

Recent progresses in *ab-initio* studies of low-energy few-nucleon reactions of astrophysical interest

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Results obtained in collaboration with:

- The FBS Pisa Group
- The astrophysics colleagues of the Dep. (E. Tognelli, S. Degl'Innocenti, P. Prada Moroni)
- R. Schiavilla and collaborators
- G. Mangano (Napoli)

Outline

- *Ab-initio* methods for few-nucleon systems
 - (Standard) phenomenological approach (PhenAp)
 - Chiral effective field theory (χ EFT)
- Tests
 - Electromagnetic structure of $A = 2, 3, 4$ nuclei
 - Muon capture on light nuclei
- Results
 - $p + p \rightarrow d + e^+ + \nu_e$
 - $p + d \rightarrow {}^3\text{He} + \gamma$
- Outlook
 - The weak sector in χ EFT
 - Recent (local) χ EFT potential WITH Δ 's

Theoretical framework: *ab-initio* studies

A-nucleon system

↔

Cross section σ [or S -factor $S(E)$]

Ingredients

- Realistic nuclear Hamiltonian
$$H = T + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk}$$
- *Ab-initio* method to solve the A -body quantum problem
(HH method for $A = 3$)
- Realistic nuclear electroweak currents j^{EW}

Nuclear Hamiltonian

- PhenAp → AV18/UIX
- χ EFT → N3LO(Idaho)/N2LO(local form) – $\Lambda = 500, 600$ MeV

Electroweak currents in PhenAp

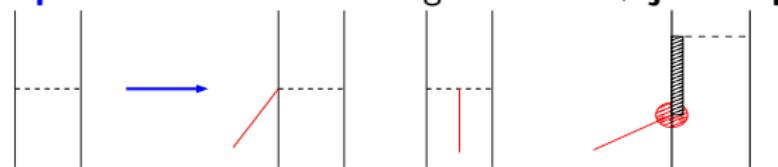
EW operators: $\rho^\gamma, \mathbf{j}^\gamma$; $\rho^V/A, \mathbf{j}^V/A$ but CVC $\Rightarrow \rho^V/\mathbf{j}^V \rightarrow \rho^\gamma/\mathbf{j}^\gamma$

- $\mathbf{j}^\gamma \rightarrow$ Current Conservation Relation (CCR) $\longrightarrow \mathbf{q} \cdot \mathbf{j}^\gamma \propto [\rho^\gamma, H]$
Realistic model

1b operators



2b operators: Meson-exchange currents + $\mathbf{j}^{MD} \perp \mathbf{q}$



CCR with T at $\mathcal{O}(\frac{1}{m})$

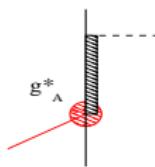
Interplay potential-current: MI

MD

CCR “exact” with AV18/UIX [L.E. Marcucci *et al.*, PRC 72, 014001 (2005)]

- $\mathbf{j}^A \rightarrow$ no CCR \Rightarrow MD

Largest contribution to $\mathbf{j}^A(\text{MD})$ from



g_A^* fit to observable: GT_{Exp} of tritium β -decay

| | 2N force | 3N force | 4N force |
|-------------------|----------|----------|----------|
| LO | | — | — |
| NLO | | — | — |
| N ² LO | | | — |
| N ³ LO | | | |

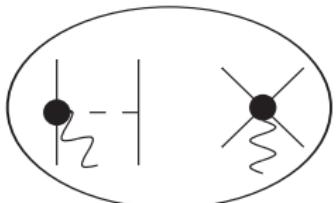
Similar power counting for the EW currents

Electroweak currents in χ EFT: power counting for \mathbf{j}^γ

$$\mathcal{O}(Q^{-2}) \quad \left| \begin{array}{c} \text{---} \\ \text{---} \end{array} \right| \quad \mathbf{j}^{(-2)} \propto [e_N(1)(\mathbf{p}'_1 + \mathbf{p}_1) + i\mu_N(1)\sigma_1 \times \mathbf{q}] \times \delta(\mathbf{p}'_2 - \mathbf{p}_2) + 1 \leftrightarrow 2$$

$$\mathcal{O}(Q^{-1}) \quad \left| \begin{array}{c} \text{---} \\ \text{---} \end{array} \right| \quad \left| \begin{array}{c} \text{---} \\ \text{---} \end{array} \right| \quad \text{"standard" one-pion-exchange}$$

$$\mathcal{O}(Q^0) \quad \left| \begin{array}{c} \blacksquare \\ \text{---} \end{array} \right| \quad \blacksquare = \text{relativistic corrections}$$

$$\mathcal{O}(Q^1) \quad \left| \begin{array}{c} \text{---} \\ \text{---} \end{array} \right| \quad \left| \begin{array}{c} \text{---} \\ \text{---} \end{array} \right|$$


Electroweak currents in χ EFT: power counting for \mathbf{j}^A

Note:

$$\mathcal{O}(Q^{-3}) \quad \left| \begin{array}{c} \text{---} \\ \text{---} \end{array} \right|$$

$$\mathcal{O}(Q^{-1}) \quad \left| \begin{array}{c} \text{---} \\ \text{---} \\ \blacksquare \end{array} \right|$$

$$\mathcal{O}(Q^0) \quad \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right| \quad \text{---} \times \text{---}$$

$$\mathcal{O}(Q^1) \quad \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right| \quad \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right| \quad \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right|$$

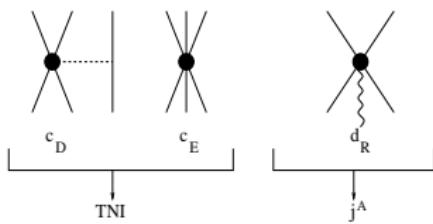
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- $\mathcal{O}(Q^1)$: two-pion-exchange
A. Baroni *et al.*, PRC **93**, 015501 (2016)

- Park *et al.* up to $\mathcal{O}(Q^0)$
→ one LEC - d_R

$$d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

- fit c_D and c_E (in TNI at N2LO) to $B(A=3)$ and GT_{Exp}



Electroweak currents: latest development

PhenAp

A new 1b term → relativistic correction (RC) $\mathcal{O}(1/m^3)$.

L. Girlanda *et al.*, PRL **105**, 232502 (2010): significant (few %) contribution to $n + d \rightarrow {}^3\text{H} + \gamma$.

χ EFT

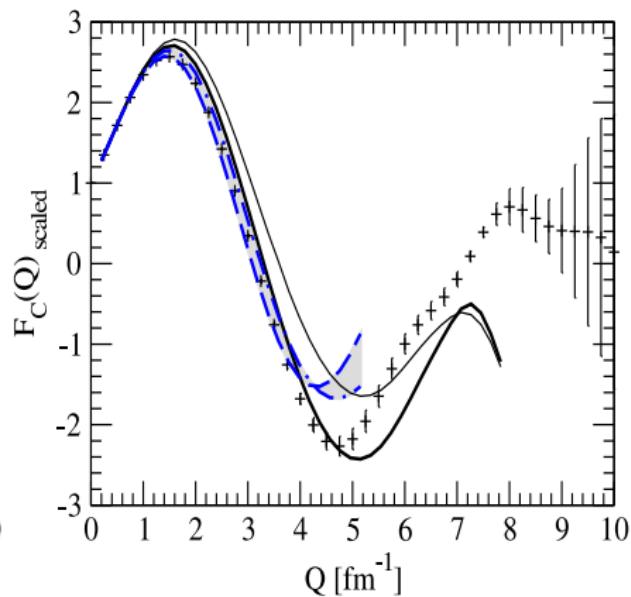
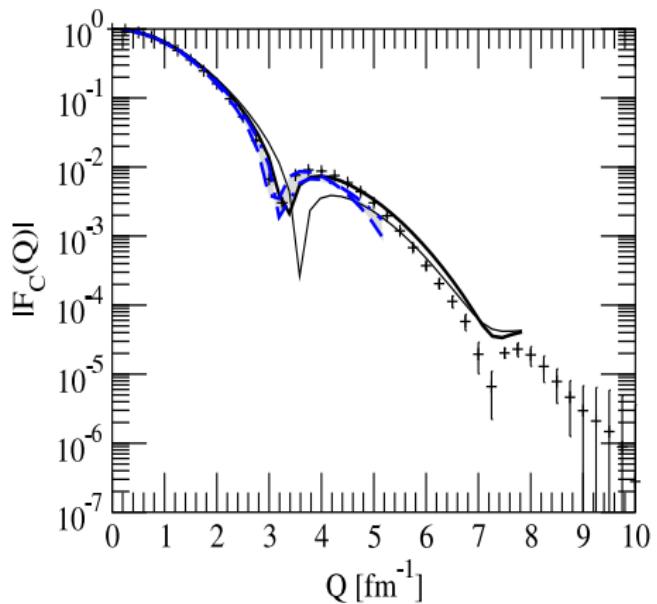
Main results in the weak sector up to $\mathcal{O}(Q^0)$ (Park *et al.*). Preliminary results also including the $\mathcal{O}(Q^1)$ contributions.

Electromagnetic structure of $A = 2, 3, 4$ nuclei

| | PhenAp | χ EFT | Exp. |
|---------------------------|--------------|---------------------------------------|---------------------------------------|
| $r_c(d)$ [fm] | 2.119 | 2.126 ± 0.004 | 2.130 ± 0.010 |
| $\mu(d)$ [n.m.] | 0.847 | <u>0.8574</u> | 0.8574 |
| $Q(d)$ [fm 2] | 0.280 | 0.2836 ± 0.0016 | 0.2859 ± 0.0003 |
| $r_c(^3\text{He})$ [fm] | 1.928 | 1.962 ± 0.004 | 1.973 ± 0.014 |
| $r_m(^3\text{He})$ [fm] | 1.909 | 1.920 ± 0.007 | 1.976 ± 0.047 |
| $\mu(^3\text{H})$ [n.m.] | 2.953 | <u>2.979</u> | 2.979 |
| $\mu(^3\text{He})$ [n.m.] | -2.125 | <u>-2.128</u> | -2.128 |
| $r_c(^4\text{He})$ [fm] | 1.639 | 1.663 ± 0.011 | 1.681 ± 0.004 |

L.E. Marcucci *et al.*, JPG **43**, 023002 (2016)

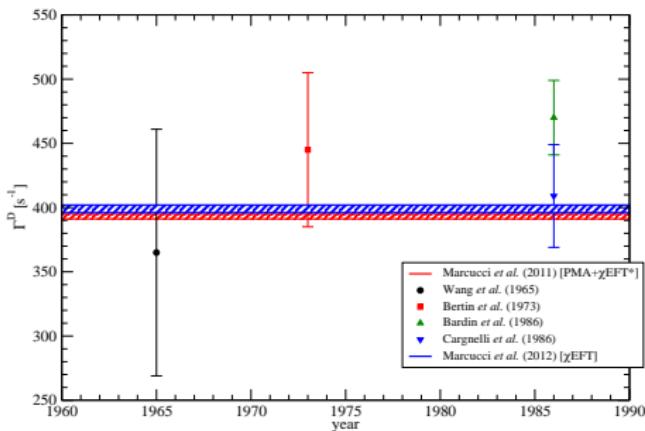
$A = 4$ form factor



Scaling function: $F_C(Q)_{\text{scaled}} = F_c(Q)/[F_0 \exp(-Q/k)]$, with $F_0 = 1$ & $k = 0.760488 \text{ fm}^{-1}$

Muon capture on $A = 2, 3$ nuclei

- $\mu^- + d \rightarrow n + n + \nu_\mu \longrightarrow$ capture rate in the doublet hyperfine state Γ^D
- $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu \longrightarrow$ total capture rate Γ_0



$$\Gamma_0(\text{PhenAp}) = 1495(11) \text{ s}^{-1}$$

$$\text{vs. } \Gamma_0(\chi\text{EFT}) = 1494(21) \text{ s}^{-1}$$

$$\text{vs. } \Gamma_0(\text{Exp}) = 1496(4) \text{ s}^{-1}$$

L.E. Marcucci *et al.*, PRC **83**, 014002 (2011)

L.E. Marcucci *et al.*, PRL **108**, 052502 (2012)

The proton-proton weak capture reaction

$S(E)$
in χ EFT and
PhenAp

$S(0)$ cumulative
contributions
(in 10^{-23} MeV fm 2)

- Energy range 2 keV – 100 keV
- PhenAp or χ EFT + FULL EM interaction
- pp $L \leq 1$ partial waves: $^1S_0 +$ all P -waves

| | 1S_0 | $\dots + ^3P_0$ | $\dots + ^3P_1$ | $\dots + ^3P_2$ |
|------------|----------|-----------------|-----------------|-----------------|
| PhenAp | 4.000(3) | 4.003(3) | 4.015(3) | 4.033(3) |
| χ EFT | 4.008(6) | 4.011(6) | 4.020(6) | 4.030(6) |

$$S(0) = (4.030 \pm 0.006) \times 10^{-23} \text{ MeV fm}^2$$

vs.

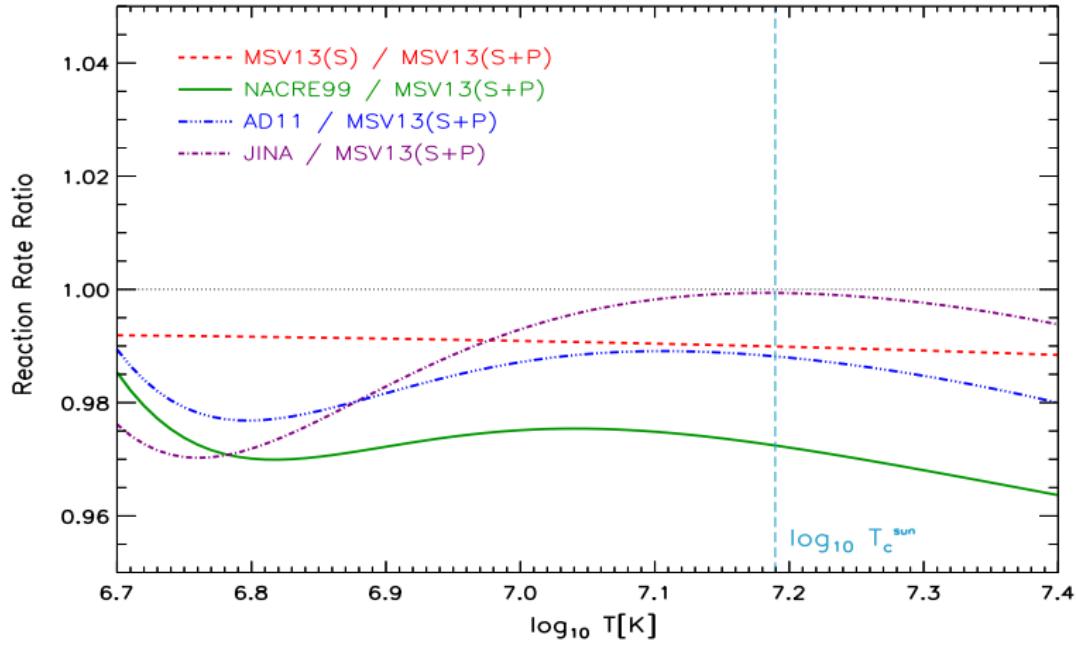
$$S(0)^{\text{SFII}} = (4.01 \pm 0.04) \times 10^{-23} \text{ MeV fm}^2$$

SFII: E.G. Adelberger *et al.*, RMP **83**, 195 (2011)

L.E. Marcucci *et al.*, PRL **110**, 192503 (2013)

Effects on

- age of mid and old stellar clusters (1-12 Gyr)
- standard solar model predictions

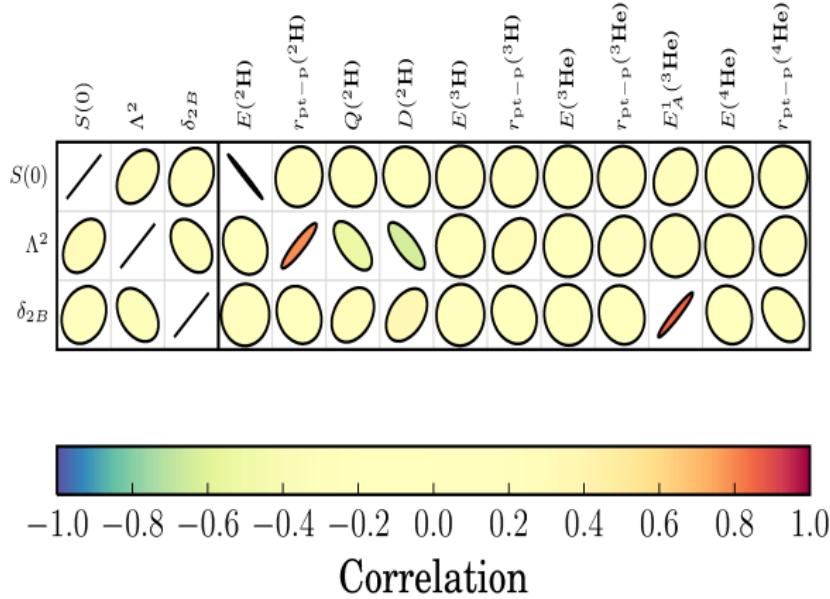


E. Tognelli *et al.*, PLB 742, 189 (2015)

Neutrino fluxes relative differences

| | MSV13(S+P) reference | MSV13(S) | NACRE99 | AD11 | JINA |
|----------------------------------|-------------------------|----------------------|---------|------|------|
| | | relative differences | | | |
| $T_c [10^7 \text{ K}]$ | 1.54794 | -1% | -3% | -2% | -1% |
| $\Phi_{\text{pp}}^\nu [10^{10}]$ | 6.020 | 1% | 2% | 2% | 1% |
| $\Phi_{\text{pep}}^\nu [10^8]$ | 1.446 | -2% | -6% | -2% | -1% |
| $\Phi_{\text{hep}}^\nu [10^3]$ | 8.584 | -1% | -3% | < 1% | 2% |
| $\Phi_{\text{Be-7}}^\nu [10^9]$ | 4.503 | -1% | -3% | -1% | -9% |
| $\Phi_{\text{B-8}}^\nu [10^6]$ | 3.694 | -3% | -7% | -4% | -2% |
| $\Phi_{\text{N-13}}^\nu [10^8]$ | 2.417 | -2% | -6% | -3% | -1% |
| $\Phi_{\text{O-15}}^\nu [10^8]$ | 1.811 | -3% | -8% | -4% | -2% |
| $\Phi_{\text{F-17}}^\nu [10^6]$ | 3.373 | -3% | -8% | -4% | -2% |

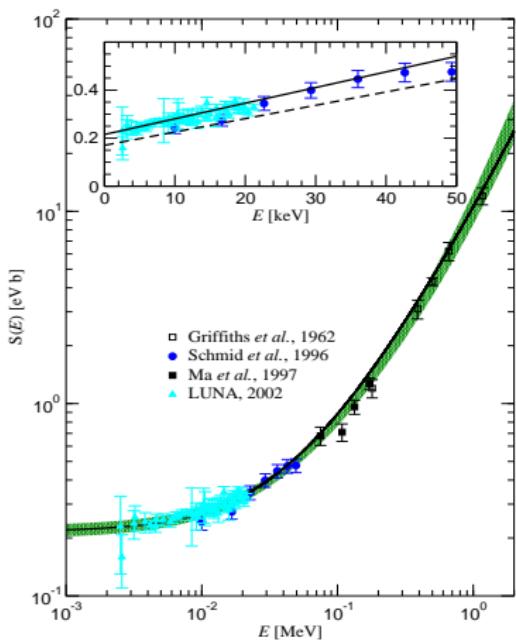
Latest calculation in χ EFT consistent up to N2LO



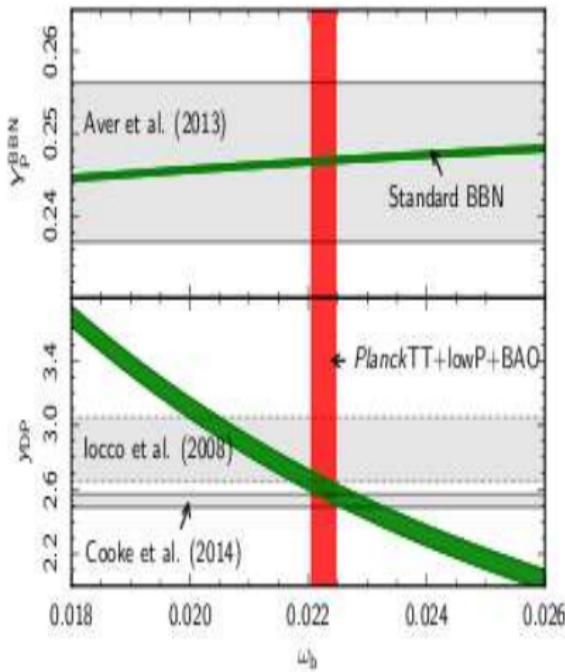
$$S_{\text{cor}}(0) = (4.047^{+0.024}_{-0.032}) \times 10^{-23} \text{ MeV fm}^2$$

B. Acharya *et al.*, arXiv:1693.01593

The $p + d \rightarrow {}^3\text{He} + \gamma$ reaction



SFII: E.G. Adelberger *et al.*, RMP 83, 195 (2011)



Planck Collab., arXiv:1501.01589 (2015)

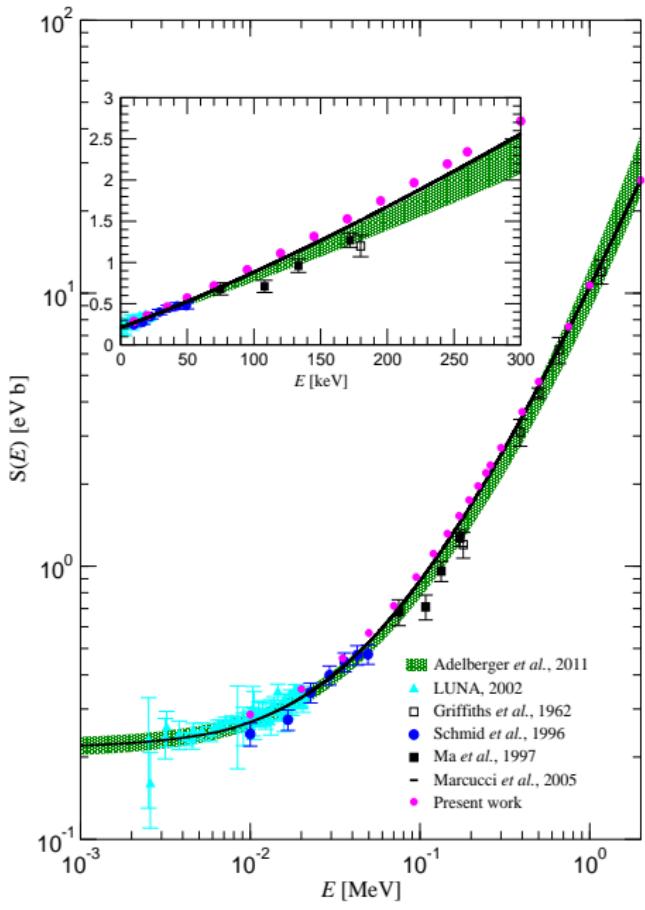
Latest news on the $p + d \rightarrow {}^3\text{He} + \gamma$ reaction

- New ongoing measurement by the **LUNA Collab.** at LNGS in the energy range of interest for BBN
- “New” calculation in **PhenAP**:
 - ➊ test of the nuclear scattering wave functions by calculating $\langle H \rangle = \langle \Psi^{LSJ} | H | \Psi^{LSJ} \rangle$ in a box with $R_{box} = 70$ fm
 - ➋ inclusion of the 1b RC term
- Preliminary calculation in **χ EFT**

PhenAp results

| E [keV] | ΔS_{WF} [%] | $\Delta j^{(RC)}$ [%] |
|-----------|---------------------|-----------------------|
| 35 | 1.1 | +1.3 |
| 70 | 0.4 | +2.1 |
| 95 | 0.3 | +2.3 |
| 120 | 0.8 | +2.4 |
| 145 | 0.4 | +2.5 |
| 170 | 0.4 | +2.6 |
| 195 | 0.4 | +2.6 |
| 220 | 0.5 | +2.8 |
| 245 | 0.4 | +2.7 |

L.E. Marcucci *et al.*, PRL 116, 102501 (2016)



Implications for Big Bang Nucleosynthesis

- $S(E) \rightarrow$ primordial ^2H abundance
- $^2\text{H}/\text{H}$ abundance as function of $\Omega_b h^2$ and N_{eff}
- Likelihood function to get the best fit on $\Omega_b h^2$ and N_{eff}

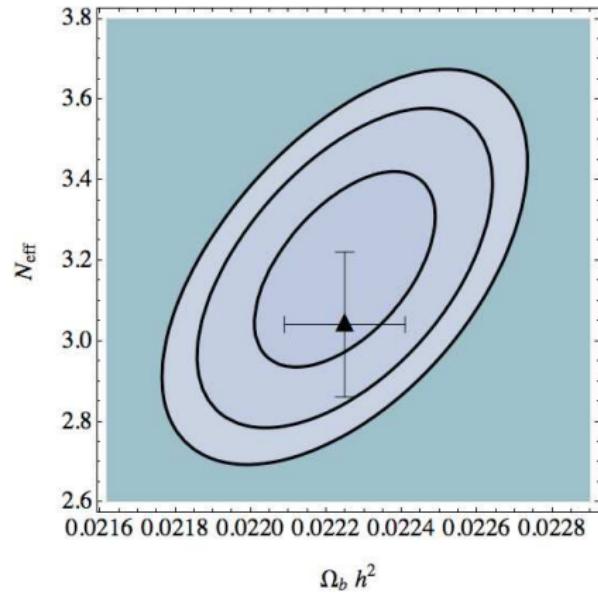
$$^2\text{H}/\text{H}|_{TH} = (2.49 \pm 0.03 \pm 0.03) \times 10^{-5}$$

$\Omega_b h^2 \rightarrow$ Planck 2015 & standard N_{eff}

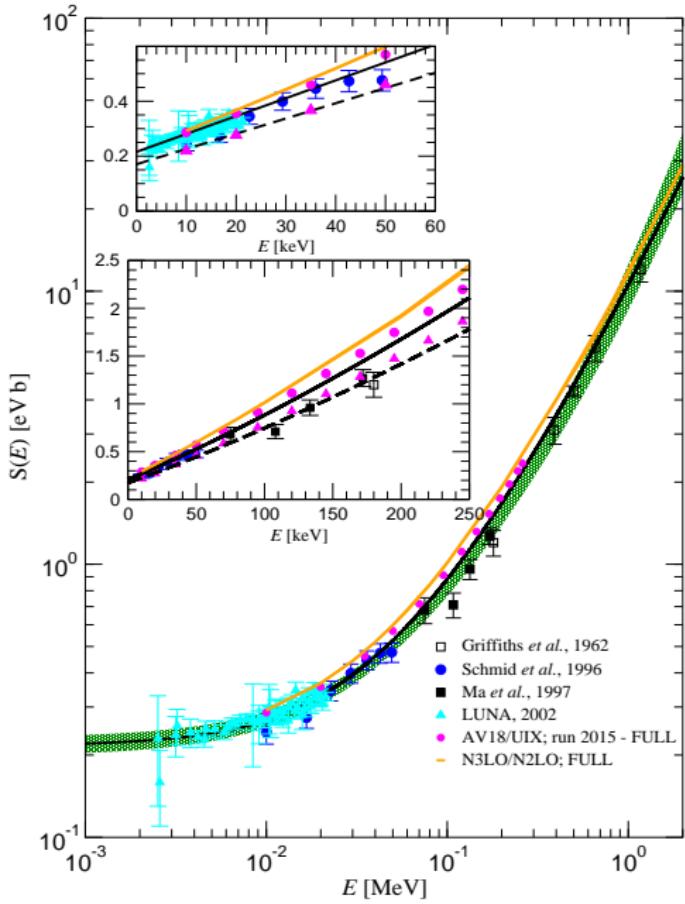
vs.

$$^2\text{H}/\text{H}|_{Exp} = (2.53 \pm 0.04) \times 10^{-5}$$

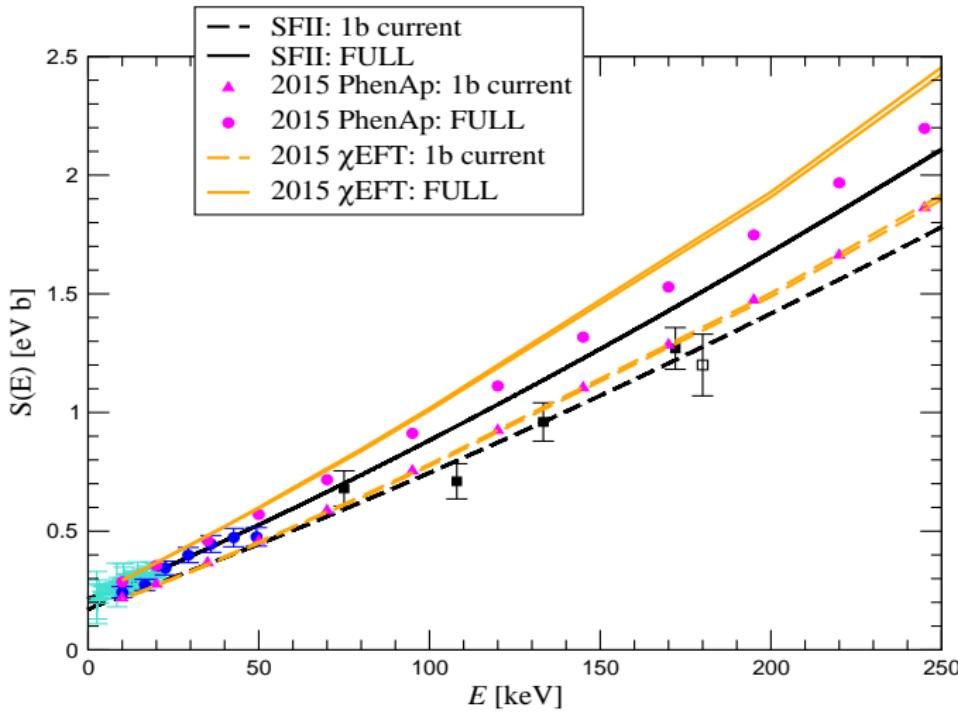
L.E. Marcucci *et al.*, PRL 116, 102501 (2016)



PRELIMINARY RESULTS in χ EFT

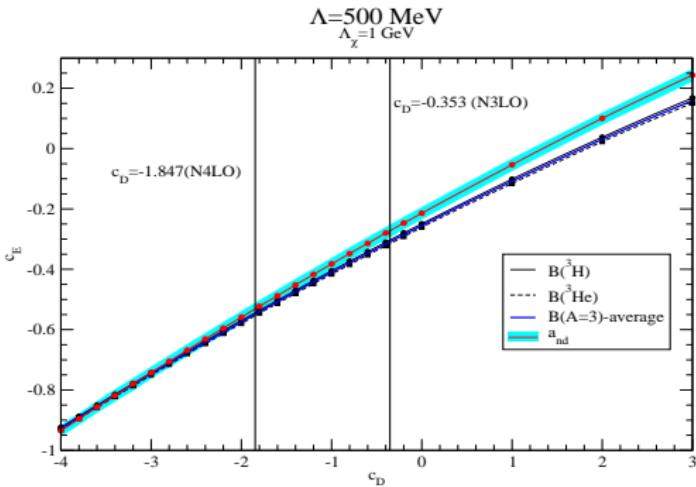


PRELIMINARY RESULTS in χ EFT: zoom for $E = 0 - 250$ keV



Outlook: the weak sector in χ EFT (I)

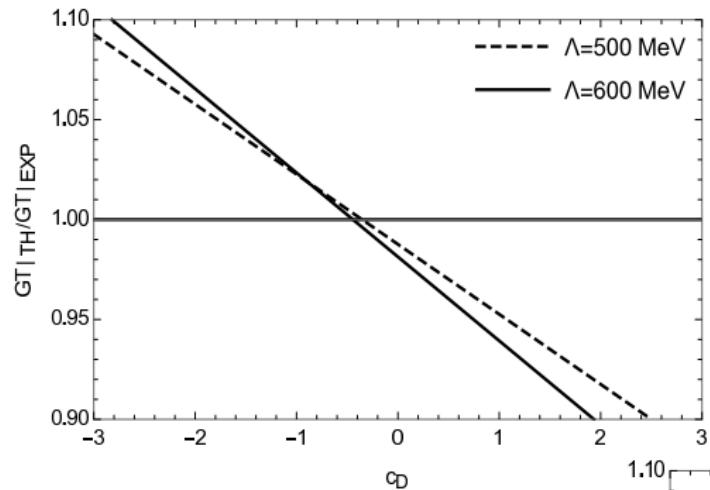
- Weak current up to $\mathcal{O}(Q^1) + \text{N3LO}(\text{Idaho})/\text{N2LO}$ (local form)
- Fit c_D using $B(A=3)$ and GT_{Exp} (**PRELIMINARY** – $\Lambda = 500$ MeV)



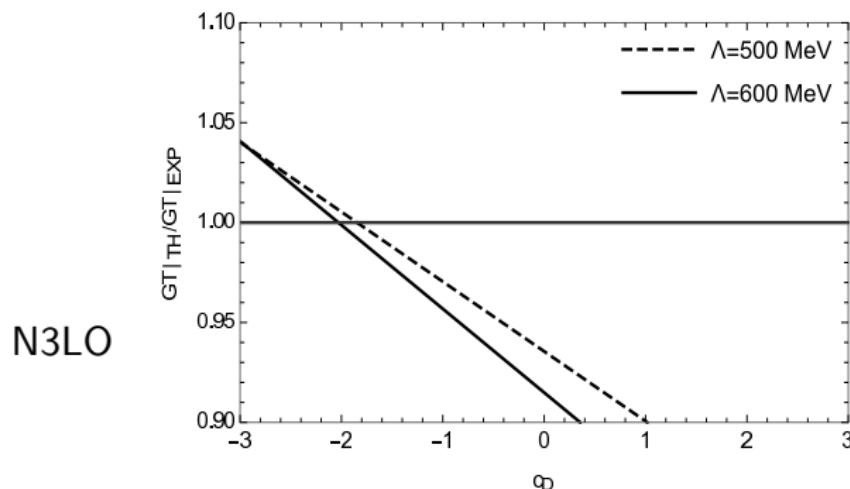
Fit of GT_{Exp} up to $\mathcal{O}(Q^1)$
 $\Rightarrow c_D = -1.847$ & $c_E = -0.548$
 $a_{nd} = 0.654$ fm

Fit of GT_{Exp} up to $\mathcal{O}(Q^0)$
 $\Rightarrow c_D = -0.353$ & $c_E = -0.305$
 $a_{nd} = 0.665$ fm

$$a_{nd}^{Exp} = 0.645 \pm 0.010 \text{ fm}$$



N2LO



N3LO

Outlook: the weak sector in χ EFT (II)

Using the χ EFT weak current up to $\mathcal{O}(Q^1)$

- μ -capture on deuteron and ${}^3\text{He}$
- $p + {}^3\text{He} \rightarrow {}^4\text{He} + e^+ + \nu_e$ – the *hep* reaction
- ν -scattering on light nuclei ($A = 2 - 4$)
- ...

Outlook: a new χ EFT potential with Δ 's

M. Piarulli *et al.*, PRC **91**, 024003 (2015)

- χ EFT with Δ 's in two-pion exchange up to $\mathcal{O}(Q^3)$ (N3LO)
- minimally non-local / coordinate space (operatorial structure \sim AV18)
- fit to the 2013 Granada database with $\chi^2/\text{datum} \sim 1.3$

Steps to be done:

- consistent TNI – work in progress
- fitting procedure of $c_D - c_E$
- EW currents with the same cutoff functions

Consistent χ EFT framework

⇒ from few- to less few-nucleon systems