Microscopic optical potential from chiral forces

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Motivation

Nuclear reaction theory relies on reducing the many-body problem to a problem with few degrees of freedom: **optical potentials**.

2 body

NN data

$$\left(\frac{d\sigma}{d\Omega}\right)_{NN}$$

$$\delta_L(E)$$

Effective interaction

$$V_{NN} = \begin{cases} G_{NN} \\ t_{NN} \end{cases}$$

N + A

$$U_{opt}(\boldsymbol{k}', \boldsymbol{k}, E)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{NA}$$

$$A_y$$

Nuclear reaction theory relies on reducing the many-body problem to a problem with few degrees of freedom: **optical potentials**.

Phenomenological

Unfortunately, currently used optical potentials for low-energy reactions are phenomenological, and primarily constrained by elastic scattering.
Unreliable when extrapolated beyond their fitted range in energy and nuclei

Microscopical

Existing microscopic optical potentials are *usually* developed in an high-energy regime (≥ 100 MeV) and not applicable for lower energy reactions.

No fitting

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The optical potential has the form: $U(r) = V(r) + iW(r)$

1. The real part of the optical potential explains the scattering (Woods-Saxon form)
2. The imaginary part provides absorption (stronger at the surface)
3. The radial dependence is rather flat in the inner region of the nucleus, falls off rapidly at the nuclear surface
4. A spin orbit term is also included which also peaks near the surface.
5. For a charged projectile a Coulomb term is also necessary.

$$
\begin{aligned}
V(r) &= -V_R f_R(r) - iW_V f_V(r) \\
&+ 4a_{VD}V_D \frac{d}{dr} f_{VD}(r) + 4ia_{WD}\frac{d}{dr} f_{WD}(r) \\
&+ \frac{\lambda_\pi^2}{r}\left[V_{SO}\frac{d}{dr} f_{VSO}(r) + iW_{SO}\frac{d}{dr} f_{WSO}(r)\right]\vec{\sigma}\cdot\vec{l}
\end{aligned}
$$

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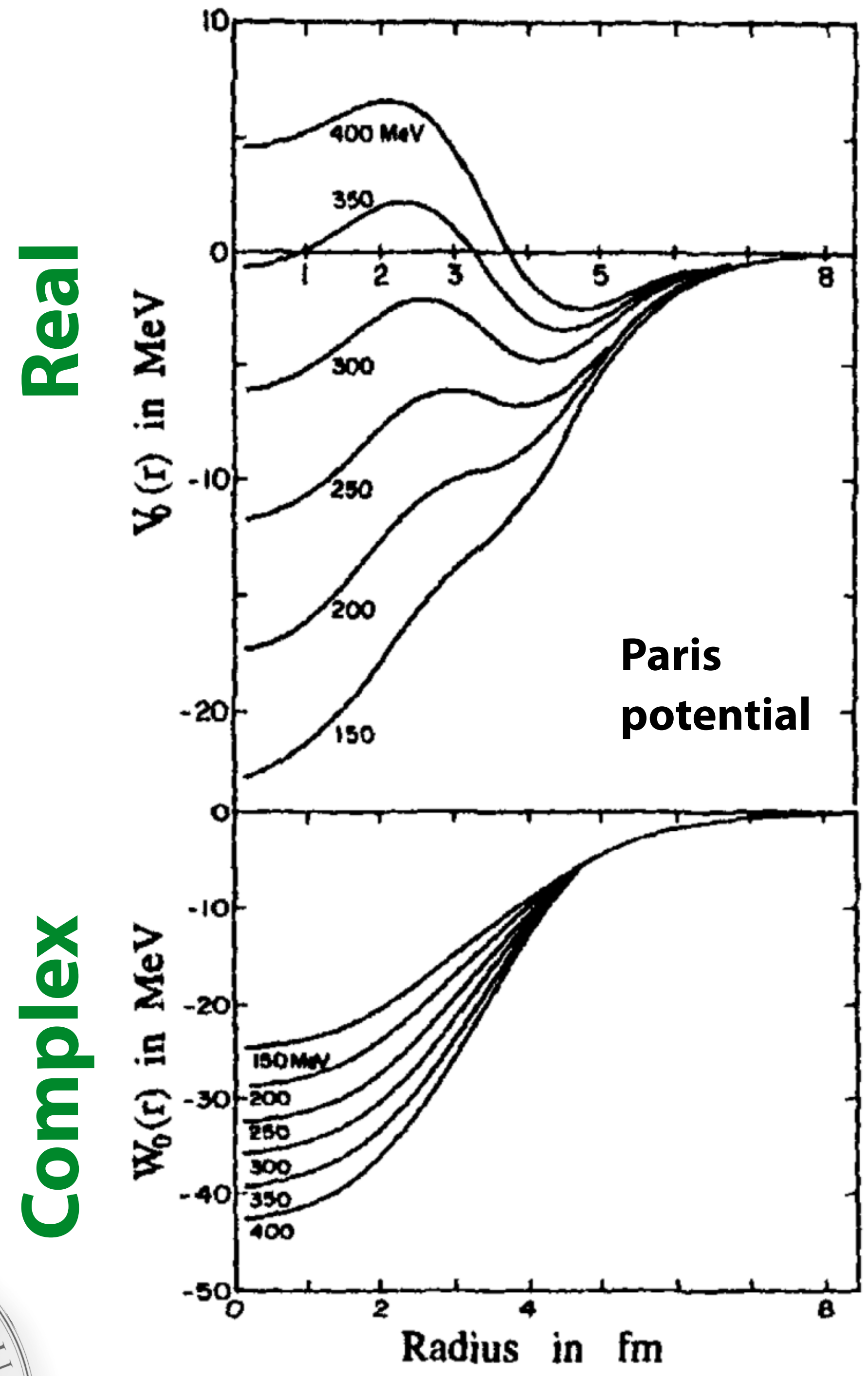

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Model

The general goal when solving the scattering problem of a nucleon from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the many-body transition amplitude T

$$T = V + VG_0(E)T$$

all two nucleon interactions

$$V = \sum_{i=1}^{A} v_{0i}$$

Green Function propagator

$$G_0(E) = \frac{1}{E - H_0 + i\epsilon}$$

where

$$H_0 = h_0 + H_A$$

$$H_A |\Phi_A\rangle = E_A |\Phi_A\rangle$$ **target Hamiltonian**

$$h_0$$ **kinetic term of the projectile**

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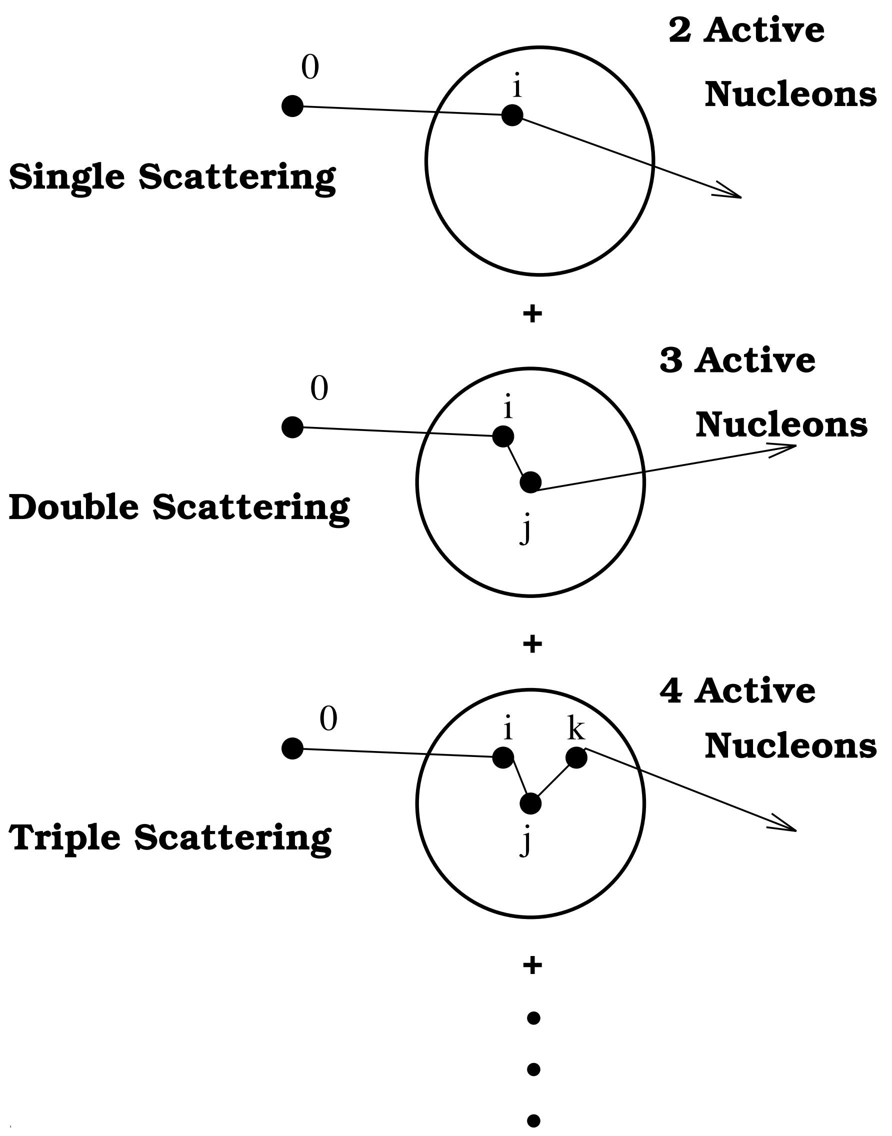

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$$T = V + VG_0(E)T$$

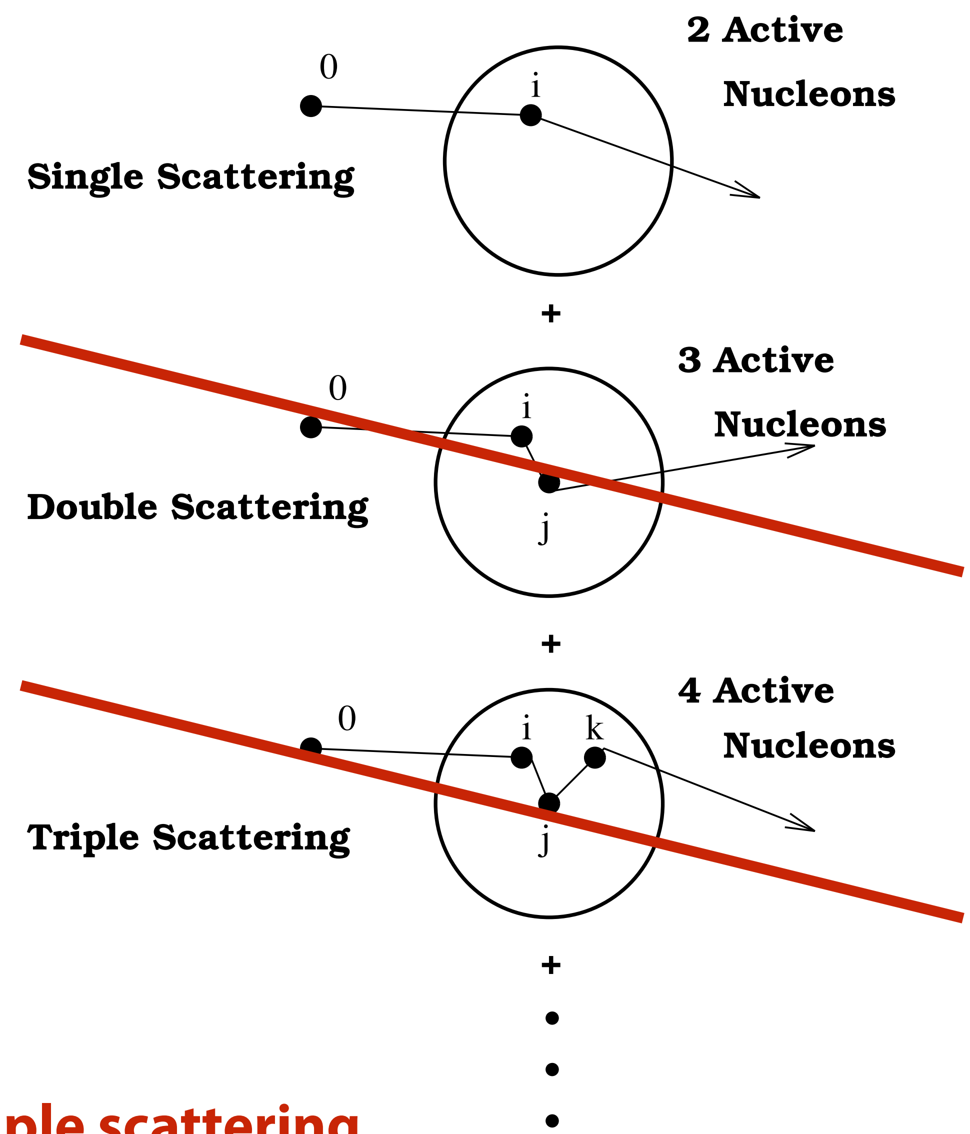
Spectator expansion
two nucleon interaction dominates the scattering process

$$T = \sum_{i=1} T_{0i}$$

$$T_{0i} = v_{0i} + v_{0i}G_0(E)T,$$

$$\begin{aligned}
T_{0i} &= v_{0i} + v_{0i}G_0(E)\sum_j T_{0j} \\
&= v_{0i} + v_{0i}G_0(E)T_{0i} + v_{0i}G_0(E)\sum_{j\neq i} T_{0j} \\
(1 - v_{0i}G_0(E))T_{0i} &= v_{0i} + v_{0i}G_0(E)\sum_{j\neq i} T_{0j}
\end{aligned}$$

$$\boxed{T_{0i} = t_{0i} + t_{0i}G_0(E)\sum_{j\neq i} T_{0j}.}$$



Watson multiple scattering

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Spectator expansion

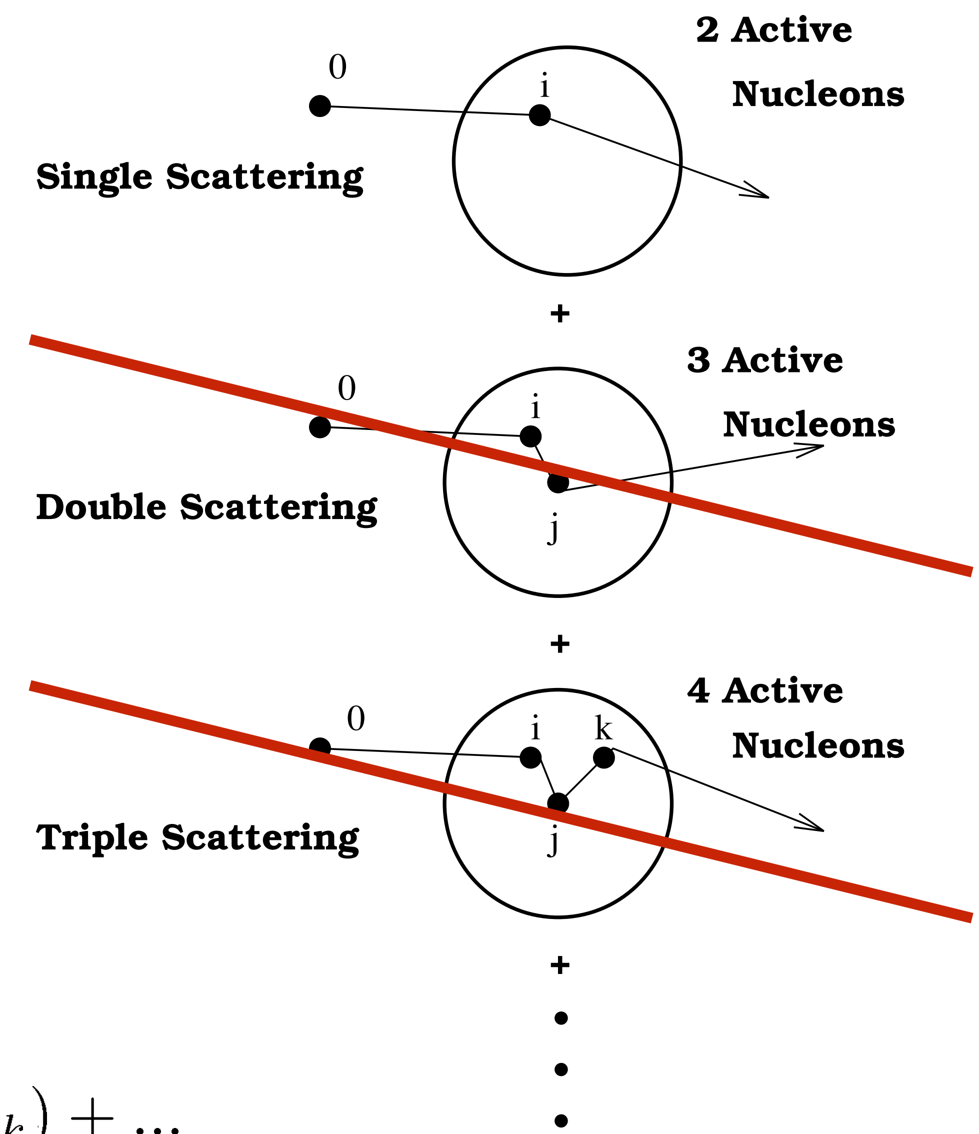
two nucleon interaction dominates the scattering process

Watson multiple scattering

$$\boxed{T_{0i} \quad = \quad t_{0i} + t_{0i} G_0(E) \sum_{j \neq i} T_{0j}}$$

$$t_{0i} = v_{0i} + v_{0i} G_0(E) t_{0i}$$

$$T = \sum_{i=1}^{A} t_{0i} \quad + \quad \sum_{i<j} (t_{ij} - t_{0i} - t_{0j})$$
$$+ \quad \sum_{i<j<k} (t_{ijk} - t_{ij} - t_{ik} - t_{jk} + t_{0i} + t_{0j} + t_{0k}) + \dots$$



The general goal when solving the scattering problem of a nucleon from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the many-body transition amplitude T

$$T = V + VG_0(E)T$$

Let's introduce the **optical potential U**

$$T = U + UG_0(E)PT$$

$$U = V + VG_0(E)QU$$

$$P + Q = 1$$

$$[G_0, P] = 0$$

In the case of elastic scattering,

P projects onto the elastic channel

$$P = \frac{|\Phi_A\rangle\langle\Phi_A|}{\langle\Phi_A|\Phi_A\rangle}$$

The general goal when solving the scattering problem of a nucleon from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the many-body transition amplitude T

$$T = V + VG_0(E)T$$

transition amplitude T for elastic scattering

$$T_{\mathrm{el}} = PUP + PUPG_0(E)T_{\mathrm{el}}$$

we need to calculate *PUP*

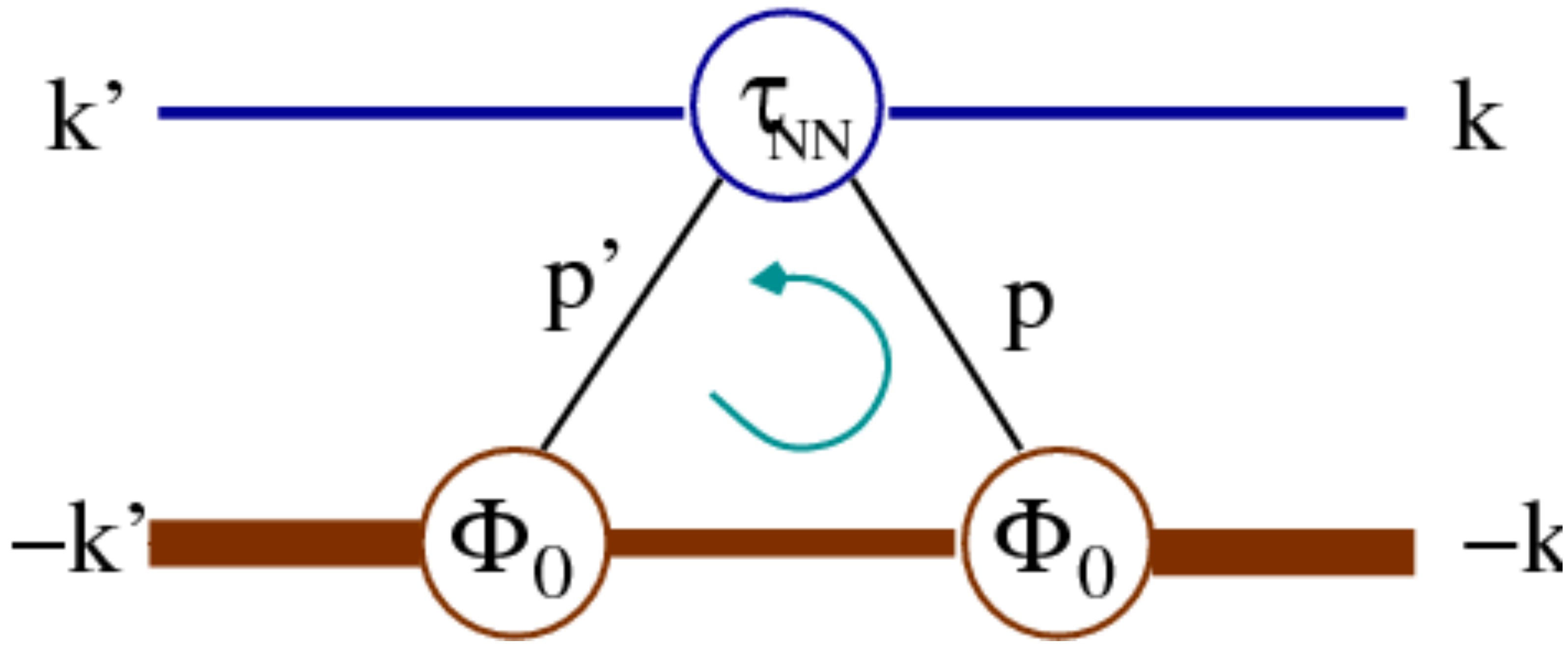
The general goal when solving the scattering problem of a nucleon from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the elastic amplitude T

$$T_{\rm el} = PUP + PUPG_0(E)T_{\rm el}$$

$$U = \sum_{i=1}^{A} \tau_i + \sum_{i,j\neq i}^{A} \tau_{ij} + \sum_{i,j\neq i,k\neq i,j}^{A}$$

$$\begin{aligned} \hat{\tau}_i &= v_{0i} + v_{0i} G_0(E) \hat{\tau}_i \\ &= \tau_{0i} + \tau_{0i} G_0(E) P \hat{\tau}_{0i}. \end{aligned}$$

$$\langle \Phi_A | \tau_i | \Phi_A \rangle = \langle \Phi_A | \hat{\tau}_i | \Phi_A \rangle - \langle \Phi_A | \hat{\tau}_i | \Phi_A \rangle \times \frac{1}{(E - E_A) - h_0 + i\epsilon} \langle \Phi_A | \tau_i | \Phi_A \rangle$$



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$$U = \sum_{i=1}^{A} \tau_i + \sum_{i,j\neq i}^{A} \tau_{ij} + \sum_{i,j\neq i,k\neq i,j}^{A}$$

$$\langle \Phi_A|\tau_i|\Phi_A\rangle = \langle \Phi_A|\hat{\tau}_i|\Phi_A\rangle - \langle \Phi_A|\hat{\tau}_i|\Phi_A\rangle \times \frac{1}{(E-E_A)-h_0+i\epsilon}\langle \Phi_A|\tau_i|\Phi_A\rangle$$

$$\hat{\tau}_i = v_{0i} + v_{0i}G_0(E)\hat{\tau}_i = \tau_{0i} + \tau_{0i}G_0(E)P\hat{\tau}_{0i}.$$

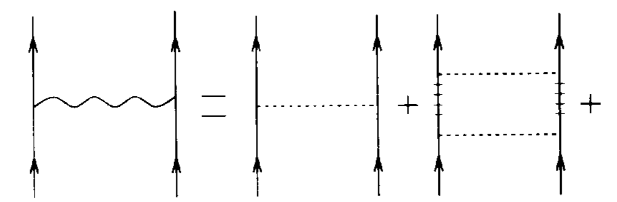
Expanding the propagator $G_i(E) = \frac{1}{(E-E^i)-h_0-h_i-W_i+i\epsilon}$

$$\hat{\tau}_i = v_{0i} + v_{0i}G_i(E)\hat{\tau}_i = t_{0i} + t_{0i}g_iW_iG_i(E)\hat{\tau}_i$$

IMPULSE APPROXIMATION $\hat{\tau}_i \approx t_{0i}$

free NN t matrix

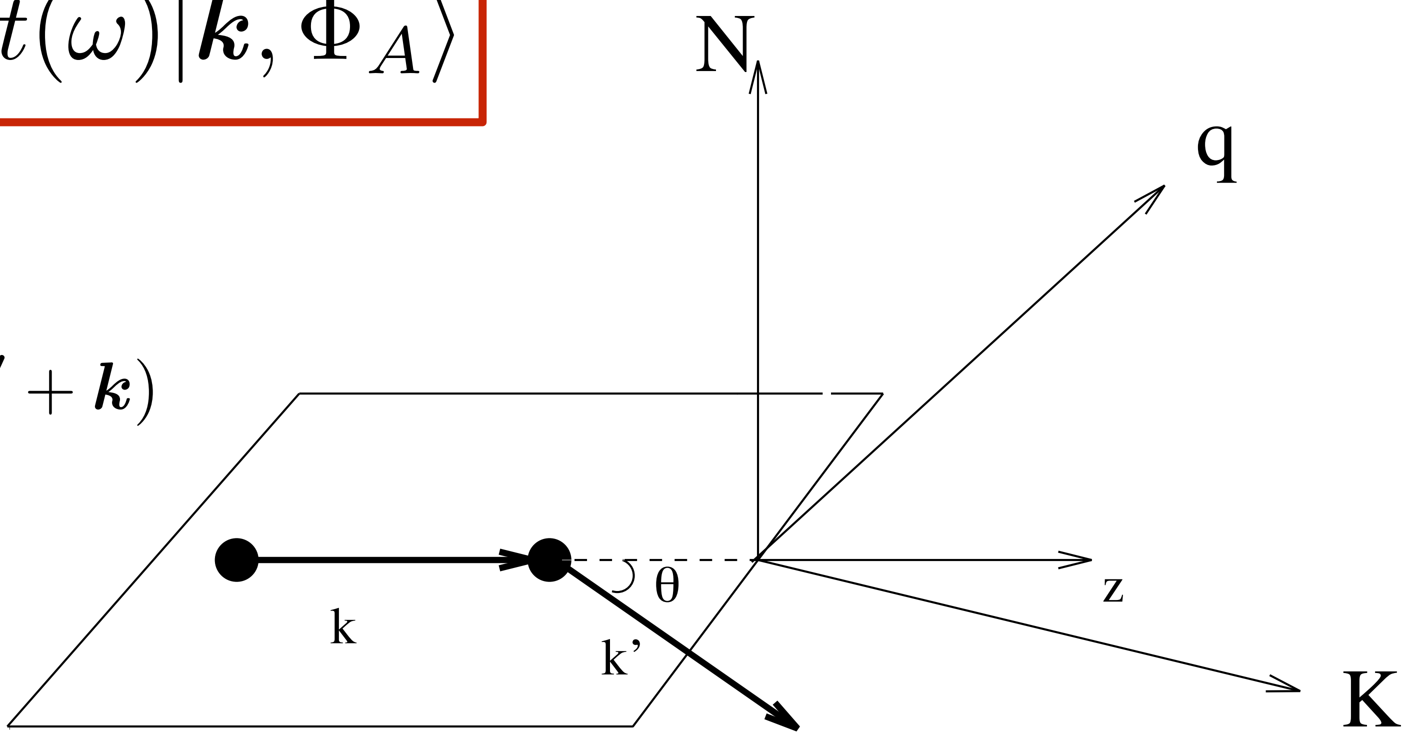
$$t_{0i} = v_{0i} + v_{0i}g_it_{0i}$$



First-order optical potential

Kerman, McManus and Thaler, Ann. Phys. **8** (1959) 551 and many others

$$\boxed{\hat{U}(\boldsymbol{k}', \boldsymbol{k}; \omega) = (A-1)\,\langle \boldsymbol{k}', \Phi_A | t(\omega) | \boldsymbol{k}, \Phi_A \rangle}$$



$$\boldsymbol{q} \equiv \boldsymbol{k}' - \boldsymbol{k}\,, \qquad \boldsymbol{K} \equiv \frac{1}{2}(\boldsymbol{k}' + \boldsymbol{k})$$

$$\hat{U}(\boldsymbol{q}, \boldsymbol{K}; \omega) = \frac{A-1}{A}\,\eta(\boldsymbol{q}, \boldsymbol{K})$$

$$\times \sum_{N=n,p} t_{pN}\left[\boldsymbol{q}, \frac{A+1}{A}\boldsymbol{K}; \omega\right] \rho_N(q)$$

Optimum factorization factor

Møller factor

$$\eta(\boldsymbol{q}, \boldsymbol{K}) = \left[\frac{E_{\rm proj}(\boldsymbol{\kappa}')\,E_{\rm proj}(-\boldsymbol{\kappa}')\,E_{\rm proj}(\boldsymbol{\kappa})\,E_{\rm proj}(-\boldsymbol{\kappa})}{E_{\rm proj}(\boldsymbol{k}')\,E_{\rm proj}\left(-\frac{\boldsymbol{q}}{2} - \frac{\boldsymbol{K}}{A}\right)\,E_{\rm proj}(\boldsymbol{k})\,E_{\rm proj}\left(\frac{\boldsymbol{q}}{2} - \frac{\boldsymbol{K}}{A}\right)}\right]^{\frac{1}{2}}$$

First-order optical potential

$$\hat{U}(\boldsymbol{q},\boldsymbol{K};\omega) = \hat{U}^c(\boldsymbol{q},\boldsymbol{K};\omega) + \frac{i}{2}\boldsymbol{\sigma}\cdot\boldsymbol{q}\times\boldsymbol{K}\,\hat{U}^{ls}(\boldsymbol{q},\boldsymbol{K};\omega)$$

Central component

$$\hat{U}^c(\boldsymbol{q},\boldsymbol{K};\omega) = \frac{A-1}{A}\,\eta(\boldsymbol{q},\boldsymbol{K}) \times \sum_{N=n,p} t^c_{pN}\left[\boldsymbol{q},\frac{A+1}{A}\boldsymbol{K};\omega\right]\rho_N(q)$$

Spin-orbit component

$$\hat{U}^{ls}(\boldsymbol{q},\boldsymbol{K};\omega) = \frac{A-1}{A}\,\eta(\boldsymbol{q},\boldsymbol{K})\left(\frac{A+1}{2A}\right) \times \sum_{N=n,p} t^{ls}_{pN}\left[\boldsymbol{q},\frac{A+1}{A}\boldsymbol{K};\omega\right]\rho_N(q)$$

NN transition matrix

$$M(\boldsymbol{\kappa}', \boldsymbol{\kappa}, \omega) = \langle \boldsymbol{\kappa}' | M(\omega) | \boldsymbol{\kappa} \rangle = -4\pi^2 \mu \, \langle \boldsymbol{\kappa}' | t(\omega) | \boldsymbol{\kappa} \rangle$$

$$M = a + c(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{\boldsymbol{n}} + m(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{n}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{n}})$$
$$+ (g+h)(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{l}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{l}}) + (g-h)(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{m}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{m}})$$

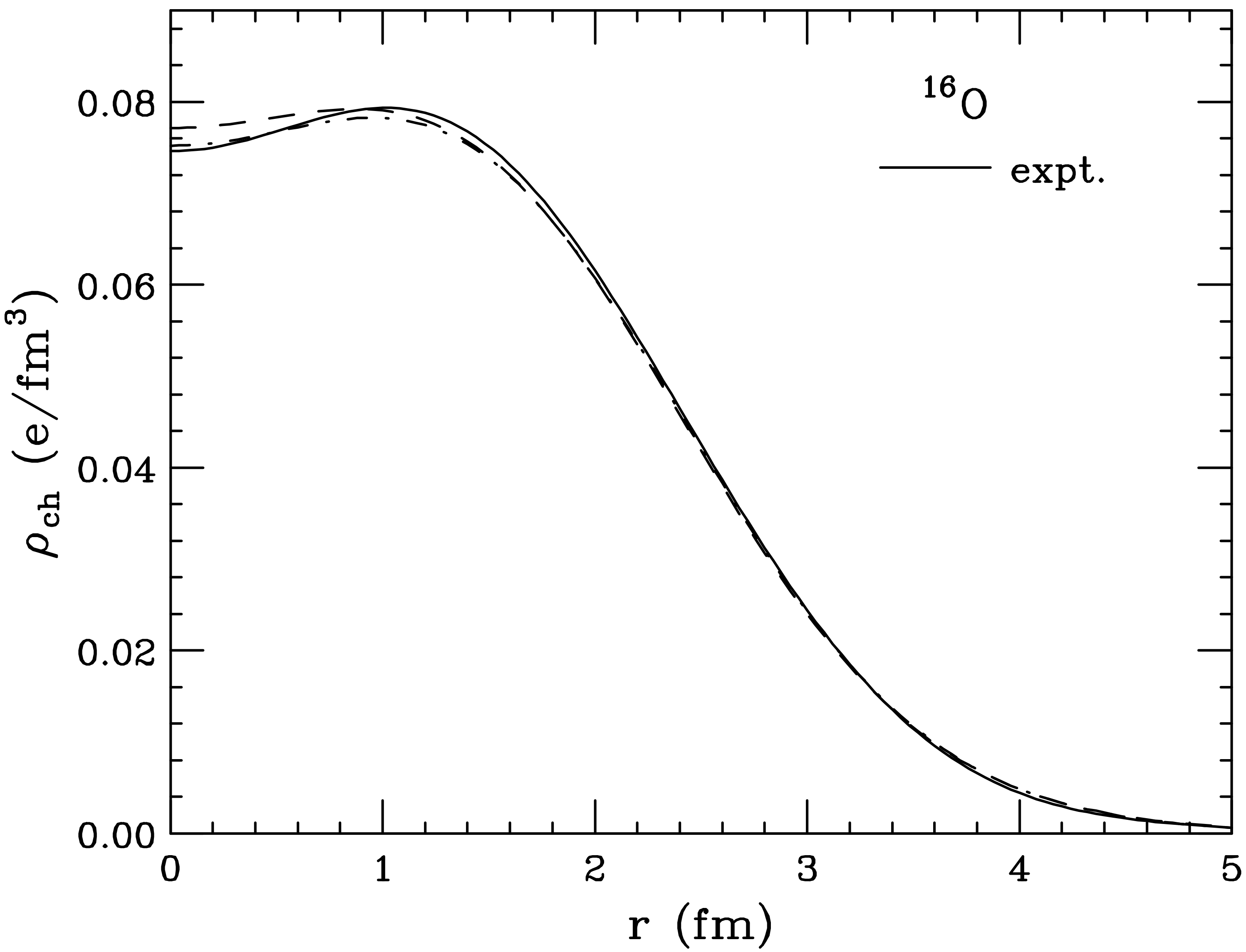
$$\boxed{\begin{aligned} c_{pN} = \frac{i}{f_{pN}\pi^2} \sum_{L=1}^{\infty} P_L^1(\cos\phi) \Bigg[\left(\frac{2L+3}{L+1}\right) M_{LL}^{L+1,S=1} \\ - \left(\frac{2L+1}{L(L+1)}\right) M_{LL}^{L,S=1} - \left(\frac{2L-1}{L}\right) M_{LL}^{L-1,S=1} \Bigg] \end{aligned}}$$

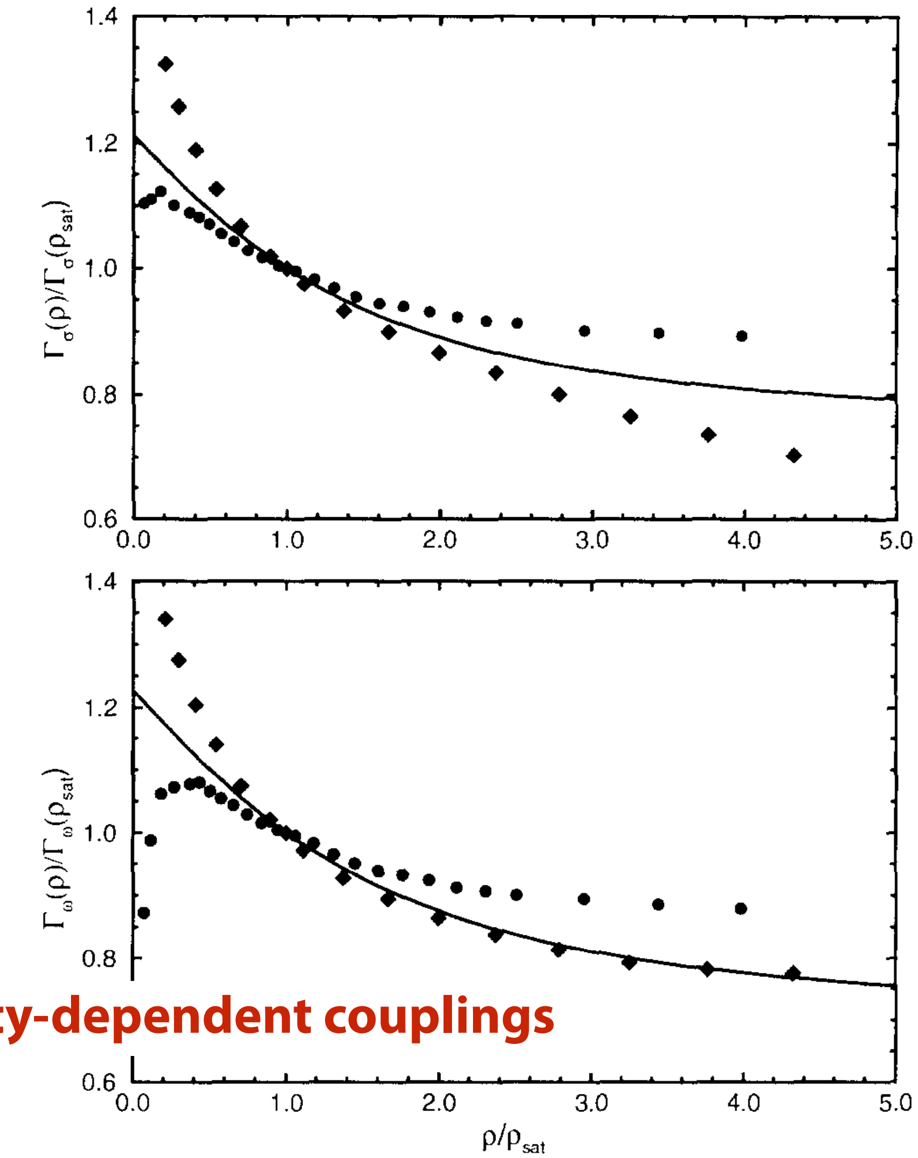
$$\boxed{\begin{aligned} a_{pN} = \frac{1}{f_{pN}\pi^2} \sum_{L=0}^{\infty} P_L(\cos\phi) \Big[(2L+1)\, M_{LL}^{L,S=0} \\ + (2L+1)\, M_{LL}^{L,S=1} + (2L+3)\, M_{LL}^{L+1,S=1} \\ + (2L-1)\, M_{LL}^{L-1,S=1} \Big] \end{aligned}}$$

Relevant components for 0+ nuclei

Matter densities

Typel and Wolter , Nuc. Phys. A **656** (1999) 331





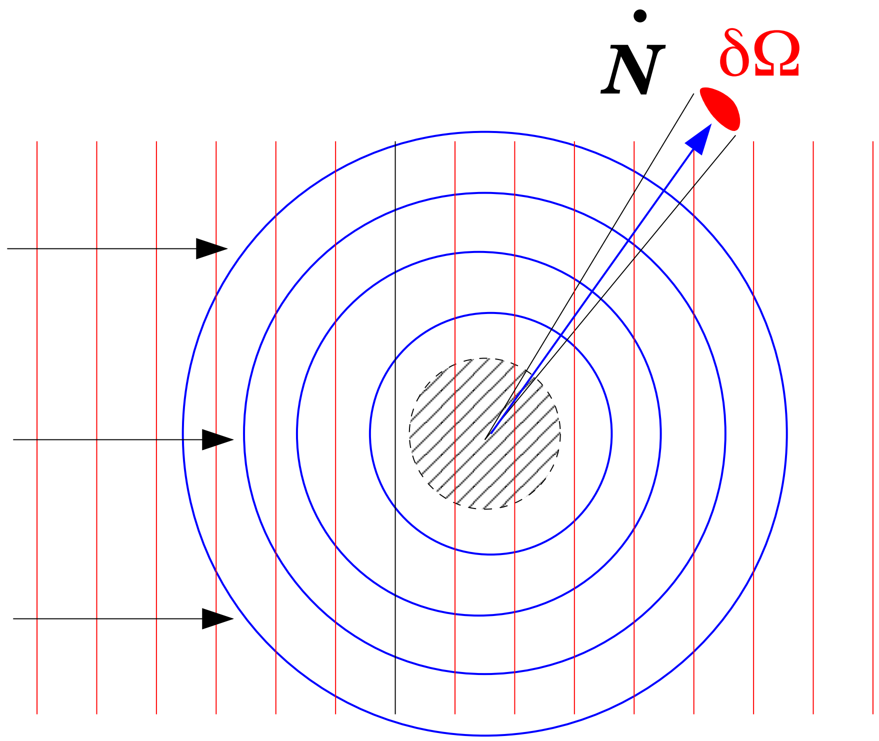
Density-dependent couplings

$$\mathcal{L} = \bar{\Psi}\left[\gamma^\mu\left(i\partial_\mu - \Gamma_\omega A_\mu^{(\omega)} - \Gamma_\varrho \frac{\boldsymbol{\tau}}{2}\cdot \boldsymbol{A}_\mu^{(\varrho)} - e\frac{1+\tau_3}{2}A_\mu^{(\gamma)}\right) - (m - \Gamma_\sigma \phi)\right]\Psi$$
$$+\frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m_\sigma^2\phi^2 - \frac{1}{2}F_{\mu\nu}^{(\omega)}F^{(\omega)\mu\nu} + m_\omega^2 A_\mu^{(\omega)}A^{(\omega)\mu}\right.$$
$$\left.-\frac{1}{2}\boldsymbol{F}_{\mu\nu}^{(\varrho)}\cdot\boldsymbol{F}^{(\varrho)\mu\nu} + m_\varrho^2\boldsymbol{A}_\mu^{(\varrho)}\cdot\boldsymbol{A}^{(\varrho)\mu} - \frac{1}{2}F_{\mu\nu}^{(\gamma)}F^{(\gamma)\mu\nu}\right]$$

DDME1 parametrization

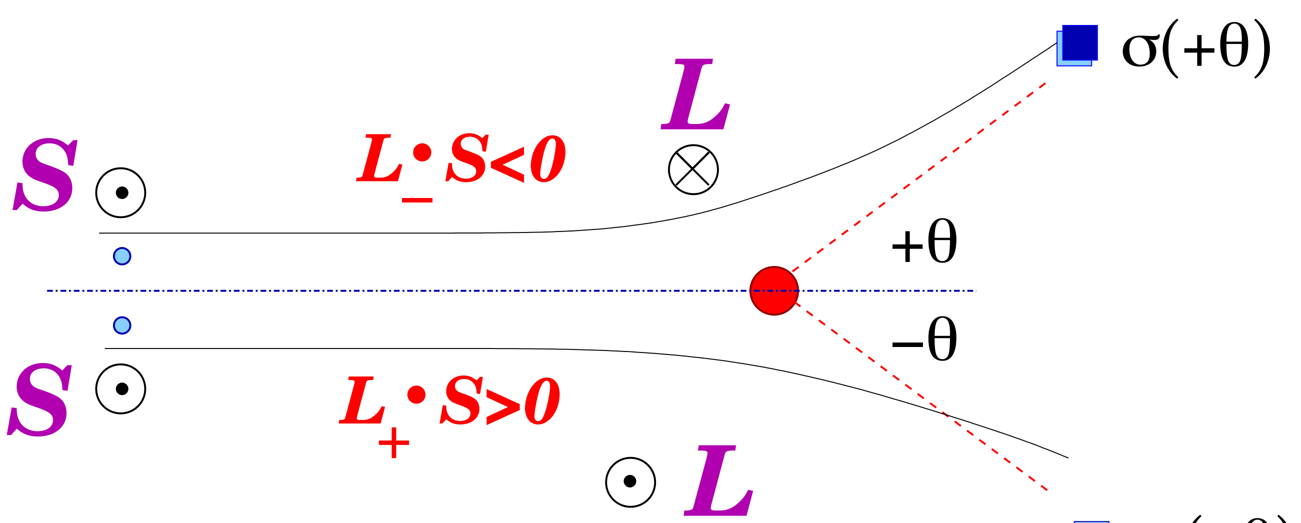
T. Nikšić, D. Vretenar, P. Finelli and P. Ring
Phys. Rev. C **66** (2002) 024306

Scattering observables



$$\dot{N} \sim \left\langle \frac{d\sigma}{d\Omega} \right\rangle \delta\Omega$$

$$\sigma(\theta) = \frac{d\sigma}{d\Omega} \sim \langle \mathbf{k}' | U | \psi_k \rangle$$



$\sigma(+\theta) \neq \sigma(-\theta)$

$U_c(r) + \mathbf{L} \cdot \mathbf{S}\, U_{so}(r)$

$$A_y(\theta) = \frac{\sigma(+\theta) - \sigma(-\theta)}{\sigma(+\theta) + \sigma(-\theta)}$$

It can be measured by sending a beam of polarised protons along ***+y*** and measure the total cross-section at angles ***θ*** and ***-θ*** in the scattering plane

Scattering observables

Spin-flip amplitude

$$M(k_0,\theta) = A(k_0,\theta) + \boldsymbol{\sigma}\cdot\hat{\mathbf{N}}\,C(k_0,\theta)$$

$$A(\theta) = \frac{1}{2\pi^2}\sum_{L=0}^{\infty}\left[(L+1)F_L^{+}(k_0) + LF_L^{-}(k_0)\right]P_L(\cos\theta)$$

$$F_{LJ}(k_0) = -\frac{A}{A-1}4\pi^2\mu(k_0)\hat{T}_{LJ}(k_0,k_0;E)$$

$$C(\theta) = \frac{i}{2\pi^2}\sum_{L=1}^{\infty}\left[F_L^{+}(k_0) - F_L^{-}(k_0)\right]P_L^1(\cos\theta)$$

Differential cross section

$$\frac{d\sigma}{d\Omega}(\theta) = |A(\theta)|^2 + |C(\theta)|^2$$

Analyzing power

$$A_y(\theta) = \frac{2\mathrm{Re}[A^*(\theta)\,C(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$

Spin rotation

$$Q(\theta) = \frac{2\mathrm{Im}[A(\theta)\,C^*(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$

Rotation of the spin vector in the scattering plane, i.e. protons polarised along the ***+x*** axis have a finite probability of having the spin polarised along the **± *z*** axis after the collision

Inclusion of the Coulomb potential

Combine phase shifts from Coulomb and nuclear

$$\boxed{\sigma_L = \arg\Gamma\left[L + 1 + i\eta(k_0)\right]}$$

The central amplitude include a Coulomb component

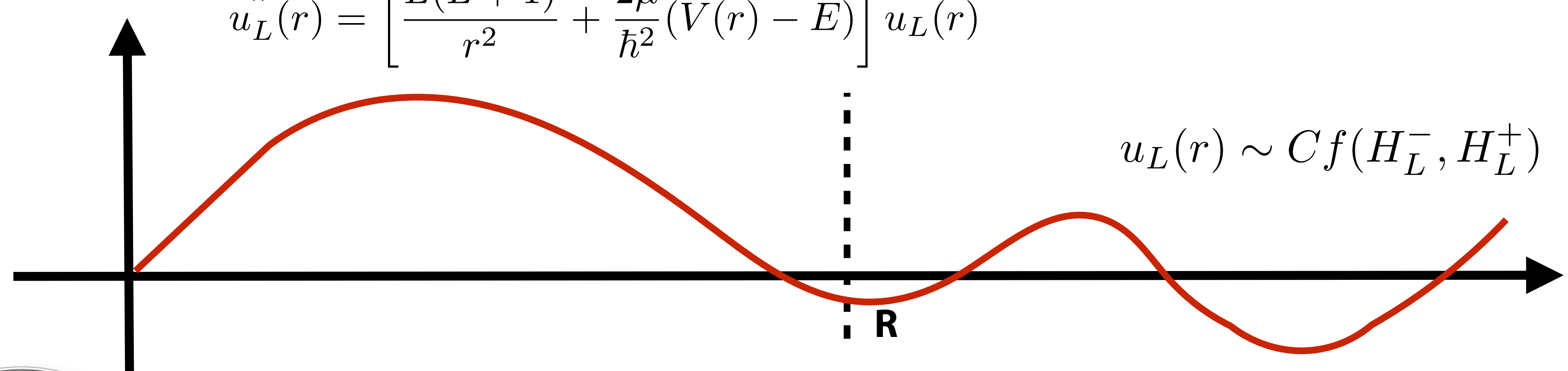
$$A(k_0,\theta) = F^c_{pt}(k_0,\theta) + \frac{1}{2\pi^2}\sum_{L=0}^{\infty} e^{2i\sigma_L}\left[(L+1)\bar{F}^+_L(k_0) + L\bar{F}^-_L(k_0)\right]P_L(\cos\theta)$$

$$F^c_{pt}(k_0,\theta) = \frac{-\eta(k_0)\,\exp\left[2i\sigma_0 - i\eta(k_0)\ln(1-\cos\theta)\right]}{k_0(1-\cos\theta)}$$

Sommerfeld parameter

$$\eta(k) = \frac{\mu Z\alpha}{k}$$

$$u''_L(r) = \left[\frac{L(L+1)}{r^2} + \frac{2\mu}{\hbar^2}(V(r)-E)\right]u_L(r)$$



Do not add nuclear and Coulomb separately!

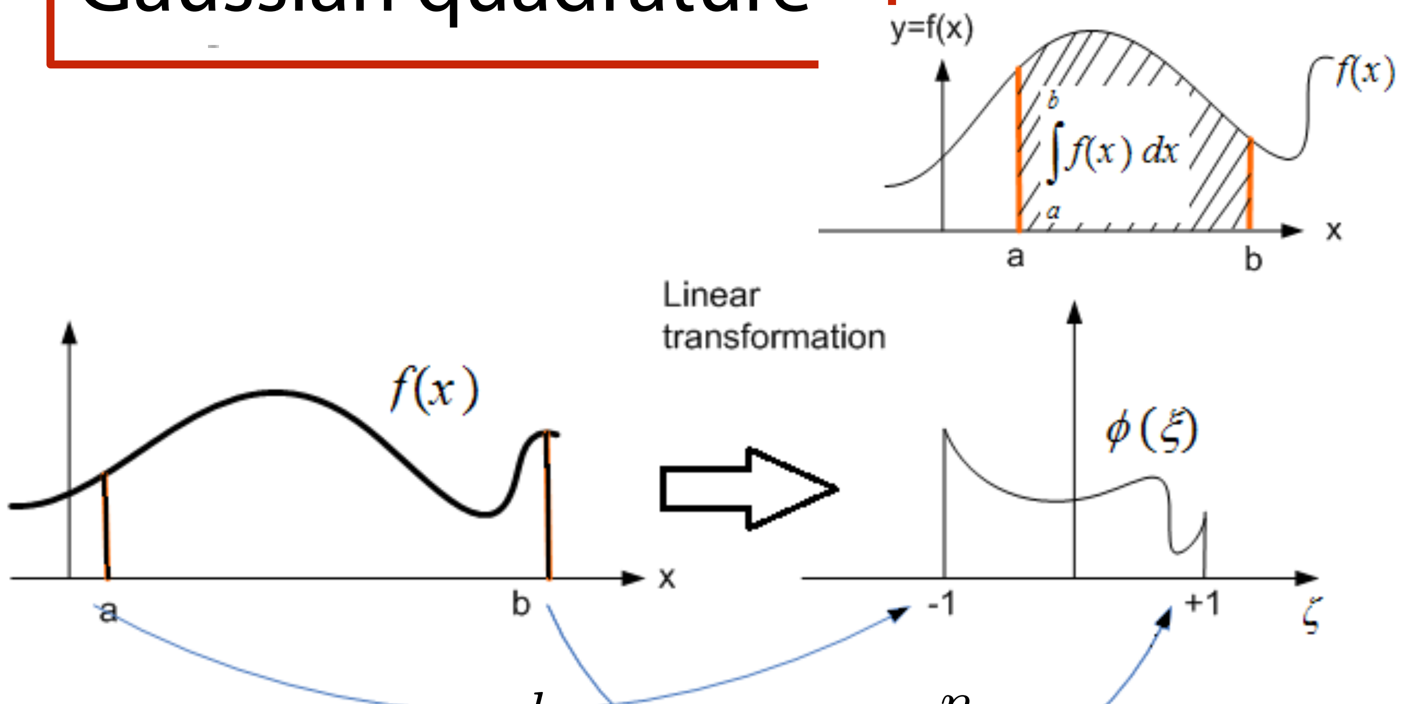
$$\bar{U}(\boldsymbol{k}',\boldsymbol{k};\omega) = \langle\boldsymbol{k}'|\bar{U}(\omega)|\boldsymbol{k}\rangle = \langle\psi^{(+)}_c(\boldsymbol{k}')|\hat{U}(\omega)|\psi^{(+)}_c(\boldsymbol{k})\rangle$$

Numerical details

Partial waves of the NN potential used to construct the three-dimensional NN t matrix

$t^{S=0,T=1}_{L,LL}$	:	${}^1S_0, {}^1D_2, {}^1G_4, {}^1I_6, {}^1K_8$
$t^{S=1,T=1}_{L-1,LL}$	:	${}^3P_0, {}^3F_2, {}^3H_4, {}^3J_6, {}^3L_8$
$t^{S=1,T=1}_{L,LL}$	:	${}^3P_1, {}^3F_3, {}^3H_5, {}^3J_7$
$t^{S=1,T=1}_{L+1,LL}$	:	${}^3P_2, {}^3F_4, {}^3H_6, {}^3J_8$
$t^{S=0,T=0}_{L,LL}$	:	${}^1P_1, {}^1F_3, {}^1H_5, {}^1J_7$
$t^{S=1,T=0}_{L-1,LL}$	:	${}^3D_1, {}^3G_3, {}^3I_5, {}^3K_7$
$t^{S=1,T=0}_{L,LL}$	:	${}^3D_2, {}^3G_4, {}^3I_6, {}^3K_8$
$t^{S=1,T=0}_{L+1,LL}$	:	${}^3S_1, {}^3D_3, {}^3G_5, {}^3I_7$

Gaussian quadrature



$$\int_a^b f(x)dx \simeq \sum_{i=1}^{n} w_i f(x_i)$$

150-200 points for $\mathbf{t}_{\mathbf{NN}}$ and ~ 100 for $\mathbf{t}_{\mathbf{NA}}$

NN potential

Chiral potentials: why?

Chiral potentials

1. QCD symmetries are consistently respected

Phenomen. potentials

1. QCD symmetries are not respected

1. Lorentz covariance
2. Chiral symmetry
3. Gauge invariance

Chiral potentials: why?

Chiral potentials	Phenomen. potentials
1. QCD symmetries are consistently respected *2. Systematic expansion (order by order you know exactly the terms to be included)* *3. Theoretical errors*	*1. QCD symmetries are not respected* *2. Expansion determined by phenomenology (add whatever you need). A lot of freedom* *3. Errors can't be estimated*

Order by order in a power expansion, the uncertainties are of order of $\mathcal{O}(k_F/\Lambda_\chi)^n$

Higher orders of

$\vec{\sigma}_1 \cdot \vec{\sigma}_2,\ S_{12}(\vec{r}),\ S_{12}(\vec{p}),\ \vec{L} \cdot \vec{S},\ (\vec{L} \cdot \vec{S})^2$ **?**

How to choose what to include?

Chiral potentials: why?

Chiral potentials

1. QCD symmetries are consistently respected
2. Systematic expansion (order by order you know exactly the terms to be included)
3. Theoretical errors
4. Two- and three- body forces belong to the same framework

Many-body !

Phenomen. potentials

1. QCD symmetries are not respected
2. Expansion determined by phenomenology (add whatever you need). A lot of freedom

3. Errors can't be estimated
4. Two- and three- body forces are not related one to each other

Chiral potentials: why?

Chiral potentials

1. QCD symmetries are consistently respected
2. Systematic expansion (order by order you know exactly the terms to be included)
3. Theoretical errors
4. Two- and three- body forces belong to the same framework

Difficult (hard calculations)

Phenomen. potentials

1. QCD symmetries are not respected
2. Expansion determined by phenomenology (add whatever you need). A lot of freedom

3. Errors can't be estimated
4. Two- and three- body forces are not related one to each other

Easy (not always...)

Chiral potentials: why?

Chiral potentials	Phenomen. potentials
Many-body data needed and many-body forces inevitable	Two-body data may be sufficient; many-body forces as last resort
Exploit divergences (cutoff dependence as tool)	Avoid (hide) divergences
Power counting determines diagrams and truncation error	Choose diagrams by intuition

© Adapted from R. Furnstahl

How to build a chiral potential

Problems with nucleons: cutoffs

Usually one regulates the integrals and then removes the dependence on the regularization parameters (scales, cutoffs) by *renormalization*. In the end, the theory and its predictions do not depend on cutoffs or renormalization scales.

In contrast, EFTs are renormalized by **counter terms** (contact terms) that are introduced order by order in increasing numbers. In the nuclear case the potential has validity only for momenta smaller than the chiral symmetry breaking scale $\mathbf{\Lambda_\chi \sim 1GeV}$.

The cutoff independence should be examined for cutoffs below the hard scale and not beyond.

How to build a chiral potential

Following Machleidt

1. Identify the soft and hard scales, and the degrees of freedom (DOF) appropriate for (low-energy) nuclear physics. Soft scale: $Q \sim m_\pi$, hard scale: $\Lambda_\chi \sim m_\rho \sim 1$ GeV; DOF: pions and nucleons.

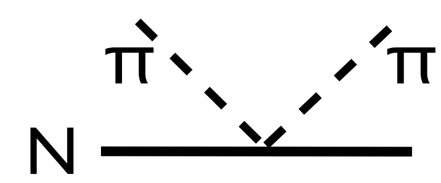
2. Identify the relevant symmetries of low-energy QCD and investigate if and how they are broken: explicitly and spontaneously broken chiral symmetry (spontaneous symmetry breaking generates the pions as Goldstone bosons).

3. Construct the most general Lagrangian consistent with those symmetries and symmetry breakings, see Ref. [13].

At first order

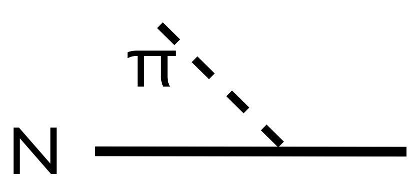
$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}\left(i\gamma^\mu D_\mu - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)N$$

$$-\frac{1}{4f_\pi^2}(\bar\Psi\gamma_\mu\vec\tau\Psi)\cdot\left(\vec\Phi\times(\partial^\mu\vec\Phi)\right)$$



Weinberg-Tomozawa coupling

$$+\frac{1}{2f_\pi}(\bar\Psi\gamma_\mu\gamma_5\vec\tau\Psi)\partial^\mu\vec\Phi$$



$$u_\mu \equiv i u^\dagger(\partial_\mu U)u^\dagger = -\frac{\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}}{F} + \mathcal{O}(\boldsymbol{\pi}^3)$$

$$D_\mu N \equiv (\partial_\mu + \Gamma_\mu)N\,, \quad \text{with} \quad \Gamma_\mu \equiv \frac{1}{2}\left(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right) = \frac{i}{4F^2}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi} + \mathcal{O}(\boldsymbol{\pi}^4)$$

How to build a chiral potential

Following Machleidt

4. Design an organizational scheme that can distinguish between more and less important contributions: a low-momentum expansion, $(Q/\Lambda_\chi)^\nu$, with ν determined by 'power counting'.

$$\frac{Q}{\Lambda_\chi}$$

$Q \rightarrow$ **Soft scale (p_π, m_π)**

$\Lambda_\chi \rightarrow$ **Hard scale ($\Lambda \sim 4\pi f_\pi$, M_N)**

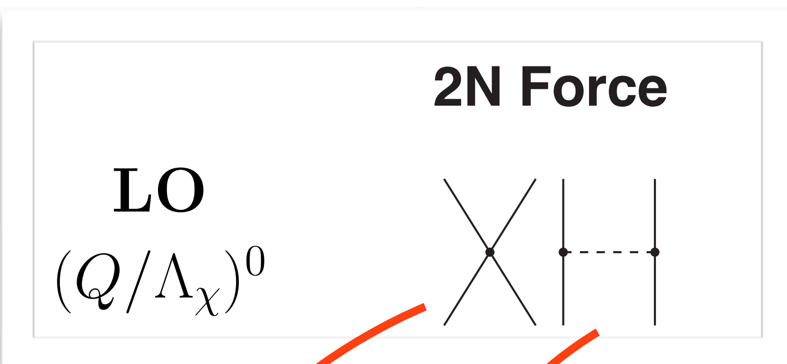
Contrary to the pion mass, the nucleon mass does not vanish in the chiral limit and introduces an additional hard scale in the problem

How to build a chiral potential

Following Machleidt

4. Design an organizational scheme that can distinguish between more and less important contributions: a low-momentum expansion, $(Q/\Lambda_\chi)^\nu$, with ν determined by 'power counting'.

$$S_\mu = i\gamma_5\sigma_{\mu\nu}v^\nu$$



$$\mathcal{L}^{(0)}_{\pi N} = \bar{N}\left[i\,v\cdot D + g_A\,u\cdot S\right]N$$

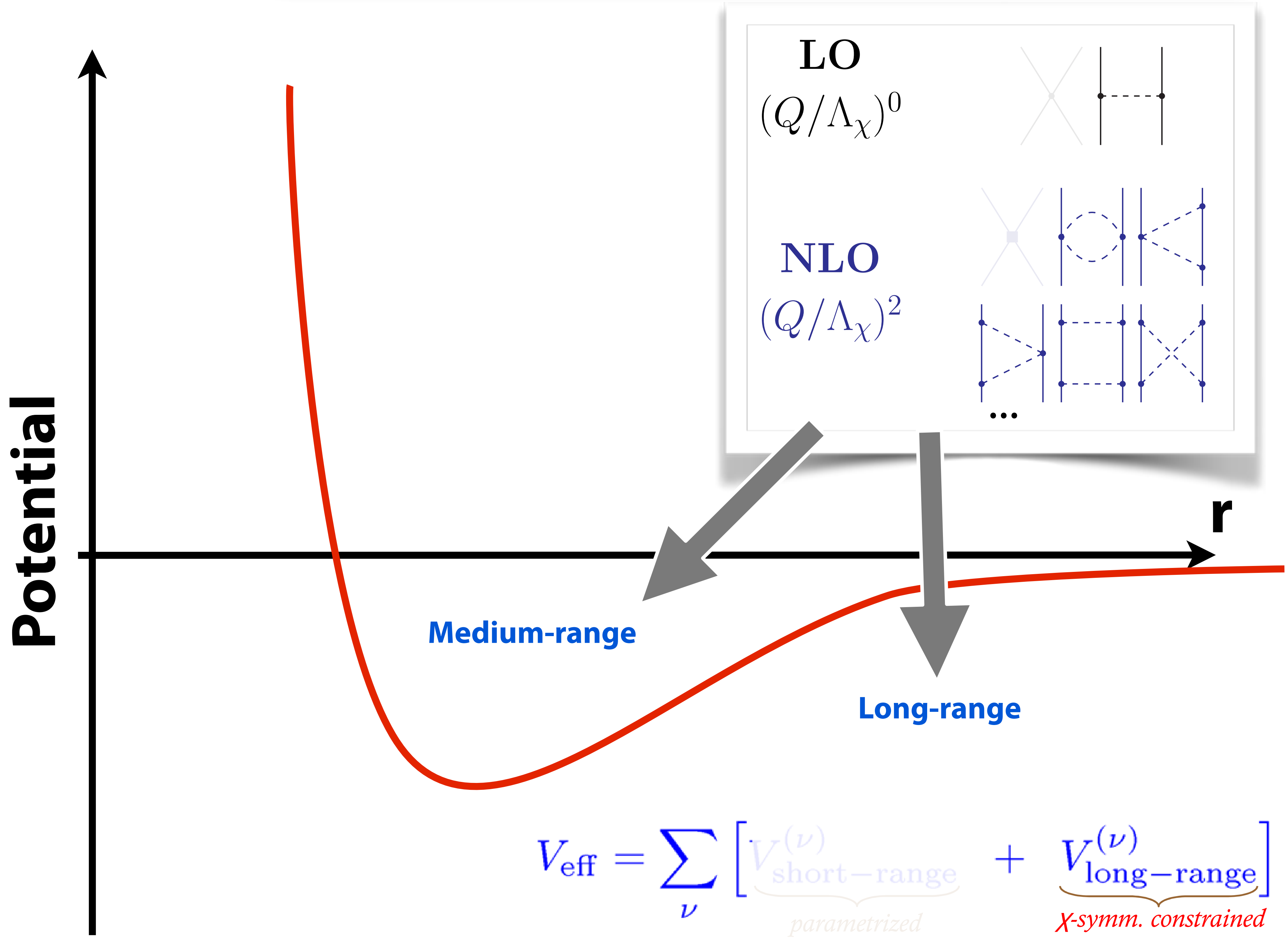
$$\mathcal{L}^{(0)}_{\pi NN} = -\frac{1}{2}C_S(\bar{N}N)(\bar{N}N) + 2C_T(\bar{N}SN)\cdot(\bar{N}SN)$$

$$\boxed{\nu = 2L + \sum_i \Delta_i \qquad \Delta_i \equiv d_i + \frac{n_i}{2} - 2}$$

$$\nu = 0 + \sum(1 + \frac{2}{2} - 2) = 0$$

$$\nu = 0 + (0 + \frac{4}{2} - 2) = 0$$

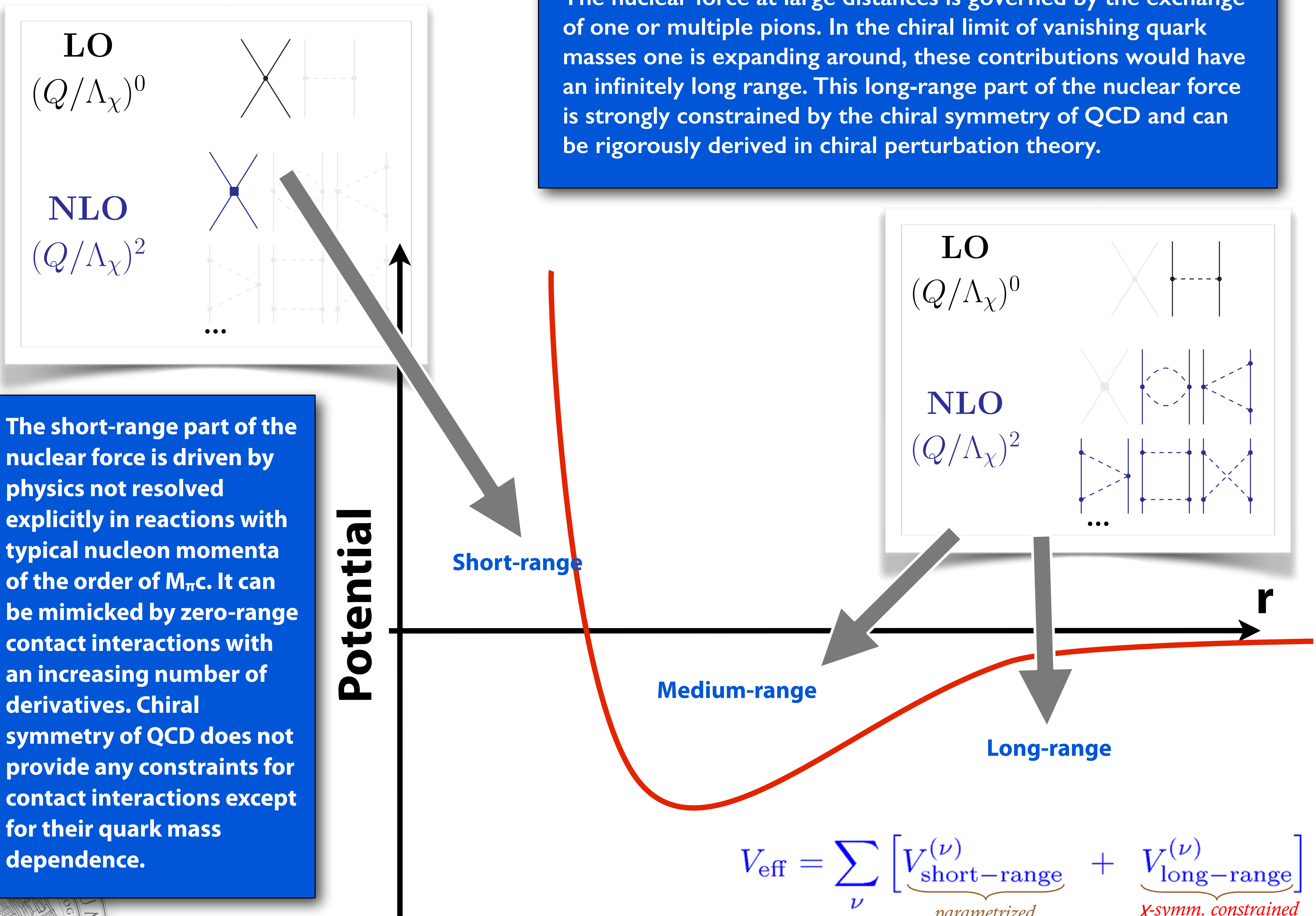
The nuclear force at large distances is governed by the exchange of one or multiple pions. In the chiral limit of vanishing quark masses one is expanding around, these contributions would have an infinitely long range. This long-range part of the nuclear force is strongly constrained by the chiral symmetry of QCD and can be rigorously derived in chiral perturbation theory.



$$V_{\rm eff} = \sum_{\nu} \Big[\underbrace{V^{(\nu)}_{\rm short-range}}_{parametrized} + \underbrace{V^{(\nu)}_{\rm long-range}}_{\chi\text{-symm. constrained}} \Big]$$

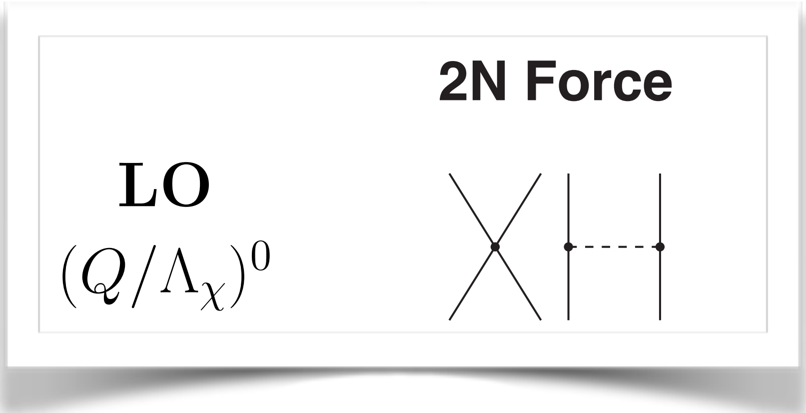
The nuclear force at large distances is governed by the exchange of one or multiple pions. In the chiral limit of vanishing quark masses one is expanding around, these contributions would have an infinitely long range. This long-range part of the nuclear force is strongly constrained by the chiral symmetry of QCD and can be rigorously derived in chiral perturbation theory.

The short-range part of the nuclear force is driven by physics not resolved explicitly in reactions with typical nucleon momenta of the order of $M_\pi c$. It can be mimicked by zero-range contact interactions with an increasing number of derivatives. Chiral symmetry of QCD does not provide any constraints for contact interactions except for their quark mass dependence.



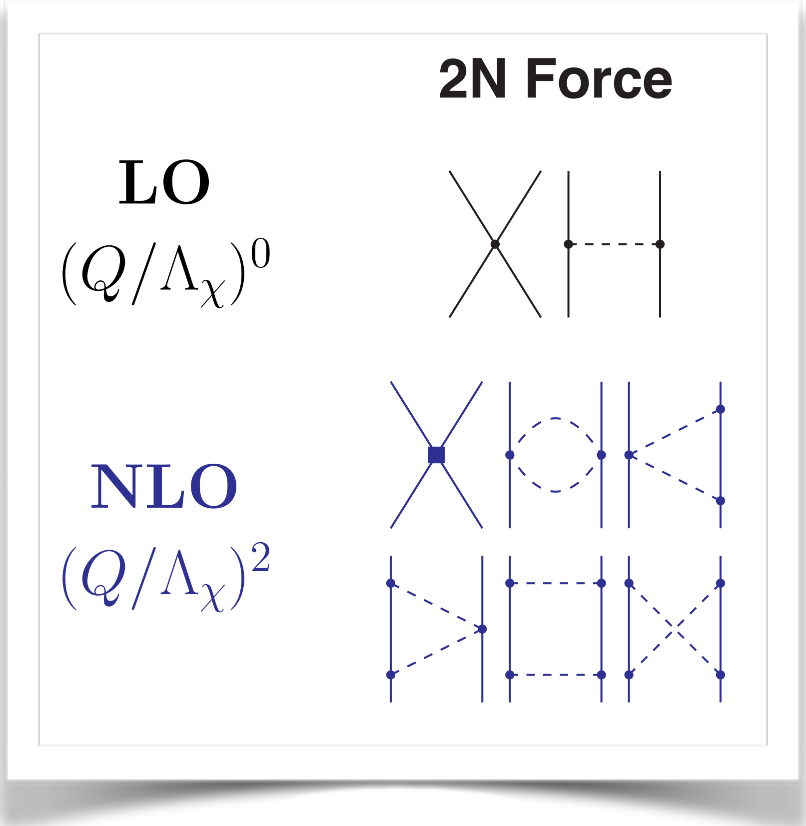
$$V_{\text{eff}} = \sum_\nu \Big[\underbrace{V^{(\nu)}_{\text{short-range}}}_{\textit{parametrized}} + \underbrace{V^{(\nu)}_{\text{long-range}}}_{\chi\textit{-symm. constrained}} \Big]$$

How to build a chiral potential



$$V_{2\mathrm{N}}^{(0)} = \underbrace{-\frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}_{\text{One-pion exchange}} + \underbrace{C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2}_{\text{Contact terms (pure nucleonic term)}}$$

How to build a chiral potential



Contact terms

$$V_{\rm ct}^{(2)}(\vec{p}\,',\vec{p}) = C_1\,q^2 + C_2\,k^2 + \left(C_3\,q^2 + C_4\,k^2\right)\vec{\sigma}_1\cdot\vec{\sigma}_2 + C_5\left(-i\vec{S}\cdot(\vec{q}\times\vec{k})\right) + C_6\,(\vec{\sigma}_1\cdot\vec{q})\,(\vec{\sigma}_2\cdot\vec{q}) + C_7\,(\vec{\sigma}_1\cdot\vec{k})\,(\vec{\sigma}_2\cdot\vec{k}).$$

Two-pion exchange

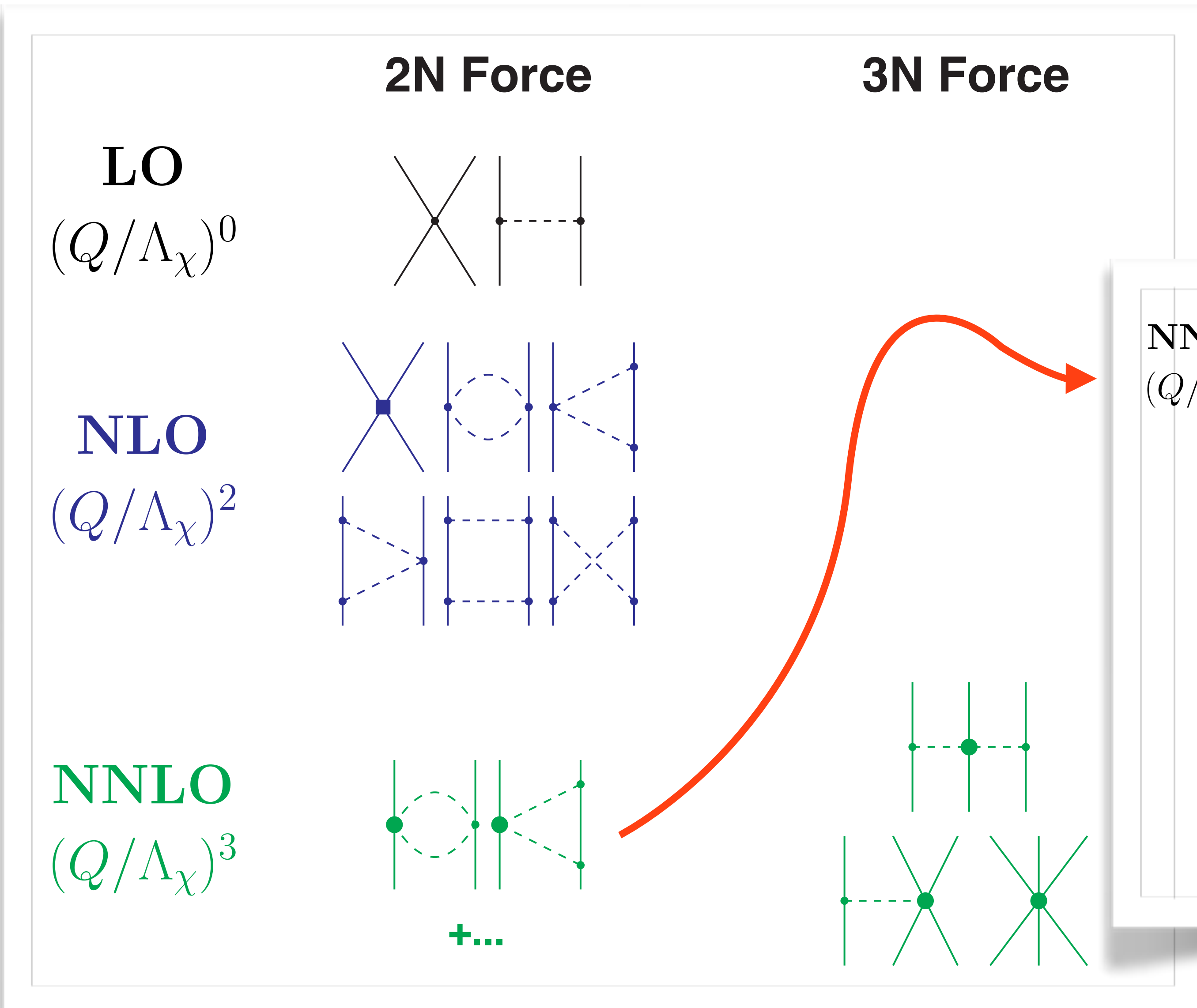
$$V(\vec{p}\,',\vec{p}) = V_C + \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,W_C + [\,V_S + \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,W_S\,]\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + [\,V_{LS} + \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,W_{LS}\,]\,\left(-i\vec{S}\cdot(\vec{q}\times\vec{k})\right) + [\,V_T + \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,W_T\,]\,\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q} + [\,V_{\sigma L} + \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,W_{\sigma L}\,]\,\vec{\sigma}_1\cdot(\vec{q}\times\vec{k})\vec{\sigma}_2\cdot(\vec{q}\times\vec{k}),$$

$$W_C = -\frac{L(q)}{384\pi^2 f_\pi^4}\left[4m_\pi^2(5g_A^4 - 4g_A^2 - 1) + q^2(23g_A^4 - 10g_A^2 - 1) + \frac{48g_A^4 m_\pi^4}{w^2}\right]$$

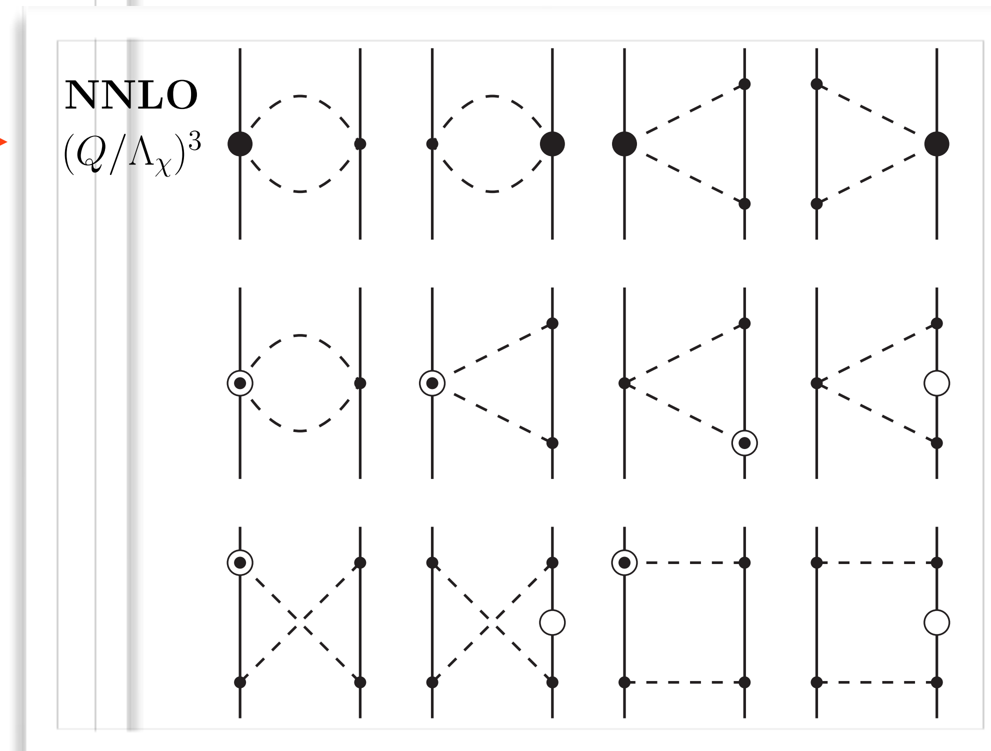
$$V_T = -\frac{1}{q^2}V_S = -\frac{3g_A^4 L(q)}{64\pi^2 f_\pi^4},$$

$$L(q) \equiv \frac{w}{q}\ln\frac{w+q}{2m_\pi} \qquad w \equiv \sqrt{4m_\pi^2 + q^2}$$

How to build a chiral potential



Two-pion exchange



$$V_C = \frac{3g_A^2}{16\pi f_\pi^4}\left\{\frac{g_A^2 m_\pi^5}{16 M_N w^2} - \left[2m_\pi^2(2c_1 - c_3) - q^2\left(c_3 + \frac{3g_A^2}{16M_N}\right)\right]\widetilde{w}^2 A(q)\right\},$$

$$W_C = \frac{g_A^2}{128\pi M_N f_\pi^4}\left\{3g_A^2 m_\pi^5 w^{-2} - \left[4m_\pi^2 + 2q^2 - g_A^2(4m_\pi^2 + 3q^2)\right]\widetilde{w}^2 A(q)\right\},$$

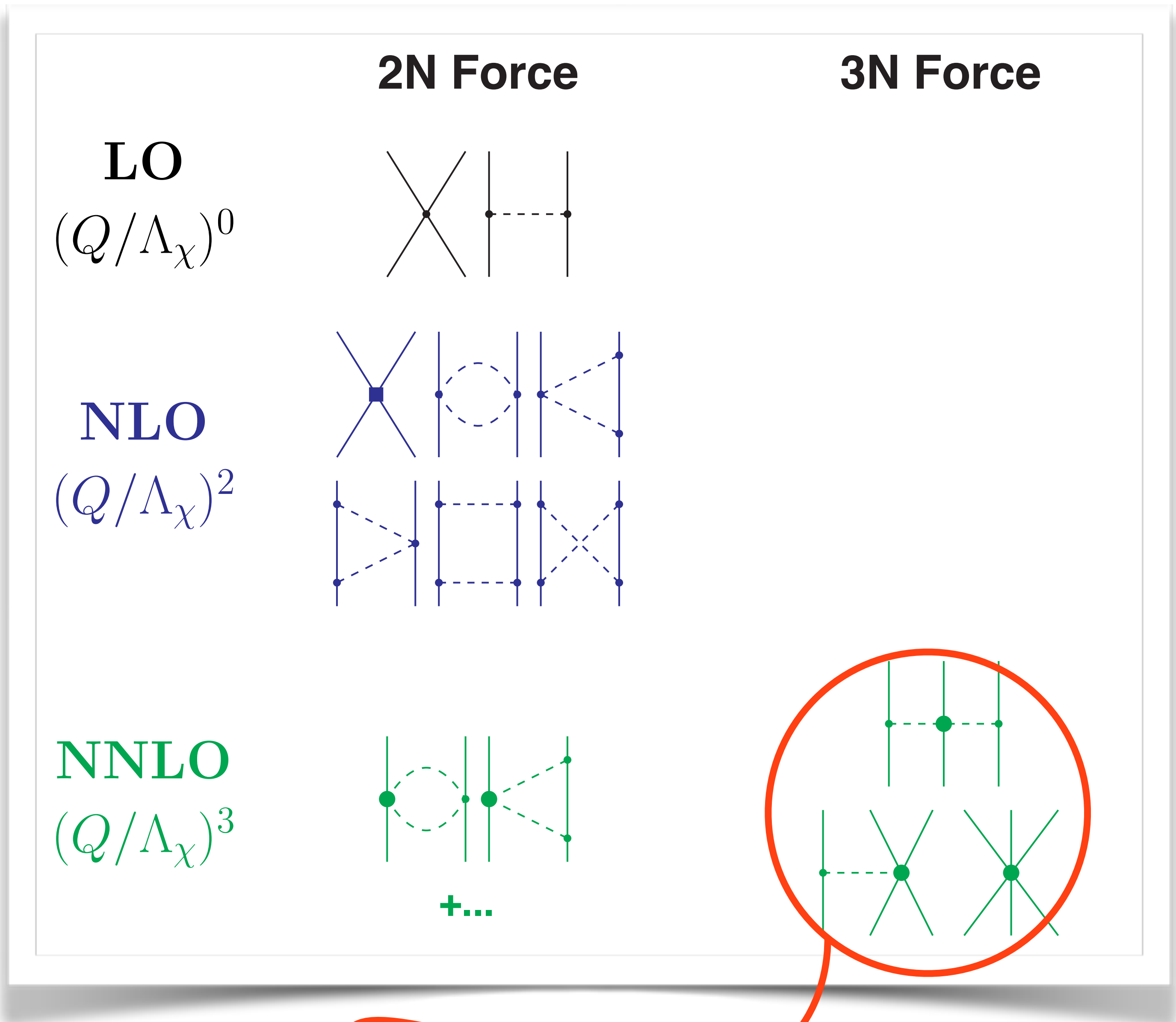
$$V_T = -\frac{1}{q^2}V_S = \frac{9g_A^4\widetilde{w}^2 A(q)}{512\pi M_N f_\pi^4},$$

$$W_T = -\frac{1}{q^2}W_S = -\frac{g_A^2 A(q)}{32\pi f_\pi^4}\left[\left(c_4 + \frac{1}{4M_N}\right)w^2 - \frac{g_A^2}{8M_N}(10m_\pi^2 + 3q^2)\right],$$

$$V_{LS} = \frac{3g_A^4\widetilde{w}^2 A(q)}{32\pi M_N f_\pi^4},$$

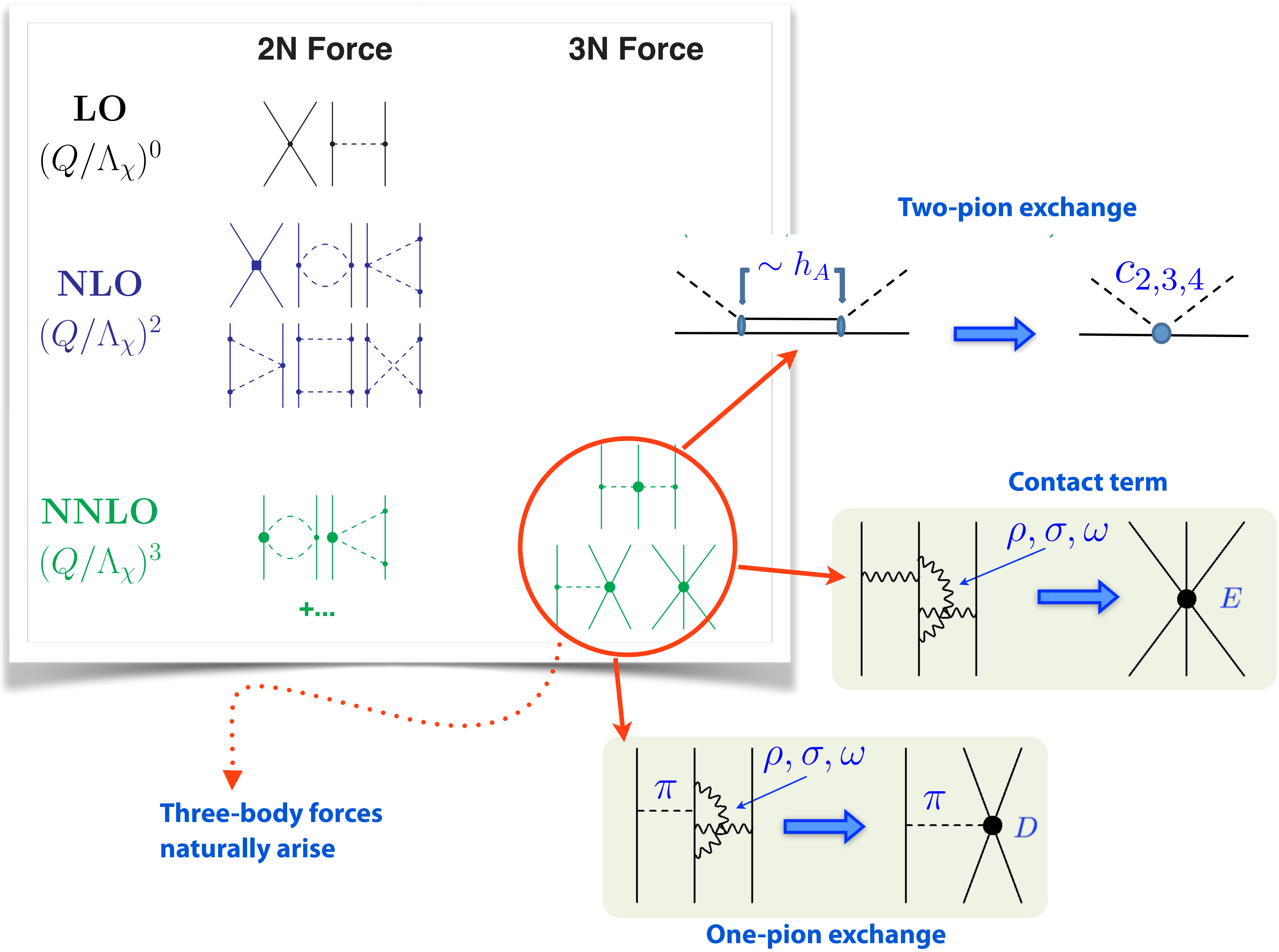
$$W_{LS} = \frac{g_A^2(1-g_A^2)}{32\pi M_N f_\pi^4} w^2 A(q),$$

How to build a chiral potential

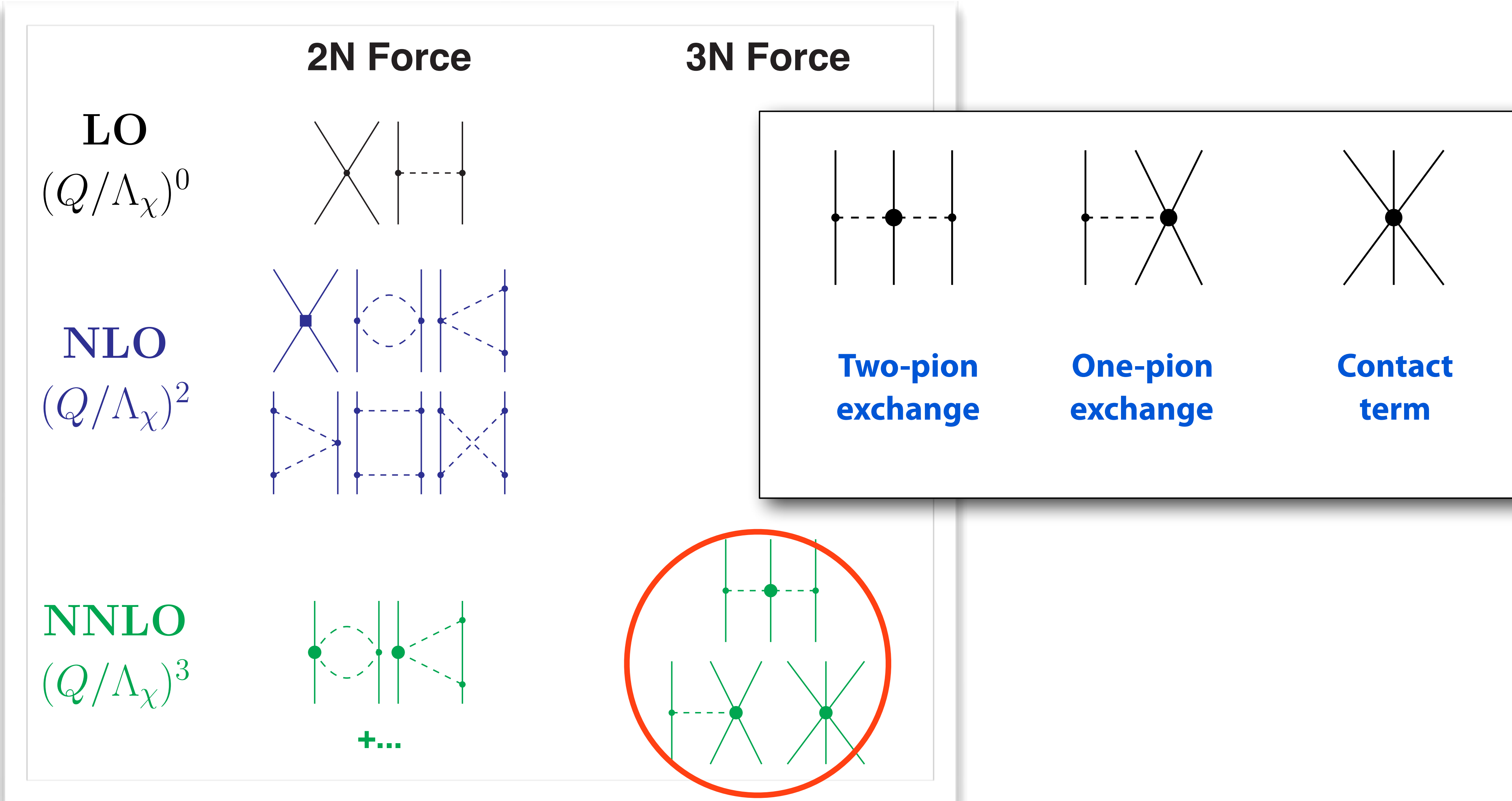


Three-body forces naturally arise

How to build a chiral potential



How to build a chiral potential



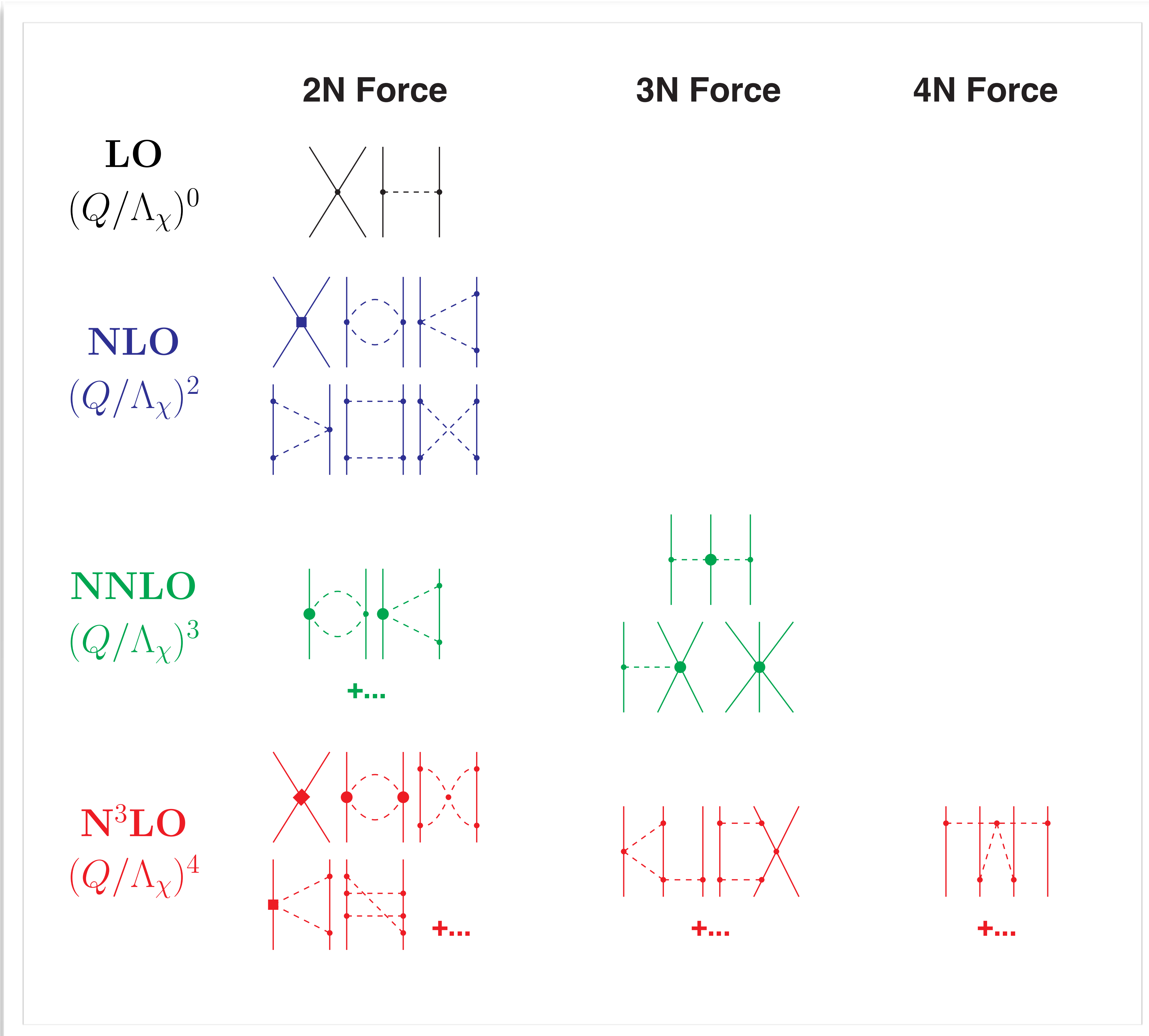
$$V^{3\mathrm{NF}}_{2\mathrm{PE}} = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{1}{2} \sum_{i\neq j\neq k} \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(q_i^2 + m_\pi^2)(q_j^2 + m_\pi^2)} F^{ab}_{ijk}\, \tau^a_i \tau^b_j$$

$$F^{ab}_{ijk} = \delta^{ab}\left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\, \vec{q}_i \cdot \vec{q}_j\right] + \frac{c_4}{f_\pi^2} \sum_c \epsilon^{abc}\, \tau^c_k\, \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j]$$

$$V^{3\mathrm{NF}}_{1\mathrm{PE}} = -D\, \frac{g_A}{8f_\pi^2} \sum_{i\neq j\neq k} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{q_j^2 + m_\pi^2} (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)(\vec{\sigma}_i \cdot \vec{q}_j)$$

$$V^{3\mathrm{NF}}_{\mathrm{ct}} = E\, \frac{1}{2} \sum_{j\neq k} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k$$

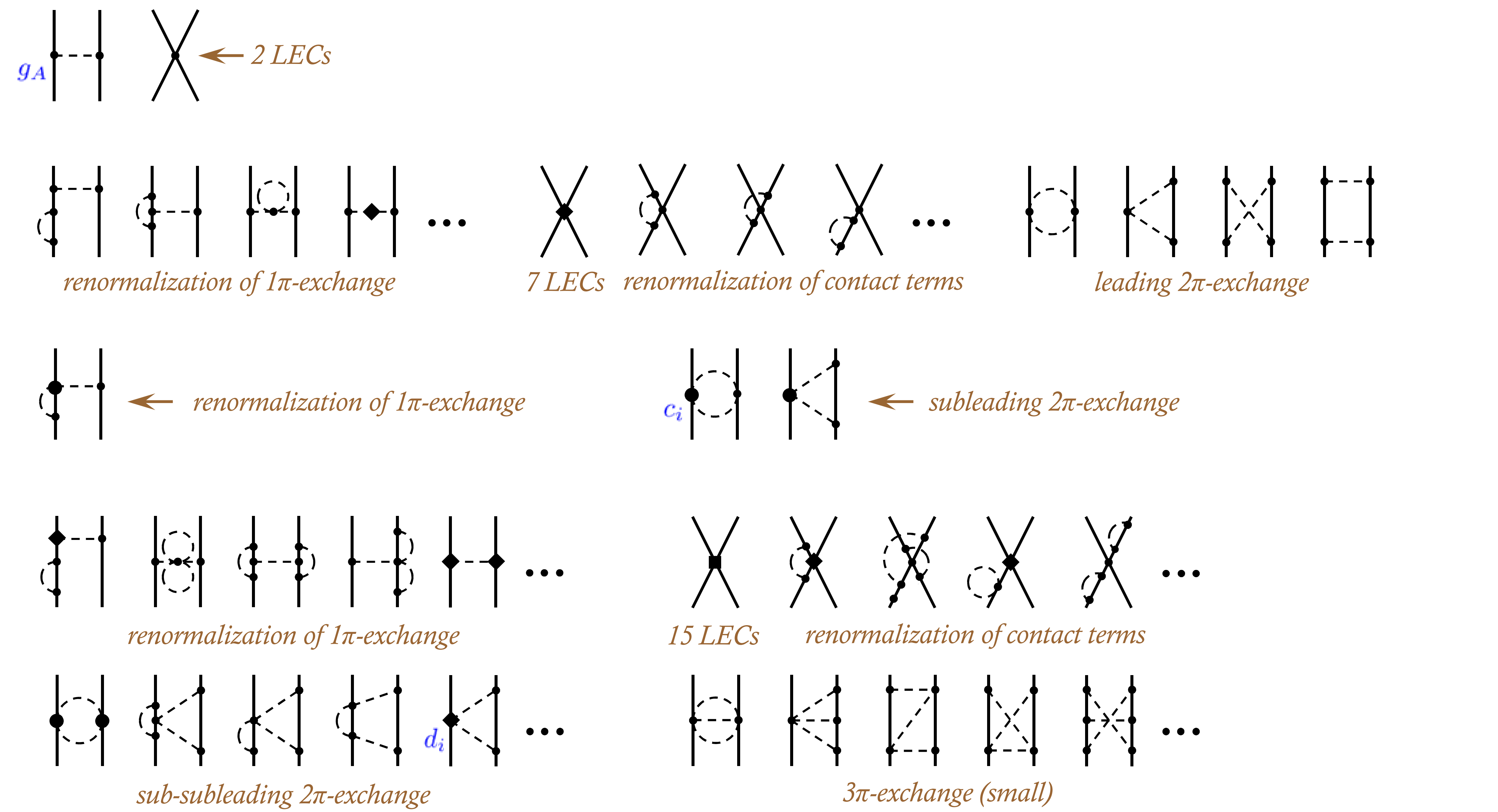
How to build a chiral potential



How to build a chiral potential

Chiral expansion of the 2N force: $V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \ldots$

- LO:
- NLO:
- N²LO:
- N³LO:



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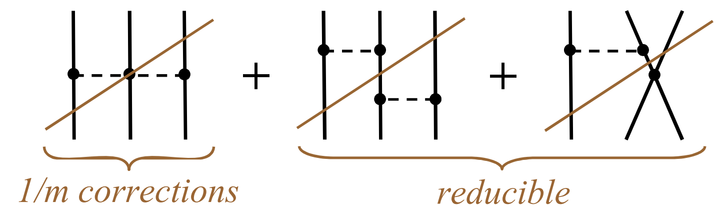
How to build a chiral potential

Three-body

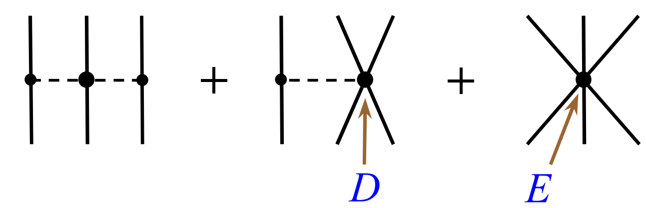
- **NLO:** does not contribute

  Weinberg '91; Coon & Friar '94; van Kolck '94; E.E. et al.,'98; ...



- **N^{2}LO:** first nonvanishing contributions

  van Kolck '94; E.E. et al. '02

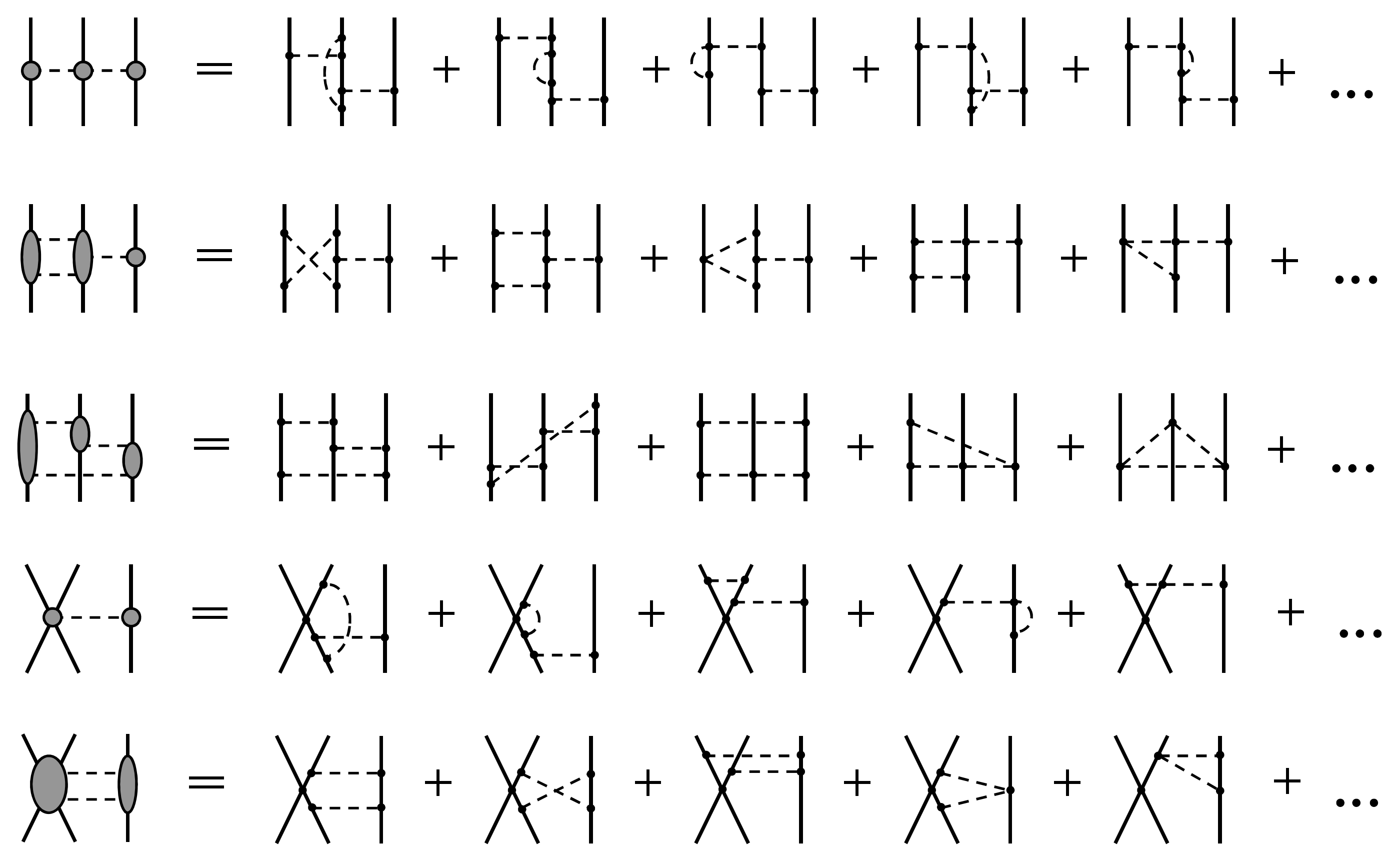


- **N^{3}LO:** work in progress

  Bernard, E.E., Krebs, Meißner '07
  Ishikawa, Robilotta '07

  — no free parameters

  — chiral symmetry plays essential role



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How to build a chiral potential

NUMBER OF PARAMETERS
for the *np* potential

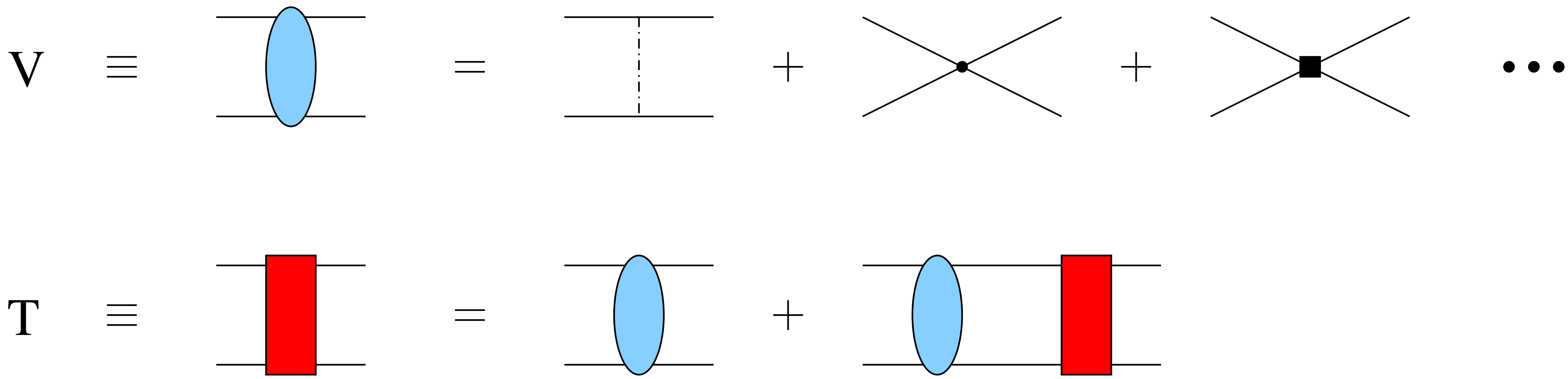
	Nijmegen PWA93	CD-Bonn "high precision"	NLO Q^2 (NNLO)	N^{3}LO Q^4 (N^{4}LO)	N^{5}LO Q^6
$^1S_0$	3	4	2	4	6
$^3S_1$	3	4	2	4	6
$^3S_1$-$^3D_1$	2	2	1	3	6
$^1P_1$	3	3	1	2	4
$^3P_0$	3	2	1	2	4
$^3P_1$	2	2	1	2	4
$^3P_2$	3	3	1	2	4
$^3P_2$-$^3F_2$	2	1	0	1	3
$^1D_2$	2	3	0	1	2
$^3D_1$	2	1	0	1	2
$^3D_2$	2	2	0	1	2
$^3D_3$	1	2	0	1	2
$^3D_3$-$^3G_3$	1	0	0	0	1
$^1F_3$	1	1	0	0	1
$^3F_2$	1	2	0	0	1
$^3F_3$	1	2	0	0	1
$^3F_4$	2	1	0	0	1
$^3F_4$-$^3H_4$	0	0	0	0	0
$^1G_4$	1	0	0	0	0
$^3G_3$	0	1	0	0	0
$^3G_4$	0	1	0	0	0
$^3G_5$	0	1	0	0	0
Total	35	38	9	24	50

Then add parameters for three-body forces:

c_2, c_3, c_4, c_E, c_D

How to build a chiral potential

Phase shifts & potential



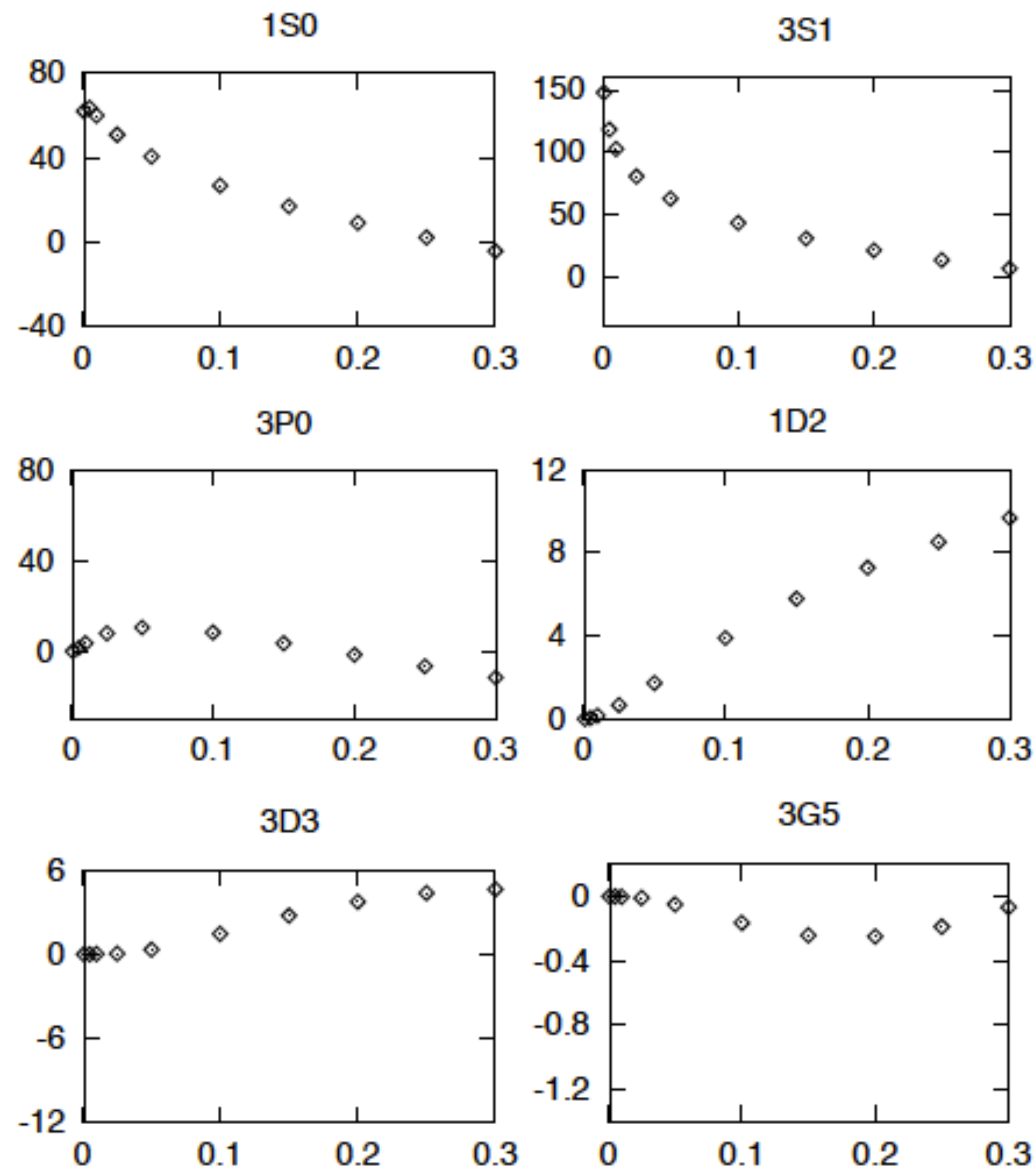
$$\widehat{T}(\vec{p}\,', \vec{p}) = \widehat{V}(\vec{p}\,', \vec{p}) + \int d^3p'' \widehat{V}(\vec{p}\,', \vec{p}\,'') \frac{M_N}{p^2 - p''^2 + i\epsilon} \widehat{T}(\vec{p}\,'', \vec{p})$$

A free available code to solve the phase-shifts problem can be found
http://folk.uio.no/mhjensen/manybody/phase.tar.gz

How to build a chiral potential

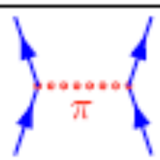
Phase shifts

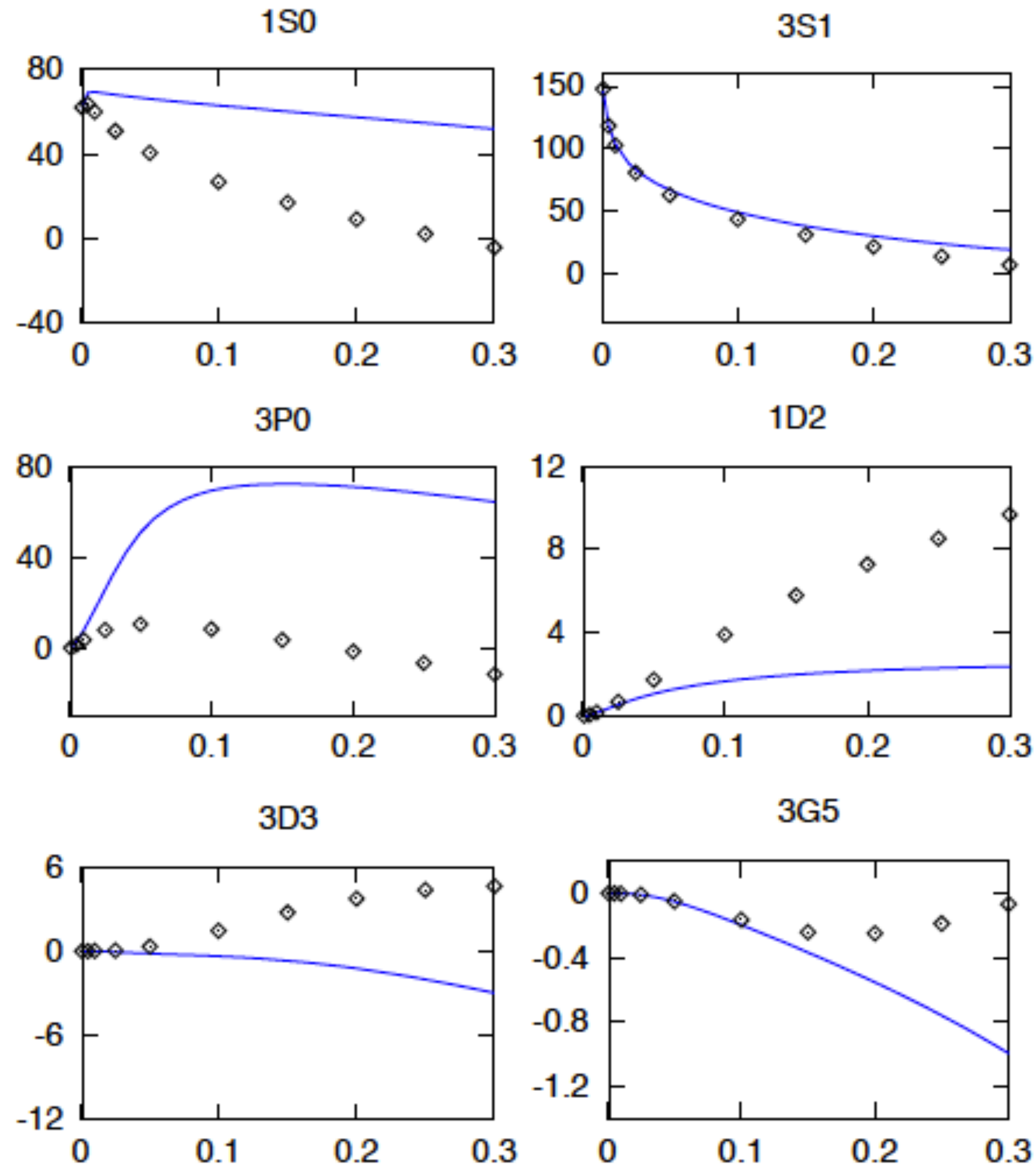
Q^{ν}	1π	2π	$4N$



How to build a chiral potential

Phase shifts

Q^ν	1π	2π	$4N$
Q^0		—	 (2)



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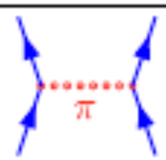
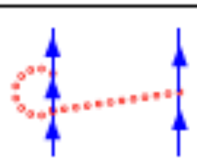
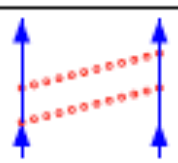
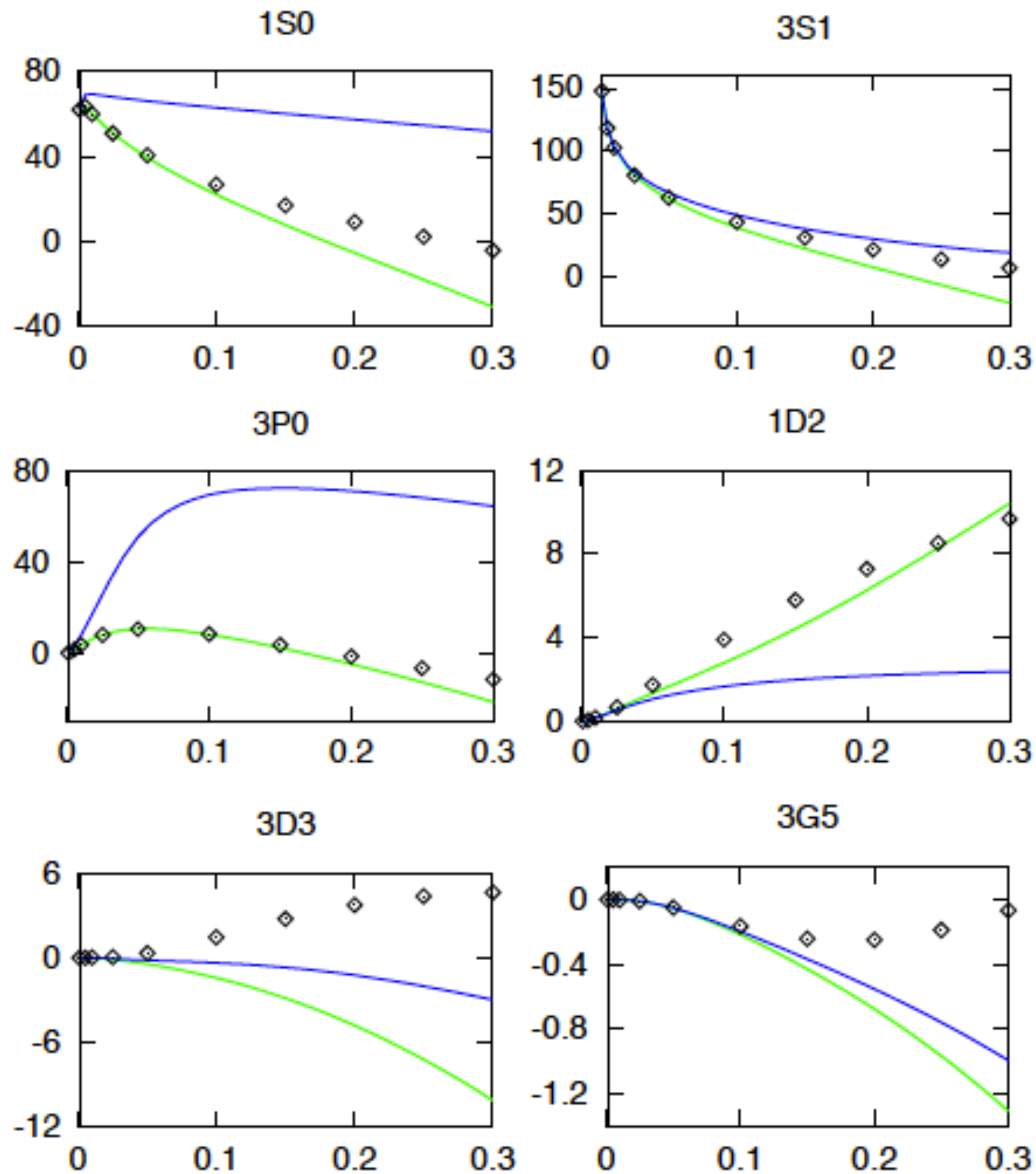
How to build a chiral potential

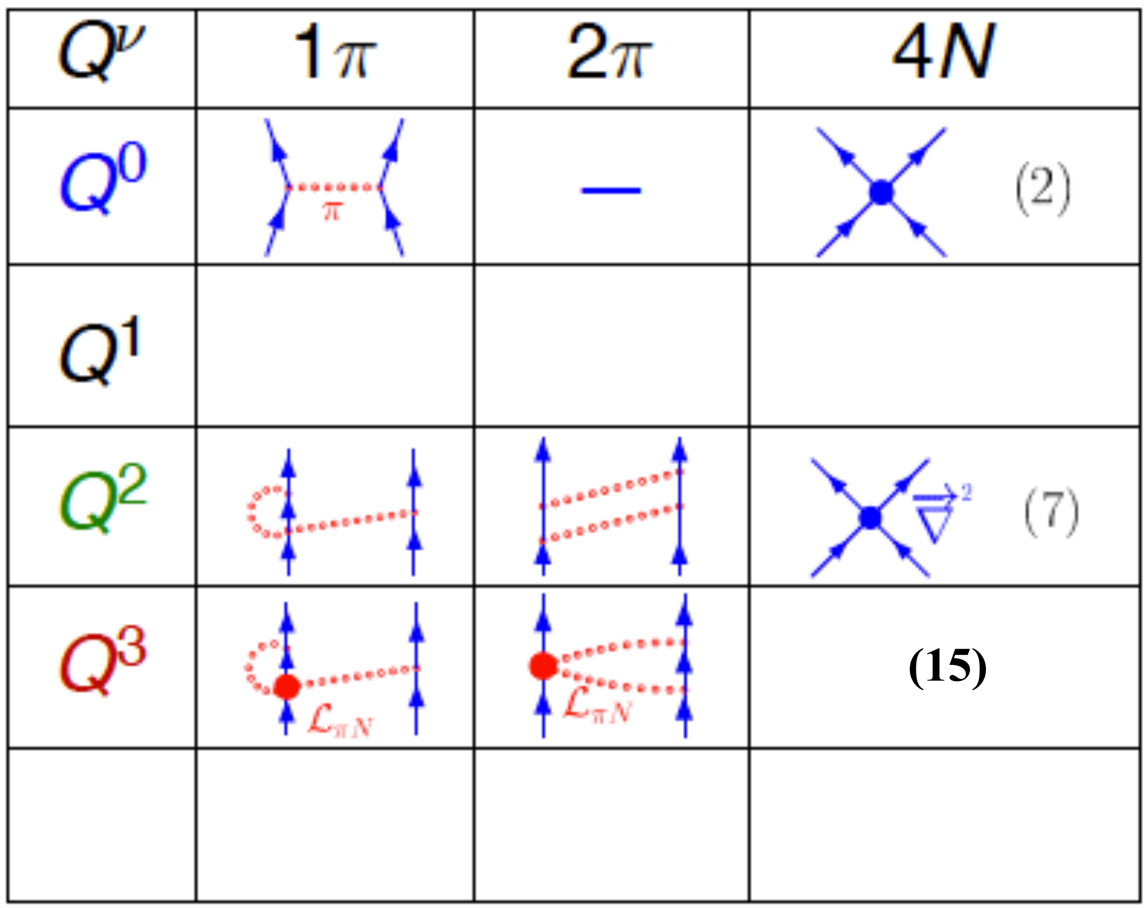
Phase shifts

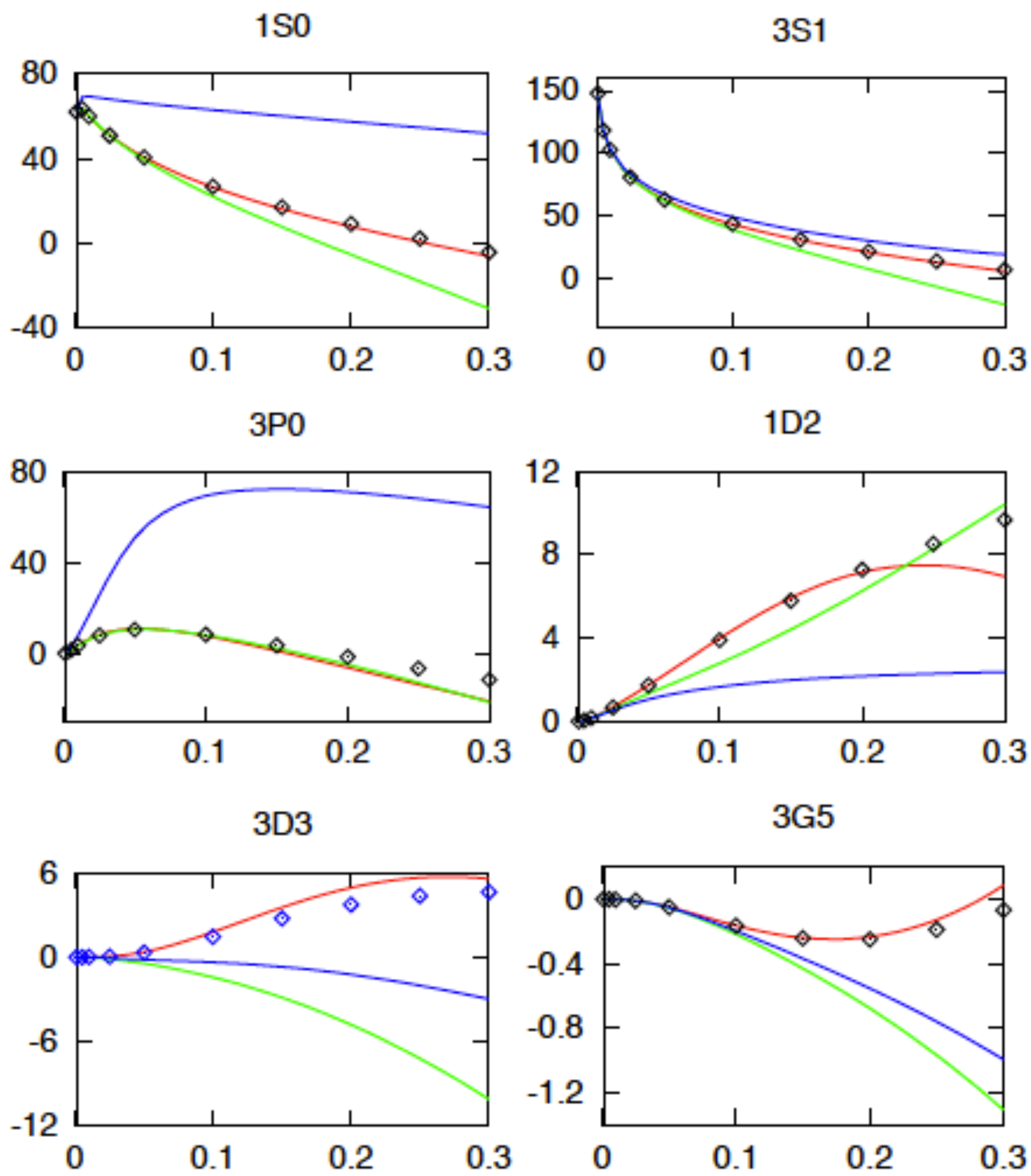
Q^ν	1π	2π	$4N$
Q^0		—	 (2)
Q^1			
Q^2			 (7)



How to build a chiral potential

Phase shifts





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How to build a chiral potential

Machleidt (N3LO) | **Epelbaum (N3LO)**

Lippmann-Schwinger cutoff

$$V(\boldsymbol{k}, \boldsymbol{k}') \to V(\boldsymbol{k}, \boldsymbol{k}') f^{\Lambda}(k, k')$$

$$f^{\Lambda} = \exp\left(-(k'/\Lambda)^{2n} - (k/\Lambda)^{2n}\right) \quad \text{with} \quad n = 2, 3$$

cutoff the short-range part of the 2PE contribution

dimensional regularization

$$\Lambda = 450, 500, 600$$

spectral function

$$\{\Lambda, \tilde{\Lambda}\} = \{450, 500\}, \{450, 700\}, \{550, 600\}, \{600, 600\}, \{600, 700\}$$

How to build a chiral potential

Phase shifts

R. Machleidt et al,
Chiral effective field theory and nuclear forces, Phys. Rep. 503 (2011)

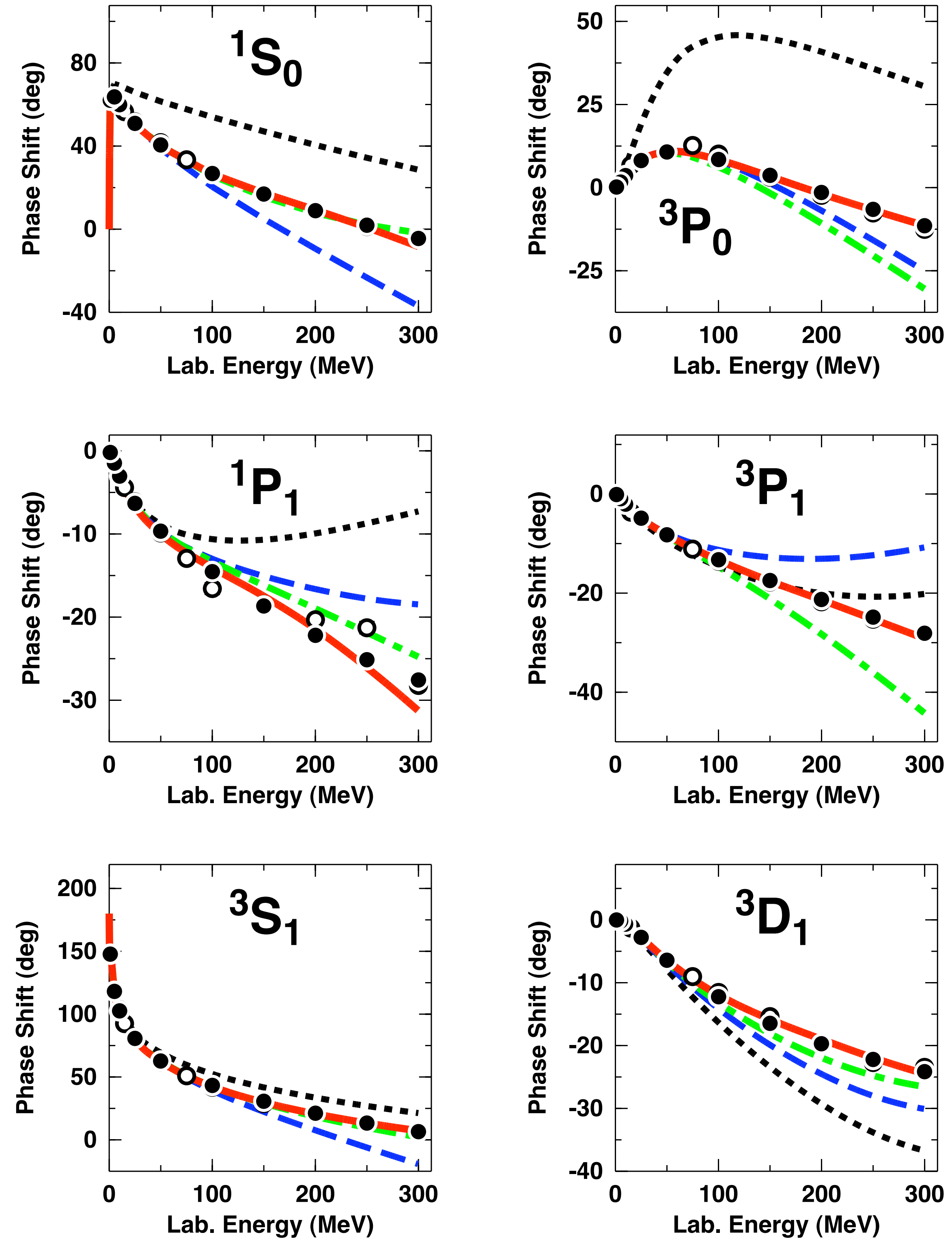
Phase shifts of np scattering as calculated from NN potentials at different orders of ChPT.

LO

NLO

NNLO

N3LO



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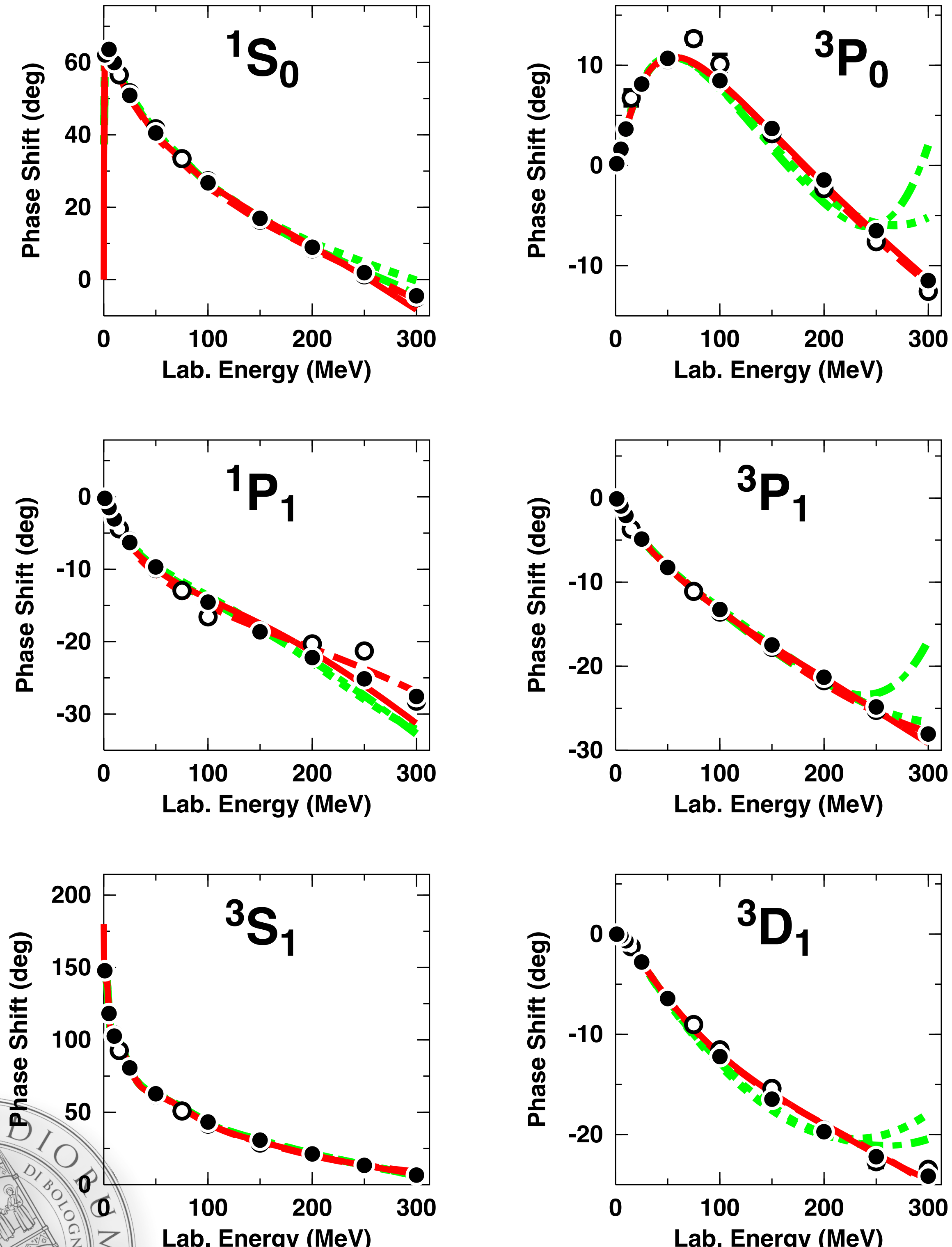
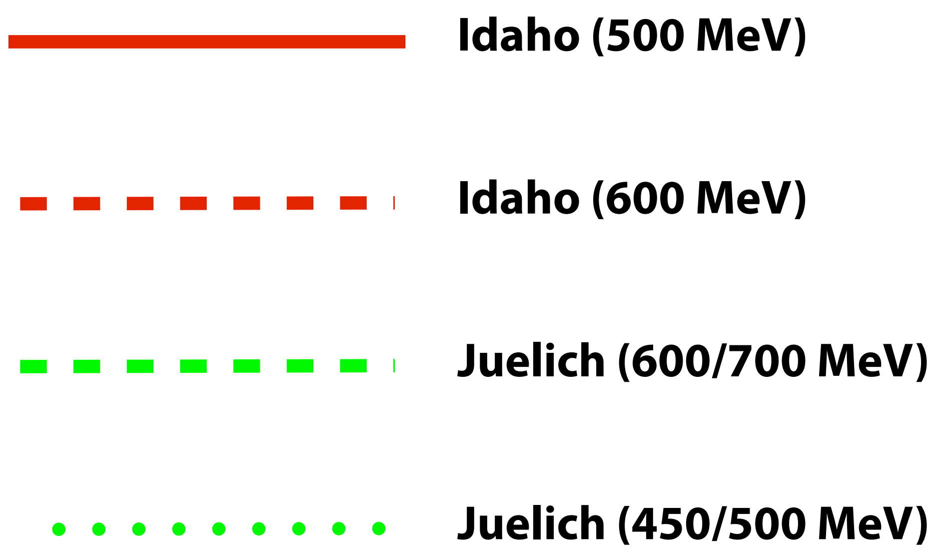


Neutron–proton phase parameters as described by various chiral potentials at N3LO



How to build a chiral potential

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R. Machleidt et al,
***Chiral effective field theory and nuclear forces*, Phys. Rep. 503 (2011)**

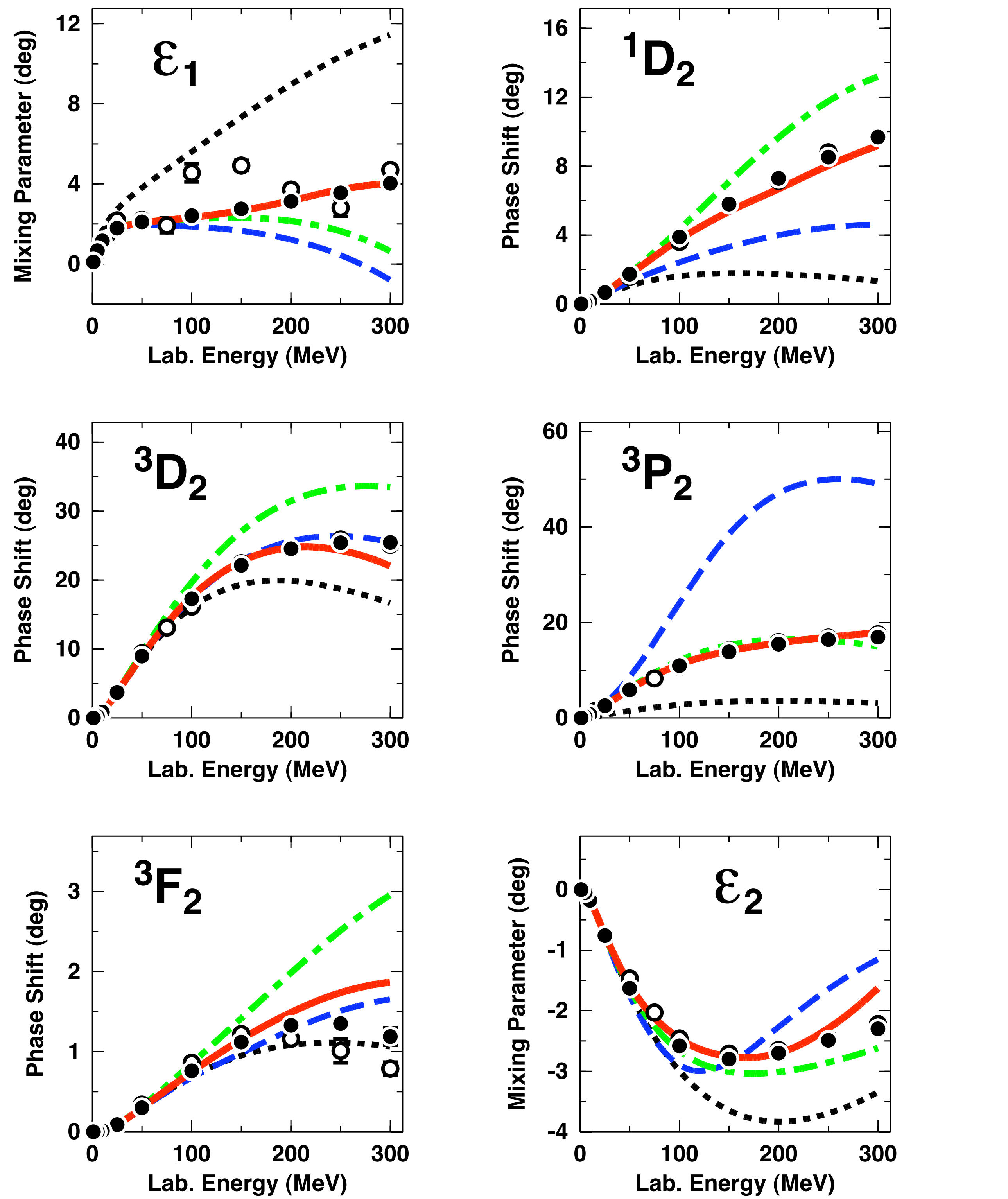
Phase shifts of np scattering as calculated from NN potentials at different orders of ChPT.

LO

NLO

NNLO

N3LO



How to build a chiral potential

Phase shifts

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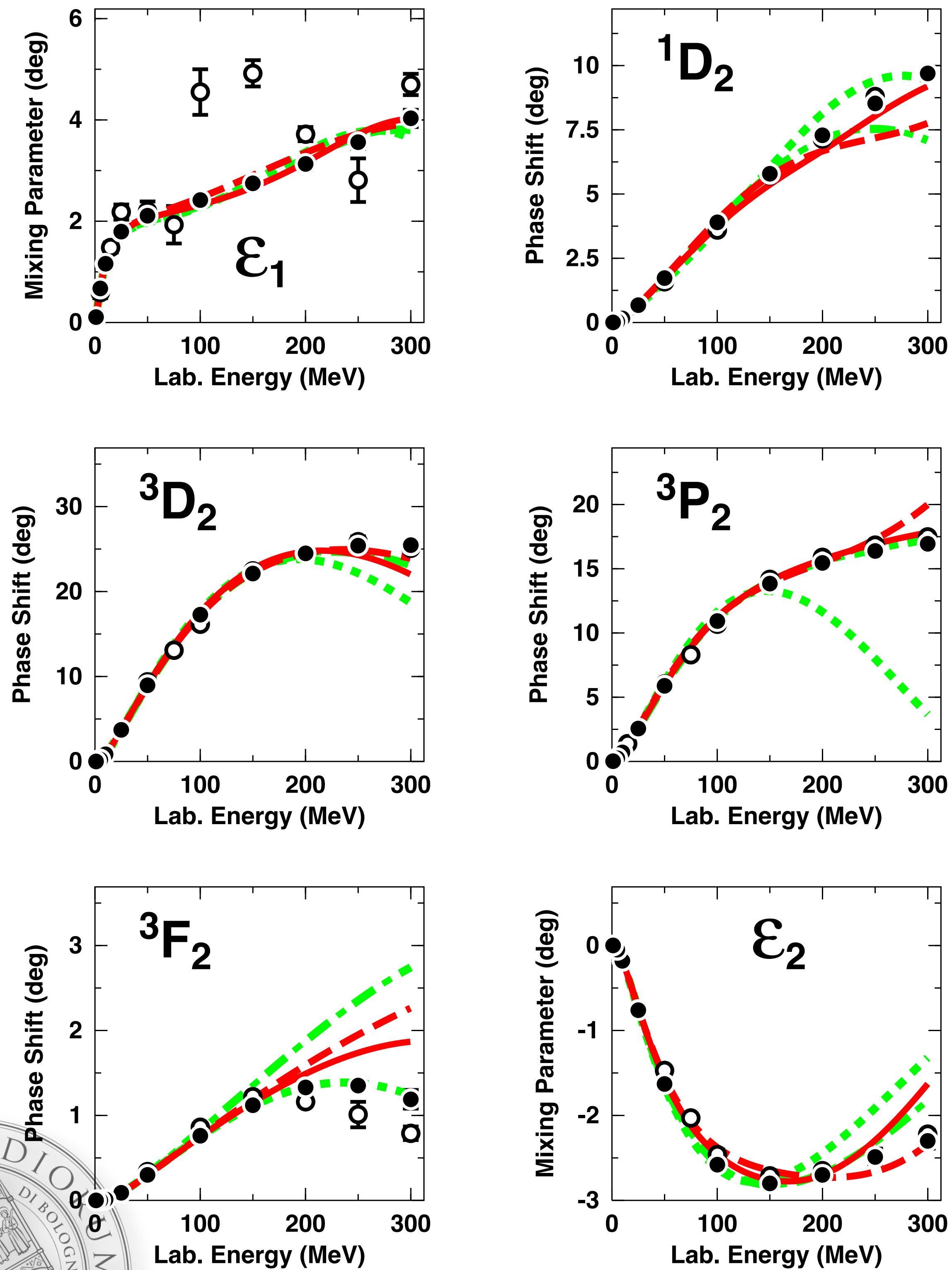

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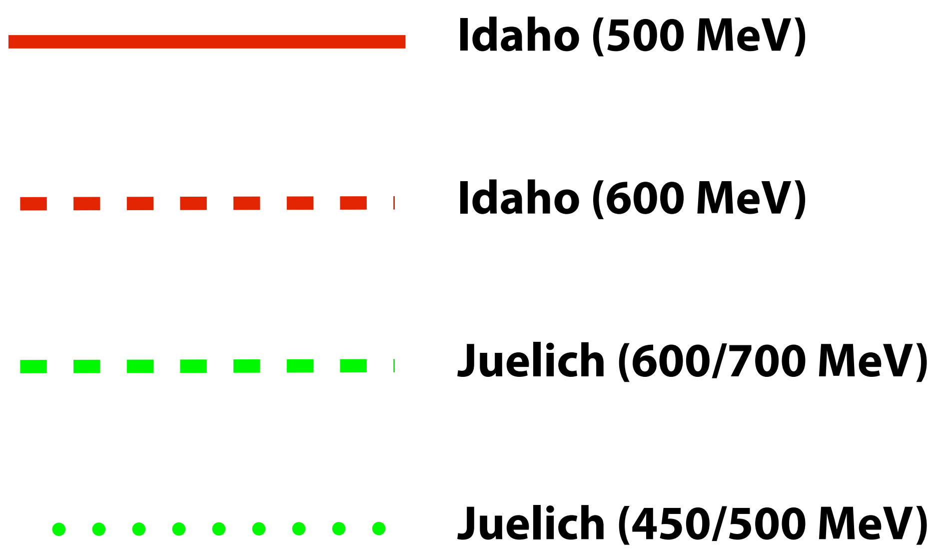
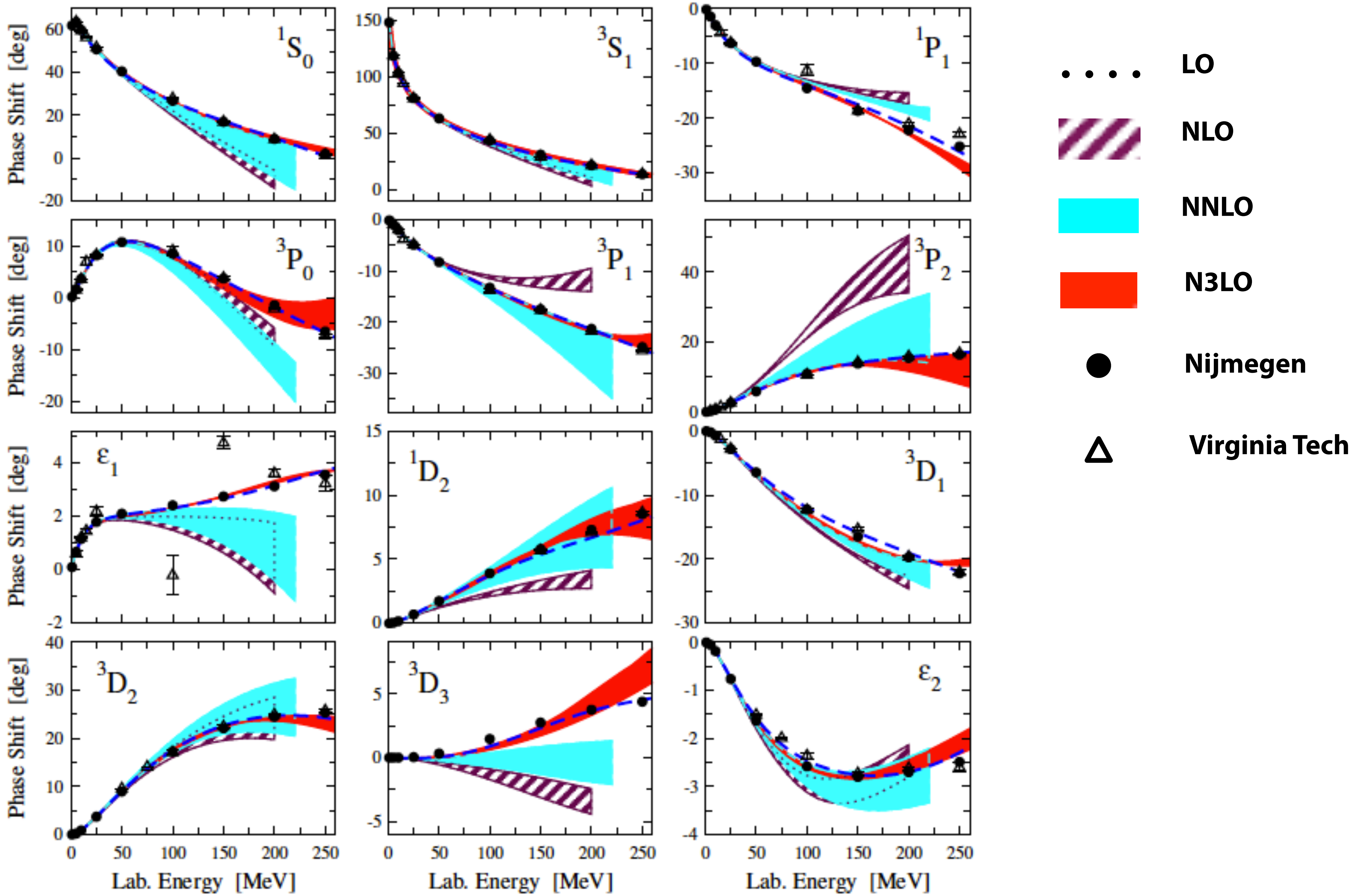


How to build a chiral potential

Exploit the cutoffs

E. Epelbaum et al, *Modern Theory of Nuclear Forces*, Rev. Mod. Phys. 81 (2009) 1773-1825



How to build a chiral potential

Exploit the cutoffs

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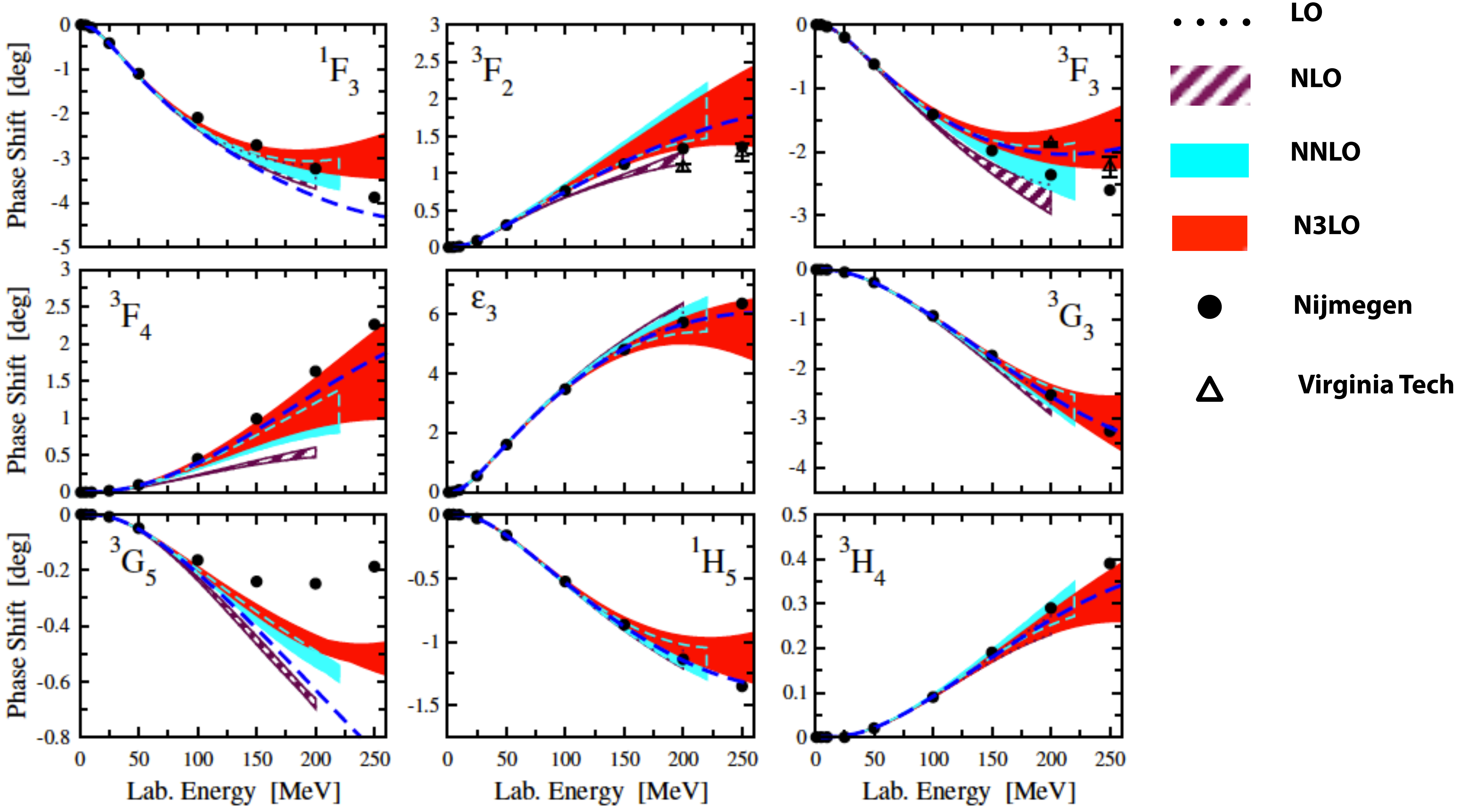


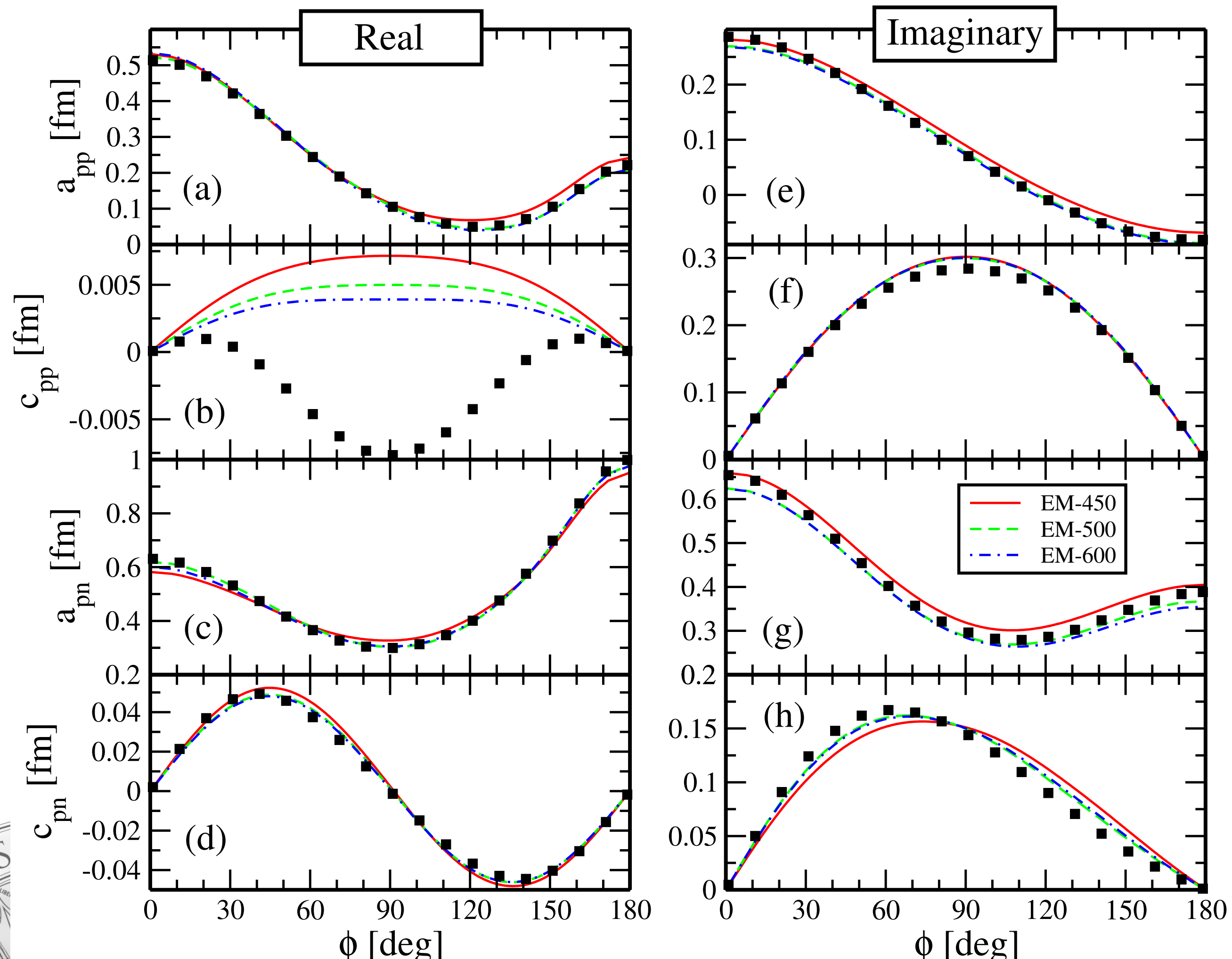
Results

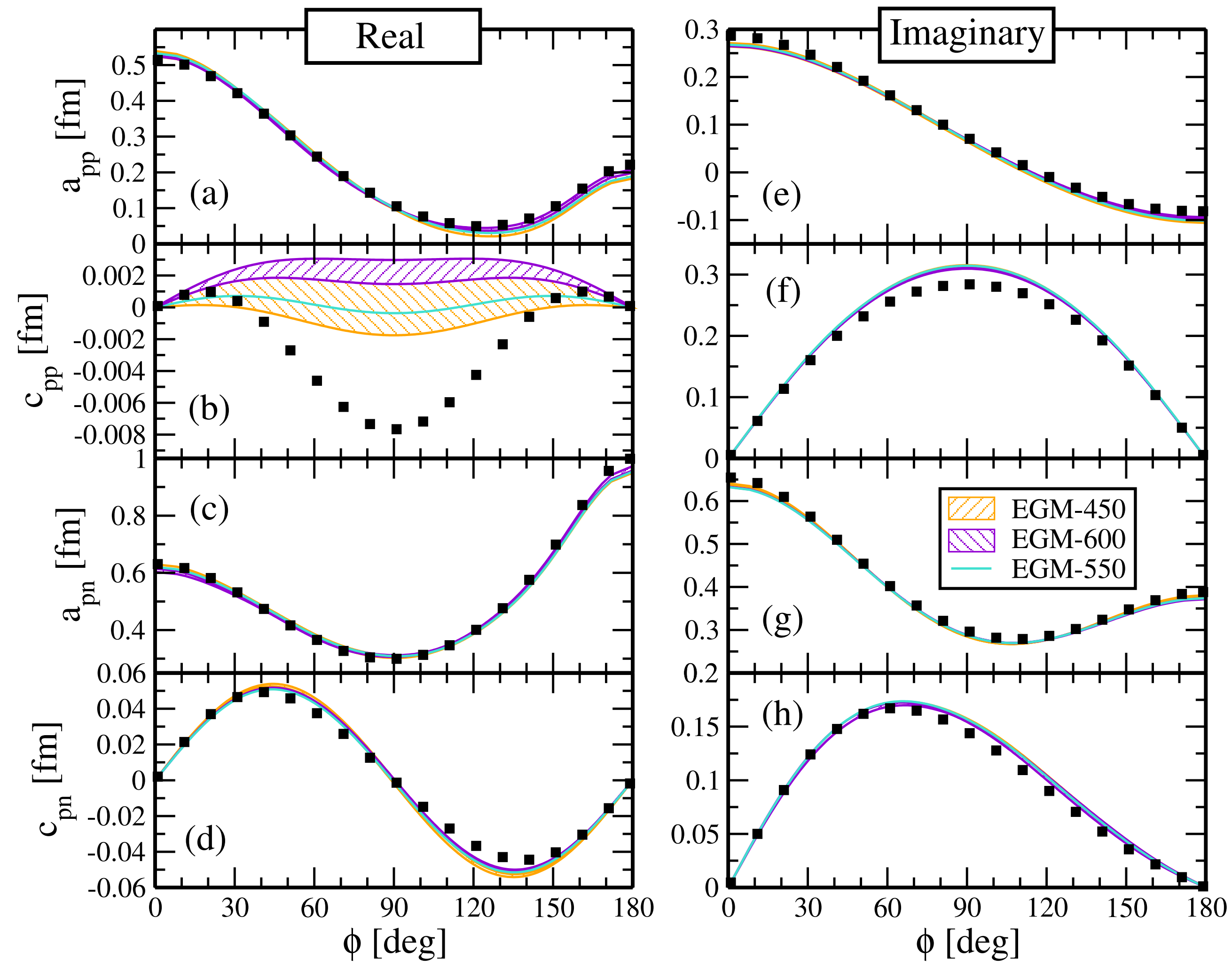
NN amplitudes - 100 MeV

$$M(\boldsymbol{\kappa}', \boldsymbol{\kappa}, \omega) = \langle \boldsymbol{\kappa}' | M(\omega) | \boldsymbol{\kappa} \rangle = -4\pi^2 \mu \langle \boldsymbol{\kappa}' | t(\omega) | \boldsymbol{\kappa} \rangle$$

$$a_{pN} = \frac{1}{f_{pN}\pi^2} \sum_{L=0}^{\infty} P_L(\cos\phi) \Big[(2L+1)\, M_{LL}^{L,S=0} + (2L+1)\, M_{LL}^{L,S=1} + (2L+3)\, M_{LL}^{L+1,S=1} + (2L-1)\, M_{LL}^{L-1,S=1} \Big]$$

$$c_{pN} = \frac{i}{f_{pN}\pi^2} \sum_{L=1}^{\infty} P_L^1(\cos\phi) \Big[\left(\frac{2L+3}{L+1}\right) M_{LL}^{L+1,S=1} - \left(\frac{2L+1}{L(L+1)}\right) M_{LL}^{L,S=1} - \left(\frac{2L-1}{L}\right) M_{LL}^{L-1,S=1} \Big]$$



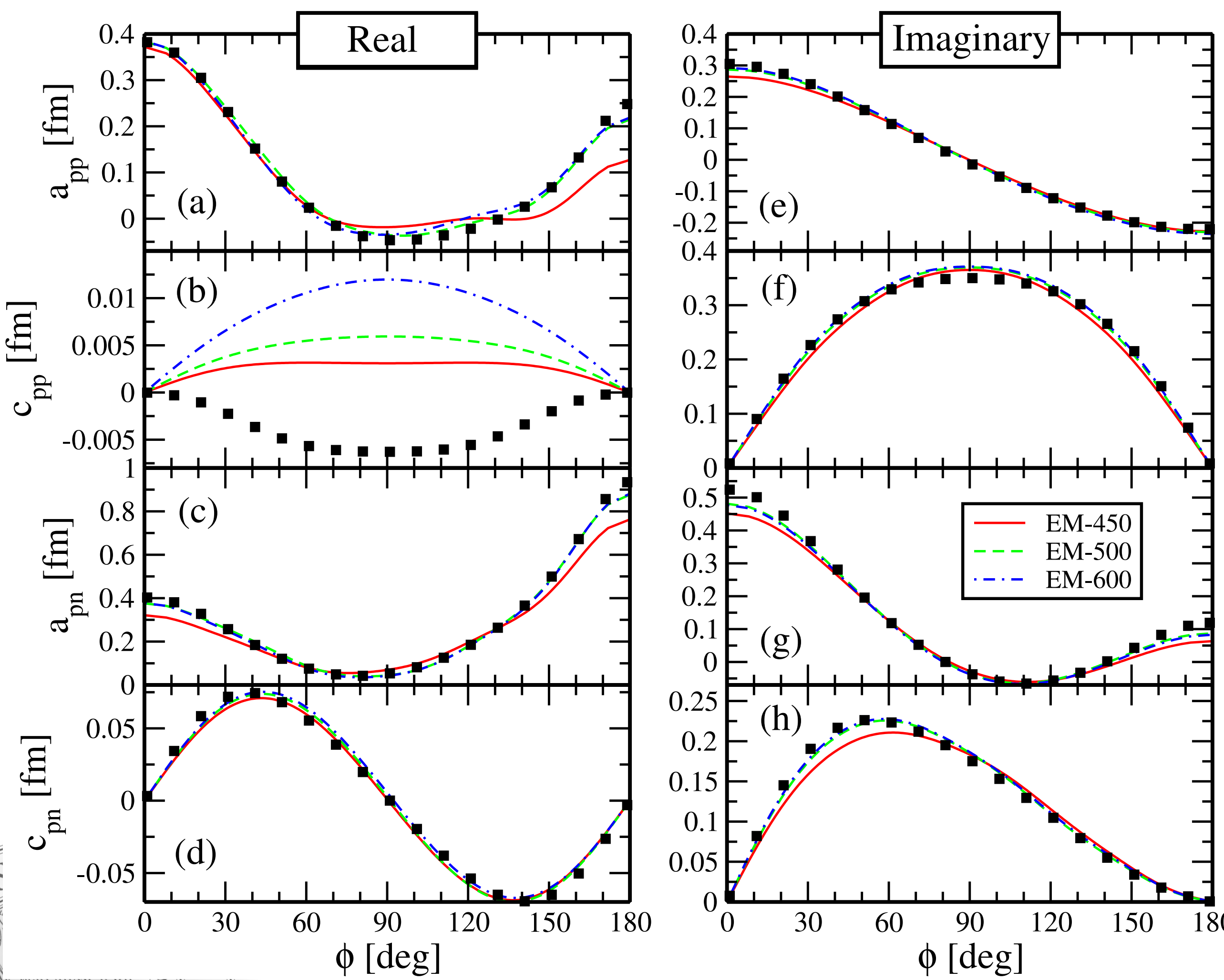


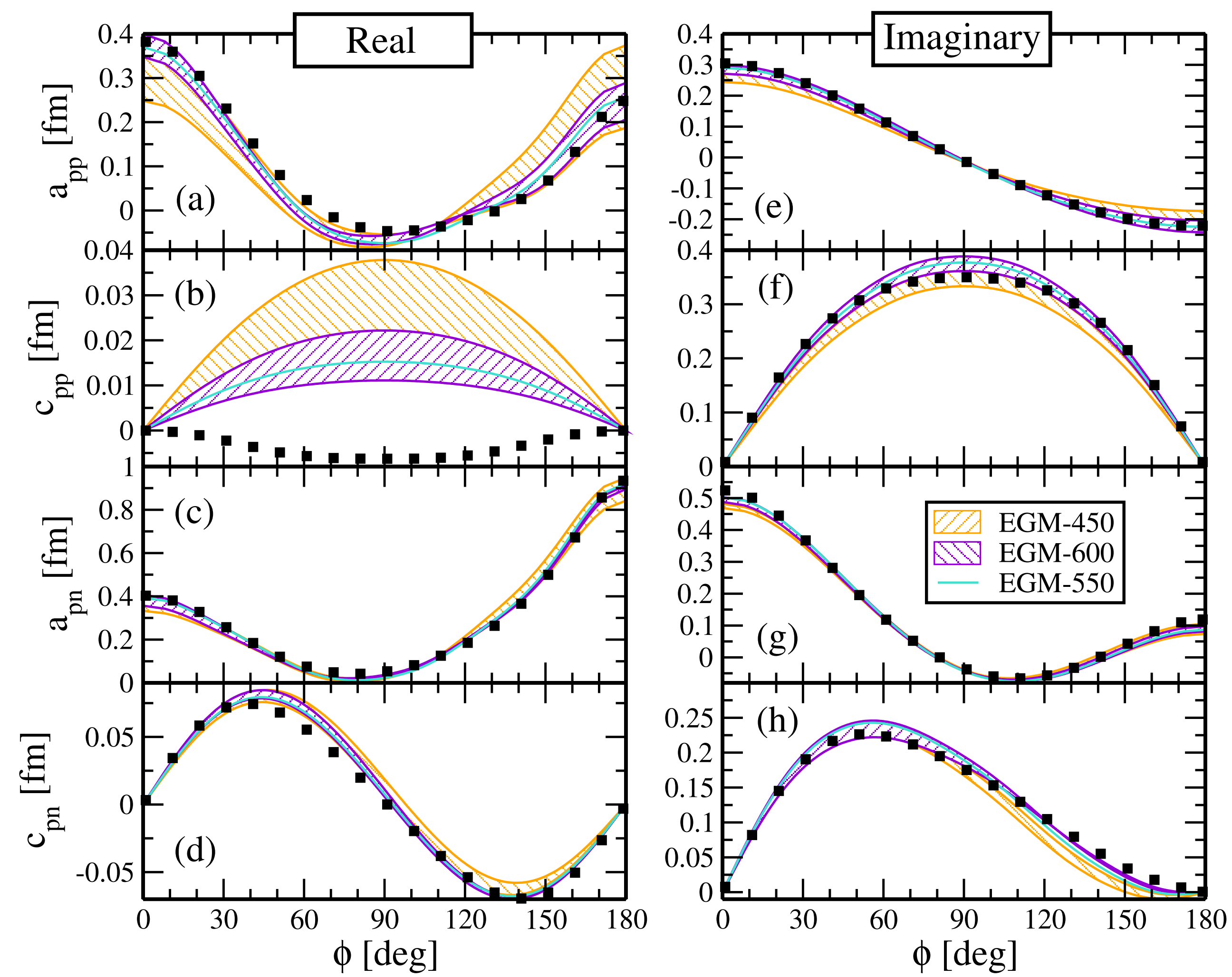
NN amplitudes - 200 MeV

$$M(\boldsymbol{\kappa}', \boldsymbol{\kappa}, \omega) = \langle \boldsymbol{\kappa}' | M(\omega) | \boldsymbol{\kappa} \rangle = -4\pi^2 \mu \langle \boldsymbol{\kappa}' | t(\omega) | \boldsymbol{\kappa} \rangle$$

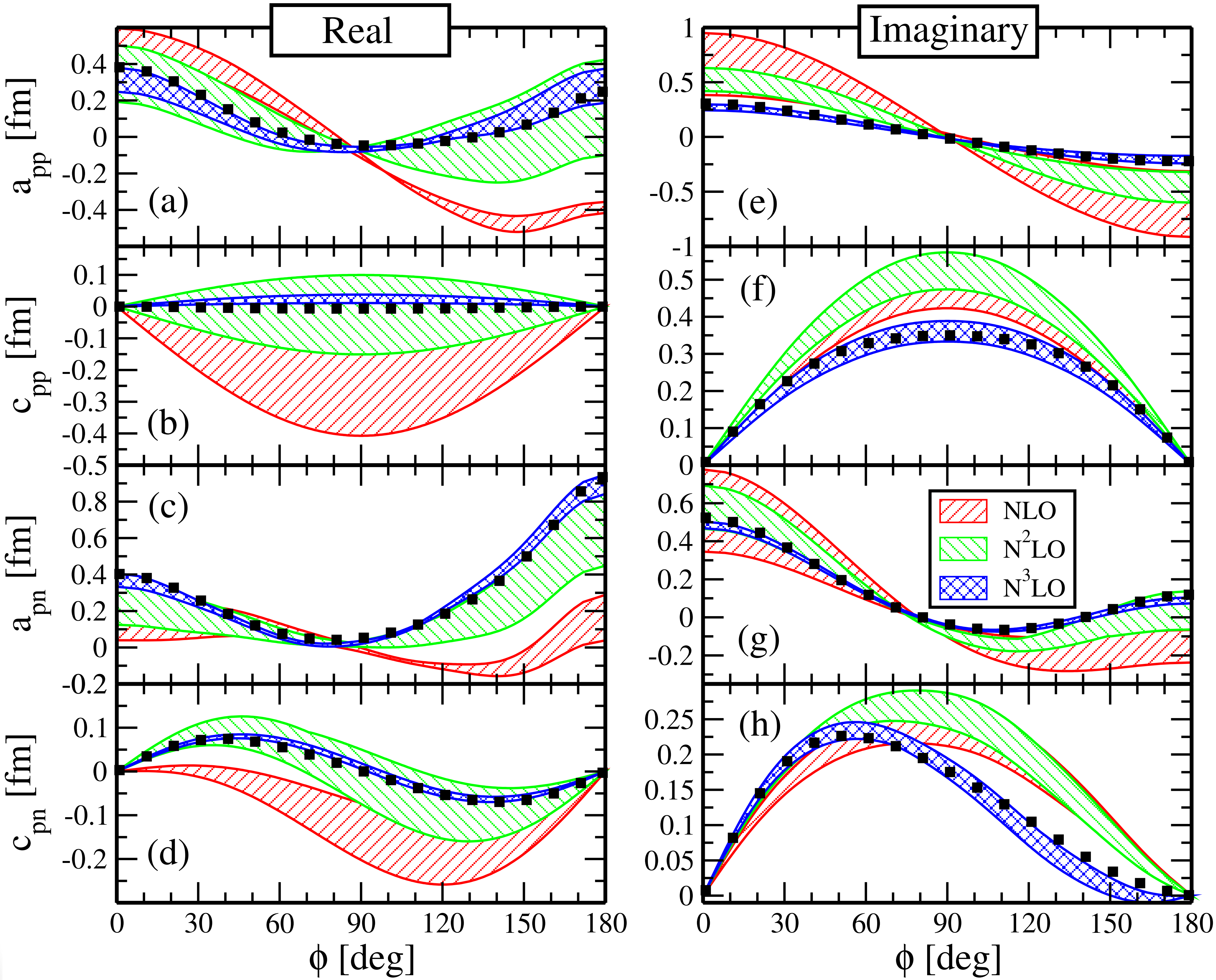
$$a_{pN} = \frac{1}{f_{pN}\pi^2} \sum_{L=0}^{\infty} P_L(\cos\phi) \Big[(2L+1)\, M_{LL}^{L,S=0} + (2L+1)\, M_{LL}^{L,S=1} + (2L+3)\, M_{LL}^{L+1,S=1} + (2L-1)\, M_{LL}^{L-1,S=1} \Big]$$

$$c_{pN} = \frac{i}{f_{pN}\pi^2} \sum_{L=1}^{\infty} P_L^1(\cos\phi) \left[\left(\frac{2L+3}{L+1}\right) M_{LL}^{L+1,S=1} - \left(\frac{2L+1}{L(L+1)}\right) M_{LL}^{L,S=1} - \left(\frac{2L-1}{L}\right) M_{LL}^{L-1,S=1} \right]$$

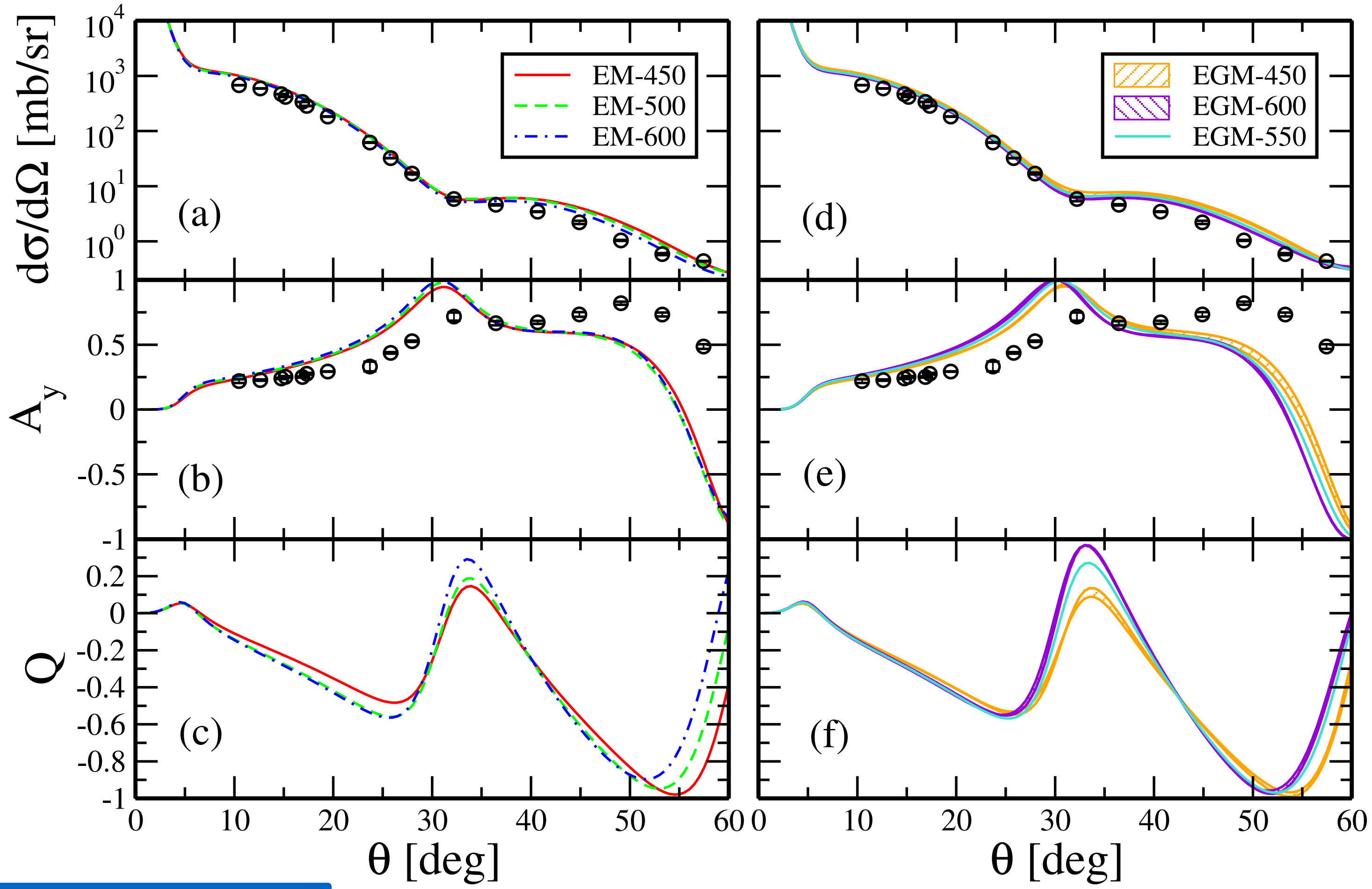




NN amplitudes - convergence (200 MeV)



Oxygen 16

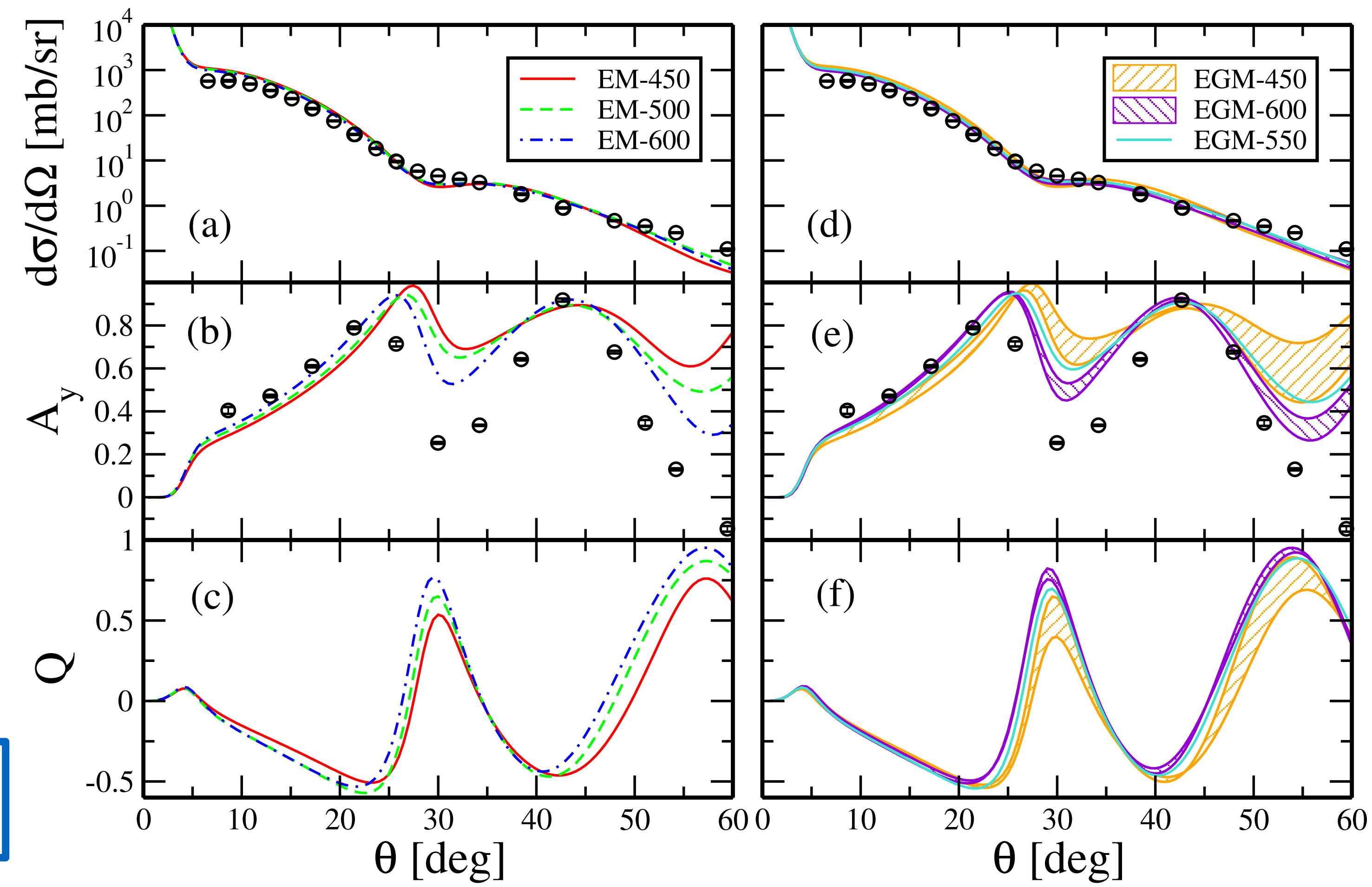


100 MeV

$$\frac{d\sigma}{d\Omega}(\theta) = |A(\theta)|^2 + |C(\theta)|^2$$

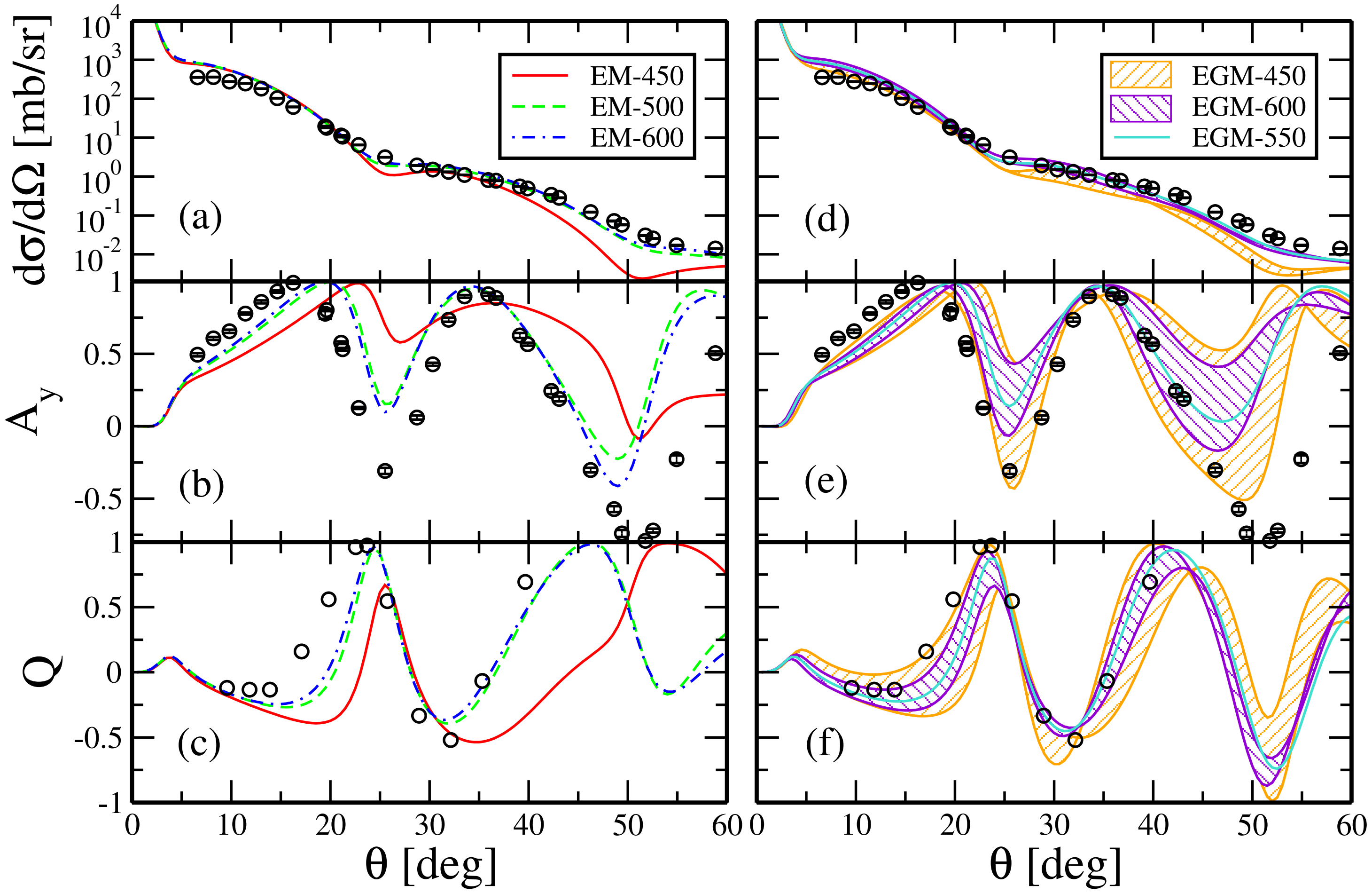
$$A_y(\theta) = \frac{2\mathrm{Re}[A^*(\theta)\, C(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$

$$Q(\theta) = \frac{2\mathrm{Im}[A(\theta)\, C^*(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$



135 MeV

Oxygen 16

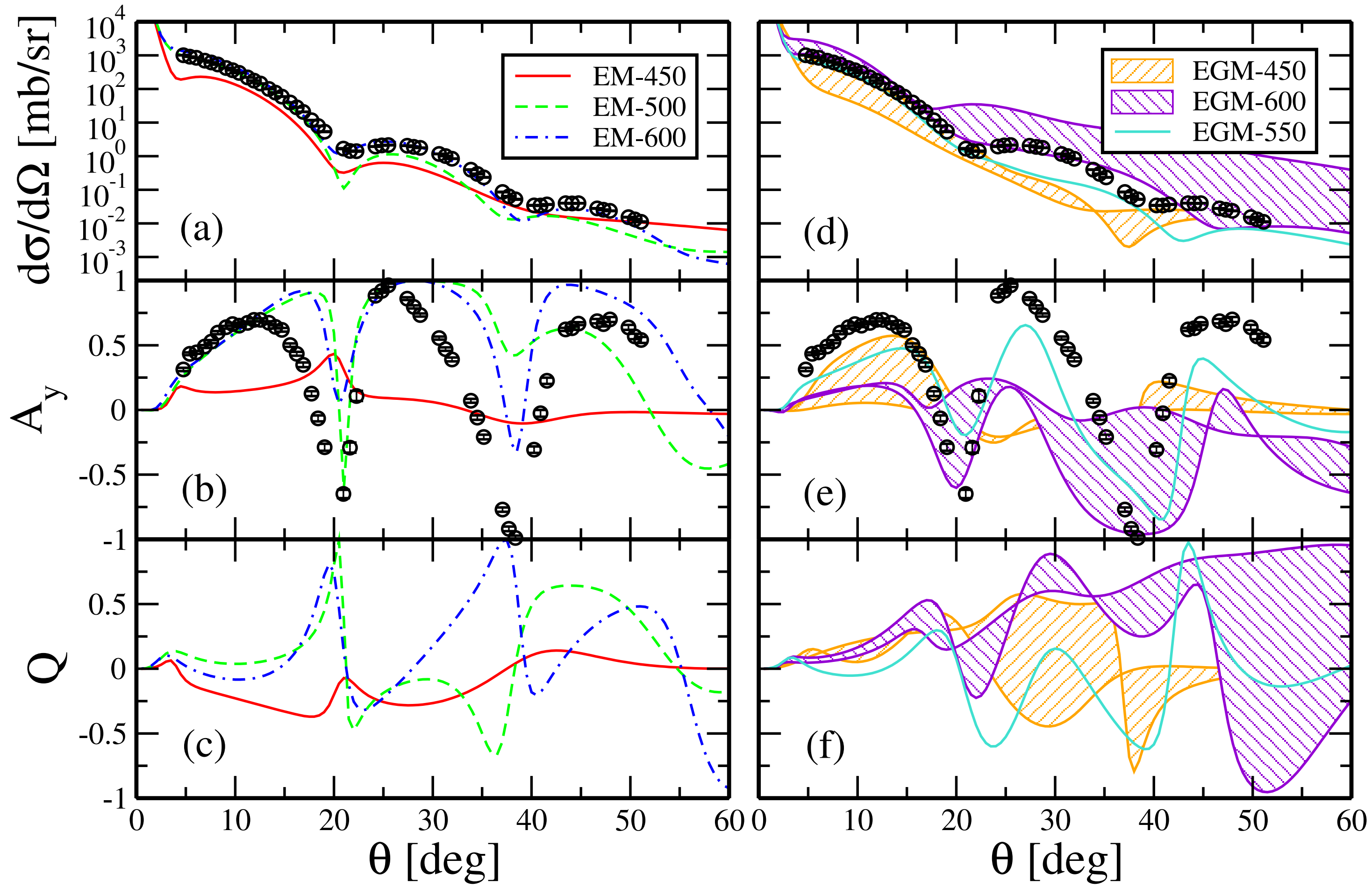


200 MeV

$$\frac{d\sigma}{d\Omega}(\theta) = |A(\theta)|^2 + |C(\theta)|^2$$

$$A_y(\theta) = \frac{2\mathrm{Re}[A^*(\theta)\,C(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$

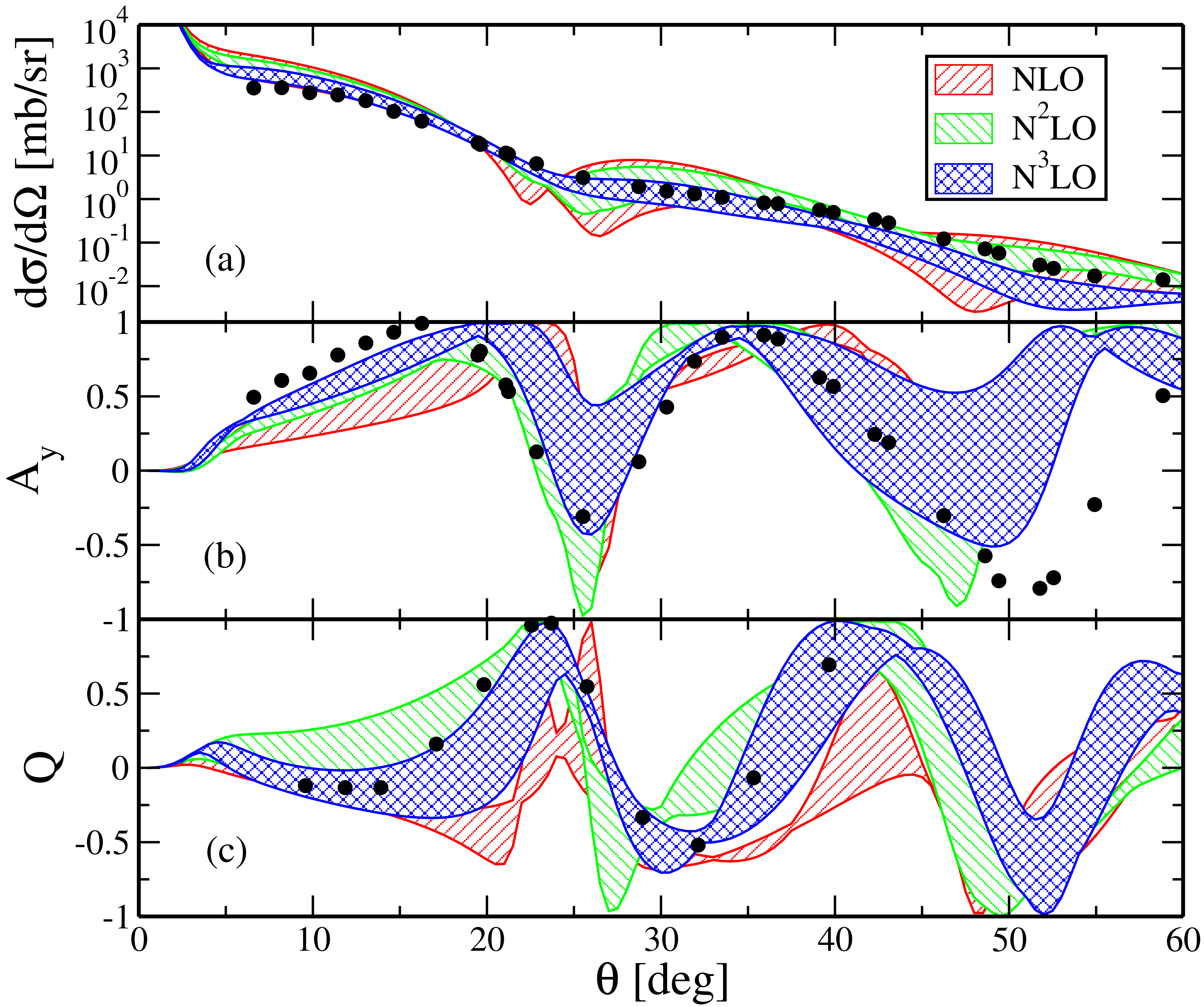
$$Q(\theta) = \frac{2\mathrm{Im}[A(\theta)\,C^*(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2}$$



318 MeV

Scattering observable - convergence

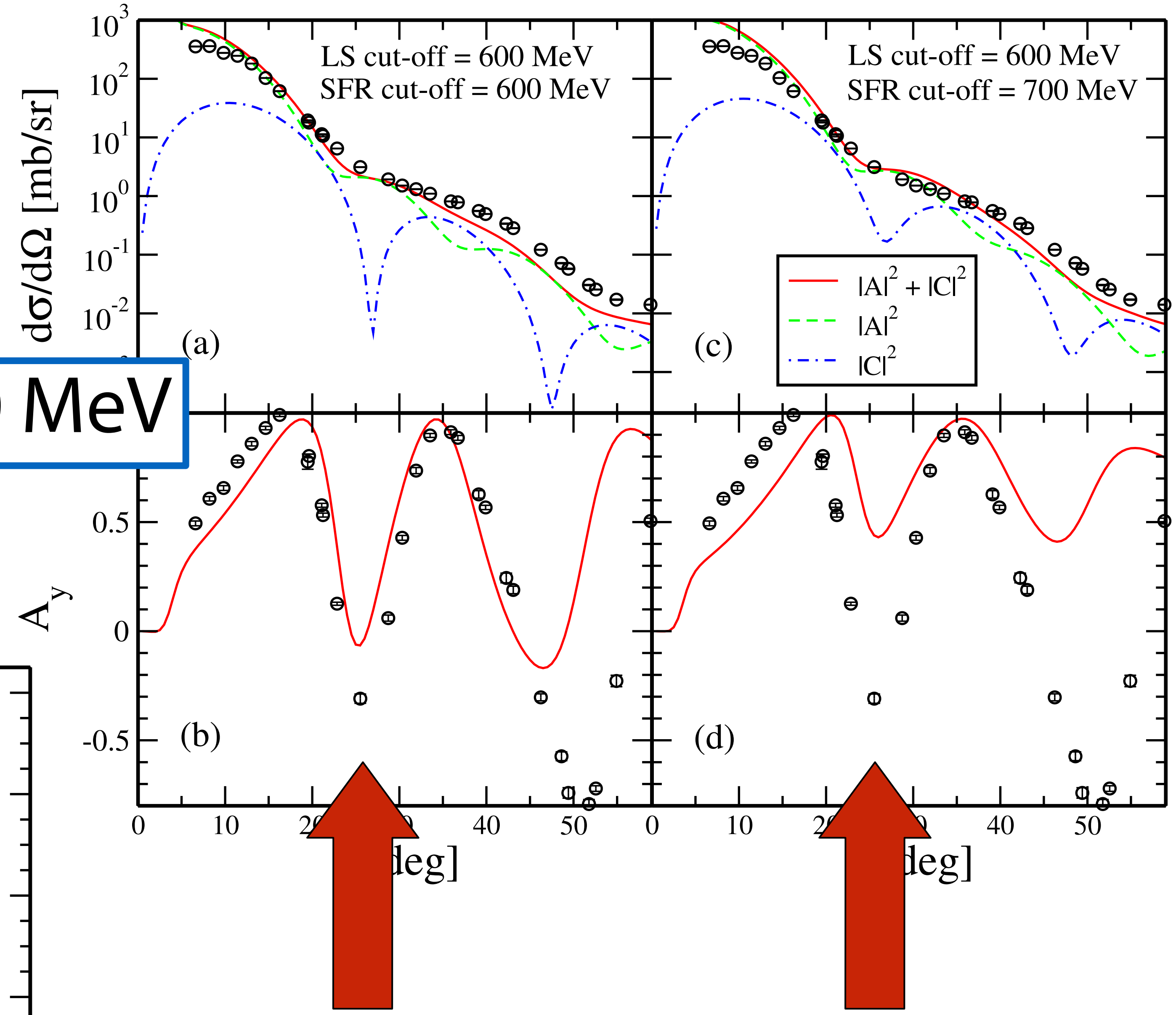
Oxygen 16



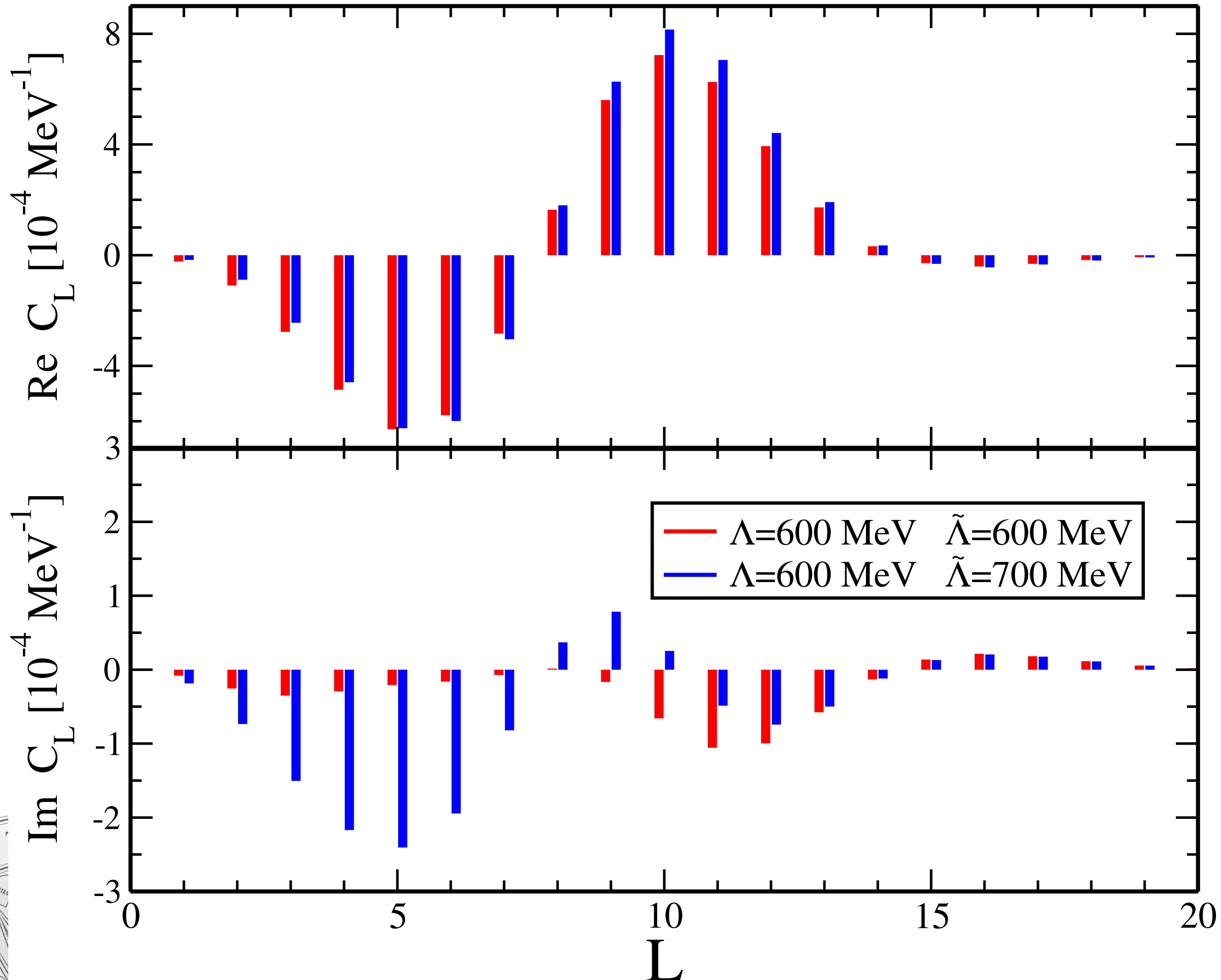
Scattering observable

analysis of the contributions

200 MeV



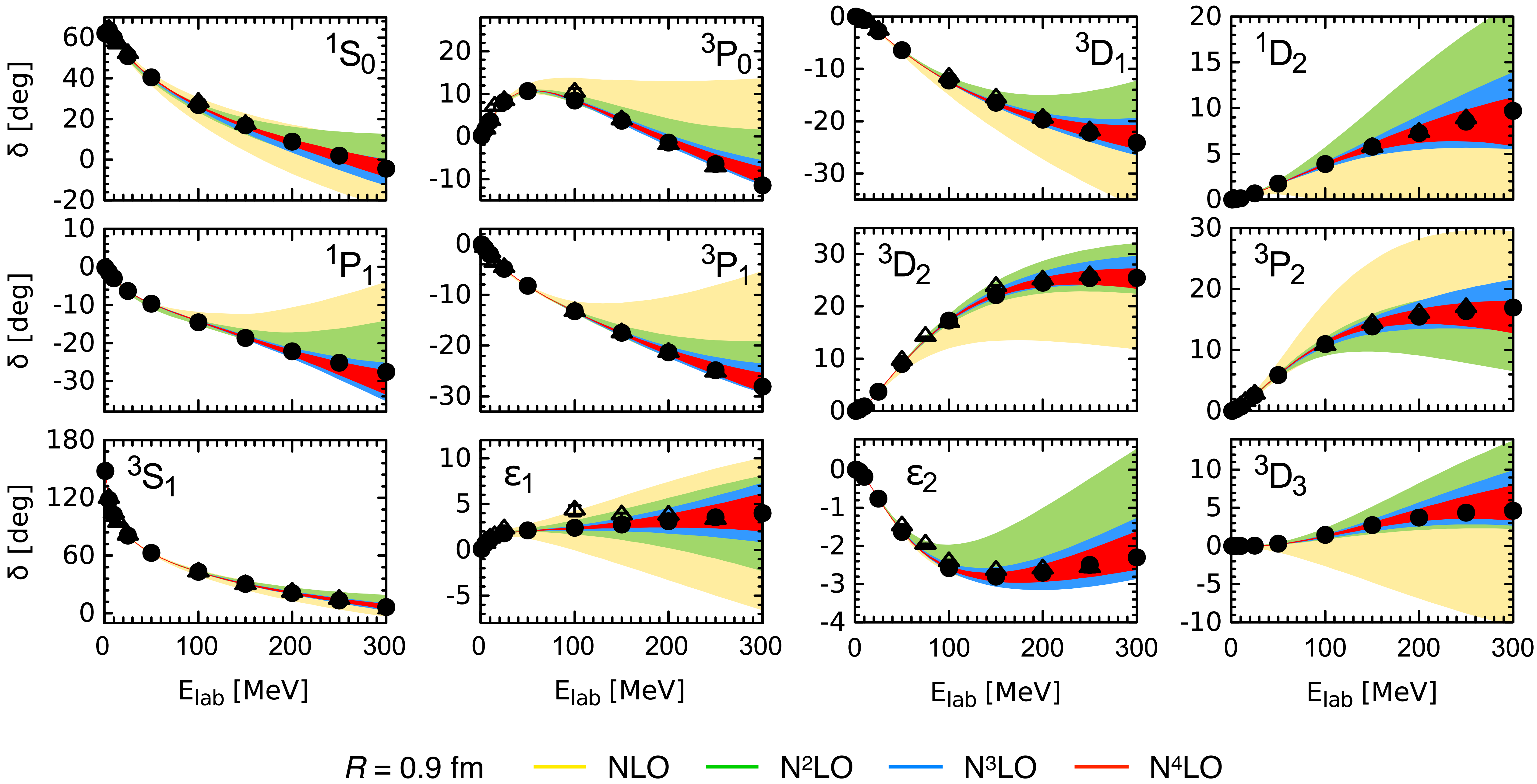
$$C_L = \left[F_L^+(k_0) - F_L^-(k_0)\right] P_L^1(\cos\theta)$$



Future

Go to N4LO

Epelbaum, HK, Meißner, arXiv: 1412.4623



New renormalisation technique in the coordinate space with the cutoff R being chosen in the range of $R = 0.8 \ldots 1.2$ *fm*. For contact interactions, they use a non- local Gaussian regulator in momentum space with the cutoff $\Lambda = 2R^{-1}$

$$f\left(\frac{r}{R}\right) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^6$$

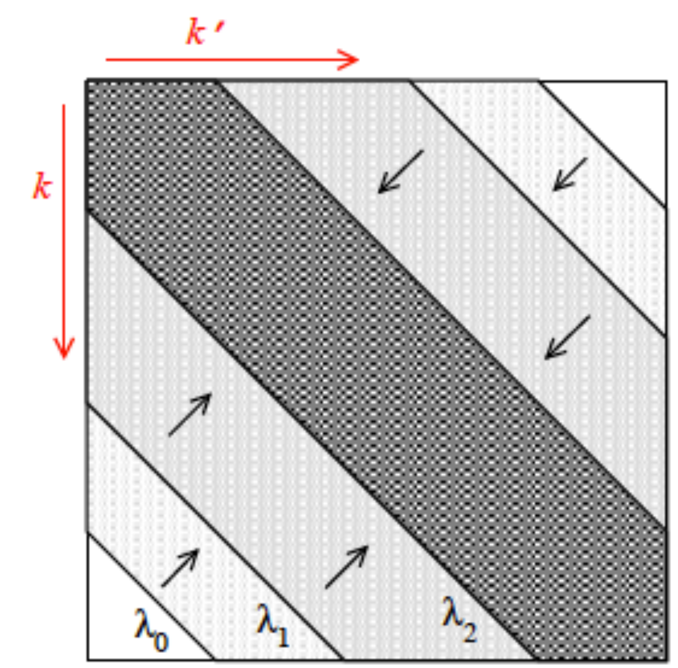
SRG - Similarity Renormalization Group

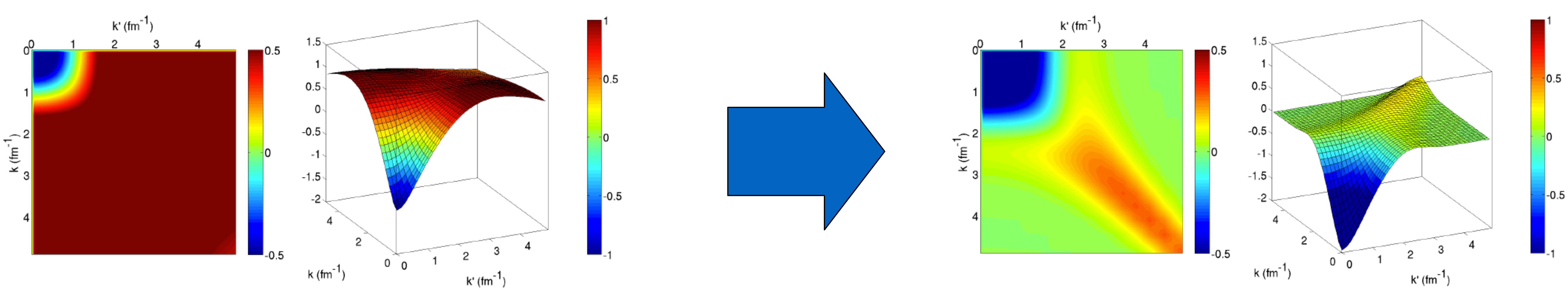
- Unitary transformation designed **to decouple** low- and high-energy states

- All **observables preserved**

- No relevant changes to low energy observables even when high momenta are removed

- Natural hierarchy of many-body forces maintained



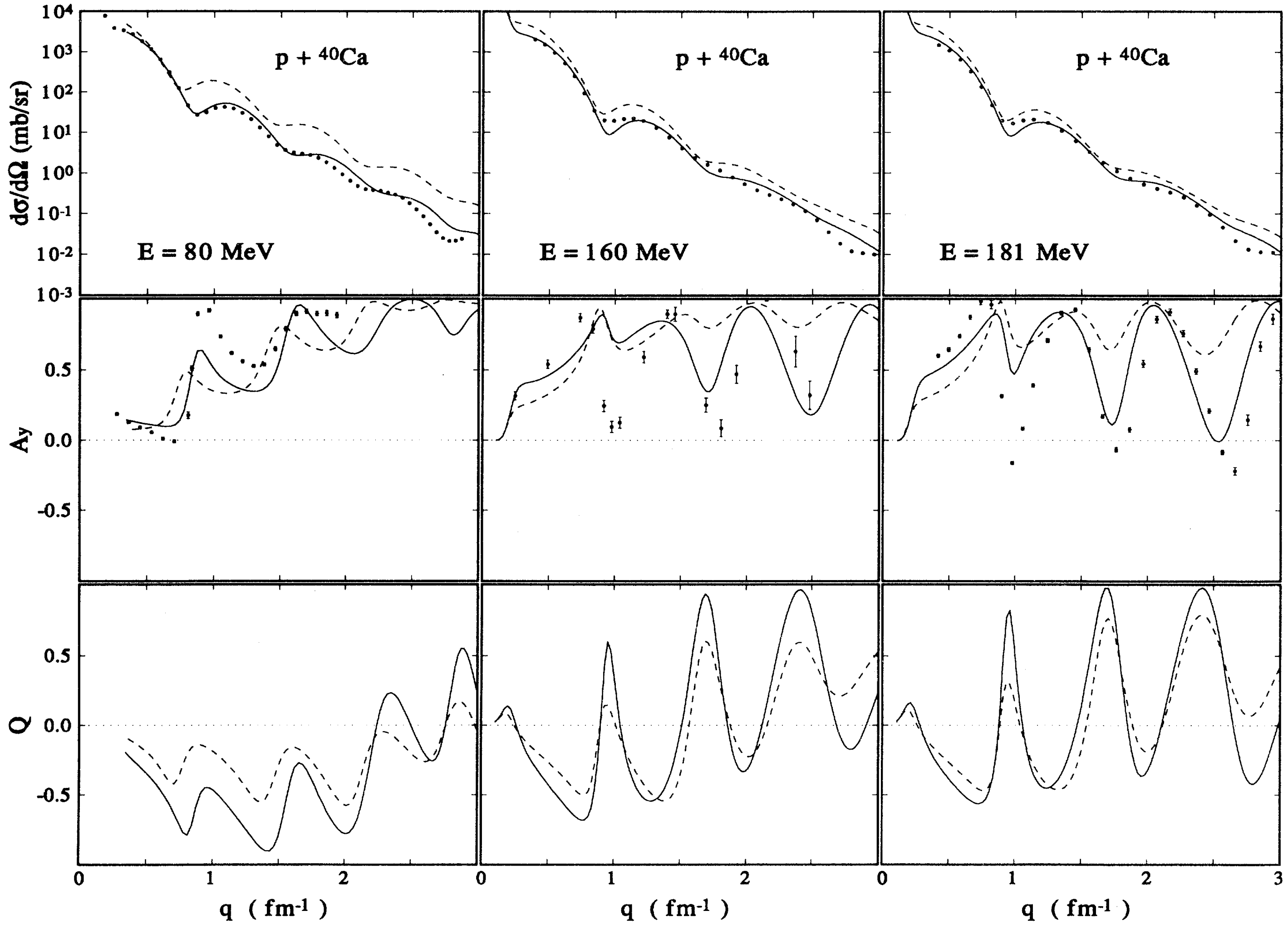


Medium effects - G matrix

$$U_{\mathrm{NM}}(k;\ k_F) = \sum_{\alpha \le \epsilon_F} \left\langle \tfrac{1}{2}(\vec{k} - \vec{k}_\alpha) \right| g_{[\,\vec{k}+\vec{k}_\alpha\,]}(\epsilon(k) + \epsilon(k_\alpha)) \times \left| \tfrac{1}{2}(\vec{k} - \vec{k}_\alpha) \right\rangle ,$$

$$\left\langle \vec{\kappa}' \left| g_{[\,\vec{P}\,;\,\vec{R}\,]}(\omega) \right| \vec{\kappa} \right\rangle = \langle \vec{\kappa}' | V | \vec{\kappa} \rangle + \int d\vec{\kappa}''\ \langle \vec{\kappa}' | V | \vec{\kappa}'' \rangle \times \lambda^{\mathrm{NM}}_{\vec{P}}(\vec{\kappa}''; \omega; k_F(R)) \times \left\langle \vec{\kappa}'' \left| g_{[\,\vec{P}\,;\,\vec{R}\,]}(\omega) \right| \vec{\kappa} \right\rangle ,$$

$$\lambda^{\mathrm{NM}}_{\vec{P}}(\vec{q}\,;\ \omega\ ;\ k_F) = \frac{\mathcal{Q}(P_+\,; P_-\,; k_F\,)}{\omega + i\eta - \epsilon(P_+; k_F) - \epsilon(P_-; k_F)} ,$$



…could be easily extended with the inclusion of three-body force (with equivalent two-body density dependent)

Arellano, Brieva and Love, Phys. Rev. C **52** (1995) 301

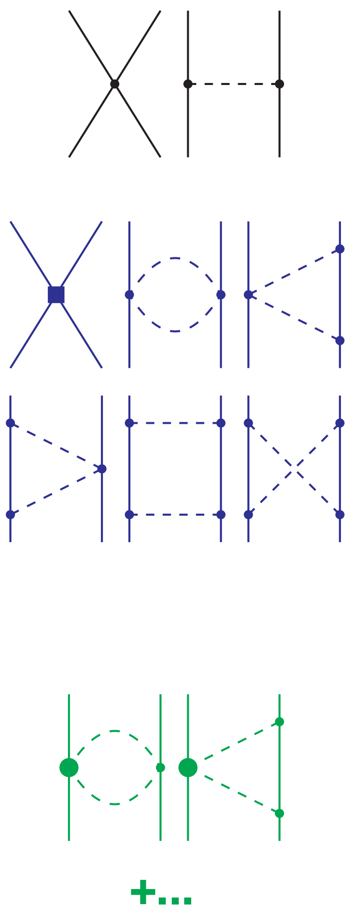
Include three-body forces

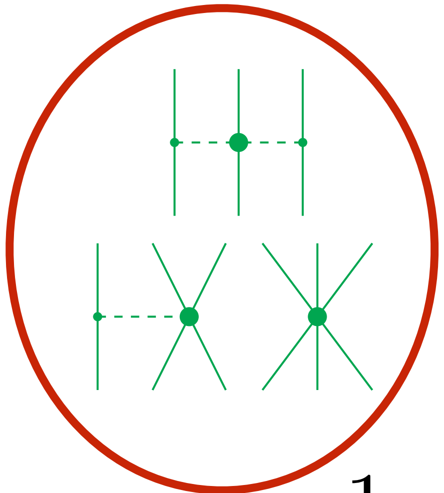
2N Force **3N Force**

LO $(Q/\Lambda_\chi)^0$

NLO $(Q/\Lambda_\chi)^2$

NNLO $(Q/\Lambda_\chi)^3$





$$U_i \equiv v_{0i} + v_{0i} G_0(E) Q U \,,$$

$$U_{ij} \equiv v_{0ij} + v_{0ij} G_0(E) Q U \,,$$

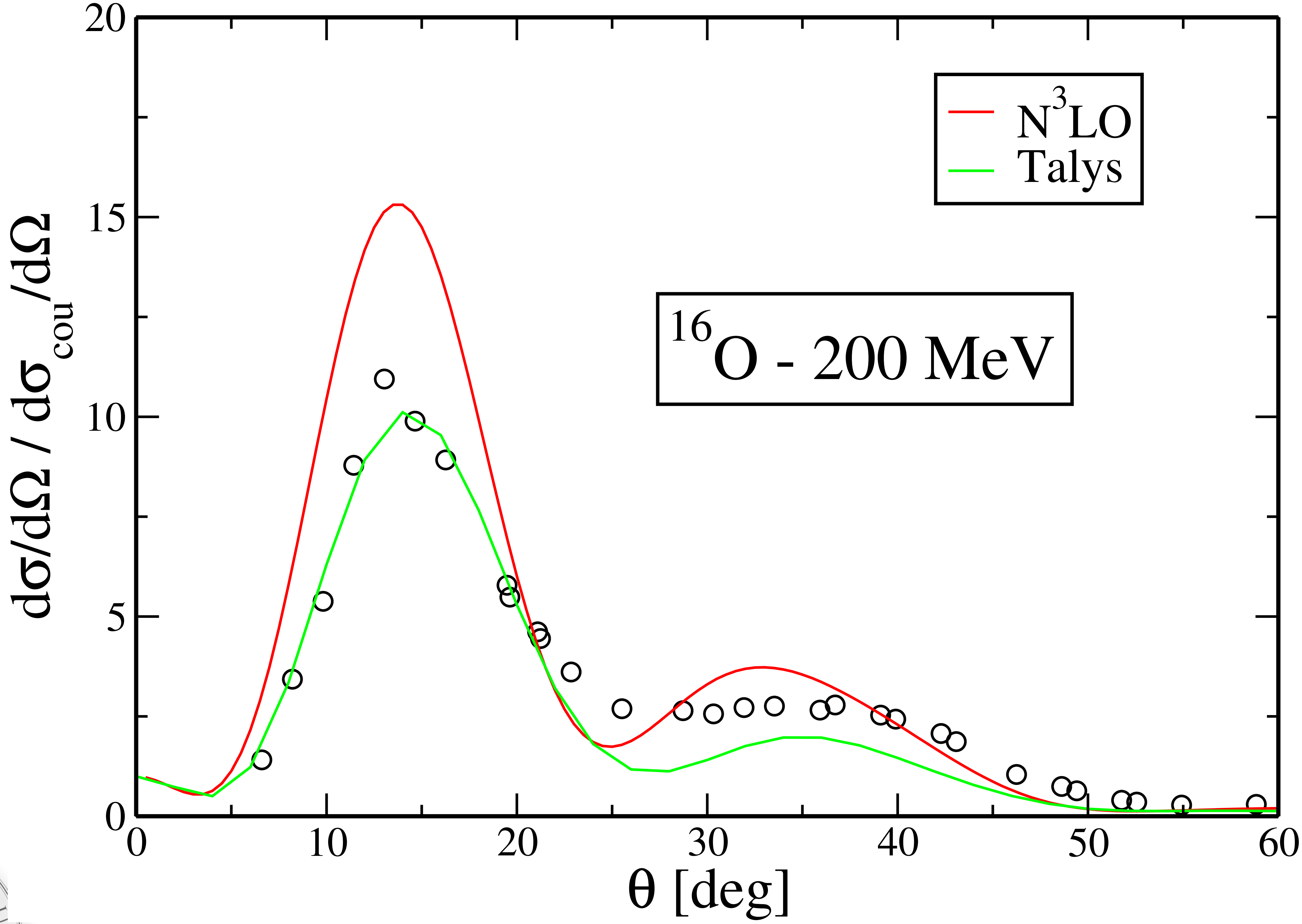
$$\boxed{U = \sum_{i=1}^{A} U_i + \frac{1}{2} \sum_{i=1}^{A} \sum_{\substack{j=1 \\ j \neq i}}^{A} U_{ij}}$$

$$\theta_{ij} = v_{0ij} + v_{0ij} G_0(E) Q \theta_{ij}$$

$$U_{ij} = \theta_{ij} + \theta_{ij} G_0(E) Q \frac{1}{2} \sum_{\substack{k=1 \\ k \neq i}}^{A} \sum_{\substack{l=1 \\ l \neq k \\ l \neq j}}^{A} U_{kl} + \theta_{ij} G_0(E) Q \sum_{k=1}^{A} U_k$$

$$U = \sum_{i=1}^{A} \tau_i + \frac{1}{2} \sum_{i=1}^{A} \sum_{\substack{j=1 \\ j \neq i}}^{A} \theta_{ij}$$

Isotope chains - micro *vs.* pheno



Thanks

The general goal when solving the scattering problem of a nucleon from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the many-body transition amplitude T

$$T = V + VG_0(E)T$$

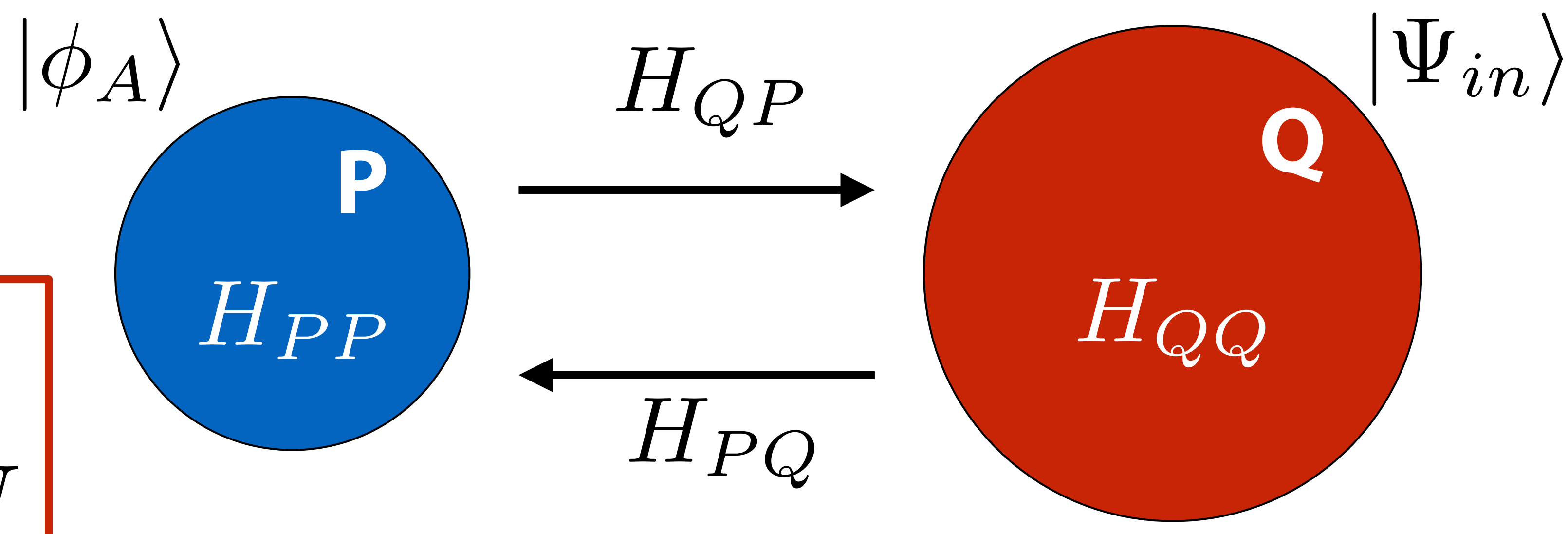
$$|\Psi\rangle = |\phi_A\rangle + |\Psi_{in}\rangle$$

$$P + Q = 1$$

$$[G_0, P] = 0$$

$$H(P+Q)|\Psi\rangle = E(P+Q)|\Psi\rangle$$

$$P = \frac{|\Phi_A\rangle\langle\Phi_A|}{\langle\Phi_A|\Phi_A\rangle}$$



$$\boxed{\begin{aligned} T &= U + UG_0(E)PT \\ U &= V + VG_0(E)QU \end{aligned}}$$

$$[E - PHP]|\phi_A\rangle = PHQ|\Psi_{in}\rangle$$