

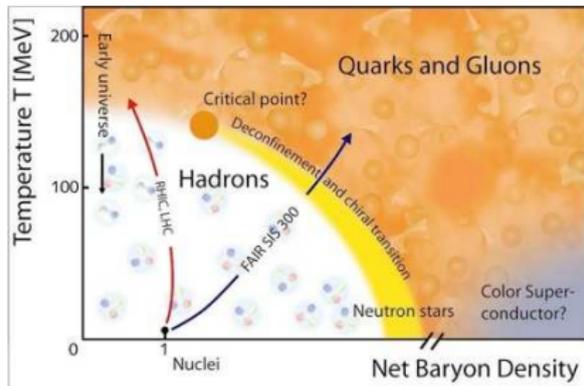
# Medium effects on heavy-flavour observables in high-energy nuclear collisions

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# Heavy-ion collisions: exploring the QCD phase-diagram



QCD phases identified through the *order parameters*

- **Polyakov loop**  $\langle L \rangle \sim e^{-\beta \Delta F_Q}$  energy cost to add an isolated color charge
- **Chiral condensate**  $\langle \bar{q}q \rangle \sim$  effective mass of a “dressed” quark in a hadron

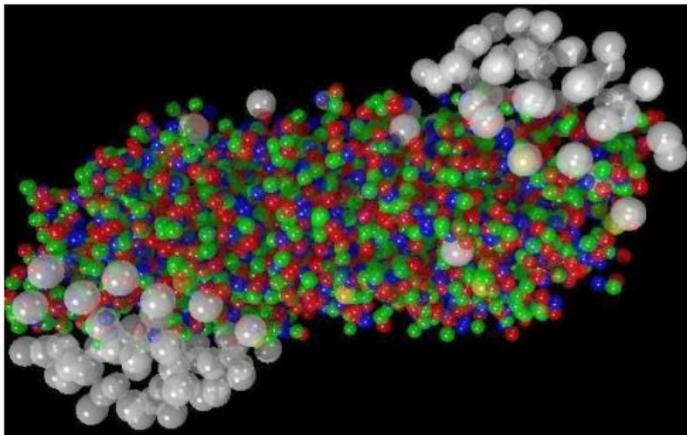
Region explored at LHC: *high-T/low-density* (early universe,  $n_B/n_\gamma \sim 10^{-9}$ )

- From **QGP** (color deconfinement, chiral symmetry restored)
- to **hadronic phase** (confined, **chiral symmetry breaking**<sup>1</sup>)

NB  $\langle \bar{q}q \rangle \neq 0$  **responsible for most of the baryonic mass of the universe: only  $\sim 35$  MeV of the proton mass from  $m_{u/d} \neq 0$**

<sup>1</sup>V. Koch, *Aspects of chiral symmetry*, Int.J.Mod.Phys. E6 (1997)

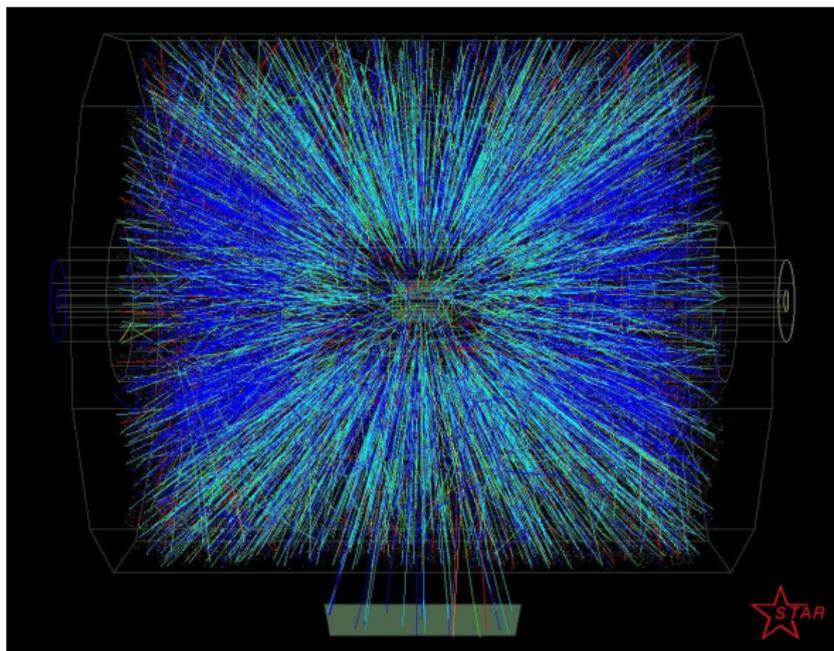
## Heavy-ion collisions: a typical event



- Valence quarks of participant nucleons act as sources of strong color fields giving rise to *particle production*
- Spectator nucleons don't participate to the collision;

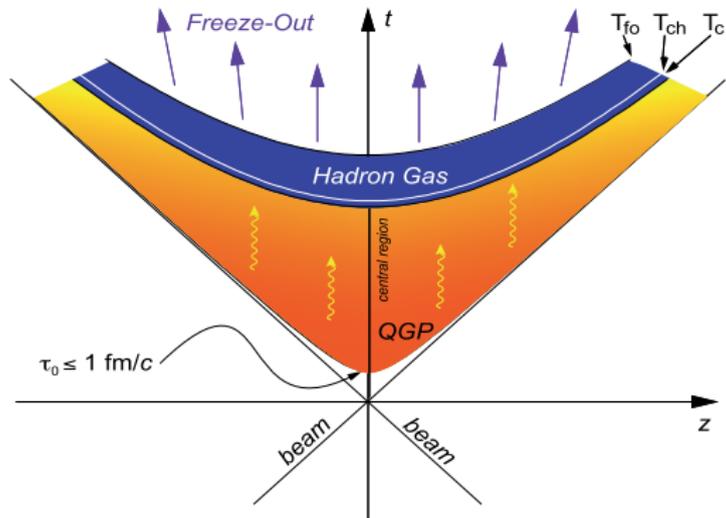
*Almost all the energy and baryon number carried away by the remnants*

## Heavy-ion collisions: a typical event



Event display of a Au-Au collision at  $\sqrt{s_{NN}} = 200$  GeV

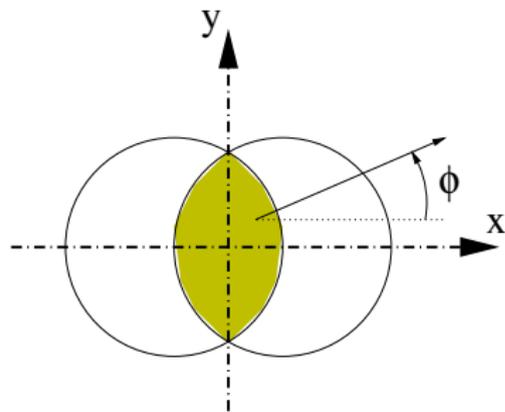
# Heavy-ion collisions: a cartoon of space-time evolution



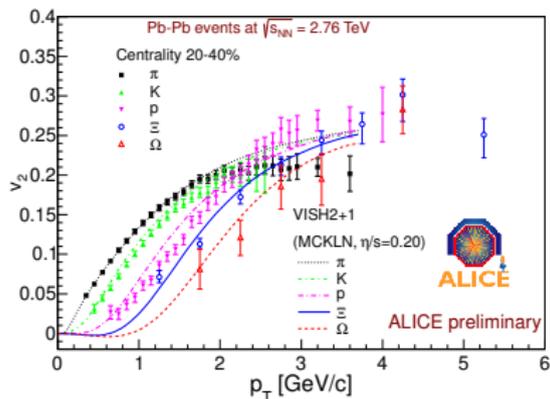
- **Soft probes** (low- $p_T$  hadrons): **collective behavior** of the *medium*;
- **Hard probes** (high- $p_T$  particles, heavy quarks, quarkonia): produced in *hard pQCD processes* in the initial stage, allow to perform a **tomography of the medium**

## Hydrodynamic behavior: elliptic flow

- In *non-central collisions* particle emission is not azimuthally-symmetric!



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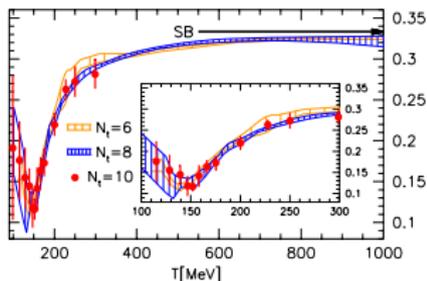
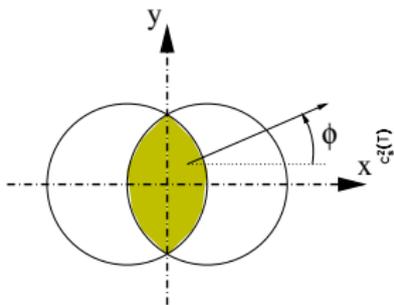
- The effect can be quantified through the *Fourier coefficient*  $v_2$

$$\frac{dN}{d\phi} = \frac{N_0}{2\pi} (1 + 2v_2 \cos[2(\phi - \psi_{RP})] + \dots)$$

$$v_2 \equiv \langle \cos[2(\phi - \psi_{RP})] \rangle$$

- $v_2(p_T) \sim 0.2$  gives a modulation **1.4** vs **0.6** for **in-plane** vs **out-of-plane** particle emission!

## Elliptic flow: physical interpretation

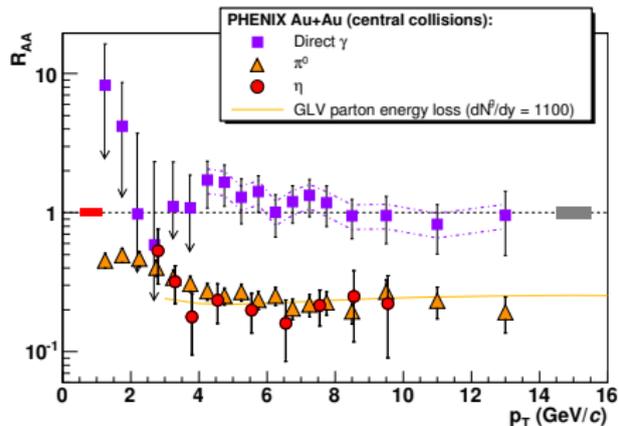


- Matter behaves like a fluid whose *expansion is driven by pressure gradients*

$$(\epsilon + P) \frac{dv^i}{dt} \Big|_{v \ll c} = - \frac{\partial P}{\partial x^i} \quad (\text{Euler equation})$$

- **Spatial anisotropy** is converted into **momentum anisotropy**;
- At freeze-out *particles are mostly emitted along the reaction-plane.*
- It provides information on the **EOS of the produced matter** (Hadron Gas vs QGP) through the *speed of sound*:  $\vec{\nabla} P = c_s^2 \vec{\nabla} \epsilon$

# The medium is opaque: jet-quenching

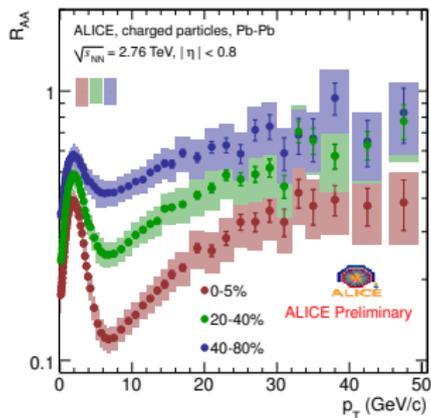


The *nuclear modification factor*

$$R_{AA} \equiv \frac{(dN^h/dp_T)^{AA}}{\langle N_{\text{coll}} \rangle (dN^h/dp_T)^{PP}}$$

quantifies the suppression of high- $p_T$  *hadron spectra*

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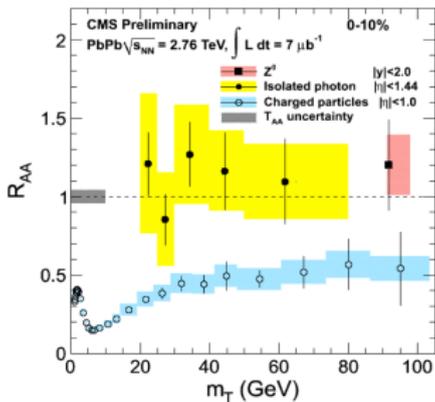


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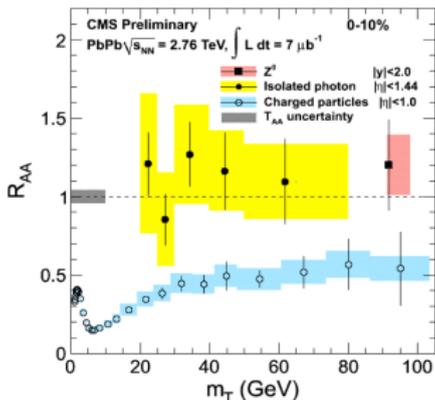


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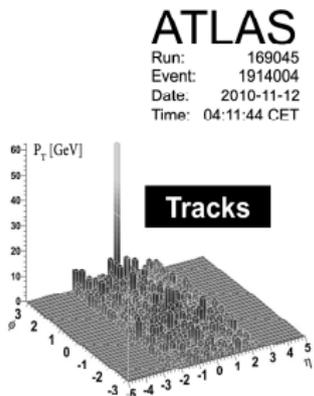
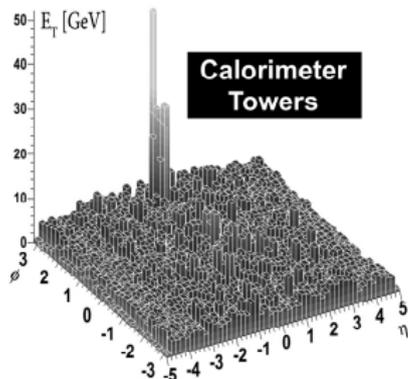
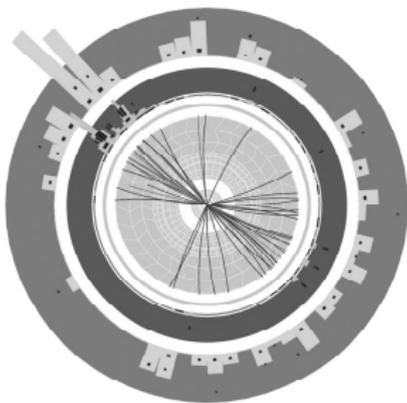
quantifies the suppression of high- $p_T$  *hadron spectra*

Hard-photon  $R_{AA} \approx 1$

- supports the Glauber picture (binary-collision scaling);
- entails that **quenching of inclusive hadron spectra** is a *final state effect due to in-medium energy loss*.

## Di-jet imbalance at LHC: looking at the event display

An important fraction of events display a *huge mismatch* in  $E_T$  between the leading jet and its away-side partner



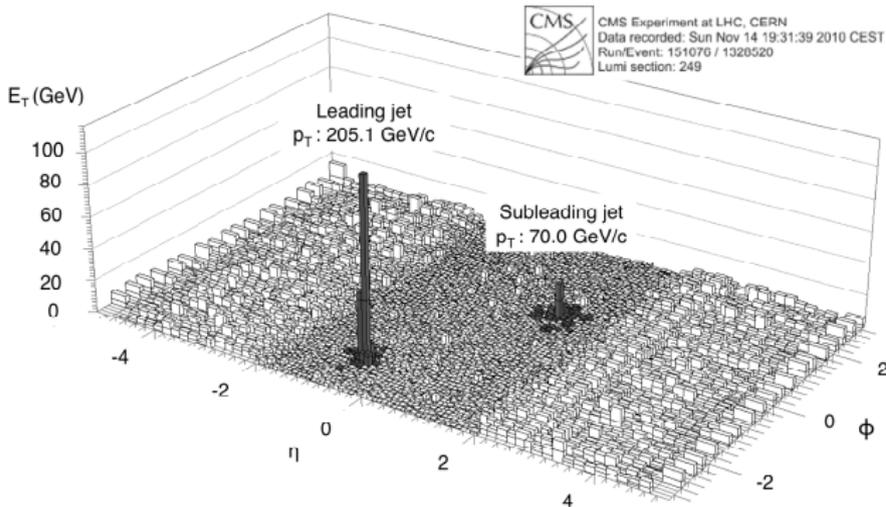
**ATLAS**

Run: 169045  
Event: 1914004  
Date: 2010-11-12  
Time: 04:11:44 CET

Possible to observe event-by-event, without any analysis!

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## Heavy Flavour in the QGP: the conceptual setup

- Description of **soft observables** based on **hydrodynamics**, assuming to deal with **a system close to local thermal equilibrium** (no matter why);
- Description of **jet-quenching** based on **energy-degradation** of **external probes** (high- $p_T$  partons);

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NB At high- $p_T$  the interest in heavy flavor is no longer related to thermalization, but to the study of the **mass** and **color charge dependence** of **jet-quenching** (not addressed in this talk)

## Why are charm and beauty considered *heavy*?

- $M \gg \Lambda_{\text{QCD}}$ : their **initial production** (as shown!) is well described by **pQCD**

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NB for realistic temperatures  $g \sim 2$ , so that one can wonder *whether a charm is really “heavy”*, at least in the initial stage of the evolution.

# Heavy quarks as probes of the QGP

A realistic study requires developing *a multi-step setup*:

- **Initial production**: pQCD + possible nuclear effects (nPDFs,  $k_T$ -broadening)  $\rightarrow$  **QCD event generators**;

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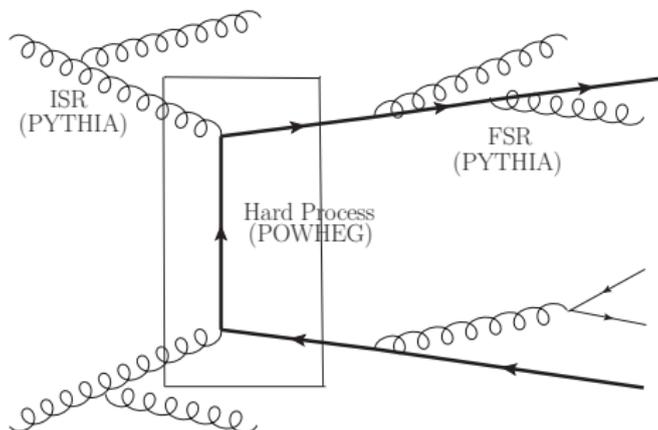
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- **Hadronization**: not well under control (fragmentation in the vacuum? recombination with light thermal partons?)
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  - However, a *source of systematic uncertainty for studies of parton-medium interaction*;
- **Final decays** ( $D \rightarrow X\nu e$ ,  $B \rightarrow XJ/\psi\dots$ )

## HQ production: NLO calculation + Parton Shower



- The strategy to simulate the initial  $Q\bar{Q}$  production is to interface the output of a **NLO event-generator** for the **hard process** with a **parton-shower** describing the **Initial** and **Final State Radiation** and modeling other *non-perturbative processes* (intrinsic  $k_T$ , MPI, **hadronization**)
- This provides a *fully exclusive information on the final state*

# Heavy flavour: experimental observables

- $D$  and  $B$  mesons
- Non-prompt  $J/\psi$ 's ( $B \rightarrow J/\psi X$ )
- Heavy-flavour electrons, from the decays

- of charm ( $e_c$ )

$$D \rightarrow X \nu e$$

- of beauty ( $e_b$ )

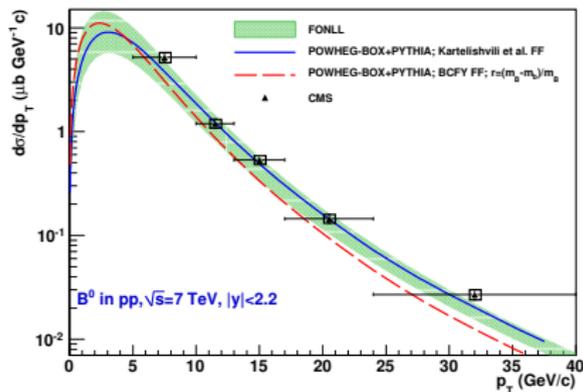
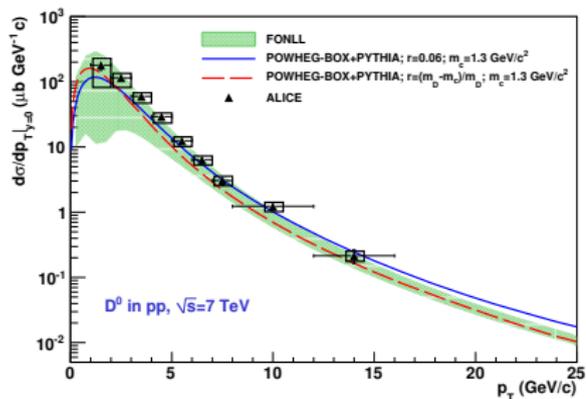
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- B-tagged jets

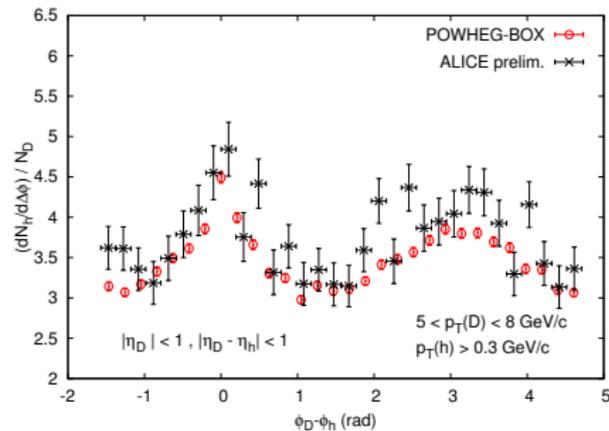
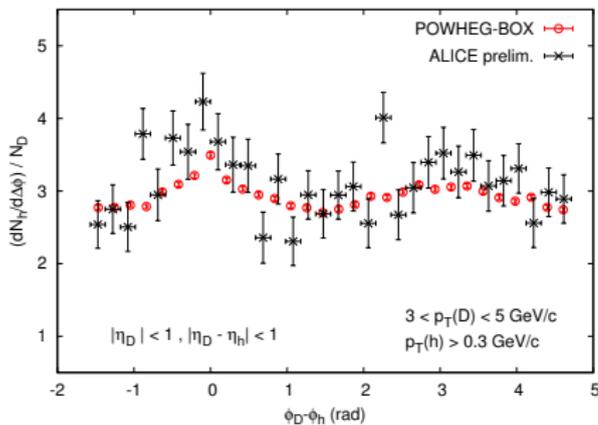
# HF production in $pp$ collisions: results



- Besides reproducing the inclusive **D-meson  $p_T$ -spectra**<sup>2</sup>
- and the heavy-flavour **electrons**

<sup>2</sup>W.M. Alberico *et al*, Eur.Phys.J. C73 (2013) 2481

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- Besides reproducing the inclusive  $D$ -meson  $p_T$ -spectra<sup>2</sup>
- and the heavy-flavour electrons
- ...the POWHEG+PYTHIA setup allows also the comparison with  $D-h$  correlation data, which start getting available.

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## Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution  $f_Q(t, \mathbf{x}, \mathbf{p})^3$ :

$$\frac{d}{dt} f_Q(t, \mathbf{x}, \mathbf{p}) = C[f_Q]$$

- **Total derivative** along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{x}} + \mathbf{F} \frac{\partial}{\partial \mathbf{p}}$$

Neglecting  $\mathbf{x}$ -dependence and mean fields:  $\partial_t f_Q(t, \mathbf{p}) = C[f_Q]$

- **Collision integral**:

$$C[f_Q] = \int d\mathbf{k} \left[ \underbrace{w(\mathbf{p} + \mathbf{k}, \mathbf{k}) f_Q(\mathbf{p} + \mathbf{k})}_{\text{gain term}} - \underbrace{w(\mathbf{p}, \mathbf{k}) f_Q(\mathbf{p})}_{\text{loss term}} \right]$$

$w(\mathbf{p}, \mathbf{k})$ : HQ transition rate  $\mathbf{p} \rightarrow \mathbf{p} - \mathbf{k}$

<sup>3</sup>For results based on BE see e.g. Catania-group papers

## From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*<sup>4</sup> (Landau)

$$C[f_Q] \approx \int d\mathbf{k} \left[ k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p}, \mathbf{k}) f_Q(t, \mathbf{p})]$$

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$$\frac{\partial}{\partial t} f_Q(t, \mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t, \mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t, \mathbf{p})] \right\}$$

where (verify!)

$$A^i(\mathbf{p}) = \int d\mathbf{k} k^i w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{A^i(\mathbf{p}) = A(p) p^i}_{\text{friction}}$$

$$B^{ij}(\mathbf{p}) = \frac{1}{2} \int d\mathbf{k} k^i k^j w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{B^{ij}(\mathbf{p}) = \hat{p}^i \hat{p}^j B_0(p) + (\delta^{ij} - \hat{p}^i \hat{p}^j) B_1(p)}_{\text{momentum broadening}}$$

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Problem reduced to the *evaluation of three transport coefficients*

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## The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark: the **Langevin equation**

$$\frac{\Delta p^i}{\Delta t} = - \underbrace{\eta_D(\mathbf{p}) p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}},$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t) \xi^j(\mathbf{p}_{t'}) \rangle = b^{ij}(\mathbf{p}_t) \frac{\delta_{tt'}}{\Delta t} \quad b^{ij}(\mathbf{p}) \equiv \kappa_{\parallel}(\mathbf{p}) \hat{p}^i \hat{p}^j + \kappa_{\perp}(\mathbf{p}) (\delta^{ij} - \hat{p}^i \hat{p}^j)$$

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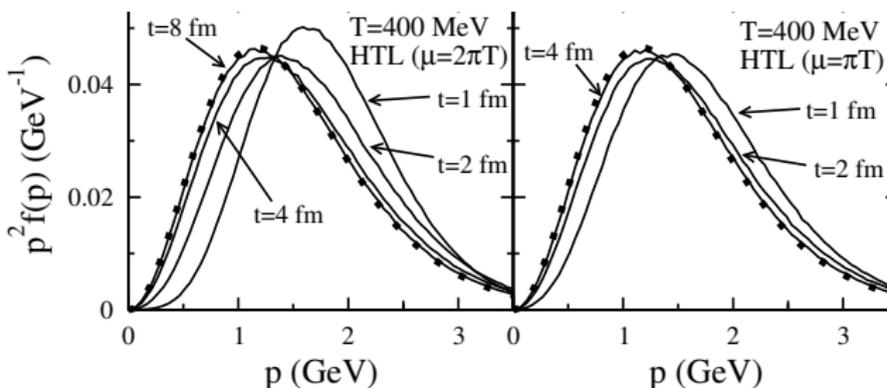
**Transport coefficients** to calculate:

- **Momentum diffusion**  $\kappa_{\perp} \equiv \frac{1}{2} \frac{\langle \Delta p_{\perp}^2 \rangle}{\Delta t}$  and  $\kappa_{\parallel} \equiv \frac{\langle \Delta p_{\parallel}^2 \rangle}{\Delta t}$ ;
- **Friction** term (dependent on the **discretization scheme!**)

$$\eta_D^{\text{Ito}}(p) = \frac{\kappa_{\parallel}(p)}{2TE_p} - \frac{1}{E_p^2} \left[ (1 - v^2) \frac{\partial \kappa_{\parallel}(p)}{\partial v^2} + \frac{d-1}{2} \frac{\kappa_{\parallel}(p) - \kappa_{\perp}(p)}{v^2} \right]$$

fixed in order to assure approach to equilibrium (Einstein relation):

## A first check: thermalization in a static medium



For  $t \gg 1/\eta_D$  one approaches a relativistic Maxwell-Jüttner distribution<sup>5</sup>

$$f_{\text{MJ}}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T K_2(M/T)}, \quad \text{with } \int d^3 p f_{\text{MJ}}(p) = 1$$

(Test with a sample of  $c$  quarks with  $p_0 = 2 \text{ GeV}/c$ )

<sup>5</sup>A.B., A. De Pace, W.M. Alberico and A. Molinari, NPA 831, 59 (2009)

## The realistic case: expanding fireball

Within our **POWLANG** setup (**POWHEG+LANG**evin) the HQ evolution in heavy-ion collisions is simulated as follows

- $Q\bar{Q}$  pairs initially **produced with** the **POWHEG-BOX** package (with nPDFs) and **distributed** in the transverse plane **according to**  $n_{\text{coll}}(\mathbf{x}_{\perp})$  from (optical) Glauber model;

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<sup>6</sup>P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99** (2007) 172301  
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- **update** of the HQ momentum and position **to be done** at each step *in the local fluid rest-frame*
  - $u^{\mu}(x)$  used to perform the boost to the **fluid rest-frame**;
  - $T(x)$  used to set the value of the **transport coefficients**with  $u^{\mu}(x)$  and  $T(x)$  fields taken from the output of **hydro codes**<sup>6</sup>;
- Procedure iterated **until hadronization**

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The Langevin equation provides a link between *what is possible to calculate in QCD* (transport coefficients) and *what one actually measures* (final  $p_T$  spectra)

### Evaluation of transport coefficients:

- **Weak-coupling** hot-QCD calculations<sup>7</sup>
- Non perturbative approaches
  - **Lattice-QCD**
  - AdS/CFT correspondence
  - Resonant scattering

---

<sup>7</sup>Our approach: W.M. Alberico *et al.*, Eur.Phys.J. C71 (2011) 1666

## Transport coefficients: perturbative evaluation

*It's the stage where the various models differ!*

We account for the effect of  $2 \rightarrow 2$  collisions in the medium

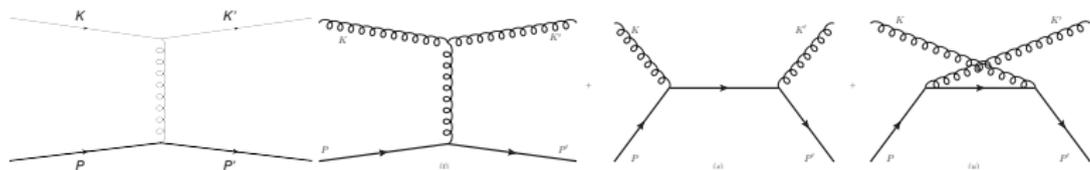
*Intermediate cutoff  $|t|^* \sim m_D^2$ <sup>8</sup> separating the contributions of*

- **hard collisions** ( $|t| > |t|^*$ ): kinetic pQCD calculation
- **soft collisions** ( $|t| < |t|^*$ ): Hard Thermal Loop approximation  
(*resummation of medium effects*)

---

<sup>8</sup>Similar strategy for the evaluation of  $dE/dx$  in S. Peigne and A. Peshier, Phys.Rev.D77:114017 (2008).

## Transport coefficients $\kappa_{T/L}(p)$ : hard contribution

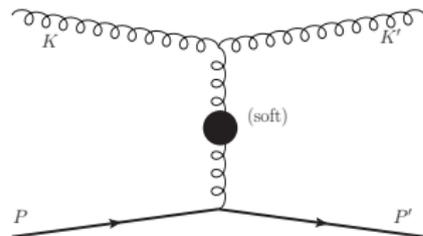
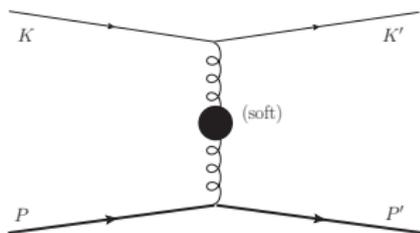


$$\kappa_T^{g/q(\text{hard})} = \frac{1}{2} \frac{1}{2E} \int_k \frac{n_{B/F}(k)}{2k} \int_{k'} \frac{1 \pm n_{B/F}(k')}{2k'} \int_{p'} \frac{1}{2E'} \theta(|t| - |t|^*) \times \\ \times (2\pi)^4 \delta^{(4)}(P + K - P' - K') |\overline{\mathcal{M}}_{g/q}(s, t)|^2 q_T^2$$

$$\kappa_L^{g/q(\text{hard})} = \frac{1}{2E} \int_k \frac{n_{B/F}(k)}{2k} \int_{k'} \frac{1 \pm n_{B/F}(k')}{2k'} \int_{p'} \frac{1}{2E'} \theta(|t| - |t|^*) \times \\ \times (2\pi)^4 \delta^{(4)}(P + K - P' - K') |\overline{\mathcal{M}}_{g/q}(s, t)|^2 q_L^2$$

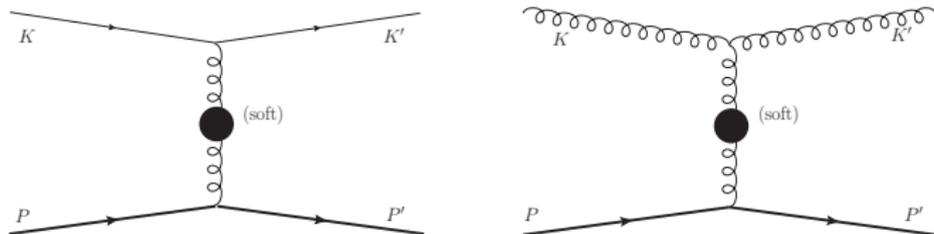
where:  $(|t| \equiv q^2 - \omega^2)$

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When the exchanged 4-momentum is **soft** the **t-channel gluon** feels the **presence of the medium** and **requires resummation**.

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When the exchanged 4-momentum is **soft** the **t-channel gluon** feels the **presence of the medium** and **requires resummation**.

The **blob** represents the **dressed gluon propagator**, which has longitudinal and transverse components:

$$\Delta_L(z, q) = \frac{-1}{q^2 + \Pi_L(z, q)}, \quad \Delta_T(z, q) = \frac{-1}{z^2 - q^2 - \Pi_T(z, q)},$$

where *medium effects* are embedded in the **HTL gluon self-energy**.

## Lattice-QCD transport coefficients: setup

One consider the non-relativistic limit of the Langevin equation:

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t) \xi^j(t') \rangle = \delta^{ij} \delta(t - t') \kappa$$

Hence, in the  $p \rightarrow 0$  limit:

$$\kappa = \frac{1}{3} \int_{-\infty}^{+\infty} dt \langle \xi^i(t) \xi^i(0) \rangle_{\text{HQ}} \approx \frac{1}{3} \int_{-\infty}^{+\infty} dt \underbrace{\langle F^i(t) F^i(0) \rangle_{\text{HQ}}}_{\equiv D^>(t)},$$

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$$\mathbf{F}(t) = g \int d\mathbf{x} Q^\dagger(t, \mathbf{x}) t^a Q(t, \mathbf{x}) \mathbf{E}^a(t, \mathbf{x})$$

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In a thermal ensemble  $\sigma(\omega) \equiv D^>(\omega) - D^<(\omega) = (1 - e^{-\beta\omega}) D^>(\omega)$  and

$$\kappa \equiv \lim_{\omega \rightarrow 0} \frac{D^>(\omega)}{3} = \lim_{\omega \rightarrow 0} \frac{1}{3} \frac{\sigma(\omega)}{1 - e^{-\beta\omega}} \underset{\omega \rightarrow 0}{\sim} \frac{1}{3} \frac{T}{\omega} \sigma(\omega)$$

## Lattice-QCD transport coefficients: results

The **spectral function**  $\sigma(\omega)$  has to be reconstructed starting from the *euclidean electric-field correlator*

$$D_E(\tau) = - \frac{\langle \text{Re Tr}[U(\beta, \tau) g E^i(\tau, \mathbf{0}) U(\tau, 0) g E^i(0, \mathbf{0})] \rangle}{\langle \text{Re Tr}[U(\beta, 0)] \rangle}$$

according to

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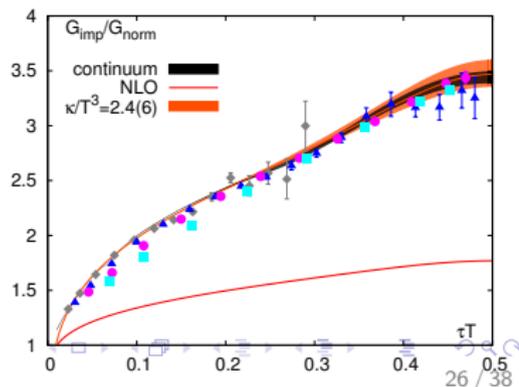
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One gets ([arXiv:1409.3724](https://arxiv.org/abs/1409.3724))

$$\kappa/T^3 \approx 2.4(6) \text{ (quenched QCD, cont.lim.)}$$

$\sim 3$ - $5$  times larger than the perturbative result (W.M. Alberico *et al*, EPJC 73 (2013) 2481).

**Challenge:** approaching the **continuum limit** in **full QCD** (see [Kaczmarek talk](#) at QM14)!



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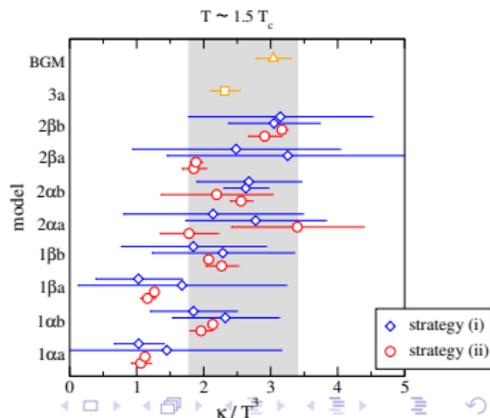
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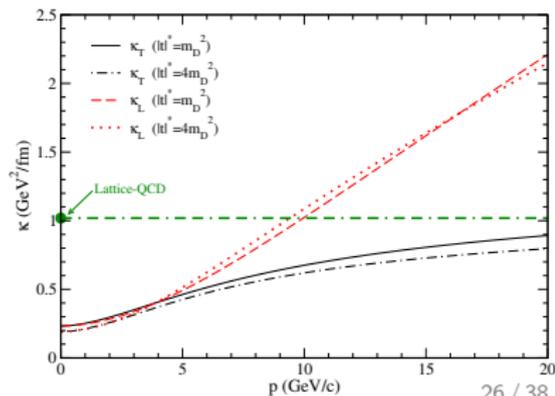
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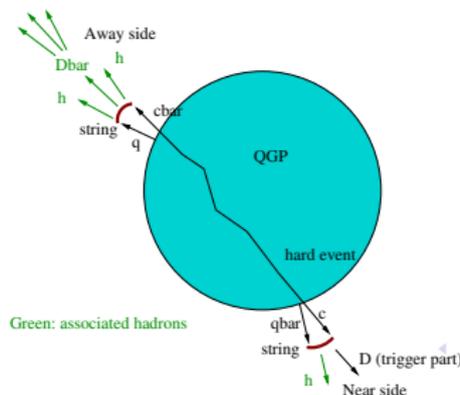


## From quarks to hadrons

In-medium hadronization may affect the  $R_{AA}$  and  $v_2$  of final D-mesons due to the *collective flow of light quarks*. We tried to estimate the effect through this *model* interfaced to our POWLANG transport code:

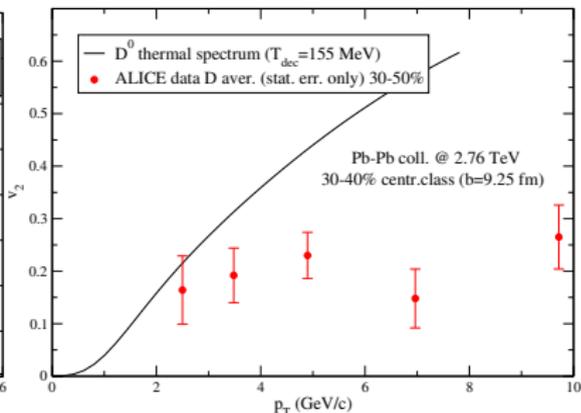
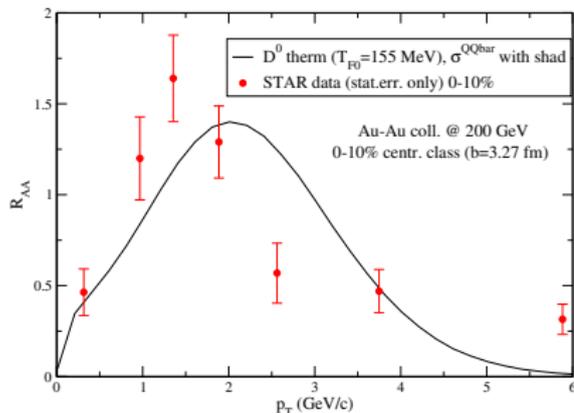
- At  $T_{dec}$  c-quarks coupled to light  $\bar{q}$ 's from a local *thermal distribution*, eventually boosted ( $u_{fluid}^\mu \neq 0$ ) to the lab frame;
- *Strings are formed* and given to PYTHIA 6.4 to simulate their fragmentation and produce the final hadrons ( $D + \pi + \dots$ )

One can address the study of  $D-h$  and  $e-h$  correlations in *AA collisions*



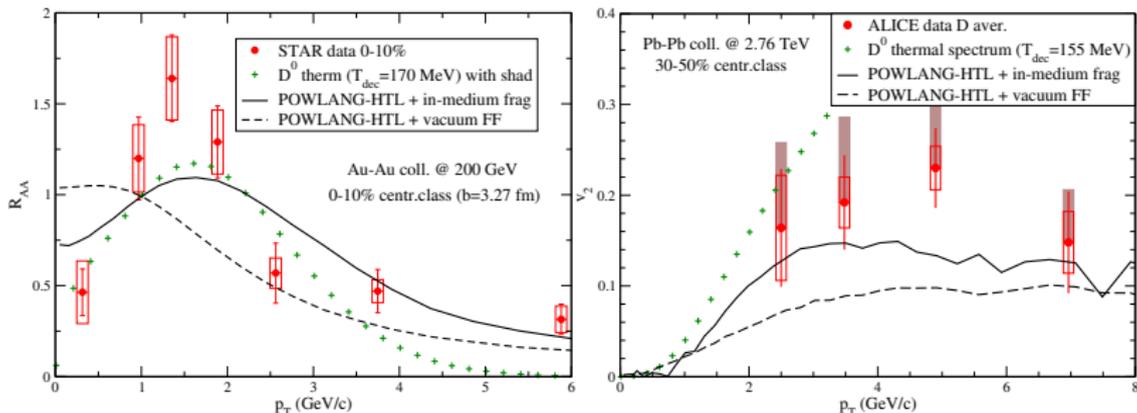
# From quarks to hadrons: effect on $R_{AA}$ and $v_2$

Experimental data display a **peak in the  $R_{AA}$**  and a **sizable  $v_2$**  one would like to interpret as a signal of *charm radial flow and thermalization*



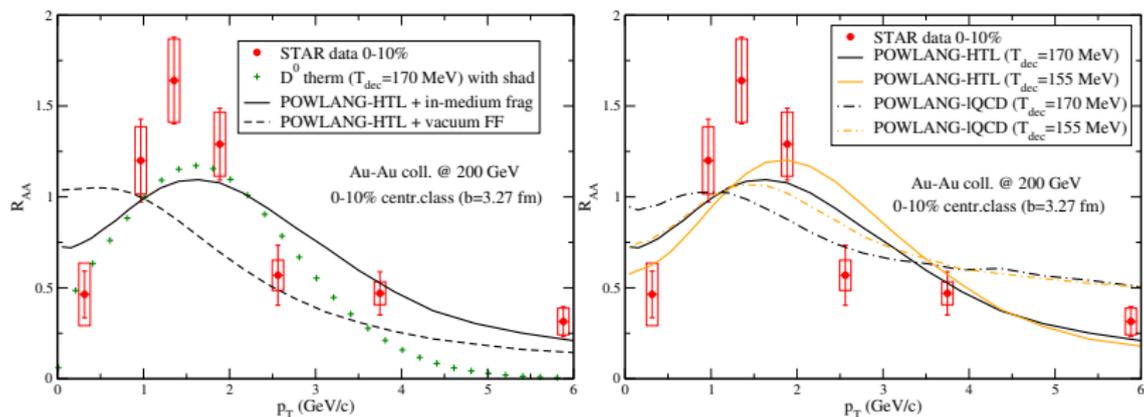
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However, comparing *transport results with/without the boost* due to  $u_{fluid}^\mu$ , at least part of the effect might be due to the **radial and elliptic flow of the light partons** from the medium picked-up at hadronization.

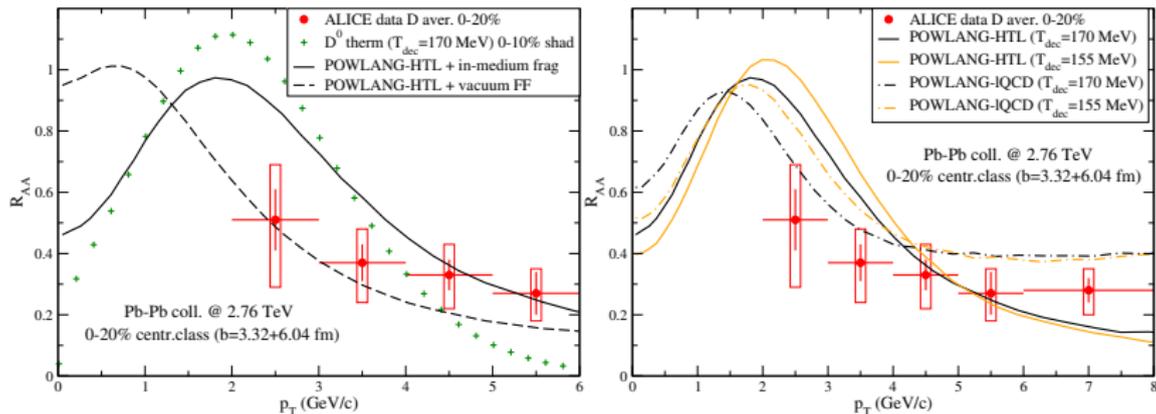
# D-meson $R_{AA}$ at RHIC



It is possible to perform a systematic study of different choices of

- **Hadronization** scheme (left panel)
- **Transport coefficients** (weak-coupling pQCD+HTL vs non-perturbative I-QCD) and **decoupling temperature** (right panel)

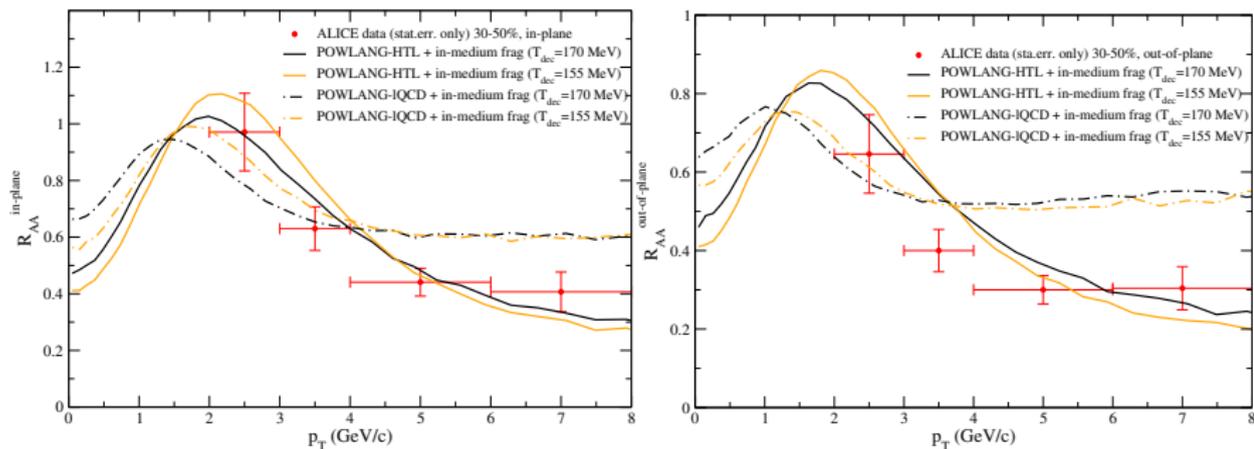
# D-meson $R_{AA}$ at LHC



Experimental data for central (0–20%) Pb-Pb collisions at LHC display a strong quenching, but – at least with the present bins and  $p_T$  range – don't show strong signatures of the bump from radial flow predicted by “thermal” and “transport +  $Q\bar{q}_{\text{therm}}$ -string fragmentation” curves.

## D meson $R_{AA}$ : in-plane vs out-of-plane

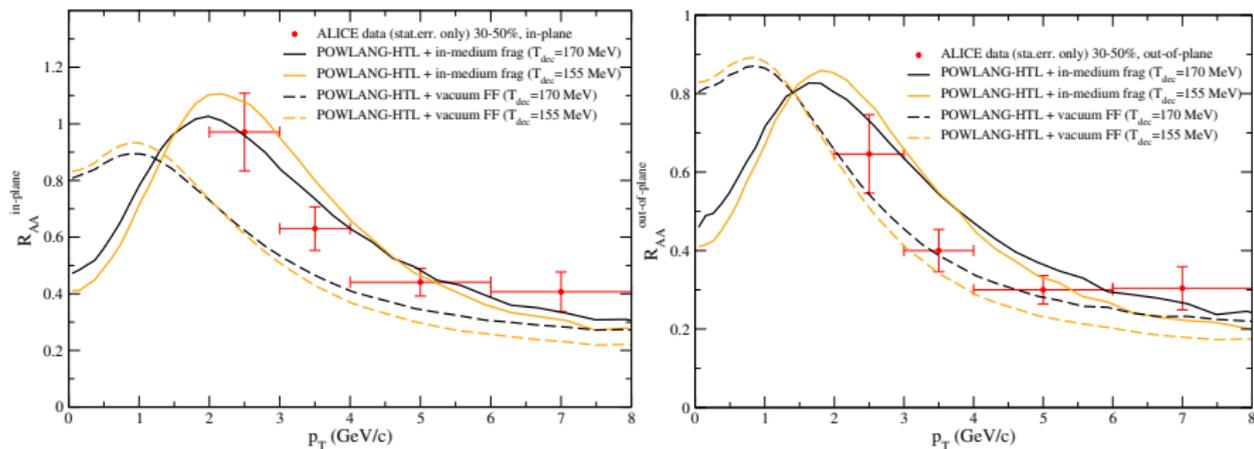
One can study di  $R_{AA}$  in- and out-of-plane in non-central (30–50%) Pb-Pb collisions at LHC:



- Data better described by weak-coupling (pQCD+HTL) transport coefficients;

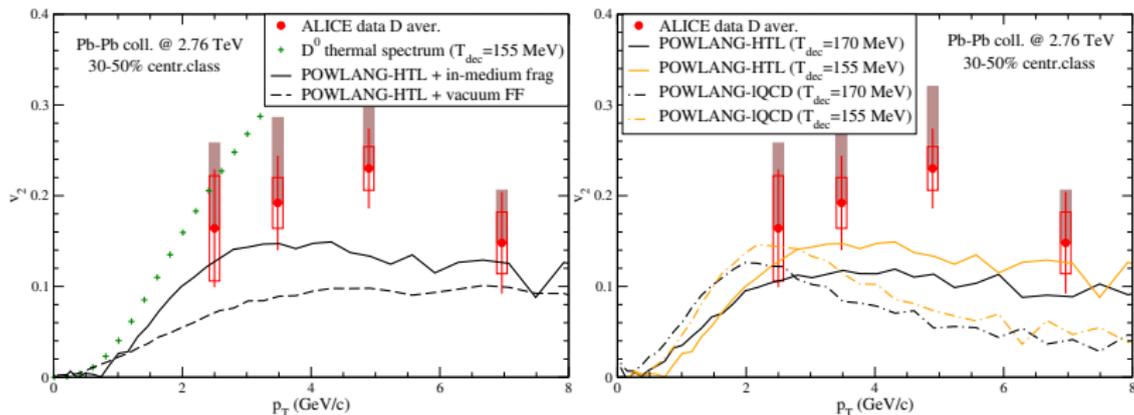
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- Data better described by weak-coupling (pQCD+HTL) transport coefficients;
- $Q\bar{q}_{therm}$ -string fragmentation describes data slightly better than in-vacuum independent Fragmentation Functions.

# D-meson $v_2$ at LHC



Concerning  $D$ -meson  $v_2$  in non-central (30–50%) Pb-Pb collisions:

- $Q\bar{q}_{therm}$ -string fragmentation routine significantly improves our transport model predictions compared to the data;
- HTL curves with a lower decoupling temperature display the best agreement with ALICE data

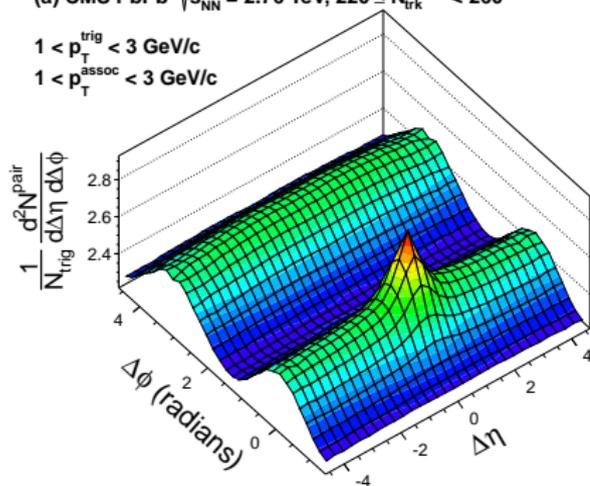
# HF in small systems

(p-Pb and d-Au collisions)

# Hydrodynamic behavior in small systems?

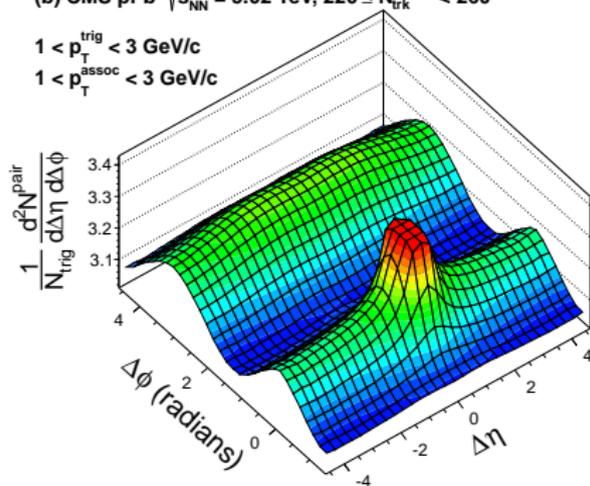
(a) CMS PbPb  $\sqrt{s_{NN}} = 2.76$  TeV,  $220 \leq N_{trk}^{offline} < 260$

$1 < p_T^{trig} < 3$  GeV/c  
 $1 < p_T^{assoc} < 3$  GeV/c



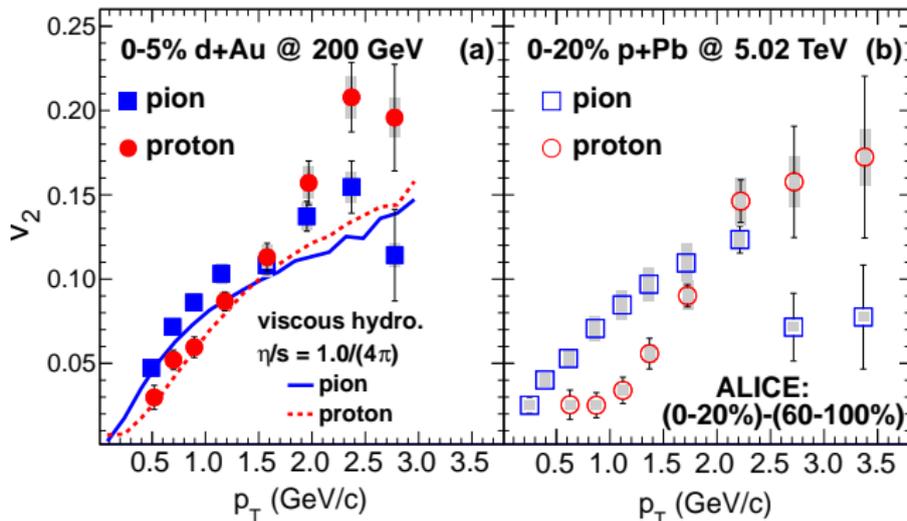
(b) CMS pPb  $\sqrt{s_{NN}} = 5.02$  TeV,  $220 \leq N_{trk}^{offline} < 260$

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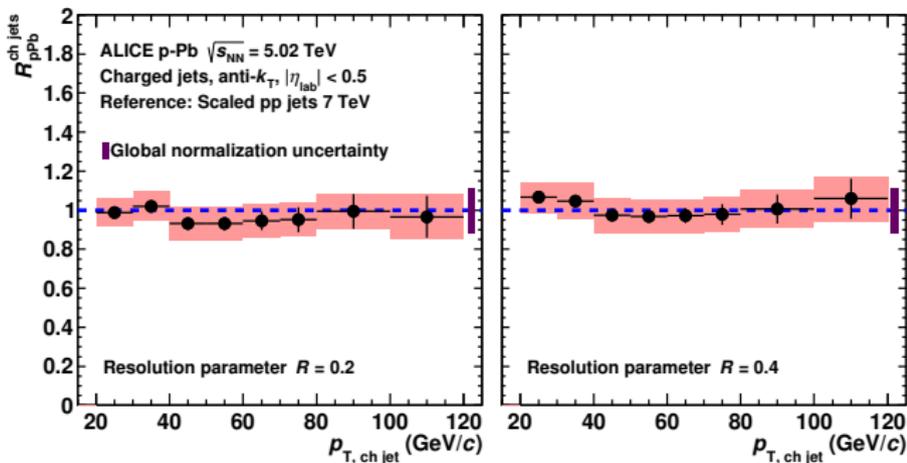
- Long-range rapidity correlations in high-multiplicity p-Pb (and p-p) events: collectiv flow?

## Hydrodynamic behavior in small systems?



- Long-range rapidity correlations in high-multiplicity p-Pb (and p-p) events: collectiv flow?
- Evidence of non-vanishing elliptic flow  $v_2$  (and mass ordering) in d-Au and p-Pb.

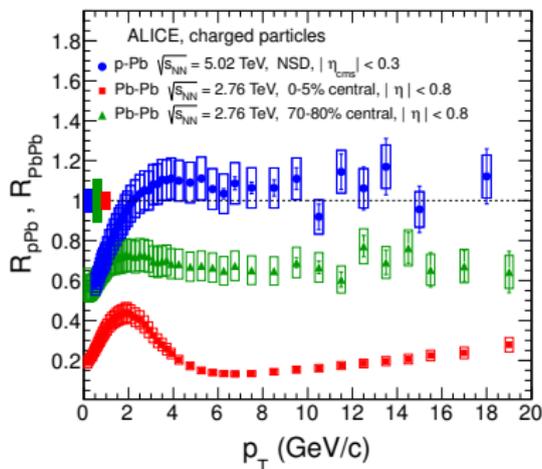
## Hard observables in p-A collisions: no medium effect?



No evidence of medium effects in the nuclear modification factor

- neither of jets

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No evidence of medium effects in the nuclear modification factor

- neither of jets
- nor of charged particles

NB Lack of a p-p reference at the same center-of-mass energy source of systematic uncertainty

## Hard and soft probes: different sensitivity to the medium

The **quenching of a high-energy parton** is described by the pocket formula

$$\langle \Delta E \rangle \sim C_R \alpha_s \hat{q} L^2 \sim T^3 L^2$$

with a strong dependence on the **temperature** and **medium thickness**.

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If one believes that also in p-A collisions **soft physics** is described by hydrodynamics ( $\lambda_{\text{mfp}} \ll L$ ), then starting from an entropy-density profile

$$s(x, y) \sim \exp \left[ -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right]$$

and employing the Euler equation (for  $v \ll 1$ ) and  $Tds = d\epsilon$

$$(\epsilon + P) \frac{d}{dt} \vec{v} = -\vec{\nabla} P \quad \xrightarrow{\vec{\nabla} P = c_s^2 \vec{\nabla} \epsilon} \quad \partial_t \vec{v} = -c_s^2 \vec{\nabla} \ln s$$

whose solution and mean square value over the transverse plane is

$$v^i = c_s^2 \frac{x^i}{\sigma_i^2} t \quad \longrightarrow \quad \overline{v^{x/y}} = c_s^2 \frac{t}{\sigma_{x/y}}$$

The result has a much **milder temperature dependence** ( $c_s^2 \approx 1/3$ ) wrt  $\hat{q}$  and, although the medium has a ( $\approx 3$  times) shorter lifetime, **radial flow develops earlier**, due to the larger pressure gradient

## Medium modeling: event-by-event hydrodynamics

Event-by-event fluctuations (e.g. in the nucleon positions) modeled by Glauber-MC calculation leads to an initial *eccentricity* (responsible for a non-vanishing elliptic flow)

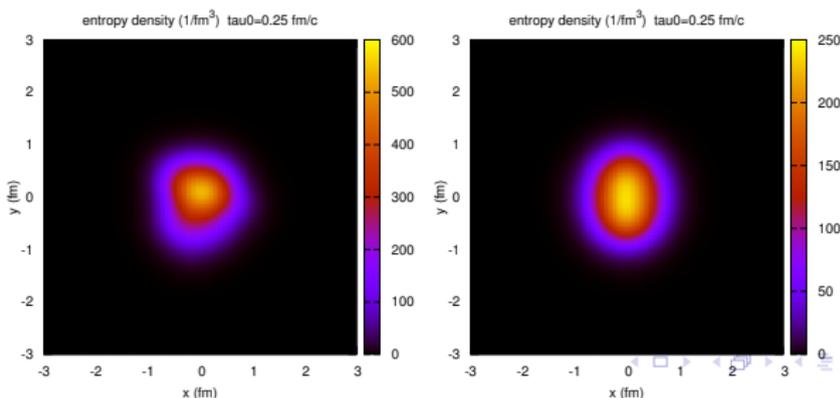
$$s(\mathbf{x}) = \frac{K}{2\pi\sigma^2} \sum_{i=1}^{N_{\text{coll}}} \exp\left[-\frac{(\mathbf{x} - \mathbf{x}_i)^2}{2\sigma^2}\right] \quad \rightarrow \quad \epsilon_2 = \frac{\sqrt{\{y^2 - x^2\}^2 + 4\{xy\}^2}}{\{x^2 + y^2\}}$$

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One can consider an *average background* obtained *summing* all the *events* of a given centrality class *rotated* by the *event-plane angle*  $\psi_2$

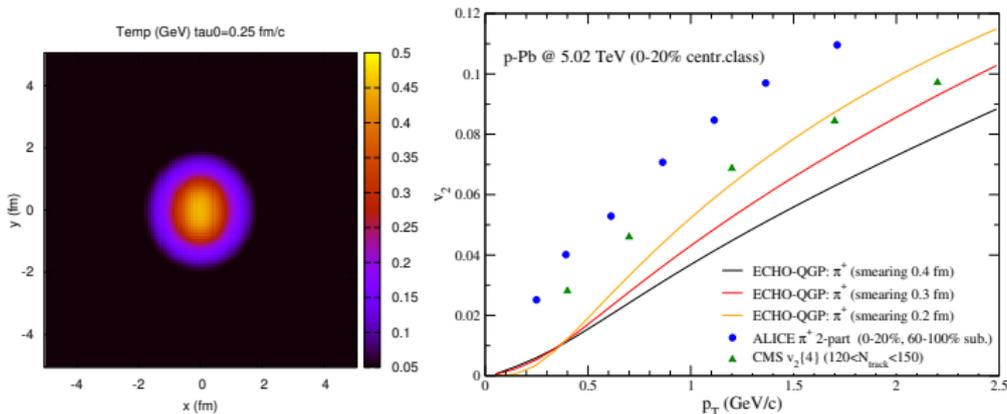


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# Transport-model predictions

