Crystalline chiral condensates

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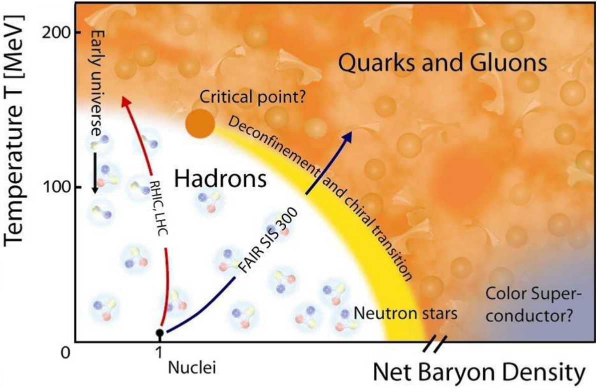
The QCD phase diagram



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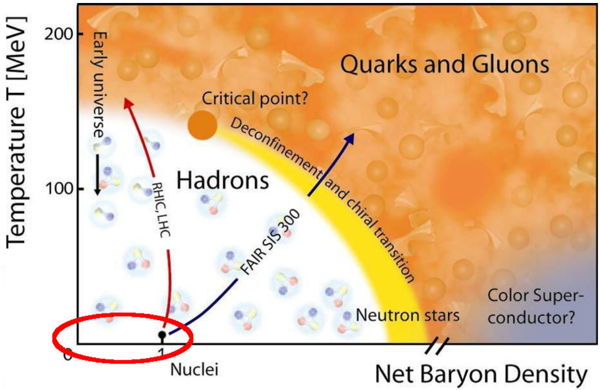

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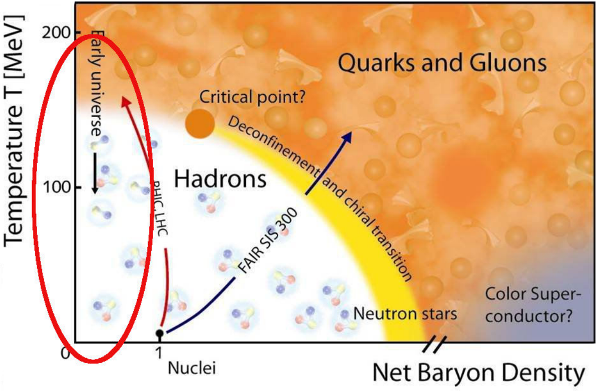

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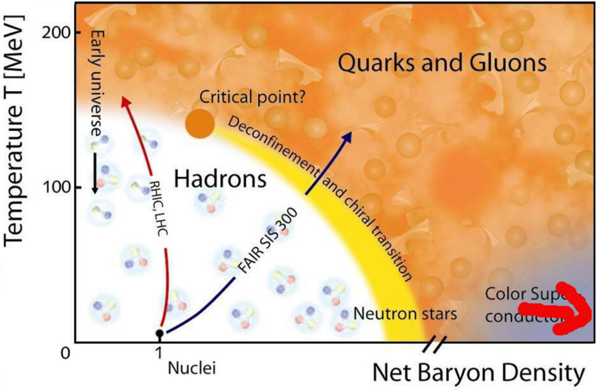

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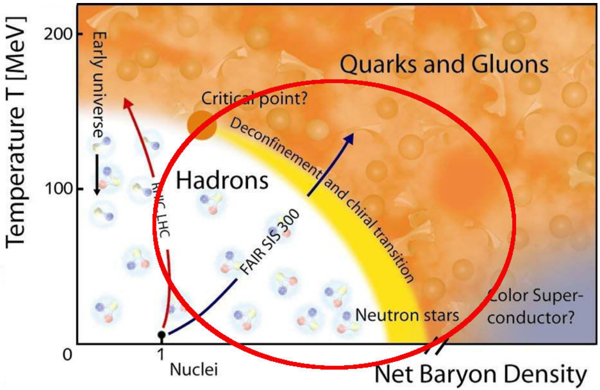

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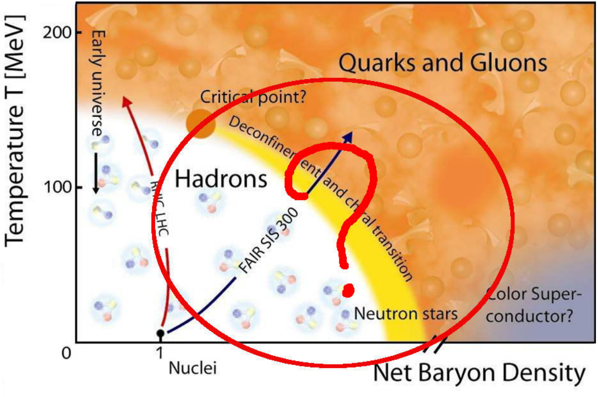

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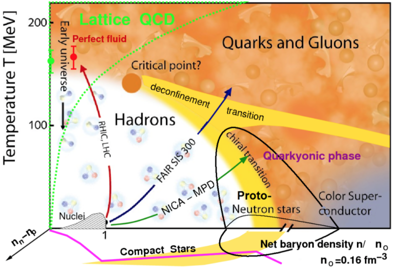
- Charge neutrality



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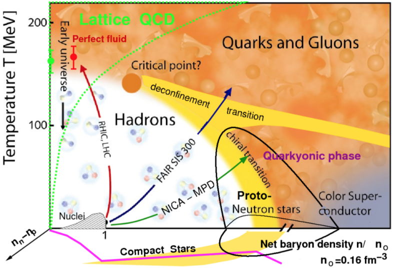

- Magnetic fields
- ....

Pairing mechanisms in quark matter

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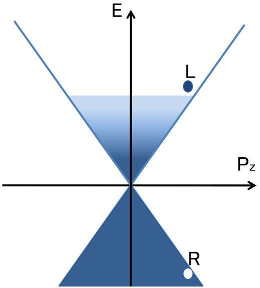
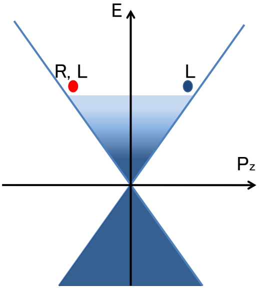
Vacuum: quark-antiquark pairing
$\rightarrow$ homogeneous chiral symmetry breaking



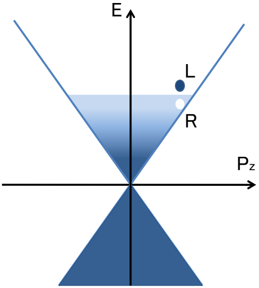
Pairing mechanisms in quark matter

High density: diquark pairing
$\rightarrow$ color-superconductivity



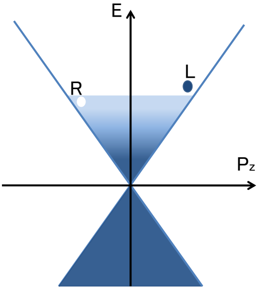
Pairing mechanisms in quark matter

Intermediate densities: quark-hole pairing (opposite momenta)
$\rightarrow$ exciton



Pairing mechanisms in quark matter

Intermediate densities: quark-hole pairing (equal momenta)
$\rightarrow$ inhomogeneous chiral symmetry breaking



Why inhomogeneous phases ?

- Popular already for quite some time...
  - Overhauser pairing in nuclear matter
  - Pion condensation
  - (Color-) Superconductivity

- Recently rediscovered and revised
  - Studies of lower-dimensional models (GN_2, NJL_2, ...)
  - Quarkyonic chiral spirals
  - ...

- May be relevant for cold dense matter !

Formalism

- Nambu–Jona-Lasinio model

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi + G\left(\left(\bar{\psi}\psi\right)^{2} + \left(\bar{\psi}i\gamma^{5}\tau^{a}\psi\right)^{2}\right)$$

- Chirally invariant four-fermion interaction
- Mean-field approximation $\rightarrow$ Thermodynamic potential

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- Chirally invariant four-fermion interaction
- Mean-field approximation $\rightarrow$ Thermodynamic potential
- Inhomogeneous phases:
  retain spatial dependence of the condensates

$$\langle\bar{\psi}\psi\rangle = S(\mathbf{x}), \qquad \langle\bar{\psi}i\gamma^{5}\tau_{3}\psi\rangle = P(\mathbf{x})$$

- Minimize thermodynamic potential $\Omega(T, \mu; S, P)$

What kind of modulation?

- Tackling an arbitrary spatial modulation is extremely complicated!

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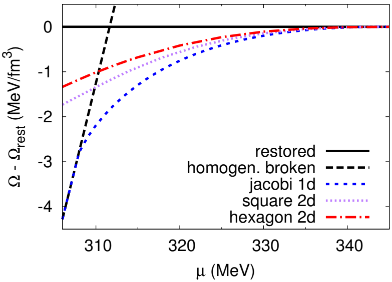
- In practice: choose a specific ansatz for the spatial dependence

- Typical examples:
  - Chiral density wave: $S(\mathbf{x}) \sim \Delta \cos(\mathbf{q} \cdot \mathbf{x})\,, \qquad P(\mathbf{x}) \sim \Delta \sin(\mathbf{q} \cdot \mathbf{x})$
  - Real cosine: $S(\mathbf{x}) \sim \Delta \cos(\mathbf{q} \cdot \mathbf{x})\,, \qquad P(\mathbf{x}) = 0$
  - Real kink crystal: $S(\mathbf{x}) \sim \Delta \nu \, \mathrm{sn}(\Delta z, \nu)\,, \qquad P(\mathbf{x}) = 0$
  - 2D solutions: $S(\mathbf{x}) \sim \Delta \cos(qx) \cos(qy)\,, \quad P(\mathbf{x}) = 0$

What kind of modulation?

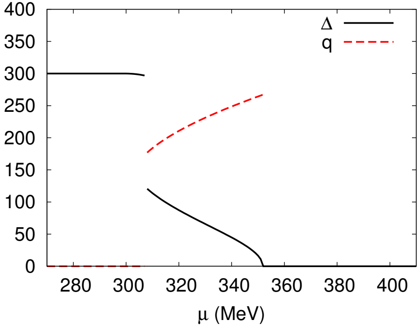


- Real kink crystal is (usually) the most favored solution
- Qualitatively similar results for order parameters / phase diagrams for all modulations

CDW order parameters at $T = 0$

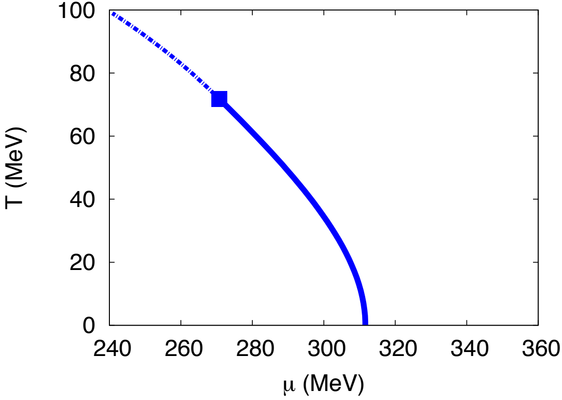
- Chiral density wave: $S(\mathbf{x}) \sim \Delta \cos(\mathbf{q} \cdot \mathbf{x})\,, \qquad P(\mathbf{x}) \sim \Delta \sin(\mathbf{q} \cdot \mathbf{x})$

$$M(\mathbf{x}) = -2G\big(S(\mathbf{x}) + iP(\mathbf{x})\big) \equiv \Delta e^{i\mathbf{q}\cdot\mathbf{x}}$$



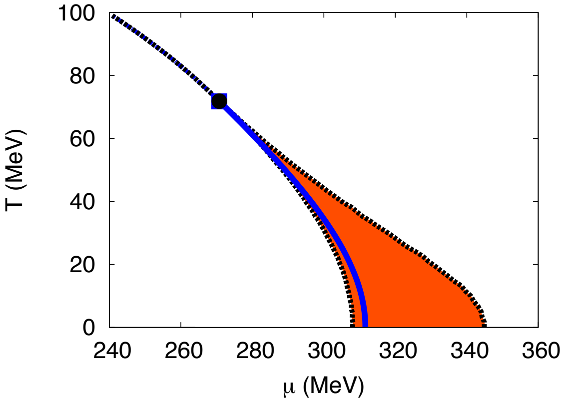
Phase diagram

Homogeneous matter only



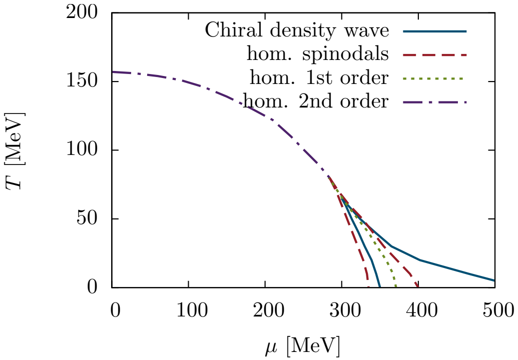
Phase diagram

Allowing for inhomogeneous matter



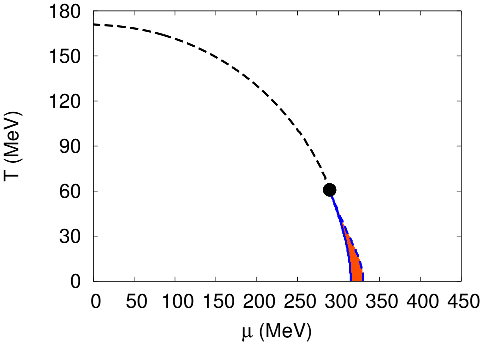
Other models

Dyson-Schwinger methods



Other models

Quark-meson model ("renormalized" MFA)



Extended NJL for realistic compact star description

- Magnetic fields
- Density effects ("vector interactions")

- Interplay with color-superconductivity
- Charge neutrality

Magnetic fields

- Effective dimensional reduction at the lowest Landau level (LLL)
- “Famous” consequence: Magnetic catalysis

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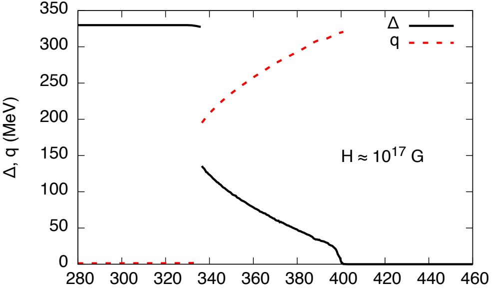
- Dimensional reduction to 1+1D $\rightarrow$ Inhomogeneous χSB

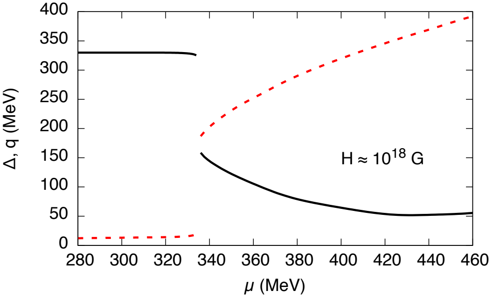
- Known examples:
  - 1+1-dimensional QCD
  - NJL_2 model
  - Quarkyonic matter ($\rightarrow$ QCD_2)

Order parameters



μ (MeV)

Order parameters



Vector interactions

- Vector interactions expected to play a role at finite densities
- Extend NJL Lagrangian by adding

$$\mathcal{L}_V = -G_V \sum_a \left(\bar{\psi}\gamma^\mu \tau_a \psi\right)^2$$

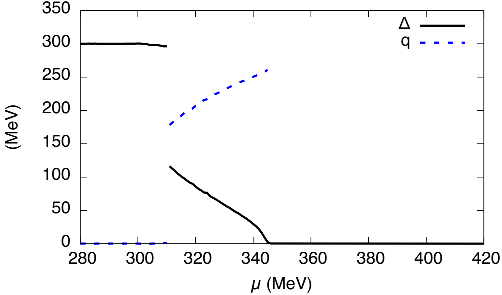
- Mean-field - amounts to a shift in the chemical potentials

$$\mu_f \;\rightarrow\; \tilde{\mu}_f = \mu_f - 4G_v\rho_f$$

- G_V free parameter

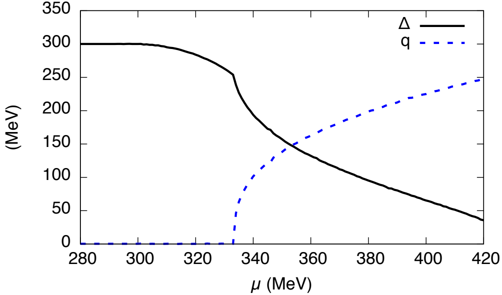
Vector interactions

$G_V = 0$

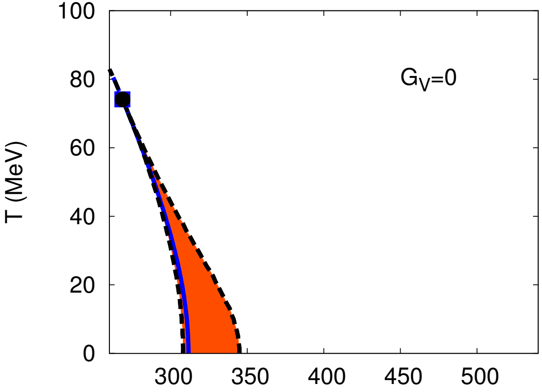


Vector interactions

$G_V = G_S$



Phase diagram - vector interactions



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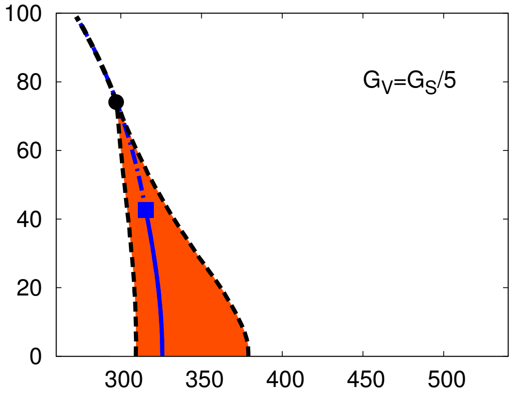
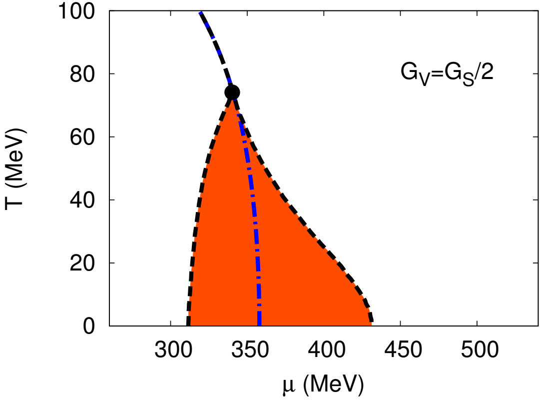


Phase diagram - vector interactions



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- Reviews:

  Prog. Part. Nucl. Phys 81 (2015) , arXiv:1406.1367
  EPJA Topical Issue "Exotic matter in NS" (2016), arXiv:1508.04361