Applications of nuclear physics to a wider context: from molecules to stars passing through hypernuclei

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CONTENTS:

- Electron screening puzzle (with **C.Spitaleri, C.Bertulani, A.Vitturi**)
- Diatomic molecules in fullerenes (with **F. Pérez-Bernal**)
- Spectrum of lambda hypernuclei (with **K.Hagino**)

AIMS:

- Discussing a few topics of theoretical nuclear physics that are under investigation in Padova
- Showing how theoretical nuclear theory fosters new interdisciplinary physics

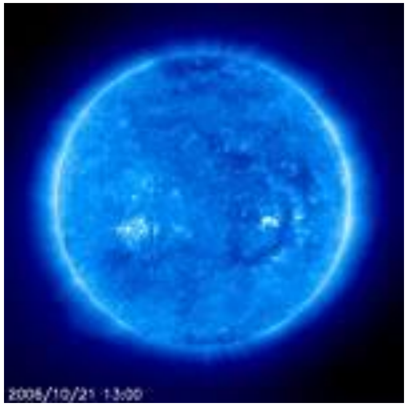
Università degli Studi di Padova **IN:Theory** project 2016-18

Dipartimento di Fisica e Astronomia Galileo Galilei

INFN Sez. Padova STRENGTH

The solution of the electron screening puzzle

C.Spitaleri, C.A.Bertulani, L.F. and A.Vitturi, Phys. Lett. B 755 (2016) 275–278



Nuclear reactions in stars happen at energies around the Gamow peak E_G << $E_{C.B.}$ at the presence of an electron plasma that create a screening effect that enhances the reaction cross-sections at these ultra-low energies.

$$\sigma_b(E) = \frac{S(E)}{E} \exp\left[2\pi\eta(E)\right]$$

Target materials in laboratory also have electrons bound in atoms, molecules or crystals. The laboratory cross-sections and those in stellar environment usually differ and bare values should be extracted.

$$f_{lab}(E) = \frac{\sigma_s(E)}{\sigma_b(E)} = \frac{S_s(E)}{S_b(E)} \sim \exp\left[\pi\eta \frac{{U_e}^{(lab)}}{E}\right]$$

where U_e is the electron screening potential in laboratory experiments.

The solution of the electron screening puzzle

Very often the direct measurements of reactions involving light nuclei show that the enhancement is significantly larger than expected from models of atomic physics.

	Reaction	U_e^{adlim} (eV)	U_e^{exp} (eV)
[1]	^{2}H$(d,t)^1$H	14	19.1±3.4
[2]	^{3}He$(d,p)^4$He	65	109±9
[3]	^{3}He$(d,p)^4$He	120	219±7
[4]	^{3}He$(^3$He,2p$)^4$He	240	305±90
[5]	^{6}Li$(d,\alpha)^4$He	175	330±120
[6]	^{6}Li$(d,\alpha)^4$He	175	330±49
[7]	^{6}Li$(p,\alpha)^3$He	175	440±150
[8]	^{6}Li$(p,\alpha)^3$He	175	355±67
[9]	^{7}Li$(p,\alpha)^4$He	175	300±160
[10]	^{7}Li$(p,\alpha)^4$He	175	363±52
[11]	^{9}Be$(p,\alpha_0)^6$Li	240	788±70
[12]	^{10}B$(p,\alpha_0)^7$	340	376±75
[13]	^{11}B$(p,\alpha_0)^8$Be	340	447±67

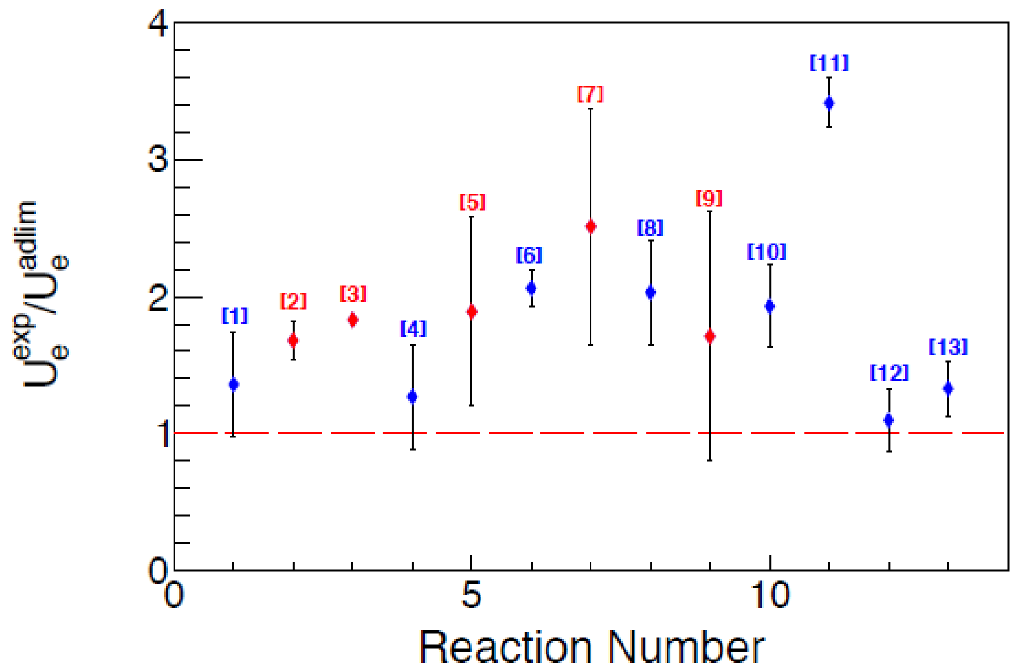
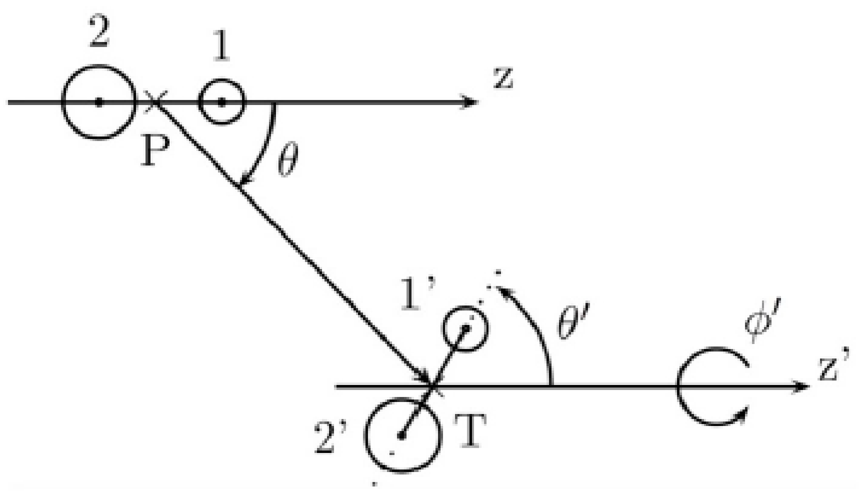


FIG. 2. Ratio of the experimental electron screening potential U_e^{exp} and the theoretical adiabatic limit of the electron screening potential U_e^{adlim} as function of the main reaction present in the literature. The vertical bars are the total uncertainties of the measurements reported in literature. The numbers in brackets correspond to those in Table I.

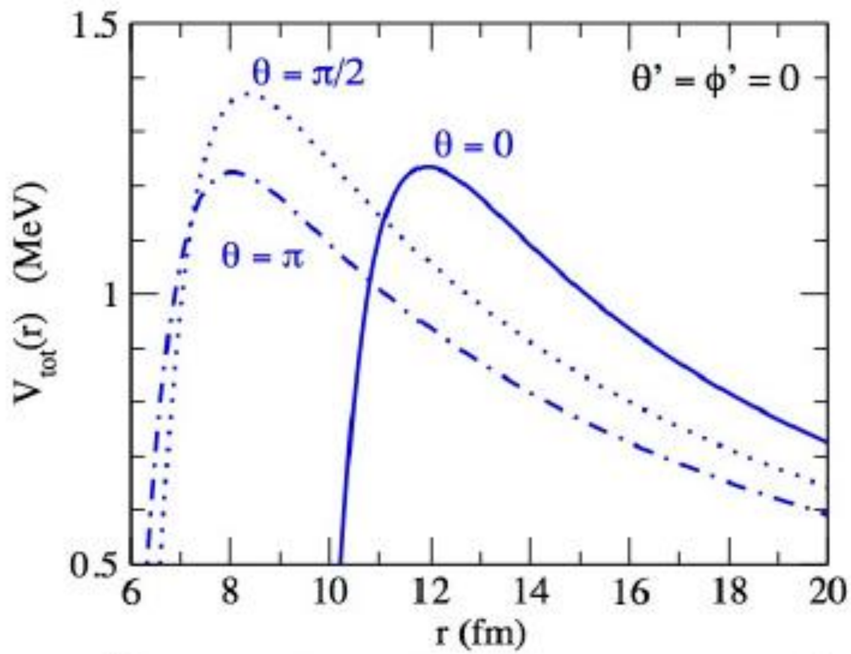
The solution of the electron screening puzzle

The disagrement is more pronounced when one (or two) reacting nuclei is the region of Z=3,4,5, that is the region where cluster effects are known to be very important. Therefore is seems that there is a correlation between cluster and fusion enhancement.

We have modeled cluster nuclei and calculated the Gamow factor, G, and penetrability of Coulomb barriers, P, discovering that is the lowering of the barrier due to clusterization that explains the anomalous enhancement !!

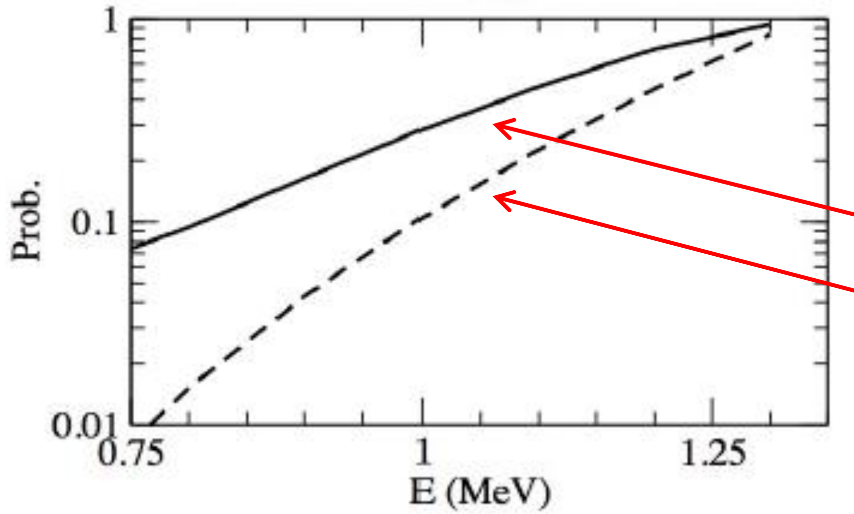


The solution of the electron screening puzzle



$$V_{tot}(r,\theta,\theta',\phi') = \sum_{i,j=1}^{2}\left(\frac{Z_i Z_j e^2}{r_{ij}} + V_N(r_{ij})\right)$$

$$G(E,\theta,\theta',\phi') = \frac{\sqrt{2m}}{\hbar}\int_a^b \sqrt{V_{tot}(r,\theta,\theta',\phi') - E}\; dr$$



$$P = e^{-2G}$$

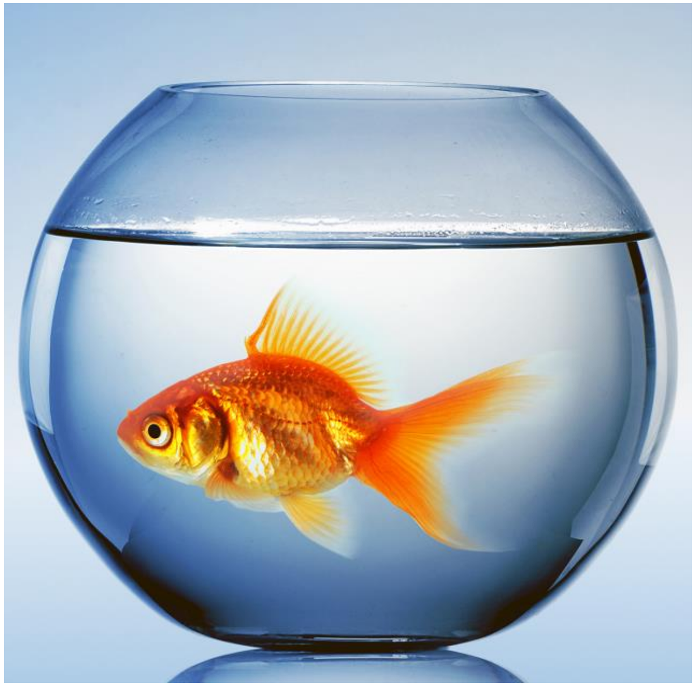
It is clear from this simple analysis that the probability is very much enhanced in the dicluster-dicluster fusion model with respect to the sphere-on-sphere model.

The solution of the electron screening puzzle

Caged molecules and hypernuclei

Lorenzo FORTUNATO Univ. Padova - ITA

In collaboration with F. Pérez-Bernal (Huelva, SPA)



In collaboration with K.Hagino (Sendai, JPN)

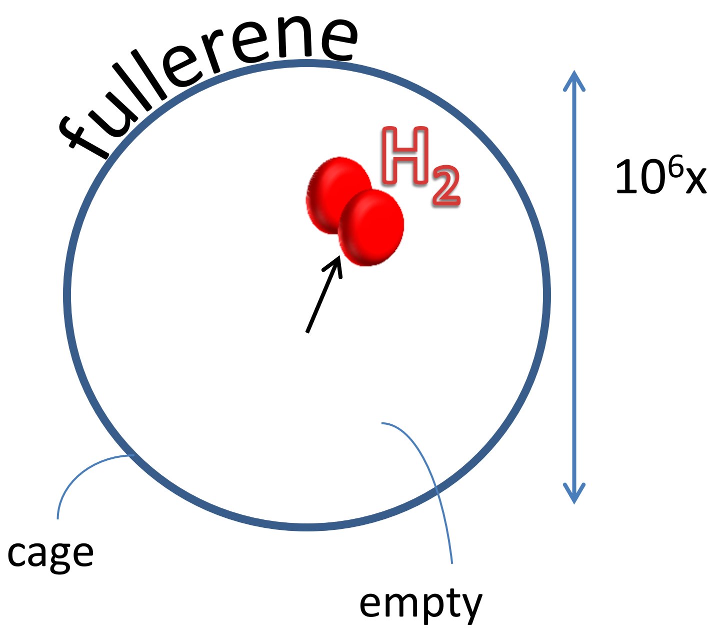
Algebraic schemes allow to unify very diverse (quantum) physical systems that share the same dynamical symmetries, or part of them, across very different length and energy scales!

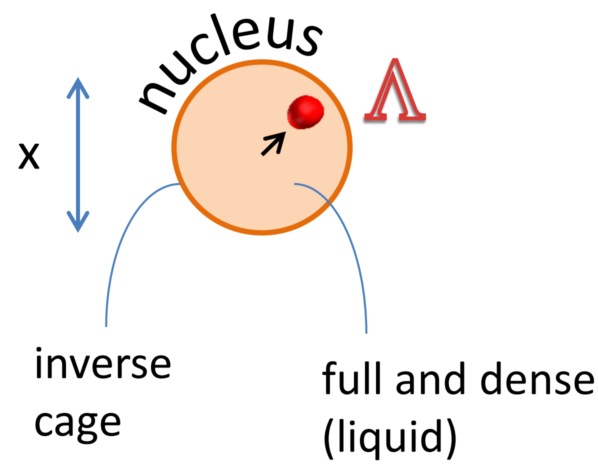
L. Fortunato

Analogies & differencies

$H_2@C_{60}$ supramolecular complex: diatomic hydrogen molecule in a fullerene cage

$^{89}Y_\Lambda$ Hypernucleus or $\Lambda@88Y$: a Lambda particle inside a heavy nucleus





Bird in a ballon:
rotates, vibrates, translates,
but never touches the walls

Fish in a water-planet:
rotates, vibrates, translates,
but cannot jump far from water

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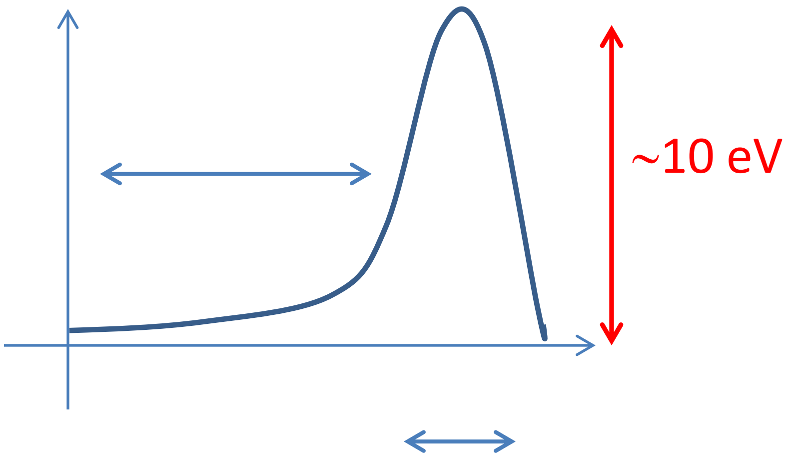
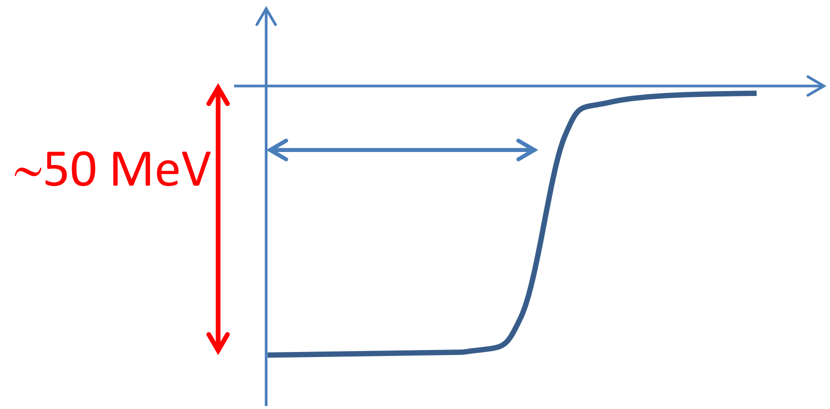
Analogies & differencies

$H_2@C_{60}$ supramolecular complex: diatomic hydrogen molecule in a fullerene cage

$^{89}Y_{\Lambda}$ Hypernucleus or Λ@88Y : a Lambda particle inside a heavy nucleus





Barrier
too thick!
G ~ 0

Decay: $\tau_{\Lambda} = 2.3\ 10^{-10}$ s

$\Rightarrow$ *Stable*

$\Rightarrow$ *Unstable*

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Analogies & differencies

$H_2@C_{60}$ supramolecular complex: diatomic hydrogen molecule in a fullerene cage

$^{89}Y_\Lambda$ Hypernucleus or $\Lambda@88Y$: a Lambda particle inside a heavy nucleus

$$U(7) \supset U(3) \oplus U(4)$$

$$U(3/2) \supset U_B(3) \oplus U_F(2)$$

Vibron model for diatomic molecule internal d.o.f.

Spin ½ fermion

Harmonic motion of a caged particle

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Molecular Surgery

- Komatsu, Murata & Murata, SCIENCE 307 (2005) 238

Spin Chemistry and Magnetic Resonance of $H_2@C_{60}$ *Turro et al.*

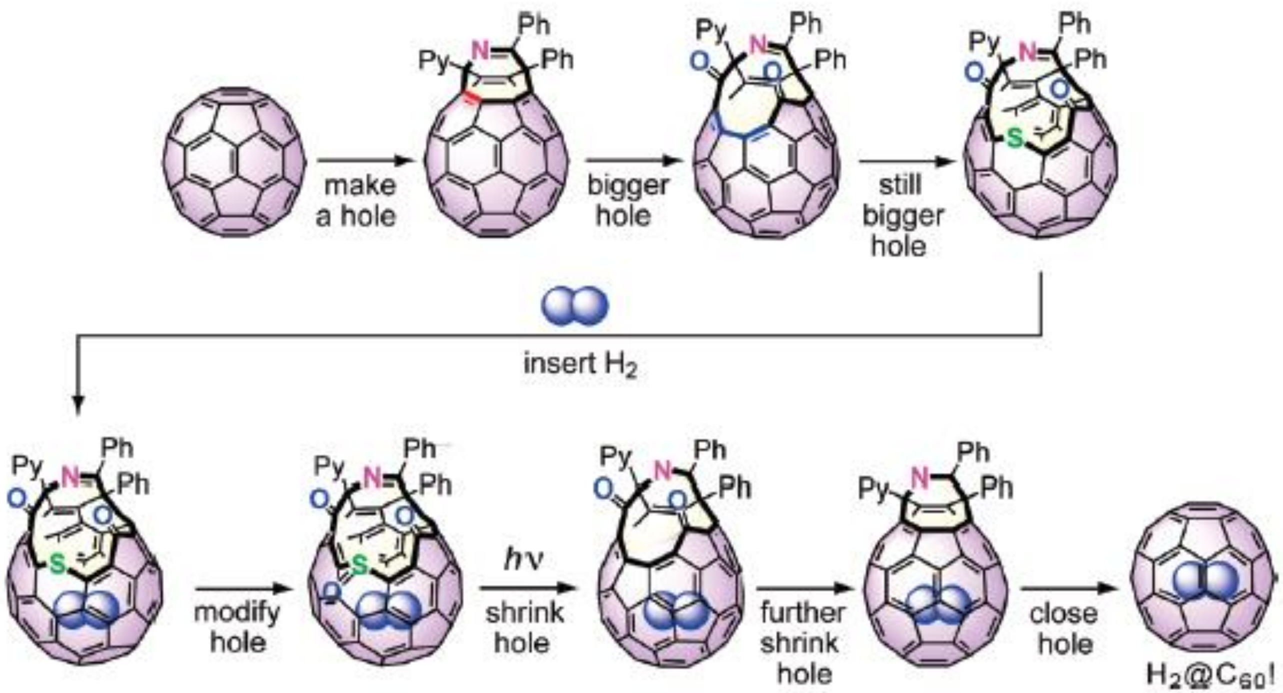

FIGURE 3. An example of molecular surgery: the synthesis of $H_2@C_{60}$. Starting with C_{60}, the surgery starts with the opening of the buckyball followed by creation of larger holes until the hole is large enough for the insertion of H_2 at high temperature and pressure. Upon return to room temperature and atmospheric pressure, the H_2 is incarcerated and stable inside the fullerene. The surgery is completed by closing the hole and regenerating C_{60}.

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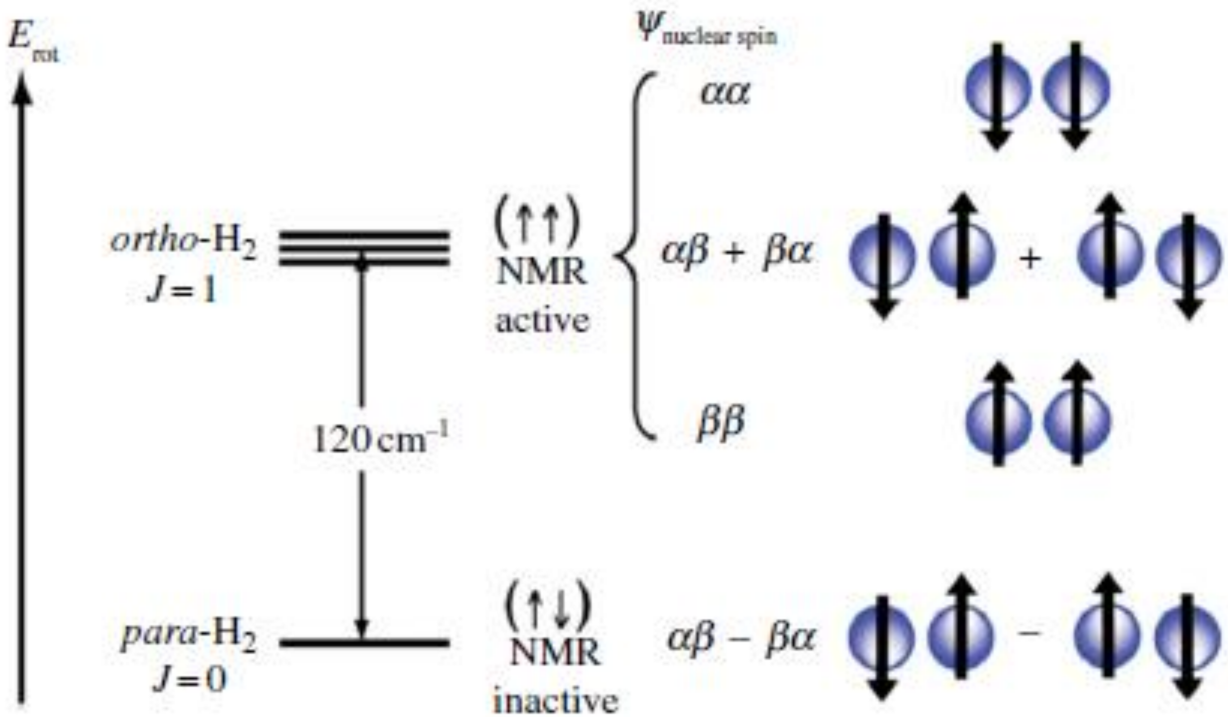
Spectrum of H_2 in a fullerene

- Komatsu, Murata & Murata, SCIENCE 307 (2005) 238
- Turro, Ge, Xu, Mamone, Bacic, Horsewill, Lewitt, et al.

(JCP, PRB, J.Am.Chem. Soc., Coord.Chem.Rev., Phil.Trans.R.Soc. A)



Experimental techniques:

1. NMR
2. IR
3. INS

Spectrum of H_2 in a fullerene

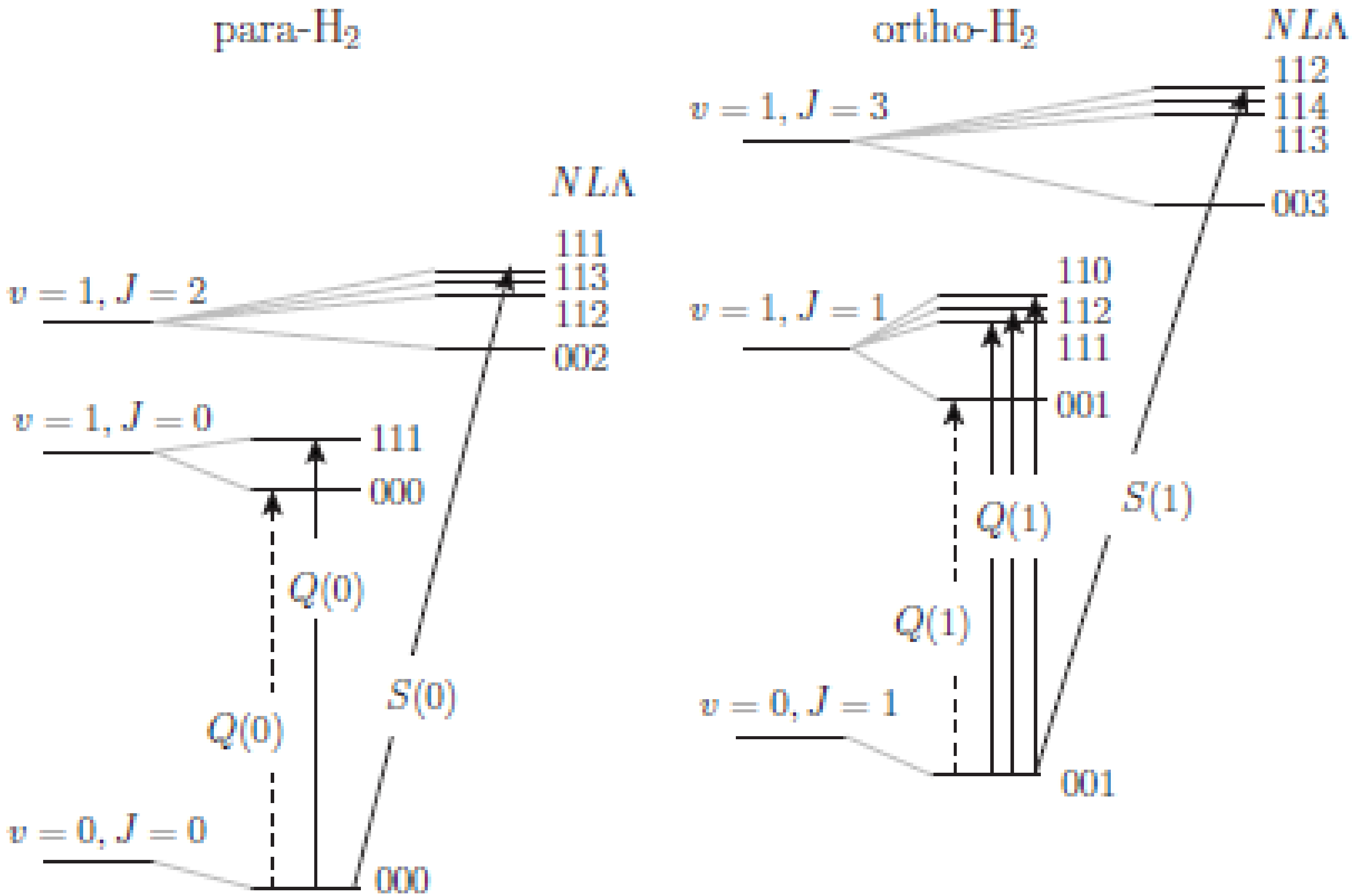


FIG. 2. Diagram of H_2 energy levels refined against low-T IR data.

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Spectrum of $H_2@C_{60}$ in the u(7) algebraic scheme

$$\begin{array}{ccccccccccccc} u(7) & \supset & u(3) & \oplus & u(4) & \supset & u(3) & \oplus & so(4) & \supset & so(3) & \oplus & so(3) & \supset & so(3) \\ N & & N & & N_b & & & & \omega & & L & & J & & \Lambda \end{array}$$

$$|\,[N]NL;N_b vJ;\Lambda\rangle$$

The total hamiltonian can be written as

$$\hat{H} = \hat{H}_{u(3)} + \hat{H}_{u(4)} + \hat{H}_{Coupl}$$

This is Iachello's Vibron model for the internal degrees of freedom of the H2 molecule + Dunham's corrections.

$$\hat{H}_{u(4)} = \hat{H}_{II} + \hat{H}_{Dun}$$

$$\hat{H}_{Dun} \to E_{Dun} = \eta\left[J(J+1)\right]^2 + \kappa\left[\omega(\omega+2)J(J+1)\right]$$

$$H_{II} = E_0'' + \beta\hat{C}_2(so(4)) + \gamma\hat{C}_2(so_J(3))$$

with eigenvalues

$$E_{u(4)} = E_0'' + \beta\omega(\omega+2) + \gamma J(J+1)$$

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Spectrum of $H_2@C_{60}$ in the u(7) algebraic scheme

$$\hat{H}_{u(3)} = a\hat{C}_1(u(3)) + b\hat{C}_2(u(3)) + c\hat{C}_2(so_L(3))$$

$$E_{u(3)} = aN + bN(N+2) + cL(L+1)$$

The u(3) part is just a harmonic oscillator (with anharmonicities collected in the quadratic term) plus a rotational splitting

F. Pérez-Bernal , a.k.a. Curro, has made extensive fits for about 55 IR lines and 16 INS transitions using the above hamiltonian, obtaining very good results

$\hat{H}_{u_p(4)}$	β	γ	κ	γ_2
F_0	$-1083.23(18)$	$58.09(17)$	$0.88(4)$	–
F_1	$-1081.72(15)$	$58.28(20)$	$0.810(25)$	$-0.032(15)$
$\hat{H}_{u_q(3)}$	a	b	c	
F_0	$178.3(8)$	$9.6(3)$	$-3.26(15)$	
F_1	$179.0(4)$	$8.46(17)$	$-3.18(8)$	
$\hat{H}_{Coupl}$	ϑ_{pq}	ϑ_{pqw}	v_{pq}	
F_0	$0.94(7)$	–	–	
F_1	$0.86(5)$	$-0.028(14)$	$-1.02(8)$	
rms	F_0	3.1	F_1	1.7

TABLE II. F_0 (Minimal) and F_1 (Finer) fits parameter values. In both cases $N_p = 34$. Hamiltonian parameters and rms are expressed in cm^{-1} units.

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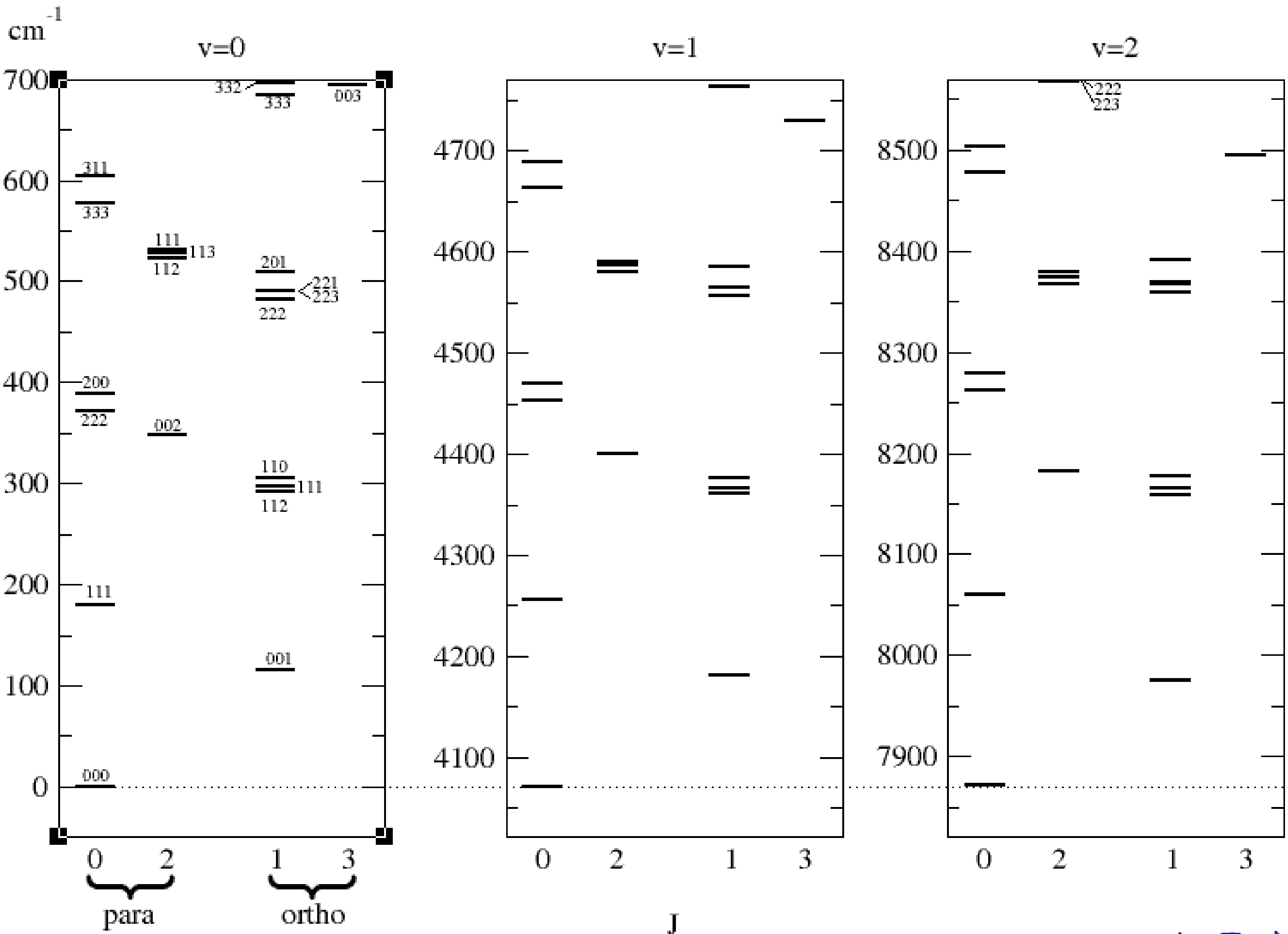
Spectrum of $H_2@C_{60}$ in the u(7) algebraic scheme



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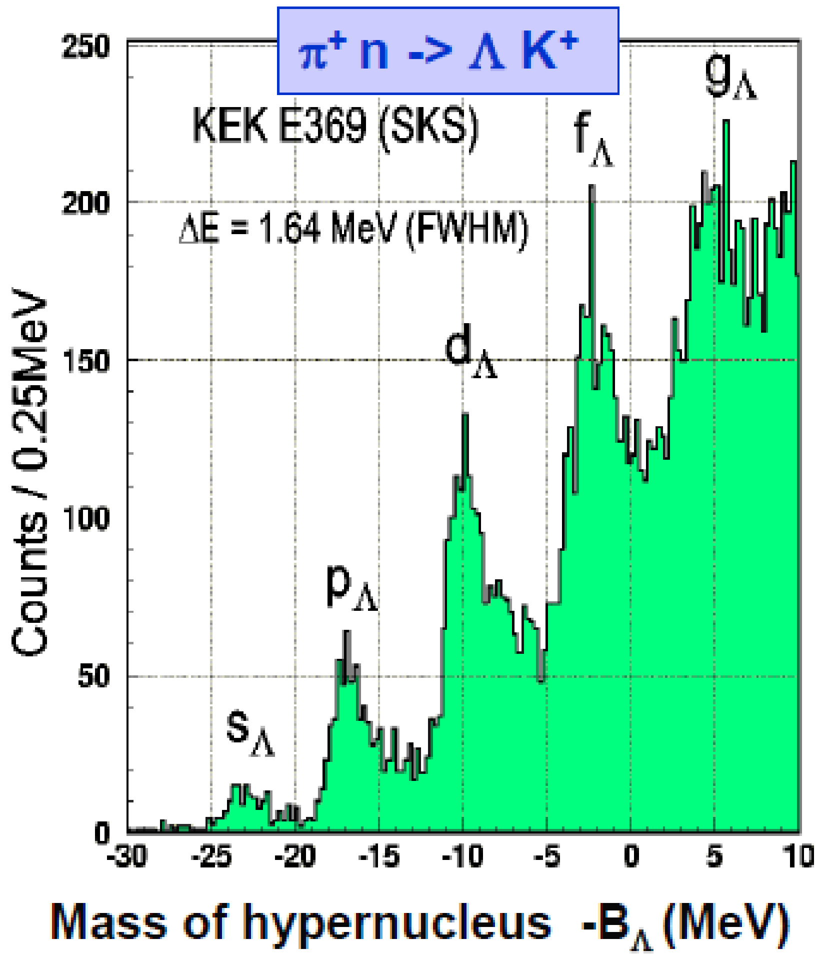
Λ hypernuclei

(π, K⁺) spectroscopy

A series of beautiful experiments at KEK-PS SuperKaonSpectrometer measured the yield of Λ particles inside medium-mass or heavy nuclei. These particles have been produced in different angular momentum states inside the nucleus, but a complete theoretical understanding was still missing.

SKS at KEK-PS

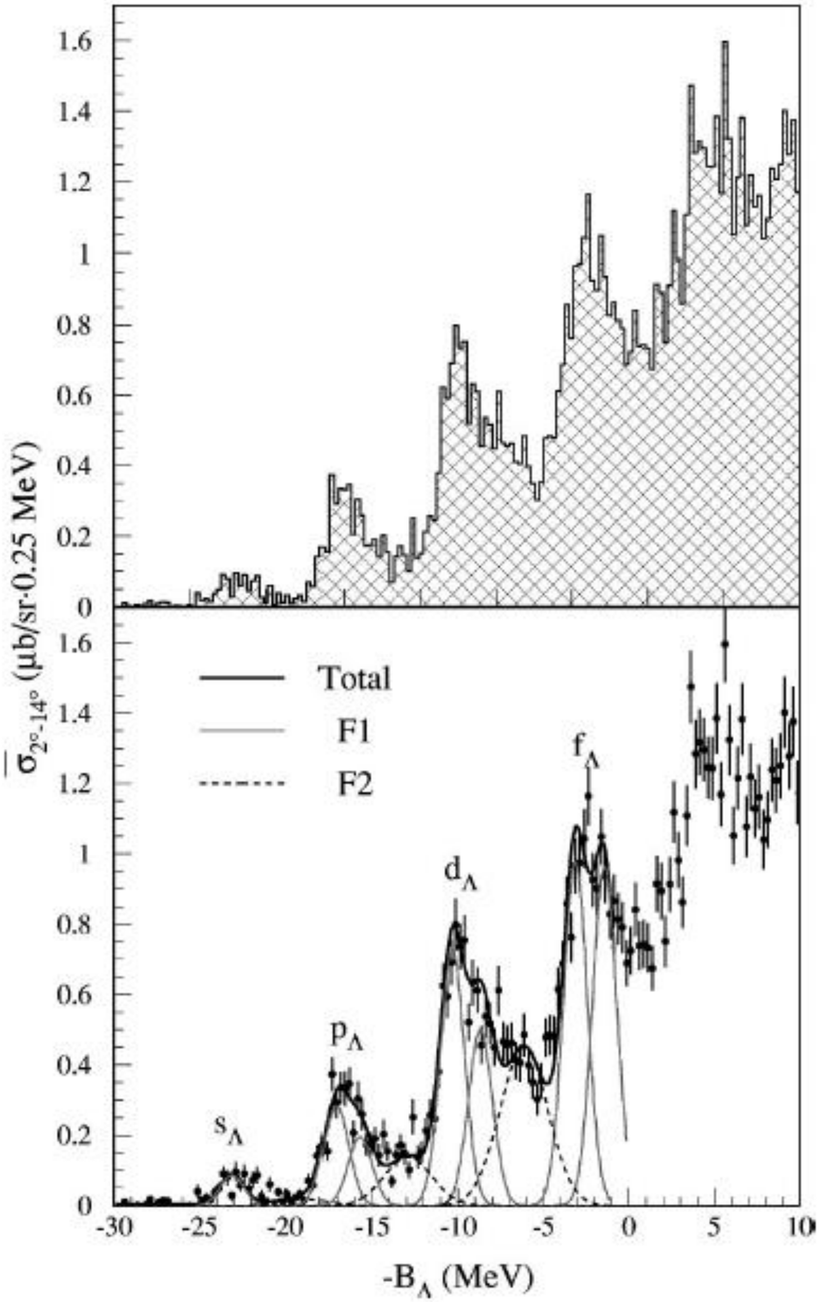
$^{89}Y\ (\pi^+, K^+)\ ^{89}{}_{\Lambda}Y$



Hotchi et al., PRC 64 (2001) 044302

Pictures taken from slides of H.Tamura (Sendai)

Spectrum of Λ particles in heavy nuclei



Hotchi et al., PRC 64 (2001) 044302

$$F_1 = G(a_0, b_0, \sigma_{\text{exp}}) + \sum_{l=1}^{3} \{G(a_l^L, b_l^L, \sigma_{\text{exp}}) + G(a_l^R, b_l^R, \sigma_{\text{exp}})\},$$

$$F_2 = G(A_0, b_0 + \delta B, \sigma) + \sum_{l=1}^{2} G(A_l, b_l^L + \delta B, \sigma), \quad (4.2)$$

where $G(a,b,\sigma) \equiv \Delta E_{\text{bin}}[(a/\sqrt{2\pi\sigma^2})\exp\{-(x-b)^2/2\sigma^2\}]$

Peaks ($F1$)	B_Λ (MeV)	FWHM (MeV)	Cross sections (μb/sr)
$l=0$	23.11±0.10	1.65(fixed)	0.60±0.06
$l=1-L$	17.10±0.08		2.00±0.22
$l=1-R$	15.73±0.18		1.38±0.19
$l=2-L$	10.32±0.06		5.10±0.31
$l=2-R$	8.69±0.13		3.52±0.25
$l=3-L$	3.13±0.07		6.87±0.33
$l=3-R$	1.43±0.07		6.79±0.31

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Spectrum of Λ particles in heavy nuclei

This problem was suggested to me by K.Hagino (Sendai, Japan) and the solution I propose is to use the u(3)x u(2) algebra

$$u(3/2) \supset u_B(3) \times u_F(2) \supset so_B(3) \times su_F(2)$$

$$\hat{H} = \hat{H}_{Nucl} + \hat{H}_{Hyp} \qquad \hat{H}_{Hyp} = \hat{H}_{u(3)} + \hat{H}_{u(2)} + \hat{V}_{int}$$

$$\hat{H}_{u(3)} = \alpha \hat{C}_1(u(3)) + \beta \hat{C}_2(u(3)) + \gamma \hat{C}_2(so_L(3))$$

where a, b, c are free parameters and the spectrum is given by

$$E_{u(3)} = \alpha N + \beta N(N+2) + \gamma L(L+1)$$

Energy formula for the centroids of the gaussians!

$$b_{NL} = \alpha N + \beta N(N+2) + \gamma L(L+1) + E_\Lambda$$

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Spectrum of Λ particles in heavy nuclei

$$b_{NL} = \alpha N + \beta N(N+2) + \gamma L(L+1) + E_\Lambda$$

$$G(a_{NL}, b_{NL}, \sigma) = \Delta E_{bin} \frac{a_{NL}}{\sqrt{2\pi\sigma^2}} e^{\{-(x-b_{NL})^2/2\sigma^2\}}$$

a_{00}	1.03458	a_{11}	4.774	a_{20}	4.26848	a_{22}	8.93652
a_{31}	9.57975	a_{33}	14.6199	a_{42}	21.4563	a_{44}	22.7715
α	5.39547	β	0.506972	γ	−0.321663	E_Λ	−22.6373

I have «ONLY» 12 parameters for yttrium, 8 of which are intensities and 4 of which allow to calculate ALL the centroids at once (and to extrapolate to higher q.n.)

The phenomenological fit in the cited PRC paper has 10 intensities and 8 fitted centroids for a total of 18 free parameters.

A sensible improvement not only in the numbers, but in the interpretation!

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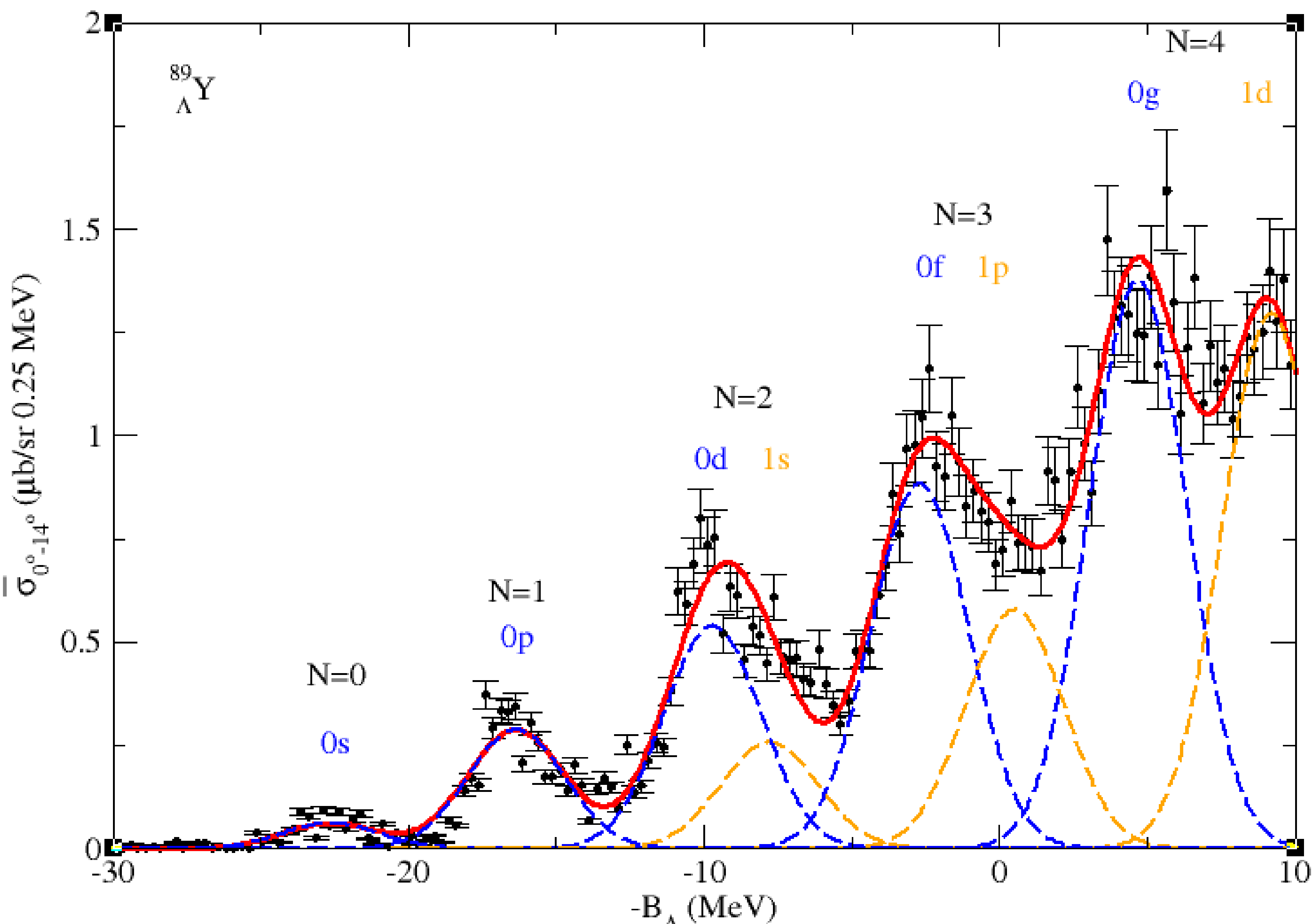
Spectrum of Λ particles in heavy nuclei -fit 1



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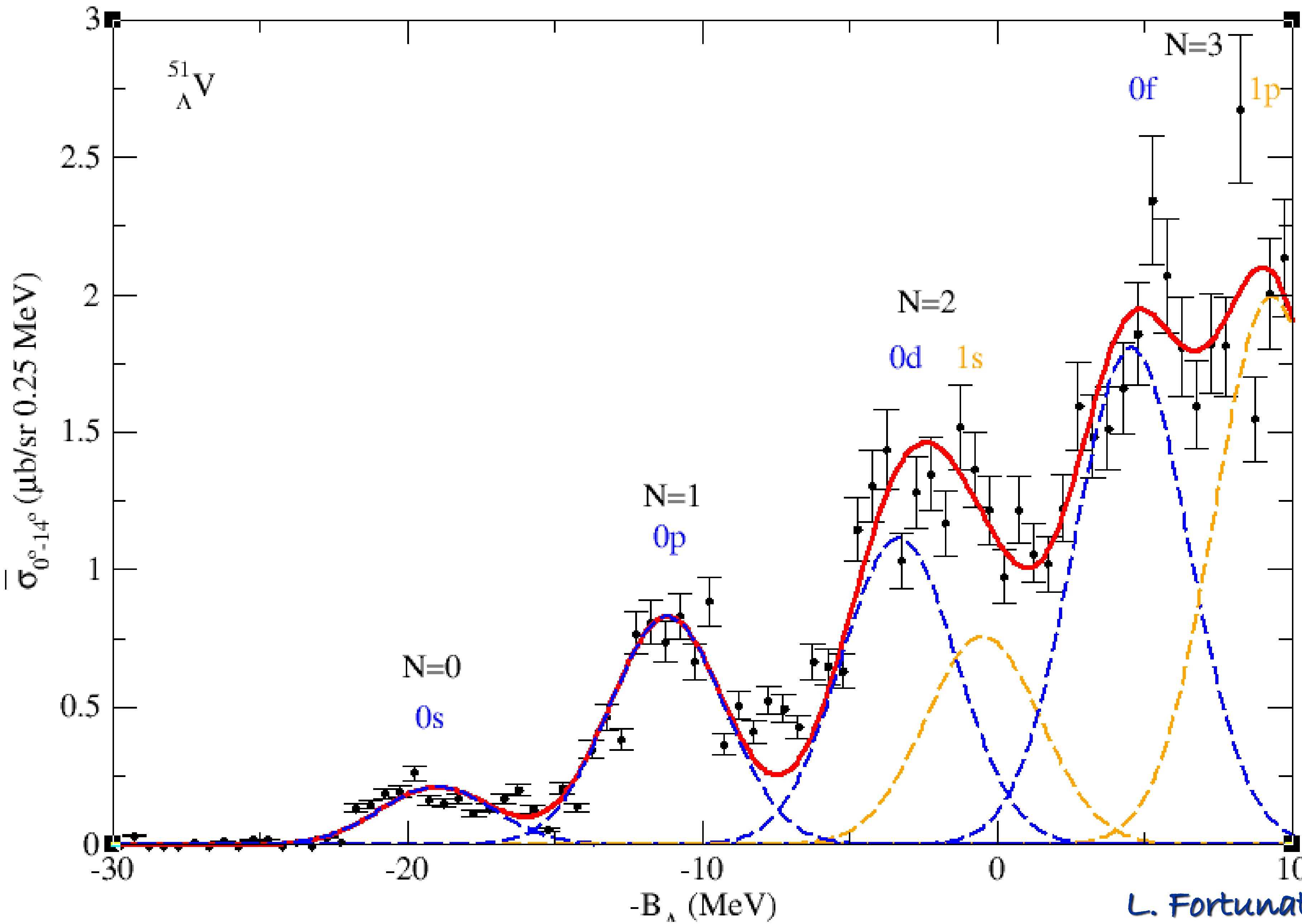
Spectrum of Λ particles in heavy nuclei -fit 2



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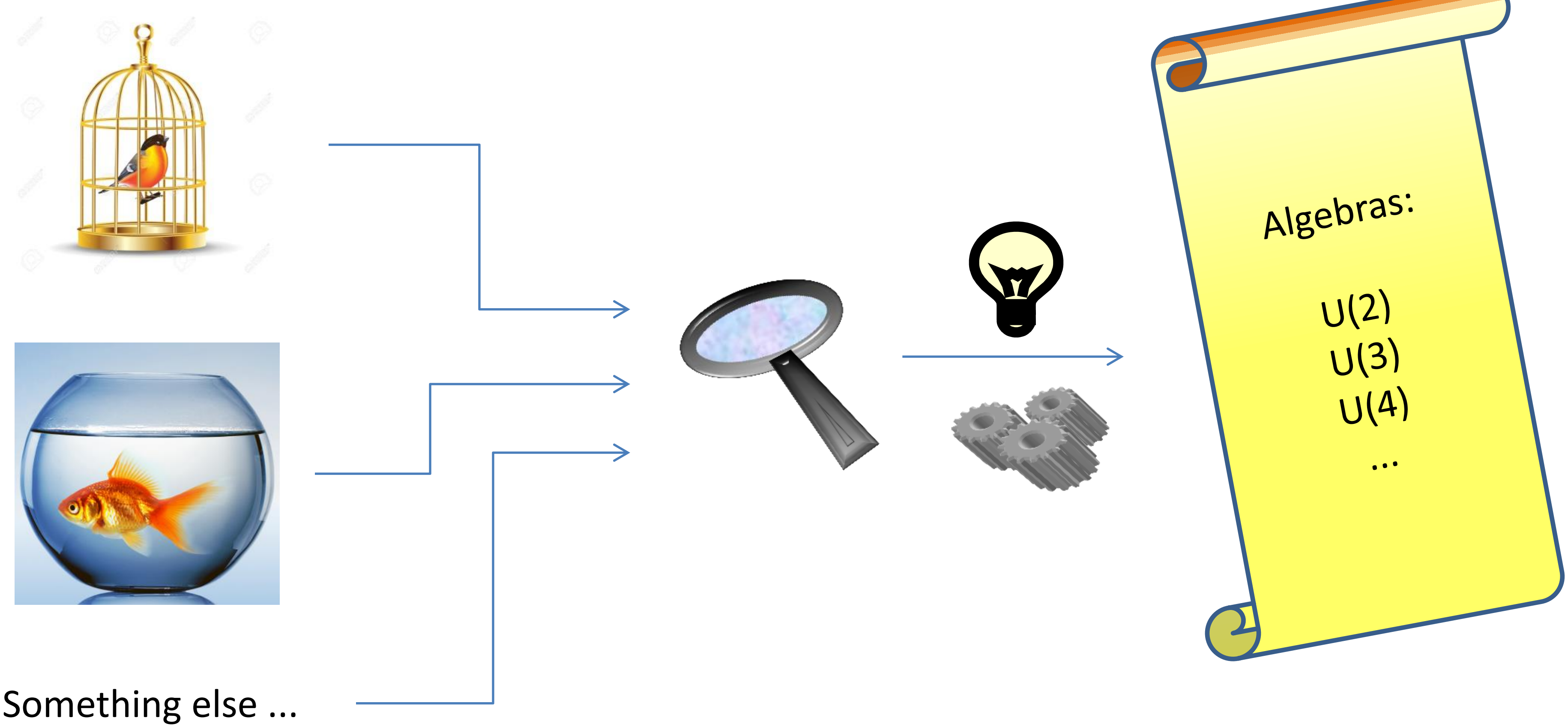
Symmetries unify very different systems!

In conclusion, caged quantum systems show similar behaviour across different scales: the internal degrees of freedom, that might be as complicated as those of a diatomic molecule u(4), or as simple as those of a spin-1/2 fermion u(2) are coupled to the harmonic motion inside the cage u(3). This allows to write energy formulas for the spectrum in both cases and the comparisons with data show remarkable agreement in both cases.



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