Direct reactions of weakly-bound nuclei within a one dimensional model

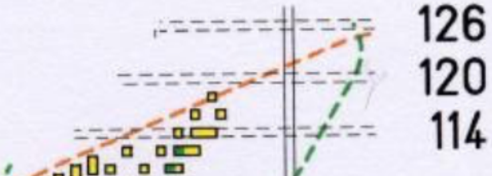
Laura Moschini

Andrea Vitturi & Antonio Moro

Outline

- Introduction
- The model
- Results
- Conclusions

Introduction



How to simplify the problem?

⚠ huge basis -> truncations
⚠ continuum discretization

=>model-dependent approximations

Let's move to one dimension!

$$\Psi(x,t) = \sum_{j=1}^{N} c_j(t)\Phi_j(x)e^{iE_jt/\hbar}$$

we can follow both the time evolutions
and understand the limitations of approximations

Study of the time evolution:

initial wavefunction $\Psi(\vec{r}, t_i)$

... time  evolution ...

final wavefunction $\Psi(\vec{r}, t_f)$

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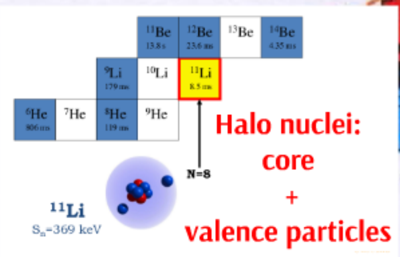
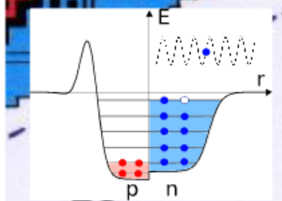
→ more complexities for weakly-bound systems!



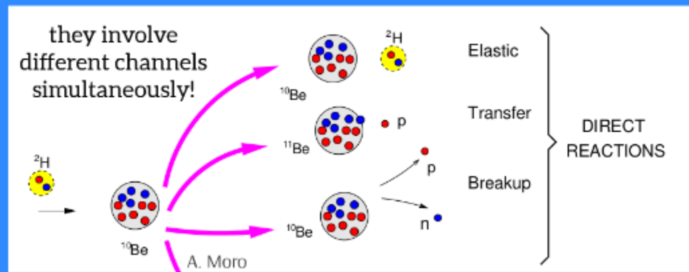
Halo nuclei:
core
+
valence particles



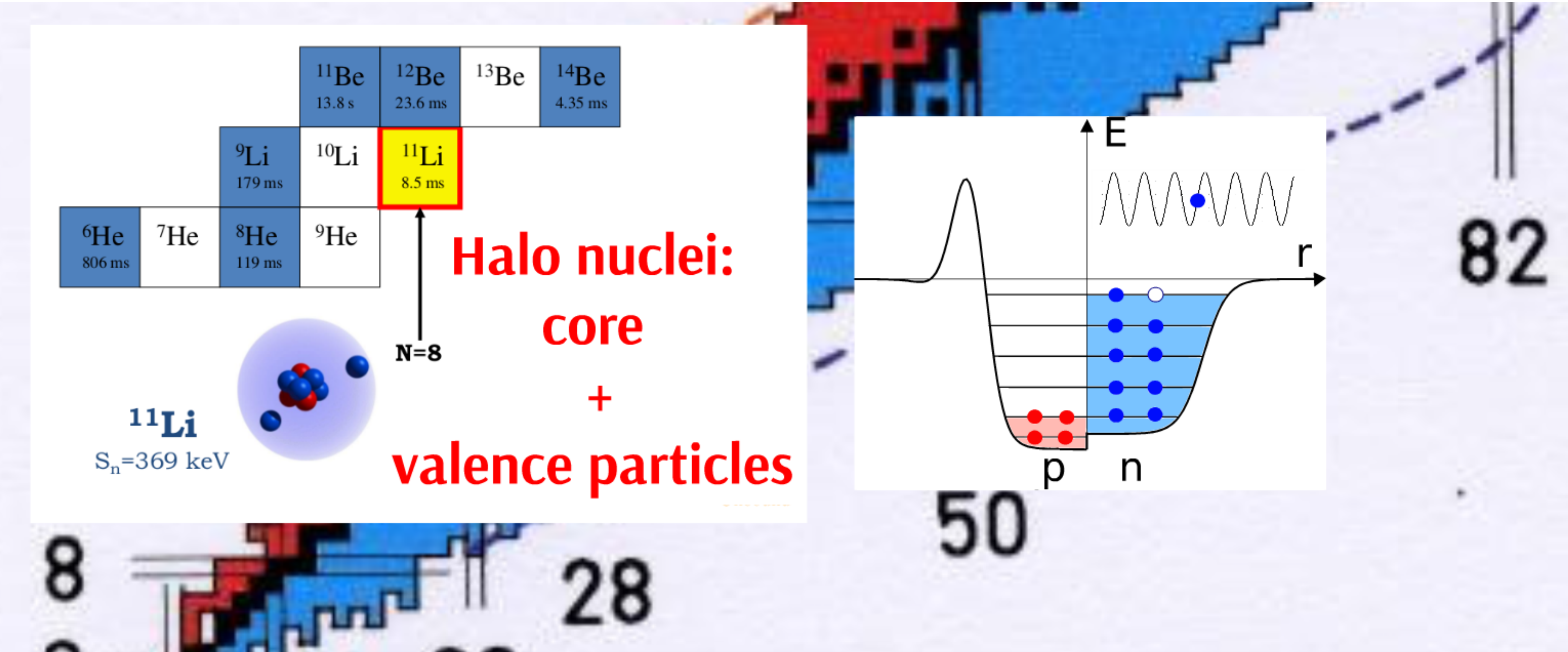
Description of structure and dynamics
of weakly-bound systems with one or more valence particle
(even considering inert core...)

One-particle halo | Two-particle halo | More particles

Study of direct reactions applied to



weakly-bound systems
with **one** valence **neutron**



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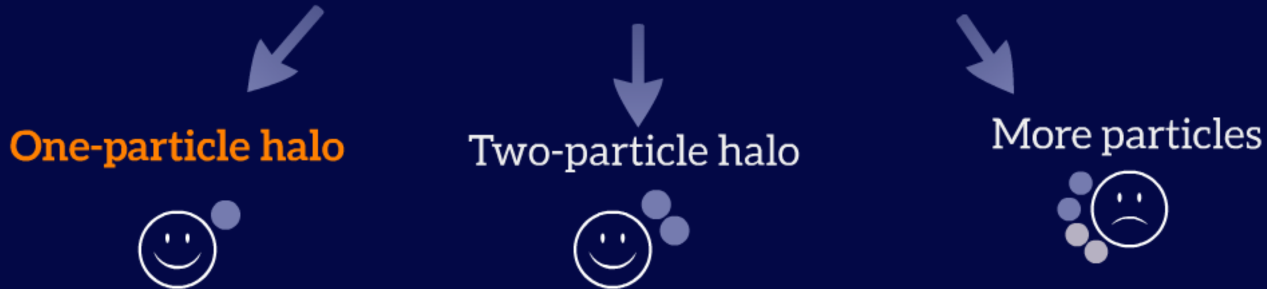
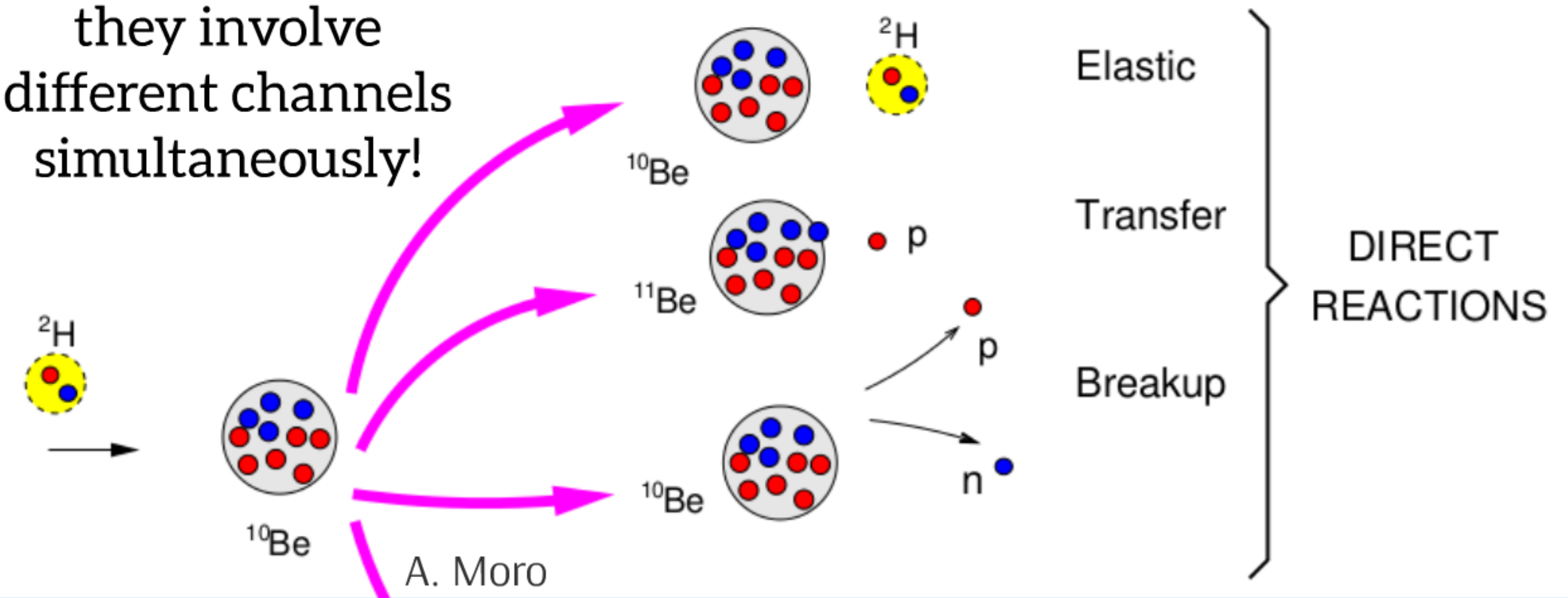


Study of **direct reactions** applied to

they involve different channels simultaneously!



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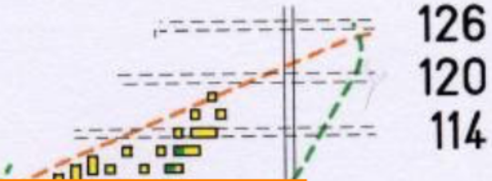

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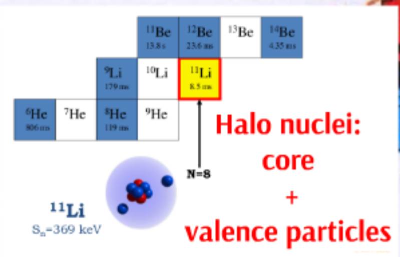

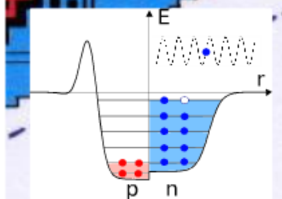

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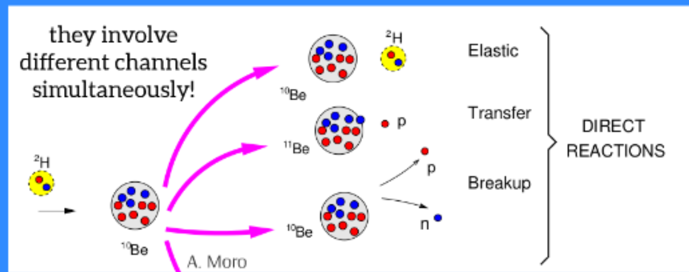

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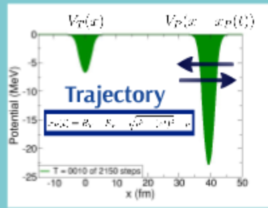

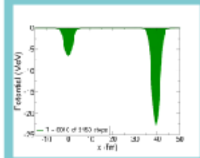

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LM, Pérez-Bernal and Vitturi, *J. Phys. G: Nucl. Part. Phys.* **43** 045112 (2016)

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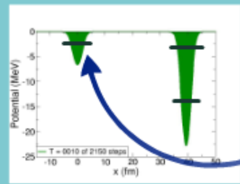

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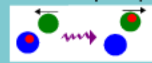

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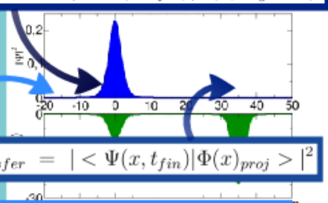

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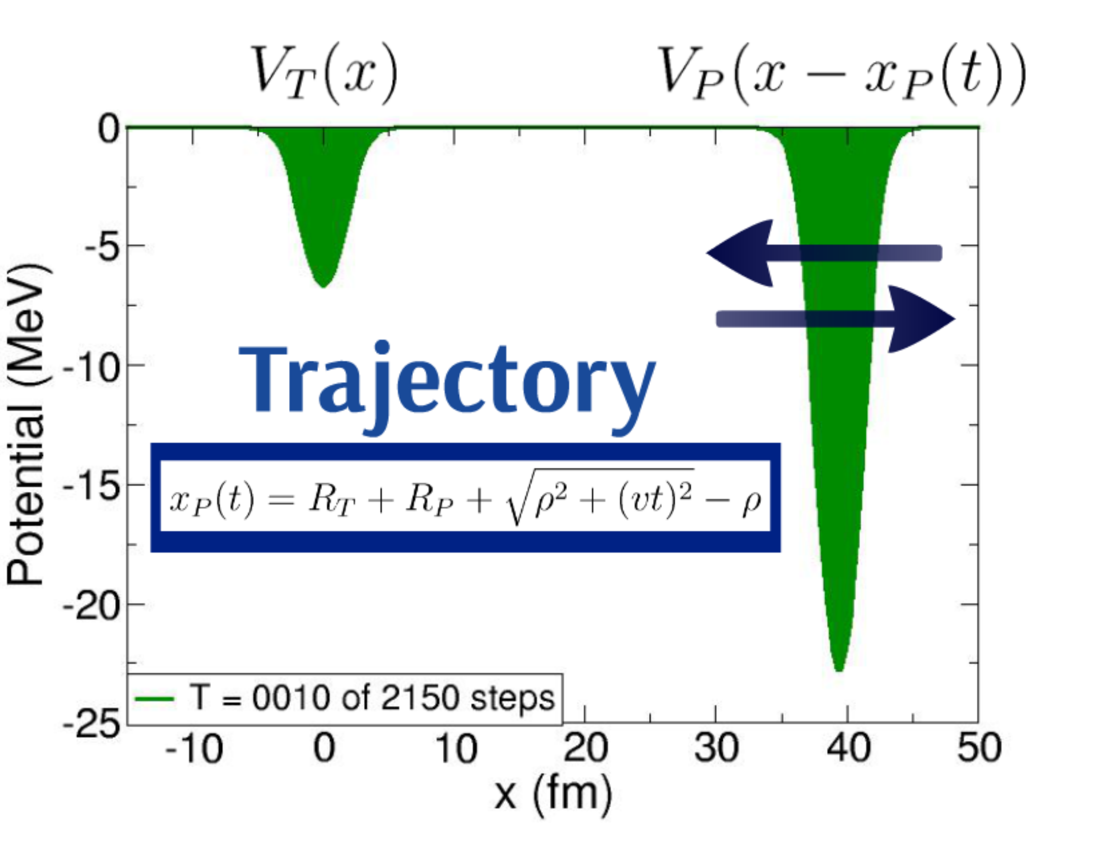
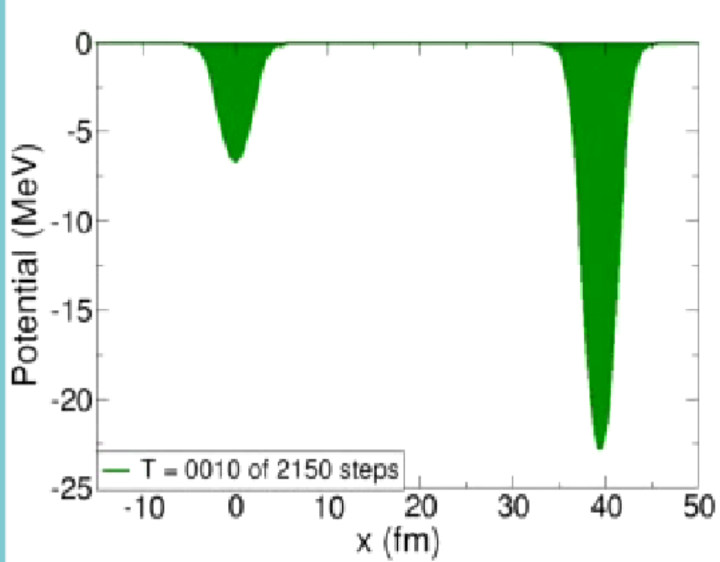


Trajectory

$$x_P(t) = R_T + R_P + \sqrt{\rho^2 + (vt)^2} - \rho$$



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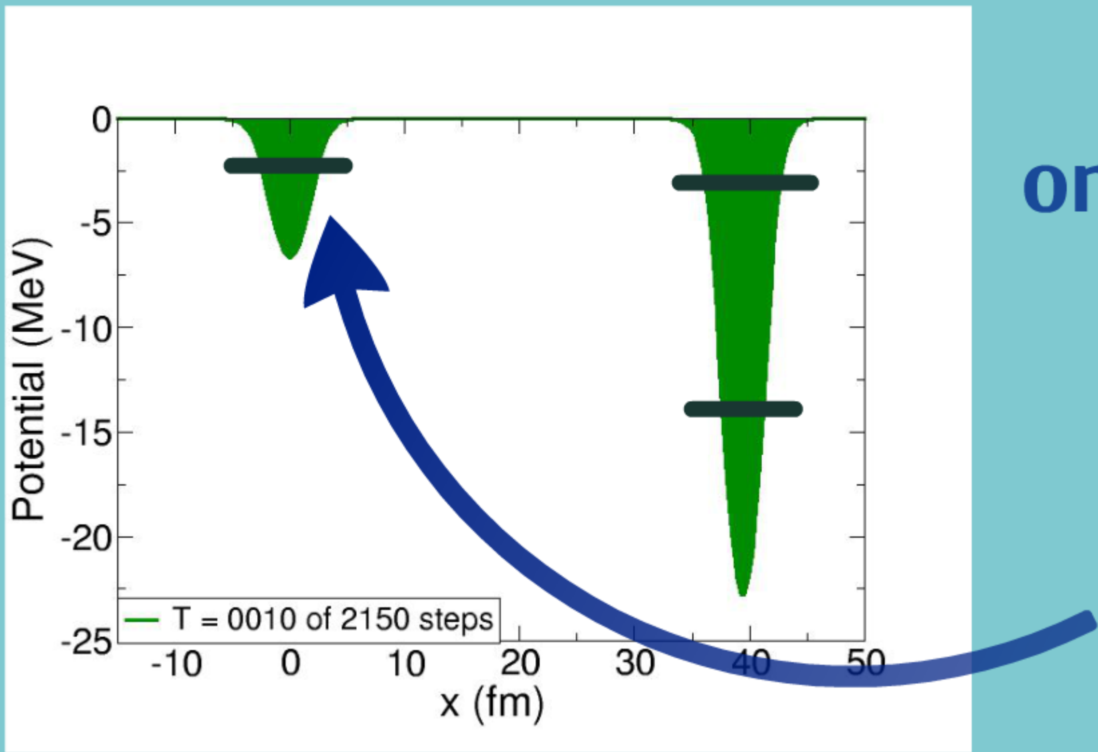

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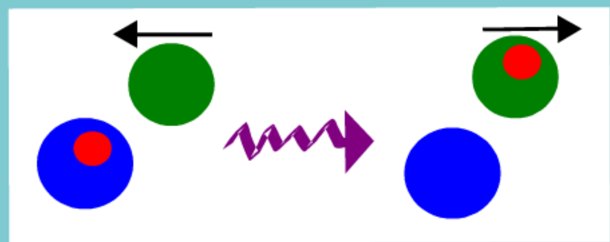

The model

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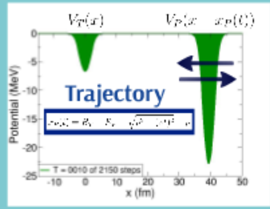

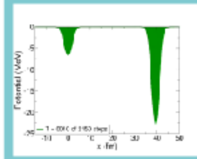

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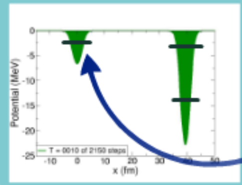

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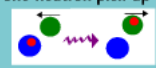

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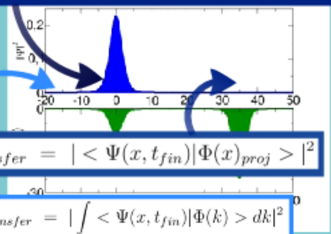

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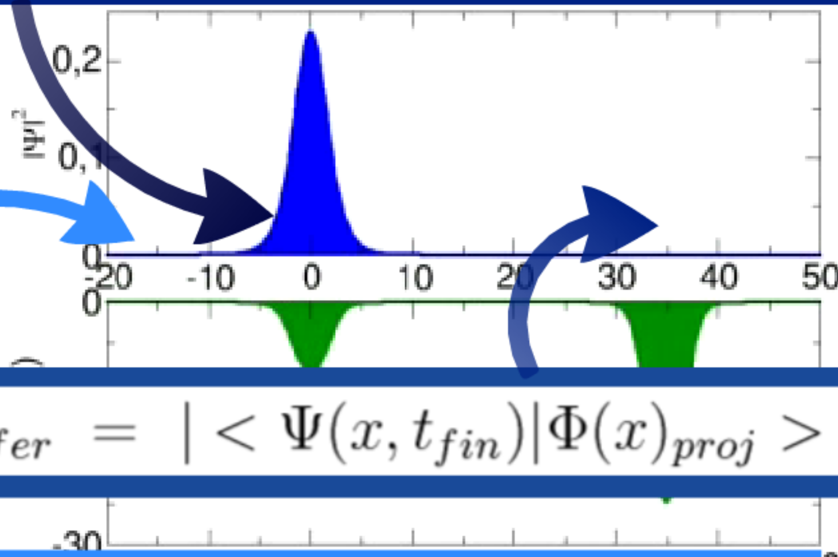

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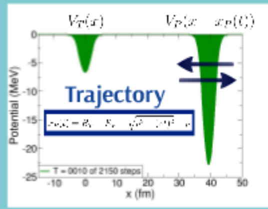

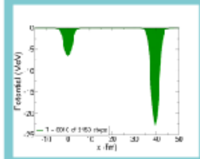

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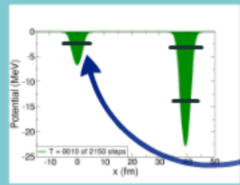

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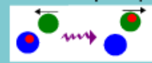

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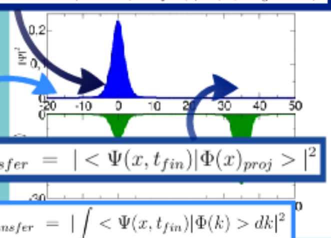

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Direct reactions of weakly-bound nuclei within a one dimensional model

Laura Moschini

Andrea Vitturi & Antonio Moro

Outline

Introduction

The model

Results

Conclusions

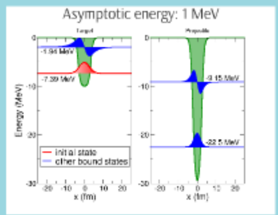
Results

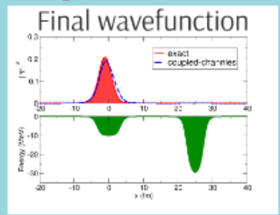
The same equation can be solved within
- the "exact" numerical method
- the coupled-channels formalism

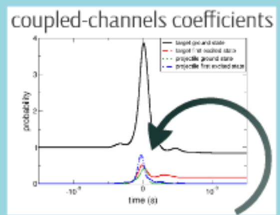
The goal is to check
- the validity of the approximation
- the necessary truncation of the two bases
- continuum discretization

This is particular relevant in the case of weakly-bound systems

Well bound system







	Exact	CC
elastic	83%	84%
inelastic	14%	16%
transfer (g.s.)	0.6%	0.002%
transfer (1st excited)	0.03%	0.2%
breakup	2.4%	-

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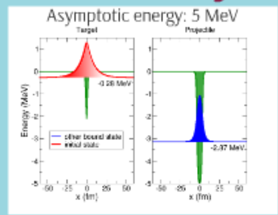
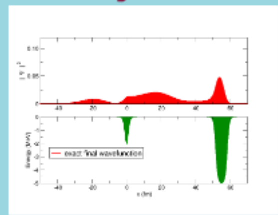
Weakly-bound system



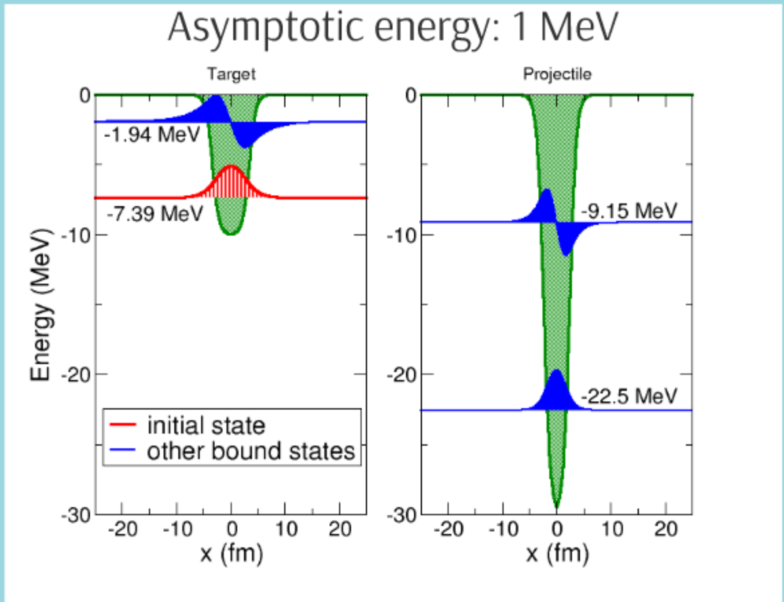


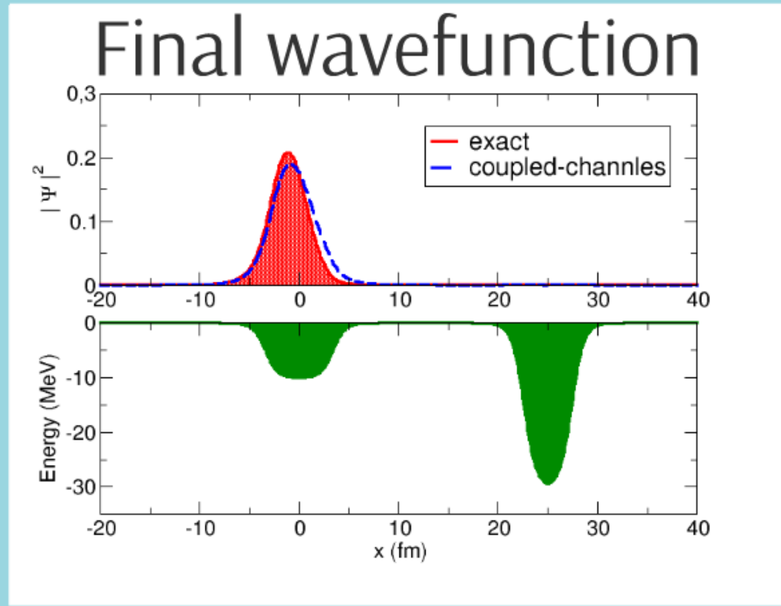
	Exact	CC (no continuum)	CC (target continuum)
elastic	21%	95%	21%
transfer	5%	5%	0.04%
breakup	74%	-	79%

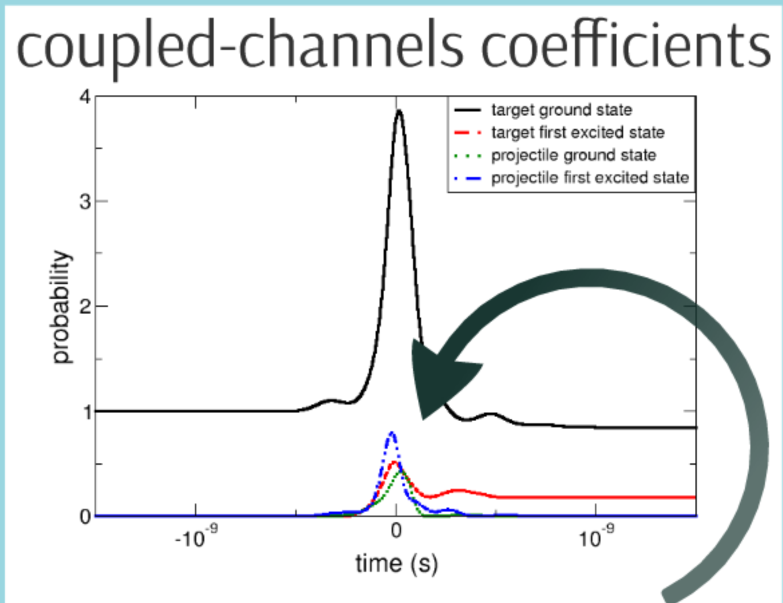
Different choices are possible for continuum:
- **pseudostates obtained by diagonalizing the potential in different bases (infinite square well, harmonic oscillator, transformed HO)**
- **bunching of continuun states into bins (CDCC style)**

=> results are model dependent!

Well bound system







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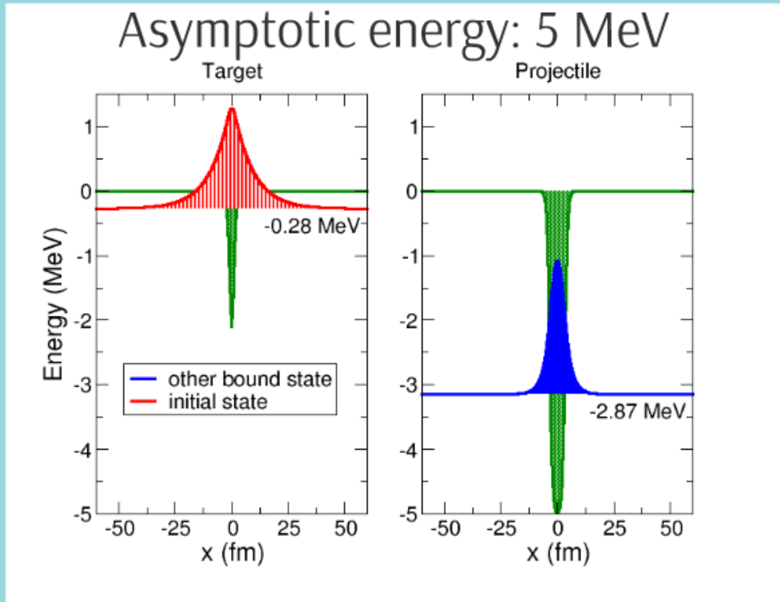

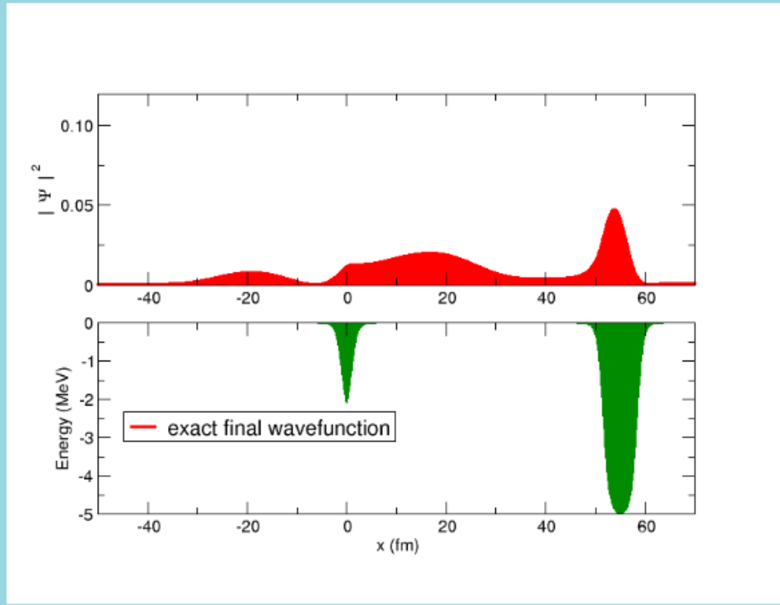

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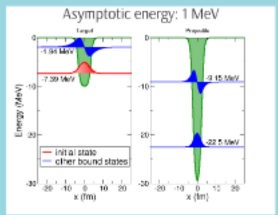

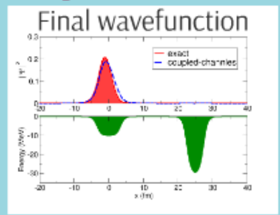

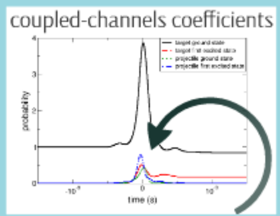

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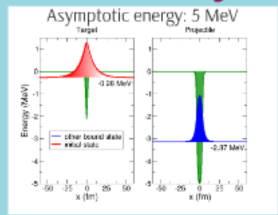

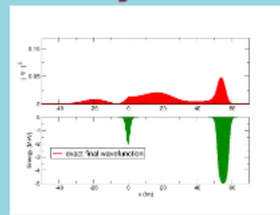

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Conclusions

- We developed a **one dimensional** model to solve **direct reactions** involving nuclei with **one valence neutron**
- We deal with the problem by solving numerically the time-dependent **Schroedinger equation** and by applying the **coupled-channels** formalism
- We understand that for **weakly-bound** systems it is mandatory to include the **continuum** and that its definition is **model dependent**