In this note the possibility to include a test beam facility, in the DAΦNE accelerator complex, is discussed.

Between two injections, the DAΦNE-LINAC can deliver the electron beam into an existing hall (see Fig. 1). This area, previously used as "Pion Test Facility", has an extension of about 100 m², it is surrounded by concrete walls, it has 20 ton crane capability and an independent entrance.

The e⁺ e⁻ DAΦNE-LINAC main features are:

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Energy</td>
<td>800 MeV</td>
</tr>
<tr>
<td>Conversion Energy</td>
<td>250 MeV</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Pulse duration</td>
<td>10 ns</td>
</tr>
<tr>
<td>Max curr./pulse</td>
<td>150 mA (10¹⁰ particles)</td>
</tr>
</tbody>
</table>

The main tasks, in order to put the test beam in operation, are:

- Transferline and diagnostic
- Civil Engineering (Hole through the concrete wall)
- Safety system upgrading.

The maximum intensity that can be used, without reinforcing the existing shielding, is under evaluation.

In the following, we describe the transport optics and, in some more details, the "single electron mode of operation" which, in our opinion, is the most interesting one for calibration purposes.
Figure 1: DAΦNE accelerator complex layout.
1- Test beam facility transport line

The complete transferline layout, from the LINAC to the Test Area Facility, is shown in Fig. 2. It consists of two DC bending magnets and three quadrupoles. It is achromatic and it is capable to transport, within a circular spot of 5 mm radius, a beam with a maximum energy of 800 MeV, an emittance \( \varepsilon = 5 \pi \) mm-mrad and an energy spread of \( \pm 1\% \). The beam energy will be monitored by the spectrometer which is upstream of \( M_1 \).

Fig. 3 shows the optical functions and the beam envelopes as calculated by the LEDA code [2], while Table 1 gives the 800 MeV maximum magnetic values of the transport line components.

In order to make the transferline tune-up easier, a beam position monitor, a beam current monitor and steering coils are located at the end of each bending magnet.

At the end of the beam line, a scintillator counter will discriminate, by pulse amplitude analysis, between single electron and many electrons.

### Table 1 - Transport line magnetics values @ 800 MeV

<table>
<thead>
<tr>
<th>Mag. length</th>
<th>( k^2 )</th>
<th>( G )</th>
<th>Aperture</th>
<th>( B_{\text{pole tip}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>(m(^2))</td>
<td>(T/m)</td>
<td>(m)</td>
<td>(T)</td>
</tr>
<tr>
<td>( Q_1 )</td>
<td>0.40</td>
<td>1.006</td>
<td>2.69</td>
<td>0.125</td>
</tr>
<tr>
<td>( Q_2 )</td>
<td>0.40</td>
<td>1.385</td>
<td>3.70</td>
<td>0.125</td>
</tr>
<tr>
<td>( Q_3 )</td>
<td>0.40</td>
<td>0.968</td>
<td>2.58</td>
<td>0.125</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mag. length</th>
<th>Bending angle</th>
<th>Radius</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>(m)</td>
<td>(m)</td>
<td>(T)</td>
</tr>
<tr>
<td>( M_1 = M_2 )</td>
<td>1.73</td>
<td>45(^\circ)</td>
<td>2.2</td>
</tr>
</tbody>
</table>
Figure 2: Transferline layout.
Figure 3: Optical functions and beam envelopes.
2 - Single electron mode.

It is convenient to start with the minimum beam current (~1 mA) which can be measured by the current monitors, in order to control the proper LINAC operation.

Being the number $N$ of particles in each LINAC pulse, for a current of 1 mA and a duration of 10 ns, given by

$$N = \frac{It}{e} \approx 6.24 \times 10^7$$

it will be necessary to reduce the number of particles, by a factor $P$ continuously variable in the range:

$$P \in [10^{-8}, 10^{-7}]$$

Afterwards, we will discuss the two alternative offered by the DAΦNE-LINAC to operate in single electron mode:

- to use the "secondary" beam produced by the electron/positron converter (max Energy ~ 550 MeV)
- to use the "primary" beam generated by the electron gun (max Energy ~ 800 MeV).

2.1 - Secondary beam option

2.1.1 - Particle number reduction method

The reduction of the particle number will be obtained by combining two systems:

- Emittance filter
- Energy selector.

a) Emittance filter

An emittance filter consists of a diaphragm intercepting the particle beam (the diaphragm admittance must be smaller than the beam emittance).

The admittance of a diaphragm, with radius $R$ and thickness $L$, is given
by:

\[ A = \frac{4}{\pi} \frac{R^2}{L} \]  \hspace{1cm} (1)

The (1) is the exact expression for a square shaped diaphragm with side 2R, while it is only an upper limit when the shape is circular.

If a longitudinal accelerating electric field is present, we have to replace in (1) L with the equivalent thickness \( L_{eq} \):

\[ L_{eq} = \frac{W_o}{eE_z} \ln \left( 1 + \frac{eE_z L}{W_o} \right) \]  \hspace{1cm} (2)

where \( W_o \) is the input beam energy, \( E_z \) the accelerating electric field and L the real diaphragm thickness.

The low energy part of the DAΦNE-LINAC, upstream of the converter, can be used as diaphragm. The admittance of this part can be varied by acting on the focusing optics.

For simplicity we just consider the case without focusing (minimum admittance value). In this case the LINAC can be considered as a pipe with radius R (equal to the minimum radius of the internal irises of the accelerating section) and length given by:

\[ L_{LINAC} = L_1 + \sum_{k=2}^{n} a_{k-1} \left( L_{o} + L_{k} \right) \]  \hspace{1cm} (3)

\[ a_k = \prod_{m=1}^{k} \left( 1 + \frac{eE_z L}{W_m} \right)^{1} \]

where \( n \) is the number of accelerating sections, L the section length, \( L_o \) the length of the drift between two contiguous sections, \( L_k \) the equivalent length of the k-th section calculated by (2), \( W_k \) the k-th section input energy and \( E_z \) the accelerating electric field.
For the low energy part of the DAΦNE-LINAC, 9 sections of 3 m length, 20 MeV/m gradient, 10 mm internal iris radius, 50 cm drift length and 10 MeV input energy (this is only an hypothetical configuration), we obtain:

\[ L_{\text{LINAC}} = 2.21 \text{ m} \]

By substituting this value in (1) one get the admittance minimum value:

\[ A_{\text{LINAC}} = 57.6 \pi \text{ mm mrad} \]

The DAΦNE-LINAC beam after the converter (energy ≅ 10 MeV) will have, at least, an emittance \( \varepsilon \approx 250 \pi \text{ mm·mrad} \). By assuming an uniform phase space particle density, and considering that the emittance "cut" is made in both planes, one can say that the percentage of beam which reaches the end of the LINAC (without focussing) will be no more than:

\[ P_{\text{ef}} \leq \left( \frac{A_{\text{LINAC}}}{\varepsilon} \right)^2 \approx 5\% \]

To conclude we can say that by acting on the LINAC focussing, one can get an emittance filter which allows to reduce the beam current of a continuously variable factor in the range:

\[ P_{\text{ef}} \in [5 \times 10^{-2}, 1] \]  \hspace{1cm} (4)

\( b) \) Energy selector

The energy selection is obtained by a bending magnet+slit system which selects the particles within a given energy interval. This system is positioned behind a metallic (extractable) target (see Fig. 2) of thickness \( \alpha \) (measured in radiation lengths) that intercepts the beam and increases its energy spread.

The probability for a particle, with initial energy \( E_0 \), to have an energy \( E \) at the target output is [3]:

\[ w \left( E_o, E, \alpha \right) = \frac{1}{E_o} \left[ \ln \left( \frac{E_o}{E} \right) \right]^{\alpha-1} \frac{\alpha}{\ln 2} \Gamma \left( \frac{\alpha}{\ln 2} \right) \]

\hspace{1cm} (5)
For $\Delta E \ll E_0$, the beam percentage of particles within the energy interval $[E_0 - \Delta E, E_0]$ can be expressed as:

$$P_{es} = \frac{1}{2 \Gamma(\alpha / \ln 2)} \left( \frac{1}{\ln \frac{1}{1 - \frac{\Delta E}{E_0}}} \right) \frac{\alpha}{\ln 2} - \frac{1}{\Delta E}$$

The $1/2$ factor derives from the mathematical average we have made between the values of (5) in $E_0 - \Delta E$ $E_0$.

Moreover, since $\Delta E \ll E_0$, one can write with good approximation:

$$P_{es} \approx \frac{1}{2 \Gamma(\alpha / \ln 2)} \left( \frac{\Delta E}{E_0} \right) \frac{\alpha}{\ln 2} \quad (6)$$

As it has already been said at the beginning of this section, the energy selection is mainly made by a bending magnet-slit system.

This system consists of a DC sector magnet $M_1$ (bending angle $\phi = 45^\circ$, curvature radius $\rho$) and a slit $S_1$ (of variable width $h$) placed along the beam axis at a distance $\rho$ from the end of the magnet.

The only particles that can cross this system are those which have energy in the neighbourhood of $E_0$:

$$\left| \frac{\Delta E}{E} \right| = \frac{h}{2\rho} + \sqrt{2} \left| x'_0 \right|_{\text{MAX}} \quad (7)$$

where $\left| x'_0 \right|_{\text{MAX}}$ is the maximum input divergence that the system accept.

If upstream the magnet there is another slit $S_2$ of thickness $D$ and (variable) width $d$, then

$$\left| x'_0 \right|_{\text{MAX}} = \frac{d}{D}$$
By substituting this last expression in (7) one obtains

\[
\frac{\Delta E}{E} = \frac{h}{2\rho} + \sqrt{2} \frac{d}{D}
\]

For \( \rho = 2.2 \text{ m}, \ h = 2 \text{ mm}, \ d = 1 \text{ mm} \) and \( D = 15 \text{ cm} \) we get

\[
\left| \frac{\Delta E}{E} \right| \cong 1\%
\]

Finally from (6), by using the value \( \alpha=2 \), one obtain the beam percentage \( P_{es} \) which crosses the energy selector:

\[
P_{es} \cong 4 \times 10^{-7}
\]

2.1.2 Secondary beam option conclusions

By combining the two described methods the attenuation of the particles number is given by

\[
P = P_{ef} P_{es}
\]

Such value can vary continuously from \( 4 \times 10^{-7} \) to \( 2 \times 10^{-8} \). This reduction is mainly made by the energy selector, while the "fine regulation" is done by the Linac optics, used as emittance filter.

2.2 - Primary beam option

In this case too we start with a current of about 1 mA. The beam emittance will be not greater than \( 25 \pi \text{ mm mrad} \) at 20 MeV (anticipated buncher energy output).

The main reduction will be made by the energy selector described in section 2.1.1. In the following paragraph we discuss how to obtain the "fine regulation" of the reduction factor \( P \).

2.2.1 Fine regulation of the reduction factor \( P \)

The LINAC admittance without focusing is now (all the LINAC must be considered):

\[
A_{\text{LINAC}} \cong 31 \pi \text{ mm mrad}
\]
This admittance is larger than the beam emittance, so in this case the LINAC is not a good emittance filter.

The necessary reduction in the particles number can be obtained by using either one or both of the following methods:
- Further reduction of the gun current.
- To use the quadrupoles (triplets or FODO system) in the LINAC high current part to defocussing or strongly focussing the beam.

2.3 - Statistical considerations.

For large N, the probability to have k electrons in a single bunch is given by the Poisson distribution:

\[ P_m(k) = e^{-m} \frac{m^k}{k!} \quad (8) \]

Where \( m \) defined by:

\[ m = P_{es} N \quad (9) \]

and \( P_{es} \) given by (6).

If all the parameters are chosen in order to have \( m = 1 \) (single electron maximum transmission efficiency), the (8) becomes:

\[ P_1(k) = e^{-1} \frac{1}{k!} \]

and we will have about 37% of 'zero electron bunches', 37% of single electron, 18% of bunches with two electrons and so on.

Let us now evaluate the sensitivity of \( m \) to the variation of the system main parameters.

From (8) we have:

\[ \frac{\Delta P_m(k)}{P_m(k)} = e^{-\Delta m} \left( 1 + \frac{\Delta m}{m} \right)^k - 1 \quad (1) \]

while \( \Delta m/m \), obtained by combining (6) and (9), and by making use of the
Stirling approximation for the $\Gamma$ function, is given, after some algebra, by:

$$\frac{\Delta m}{m} = \frac{\Delta N}{N} + \frac{\alpha}{\ln 2} \left[ \frac{\Delta E_0}{E_0} + \frac{\Delta(\Delta E)}{\Delta E} + \ln \left( \frac{\Delta E/E_0}{\alpha/\ln 2 - 1} \right) - \frac{1}{2} \left( \frac{\alpha/\ln 2 - 1}{\alpha/\ln 2} \right) \Delta \alpha \right]$$  \hspace{1cm} (11)

where:

$\Delta N$ = number of particles variation. Fluctuations of the linac components parameters (gun, power supplies, etc).

$\Delta \alpha$ = target thickness variation. Mechanical tolerances and thermal variation.

$\Delta E_0$ = energy spread and fluctuations of the beam energy.

$\Delta(\Delta E)$ = fluctuations of the energy range accepted by the energy selector. Width variation of the slits, fluctuation of the field inside the bending magnets.

If the fluctuations (tolerances) of the system are such that $\Delta m/m < 15\%$ for $k=1$, the (11) can be approximated by:

$$\frac{\Delta P_m(1)}{P_m(1)} \approx -\frac{1}{2} \left( \frac{\Delta m}{m} \right)^2$$

The expression (12) shows that the (k=1) Poisson distribution is not very sensitive to $m$ variations.

REFERENCES

