Efimov physics with 1/2 spin-isospin symmetry

A. Kievsky

INFN, Sezione di Pisa (Italy)

TNPI2016 - XV Conference on Theoretical Nuclear Physics in Italy
Pisa 20-22 April 2016

Collaborators

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- M. Viviani - *INFN & Pisa University, Pisa (Italy)*
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Preliminaries

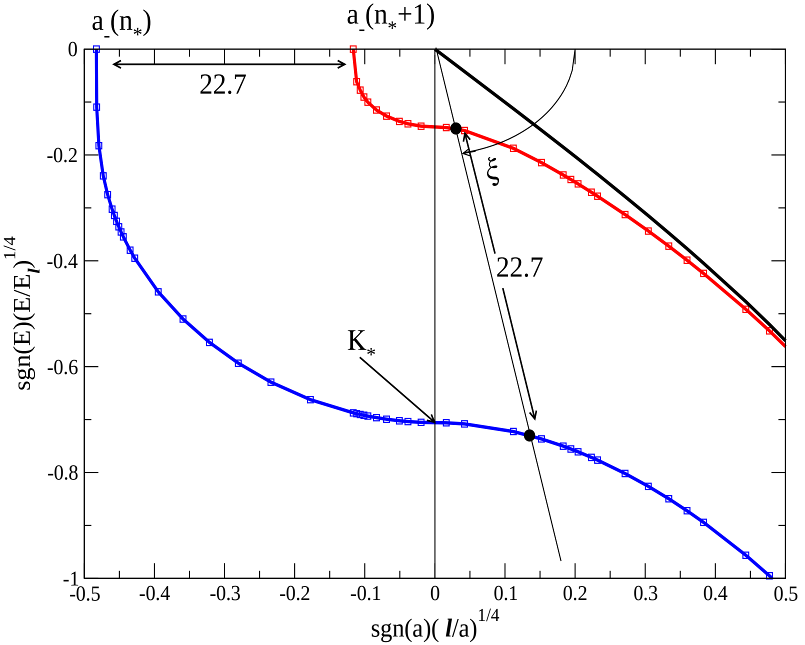
Efimov physics for three bosons (zero-range theory)

The spectrum in terms of the two-body scattering length a is:

$$
\begin{aligned}
K_3^n a &= \tan\xi \\
\kappa_* a &= e^{(n-n^*)\pi/s_0} \frac{e^{-\Delta(\xi)/2s_0}}{\cos\xi} \\
\text{or} \quad K_3^n &= \kappa_*\, e^{-(n-n^*)\pi/s_0}\, e^{\Delta(\xi)/2s_0} \sin\xi
\end{aligned}
$$

- $\hbar^2 (K_3^n)^2/m = E_3^n$
- $e^{-\Delta(\xi)/2s_0}$ is a universal function obtained for example solving the zero-range three-boson problem (STM equation).

Knowing the universal function the spectrum is completely solved by fixing the value of κ_* (called the three-body parameter). Accordingly the above equation is a one-parameter equation.


a_(n*)
a_(n*+1)
22.7
22.7
ξ
K*
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
sgn(E)(E/E_l)^1/4
sgn(a)(l/a)^1/4

Zero-Range vs. Finite-Range (two-body system)

Defining $E_2 = \frac{\hbar^2}{m a_B^2}$

The zero-range theory implies $\longrightarrow a - a_B = 0$

In a finite-range theory we can define $\longrightarrow a - a_B = r_B$

Inside the Efimov window ($a, a_B << r_B$) r_B has a well define meaning:

$$r_B \approx \frac{r_{eff}}{2}\frac{a}{a_B}$$

moving around the unitary limit

Defining $V_\lambda = \lambda V$, varying λ close to the unitary limit the Scrödinger can be solved $H_\lambda \Psi = E\Psi$ for the shallow state (bound or virtual) and the zero-energy state $E = 0$. For the different λ values it results:

$$r_B^\lambda \approx \frac{r_{eff}^\lambda}{2}\frac{a}{a_B} \approx constant = \frac{r_u}{2}$$

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Zero-Range vs. Finite-Range (two-body system)

Defining

$$r_u = r_{eff}(1/a = 0) = 2r_B$$

and assuming

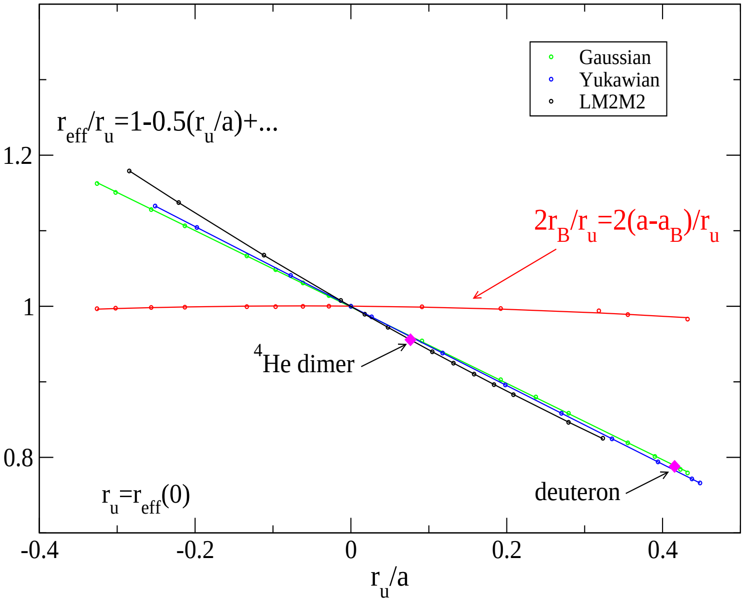
$$r_B = \frac{r_{eff}}{2}\frac{a}{a_B} = constant$$

we obtain

$$\frac{r_{eff}}{r_u} = \frac{2r_B}{r_u}\frac{a_B}{a} = \frac{2r_B}{r_u}\frac{a - r_B}{a} = 1 - 0.5\frac{r_u}{a}$$

universal function for the effective range

$$\frac{r_{eff}}{r_u} = 1 - 0.5\frac{r_u}{a}$$



Gaussian
Yukawian
LM2M2
r_eff/r_u=1-0.5(r_u/a)+...
2r_B/r_u=2(a-a_B)/r_u
4He dimer
deuteron
r_u=r_eff(0)
1.2
1
0.8
-0.4
-0.2
0
0.2
0.4
r_u/a

Zero-Range vs. Finite-Range Effects

Zero-Range Equations: $E_2 = \hbar^2/ma^2$

$$E_3^n/(\hbar^2/ma^2) = \tan^2\xi$$

$$\kappa_* a = e^{\pi(n-n_*)/s_0} e^{-\Delta(\xi)/2s_0}/\cos\xi$$

$\Delta(\xi)$ calculated from the Skorniakov-Ter-Martirosian (STM) equation

Finite-Range Equations: $E_2 = \hbar^2/ma_B^2$ from $V(r)$

$$E_3^n/E_2 = \tan^2\xi$$

$$\kappa_*^n a_B = e^{-\widetilde{\Delta}_n(\xi)/2s_0}/\cos\xi$$

$$\widetilde{\Delta}_n(\xi) = s_0 \ln\left(\frac{E_3^n + E_2}{\hbar^2(\kappa_*^n)^2/m}\right)$$

$\widetilde{\Delta}_n(\xi) \rightarrow \Delta(\xi)$ for $n > 0$

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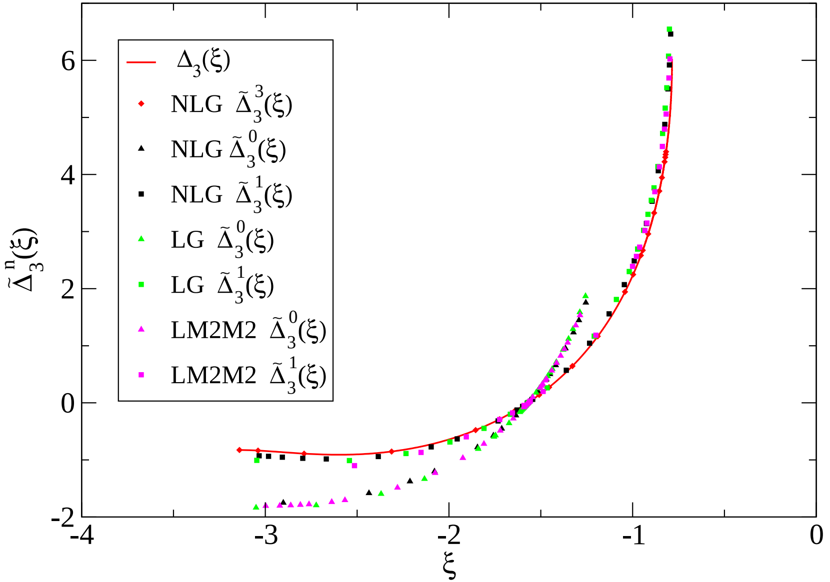
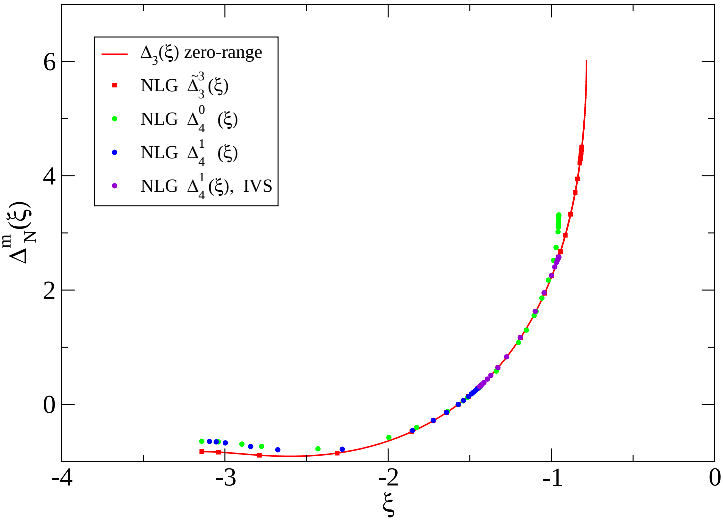


Δ3(ξ)
NLG Δ̃3^3(ξ)
NLG Δ̃3^0(ξ)
NLG Δ̃3^1(ξ)
LG Δ̃3^0(ξ)
LG Δ̃3^1(ξ)
LM2M2 Δ̃3^0(ξ)
LM2M2 Δ̃3^1(ξ)
Δ̃3^n(ξ)
ξ
6
4
2
0
-2
-4
-3
-2
-1
0

Extension to N=4



Zero-Range Equations for $N = 4$

Finite-Range Equations: $E_2 = \hbar^2/ma_B^2$ from $V(r)$

$$E_4^{n,m}/E_2 = \tan^2\xi$$

$$\kappa_4^{n,m} a_B = \mathrm{e}^{-\widetilde{\Delta}_4^{n,m}(\xi)/2s_0}/\cos\xi$$

$$\widetilde{\Delta}_4^{n,m}(\xi) = s_0 \ln\left(\frac{E_4^{n,m} + E_2}{\hbar^2(\kappa_4^{n,m})^2/m}\right)$$

Zero-Range Equations: $E_2 = \hbar^2/ma^2$

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with $\kappa_*^0/\kappa_*^1 = 4.6003$

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1/2-spin 1/2-isospin fermions close to the unitary limit

The 2N system in s-wave

This is a two-channel system with spin $S = 0$ and $S = 1$. For two nucleons the physical values are:

$E_d = -2.2245$ MeV, $a_B = 4.318$ fm

$a_1 = 5.424 \pm 0.003$ fm $r_1^{eff} = 1.760 \pm 0.005$ fm

$a_0 = -23.740 \pm 0.020$ fm $r_0^{eff} = 2.77 \pm 0.05$ fm

moving the system to the unitary limit

- The $S = 1$ channel:

  a gaussian $V_1 e^{-r^2/r_1^2}$ with V_0 and r_1 fixed to describe a_1 and a_B. V_1 is varied: this path has the value $r_B = a_1 - a_B$ almost constant. For nuclear physics we have $r_B \approx 1.2$ fm

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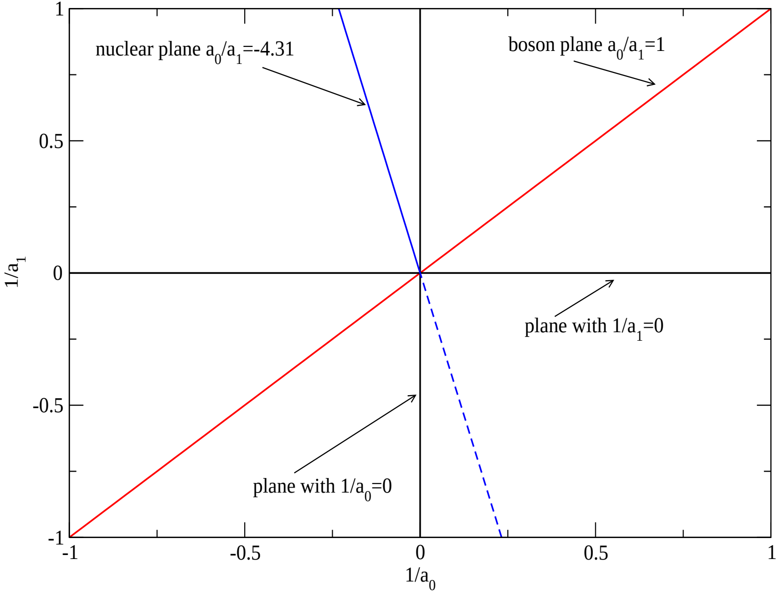

nuclear plane a0/a1=-4.31
boson plane a0/a1=1
plane with 1/a1=0
plane with 1/a0=0
1/a1
1/a0
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

Three-body spectrum with spin-isospin symmetry

Finite-Range Equations: $E_2 = \hbar^2/ma_B^2$ from $V_1(r)$

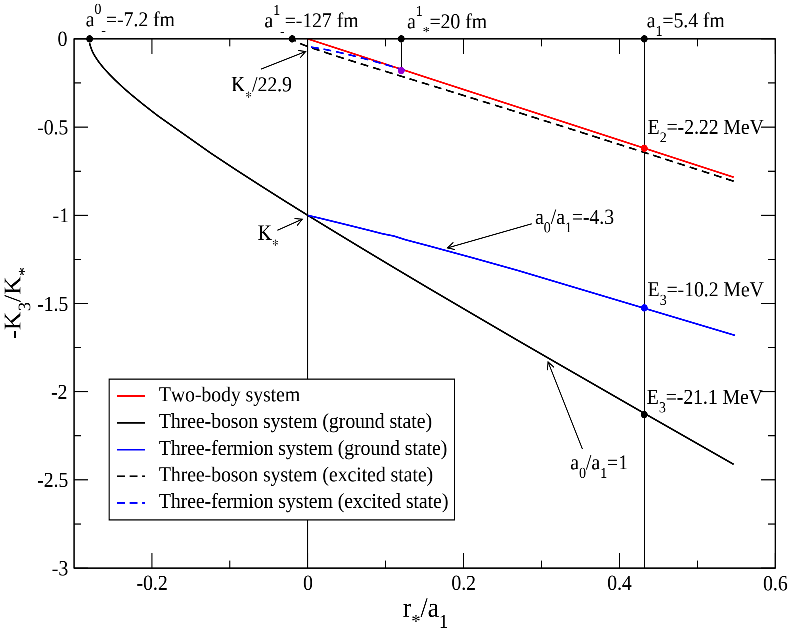
$$E_3^n/E_2 = \tan^2\xi$$

$$\kappa_*^n a_B = \mathrm{e}^{-\widetilde{\Delta}_3(\xi,\phi)/2s_0}/\cos\xi$$

$$\widetilde{\Delta}_n(\xi,\phi) = s_0 \ln\left(\frac{E_3^n + E_2}{\hbar^2(\kappa_*^n)^2/m}\right)$$

$$\frac{a_1}{a_0} = \tan\phi$$

For $n > 0$ $\qquad \widetilde{\Delta}_n(\xi,\phi) \to \Delta(\xi,\phi)$



a^0_-=-7.2 fm
a^1_-=-127 fm
a^1_*=20 fm
a_1=5.4 fm
$K_*/22.9$
E_2=-2.22 MeV
a_0/a_1=-4.3
K_*
E_3=-10.2 MeV
E_3=-21.1 MeV
a_0/a_1=1
$-K_3/K_*$
r_*/a_1
0
-0.5
-1
-1.5
-2
-2.5
-3
-0.2
0
0.2
0.4
0.6
Two-body system
Three-boson system (ground state)
Three-fermion system (ground state)
Three-boson system (excited state)
Three-fermion system (excited state)

Comments on the two-channel plot

- Studying a three-boson system using finite-range potentials, the first excited state does not dispapear onto the two-body threshold
- In the two-channel system the excited state disappears on the two-body threshold as the ratio a_0/a_1 varies.
- The analysis of the nuclear plane produces a binding energy at the unitary limit of $E_u \approx 3.6$ MeV.
- However at the nuclear point the binding energy of $E_3 \approx 10.2$ MeV is far from the experimental value of 8.5 MeV
- A three-body force has to be included
- using a more realistic potential model and varying the depth, the unitary limit can be reached.
- The value obtained has been $E_u \approx 2.8$ MeV.

Working on the nuclear point

The 2N sector

Low Energy data:

$E_d = -2.2245$ MeV

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Constructing LO 2N potential

Two parameters corresponding to the $I = 0$ partial waves with $S = 0, 1$:

$V_0(r) = -V_0 e^{-r^2/r_0^2}$, $V_1(r) = -V_1 e^{-r^2/r_1^2}$

V_0[MeV]	r_0[fm]	a_0[fm]	r_0^{eff}[fm]	V_1[MeV]	r_1[fm]	a_1[fm]	r_1^{eff}[fm]
53.255	1.40	−23.741	2.094	79.600	1.40	5.309	1.622
42.028	1.57	−23.745	2.360	65.750	1.57	5.423	1.776
40.413	1.60	−23.745	2.407	63.712	1.60	5.447	1.802
37.900	1.65	−23.601	2.487	60.575	1.65	5.482	1.846
33.559	1.75	−23.745	2.644	55.036	1.75	5.548	1.930
30.932	1.82	−23.746	2.756				

Working on the nuclear point

The 3N sector

V_0[MeV]	r_0[fm]	V_1[MeV]	r_1[fm]	E_3^0[MeV]	E_3^1[MeV]	${}^2a_{nd}$[fm]
53.255	1.40	79.600	1.40	−12.40	−2.191	−2.175
42.028	1.57	65.750	1.57	−10.83	−2.199	−1.236
40.413	1.60	63.712	1.60	−10.59	−2.197	−1.097
37.900	1.65	60.575	1.65	−10.22	−2.199	−0.860
33.559	1.75	55.036	1.75	−9.584	−2.201	
30.932	1.82	65.750	1.57	−9.715		−0.285
Exp.				−8.482		0.645 ± 0.010

Introducing a Three-Body Force

We choose a simple (two-parameter) form:

$$W(\rho) = W_0 e^{-\rho^2/\rho_0^2}$$

with $\rho^2 = \frac{2}{3}(r_{12}^2 + r_{23}^2 + r_{31}^2)$

Working on the nuclear point

The 3N sector

V_0[MeV]	r_0[fm]	V_1[MeV]	r_1[fm]	E_3^0[MeV]	E_3^1[MeV]	$^2a_{nd}$[fm]
53.255	1.40	79.600	1.40	−12.40	−2.191	−2.175
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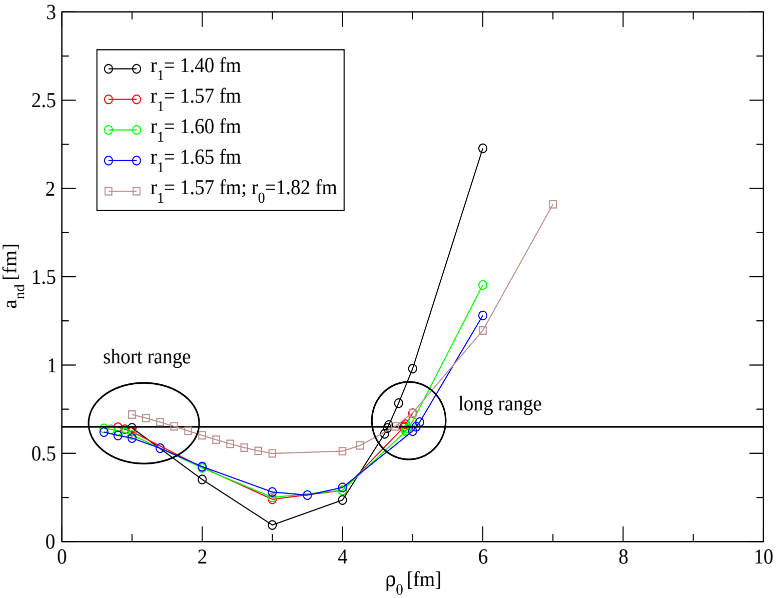
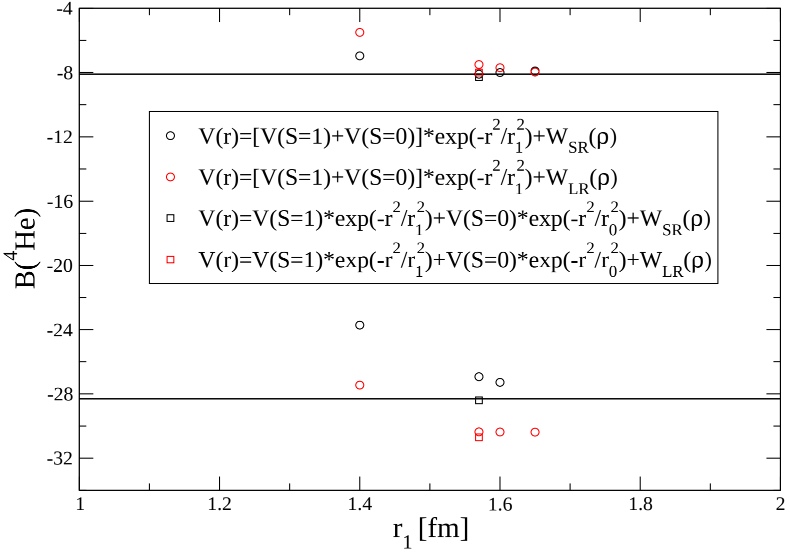
$$V(r)=[V(S=1)+V(S=0)]*\exp(-r^2/r_1^2)+W_0*\exp(-\rho^2/\rho_0^2)$$

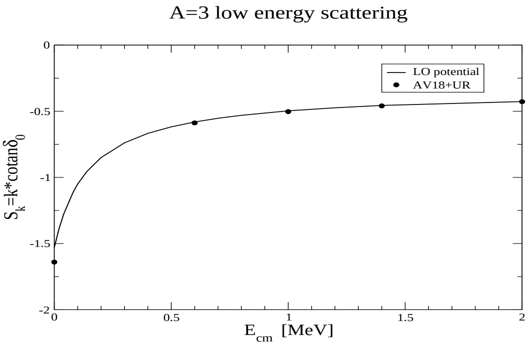


The N=4 ground and excited state



Summary of the LO potential

LO	E_d	$B(^3\mathrm{H})$	$B(^4\mathrm{He})$	$B(^4\mathrm{He}^*)$	$^2a_{nd}$
	-2.225	-8.480	-28.41	-8.29	0.652
Exp.	-2.225	-8.482	-28.296	-8.10	0.645



No bad for a 4-parameter 2N potential + 2-parameter 3N potential!
next step (in progress)→ ^{6}He and ^{6}Li ground states

Conclusions

- A path matching a physical point to the unitary limit has been analyzed
- Varying the depth of the potential the quantity $r_B = a - a_B$ remains almost constant
- Along this path different scale can be joined
- Finite-range effects have been analyzed
- Using this procedure a 1/2-spin 1/2-isospin fermion system has been studied
- A detailed study on the nuclear physics point has been performed with gaussian potentials
- Including a three-body force the doublet $n-d$ scattering length and the four-nucleon system have been studied
- Work in progress: extension to $A > 4$

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