightarrow$ if Ω_N is maximum frequency in $x_c(t)$ then we want

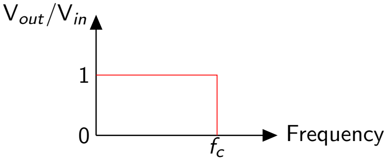
$$\Omega_s - \Omega_N \geq \Omega_N \Longrightarrow \Omega_s \geq 2\Omega_N$$



Shannon Theorem: the sinc function

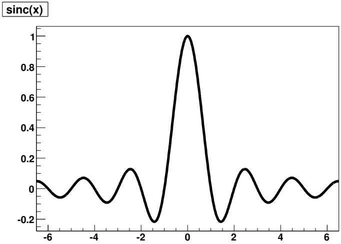
The sinc is the inverse FT of (the response of) the perfect reconstruction filter: the brick wall filter:



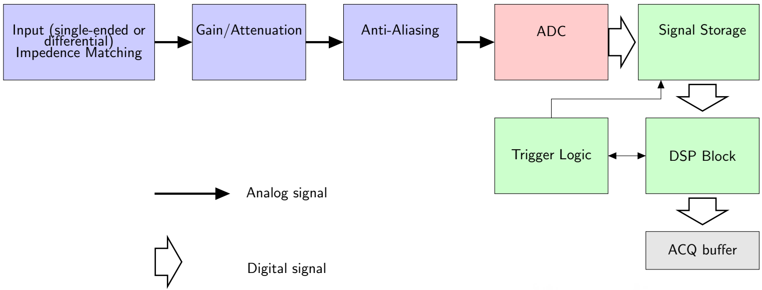


The sinc function:

1. extends to $\pm\infty$;
2. it is $= 1$ in $x = 0$;
3. it is $= 0$ for integer x.



Block diagram of a digitizer



- input stage: can be single ended or differential, matches transmission line characteristic impedance
- gain: adjust overall dynamic range, variable gain to change max non-saturating amplitude (ADC range is fixed)
- antialias: LPF needed to cope with Sampling Theorem, more about it later
- ADC: produces one binary value every T_s (sampling period, $F_s = 1/T_s$ sampling frequency)
- Storage: here we keep digital signal until we have processed it as needed
- DSP and ACQ buffer: signal elaboration, information extraction (on-board μprocessor or FPGA)
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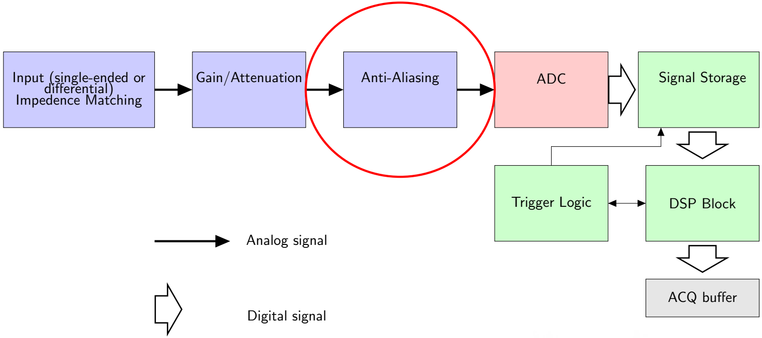
Block diagram of a digitizer

SOME WORDS ABOUT THE ANTIALIASING FILTER...



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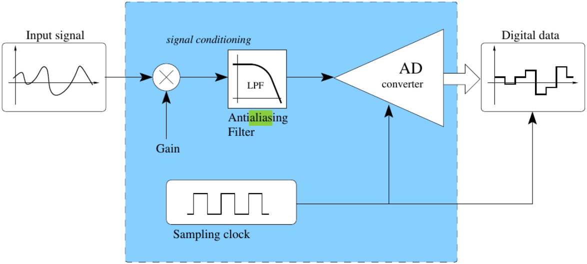
Antialiasing filter



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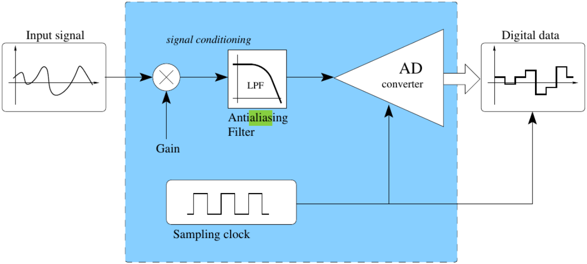
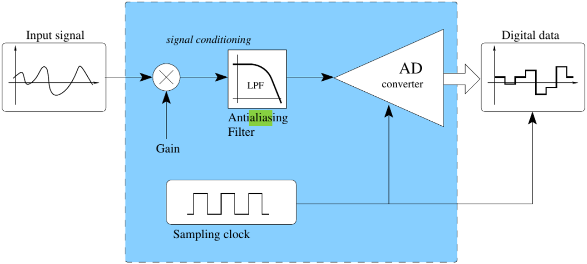
Antialiasing filter



- low pass filter: to limit the bandwidth before sampling;
- a component at $F_s/2 + \Delta f$ appears at $F_s/2 - \Delta f$ after sampling (*aliasing*);

Antialiasing filter



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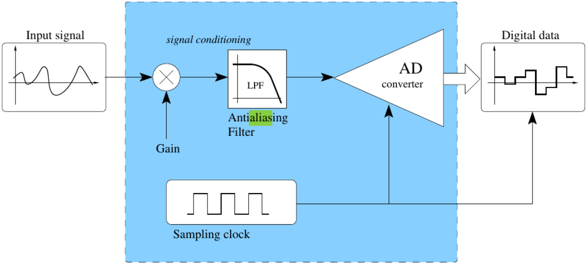

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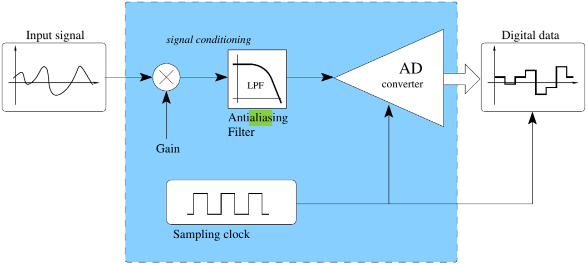

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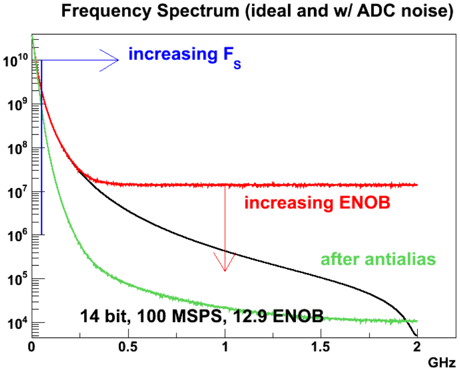
- importance of good t-domain response (e.g. step response);
- a real antialias filter can't eliminate $f > F_s/2$ completely;
- aliasing will worsen the *SNR* of the digitizer (must be always compared to the *SNR* of the original signal);

Pulse f-spectrum, ideal, w/ noise, w/ anti-alias

Example: Analog f-spectrum (i.e. no mirror images) of PMT pulse lasting about 50 ns (black curve). Detector signals have vanishing frequency content at high freq. (easier job for antialias filter).



In red: ADC noise added. In green: with antialias filter.

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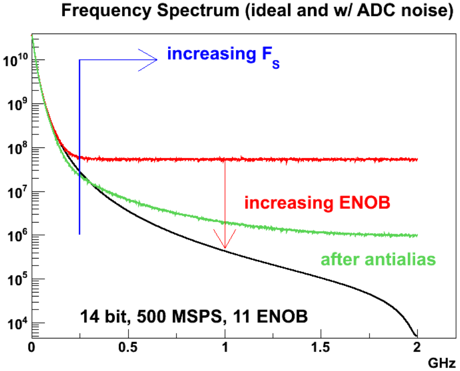

Only 11 ENOB (though N=14 bit resol): noise > signal near Nyquist.

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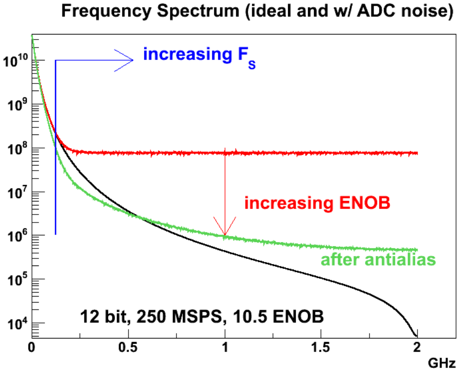

Only 12 bit resol. However 10.5 ENOB $\Longrightarrow$ reasonable SNR.

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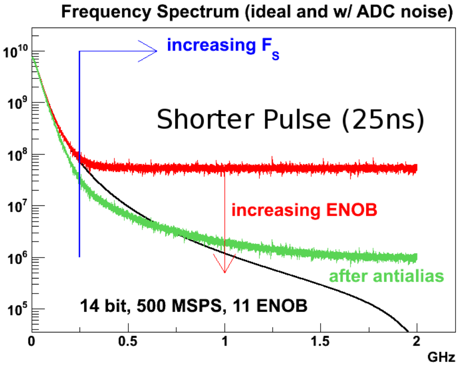


Shorter pulse! More high frequency content. Now 500 MSPS probably better!

Shannon Theorem: additional remarks/caveats

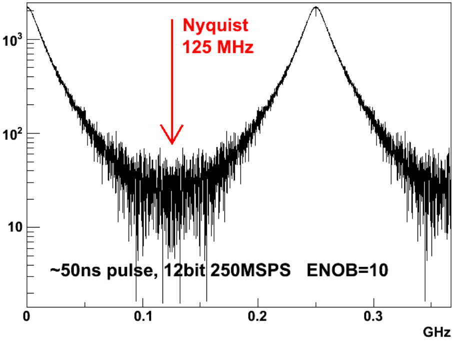
- equation (1) is a particular instance of *interpolation formula*

$$f(t) = \sum_{n=-\infty}^{+\infty} c_n \cdot g\left(\frac{t}{T_s} - n\right) \tag{2}$$

  where $g(t)$ is called *interpolation kernel*;
- "interpolation is the process of estimating the intermediate values of a continuous event from discrete samples" [Keys1981].
- usually $c_n = x[n]$, i.e. eq.(2) is a discrete convolution $(x * g)$
- you want to get back the original signal...
- ...in real life you never sample the *original* signal (artifacts added by input circuit, antialias filter, ADC noise,...)
- "original" signal already has fluctuations/noise (e.g. statistical fluctuations in carrier production, read-out circuit noise,...)
- interpolation-induced deviations of the same order of the ones already present would not be much of a problem...

Interpolation and frequency domain

In discrete-time, as in continuous time, t-domain convolution of signals $\equiv$ product of their Fourier Transforms. Convolution with sinc removes all frequencies above Nyquist frequency ($F_s/2$).



Spectrum of 50 ns pulse+ADC noise sampled at 12 bit 250 MSPS.

Alternative interpolation kernels...

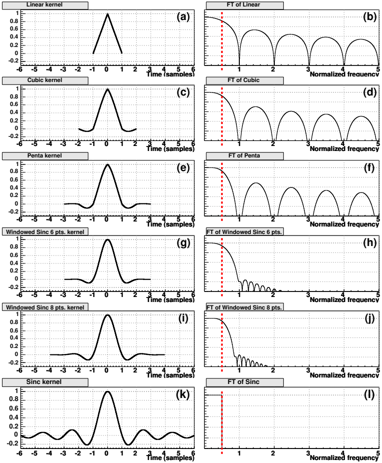


Figure 2: Interpolation kernels and their Fourier Transforms (from [Bardelli2005])

- Alternative kernels (left) with their Fourier Transforms (right)

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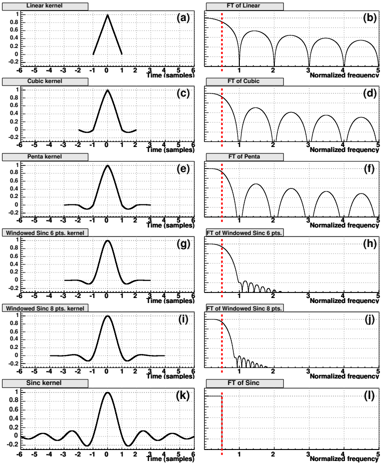

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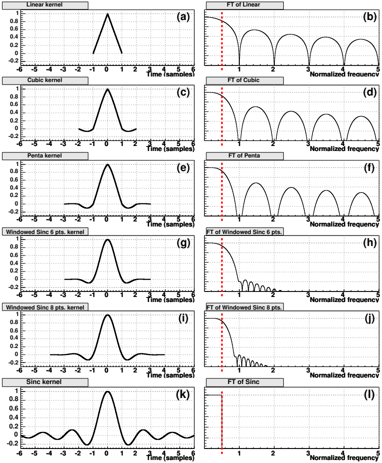

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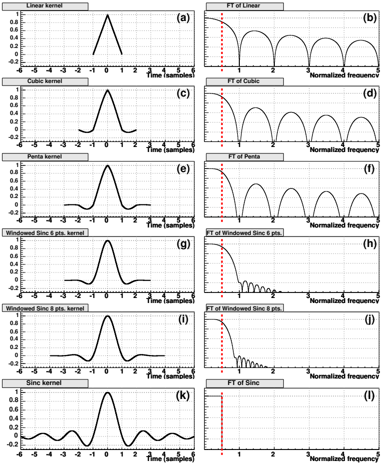

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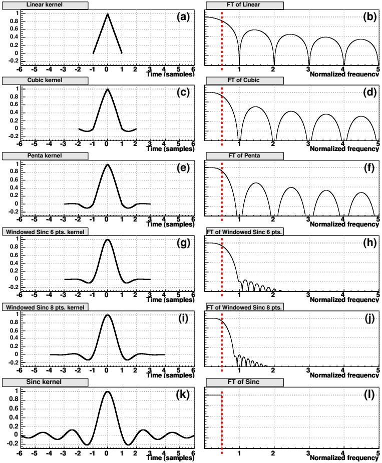

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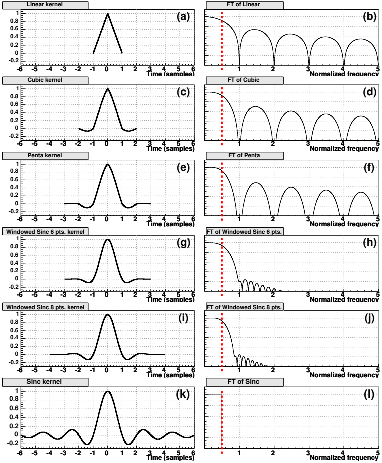

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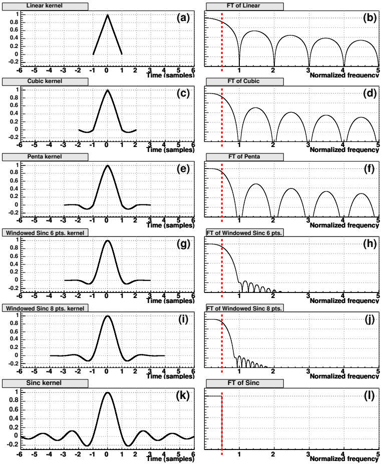

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  1. needs ∞ number of samples (sinc not limited in t-domain);
  2. contribution from "distant" samples $\leq$ finite numerical precision; (bad: no exact calc.; good: $\#$ of terms $< \infty$);

Interpolation: alternative kernels

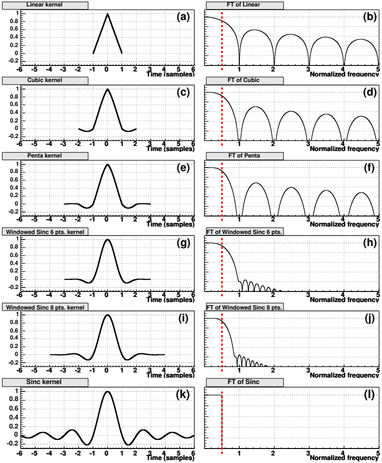

Figure 3: Interpolation kernels and their Fourier Transforms (from [Bardelli2005])

- What happens with the alternatives?

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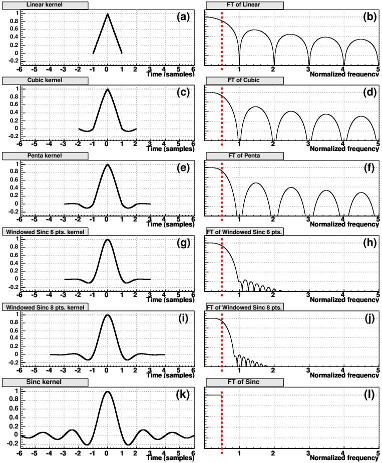

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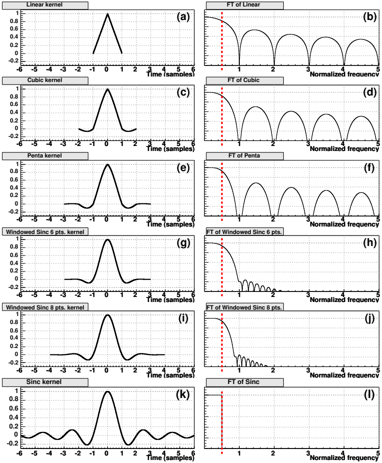

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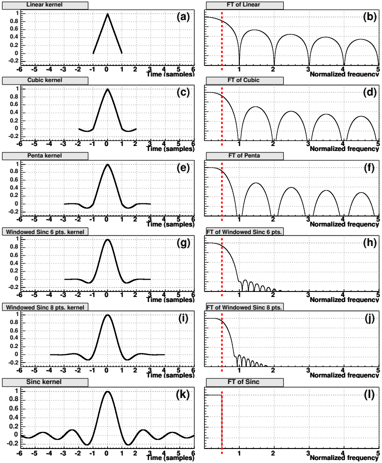

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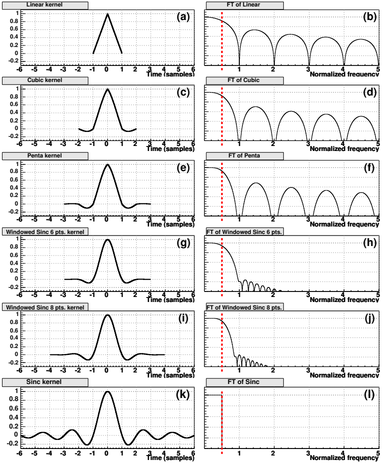

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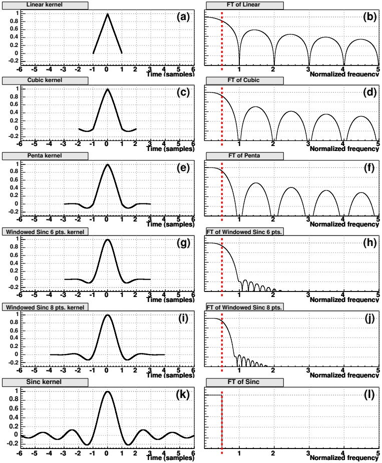

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- other kernels (a, c, e) much more extended in f-domain
- linear (tent) kernel (a) amounts to linear interpolation (connects points with segments)

Interpolation: alternative kernels...

Other interpolation kernels (from [Keys1981]):

- kernel b) corresponds to a) in previous slide;
- kernel d) is [Keys1981] version of a cubic kernel;

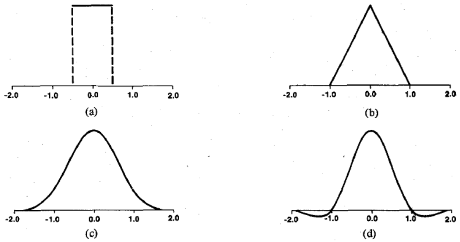


Fig. 1. Interpolation kernels. (a) Nearest-neighbor. (b) Linear interpolation. (c) Cubic spline. (d) Cubic convolution.

- a), a.k.a. "box interpolation" kernel, produces a stepped result;
- c) is cubic box-spline (a case for which $c_n \neq x[n]$)

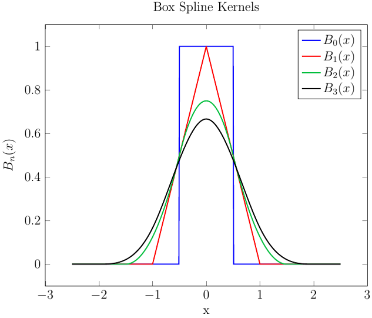
Generic interpolation and cubic splines

Starting from samples $x[n] = x(n\,T_s)$, look for $f(t)$ in

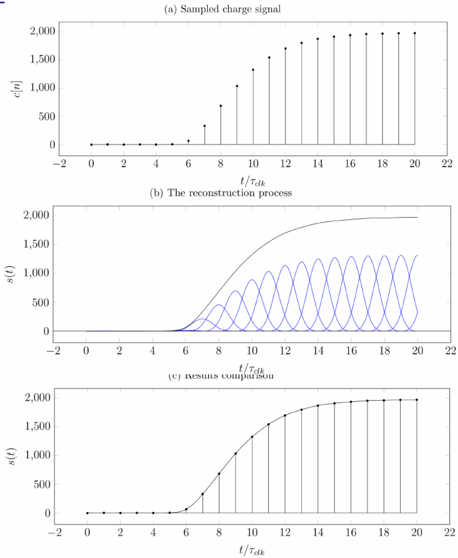
$$K(g,\ T_s) = \left\{ f(t) \mid f(t) = \sum_{m=-\infty}^{+\infty} c_m\, g(t/T_s - m) \right\}$$

Find member of K passing through the samples (i.e. find c_m):
$\sum_{m=-\infty}^{+\infty} c_m\, g(n-m) = x[n] \quad \forall n \in \mathbb{Z}$

E.g.: $g(t)$ could be a box-spline (a.k.a. B-spline):
B-spline kernels obtained from box function by multiple convolution with itself (Picture from [Ottanelli2016]).

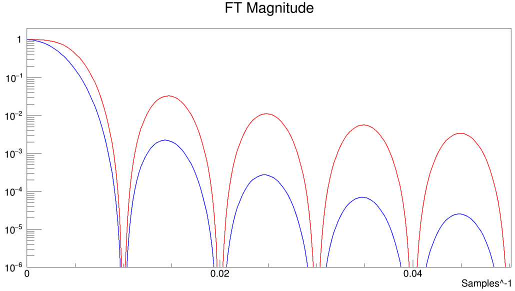


Interpolation at work (from [Ottanelli2016])



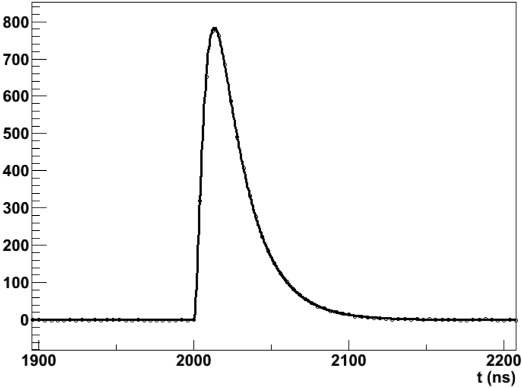
Cubic kernels compared

Cubic kernel c) of Fig.3 [Bardelli2004]
Cubic B-spline [Hou1978]



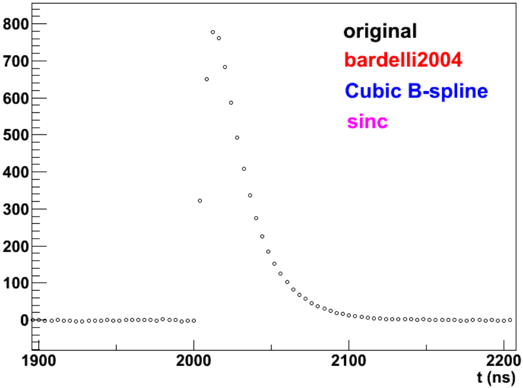
Faster and simpler: [Bardelli2004] (c_m are just signal samples)
More powerful: cubic B-splines (c_m cumbersome to calculate but... a fast approximated calc possible using IIR or FIR filters [Unser1993, Ottanelli2016]!)

Interpolating a detector pulse



Original signal (note the "sudden" start) and its samples.
Sampling includes ADC noise (12 bit, 250 MSPS, 10 ENOB).

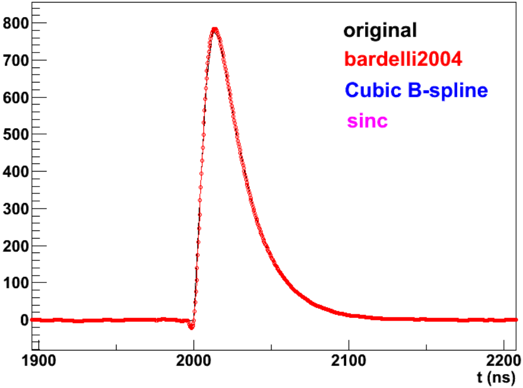
Interpolating a detector pulse



This is what we know after sampling.

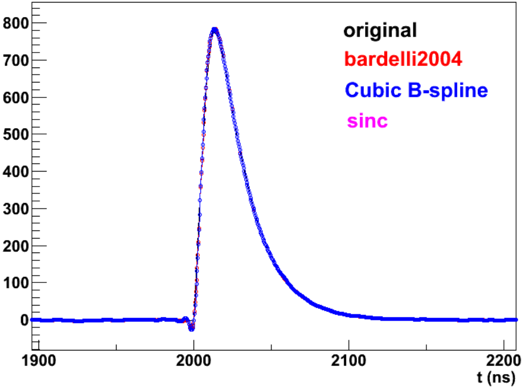
Perfect reconstruction not possible (noise, aliasing,...).

Interpolating a detector pulse



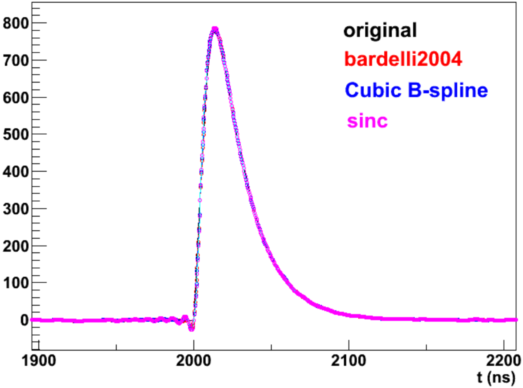
Cubic interpolation from [Bardelli2004], $\times 10$ oversampling.
Exploits the unique polynomial through 4 consecutive samples.

Interpolating a detector pulse



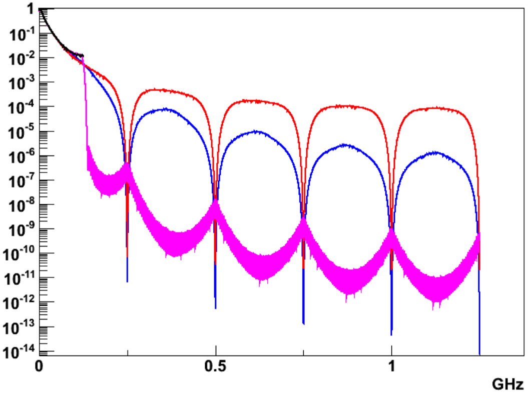
Now we show Cubic Spline interpolation, $\times 10$ oversampling.
Note different behaviour where signal varies rapidly.

Interpolating a detector pulse



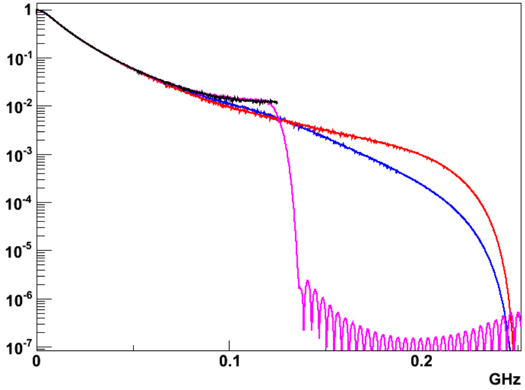
...and finally sinc interpolation. Note: all similar where signal varies slowly. Artifact and different behaviour at fast transition. Why?

Interpolating a detector pulse



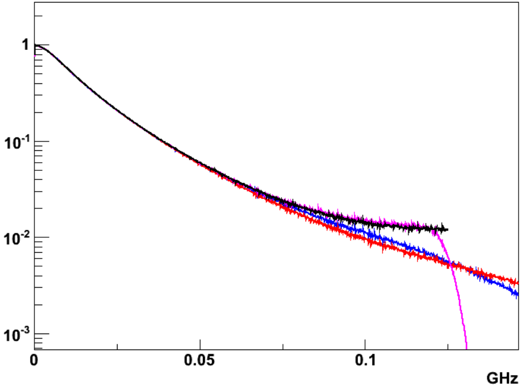
Original spectrum (black) ends at original Nyquist freq. Interpolated signal spectrum extended beyond Nyquist (black is periodic!).

Interpolating a detector pulse



New signal spectrum distorted below Nyquist. Sinc interpolation: less in-band attenuation and less artifacts beyond Nyquist (good?)

Interpolating a detector pulse



Expanded view: frequencies below Nyquist ($F_s/2 = 125\,\mathrm{MHz}$). Sinc interpolation preserves amplitude where noise could dominate.

Interpolation error and sampling clock phase

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- it's a kind of noise: *interpolation noise*!

Simulation: interpolation "noise"

- sampling of noiseless signal $S(t)$ adding a random time shift $0 < \delta < T_s$, i.e. $S[n] = S(n\,T_s + \delta)$

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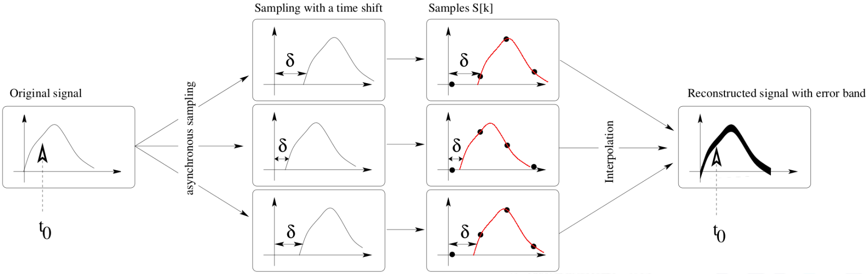
- amplitude variance at fixed $t = t_0$ (with respect to true value $S(t_0) = S_0$):

$$\sigma^2 = \frac{1}{T_s}\int_0^{T_s} \left[S_0 - S_\delta(t_0+\delta)\right]^2 \,\mathrm{d}\delta$$

Interpolation "noise" simulation: a picture

Variance formula with explicit kernel dependence [Bardelli2005]:

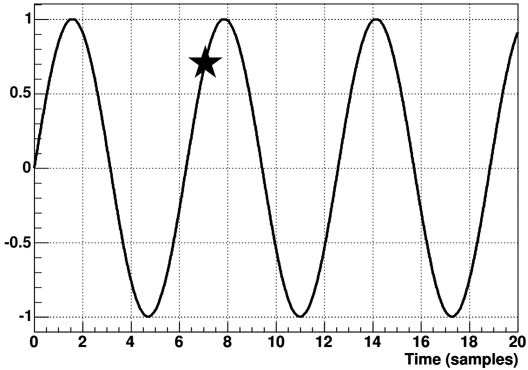
$$\sigma^2 = \frac{1}{T_s}\int_0^{T_s}\left[S_0 - \sum_{n=-\infty}^{+\infty} S(kT_s-\delta)\cdot g\left(\frac{t_0+\delta}{T_s}-n\right)\right]^2 \mathrm{d}\delta$$



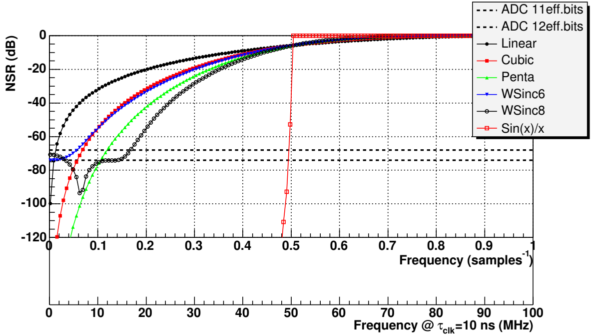
N.B. generally, red (reconstructed) signals have different shapes!

Sine simulation: select point as t_0

Simulation for sinusoidal signal: we will study amplitude fluctuations at fixed t_0, point marked with $\star$, also trying different kernels.



Sine simulation for $T_s = 10\,\text{ns}$: fluctuations at t_0



- sine is BW limited $\Longrightarrow$ sinc is the best kernel!

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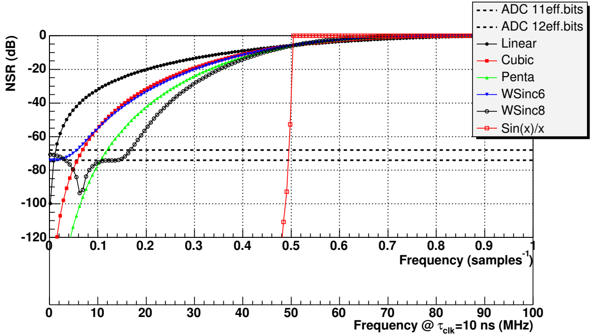

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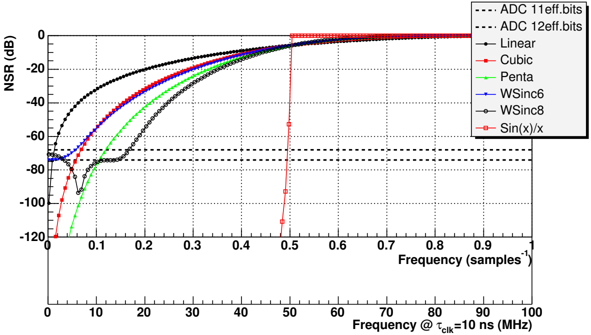
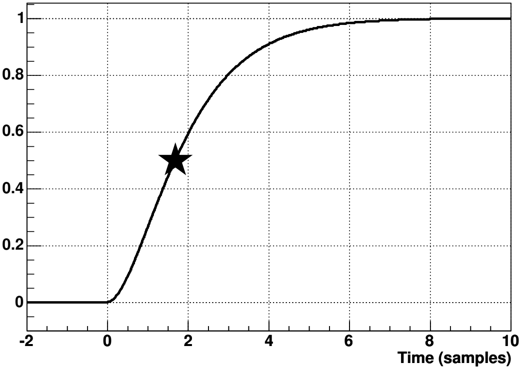


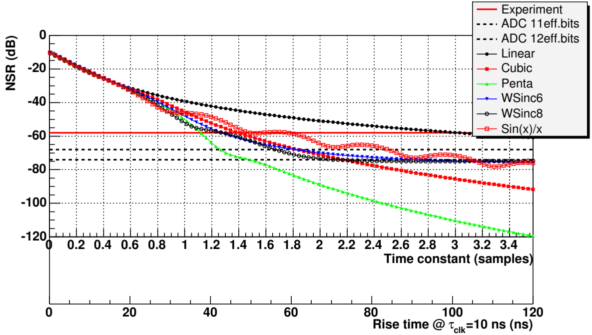
- sine is BW limited $\Longrightarrow$ sinc is the best kernel!
- note the reference ADC noise levels (two ADC's, different ENOB);
- when interpolation noise$<$ADC noise, we can forget about it

Preamp simulation: select point as t_0

Simulation for charge preamp signal: we will study amplitude fluctuations at fixed t_0, point marked with $\star$, also trying different kernels.

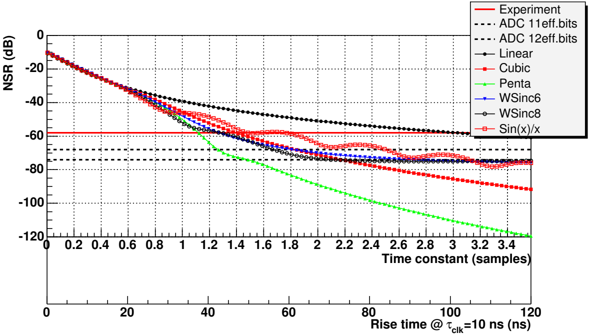


Preamp simulation for $T_s = 10\,\text{ns}$: fluctuations at t_0



- three reference noise levels (added measured noise in actual FEE);

Preamp simulation for $T_s = 10\,\text{ns}$: fluctuations at t_0



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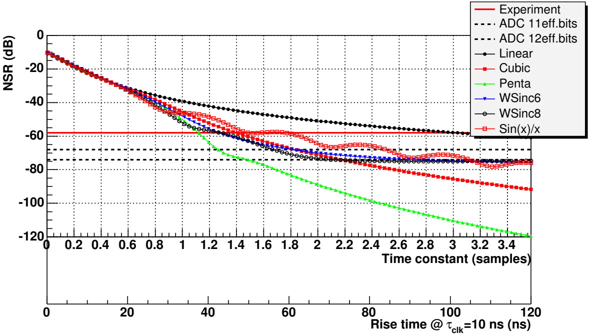

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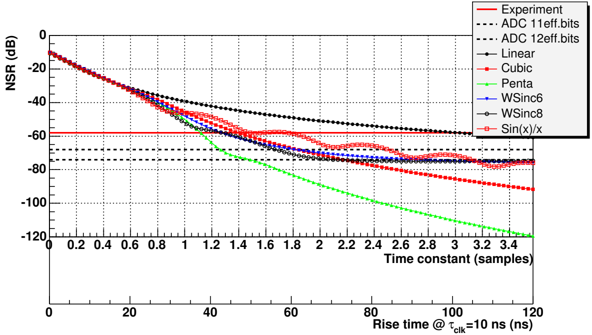

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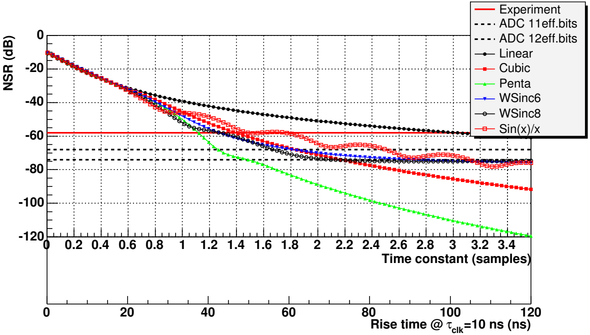

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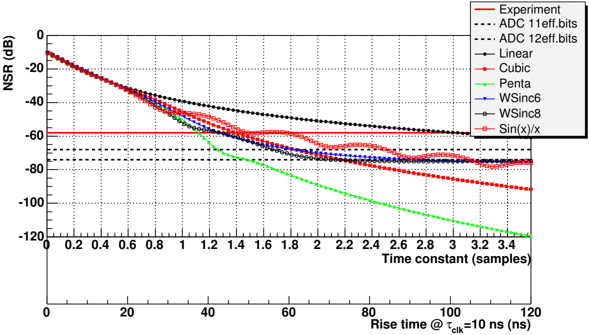

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- cubic interpolation: good performance/complexity ratio

End of First Lesson (2016-10-25)