SNRI V

Introduction to lab.2
γ–ray spectroscopy with GALILEO

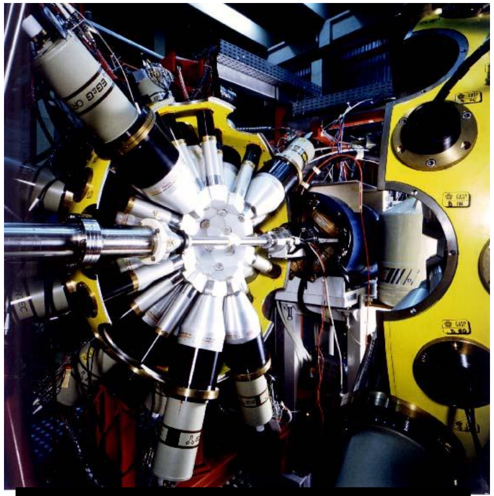
Daniele Mengoni

Dipartimento di Fisica e Astronomia "G.Galilei"
Università di Padova
INFN - Padova

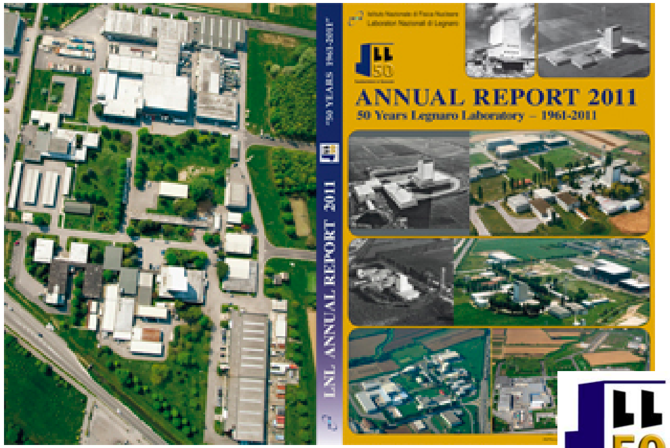
γ-ray spectroscopy at LNL

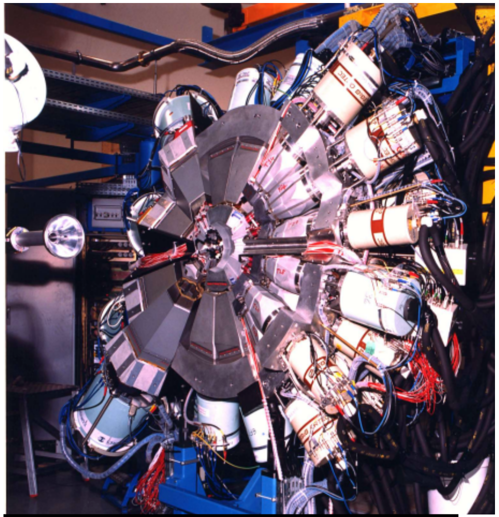
Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Legnaro

- 80% nuclear physics research
- 50% γ-ray spectroscopy
- Proton- and neutron-rich nuclei

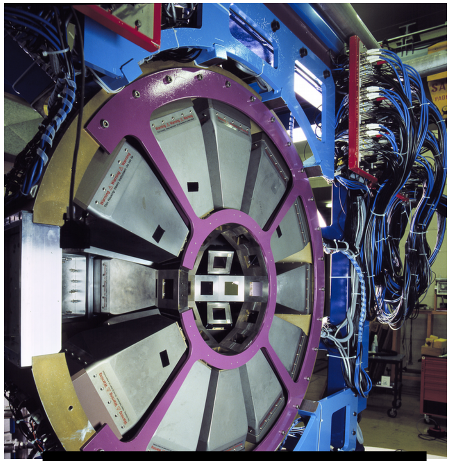


GASP 1992

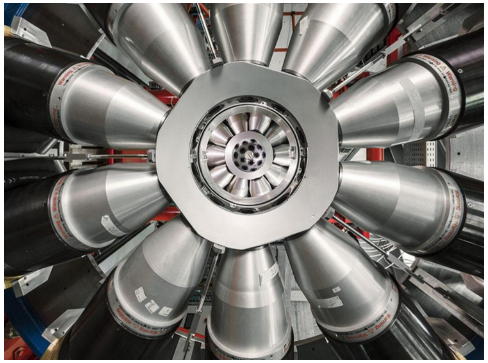




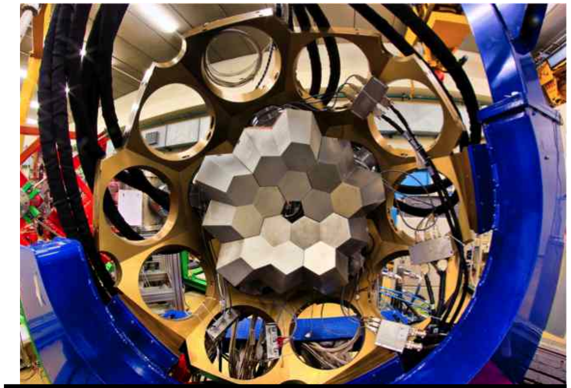
EUROBALL 1998



CLARA 2004



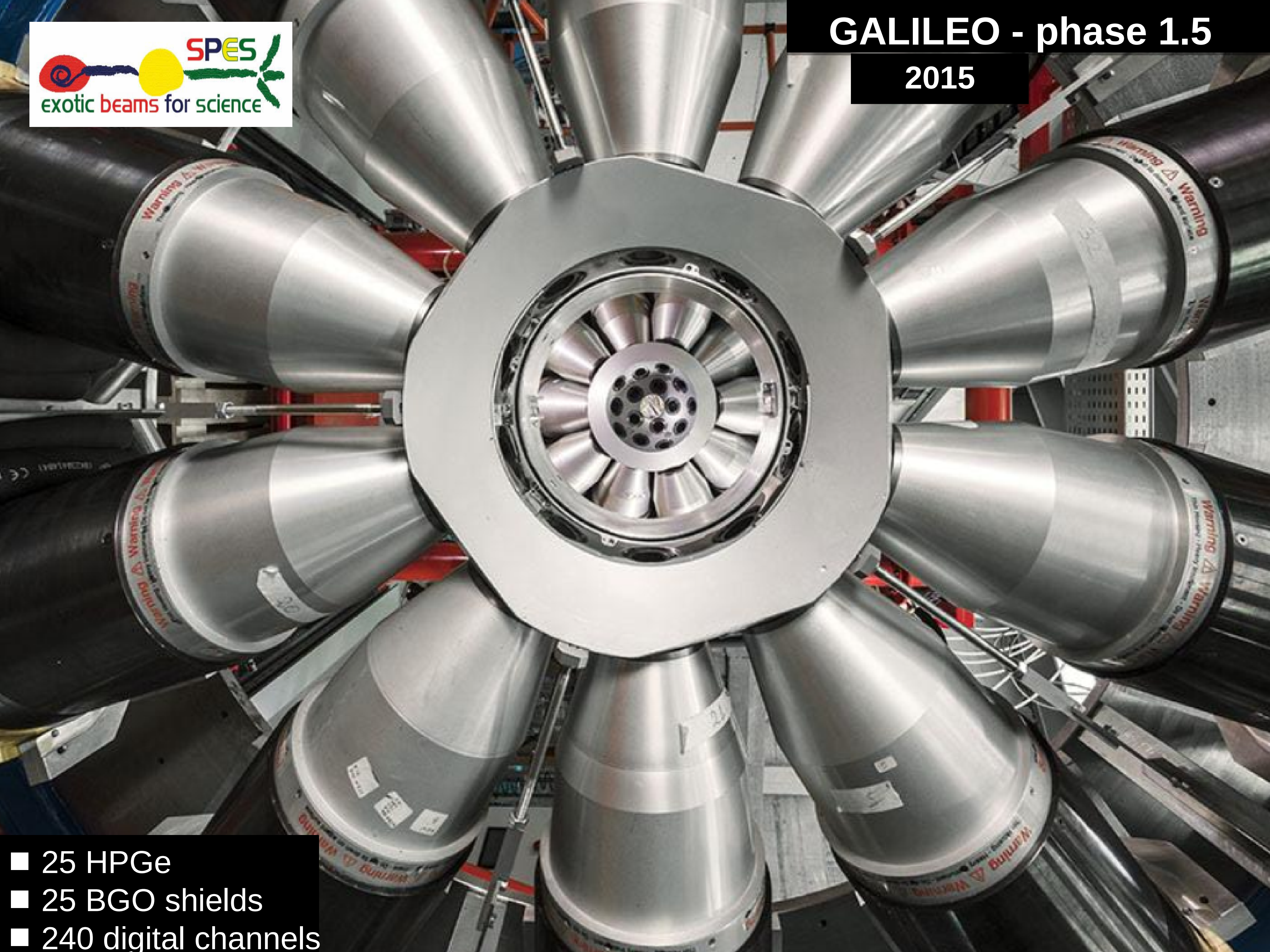
2015 - ...



AGATA 2008 → 2019..

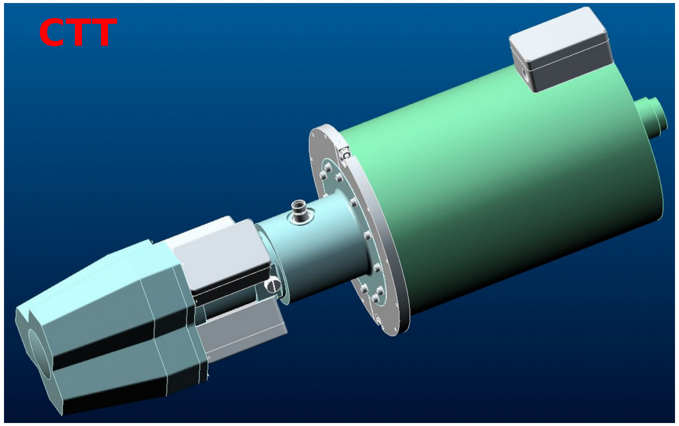
GALILEO - phase 1.5

2015

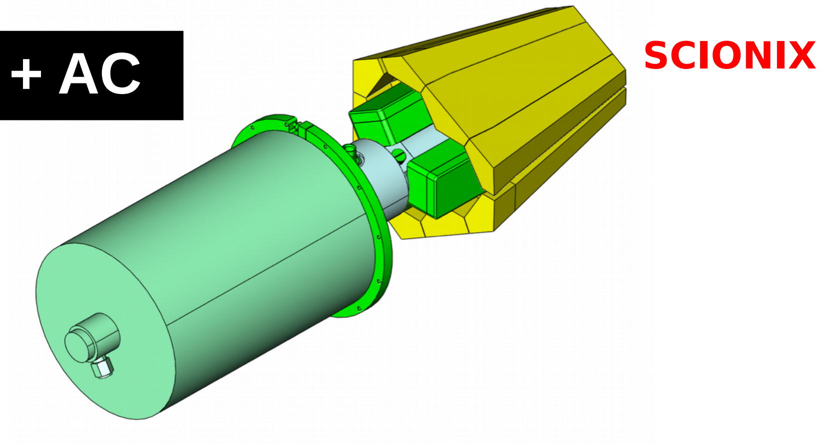


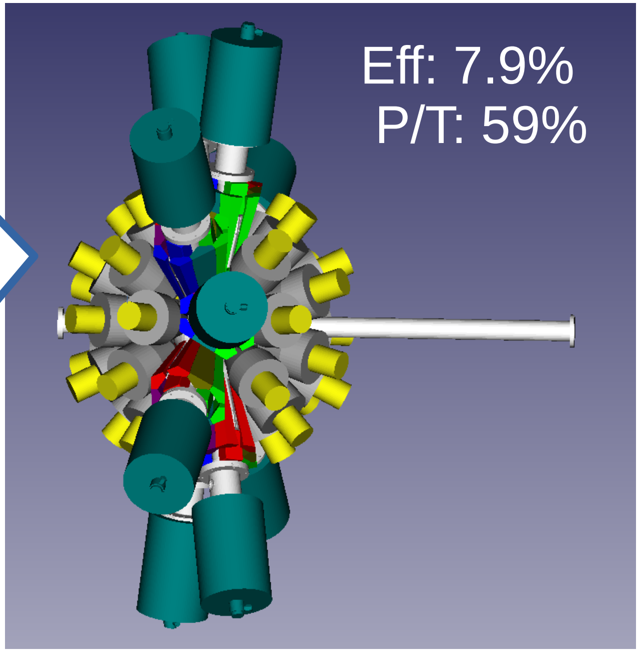
- 25 HPGe
- 25 BGO shields
- 240 digital channels

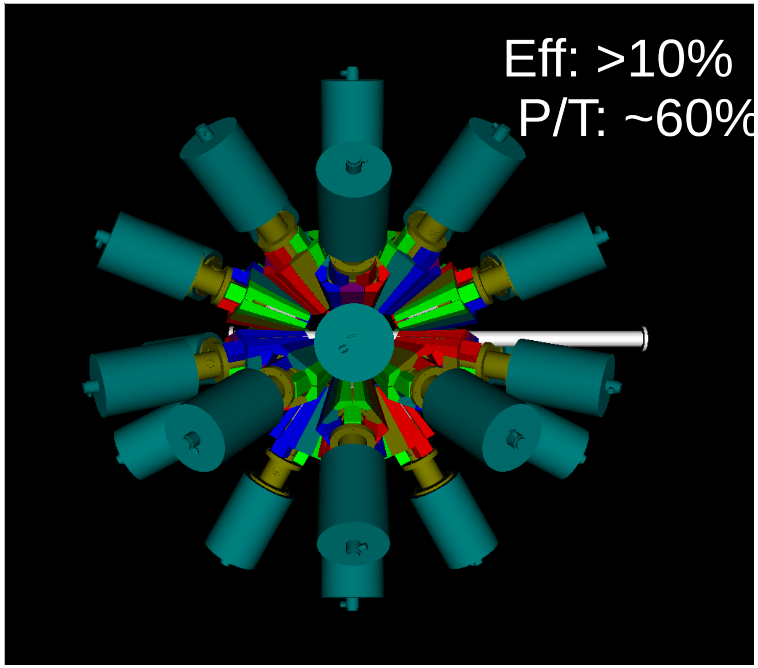
The GALILEO – phase 1 → 2 → 3



TC + AC

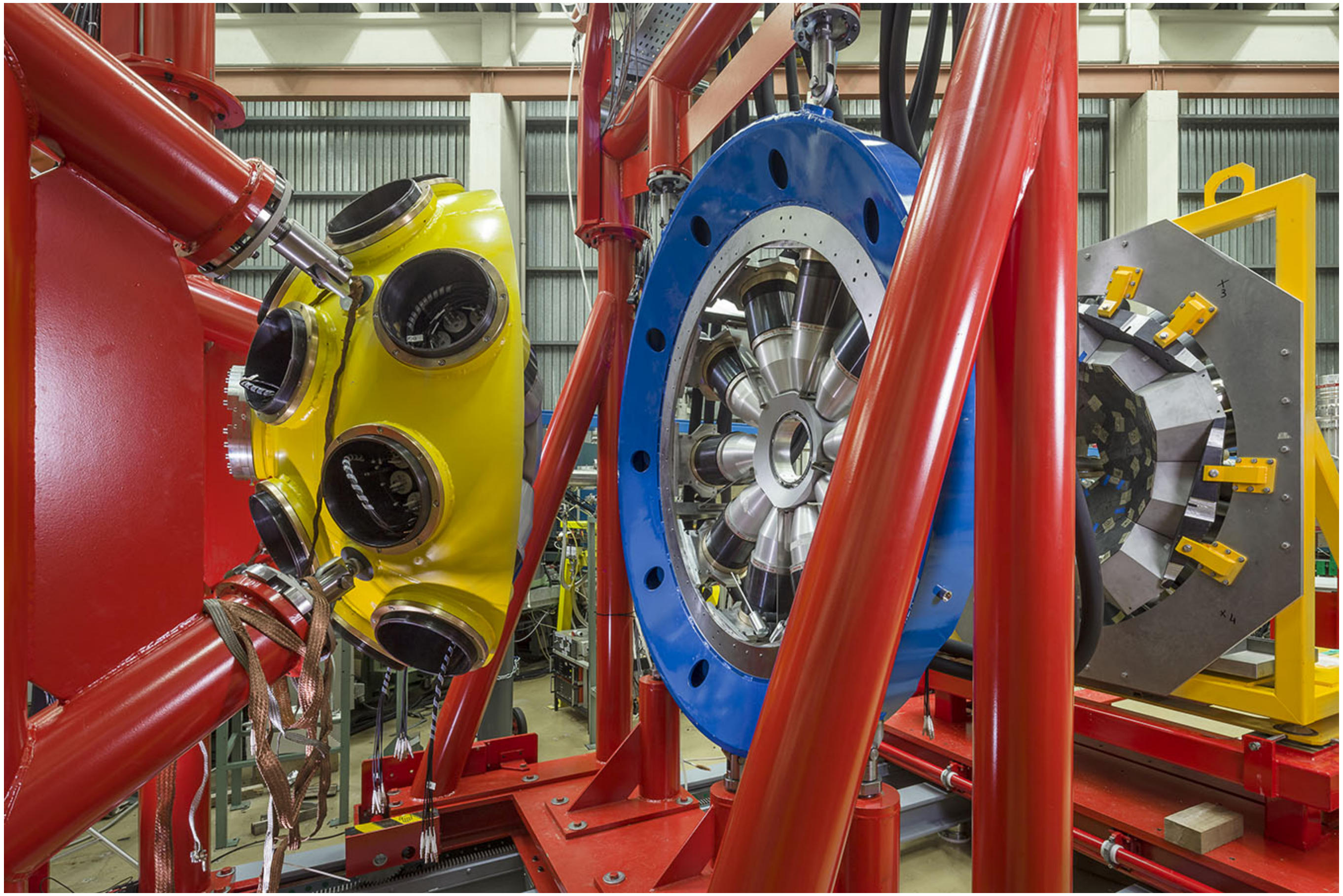




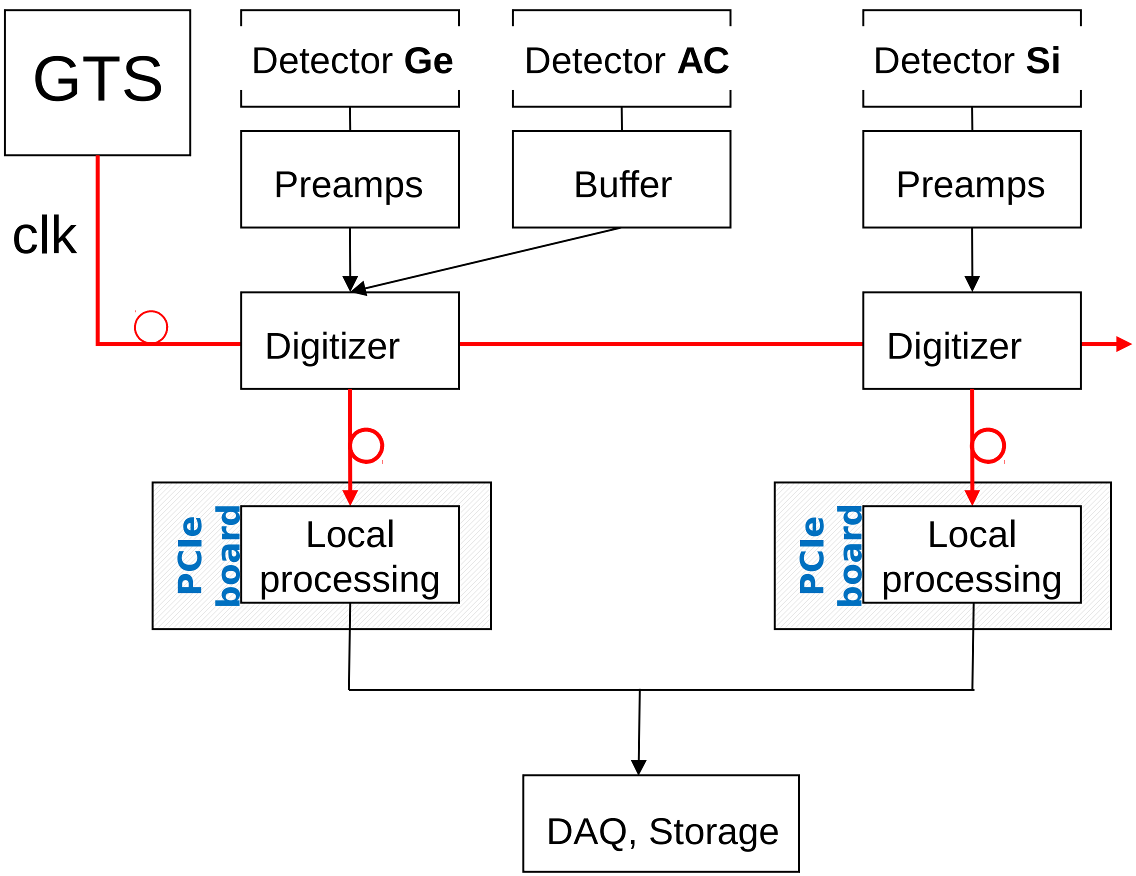


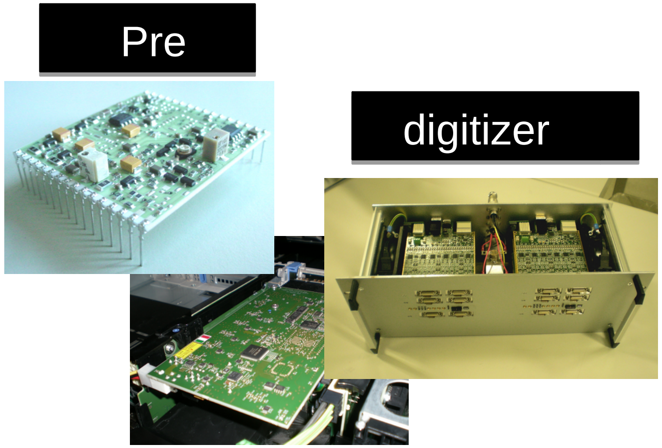
Phase 1 - The present implementation

■ 25 HPGe + AC + NW



GALILEO digital electronics





- **HPGe,AC, Anc. digitized**
- **Branches are sync by GTS.**
- **Trigger-less operation**
- **240 channels available**
- **Typical rate ~ 20 kHz/det**
- **Max rate ~ 50 kHz/det**

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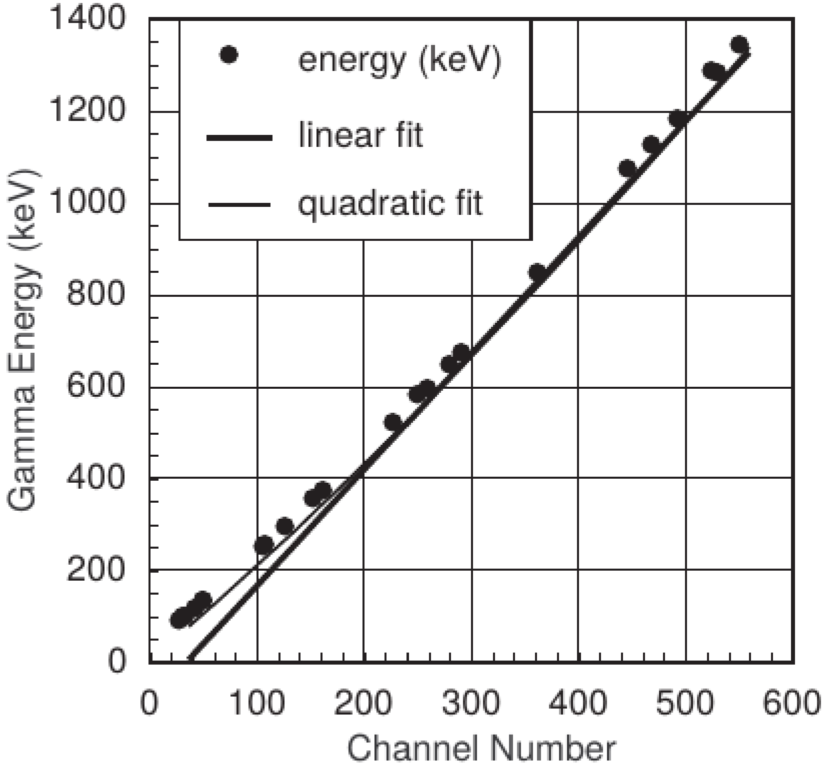
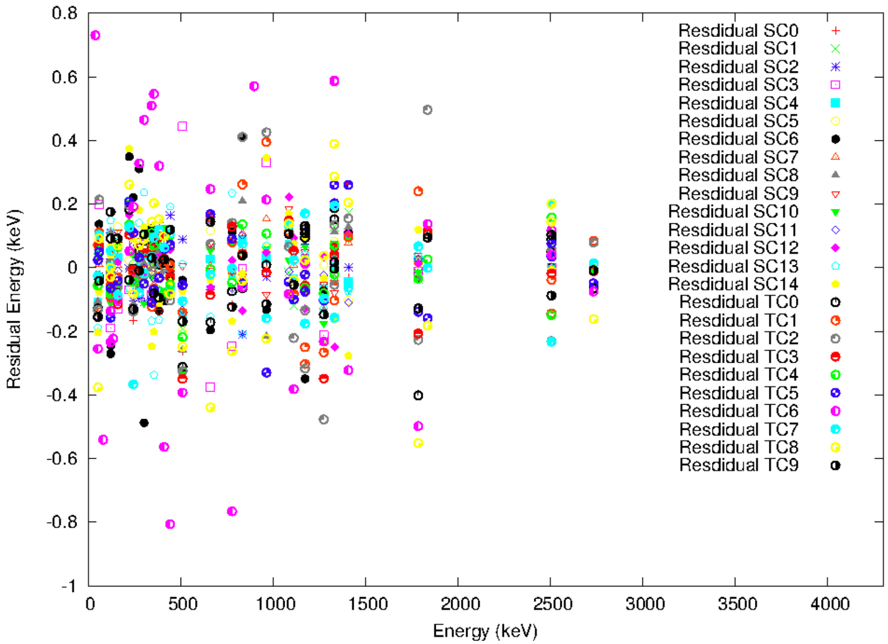
- Energy calibration [133Ba, 60Co, 152Eu]
- Relative efficiency calibration
- $\gamma\gamma$ correlation matrix [152Eu]
- Angular correlations [60Co]

Energy Calibration



- Gaussian Fit full energy peaks → centroids
- Polynomial fit of the centroids positions
- Check residues
- …



- Non linearities (INL, DNL)
- Cross talk (integral and differential)

(Some) calibration sources

Primordial radioisotopes that emit gammas

^{232}Th series	1.40 x 10^{10} yrs	α, β^-, γ	rich gamma spectrum
^{238}U series	7.04 x 10^{8} yrs	α, β^-, γ	Rn daughters produce gammas above 200 keV
^{235}U series	4.47 x 10^{9} yrs	α, β^-, γ	almost all gammas less than 220 keV
^{40}K	1.28 x 10^{9} yrs	α, β^-, γ	511 keV (11%), 1461 keV (11%)
^{138}La	1.05 x 10^{11} yrs	α, β^-, γ	1436 keV (70%), 789 keV (30%)
^{176}Lu	3.6 x 10^{10} yrs	α, β^-, γ	307 keV (100%), 202 keV (100%), 88 keV (100%)

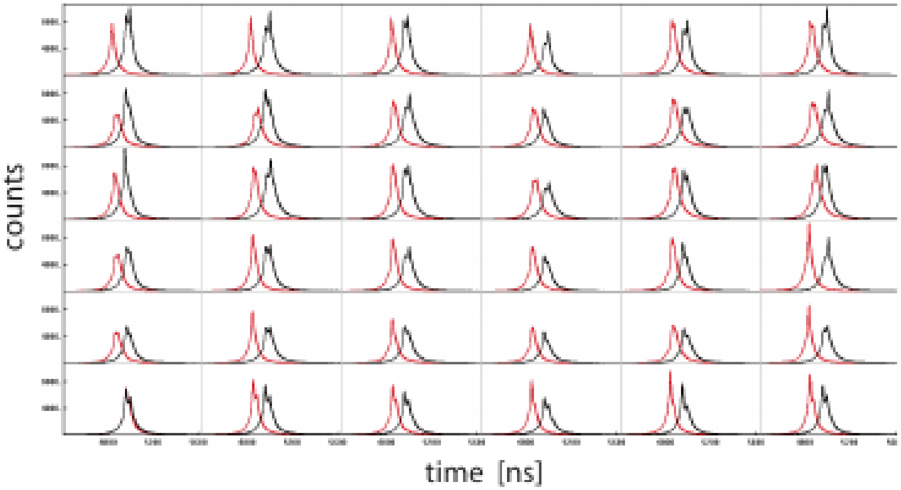
Some radioisotopes from fission events that emit gammas

^{54}Mn	313 days	β^-, γ	834.8 keV
^{60}Co	5.27 yrs	β^-, γ	1332.5 keV, 1173.2 keV
^{95}Zr	64.02 days	β^-, γ	756.7 keV,724.2 keV
^{95}Nb	34.97 days	β^-, γ	765.8 keV
^{131}I	8.040 days	β^-, γ	364.5 keV
^{134}Cs	2.05 yrs	β^-, γ	795.8 keV, 604.6 keV
^{137}Cs	30.17 yrs	β^-, γ	661.6 keV
^{133}Ba	10.53 yrs	β^-, γ, EC	356 keV, 81 keV, 303 keV
^{141}Ce	32.5 days	β^-, γ	145.4 keV
^{144}Ce	284.6 days	β^-, γ	133.5 keV, 80.1 keV

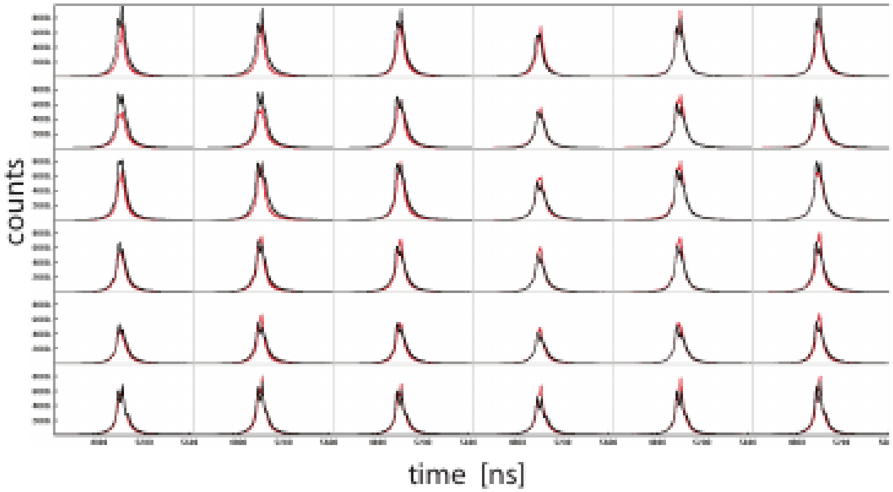
^{133}Ba	10.53 yrs	β^-, γ, EC	356 keV, 81 keV, 303 keV
^{141}Ce	32.5 days	β^-, γ	145.4 keV
^{144}Ce	284.6 days	β^-, γ	133.5 keV, 80.1 keV

Time alignment

Two different dets



Same but aligned



- Time info is fundamental: background reduction, PSA..
- Good reference time (RF, dets with good timing). It depends on the available dets → 1ns down to 100ps
- Alignment

Efficiency: measured / produced

- Absolute Efficiency (INTR X GEOM): The ratio of the number of counts produced by the detector to the number of gamma rays emitted by the source (in all directions).

- Geometrical efficiency: the ration between the solid angle subtended by the source and the total solid angle

- Intrinsic Efficiency: The ratio of the number of pulses produced by the detector to the number of gamma rays striking the detector.

- Relative Efficiency: HPGe detectors are almost universally specified in terms of their relative full-energy peak efficiency compared to that of a 3 in. x 3 in. NaI(Tl) Scintillation detector at a detector to source distance of 25 cm at 1.33 MeV..

- Full-Energy Peak (or Photopeak) Efficiency: The efficiency for producing full-energy peak pulses only.

Efficiency: consideration

Problems

- Activity not reliable, Dead Time, Background …
- Best way is to count the available events with an external detector in coincidence used to trigger the acquisition
- Can use internal coincidences

Corrections:

- Multiple decays within the event time window
- Angular correlation between the 2 γ transitions (..^{60}Co)

Conventional technique to measure ε_p

1. **Activity** (A):

   Peak integral (measured) = $A \times BR_\gamma \times \varepsilon_{int}\varepsilon_{gem}$

2. **Sum spectrum** (singles):

$R_1 = A(\varepsilon_{1p}\varepsilon_{2p})/A(\varepsilon_{2p}) = \varepsilon_{1p}\varepsilon_{2p}/\varepsilon_{V1}\varepsilon_{2p} = \varepsilon_{1p}/\varepsilon_{V1} = \varepsilon_{1p}/(1-\varepsilon_{V1}/PT_1)$;

$PT=\varepsilon_{1p}/(\varepsilon_{1p}+\varepsilon_B)$ is the peak/total ratio, ε_{V1} is the prob not to detect anything

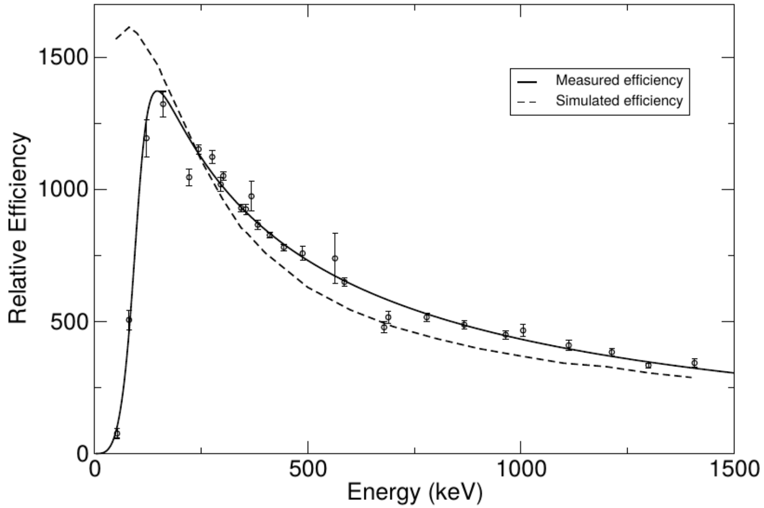
$\varepsilon_{1p} = R_1/(1+R_1/PT_1)$

$\varepsilon_{2p} = R_2/(1+R_2/PT_2)$

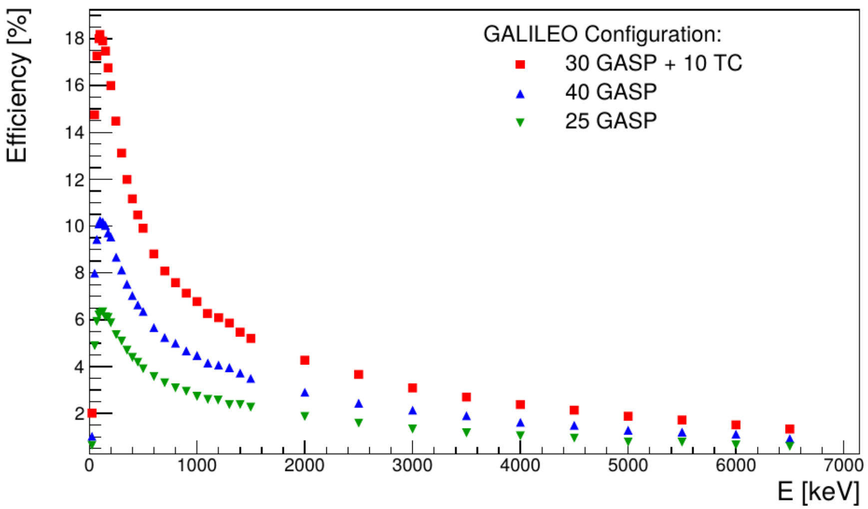
3. **Coincidences** (int/ext) to count the total number of counts

Efficiency curve

CLARA γ–ray spectrometer:
Measured vs simulations



Galileo γ–ray spectrometer:
simulations



A common model:

$$eff = exp\left(\left((A+B*x+C*x*x)*(-G)+(D+E*y+F*y*y)*(-G)\right)*\left(-\frac{1}{G}\right)\right)$$

$$x = ln\left(\frac{EG}{E1}\right)$$

E1~100keV

$$y = ln\left(\frac{EG}{E2}\right)$$

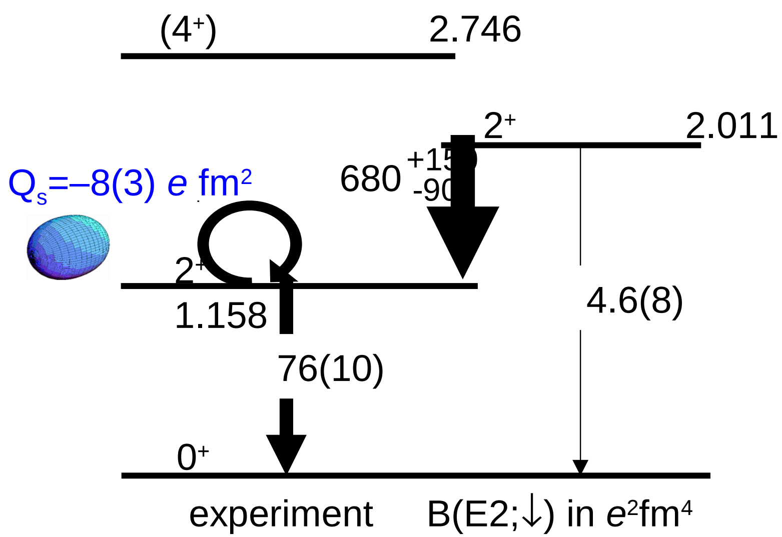
E2~1000keV

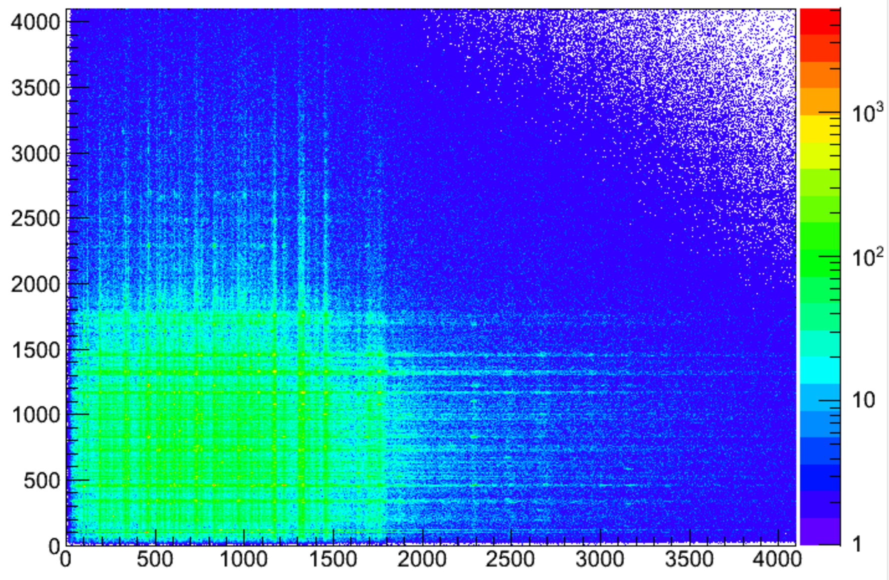
EG: gamma-ray energy

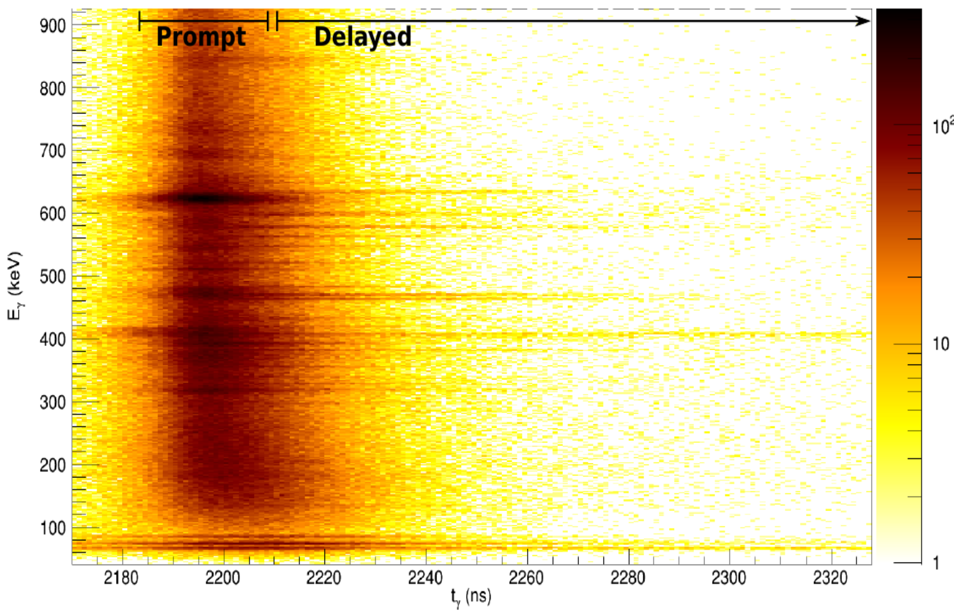
γγ matrices

- γγ promp coincidences
- Promp delayed possible

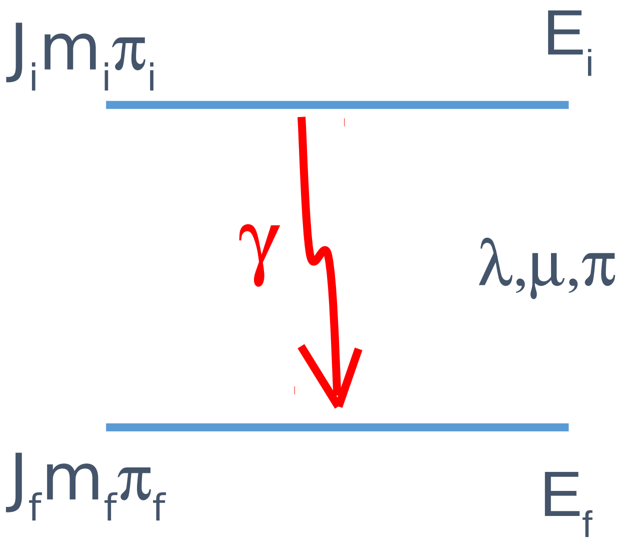
→ level scheme reconstruction







Angular distribution (AD)



Imagine the situation of a gamma ray that decays between two states, the initial one has a J^π value and the final one a J^π .

$$J_i = \mathbf{J} + J_f$$

$$\pi_i \times \boldsymbol{\pi} = \pi_f$$

Excited levels seen as charge distribution that emit radiation of multipolarity 2^λ

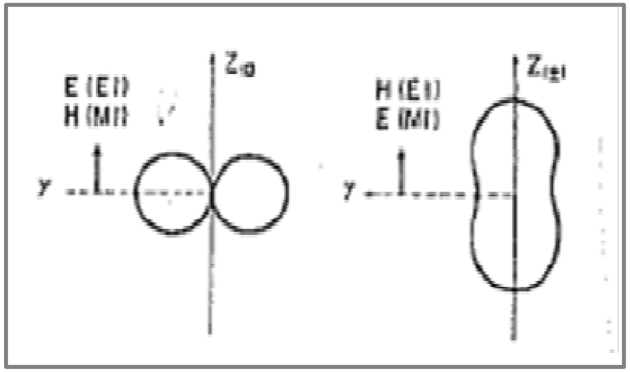
Angular distribution (AD)

Intensity of the emitted photons depends on the intensity of the Poynting vector either ~ $|\mathbf{E}|^2$ or ~ $|\mathbf{H}|^2$

For a given combination of λ,μ the angular distributions are identical for E or M transitions.
(recall properties of $X_{\lambda l m}$):

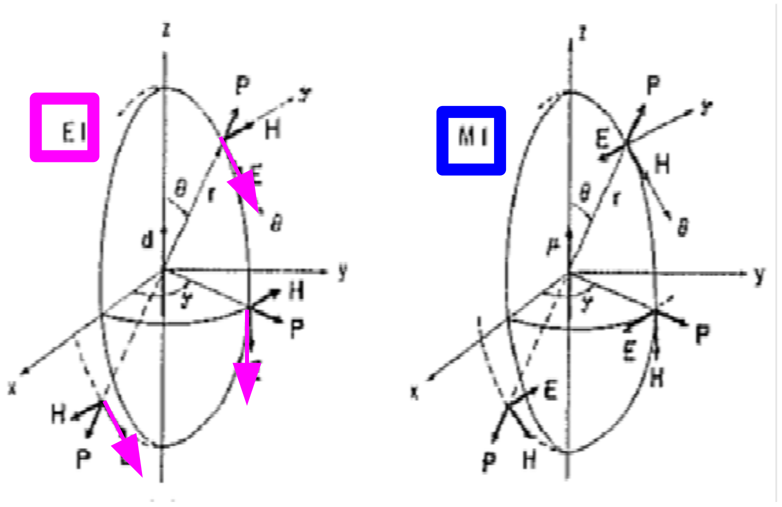
$$Z_{\lambda\mu}(\theta,\varphi) = | X_{\lambda\lambda\mu} |^2$$

1. Independent on azimuthal angle
2. Isotropic if μ takes equal weight
3. $Z_{\lambda\mu}(0)=0$
4. Symmetry with respect to π rotation



$Z_{1,0}$ $Z_{1,\pm1}$

Linear polarization (P)



Polarization depends on the character of the transition

Electric field polarized in the plane of dipole emission.
Similarly of the magnetic field but out of the plane

For $\theta = \pi/2$ the polarization can be measured at best.

P, AD from oriented states

Orientation: excited states formed in nuclear reactions are in general oriented with respect to the direction of projectiles. The degree of orientation depends on the formation process.

Nuclear state: j, m: -j,..., j, **P(m)**   [population parameter representation]

$$a_{\lambda\mu} = \sum_{m_i} |< j_i m_i \lambda\mu \mid j_f m_f >|^2 \, P(m_i)$$

γ–ray emission prob from a state with population parameter $P(m_i)$

γ–ray angular distribution:

$$W(\theta) = \sum_{m_i,\mu} |< j_i m_i \lambda\mu \mid j_f m_f >|^2 \, P(m_i) \, Z_{\lambda\mu}(\theta,\varphi)$$

Orientation can be experimentally obtained by a reaction, a magnetic field and a low temperature, or the observation of a gamma ray

Example

$J^\pi = 1^+$, m=0 decays to
$J^\pi = 0^+$, m=0 with a
$\sin^2\theta$ distribution.

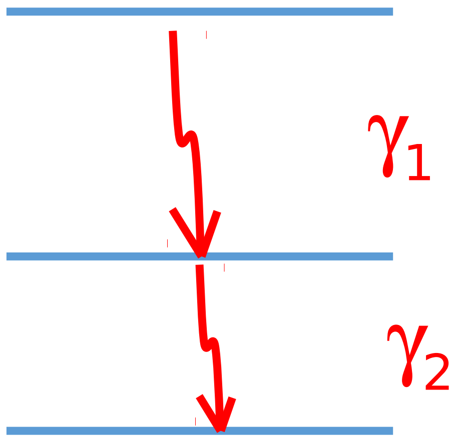
$J^\pi = 1^+$, m=±1 decays to
$J^\pi = 0^+$, m=0 with a
$\frac{1}{2}(1+\cos^2\theta)$ distribution.

So the total distribution: $\frac{1}{2}(1+\cos^2\theta) + \sin^2\theta + \frac{1}{2}(1+\cos^2\theta)$

$= 1+\cos^2\theta + \sin^2\theta$

$=2$ …flat, no ang dependence

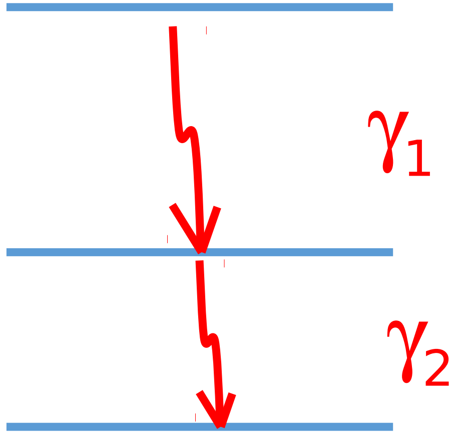
Example



Let's imagine we have two γ-rays which follow immediately after each other in the level scheme.

If we measure γ_1 or γ_2 in singles then the distribution will be isotropic (same intensity at all angles)… there is no preferred direction of emission.

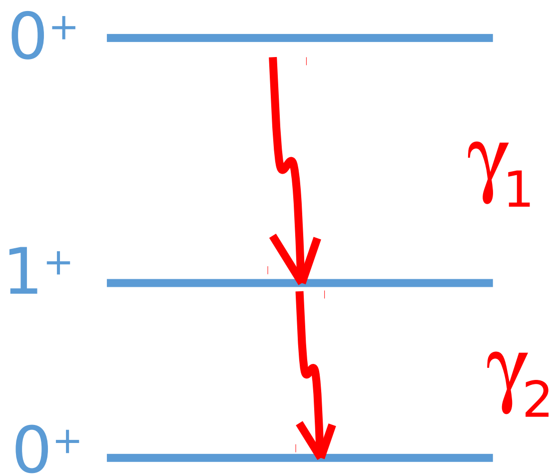
Example



Now imagine that we measure γ_1 or γ_2 in coincidence. We say that measuring the γ_1 causes the intermediate state to be aligned. We define the z direction as the direction of γ_1.

The angular distribution of the emission of γ_2 then depends on the spin/parities of the states involved and on the multipolarity of the transition.

Example



$J^\pi = 0^+, m=0$

$J^\pi = 1^+, \cancel{m=0}, m=\pm 1$

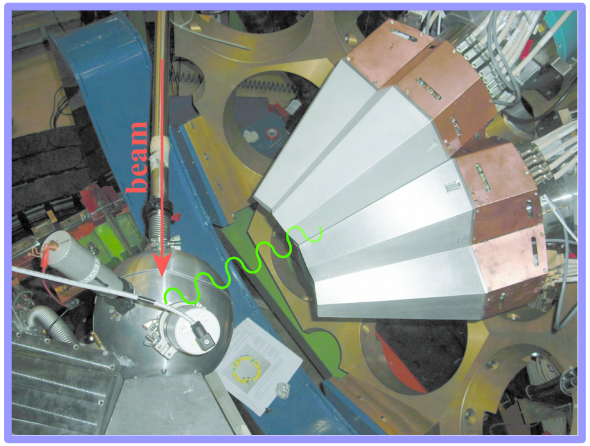
$J^\pi = 0^+, m=0$

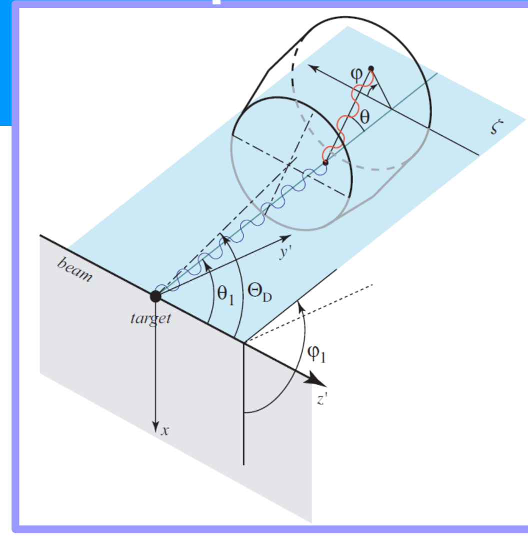
Since γ-ray transitions have angular momentum 1, γ_1 can only populate the $m = \pm 1$ substates of the $J^\pi = 1^+$ state.

Hence for γ_2 we only see the $m = \pm 1$ to $m=0$ part of the distribution i.e we see that the intensity measured as a function of angle (w.r.t γ_2) follows a $1+\cos^2\theta$ distribution.

AGATA modules as Compton Polarimeters

Partially-polarized 555.8-keV and 433.9-keV lines
in ^{104}Pd and ^{108}Pd [+unpolarized ^{137}Cs source].

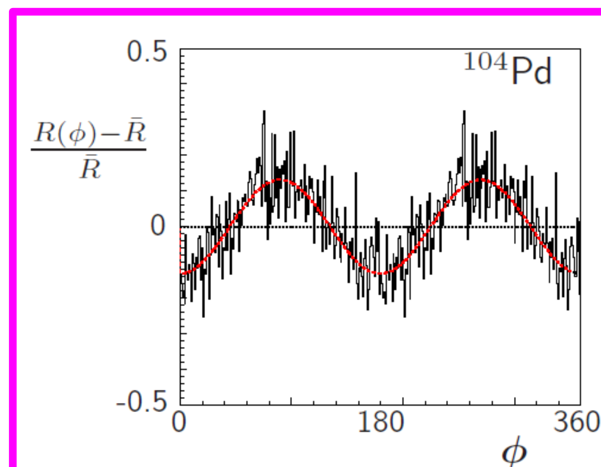




$$\bar{\sigma}_C(\theta,\varphi) = \frac{r_0^2}{4}\left(\frac{E'_\gamma}{E_\gamma}\right)^2\left[\frac{E_\gamma}{E'_\gamma} + \frac{E'_\gamma}{E_\gamma} - \sin^2\theta\,\underline{(1+P\cos 2\varphi)}\right]$$

GOSIA

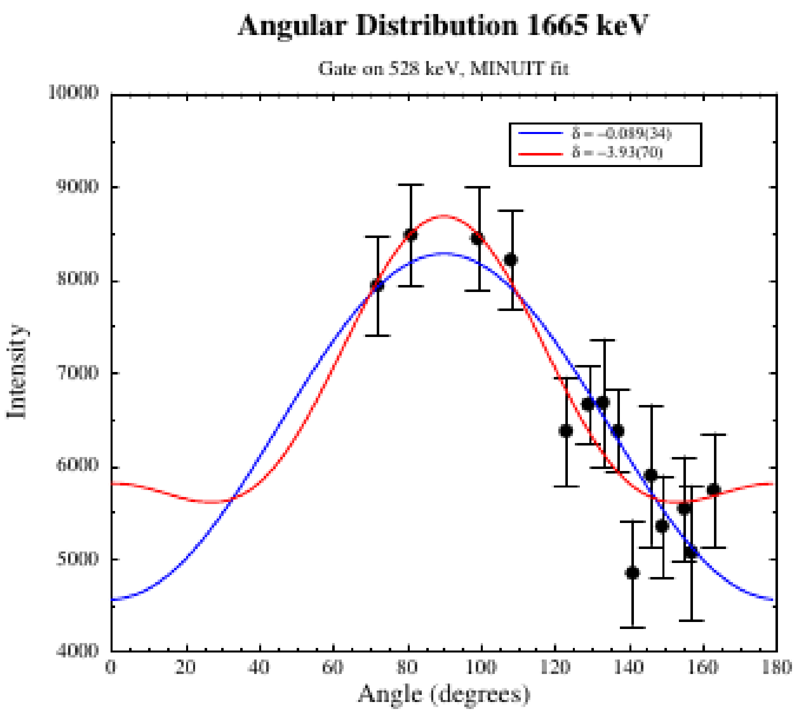
$$\boxed{\frac{\mathrm{d}N}{\mathrm{d}\varphi}} = a_0 + \boxed{a_2}\cos(2\varphi)\ .$$

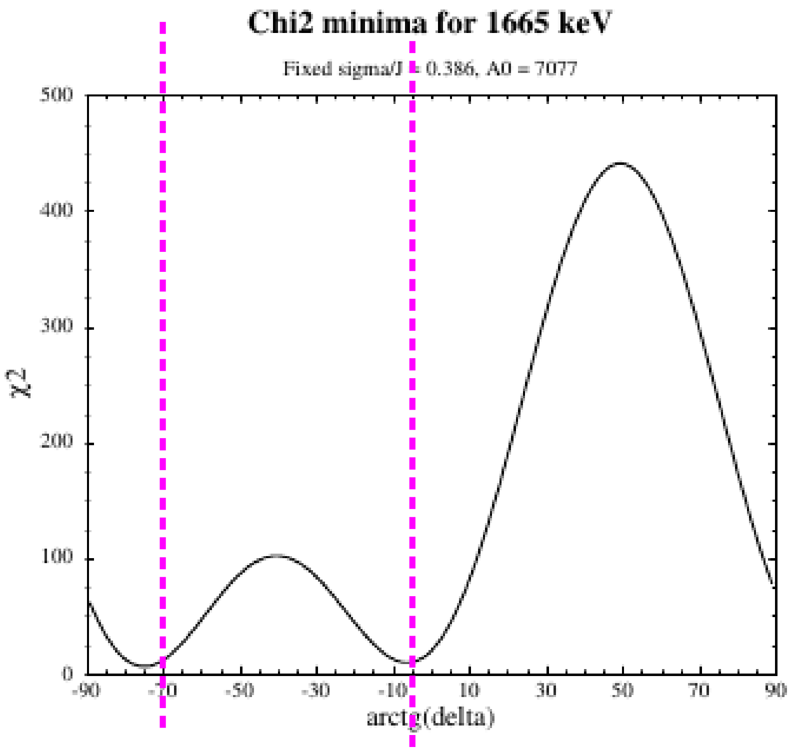


Analyzing power: 0.48

P.G. Bizzeti et al., Eur.Phys.J. A51 (2015) no.4, 49

AD and mixing ratio, ^{64}Ge (N=Z)



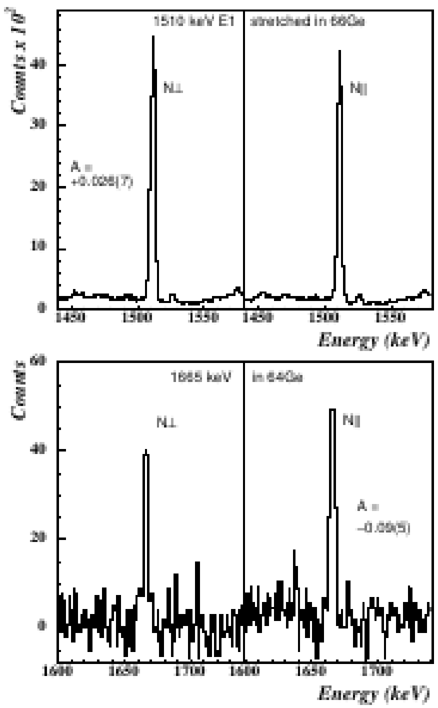


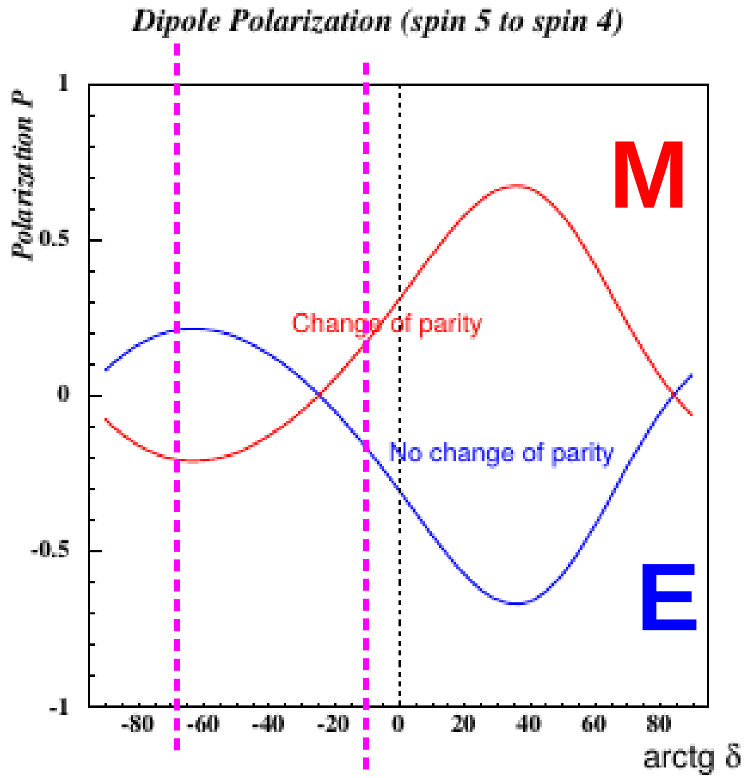
Si trovano due minimi del χ^2, per $\delta = -0.089$ e per $\delta = -3.93$.
Il secodo minimo è piú profondo, ma entrambi i valori di δ sono compatibili coi risultati sperimentali.

Polarization

^{66}Ge

^{64}Ge





Asimmetria negativa A=-0.09(5).

Solo la combinazione E1/M2 per $\delta = -3.9$

è compatibile con la polarizzazione lineare osservata.

La transizione $5^- \rightarrow 4^+$ è quasi tutta M2

e $B(E1) = 2.47 \times 10^{-7}$ W.u.

Recap

- Energy calibration [133Ba, 60Co, 152Eu]
- Relative efficiency calibration
- $\gamma\gamma$ correlation matrix [152Eu]
- Angular correlations [60Co]

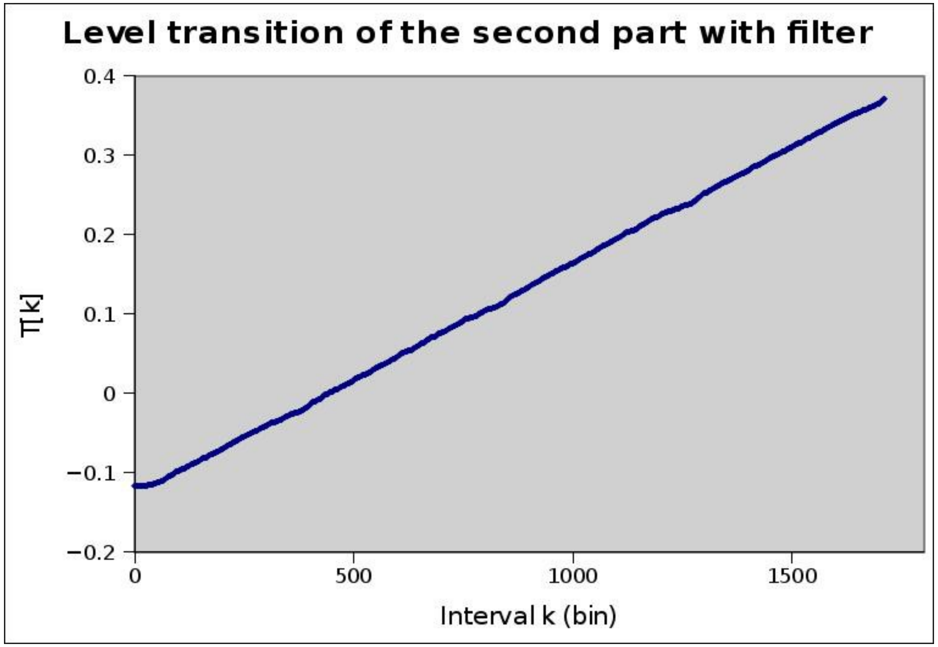
Bibliography

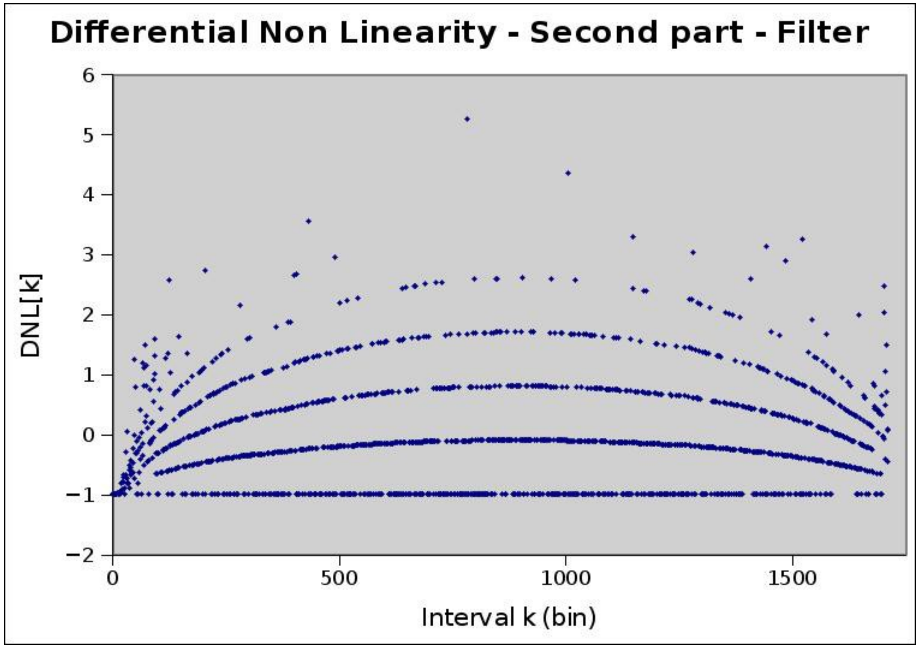
- K.S.Krane, Chapter 10, “Introductory Nuclear Phyisics”
- H.Morinaga, T.Yamazaki, Chapter 2 “In-beam gamma-ray spectroscopy”
- W.D.Hamilton: Chapters 12,14 and 15 in “The electromagnetic interaction in nuclear spectroscopy”
- P.G. Bizzeti et al., Eur.Phys.J. A51 (2015) no.4, 49

Backup slides

Noise and distortion: INL & DNL

Deviation between two analog values corresponding to adjacent input digital values.





$$\mathrm{DNL(i)} = \frac{V_{\mathrm{out}}(i+1) - V_{\mathrm{out}}(i)}{\text{ideal LSB step width}} - 1$$

Crosstalk model

AC equivalent detector

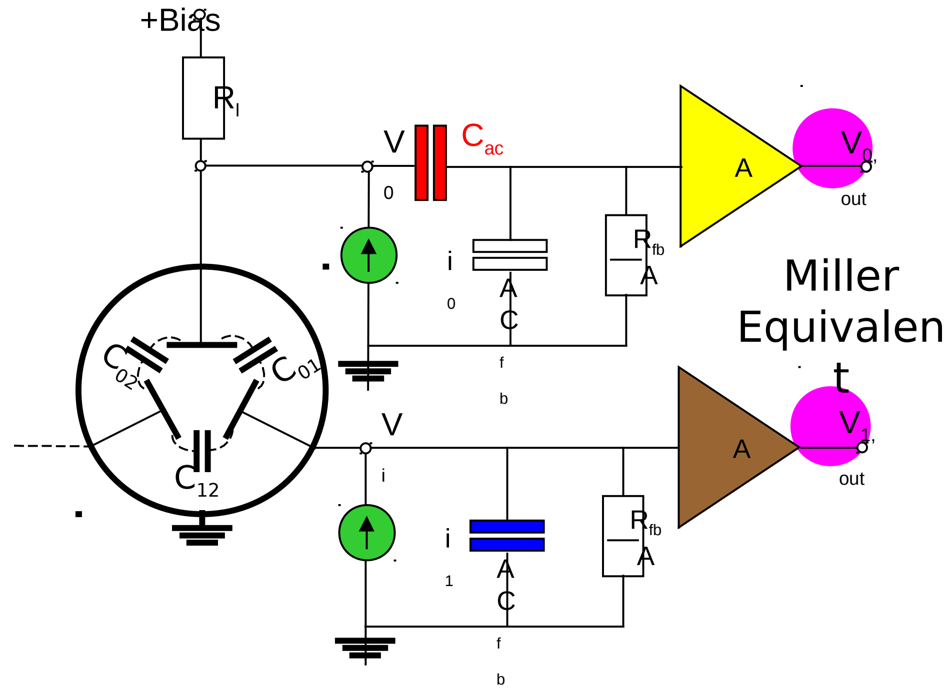
$$i_h = \underbrace{\sum_{i=1}^{N} q_i \mathbf{v}_i(\mathbf{r}_i)\cdot \mathbf{F}'_{1h}(\mathbf{r}_i)}_{\text{Ramo theoreme}} - \underbrace{\sum_{k=1}^{n} C_{hk}\frac{\partial V_k}{\partial t}}_{\text{Extension}}$$

Ramo theoreme - Extension

B. Pellegrini – Phys Rev B 34,8 (86) p. 5921

E. Gatti et al – NIM 193 (82) p. 651

Crosstalk is intrinsic property of segmented detectors !

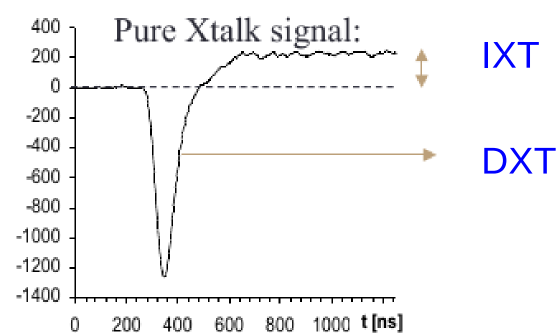


Segment-to-Core

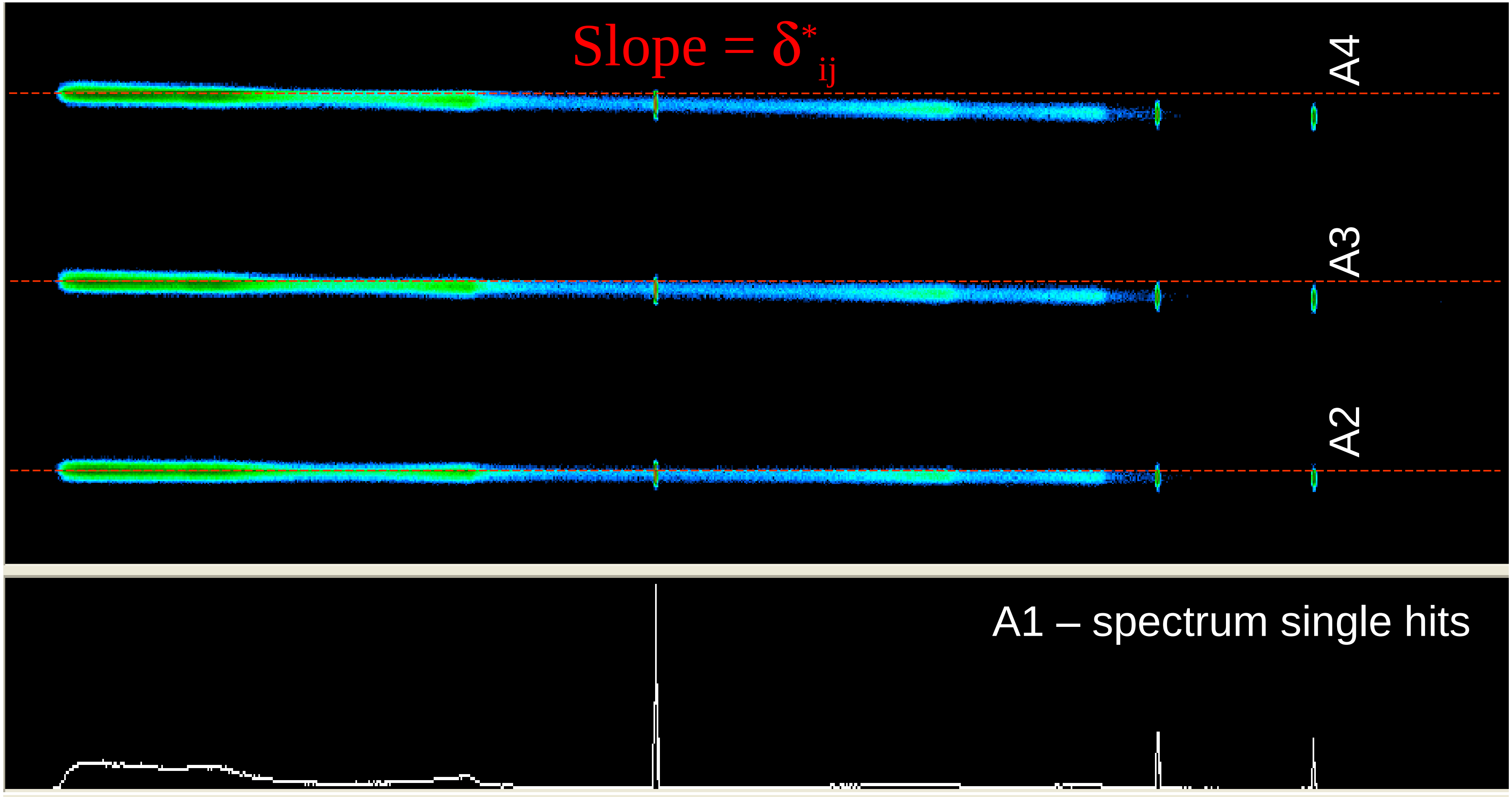
$$\vec{V}_{out} \cong \frac{1}{sC_{fb}} \begin{pmatrix} 1 & -C_{01}/AC_{fb} & -C_{02}/AC_{fb} \\ -C_{01}/C_{ac} & 1 & -C_{12}/AC_{fb} \\ -C_{02}/C_{ac} & -C_{12}/AC_{fb} & 1 \end{pmatrix} \vec{i}$$

Core-to-Seg ~ 1pF/1000pF

Segment-to-Segment ~1pF/(10000 · 1pF)

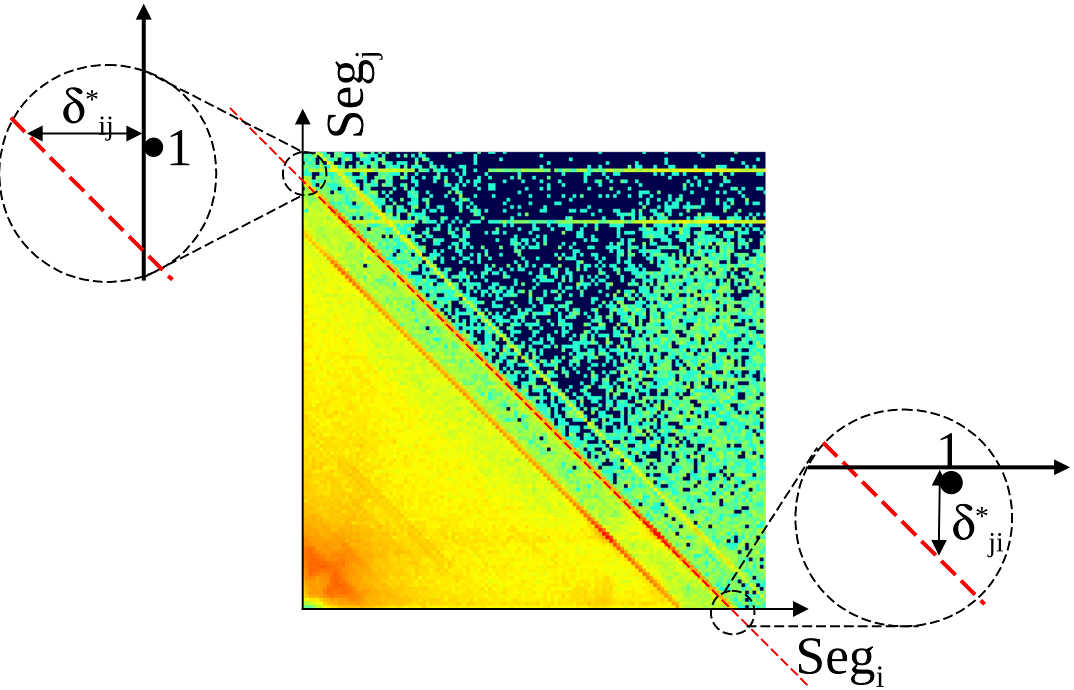


Crosstalk parameters from singles



Crosstalk parameters from doubles

Correlation matrix between each pair of segment ij



Ideal system – no cross talk

$$\begin{bmatrix} E_{core} \\ \hdashline E_{seg1} \\ E_{seg2} \\ E_{seg3} \end{bmatrix}_{meas} = \begin{bmatrix} 1 & 1 & 1 \\ \hdashline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} E_{seg1} \\ E_{seg2} \\ E_{seg3} \end{bmatrix}_{true}$$

identity

Cross talk correction

$$\begin{bmatrix} E_{core} \\ \hdashline E_{seg1} \\ E_{seg2} \\ E_{seg3} \end{bmatrix}_{meas} = \begin{bmatrix} 1+\delta^*_{01} & 1+\delta^*_{02} & 1+\delta^*_{03} \\ \hdashline 1 & \delta^*_{12} & \delta^*_{13} \\ \delta^*_{21} & 1 & \delta^*_{23} \\ \delta^*_{31} & \delta^*_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} E_{seg1} \\ E_{seg2} \\ E_{seg3} \end{bmatrix}_{true}$$

Multipole expansion

Potential from a charge distribution $\varrho(\boldsymbol{x}')$

$$\Phi(\boldsymbol{x}) = \frac{1}{\varepsilon_0}\sum_{l,m}\frac{1}{2l+1}\left[\int Y^*_{lm}(\theta',\varphi')\, r'^l \varrho(\boldsymbol{x}')\, d^3x'\right]\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}.$$

q_{lm} multipole moments: q_{00}, q_{11}, q_{10} etc

Potential depends on the charge distribution

em field

The emission of a photon is equivalent to generate an em wave that respects the Maxwell equation

$$\begin{cases} \nabla \cdot \vec{D} = \rho \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{cases} \Rightarrow$$

$$\boldsymbol{E}\left(\boldsymbol{r},t\right) = \boldsymbol{E}\left(\boldsymbol{r}\right)e^{-i\omega t} + \boldsymbol{E}^{*}\left(\boldsymbol{r}\right)e^{i\omega t},$$

$$\boldsymbol{H}\left(\boldsymbol{r},t\right) = \boldsymbol{H}\left(\boldsymbol{r}\right)e^{-i\omega t} + \boldsymbol{H}^{*}\left(\boldsymbol{r}\right)e^{i\omega t}.$$

Poynting vector

$$\boldsymbol{P} = \left(\frac{c}{4\pi}\right)\boldsymbol{E}\times\boldsymbol{H}.$$

em field angular momentum and parity

J_z operator (two components), infinitesimal rotation about the z axis

$$J_z = \underbrace{-i\left(\boldsymbol{r}\times\nabla\right)_z}_{\text{Angular momentum L}} + \underbrace{\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{Intrinsic spin S}}$$

S_γ=1, intrinsic spin of the photon

Parity π (opposite for M and E):

$$\pi\left(\boldsymbol{E}\right)\times\left(-1\right)=\pi_i\pi_f,$$

dipole

$$\pi\left(\boldsymbol{H}\right)=\pi_i\pi_f.$$

general

Electric radiation(Eλ): $\pi_i\pi_f = (-)^{\lambda}$

Magnetic radiation(Mλ): $\pi_i\pi_f = (-)^{\lambda+1}$

em field: intrinsic spin

$S=1$ $S_z=-1,0,1$

eigenvalues	eigenfunctions
$h = +1$	$\xi_1 = N(e_x + ie_y)$
$h = 0$, non physical	$\xi_0 = e_z$
$h = -1$	$\xi_{-1} = N(e_x - ie_y)$

$E(r,t;\xi_1) = \xi_1 e^{-iwt} + \xi_1^* e^{iwt} \sim e_x \cos(wt) + e_y \sin(wt)$: elec. dip.

$E(r,t;\xi_{-1}) \sim$ magnetic dipole oscillation

em field: total angular momentum

J = L + S, total angular momentum

eigenvalues

$$\boldsymbol{L}^2 Y_{lm} = l(l+1)Y_{lm},$$

$$L_z Y_{lm} = m Y_{lm};$$

$$\boldsymbol{J}^2 X_{\lambda l \mu} = \lambda(\lambda+1) X_{\lambda l \mu},$$

$$J_z X_{\lambda l \mu} = \mu X_{\lambda l \mu}.$$

eigenfunctions

$$Y_{lm}(\theta,\varphi) = e^{im\varphi} N_{lm} P_{lm}(\cos\theta),$$

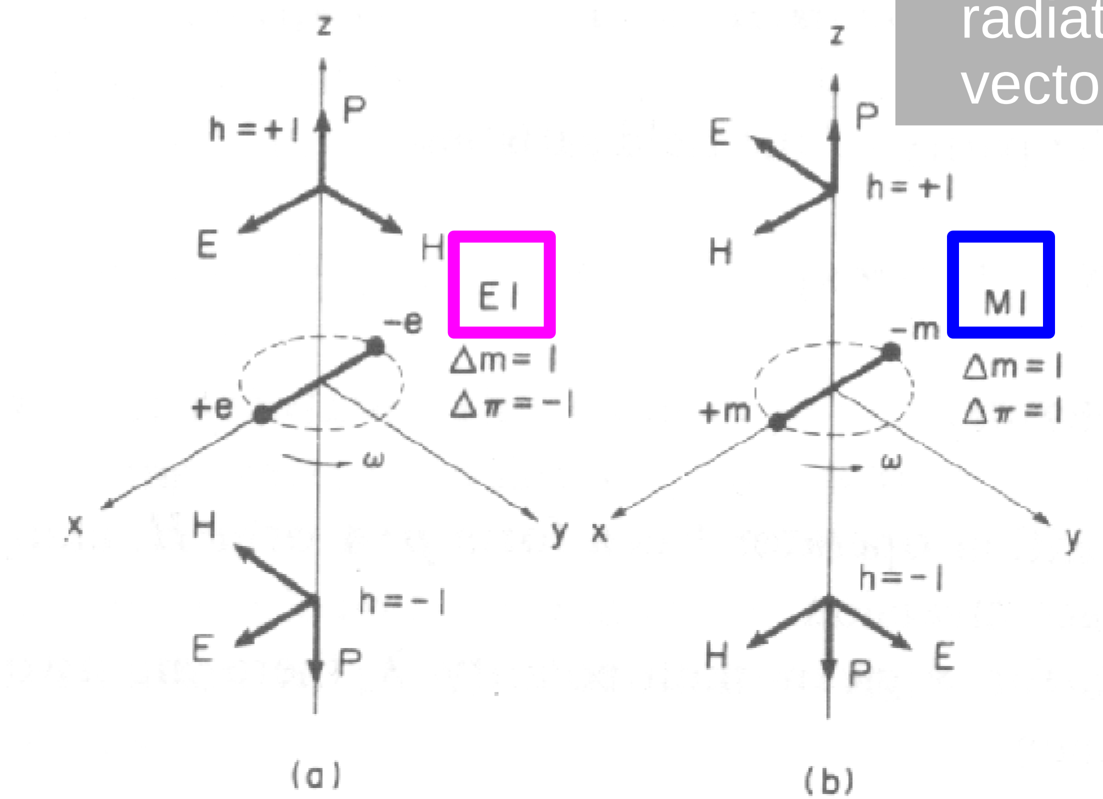
$$X_{\lambda lm} = \sum_m < l\mu - m 1m \mid \lambda\mu > Y_{l\mu - m}\xi_m;$$

θ,φ angular coordinates, in a (r,θ,φ) s.o.r.
Y_{lm} spherical harmonics
N_{lm} normalization constants
P_{lm} spherical functions based on Legendre polynomials
ξ eigenfunction of intrinsic spin

em field angular momentum and parity

$$\boldsymbol{E}_{\lambda\mu}(M\lambda) = -\boldsymbol{H}_{\lambda\mu}(E\lambda) = \boldsymbol{X}_{\lambda l\mu}.$$

- X satisfies the transverse condition and has a $(-1)^{\lambda}$ parity
- Electric field in a magnetic radiation is opposite to the H vector and viceversa



Linear polarization (P)

Assume photon emitted along *x* (*z* quantization axis), σ=E,M

$$P^{||}_{\lambda\mu}(\sigma\lambda) = \frac{|\,\underline{\boldsymbol{e}_z \cdot} \boxed{\boldsymbol{E}}_{\lambda\mu}(\sigma\lambda)\,|^2}{|\,\boldsymbol{E}_{\lambda\mu}(\sigma\lambda)\,|^2} = \frac{|\,\underline{\boldsymbol{e}_y \cdot} \boxed{\boldsymbol{H}}_{\lambda\mu}(\sigma\lambda)\,|^2}{|\,\boldsymbol{H}_{\lambda\mu}(\sigma\lambda)\,|^2}$$

$$P^{\perp}_{\lambda\mu}(\sigma\lambda) = \frac{|\,\boldsymbol{e}_y \cdot \boldsymbol{E}_{\lambda\mu}(\sigma\lambda)\,|^2}{|\,\boldsymbol{E}_{\lambda\mu}(\sigma\lambda)\,|^2} = \frac{|\,\boldsymbol{e}_z \cdot \boldsymbol{H}_{\lambda\mu}(\sigma\lambda)\,|^2}{|\,\boldsymbol{H}_{\lambda\mu}(\sigma\lambda)\,|^2}$$

Properties:

$$P^{||}_{\lambda\mu}(\sigma\lambda) + P^{\perp}_{\lambda\mu}(\sigma\lambda) = 1.$$

$$P^{||}_{\lambda\mu}(E\lambda) = P^{\perp}_{\lambda\mu}(M\lambda) = 1 - P^{||}_{\lambda\mu}(M\lambda)\,,$$

eg dipole:

$$P^{||}_{10}(M1) = P^{||}_{11}(E1) = 0,$$

$$P^{||}_{10}(E1) = P^{||}_{11}(M1) = 1.$$

In general , if many μ components are present, the polarization is the weighted sum over each $P_{\lambda\mu}$

P, AD from oriented states

γ multipolarity: $|j_i-j_f|<j<|j_i+j_f|$ [Statistical tensor representation]

statistical tensor account for a different population:

$$\rho_k(j) = \sqrt{2j+1}\sum_m (-)^{j-m} < jmj-m \mid k0 > P(m)$$

γ–ray angular distribution:

$$W(\theta) = \sum_k A_k(j_i\lambda\lambda' j_f) P_k(\cos\theta)$$

A_k depends on the statistical tensor and the mixing ratio δ, P_k Legendre polynomials

Mixing ratio:

$$\delta = \frac{< j_f \parallel \lambda' \parallel j_i >}{< j_f \parallel \lambda \parallel j_i >}.$$

Ratio between the transition matrix element for two multipolaries

While a an orientation of states ji is represented by 2ji+1 population parameters, only few ρ_k suffice to determine the AD

Angular correlation

γ cascade of 2 transitions, first randomly oriented

statistical tensor account for a different population:

$$\rho_k(j) = \sqrt{2j+1} \sum_m (-)^{j-m} < jmj-m \mid k0 > P(m)$$

γ–ray angular distribution:

$$W(\theta) = \sum_k A_k(j_i \lambda \lambda' j_f) P_k(\cos\theta)$$

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Ratio between the transition matrix element for two multipolaries

While a an orientation of states ji is represented by 2ji+1 population parameters, only few ρ_k suffice to determine the AD

Linear polarization from oriented states

γ with fixed σ,λ,μ observed at θ:π/2 with respect to the orientation axis

Pol. vector

$$P^{\|}(\sigma\lambda) = N \sum_{\mu} \underbrace{a_{\lambda\mu}}_{\text{Emission prob}} \underbrace{Z_{\lambda\mu}\left(\frac{\pi}{2}\right)}_{\text{AD}} \boxed{P^{\|}_{\lambda\mu}(\sigma\lambda)}$$

For γ–ray angular distribution:

$$\sum_{k} A_k P_k(\cos\theta) = \sum_{\mu} a_{\lambda\mu} Z_{\lambda\mu}(\theta)$$

$a_{\lambda,\mu}$ can be expressed in terms of A_k. From where we can get the ratio between the parallel and orthogonal pol. Vectors. For λ=1:

$$\frac{P^{\|}(E1)}{P^{\perp}(E1)} = \frac{P^{\perp}(M1)}{P^{\|}(M1)} = \frac{1 + A_2}{1 - 2A_2}$$

AD+POLARIZATION → UNIQUE DETERMINATION OF SPIN, PARITIES, MULTIPOLARITIES

Experimentally the linear polarization are measured in the terms of Compton scattering, with a polarimeter of polarization sensitivity ($N^{\|}/N^{\perp}$)