Universality of strange particle production in high energy proton-proton and heavy ion collisions

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XV Conference on Theoretical Nuclear Physics in Italy
Pisa, April 20 – 22, 2016

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Universal Strangeness Production
in Hadronic and Nuclear Collisions

arXiv:1603.06529

- Strangeness enhancement/suppression in high energy collisions
  - statistical model

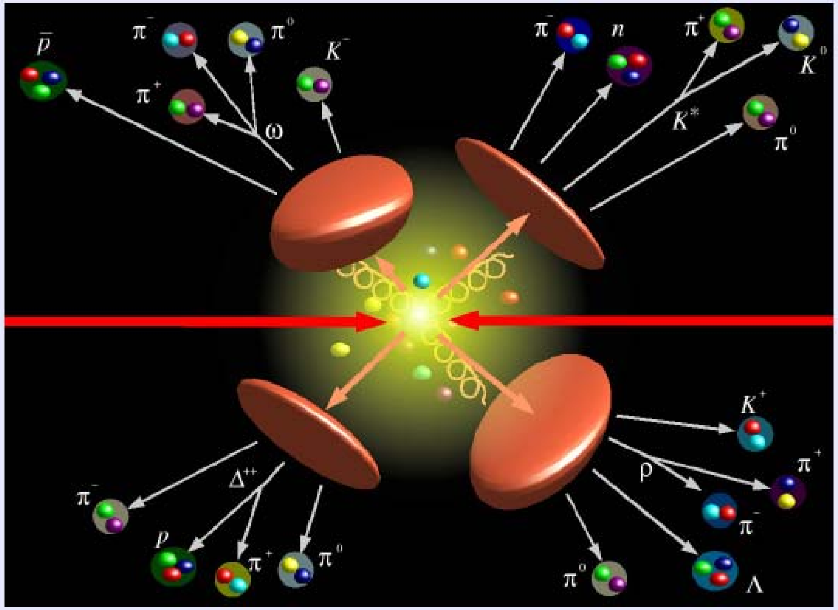
- Strangeness correlation lenght and causality constraints
  in hadron production

- The emergent, model independent, scaling behavior
  in strangeness production

- Results

Statistical Model

In modern view, the statistical model is a model of hadronization, describing the process of hadron formation at the scale where QCD is no longer perturbative



arXiv:0901.3643
An introduction to the Statistical Hadronization Model
F. Becattini

basic observation in all high energy multihadron production

$\boxed{\text{thermal production pattern}}$

- species abundances $\sim$ ideal resonance gas at T_H
- universal $T_H \simeq 165 \pm 15\ Mev$ for all (large) $\sqrt{s}$

caveats

- strangeness suppression in elementary collisions
- strangeness suppression weakened/removed in nuclear collisions

F. Becattini, Z. Phys. C69 (1996) 485.

F. Becattini, *Universality of thermal hadron production in pp, $p\bar{p}$ and e^+e^- collisions*, in *Universality features in multihadron production and the leading effect*, Erice 1966, World Scientific, Singapore (1998) 74-104; arXiv:hep-ph/9701275.

F. Becattini and G. Passaleva, Eur. Phys. J. C23 (2002) 551.

F. Becattini and U. Heinz, Z. Phys. C76 (1997) 268.

J. Cleymans et al., Phys. Lett. B 242 (1990) 111.

J. Cleymans and H. Satz, Z. Phys. C57 (1993) 135.

K. Redlich et al., Nucl. Phys. A 566 (1994) 391.

P. Braun-Munzinger et al., Phys. Lett. B344 (1995) 43.

F. Becattini, M. Gazdzicki and J. Sollfrank, Eur. Phys. J. C5 (1998) 143.

F. Becattini et al., Phys. Rev. C64 (2001) 024901.

P. Braun-Munzinger, K. Redlich and J. Stachel, in *Quark-Gluon Plasma 3*, Hwa and X.-N Wang (Eds.), World Scientific, Singapore 2003.

1. Thermal Hadron Production

what is "thermal"?

- equal *a priori* probabilities for all states in accord with given overall average energy $\Rightarrow$ temperature T;

- partition function of ideal resonance gas

$$\ln Z(T) = V \sum_i \frac{d_i}{(2\pi)^3} \phi(m_i, T)$$

  Boltzmann factor $\phi(m_i, T) = 4\pi m_i^2 T K_2(m_i/T) \sim \epsilon^{-m_i/T}$;

- relative abundances $\frac{N_i}{N_j} = \frac{d_i \phi(m_i, T)}{d_j \phi(m_j, T)} \sim \epsilon^{-(m_i - m_j)/T}$

  predicted in terms of temperature T

In the grand-canonical formulation of the statistical model, the mean hadron multiplicities are defined as

$$\langle N_i \rangle = (2J_i + 1)\frac{V}{(2\pi)^3}\int \mathrm{d}^3p \, \frac{1}{\gamma_s^{-s_i}\exp[(E_i - \mu \cdot \mathrm{q_i})/T_{\mathrm{ch}}] \pm 1}$$

Fireball volume

Strangeness suppression

Number of s or anti-s

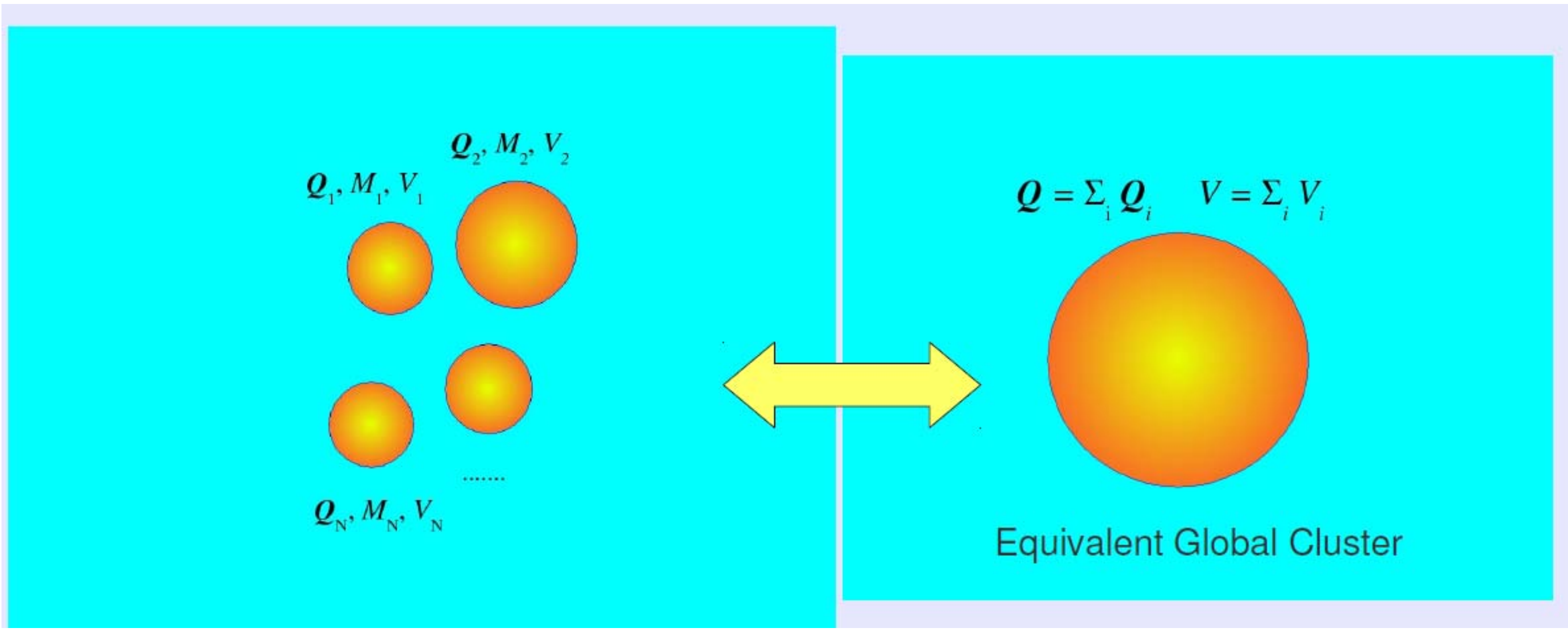
chemical potentials

Che**mi**cal Freeze –out Temperature = hadronic abundances get frozen

J. Letessier, J. Rafelski and A. Tounsi, Phys. Rev. C64 (1994) 406.

- massive colorless clusters distributed over rapidities,
  each decays statistically
- mass and charge distributions of clusters again statistically

  $\Rightarrow$ equivalent global cluster
- $V = \Sigma V_i$, $Q = \Sigma Q_i$; large enough for thermodynamics

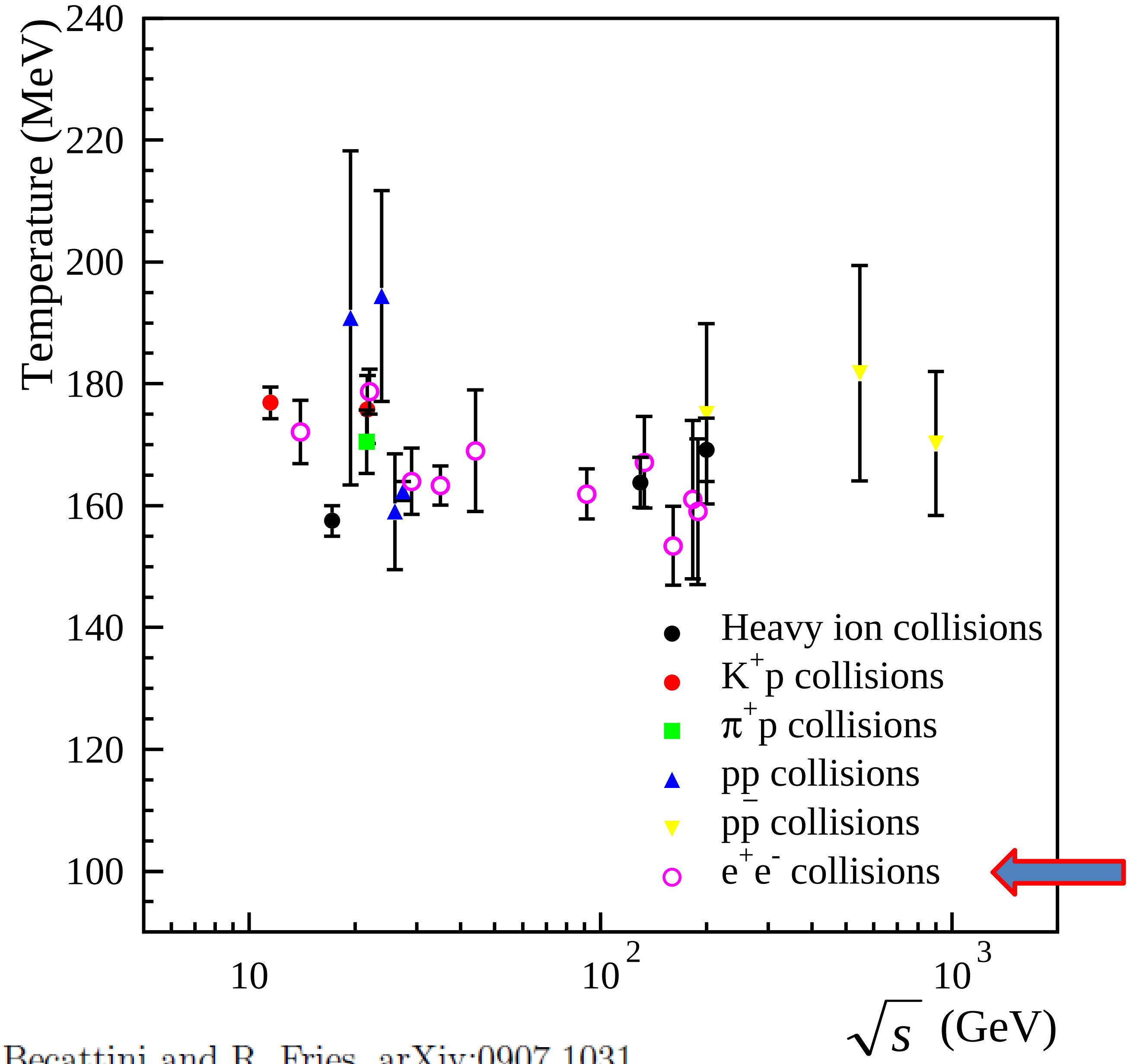


First, a primary hadron yield $\langle n_j \rangle^{\text{primary}}$ is calculated using previous equations.

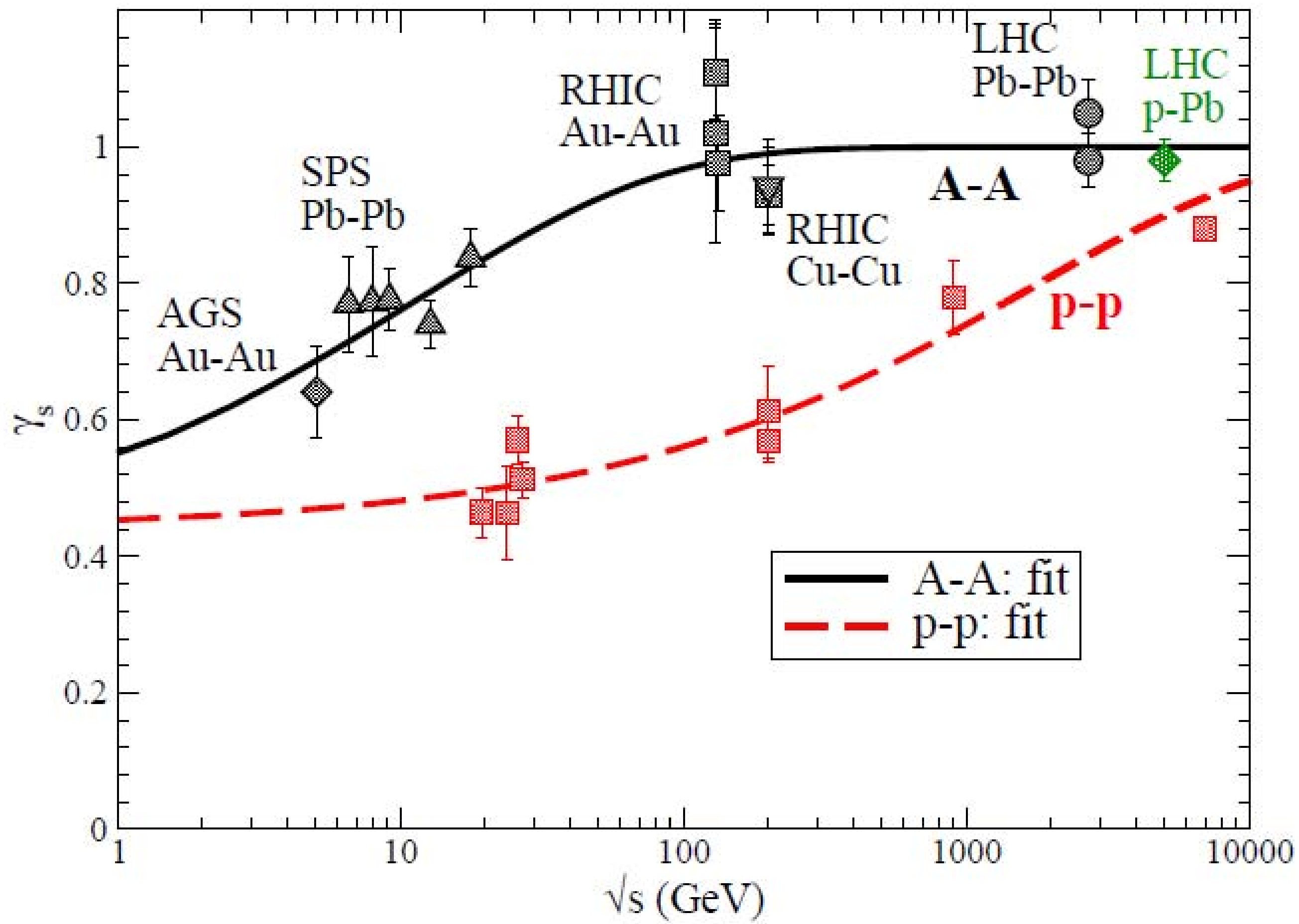
As a second step, all resonances in the gas which are unstable against strong decays are allowed to decay into lighter stable hadrons, using appropriate branching ratios (B) for the decay k → j published by the PDG. The abundances in the final state are thus determined by

$$\langle n_j \rangle = \langle n_j \rangle^{\text{primary}} + \sum \langle n_k \rangle BR(k \to j).$$

$$T, V, \gamma_s, \mu_b$$



F. Becattini and R. Fries, arXiv:0907.1031



LHC
Pb-Pb
LHC
p-Pb
RHIC
Au-Au
SPS
Pb-Pb
A-A
RHIC
Cu-Cu
AGS
Au-Au
p-p
A-A: fit
p-p: fit
γ_s
0
0.2
0.4
0.6
0.8
1
1
10
100
1000
10000
√s (GeV)

*1**) Why is strangeness production universally suppressed*** **in elementary collisions?**

*2) **Why (almost) no strangeness suppression in nuclear collisions?***

Universal behavior in Strangeness production

Canonical suppression

Statistical Thermodynamics in Relativistic Particle and Ion Physics: Canonical or Grand Canonical?
R. Hagedorn and K. Redlich

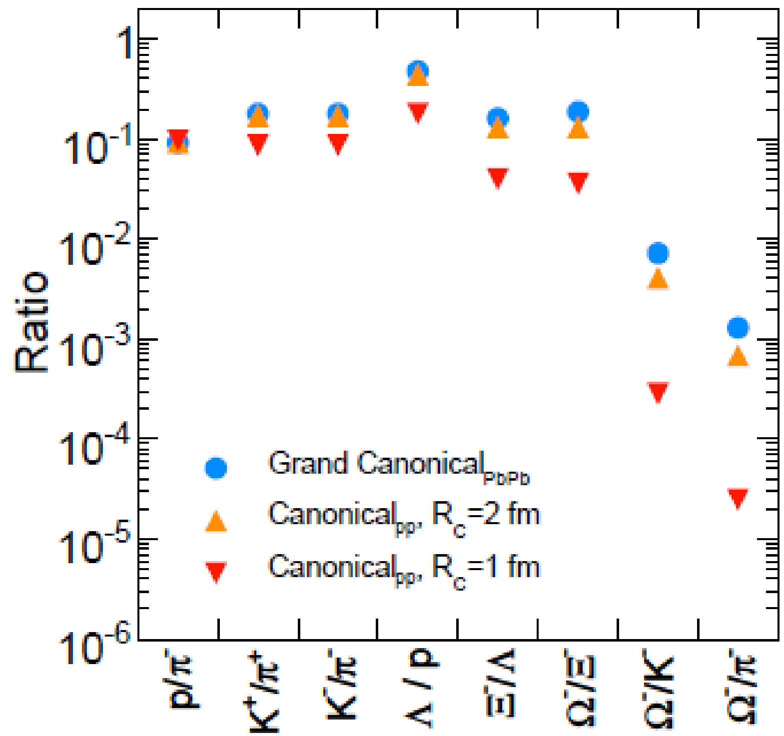
Exact conservation of strangeness

not only V but also a strangeness correlation volume V_c:

strangeness conservation should be enforced not only exactly (canonical instead of grand canonical), but moreover on a local level, within a strangeness correlation volume Vc < V : the production of a single strange particle would require that of an antiparticle nearby, not somewhere in some large equivalent global volume V.

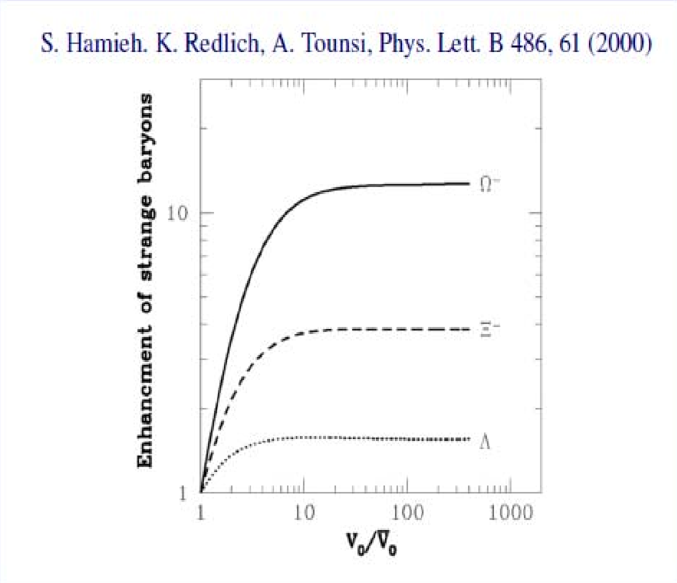
Canonical aspects of strangeness enhancement
A. Tounsia, A. Mischkeb and K. Redlich hep-ph/0209284

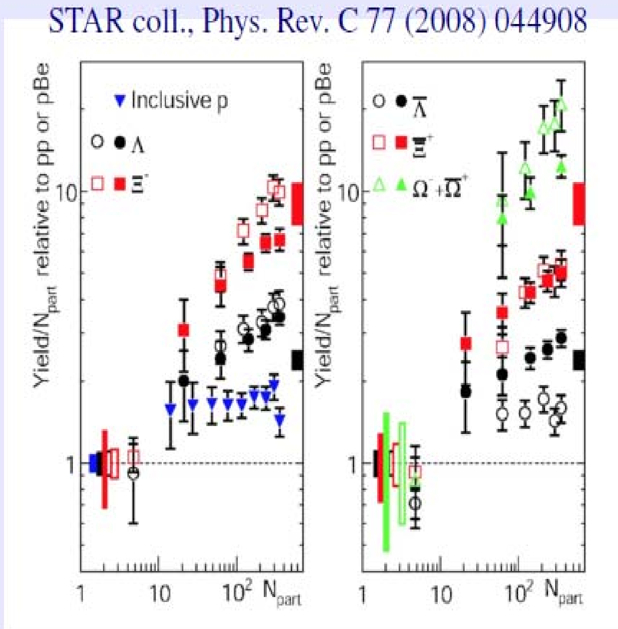
for $V_c/V \to 1$, the corresponding resonance gas predictions converge to those of an equilibrium grand canonical formulation.

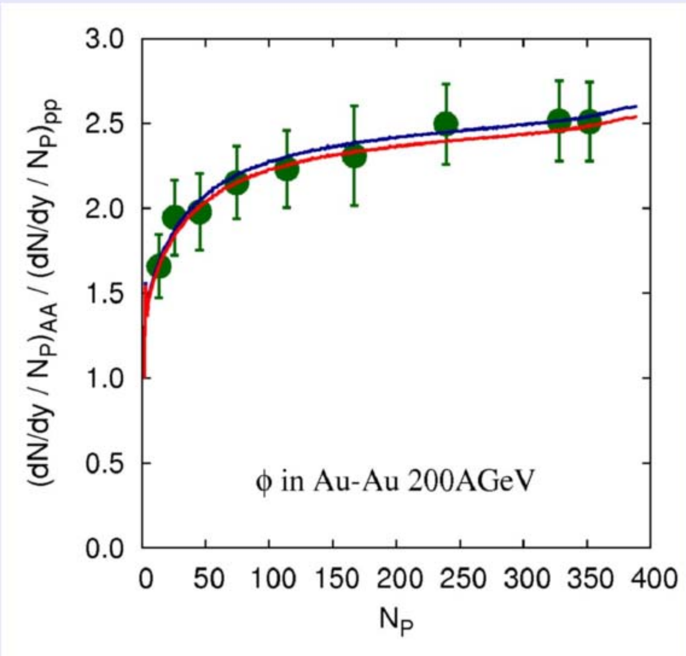


Comparison of canonical and grand-canonical results for different size of the strangeness correlation volume I. Kraus et al., J. Phys. G 37 (2010) 09421.

Canonical suppression could be not enough…





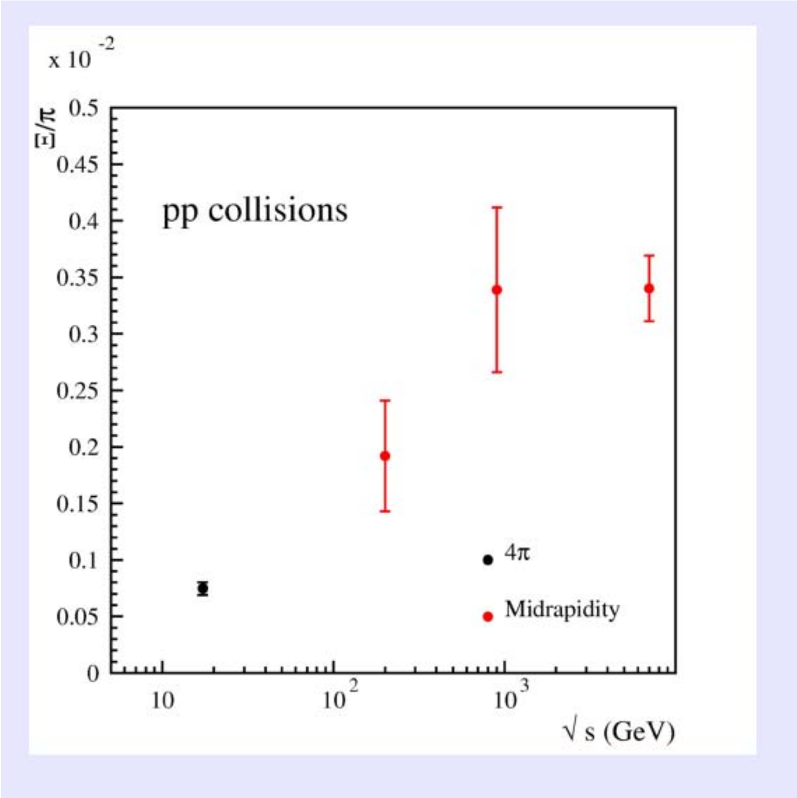


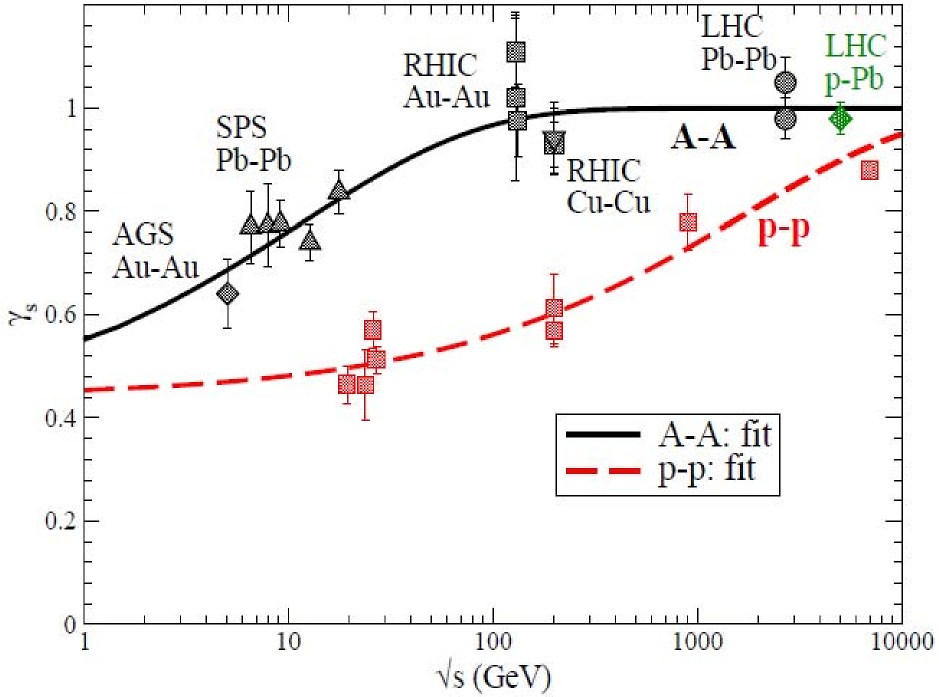
ϕ meson

no canonical suppression

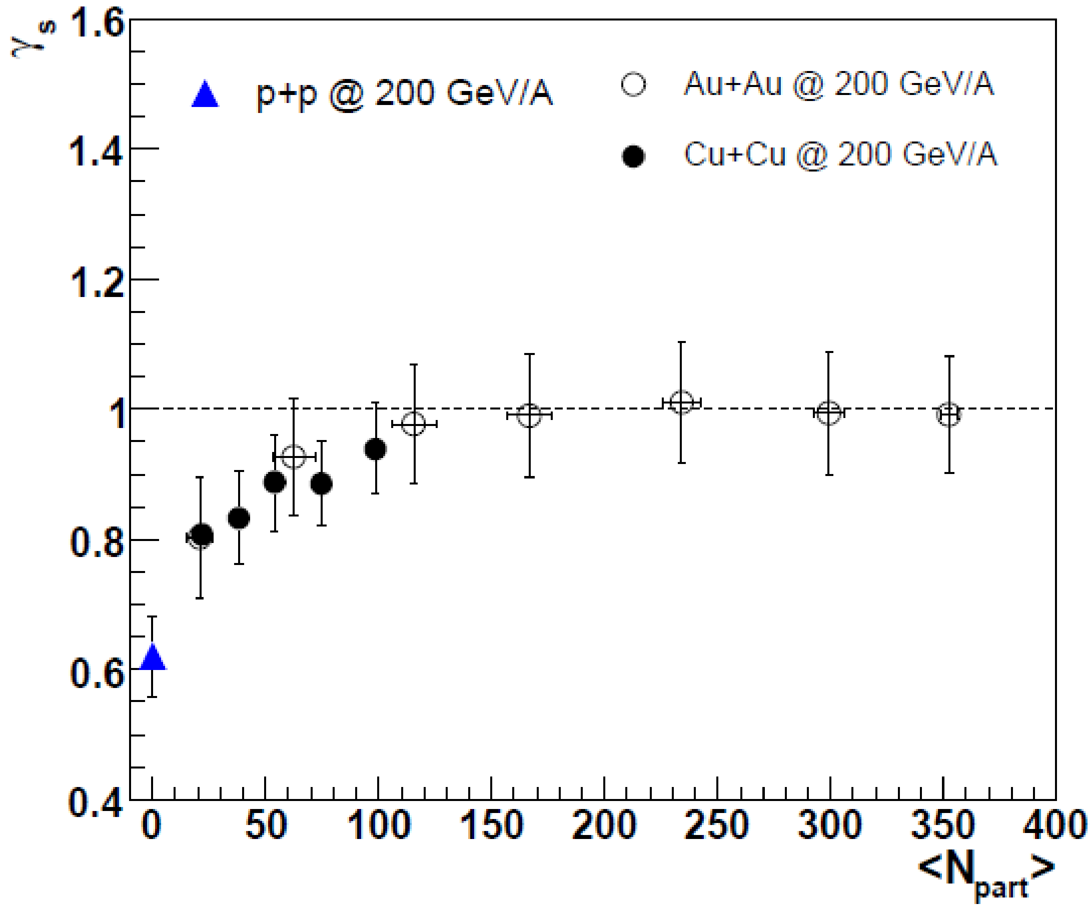
…. but it is in the data

If in pp the energy is large enough to produce fireball with a large number of particles the volume increases and the canonical suppression decreases





Size dependence not only in V_c
but also in γ_s



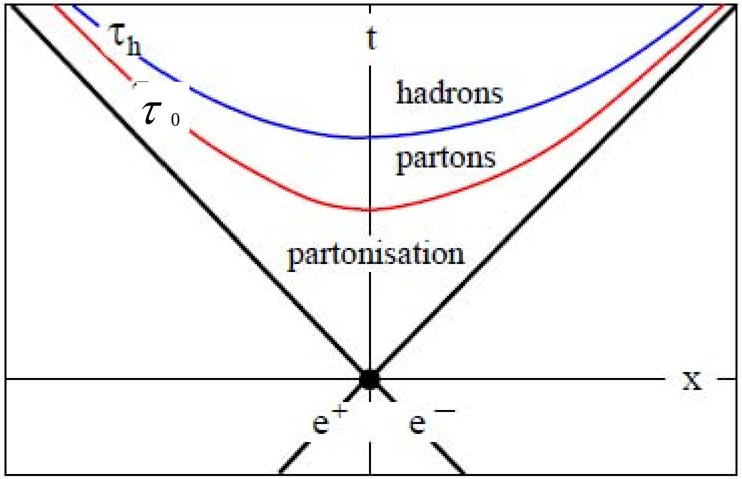
γ_s
1.6
1.4
1.2
1
0.8
0.6
0.4
p+p @ 200 GeV/A
Au+Au @ 200 GeV/A
Cu+Cu @ 200 GeV/A
0
50
100
150
200
250
300
350
400
<N$_{part}$>

Dynamical origin of the strangeness correlation volumeV_c

In a boost-invariant production scenario one finds that the existence of a deconfined quark-gluon plasma is partioned into causally disconnected space-time regions, with no communication possible between different regions. Hadrons produced at large rapidity arise through the confinement transition of a QGP fireball which is causally disjoint from a fireball leading to low rapidity hadrons∗ , and so one cannot expect strangeness conservation to occur through interaction between the relevant bubbles. The concept of a global equivalent cluster thus cannot be applied here: exactly conserved quantum numbers have to be conserved within (smaller) causally connected volumes.

τ_0

It specifies a boost-invariant proper time at which local volume elements experience the transition from an initial state of frozen virtual partons to the partons which will eventually form hadrons



P. Castorina and H. Satz, Int. J. Mod. Phys. E23 (2014) 4, 1450019.

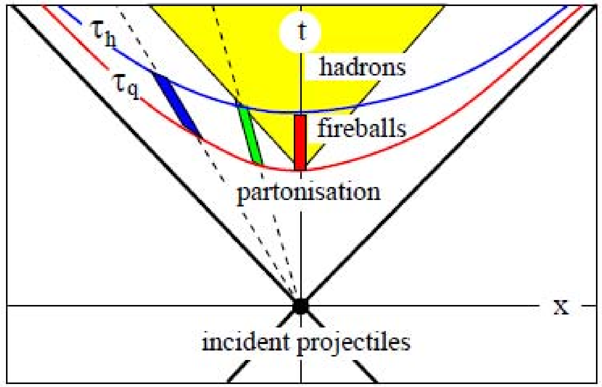
P. Castorina and H. Satz, arXiv:1601.01454.

bubble of partonic medium of proper time τ
with $\tau_0 < \tau < \tau_h$: fireball;
fireballs at different spatial
rapidities η

$t = \tau \cosh \eta, \quad x = \tau \sinh \eta,$

with transition lines

$t^2 - x^2 = \tau^2$



red fireball ($\eta = 0$) - causality region yellow
green fireball ($\eta = \eta_d$) - one common x-t point with red
blue fireball ($\eta > \eta_d$) - outside causality region of red

for $\eta > \eta_d$, with $\tanh \eta_d = (\tau_h^2 - \tau_0^2)/(\tau_h^2 + \tau_0^2)$

forward and backward fireballs are out of communication
with central fireball

examples:

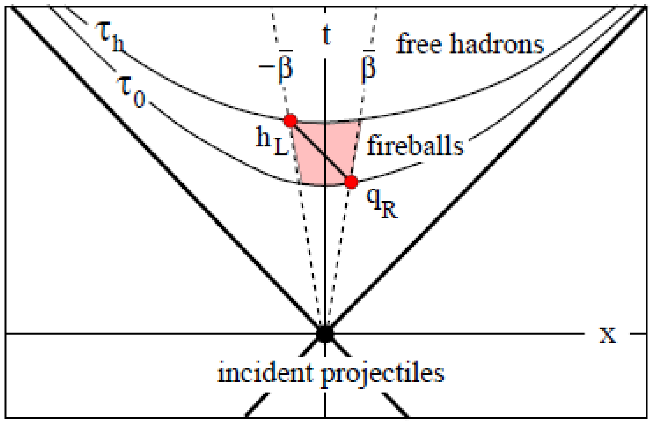
$\tau_0 = 1$ fm, $\tau_h = 2$ fm $\rightarrow \eta_d = 0.7$

$\tau_0 = 1$ fm, $\tau_h = 7$ fm $\rightarrow \eta_d = 2$

at RHIC and LHC, hadronisation occurs through causally disjoint fireballs

so far, have neglected spatial size:
what is the size of a fireball?
define through causal connectivity

require: the most separate points can still communicate



spatial diameter d of fireball in cms at hadronisation time

$$\frac{d}{\tau_0} = \sqrt{\frac{\tau_h}{\tau_0}\left(\frac{\tau_h}{\tau_0} - 1\right)}$$

causal connection (and hence correlations) for hadron production at large rapidity intervals; ⇒ means that any correlations originated in the earlier partonisation stage.

examples for different hadronisation times:

τ_h [fm]	β	η	$r = d/2$ [fm]
2	0.33	0.35	0.7
3	0.50	0.55	1.7
4	0.60	0.69	3.0
5	0.67	0.81	4.5

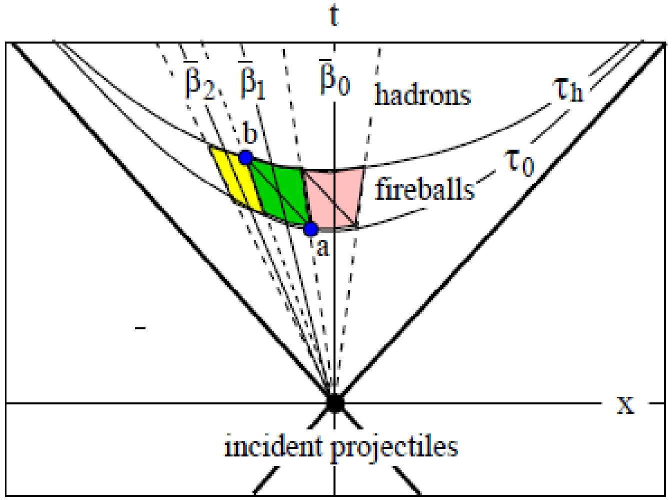
We now assume complete boost invariance: the collision leads to the production of identical fireballs at all rapidities, with identical formation and hadronisation times τ_q, τ_h in their respective rest frames.

denote average cms velocity of of central fireball by $\bar{\beta}_0 = 0$

partition production region into successive causally disjoint fireballs, of velocities

$$\bar{\beta}_n = \frac{\tau_h^{2n} - \tau_q^{2n}}{\tau_h^{2n} + \tau_q^{2n}}$$

$$n = 0, 1, 2, \ldots$$



one fireball - require that the spatially right-most point q_R at formation can send a signal to the spatially left-most point h_L at hadronisation; i.e., we require that the most separate points of the fireball can still communicate.

$$\frac{d}{\tau_0} = \sqrt{\frac{\tau_h}{\tau_0}\left(\frac{\tau_h}{\tau_0} - 1\right)} \qquad \frac{d\epsilon}{d\tau} = -\frac{(\epsilon + p)}{\tau},$$

$$p = \epsilon/3 \qquad \frac{\tau_h}{\tau_0} = \left(\frac{\epsilon_0}{\epsilon_h}\right)^{3/4}$$

initial energy density

universal hadronisation energy density

$\epsilon_h \simeq 0.4 - 0.6\ \mathrm{GeV/fm^3}$

lattice QCD

$$p = 0. \qquad \frac{\tau_h}{\tau_0} = \left(\frac{\epsilon_0}{\epsilon_h}\right)$$

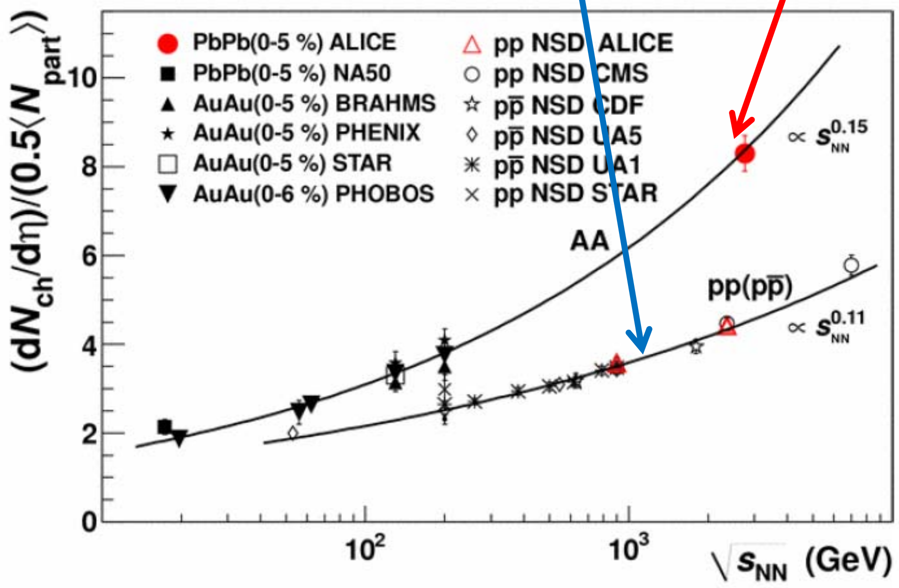
$$p = a\epsilon,\ 0 < a < 1/3, \quad \frac{\tau_h}{\tau_0} = \left(\frac{\epsilon_0}{\epsilon_h}\right)^{1/(1+a)}$$

$$\frac{d}{\tau_0} = \sqrt{\frac{\tau_h}{\tau_0}\left(\frac{\tau_h}{\tau_0} - 1\right)} \Rightarrow$$

function of the scattering energy through the initial energy density

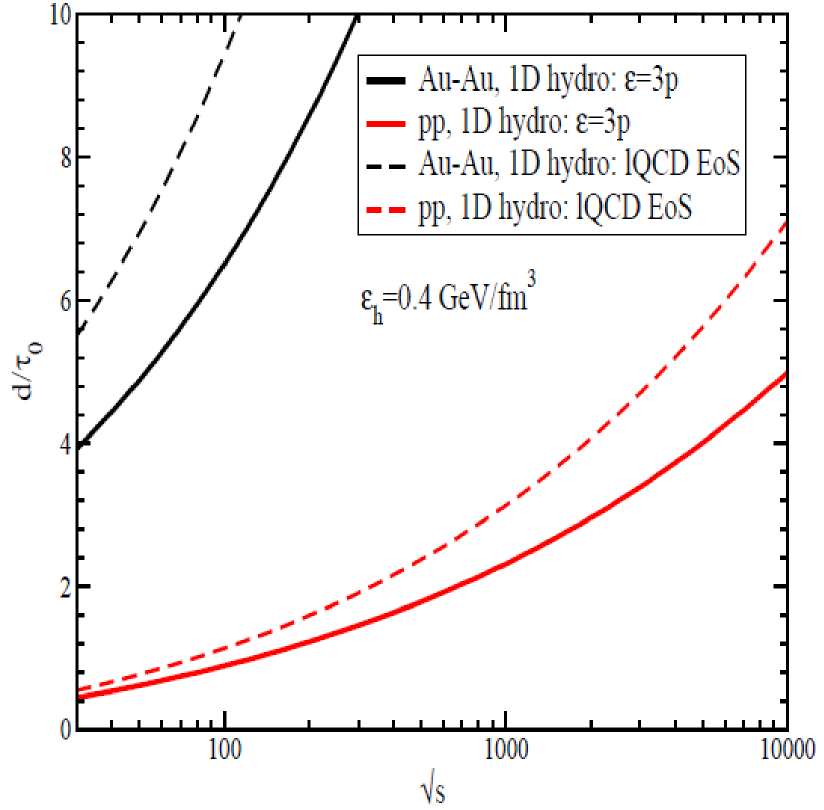
$$\epsilon_0^{AA} = \frac{m_T}{\tau_0 \pi R_A^2}\left(\frac{dN_{AA}}{dy}\right)_0 = \frac{m_T A^{1/3}}{\tau_0 \pi R_0^2}\left(\frac{dN_{AA}}{dy}\right)_0 .$$

$$\epsilon_0^{pp} = \frac{m_T}{\tau_0 \pi R_p^2}\left(\frac{dN_{pp}}{dy}\right)_0$$



K. Aamodt et al. (ALICE Collaboration), Phys. Rev. Lett. 105 (2010) 252301; arXiv nucl-exp 1011.3916.

$$\frac{d}{\tau_0}(\sqrt{s})$$ ⇒ MODEL DEPENDENT



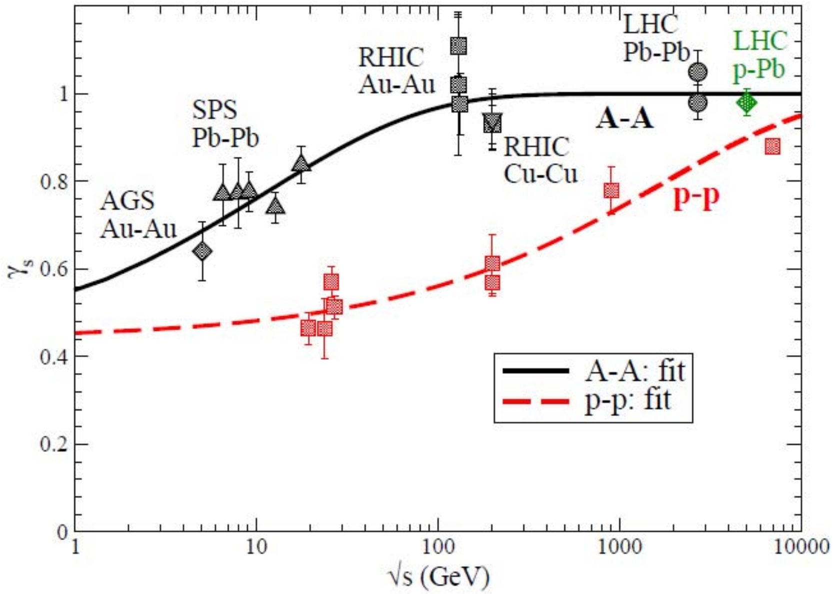
BUT : the size dependence suggests a universal MODEL INDEPENDENT behavior

Find the correct dynamical variable

The scaling law is obtained for

$$\gamma_s \quad \text{Versus} \quad \frac{dN}{Ady}$$

and is model independent



F. Becattini, J. Manninen and M. Gazdzicki, Phys. Rev. C 73 (2006) 044905.

$$\gamma_s^A(s) = 1 - a_A \exp\left(-b_A\sqrt{A\sqrt{s}}\right)$$

$$\gamma_s^p(s) = 1 - a_p \exp\left(-b_p s^{1/4}\right),$$

$$a_A = 0.606,\ \ a_p = 0.5595$$

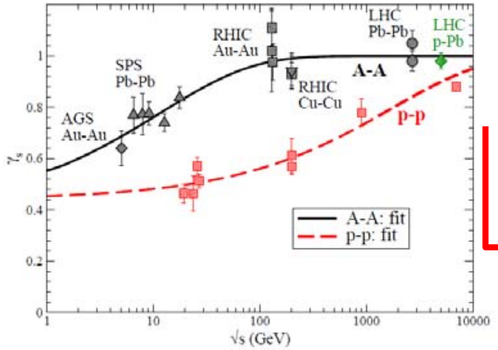
$$b_A = 0.0209,\ \ b_p = 0.0242$$

$$\epsilon_0\,\tau_0 = \epsilon_{Bj}^A\,\tau_0 = \frac{1.5\,m_T A}{\pi R_A^2}\left(\frac{dN}{dy}\right)_{y=0}^{AA},\ \left(\frac{dN}{dy}\right)_{y=0}^{AA} = a(\sqrt{s})^{0.3} + b,$$

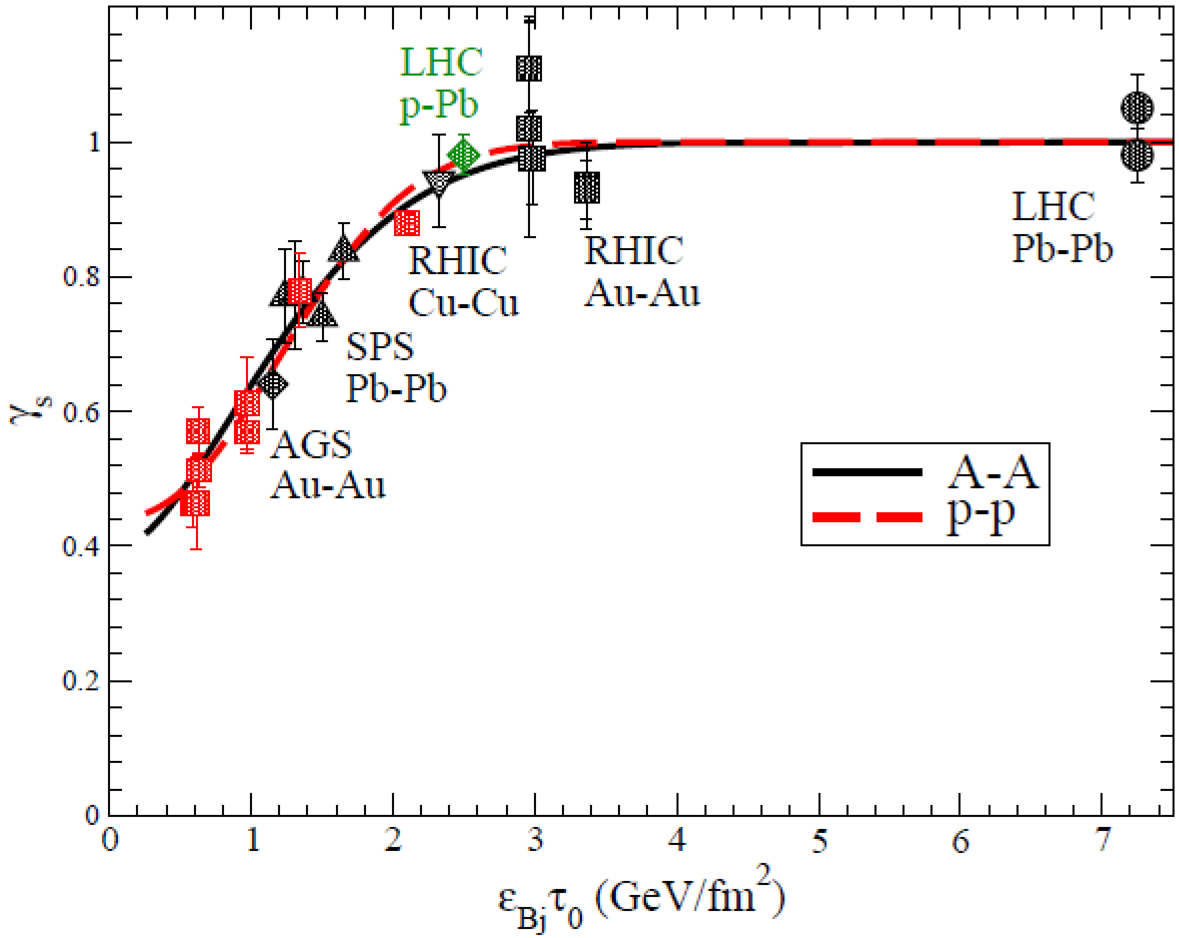
with a=0.7613 and b= 0.0534

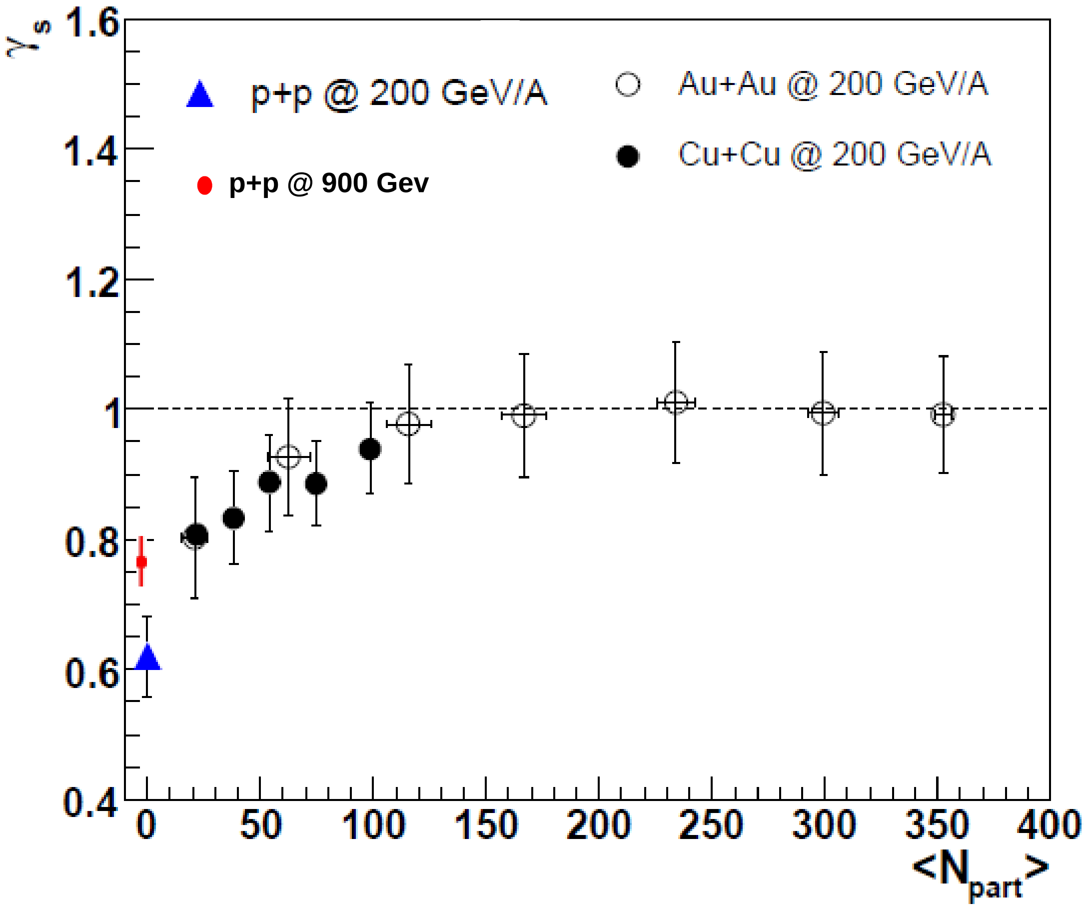
$$\epsilon_{Bj}^p\,\tau_0 = \frac{1.5\,m_T}{\pi R_p^2}\left(\frac{dN}{dy}\right)_{y=0}^{pp},\qquad \left(\frac{dN}{dy}\right)_{y=0}^{pp} = a(\sqrt{s})^{0.22} + b,$$

a=0.797 and b= 0.04123



There is a model independent scaling
Of strangeness production with
the transverse energy/transverse area



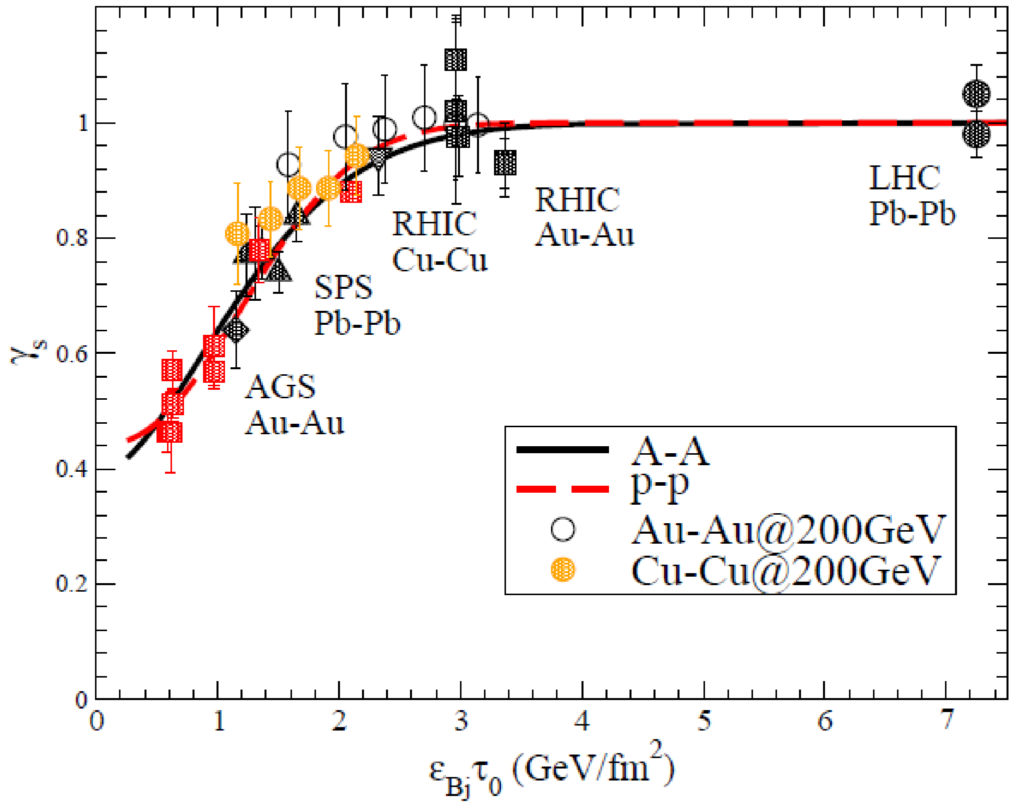


γ_s
1.6
1.4
1.2
1
0.8
0.6
0.4
p+p @ 200 GeV/A
p+p @ 900 Gev
Au+Au @ 200 GeV/A
Cu+Cu @ 200 GeV/A
0
50
100
150
200
250
300
350
400
$<N_{part}>$

initial energy density for non-central collisions.

$$\epsilon_{Bj}^{N_p}\,\tau_0 = \frac{1.5\,m_T(0.5N_p)}{\pi R_{N_p}^2}\left(\frac{dN}{dy}\right)_{y=0}^{AA},$$

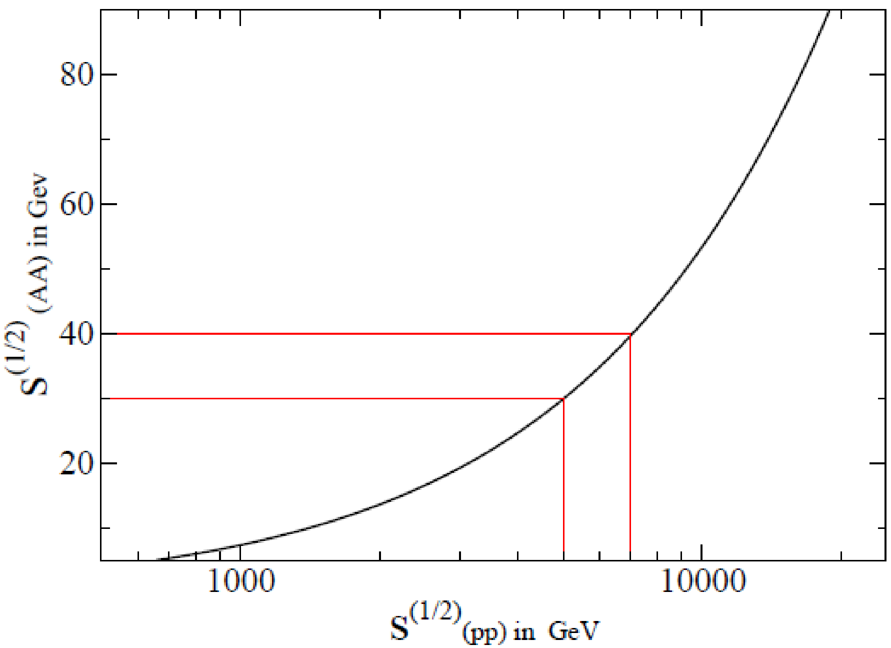
with $R_{N_p} = 1.25(0.5N_p)^{1/3}$ as an estimate of the energy density as function of the number of participants N_p. This then allows us to enter recent data for γ_s as function of N_p in $Au-Au$ and $Cu-Cu$ collisions at 200 GeV



For a given strangeness enhancement/suppression in AA one evaluates the pp collision energy to have the same behavior.

$$\epsilon_{Bj}^{A}\,\tau_0 = \epsilon_{Bj}^{p}\,\tau_0$$

$$\frac{1.5\,m_T A}{\pi R_A^2}\left(\frac{dN}{dy}\right)_{y=0}^{AA} = \frac{1.5\,m_T}{\pi R_p^2}\left(\frac{dN}{dy}\right)_{y=0}^{pp}$$



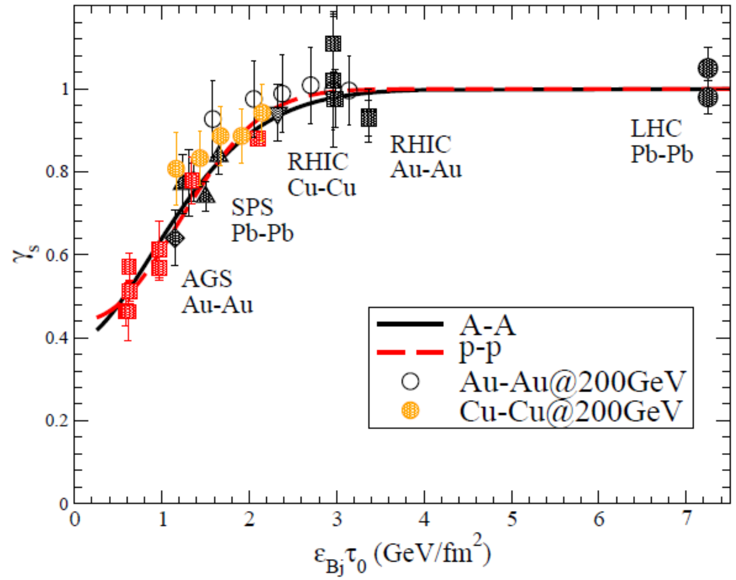
$R_p \simeq 0.8$ fm $\qquad R_A \simeq A^{1/3} R_0 \qquad R_0 = 1.25$ fm,

Conclusions

The observed scaling of γ_s with ϵ_{Bj} is thus an observable consequence of our basic causality correspondence.

Nevertheless, if one were to just *ad hoc* assume a $\gamma_s(s) - \epsilon(s)$ correlation, the results would remain.

MODEL INDEPENDENT



strangeness suppression in hadronic and nuclear collisions is fully determined by the initial energy density. → Transverse energy/transverse area

Any model of the strangeness production has to explain the observed scaling behavior

p-Pb

we had also included the LHC point for $p - Pb$ collisions at $\sqrt{s} = 5.02$ TeV. To determine the corresponding energy density, we use in eq. (6)

$$R_T = R_p(0.5\bar{N}_{\rm part})^{1/3}$$

for the transverse radius, with $\bar{N}_{\rm part} \simeq 8$, as given by

B. Abelev et al. (ALICE Coll.), Phys. Rev. Lett. 110 (2013) 032301.

Peter F. Kolb[1] and Ulrich Heinz[2]

http://arxiv.org/abs/nucl-th/0305084

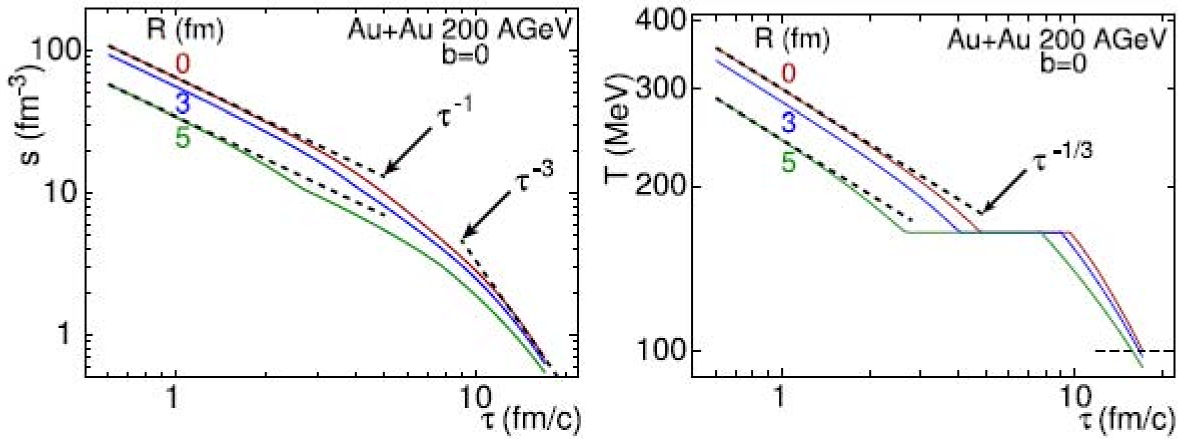


Fig. 4. Left panel: Time evolution of the entropy density at three different points in the fireball (0, 3, and 5 fm from the center). Dashed lines indicate the expectations for pure one-dimensional and three-dimensional dilution, respectively. Right panel: Time evolution of the temperature at the same points. The plateau at $T = 164\,\mathrm{MeV}$ results from the transition of the corresponding fluid cells through the mixed phase.

Causal viscous hydrodynamics in 2+1 dimensions for relativistic heavy-ion collisions
H. Song and Ulrich Heinz

http://arxiv.org/pdf/0712.3715

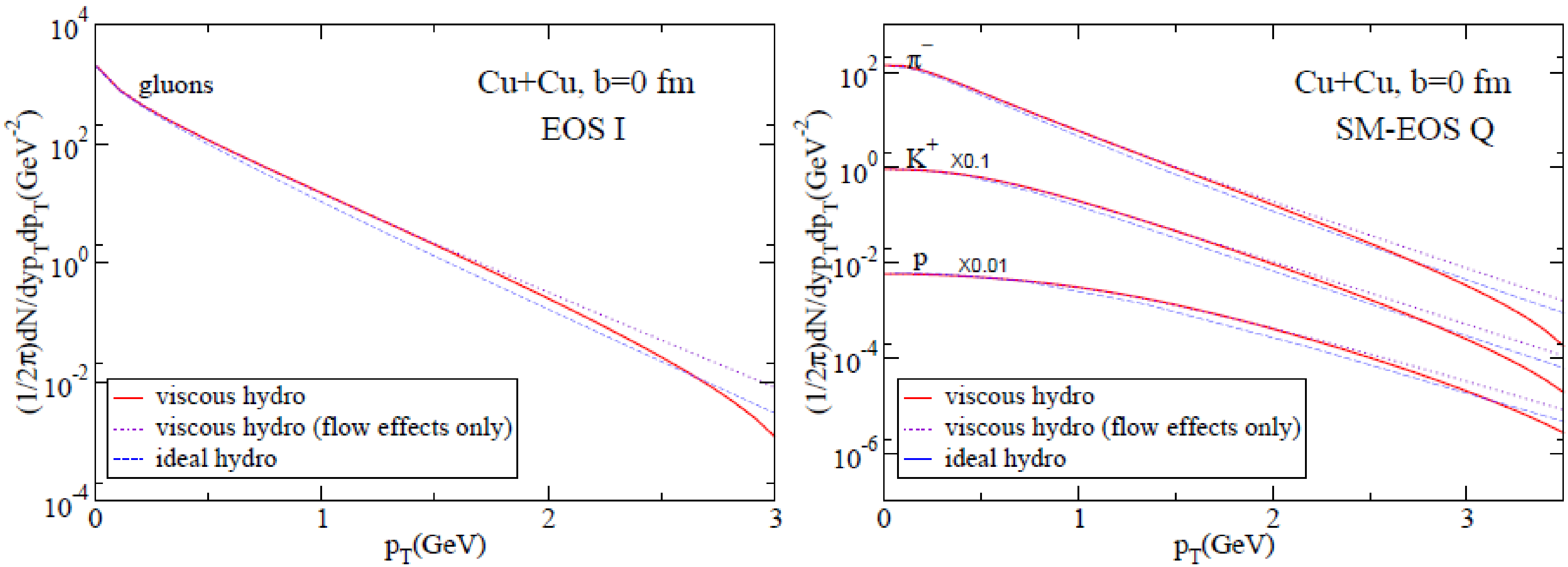


FIG. 7: (Color online) Mid-rapidity particle spectra for central Cu+Cu collisions, calculated with EOS I (left, gluons) and with SM-EOS Q (right, π^-, K^+ and p). The solid blue (red dashed) lines are from ideal (viscous) hydrodynamics. The purple dotted lines show viscous hydrodynamic spectra that neglect the viscous correction δf_i to the distribution function in Eq. (12), i.e. include only the effects from the larger radial flow generated in viscous hydrodynamics.

Canonical versus grancanonical

First, one imposes exact strangeness conservation, which leads to a volume-dependent strangeness reduction [12, 13]; the ratio of canonical to grand-canonical partition functions,

$$\frac{Z_{\rm can}(T,V,S)}{Z_{\rm gcan}(T,V,\langle S\rangle)} < 1$$

approaches unity only in the limit of large volumes.

To simplify matters, let us assume that there are only two hadron species: scalar and electrically neutral mesons, "pions" of mass m_π, "kaons" of mass m_K and strangeness $s=1$ together with "antikaons" of the same mass but strangeness $s=-1$. In this case, the grand canonical partition function for a system of of volume V and temperature T has the form

$$Z_{GC}(T,V,\mu) = \frac{VT}{2\pi^2}\left[m_\pi^2 K_2(m_\pi/T) + m_K^2 K_2(m_K/T)e^{\mu/T} + m_K^2 K_2(m_K/T)e^{-\mu/T}\right], \quad (24)$$

where μ denotes the chemical potential for strangeness. If the overall strangeness is zero, $\mu=0$ and the average density of mesons of type i ($i=\pi,K,\bar{K}$) is given by

$$n_i(T) = \frac{Tm_i^2}{2\pi^2}K_2(m_i/T), \quad (25)$$

while the ratio of kaon to pion multiplicities becomes

$$\frac{N_K}{N_\pi} = \left(\frac{m_K}{m_\pi}\right)^2 \frac{K_2(m_K/T)}{K_2(m_\pi/T)} \simeq \left(\frac{m_K}{m_\pi}\right)^{3/2} \exp\{-\frac{(m_K-m_\pi)}{T}\}. \quad (26)$$

The grand canonical form assures that the average overall strangeness is zero, but only the *average*;

$$\left(\frac{\partial^2 \ln Z_{GC}}{\partial \mu^2} \right) \sim \langle S^2 \rangle$$

The grand canonical ensemble effectively corresponds to an average over all possible strangeness configurations, with $\exp(\pm\mu/T)$ as weights. If instead we insist that the overall strangeness is *exactly* zero, we have to project out that term of the sum. This canonical ensemble can lead to a severe restriction of the available phase space and hence of the production rate. Thus the canonical density of kaons becomes

$$\tilde{n}_K(T,V) = n_K(T)\frac{I_1(x_K)}{I_0(x_K)},$$

where $I_n(X)$ is the $n-th$ order Bessel function of imaginary argument and $n_K(T)$ is given by eq. (25) and

$$x_K = \frac{VTm_K^2}{2\pi^2}K_2(m_K/T).$$

The canonical density, in contrast to the grand canonical form, thus depends on the volume V of the system. Since $I_n(x) \sim x^n$ for $x \to 0$ and $I_n(x) \to e^x$ for $x \to \infty$, we see immediately that in the large volume limit,

$$\tilde{n}_K(T,V) \to n_K(T),$$

the canonical form converges to the grand canonical one, as expected. In the small volume limit, however, the Bessel function ratio results in a strong suppression of canonical relative to grand canonical form, with $I_1(x)/I_0(x) \to 0$ for $x \to 0$.

$$Z(T,V,V_c) = \frac{VT}{2\pi^2}\left[m_\pi^2 K_2(m_\pi/T) + 2m_K^2 K_2(m_K/T)\left(\frac{I_1(x_K(T,m_K,V_c))}{I_0(x_K(T,m_K,V_c))}\right)\right],$$