A self-consistent equation of motion multiphonon method for even and odd mass nuclei

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Semiclassical	Microscopic
$\{\alpha_\lambda, \pi_\lambda\} \longrightarrow \{O_\lambda, O_\lambda^\dagger\}$ EofM $[H, O_\lambda^\dagger] = \hbar\omega_\lambda O_\lambda^\dagger$	**TDA mapping** $O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h$ EoM $[H, O_\lambda^\dagger]\, \vert\, > = \hbar\omega_\lambda\, O_\lambda^\dagger\, \vert\, >$
Collective modes	**RPA mapping** $O_\lambda^\dagger = \Sigma_{ph}\, [X_{ph}(\lambda)\, a^\dagger_p\, a_h - Y_{ph}(\lambda)\, a^\dagger_h\, a_p]$ EoM $[H, O_\lambda^\dagger]\, \vert 0> = \hbar\omega_\lambda\, O_\lambda^\dagger\, \vert 0>$ $\vert 0> \equiv$ correlated g.s

Beyond mean field: *Adopted Methods*

Non relativistic

- qp-Phonon

P. F. Bortignon et al. (Milano group)

- 2nd RPA

R. Roth et al.

D. Gambacurta et al.

Phenomenological

- QPM *(Soloviev School (Dubna))*

Relativistic: RTBA

E. Litvinova, P. Ring, D. Vretenar…..

Our proposal: EMPM

*D. Bianco, F. Knapp, N. Lo Iudice, F. Andreozzi, A. Porrino, Phys. Rev. C **85**, 014313 (2012).*

Equation of motion phonon model (EMPM)

Eigenvalue problem

$$H \,|\, \Psi_\nu > = E_\nu \,|\, \Psi_\nu >$$

$| \Psi_\nu > \ni \mathcal{H} = \Sigma_n \oplus \mathcal{H}_n$ $\qquad \mathcal{H}_n \in | n; \beta > \equiv$ n-phonon **basis states**

An obvious, but unmanageable, multiphonon basis

$$|\lambda_1, \ldots, \lambda_i, \ldots \lambda_n> \;=\; O^\dagger_{\lambda_1} \ldots O^\dagger_{\lambda_i} \ldots O^\dagger_{\lambda_n} |0>$$

$$O^\dagger_\lambda = \Sigma_{ph}\, c^\lambda_{ph}\; a^\dagger_p\, a_h$$

A viable route

$$|\alpha_n> \;=\; \Sigma_{\lambda\alpha}\, C^\beta_{\lambda\alpha}\, O^\dagger_\lambda \,|\, \alpha_{n-1} >$$

Construction of $|n; \beta>$: EoM

Assuming $|\alpha_{n-1}>$ **known,** we solve the **Eq. of Motion**

$$< \alpha_n \,|[H, O^\dagger_\lambda]|\alpha_{n-1}> = (E_\beta^{(n)} - E_\alpha^{(n-1)}) < \alpha_n |\, O^\dagger_\lambda \,|\alpha_{n-1}>$$

$$\mathcal{A}\, \mathcal{X} = \mathcal{E}\, \mathcal{X}$$

$$\mathcal{X}$$

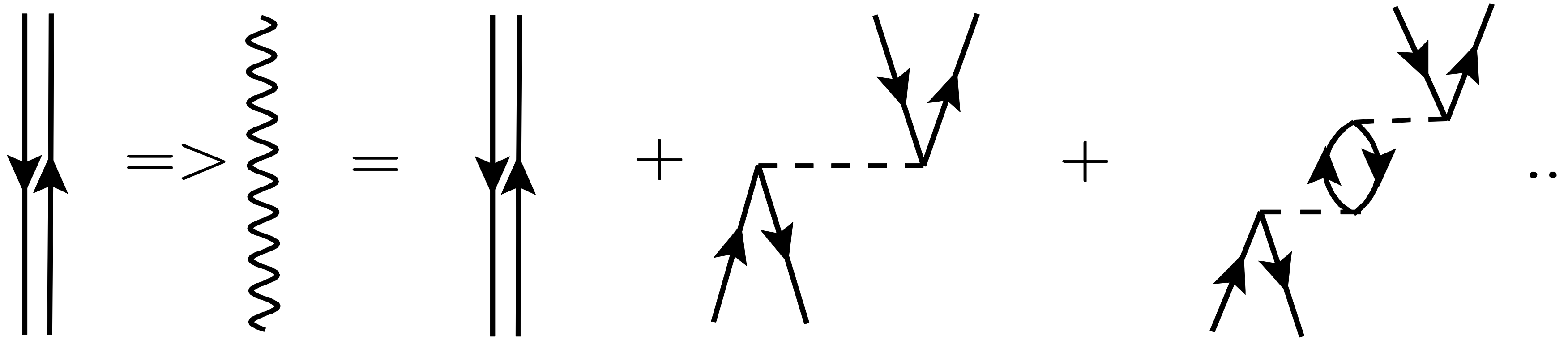
Where

$$\mathcal{A}_{(\mu\alpha)(\nu\gamma)} = (E_\mu + E_\alpha)\delta_{\alpha\gamma}\delta_{\mu\nu} + \mathcal{V}_{(\mu\alpha)(\nu\gamma)}$$

TDA matrix

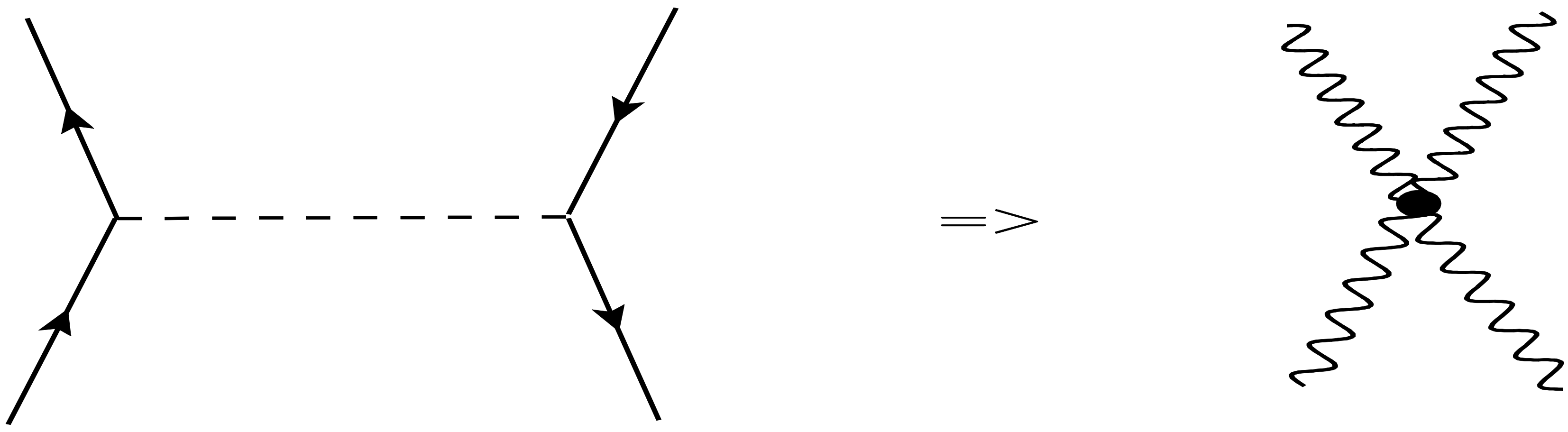
$$A_{(ph)(p'h')} = (\varepsilon_p - \varepsilon_h)\ \delta_{pp'}\, \delta_{hh'} + V_{ph'hp'}$$

From p-h to TDA to EMPM



$\varepsilon_p - \varepsilon_h \Rightarrow E_\mu$

$|ph> \Rightarrow |\mu> = \Sigma_{ph} c^{\mu}_{ph} a^{\dagger}_p a_h |>$



$V_{ph'hp'} \Rightarrow \mathcal{V}_{(\mu\nu)(\mu'\nu')}$

Construction of |n; β> : EoM

Problem

$$\mathcal{A}\,\mathcal{X} = \mathcal{E}\,\mathcal{X}$$

is **not** a true Eigenvalue Eq.!

$\{O^{\dagger}_{\lambda}\ |n\text{-}1, \alpha\rangle\}$ form a **non-orthogonal redundant** basis

⇓

$\mathcal{X} = \langle n, \beta|\ O^{\dagger}_{\lambda}\ |n\text{-}1, \alpha\rangle$ is **not** a **true** expansion coefficient

Recipe for solving the problem

1° step

$|\alpha_n> = \Sigma_{\lambda\alpha} C^{\beta}_{\lambda\alpha} \; O^{\dagger}_{\lambda} \; |\alpha_{n-1}> \}$

$| \lambda\alpha > = O^{\dagger}_{\lambda} \; |\alpha_n>$

$(\mathcal{X} = \; < \alpha_n \,|\, \lambda \; \alpha_{n-1} >)$

$\mathcal{D} \equiv \{< \lambda'\alpha_n' \,|\, \lambda\alpha_n>\}$

$$\mathcal{A} \, \mathcal{X} \; = \mathcal{E} \, \mathcal{X}$$

$\mathcal{X} = \; \mathcal{D} \, \mathcal{C}$

$$[\mathcal{H} - \; \mathcal{E}\mathcal{D}] \, \mathbf{C} = 0$$

where $\mathcal{H} = \; \mathcal{A}\mathcal{D}$ *But* $\mathcal{D}$ is **singular !**

2° conclusive step: Choleski

$$[\mathcal{H} - \mathcal{E}\mathcal{D}]\, C = 0$$

$$\mathcal{H} = \mathcal{A}\mathcal{D}$$

Cholesky

$$\mathcal{D} \rightarrow D$$

$$\mathcal{D} \equiv \{ \langle \lambda'\alpha' \mid \lambda\alpha \rangle \}$$

$$[H - E]\, C = 0$$

$$H = D^{-1}\mathcal{A}\mathcal{D}$$

$$|n, \beta\rangle = \Sigma_{\lambda\alpha}\, C^{\beta}_{\lambda\alpha}\, O^{\dagger}_{\lambda}\, |\alpha\rangle$$

$$\langle n',\alpha | n; \beta\rangle = \delta_{nn'}\ \delta_{\alpha\beta}$$

Iterative Generation of n-phonon states

$$\mathbf{A\,c} = \hbar\omega\ \mathbf{c} \Rightarrow |\lambda\rangle = \Sigma_{ph}\, \mathbf{c}^{\lambda}_{ph}\, a^{\dagger}_{p}\, a_{h}\, |\rangle$$

$$\Downarrow$$

$$\mathbf{H\,C} = \mathbf{E\,C} \Rightarrow |\alpha_n\rangle = \Sigma_{\lambda\alpha(n)}\, C^{\alpha(n)}_{\lambda\alpha(n)}\, O^{\dagger}_{\lambda}\, |\alpha_{n-1}\rangle$$

- **No approximations except for truncation!**
- **Pauli** principle fully accounted for:
- **No redundant** states!
- $|\alpha_n\rangle$ form an **orthonormal** basis

$n = 2, 3, \ldots\ldots$

Eigenvalue problem in Multiphonon basis $\{|\alpha_n>\} \equiv \{|\alpha_0>, |\alpha_1>, .. \}$

$$\Sigma_{n' \beta(n')} [(E_{\alpha(n)} - \varepsilon_\nu) \delta_{nn'} \delta_{\alpha(n) \beta(n')} + V_{\alpha(n) \beta(n')}] C^\nu_{\beta(n')} = 0$$

$$|\Psi_\nu> = \Sigma_{\alpha(n)} C^\nu_{\alpha(n)} |\alpha_n>$$

where

$$|\alpha_n> = \Sigma_{\lambda\alpha(n-1)} C^\beta_{\lambda\alpha(n-1)} O^\dagger_\lambda | \alpha_{n-1}>$$

EMPM in q-p scheme

One proceeds as in p-h scheme

$$< \alpha_n ||[H, O^\dagger_\lambda]|\alpha_{n-1}> = (E_\beta^{(n)} - E_\alpha^{(n-1)}) < \alpha_n | O^\dagger_\lambda |\alpha_{n-1}>$$

namely with the replacement

$$O_\lambda^\dagger = \Sigma_{ph} c_{ph}(\lambda) a^\dagger_p a_h \quad \Longrightarrow \quad O_\lambda^\dagger = \Sigma_{r \leq s} c_{rs}(\lambda) \alpha^\dagger_r \alpha^\dagger_s$$

G. De Gregorio,F knapp, N. Lo Iudice, P. Vesely Phys. Rev. C 044314 (2016)

Implementation

1° STEP : Intrinsic Hamiltonian

$$H = T_{int} + V_{NN}$$

where

$$T_{int} = \frac{1}{2m}\sum_i p_i^2 - \frac{P^2}{2M_{cm}}$$

$$V_{NN} = V_{\chi} = \mathrm{NNLO}_{opt}$$

2° STEP : HF(B) Self-consistent basis

3° STEP : Construction of TDA phonons

(free of spurious admixtures induced by CM and particle number violation)

$$O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h \qquad O_\lambda^\dagger = \Sigma_{r\leq s}\, c_{rs}(\lambda)\, \alpha^\dagger_r\, \alpha^\dagger_s$$

A. Ekstrom *et al.*, PRL **110**, 192502 (2013)

Implementation

4° STEP : Generation of Multiphonon basis

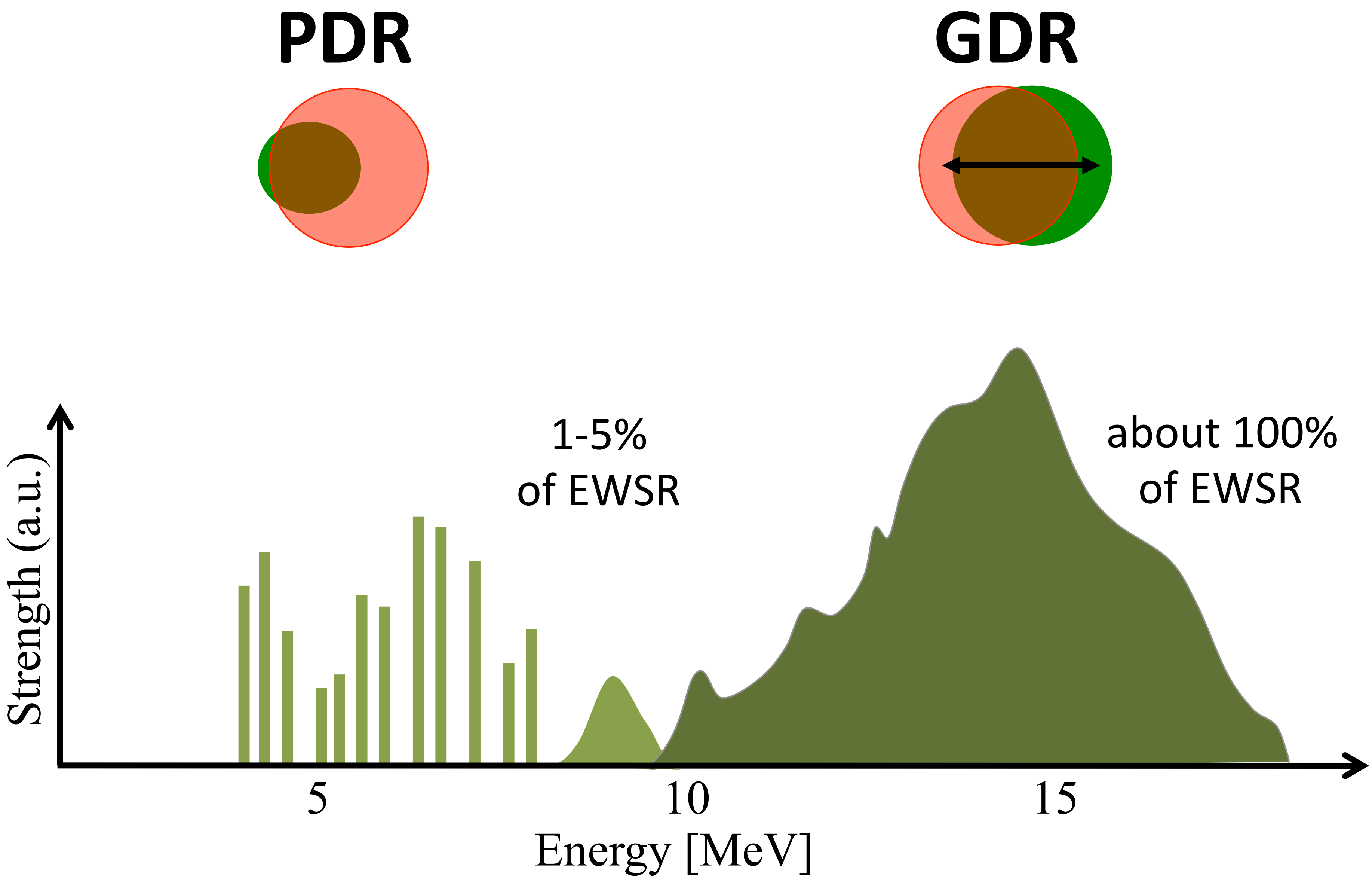
$$\{|\alpha_n>\} \equiv \{|\alpha_0>, \quad |\alpha_1>, \quad |\alpha_2>, \ldots\ldots \}$$

5° STEP : Solution of the eigenvalue problem in Multiphonon basis

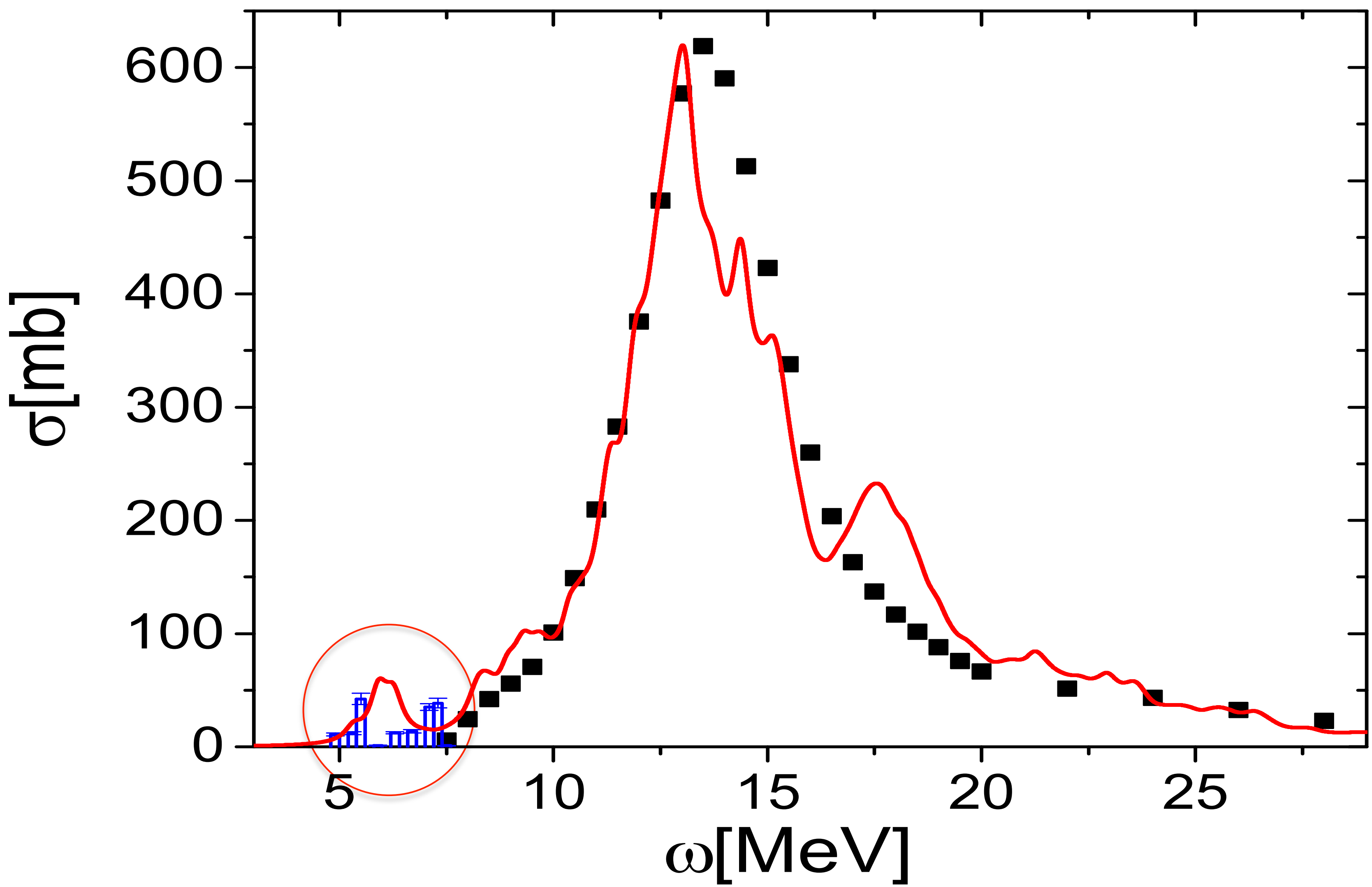
$$| \Psi_\nu \rangle = \sum_{n\alpha} C_\alpha^{(\nu)} \mid n;\alpha \rangle$$

6° STEP : Transition Probabilities and cross section

Applications: Selfconsistent study of Dipole response

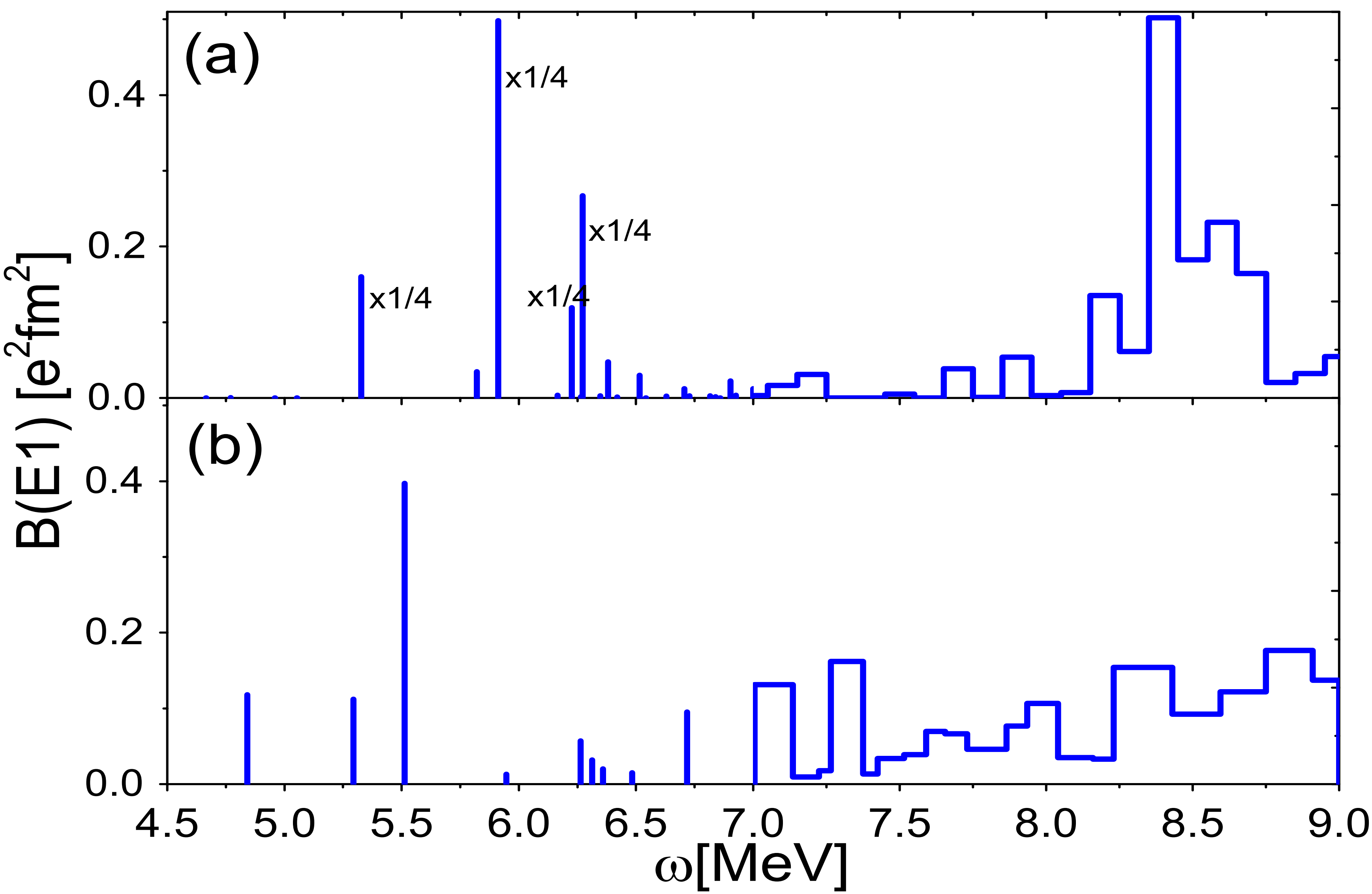


EMPM in p-h scheme: Application to neutron rich ^{208}Pb



F. Knapp,N. Lo Iudice,P. Vesely,F. Andreozzi, G. De Gregorio,A. Porrino PRC **92**, 054315(2015)

Low-lying Dipole strength in ^{208}Pb



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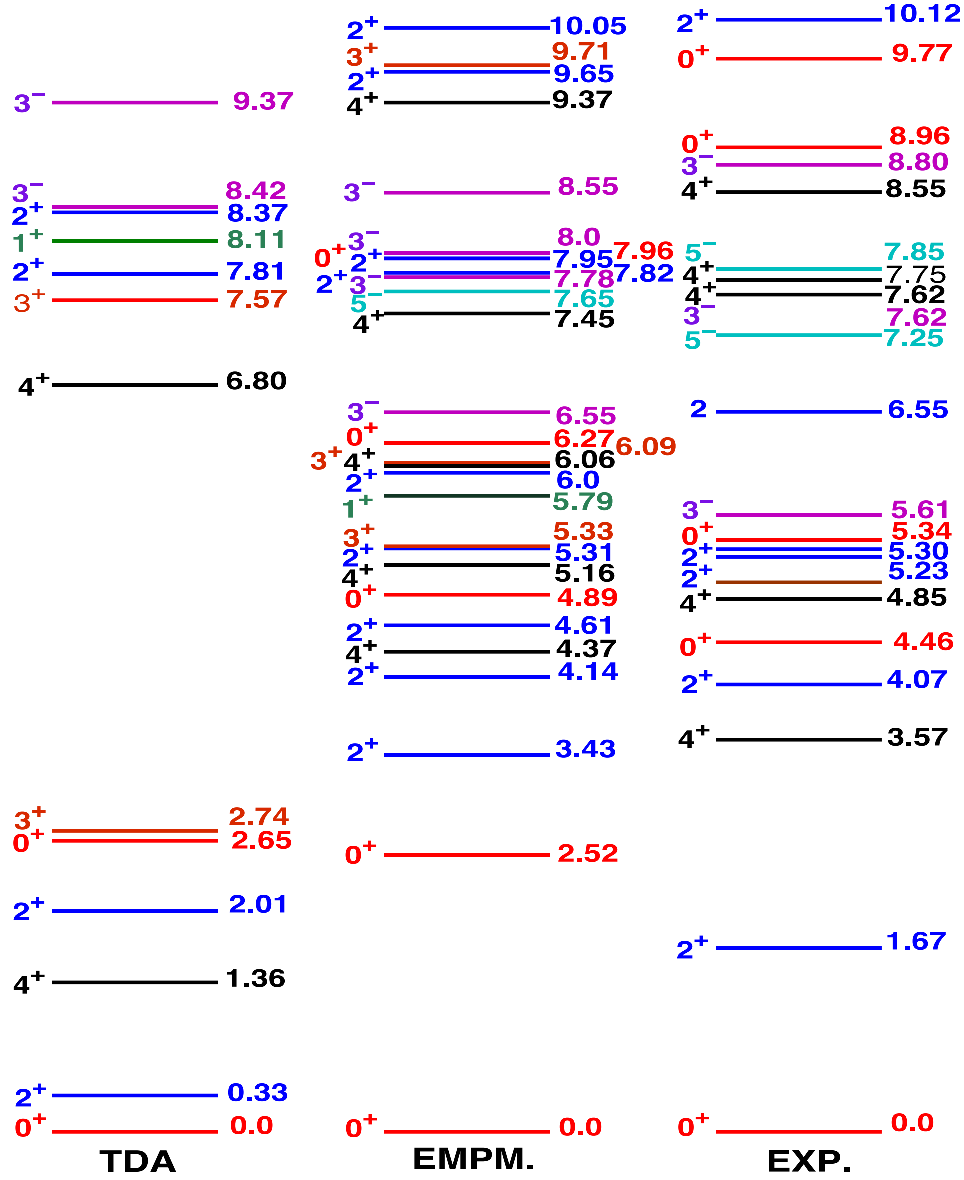
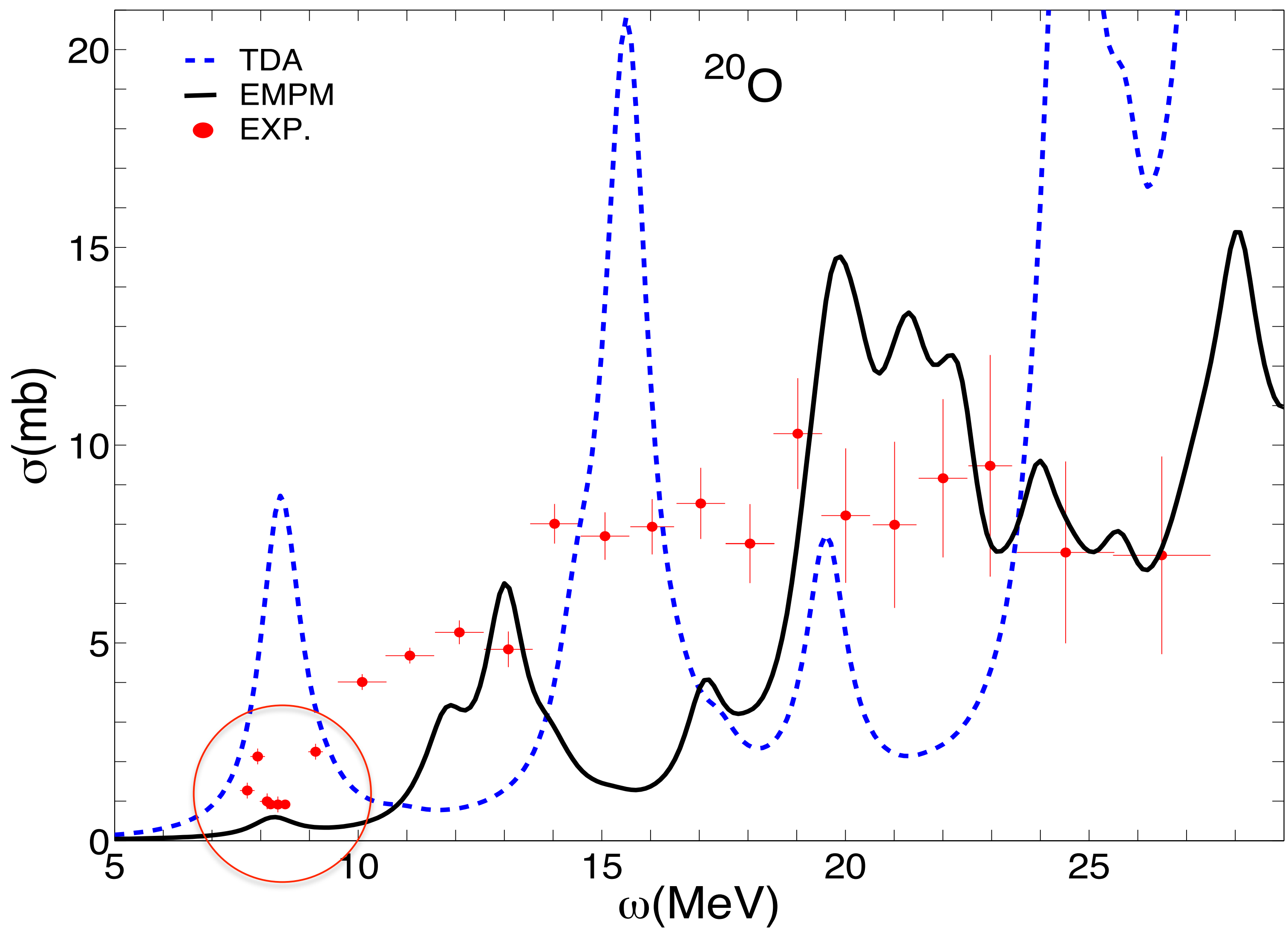
EMPM in q-p scheme: Application to neutron rich ^{20}O



G. De Gregorio, F knapp, N. Lo Iudice, P. Vesely Phys. Rev. C 044314 (2016)

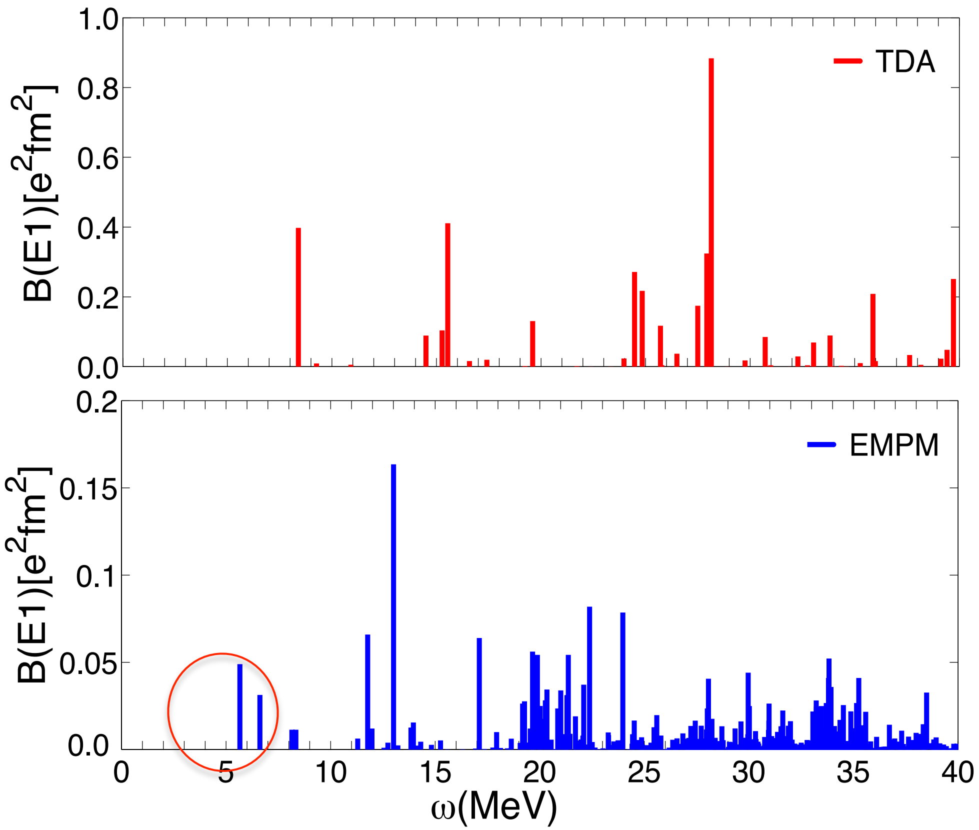
Cross section in ^{20}O



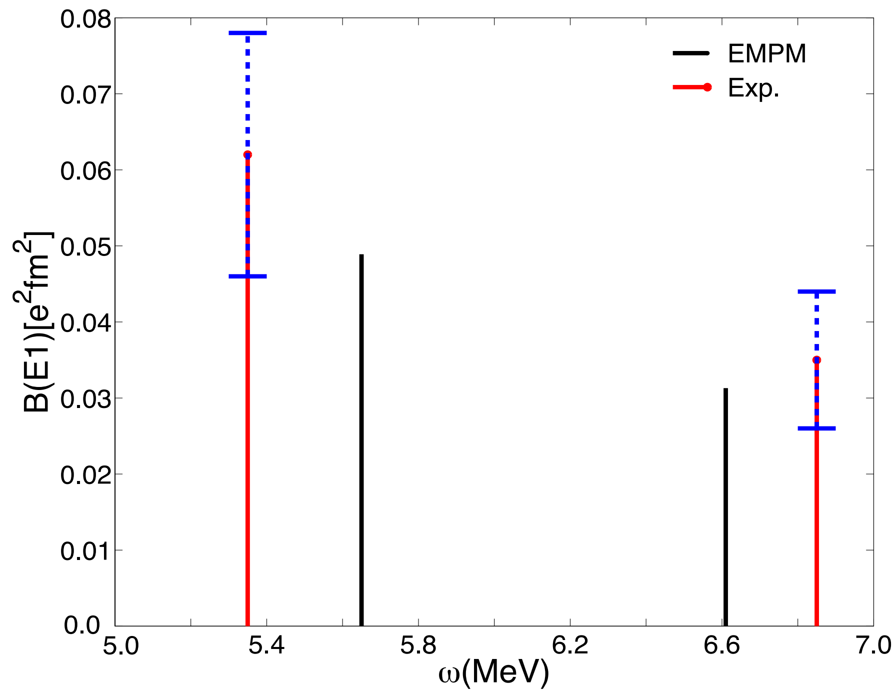
G. De Gregorio,F knapp, N. Lo Iudice, P. Vesely
Phys. Rev. C 044314 (2016)

E. Tryggestad, et al., Phys Rev. C 67, 064309 (2003).

Dipole strength in ^{20}O



E. Tryggestad, et al., Phys Rev. C 67, 064309 (2003).



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Even Nuclei: Summary

EMPM

- Extends Mean Field without **approximations,** except for truncation, for any potential

- Includes **multiphonon states** **explicitly**

Results

2-phonon configurations are **needed** for

- describing the fine structure of the GDR and PDR
  - obtaining a complete low-energy spectrum

EMPM: Odd nuclei

$$| \lambda\, \alpha > = O^{\dagger}_{\lambda} \, | \alpha > \;\Rightarrow\; | p\, \alpha > = a^{\dagger}_{p} \, | \alpha >$$

$$< \alpha_n \, || [H, O^{\dagger}_{\lambda}] || \alpha_{n-1} > \;\Rightarrow\; < \nu_n \, || [H, a^{\dagger}_{p}] || \alpha_n >$$

$$\mathcal{X} = \mathcal{D}\, \mathcal{C}$$

$$\mathcal{A}\, \mathcal{X} = \mathcal{E}\, \mathcal{X} \;\Rightarrow\; [\mathcal{H} - \mathcal{E}\mathcal{D}]\, \mathbf{C} = 0$$

$$\mathcal{D} \equiv \{ < p\, \alpha \,|\, p'\, \alpha' > \}$$

Cholesky

$$\mathcal{D} \rightarrow \mathbf{D}$$

EMPM: Odd nuclei

$$[H - E]\, C = 0$$

$$H = D^{-1} \mathcal{A} \mathcal{D}$$

$$|\nu_n> = \Sigma_{p\alpha}\ C^{\nu}_{p\alpha}\ a^{\dagger}_{p}\ |\alpha>$$

$$<\nu'|\nu> = \delta_{\nu\nu'}$$

$$|\Psi_\mu> = \Sigma_i\ C^{(\mu)}_{\nu_i}\ |\nu_i>$$

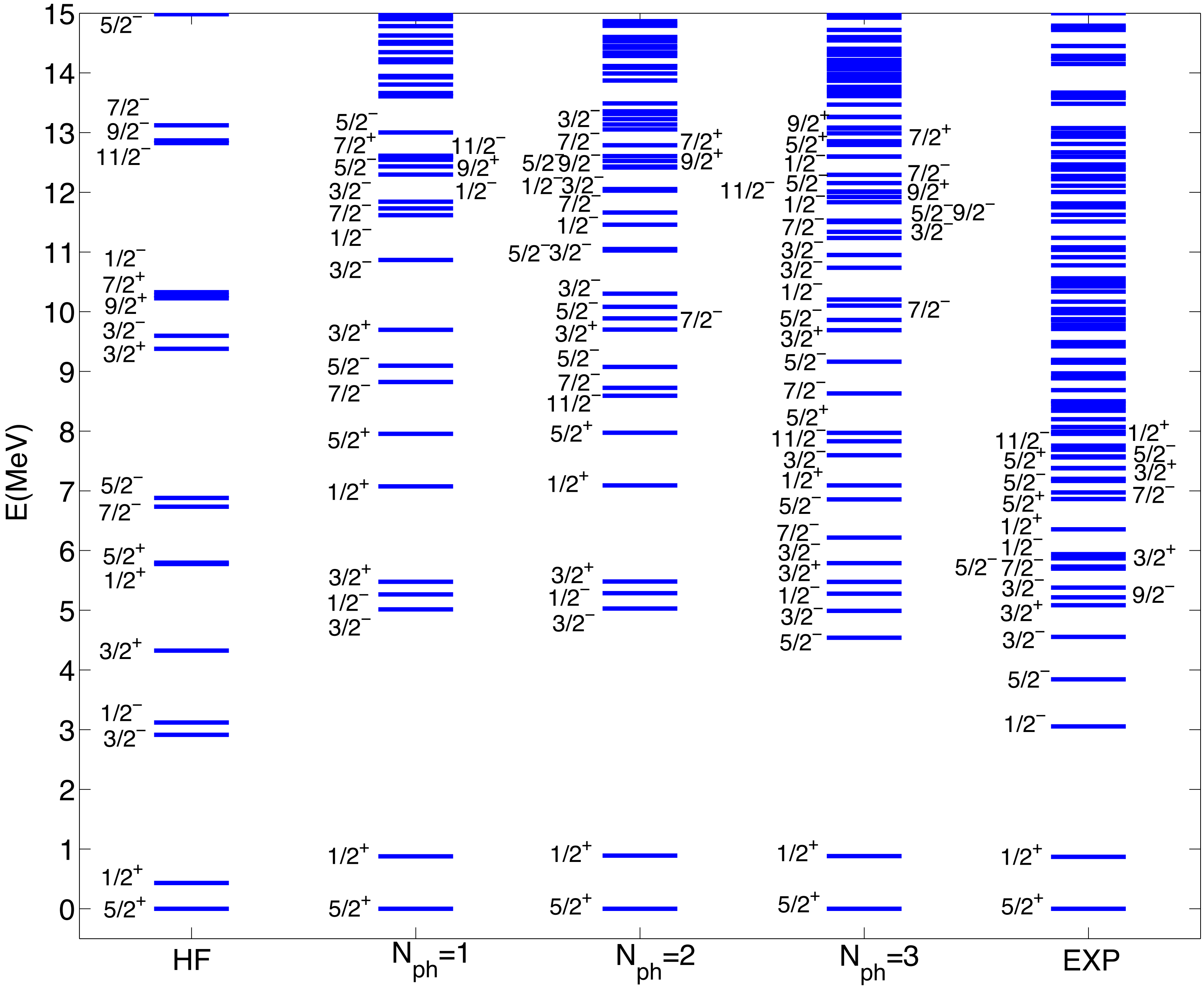
- **No approximations except for truncation !**
- **Pauli** principle fully accounted for:
- **No redundant** states!
- $|\nu_n>$ form an **orthonormal** basis

Implementation

Potential: **$NNLO_{opt}$**

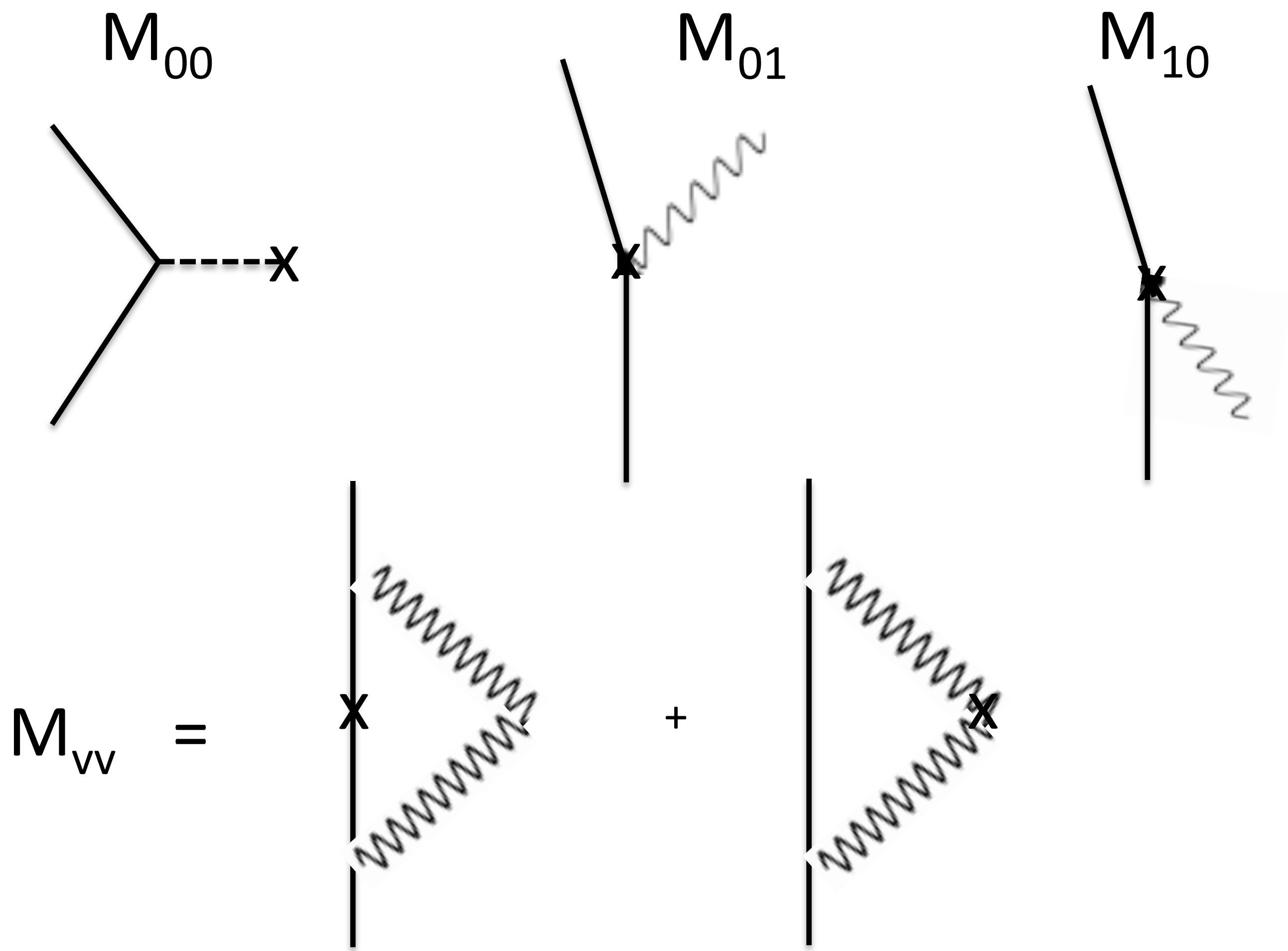
- Perform **HF**
- Construct **TDA** phonons (**free** of **CM** spurious admixtures)
- Generate the multiphonon basis $\{|\alpha_n>\} \equiv \{|\alpha_0>, \ |\alpha_1>, \ |\alpha_2>, \ |\alpha_3>\}$
- Generate the **orthornormal** particle-phonon basis $\{|\nu_n>\}$
- Full diagonalization

^{17}O spectrum

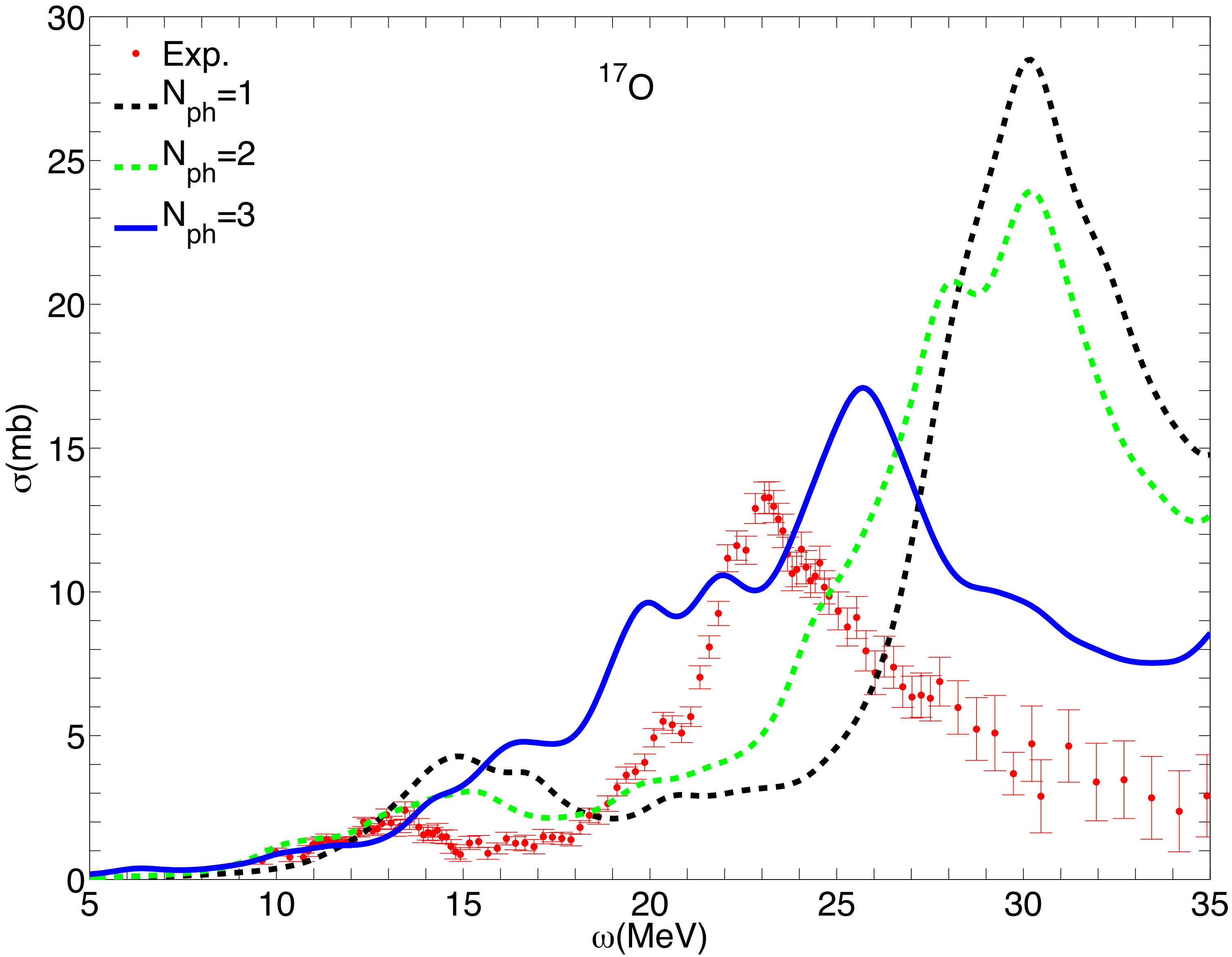


Transitions Amplitudes

$$\left\langle \Psi_{\Omega_f} \| M(\lambda) \| \Psi_{\Omega_i} \right\rangle = M_{00}(\lambda) + M_{01}(\lambda) + M_{10}(\lambda) + M_{11}(\lambda) + M_{12}(\lambda) + M_{21}(\lambda) + M_{22}(\lambda)$$

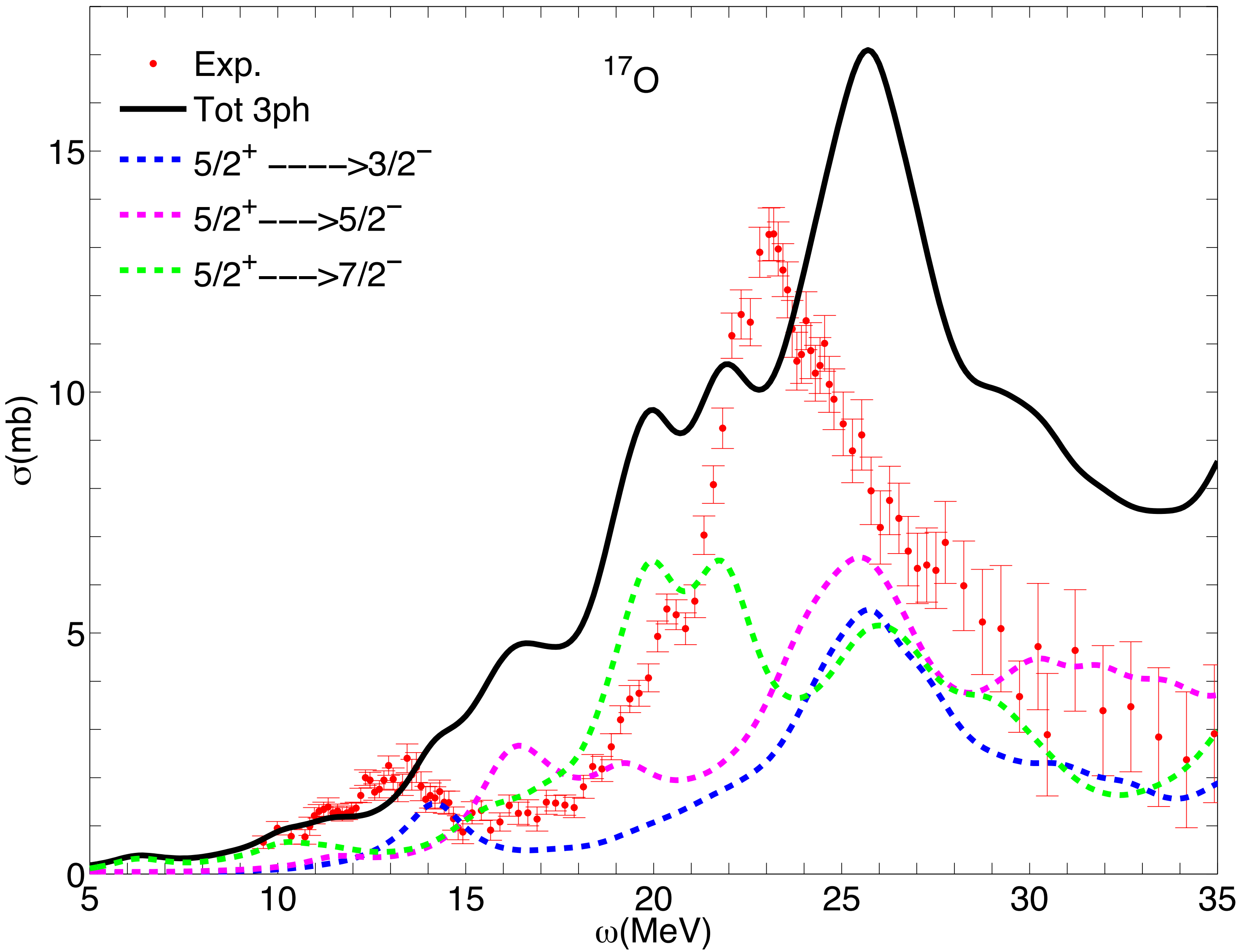


Cross section



J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

Cross section: Preliminary



J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

Mean Values

^{17}O	HF	N_{ph}=1	N_{ph}=2	N_{ph}=3	Exp
Q(barn)	0	-0.00825	-0.00830	-0.00834	-0.025[1]
μ (nm)	-1.9130	-1.8376	-1.834	-1.833	-1.893[1]
log ft	3.2944	3.3891	3.3905	3.3913	3.358[2]

[1] N.P. Stone, Atomic Data and Nuclear Data Tables **90** (2005) 75–176

[2] D.R. Tilley Nuclear Physics A **636** (1998) 249-364

Odd nuclei : Concluding remarks

•One-and two-phonon states enhances greatly the density of levels consistently with experiments.

• Three-phonon states play an important role, their (strong) coupling to particle-phonon states **$|\nu_1>$**

a) shift downwards the energies of **$|\nu_1>$**
b) improve the cross section
c) increase the density of levels in low-energy part of the spectrum

Thank you