

### UNIVERSITÀ DEGLI STUDI DI CATANIA INFN-LNS



### Anisotropic flows from initial state fluctuations in ultra-relativistic heavy ion collisions

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### Outline

- Transport approach at fixed η/s
- Abelian flux tube model for early stages of HIC
- Initial state fluctuations
  - η/s and generation of v<sub>n</sub>(pT): from RHIC to LHC
  - Correlations between ε<sub>n</sub> (space eccentricities) and ν<sub>n</sub> (collective flows)
- Conclusions

### sketch of evolution of a HIC





Initial out-of-equilibrium state: Glasma, namely, a configuration of longitudinal color–electric and color-magnetic flux tubes.

### $\eta/s(T)$ around to a phase transition

Quantum mechanism

$$\Delta E \cdot \Delta t \ge 1 \rightarrow \eta / s \approx \frac{1}{15} \langle p \rangle \cdot \tau > \frac{1}{15}$$

- AdS/CFT suggest a lower bound  $\eta/s = 1/(4 \pi) \sim 0.08$
- From pQCD:  $\eta/s \sim \frac{1}{g^4 \ln(1/g)} \Rightarrow \eta/s \sim 1$

P.Arnold et al., JHEP 0305 (2003) 051.



S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013). arXiv:1209.0601.

P. Kovtun et al., Phys.Rev.Lett. 94 (2005) 111601.
L. P. Csernai et al., Phys.Rev.Lett. 97 (2006) 152303.
R. A. Lacey et al., Phys.Rev.Lett. 98 (2007) 092301.



- LQCD some results for quenched approx. large error bars
- Quasi-Particle models seem to suggest a  $\eta/s \sim T^{\alpha} \alpha \sim 1 1.5$ .
- Chiral perturbation theory  $\rightarrow$  Meson Gas
- Intermediate Energies IE ( μ<sub>B</sub>>T)

### Information from non-equilibrium: elliptic flow



t (fm/c)

to be zero ... (but event by event fluctuations)

### Information from non-equilibrium: $v_{n}(p_{T})$







 $\epsilon_{n} = \frac{\langle r_{\perp}^{n} \cos[n(\phi - \Phi_{n})] \rangle}{\langle r_{\perp}^{n} \rangle}$ 

The  $v_{2}/\epsilon$  measures efficiency in converting the eccentricity from **Coordinate to Momentum space** 

 $\langle \mathbf{v}_n \rangle = \langle \cos[n(\phi - \Psi_n)] \rangle$ 

**n=2** 





**n=6** 

**n=3** 







Can be seen also as Fourier expansion

$$E\frac{d^{3}N}{dp^{3}} = \frac{d^{2}N}{2\pi p_{T}dp_{T}d\eta} \Big[1 + 2v_{2}\cos 2(\phi - \Psi_{2}) + 2v_{3}\cos 3(\phi - \Psi_{3}) + \dots$$

by symmetry v<sub>n</sub> with n odd expected to be zero ... (but event by event fluctuations)

### Information from non-equilibrium: $v_{n}(p_{T})$



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The  $v_2/\epsilon$  measures efficiency in converting the eccentricity from **Coordinate to Momentum space** 

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n=2 **n=3** 

## **n=4**







0.03

### Can be seen also as Fourier expansion

$$E \frac{d^{3}N}{dp^{3}} = \frac{d^{2}N}{2\pi p_{T}dp_{T}d\eta} \left[ 1 + 2v_{2}\cos 2(\phi - \Psi_{2}) + 2v_{3}\cos 3(\phi - \Psi_{3}) + \dots \right] \mathbf{0}.$$

by symmetry v<sub>n</sub> with n odd expected to be zero ... (but event by event fluctuations)



### Motivation for a kinetic approach:

$$\{p^{\mu}\partial_{\mu} + [p_{\nu}F^{\mu\nu} + M\partial^{\mu}M]\partial_{\mu}^{p}\}f(x,p) = S_{0} + C_{22} + \dots$$
Free Source term: Collisions streaming

- Starting from 1-body distribution function and not from T<sup>μν</sup>:
   possible to include f(x,p) out of equilibrium.
  - M. Ruggieri, F. Scardina, S. Plumari, V. Greco PLB 727 (2013) 177. M. Ruggieri, A. Puglisi, L. Oliva, S. Plumari, F. Scardina, V. Greco PRC 92 (2015) 064904.
  - extract information about the viscous correction δf to f(x,p)
  - S.Plumari,G.L. Guardo, V. Greco, J.Y. Ollitrault NPA (2015) 87
  - It is not a gradient expansion in  $\eta/s$ .
  - Valid at intermediate  $p_{\tau}$  out of equilibrium.
  - Valid at high η/s (cross over region): + self consistent kinetic
     freeze-out

### **Boltzmann Transport Equation**

$$\{p^{\mu}\partial_{\mu}+[p_{\nu}F^{\mu\nu}+M\partial^{\mu}M]\partial^{p}_{\mu}\}f(x,p)=S_{0}+C_{22}+\ldots$$

To solve numerically the Boltzmann-Vlasov eq. we use the test particle method

$$C_{22} = \frac{1}{2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{1}{\nu} \int \frac{d^3p'_1}{(2\pi)^3 2E'_1} \frac{d^3p'_2}{(2\pi)^3 2E'_1} f'_1 f'_2 |\mathbf{M}_{1'2' \to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

For the numerical implementation of the collision integral we use the stochastic algorithm. (Z. Xu and C. Greiner, PRC 71 064901 (2005))

$$\eta(\vec{x},t)/s = \frac{1}{15} \langle p \rangle \tau_{\eta} \longrightarrow \sigma_{tot}^{\eta/s} = \frac{1}{15} \frac{\langle p \rangle}{g(m_D/2T)n} \frac{1}{\eta/s}$$

 $\sigma$  is evaluated in such way to keep fixed the  $\eta$ /s during the dynamics according the Chapman-Enskog equation. (similar to D. Molnar, arXiv:0806.0026[nucl-th])

- We know how to fix locally  $\eta/s(T)$
- We have checked the Chapmann-Enskog (CE):
  - CE good already at  $1^{st}$  order  $\approx 5\%$
  - Relaxation Time Approx. severely understimates η
     S. Plumari et al., PRC86 (2012) 054902.



From fields (Glasma) to particles (QGP): (1+1D evolution)

$$\left[ p^{\mu} \partial_{\mu} + p_{\nu} F^{\mu\nu} \partial_{\mu}^{p} \right] f(x, p) = \frac{dN}{d\Gamma} + C_{22} + \dots$$

We solve self-consistently Boltzmann-Vlasov and Maxwell eq. (abelian approx.)

### **SCHWINGER MECHANISM**

Classical fields decay to particles pairs via tunneling due to vacuum instability

$$\frac{dN_{jc}}{d\Gamma} \equiv p_0 \frac{dN_{jc}}{d^4 x d^2 p_T dp_z} = \mathcal{R}_{jc}(p_T) \delta(p_z) p_0$$
$$\mathcal{R}_{jc}(p_T) = \frac{\mathcal{E}_{jc}}{4\pi^3} \left| \ln \left( 1 \pm e^{-\pi p_T^2 / \mathcal{E}_{jc}} \right) \right|$$
$$\mathcal{E}_{jc} = (g|Q_{jc}E| - \sigma_j) \theta (g|Q_{jc}E| - \sigma_j)$$

#### LONGITUDINAL CHROMO-ELECTRIC FIELDS DECAY IN GLUON PAIRS AND QUARK-ANTIQUARK PAIRS

Casher, Neuberger and Nussinov, PRD 20, 179 (1979) Glendenning and Matsui, PRD 28, 2890 (1983)



#### **ABELIAN FLUX TUBE MODEL**

- negligible chromo-magnetic field
- abelian dynamics for the chromoelectric field
- Iongitudinal initial field
- Schwinger mechanism

### From Glasma to Quark Gluon Plasma: (1+1D evolution)

### 1+1 D expansion

#### <u>For 4πη/s=1:</u>

- Field decays quickly with a power law
- Fast thermalization in about 1 fm/c
- Pressure isotropization in about 1 fm/c

### For large n/s:

- Field decays faster in about 0.5 fm/c and for t > 0.5 fm/c plasma oscillations
- Particle spectra different from a thermal one
- Less efficient isotropization







### **Applying kinetic theory to A+A Collisions....**



$$\{p^{\mu}\partial_{\mu} + M\partial^{\mu} M\partial^{p}_{\mu}\}f(x,p) = C_{22} + \dots$$

- Impact of  $\eta/s(T)$  on the build-up of  $v_n(p_T)$  vs. beam energy
- role of EoS on the  $v_n(p_T)$

- including the Initial state fluctuations



### **Initial State Fluctuations**



### Initial State Fluctuations: time evolution of $\langle v_n \rangle$ and $\varepsilon_n$



- The time evolution for  $\varepsilon_n$  is faster for large n. At very early times  $\varepsilon_n$  (t)= $\varepsilon_n$ (t<sub>0</sub>)- $\alpha_n$  t<sup>n-2</sup>.
- <v<sub>n</sub>> shows an opposite behaviour: <v<sub>n</sub>> develops later for large n. At very early times <v<sub>n</sub>>∝t<sup>n+1</sup>.
- Different v<sub>n</sub> can probes different value of η/s(T) during the expansion of the fireball.

### Initial State Fluctuations: role of the EoS on $\langle v_n \rangle$ and $v_n(p_T)$





- For massless case the system is more efficient in converting the initial anisotropy in coordinate space.
- The effect of the EoS is to reduce the <v,>.
- The elliptic flow show a mass ordering typical of hydro expansion where at low  $p_{T}$  the
  - $v_2(p_T) \propto p_T <\beta_T > m_T$
- Different <v<sub>n</sub>> probes different value of p/ε during the expansion of the fireball.

### Initial State Fluctuations: $v_n(p_T)$ and $\eta/s$



- $v_n(p_T)$  at RHIC is more sensitive to the value of the  $\eta$ /s at low temperature.  $v_a(p_T)$  and  $v_3(p_T)$  are more sensitive to the value of  $\eta$ /s than the  $v_2(p_T)$ .
- At LHC energies v<sub>n</sub>(p<sub>τ</sub>) is more sensitive to the value of η/s in the QGP phase (compare solid and dot-dashed lines).

### Initial State Fluctuations: $v_n(p_T)$ and $\eta/s$



- $v_n(p_T)$  at RHIC is more sensitive to the value of the  $\eta$ /s at low temperature.  $v_4(p_T)$  and  $v_3(p_T)$  are more sensitive to the value of  $\eta$ /s than the  $v_2(p_T)$ .
- At LHC energies v<sub>n</sub>(p<sub>τ</sub>) is more sensitive to the value of η/s in the QGP phase (compare solid and dot-dashed lines).

### Initial State Fluctuations: $v_n(p_T)$ for central collisions





- At low  $p_{\tau} v_n(p_{\tau}) \propto p_{\tau}^n \cdot v_2$  for higher  $p_{\tau}$  saturates while  $v_n$  for n>3 increase linearly with  $p_{\tau}$ .
- For central collisions viscous effect are more relevant. For n>2 the  $v_n(p_T)$  are more sensitive to the  $\eta$ /s ratio in the QGP phase.



### Initial State Fluctuations: ν vs ε



$$C(n,m) = \left\langle \frac{(v_n - \langle v_n \rangle)(\epsilon_m - \langle \epsilon_m \rangle)}{\sigma_{v_n} \sigma_{\epsilon_m}} \right\rangle$$

B.H. Alver, C. Gombeaud, M. Luzum and J.-Y. Ollitrault, Phys.Rev. C82 (2010) 034913.

H. Petersen, G.-Y. Qin, S.A. Bass and B. Muller, Phys.Rev. C82 (2010) 041901.

Z. Qiu and U. W. Heinz, Phys.Rev. C84 (2011) 024911.

H. Niemi, G.S. Denicol, H. Holopainen and P. Huovinen, Phys.Rev. C87 (2013) 5, 054901.

S. Plumari, G. L. Guardo, F. Scardina, V. Greco Phys.Rev. C92 (2015) no.5, 054902.

- At LHC  $v_n$  are more correlated correlated to  $\varepsilon_n$  than at RHIC.
- v<sub>2</sub> and v<sub>3</sub> linearly correlated to the corresponding eccentricities ε<sub>2</sub> and ε<sub>3</sub> rispectively.
- C(4,4) < C(2,2) for all centralities. v<sub>4</sub> and ε<sub>4</sub> weak correlated similar to hydro calculations:

F.G. Gardim, F. Grassi, M. Luzum and J.Y. Ollitrault NPA904 (2013) 503.

H. Niemi, G.S. Denicol, H. Holopainen and P. Huovinen PRC87(2013) 054901.

• For central collisions  $v_n$  are strongly correlated to  $\varepsilon_n$ :  $v_n \propto \varepsilon_n$  for n=2,3,4.

### Initial State Fluctuations: v<sub>n</sub> vs ε<sub>n</sub>



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• For central collisions  $v_n$  are strongly correlated to  $\varepsilon_n$ :  $v_n \propto \varepsilon_n$  for n=2,3,4.



### **Initial State Fluctuations: v**<sub>n</sub> vs ε<sub>n</sub>



S. Plumari, G. L. Guardo, F. Scardina, V. Greco Phys.Rev. C92 (2015) no.5, 054902.

- Equation of State and collision energy play a role on the build up of <v<sub>n</sub>> in central collisions. The effect of the EoS is to reduce the final <v<sub>n</sub>>.
- At RHIC energies the <v<sub>n</sub>> are smaller than those LHC and for more realistic η/s(T) v<sub>2</sub> > v<sub>3</sub> > v<sub>4</sub> ...
- At LHC energies and ultra-central collisions the <v<sub>n</sub>> keep more information about the initial eccentricities <ε<sub>n</sub>>.



### Conclusions

**Transport at fixed \eta/s:** 

- Relativistic transport theory permit to study early dynamics of HIC
  - Initial color-electric field decays in about 1 fm/c
  - Thermalization and Isotropization in about 1 fm/c
- Enhancement of n/s(T) in the cross-over region affect differently the expanding QGP from RHIC to LHC. LHC nearly all the v<sub>n</sub> from the QGP phase.
- At LHC stronger correlation between v<sub>n</sub> and ε<sub>n</sub> than at RHIC for all n. <u>Ultra central collisions:</u>
  - $v_n \propto \varepsilon_n$  for n=2,3,4 strong correlation C(n,n)~1
  - $v_n(p_T)$  much more sensitive to  $\eta/s(T)$
  - degree of correlation increase with the collision energy and the relative strenght of  $\langle v_n \rangle$  depend on the colliding energies.
  - correlations in  $(v_n, v_m)$  reflect the initial correlations in  $(\varepsilon_n, \varepsilon_m)$



From Glasma to Quark Gluon Plasma: (1+1D evolution)

$$\left[\left[p^{\mu}\partial_{\mu}+p_{\nu}F^{\mu\nu}\partial_{\mu}^{p}\right]f(x,p)=\frac{dN}{d\Gamma}+C_{22}+\ldots\right]$$

We solve self-consistently Boltzmann-Vlasov and Maxwell eq. (abelian approx.) Vacuum with electric field

Field interaction + source term Link between parton distribution function and classical color fields



Conductive current due to charge movement

Polarization current Dipole moment formed in vacuum by Schwinger effect



$$\frac{dN_{jc}}{d\Gamma} \equiv p_0 \frac{dN_{jc}}{d^4 x d^2 p_T dp_z} = \mathcal{R}_{jc}(p_T) \delta(p_z) p_0$$
$$\mathcal{R}_{jc}(p_T) = \frac{\mathcal{E}_{jc}}{4\pi^3} \left| \ln \left( 1 \pm e^{-\pi p_T^2 / \mathcal{E}_{jc}} \right) \right|$$
$$\mathcal{E}_{jc} = (g|Q_{jc}E| - \sigma_j) \,\theta \left( g|Q_{jc}E| - \sigma_j \right)$$

### From Glasma to Quark Gluon Plasma: (3+1D evolution)



# From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$ . (work in collaboration with J.Y. Ollitrault)

$$f(x,p)=f^{(0)}(x,p)+\delta f(x,p)$$

$$T^{\mu\nu} = T^{(0)\mu\nu} + \delta T^{\mu\nu} \leftarrow f^{(0)} + \delta f$$

A common choice for  $\delta f$  – the Grad ansatz  $\delta f \propto \Gamma_s f^{(0)} p^{\alpha} p^{\beta} \langle \nabla_{\alpha} u_{\beta} \rangle \propto p_T^2$ 



BUT it doesn't care about the microscopic dynamics

In general in the limit  $\sigma \rightarrow \infty$ , f( $\sigma$ ) can be expanded in power of 1/ $\sigma$ .

$$f(\sigma)_{\sigma \to \infty} f^{(0)} + \frac{1}{\sigma} \delta f + O\left(\frac{1}{\sigma^2}\right) \longrightarrow v_n(p_T) \underset{\sigma \to \infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma} \delta v_n + O\left(\frac{1}{\sigma^2}\right)$$

PURPOSE: evaluate the ideal hydrodynamics limit  $f^{(0)}$ ,  $v_n^{(0)}$  and the viscous corrections  $\delta f$  and  $\delta v_n$  solving the Relativistic Boltzmann eq for large values of the cross section  $\sigma$ 



# From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$ . (work in collaboration with J.Y. Ollitrault)



### For $\sigma \rightarrow \infty$ we find the ideal Hydro limit:

- f<sup>(0)</sup> is an exponential decreasing function.
- f<sup>(0)</sup> doesn't depends on microscopical details (i.e. mD).
- Universal behavior of  $v_n^{(0)}(p_T)$
- $v_n^{(0)}(p_T)/\epsilon_n$  is approximatively the same for all n and  $p_T$ .



### From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$ . (work in collaboration with J.Y. Ollitrault)

#### S.Plumari,G.L. Guardo, V. Greco, J.Y. Ollitrault NPA2015



In  $\delta f$  and  $\delta v_n$  it is encoded the information about the microscopical details

- $\delta f(p_T)/f^{(0)} \propto p_T^{\alpha}$  with  $\alpha = 1. 2.$  and  $\alpha(m_D)$ . For isotropic  $\sigma$  similar to R.S. Bhalerao et al. PRC 89, 054903 (2014)
- Larger is n larger is the viscous correction to  $v_n(p_T)$
- Scaling: for  $p_T > 1.5 \text{ GeV} \rightarrow -\delta v_n(p_T)/v_n^{(0)} \propto n$

### Initial State Fluctuations: $v_n(p_T)$ and $\eta/s$



- The initial state fluctuations reduce the  $v_2(p_T)$ .
- $v_4(p_T)$  increase by the initial state fluctuations and it becomes more sensitive to the viscosity of the QGP. Larger  $\varepsilon_4$  gives larger  $v_4$ .