Meson Properties in Asymmetric Matter

Andrea Mammarella

CSQCD V

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Based on:

AM, M. Mannarelli, Phys. Rev. D **92**, 085025
S. Carignano, AM, M. Mannarelli, Phys. Rev. D **93**, 051503

Motivation

Meson properties in an isospin and/or strangeness rich medium are important in many phenomena, such as the astrophysics of compact stars, heavy-ion collision and nuclear reactions.

It is important to have a model that incorporates isospin chemical potential μ_I and strangeness chemical potential μ_S.

The presence of chemical potentials can drastically change properties of mesons like their mass spectrum and life-time.

Approach

Inspiration

J. Kogut and D. Toublan, "QCD at small nonzero quark chemical potentials," *Phys.Rev*, vol. D64, 034007, 2001

Chiral Perturbation Theory

Effective Field Theory ⇒ analytic approach
Basic Lagrangian at p^2 order:

$$\mathcal{L} = \frac{F_0^2}{4}\mathrm{Tr}(\partial_\nu\Sigma\partial^\nu\Sigma^\dagger) + \frac{F_0^2 B_0}{2}\mathrm{Tr}[M(\Sigma+\Sigma^\dagger)] - \frac{F_0^2}{16}\mathrm{Tr}[v^\mu,\Sigma][v_\mu,\Sigma^\dagger] - \frac{iF_0^2}{4}\mathrm{Tr}\ \partial^\mu\Sigma[v_\mu,\Sigma]$$

Model

External currents

$$v^\mu = 2\mu\delta^{\mu 0}$$

Masses and chemical potentials are given by:

$$M = diag(m, m, m_s)$$

$$\mu = diag\left(\frac{1}{3}\mu_B + \frac{1}{2}\mu_I, \frac{1}{3}\mu_B - \frac{1}{2}\mu_I, \frac{1}{3}\mu_B - \mu_S\right)$$

Model

Ground State

Ground state parametrization:

$$\Sigma = e^{i\lambda_a\phi_a} \to \Sigma = u\bar{\Sigma}u \quad \text{with} \quad u = e^{i\lambda_a\phi_a/2}$$

Phase diagram

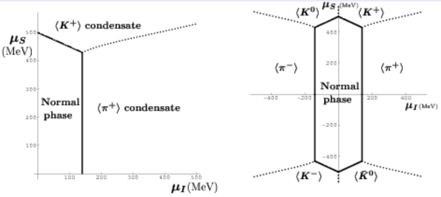


Figure: Kogut, Toublan, *Phys.Rev.*, vol. D64, 2001.

Phases

$$\cos\alpha_N = 1, \qquad \bar{\Sigma}_N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\cos\alpha_\pi = \left(\frac{m_\pi}{\mu_I}\right)^2, \qquad \bar{\Sigma}_\pi = \begin{pmatrix} \cos\alpha_\pi & \sin\alpha_\pi & 0 \\ -\sin\alpha_\pi & \cos\alpha_\pi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\cos\alpha_K = \left(\frac{m_K}{\frac{1}{2}\mu_I + \mu_S}\right)^2, \qquad \bar{\Sigma}_K = \begin{pmatrix} \cos\alpha_K & 0 & \sin\alpha_K \\ 0 & 1 & 0 \\ -\sin\alpha_K & 0 & \cos\alpha_K \end{pmatrix}$$

Masses and Mixing

Approach

We use group theory to study the symmetry breaking pattern and to constraint the mixing possibilities of mesons $\Rightarrow$ simplified calculation

Breaking pattern

$$
\begin{array}{ccc}
SU(3)_L \times SU(3)_R & & \\
\downarrow & \longleftarrow & \text{Masses} \\
SU(3)_V & & \\
\downarrow & \longleftarrow & v^{\nu} = v^{\mu} = 2\mu\delta^{\mu 0} \\
U(1)_{L+R} \times U(1)_{L+R} & &
\end{array}
$$

Symmetries and mixing

SU(2) subgroups of SU(3)

T-spin, U-spin, V-spin label the meson states also in the broken phases

Mixing states

Mixing states	(T, U)
π_+, π_-	$(1, 1/2)$
K_+, K_-	$(1/2, 1/2)$
$K_0, \bar{K}_0$	$(1/2, 1)$

π^0 and η mixing

Pion condensation phase

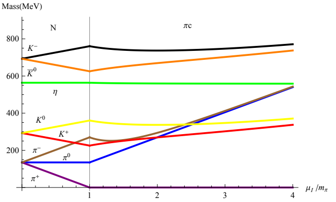
The vacuum has a charge proportional to λ_2.
$[\lambda_2, T] = 0 \Rightarrow$ the eigenstates are $|T = 1, T_3 = 0\rangle = |\pi_0\rangle$ and $|T = 0, T_3 = 0\rangle = |\eta\rangle \Rightarrow$ no mixing.

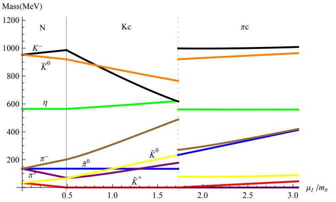
Kaon condensation phase

The vacuum has a charge proportional to λ_5.
$[\lambda_5, U] = 0 \Rightarrow$ the eigenstates are $|U = 1, U_3 = 0\rangle = \frac{|\pi_0\rangle + \sqrt{3}|\eta\rangle}{2}$ and $|U = 0, U_3 = 0\rangle = \frac{\sqrt{3}|\pi_0\rangle - |\eta\rangle}{2} \Rightarrow$ mixing.

Masses

Masses for $\mu_S = 200$ MeV and $\mu_S = 460$ MeV





Leptonic Decays

Decays

The main pion decay channels at 0 chemical potential are:

$$\pi^+ \to \ell^+ \nu_\ell \qquad \pi^- \to \ell^- \bar{\nu}_\ell$$

$$\Gamma^0_{\pi\to\ell\nu_\ell} = \frac{G_F^2 F_0^2 V_{ud}^2 m_\ell^2 m_\pi}{4\pi} \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2$$

Normal phase

$$m_\pi^\pm \to m_\pi \mp \mu_I \Rightarrow \Gamma^0_{\pi^+\to\ell^+\nu_\ell} = 0 \ \text{ for } \mu_I = m_\pi - m_\ell$$

Leptonic decays

Pion condensation phase

$\tilde{\pi}^+$ is massless, but $\tilde{\pi}^-$ is a combination of π^+ and π^-

$$\frac{\Gamma_{\tilde{\pi}_- \to \ell^+ \nu_\ell}}{\Gamma^0_{\pi \to \ell \nu_\ell}} = \frac{|U^*_{21} \cos \alpha_\pi + i U^*_{22}|^2}{2} \frac{m_{\tilde{\pi}^-}}{m_\pi} \left(\frac{1 - m_\ell^2 / m_{\tilde{\pi}^-}^2}{1 - m_\ell^2 / m_\pi^2} \right)^2$$

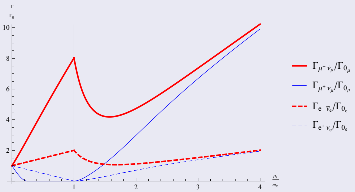
$$\frac{\Gamma_{\tilde{\pi}_- \to \ell^- \bar{\nu}_\ell}}{\Gamma^0_{\pi \to \ell \nu_\ell}} = \frac{|U^*_{21} \cos \alpha_\pi - i U^*_{22}|^2}{2} \frac{m_{\tilde{\pi}^-}}{m_\pi} \left(\frac{1 - m_\ell^2 / m_{\tilde{\pi}^-}^2}{1 - m_\ell^2 / m_\pi^2} \right)^2$$

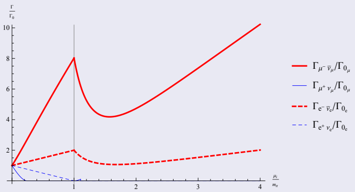
Leptonic decays

Pauli blocking

$\mu_{\ell^+} = \mu_I \Rightarrow$ the decay in ℓ^+ can be Pauli blocked

Plots





Lagrangian and Thermodynamics

LO Pressure

$\Sigma = \bar{\Sigma} \to \mathcal{L} \Rightarrow$ Pressure at 0-Temperature

$$p_{\mathrm{LO}}^{\pi c} = \frac{F_0^2 \mu_I^2}{2} \left(1 - \frac{m_\pi^2}{\mu_I^2}\right)^2 \qquad p_{\mathrm{LO}}^{Kc} = \frac{F_0^2 \mu_K^2}{2} \left(1 - \frac{m_K^2}{\mu_K^2}\right)^2$$

with $\mu_K = \mu_I/2 + \mu_S$.

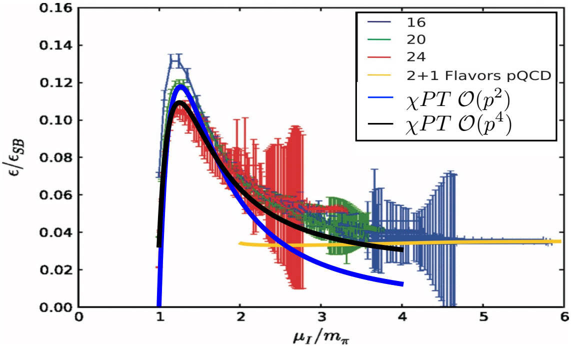
NLO Pressure

$$p_{\mathrm{NLO}}^{\pi c} = \frac{F_0^2 \mu_I^2}{2} \left(1 - \frac{m_\pi^2}{\mu_I^2}\right)^2$$

$$+ \frac{2}{\mu_I^4}(m_\pi^4 - \mu_I^4) \left[a_0(m_\pi^4 - \mu_I^4) - 2(b_0 - 2c_0)m_\pi^4\right] \epsilon$$

χPT vs Lattice

Observable: $\epsilon^{\pi^C}/\epsilon_{SB}$, with $\epsilon_{SB} = 9\mu_I^4/(4\pi^2)$



Conclusion

Model

Meson physics in an isospin and strangeness rich medium can be described by appropriate external sources in Chiral Perturbation Theory

Phases

Chemical potentials imply the existence of a normal phase, a pion condensation phase and a kaon condensation phase

Conclusion

In the normal phase only linear shifts in the masses are permitted. In the condensed phases the ground state aquires a charge. The masses are complicated functions of the chemical potentials.

Chemical potentials can greatly enhance or suppress the decay channels.
In the condensed phases the mixing factors gives contributions to the decay widths and open channels like $\Gamma_{\tilde{\pi}_- \to \ell^+ \nu_\ell}$.

χPT permits to calculate all the thermodynamic observables.
It is in good agreement with predictions made by lattice methods.

Conclusions

Applications

- Kaon decays
- Heavy-ion physics
- Astrophysics (compact stars)
- Nuclear physics (nuclear decays)

Extension

- Including baryons in the model (work in progress)