Bridging the gap: effective interactions from realistic nuclear hamiltonians

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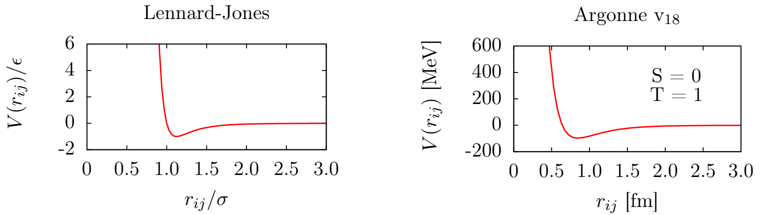
Outline

- The trouble with stongly correlated many-body systems
- Effective interactions derived from microscopic dynamics
- The CBF effective interaction
- The Fermi hard-sphere systemn as a testing ground
- Equilibrium and non-equilibrium properties of nuclear matter
- Summary & Outlook

Strongly Correlated Many-Body Systems

- ⋆ The presence of a strongly repulsive core is a prominent feature of the potentials describing the dynamics of a variety of quantum many-body systems, such as liquid helium and nuclear matter



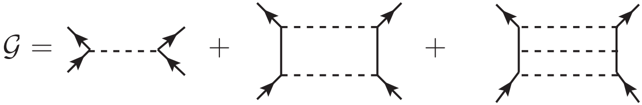
- ⋆ Perturbation theory in the basis of eigenstates of the non interacting system—Fermi gas in translationally invariant systems—is not doable

The matrix elements $\langle m_{\rm FG}|V|n_{\rm FG}\rangle$ are large, or even divergent

THE *ab initio* APPROACH

⋆ The matrix elements of the bare potential are modified by either renormalising the interaction or performing a transformation of the basis states

- G-matrix



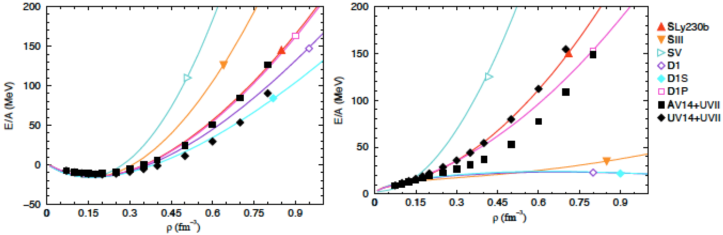
- Correlated Basis Functions (CBF)

$$|n_{\rm FG}\rangle \to |n\rangle = F|n_{\rm FG}\rangle \ \ , \ \ F = \mathcal{S}\prod_{j>i} f_{ij}$$

⋆ Both procedures allow to carry out accurate calculations of the zero-temperature Equation of State (EoS) based on realistic microscopic hamiltonians

The effective interaction approach

- In nuclear structure calculations, the *bare* potential is often replaced by an *efective* potential suitable for use in perturbation theory



- EoS of isospin-symmetric nuclear matter and pure neutron matter computed using different Skyrme- and Gogny-type effective interactions, compared to the results of the *ab initio* approach
- Effective interactions, while being capable to provide a reasonable description of the EoS, lack a direct connection with the underlying nucleon-nucleon interactions

The CBF effective interaction

- Using the cluster expansion techinque the expectation value of the Hamiltonian in the correlated ground state can be written in the form

$$\langle H \rangle = T_{\rm FG} + \sum_{n>2} \langle \Delta H \rangle_n$$

- Accurate variational estimates of the ground state energies can be obtained exploiting the FHNC summation scheme and its extensions. The shape of the correlation function is determined requiring

$$\frac{\delta \langle H \rangle}{\delta f_{ij}} = 0$$

- The CBF effective interaction is defined through

$$\langle H \rangle = \langle 0 | F^\dagger (T + V) F | 0 \rangle = \langle 0_{FG} | T + V_{\rm eff} | 0_{FG} \rangle$$

$$V_{\rm eff} = \sum_{j>i} {\rm v}_{\rm eff}(ij)$$

- ⋆ The expectation value $\langle H \rangle$, appearing in the left hand side is computed using FHNC (or any alternative technique providing a precise evaluation of the ground state energy)

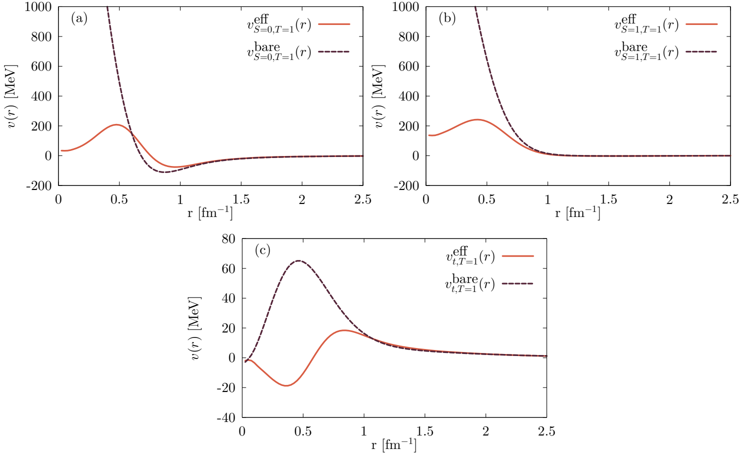
- ⋆ The right hand side is expanded at low order of the cluster expansion. At two body level, this procedure yields

$$\mathrm{v}_{\mathrm{eff}}(ij) = f_{ij}^{\dagger} \left[-\frac{1}{m}(\nabla^2 f_{ij}) - \frac{2}{m}(\nabla f_{ij}) \cdot \nabla + \mathrm{v}_{ij} f_{ij} \right]$$

- ⋆ The correlation function is adjusted in such a way as to reproduce the value of $\langle H \rangle$ appearing in the left-hand side

CBF v_{eff} AT SNM EQUILIBRIUM DENSITY

- Effective interaction obtained from the ANL v_6 + UIX nuclear Hamiltonian including two- and three-body cluster terms

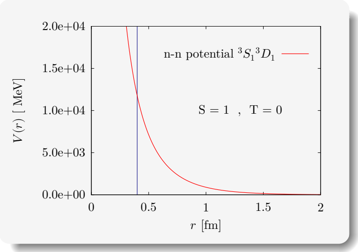


- Three-body forces consistently taken into account

The Fermi hard-sphere model: point-like spin one-half particles

$$v(r) = \begin{cases} \infty & r < a \\ 0 & r > a \end{cases}$$

- ⋆ Valuable model to study properties of nuclear matter.
- ⋆ Purely repulsive potential to prevent the possibility of Cooper pairs formation.
- ⋆ A simple many-body system to investigate the validity and robustness of the assumptions of CBF effective interaction approach.

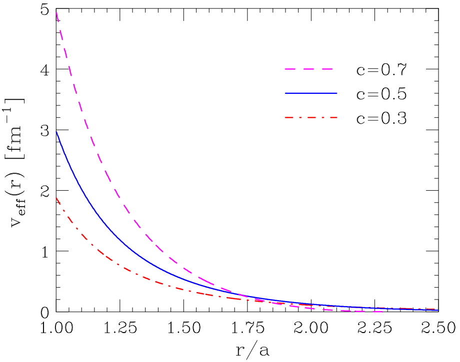


DETERMINATION OF v_{eff}

For the hard-sphere system (HS) $f(r \leq a) = 0\,,\ \lim_{r\to\infty} f(r) = 1$

$$v_{\text{eff}}(r) = \frac{1}{m}\left[\boldsymbol{\nabla} f(r)\right]^2 \quad , \quad r > a$$

We adjust the range of $f(r)$ in order to reproduce the ground state energy (FHNC/DMC) at two-body cluster level.

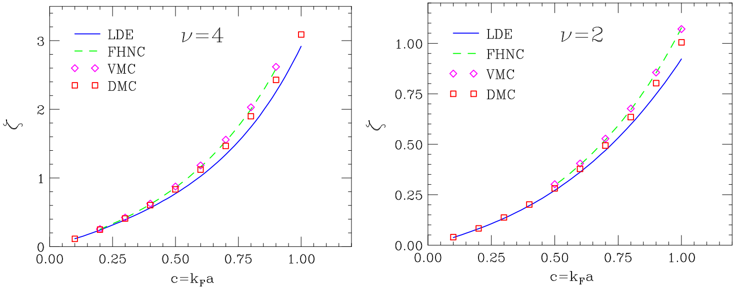


v_{eff}

- ⋆ defined from $\langle H \rangle$ in the correlated ground state
- ⋆ employed in calculations of matrix elements involving excited states.

THE GROUND-STATE ENERGY

$$E_0 = \frac{3k_F^2}{10m}\left(1+\zeta\right)$$



- The accuracy of the variational results depends on the quality of the trial wave function.
- Long-range statistical correlations effects in $f(r)$ much larger for $\nu = 2$ than for $\nu = 4$.
- DMC overcomes the limitations of the variational approach by using a projection technique on the trial wave function.

Two–point Green's function

Dyson's equation

$$G(k,E) = G_0(k,E) + G_0(k,E)\Sigma(k,E)G(k,E)$$

Non interacting Green's function

$$G_0(k,E) = \frac{\theta(k-k_F)}{E - e_0(k) + i\eta} + \frac{\theta(k_F - k)}{E - e_0(k) - i\eta}$$

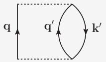
The irreducible (proper) self-energy $\Sigma(\mathbf{k}, E)$ (mass operator) takes into account the effect of interactions.
The spectrum is determined by the singularities of $G(k,E)$

$$G(k,E) = \frac{1}{E - e_0(k) - \Sigma(k,E)}$$

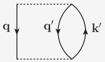
In perturbation theory

$$\Sigma(k,E) = \Sigma^{(1)}(k) + \Sigma^{(2)}(k,E) + \ldots$$

$\Sigma_{HF}(\mathbf{k})$



$\Sigma_{2p1h}(\mathbf{k}, E)$



$\Sigma_{2h1p}(\mathbf{k}, E)$

THE ELEMENTARY EXCITATION SPECTRUM

- The self energy is responsible for shifting the pole of the Green's function.
- The new poles determine energy $e(k)$ and the damping Γ_k of the quasiparticles state
- For small Γ_k, the propagation of quasiparticle states is described by

$$G(k,E) = \frac{Z_k}{E - e(k) + i\Gamma_k}$$

The energy of quasiparticle

$$e(k) = e_0(k) + \mathrm{Re}\Sigma[k, e(k)]$$

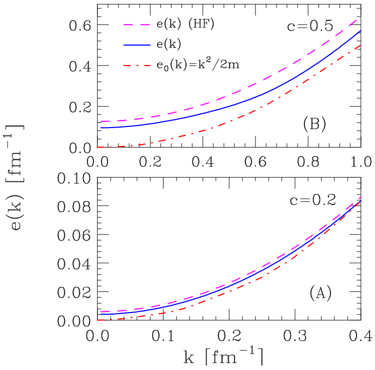
Quasiparticle lifetime

$$\tau_k^{-1} = \Gamma_k = Z_k \mathrm{Im}\Sigma[k, e(k)]$$

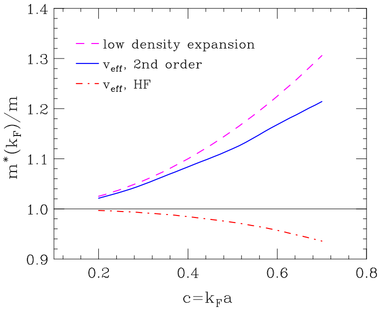
The residue of the Green's function

$$Z_k = \left[1 - \frac{\partial}{\partial E}\mathrm{Re}\Sigma[k,E]\right]^{-1}_{E=e(k)}$$

Quasiparticle spectrum



$$m^\star = \left[\frac{1}{k}\frac{de(k)}{dk}\right]^{-1}$$



$$\frac{de(k)}{dk} = \left[\frac{k}{m} + \frac{\partial}{\partial k}\mathrm{Re}\Sigma\left(k,E\right)\right]\left[1 - \frac{\partial}{\partial E}\mathrm{Re}\Sigma\left(k,E\right)\right]^{-1}_{E=e(k)}$$

Momentum distribution

- Momentum distribution describes the occupation probability of the quasiparticle state of momentum k (see Källén-Lehman representation of $G(k,E)$)

$$G(k,E) = \int_0^\infty dE' \left[\frac{P_p(k,E)}{E - E' - \mu + i\eta} + \frac{P_h(k,E)}{E + E' - \mu - i\eta} \right] \ , \ \mu = e(k_F)$$

- In term of the quasiparticle (hole) spectral functions

$$n(k) = \int_0^\infty dE P_h(k,E) = 1 - \int_0^\infty dE P_p(k,E)$$

- Is related to one body Green's function through an integration in complex variable ω on an closed contour in upper half-plane ($\mathrm{Im}\omega > 0$)

$$n(k) = \frac{1}{2\pi i} \int_C d\omega \, G(k,\omega)$$

Momentum distribution

Exploiting Dyson's equation, $n(k)$ can be determined through the knowledge of the self-energy $\Sigma(k, E)$, computed at the second order

The discontinuity at $k = k_F$ is given by

$$n(k_F - \eta) - n(k_F + \eta) = Z_{k_F} = Z$$

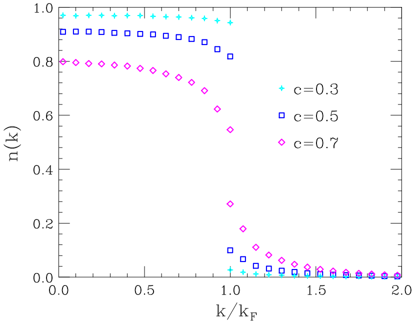
$$n(k) = n_<(k) + n_>(k)$$

with

$$n_<(k > k_F) = n_>(k < k_F) = 0$$

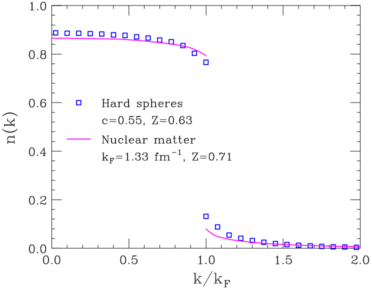
$$n_<(k < k_F) = 1 + \left[\frac{\partial}{\partial E} \mathrm{Re}\Sigma_p(k, E)\right]_{E=e_0(k)}$$

$$n_>(k > k_F) = -\left[\frac{\partial}{\partial E} \mathrm{Re}\Sigma_h(k, E)\right]_{E=e_0(k)}$$



Momentum distribution $\nu = 4$

In comparison with non orthogonal CBF perturbation theory



S. Fantoni and V. R. Pandharipande, Nucl. Phys. A **427**(1984)

Momentum distribution of HS

$$c \equiv k_F a = 0.55$$

corresponds to $n(k)$ of nuclear matter

$$\rho_{NM} = 0.16 \text{ fm}^{-3}$$
$$k_F = 1.33 \text{ fm}^{-1}$$

Nucleons in nuclear matter $\sim$ HS of radius $a = 0.55/1.33 \sim 0.4$ fm.

Virtual scattering processes between strongly correlated particles are mainly driven by the short-range repulsive core of the nucleon-nucleon interaction.

Boltzmann-Landau equation

Shear viscosity η and thermal conductivity κ measure momentum and energy fluxes in response to a gradient of velocity and temperature.

Boltzmann equation for a Fermi liquid:

$$\frac{\partial n_{\mathbf{k}}}{\partial t} + \frac{\partial n_{\mathbf{k}}}{\partial \mathbf{r}} \cdot \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} - \frac{\partial n_{\mathbf{k}}}{\partial \mathbf{k}} \cdot \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{r}} = I[n_{\mathbf{k}}]$$

- $n_{\mathbf{k}}$ is the distribution function
- $\epsilon_{\mathbf{k}}$ is the energy of a quasiparticle carrying momentum $\mathbf{k}$
- $I[n_{\mathbf{k}}]$ is the collision integral, defined in terms of the scattering probability W

Taking into account small deviations from local equilibrium, transport coefficients determined from the collision integral $I[n_{\mathbf{k}}]$.

Abrikosov–Khalatnikov solution

The lifetime

$$\tau = \frac{1}{T^2}\,\frac{8\pi^4}{m^{\star 3}}\,\frac{1}{\langle W\rangle}$$

The transport coefficients

$$\eta = \frac{16}{15}\frac{1}{T^2}\frac{k_F^5}{m^{\star 4}}\,\frac{1}{\langle W\rangle(1-\lambda_\eta)} \ , \ \ \kappa = \frac{16}{3}\frac{1}{T}\frac{\pi^2 k_F^3}{m^{\star 4}}\,\frac{1}{\langle W\rangle(3-\lambda_\kappa)}$$

τ, η, κ are expressed in terms of angular averages of W

$$\langle W\rangle \ , \ \ \lambda_\eta = \frac{\langle W[1-3\sin^4(\theta/2)\sin^2\phi]\rangle}{\langle W\rangle} \ , \ \ \lambda_\kappa = \frac{\langle W[1+2\cos\theta]\rangle}{\langle W\rangle}$$

The angular average is defined as

$$\langle f\rangle \equiv \int \frac{d\Omega}{2\pi}\,\frac{f(\theta,\phi)}{\cos\theta/2}$$

THE SCATTERING PROBABILITY

W is related to the scattering cross section

$$W(\theta, \phi) = \frac{16\pi^3}{m^2} \left(\frac{d\sigma}{d\Omega} \right)$$

- The AK formalism is derived in the frame in which the Fermi sphere is at rest (AK)
- $\frac{d\sigma}{d\Omega}$ expressed in the laboratory or in the center of mass reference frame
- the relative kinetic energy is the same E_{cm} $\forall$ frame

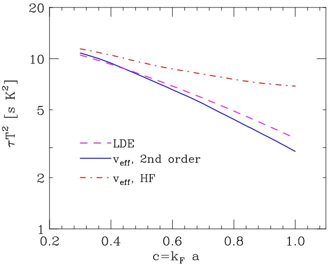
$$E_{\text{cm}} = E_{\text{rel}}^{AK} = \frac{k_F^2}{2m}(1 - \cos\theta)$$
$$\Theta_{\text{cm}} = \phi$$

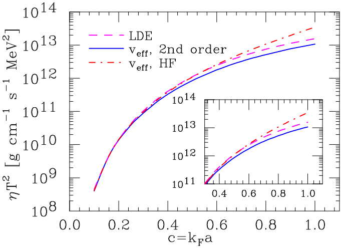
The in medium scattering probability has been computed within the Born approximation using v_{eff}

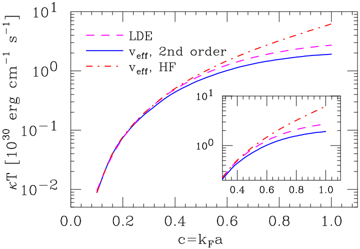
$$W(\theta, \phi) = \pi \left| [\mathbf{k}_1', \mathbf{k}_2' | v_{\text{eff}} | \mathbf{k}_1, \mathbf{k}_2] \right|^2$$

Lifetime and Transport coefficients

The second order contributions lead to a sharp increase of $m^\star$, which in turn implies a decrease of the shear viscosity coefficient η and the thermal conductivity κ.





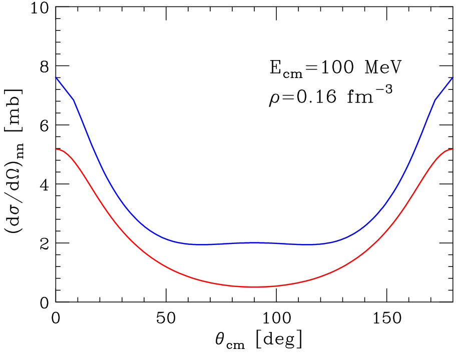


Back to nuclear matter

- In medium neutron-neutron cross section
- From Fermi's golden rule

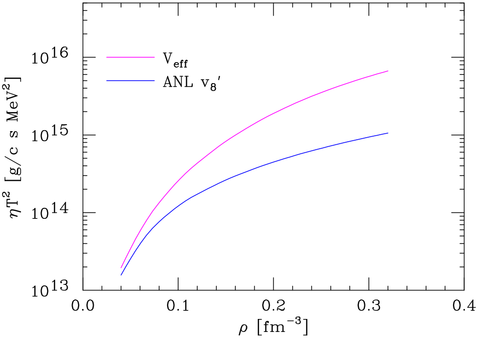
$$W(\mathbf{p},\mathbf{p}') = 2\pi \left|\hat{\mathrm{v}}_{\mathrm{eff}}(\mathbf{p}-\mathbf{p}')\right|^2 \rho(\mathbf{p}')$$

$$\frac{d\sigma}{d\Omega_{\mathbf{p}'}} = \frac{m^{\star 2}}{16\pi^2} \left|\hat{\mathrm{v}}_{\mathrm{eff}}(\mathbf{p}-\mathbf{p}')\right|^2$$



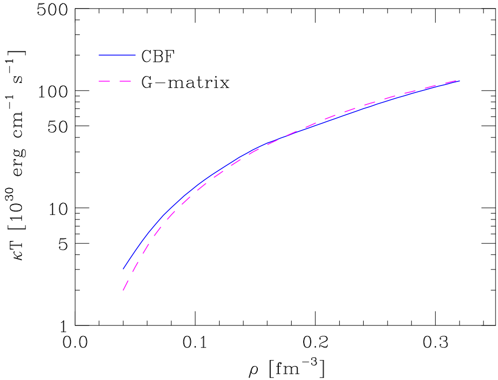
Shear viscosity of neutron matter

- ⋆ Critical to the occurrence of the CFS instability of rapidly rotating neutron stars



- ⋆ Note: the SLya effective interaction, adjusted to reproduce the microscopic EoS, predicts $\eta T^2 \sim 6 \times 10^{13}$ g cm^{-1} s^{-1} MeV^2 at nuclear matter equilibrium density, to be compared with the result obtained from the CBF effective interaction $\eta T^2 \sim 1.4 \times 10^{15}$ g cm^{-1} s^{-1} MeV^2

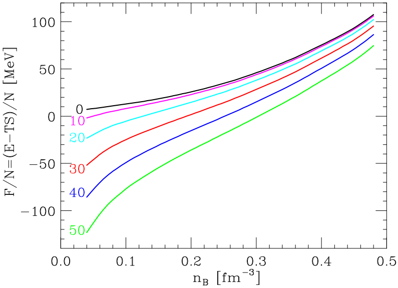
Thermal conductivity of neutron matter

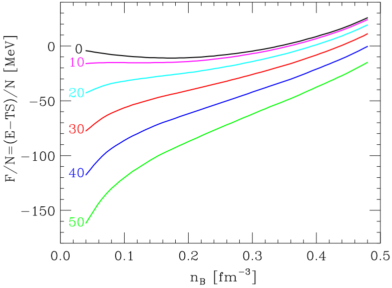


⋆ The transport coefficients computed using the CBF effective interaction is remarkably close to the result obtained within the G-matrix approach using the same bare NN potential. Note: three-body interactions not taken into account.

Future developments

- ⋆ A realistic and *consistent* description of the properties of hot nuclear matter will be needed to perform systematic studies of gravitational-wave emission from protoneutron stars
- ⋆ Free energy of PNM (left) and SNM (right) at $0 \leq T \leq 50$ MeV





SUMMARY & OUTLOOK

- ⋆ Effective interactions *obtained from realistic nuclear hamiltonians* provide a powerful tool to carry out *consistent* calculations of a variety of of properties of strongly interacting many-body systems, ranging from the EoS to quasi particle spectra, in medium scattering probabilities and transport coefficients

- ⋆ The result of systematic studies of the Fermi hard-sphere system performed using the CBF effective interaction are quite encouraging, and suggest that the same formalism can be safely employed in nuclear matter

- ⋆ Future applications to neutron star matter will include the calculation of transport coefficients, superfluid gaps and neutrino emission absorption rates