Nucleon 3D structure from double parton scattering: a Light-Front quark model analysis

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Outlook

- Double parton scattering (DPS) and double parton distribution functions (dPDFs)

- The 3D proton structure in single & double parton scatterings

- Double parton correlations (DPCs) in double parton distribution functions

- dPDFs in constituent quark models

  M.R., S. Scopetta and V. Vento, PRD 87, 114021 (2013)
  M. R., S. Scopetta, M. Traini and V.Vento, JHEP 1412, 028 (2014)
  M. R., S. Scopetta, M. Traini and V.Vento, in preparation

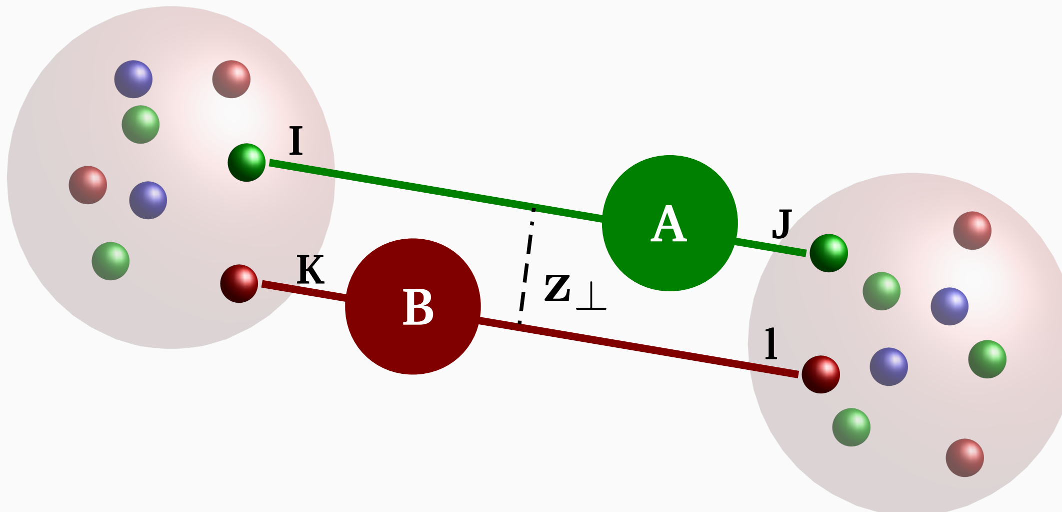
- Calculation of the "effective X-section"

  M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

- Conclusions

DPS and dPDFs from multi parton interactions

Multi parton interaction (MPI) can contribute to the, *pp* and *pA,* cross section @ the LHC:



The cross section for a DPS event can be written in the following way:

(N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982))

dPDF

Momentum scale

$$d\sigma = \frac{1}{S}\sum_{i,j,k,l}\hat{\sigma}_{ij}(x_1,x_3,\mu_A)\hat{\sigma}_{kl}(x_2,x_4,\mu_B)\int d\tilde{z}_\perp F_{ik}(x_1,x_2,z_\perp,\mu_A,\mu_B)F_{jl}(x_3,x_4,\boxed{z_\perp},\mu_A,\mu_B)$$

Momentum fraction carried by the parton inside the hadron

Transverse distance between the two partons

DPS processes are important for fundamental studies, *e.g.* the background for the research of new physics and to grasp information on the 3D PARTONIC STRUCTURE OF THE PROTON

How 3-Dimensional structure of a hadron can be investigated?

The 3D structure of a strongly interacting system (e.g. nucleon, nucleus..) could be accessed through different processes (e.g. SIDIS, DVCS, double parton sattering …), measuring different kind of Parton Distributions, providing different kind of information:

DVCS *Generalized Parton Distributions in impact parameter space*

$$\mathcal{H}(x_1,\boldsymbol{b}_\perp) \quad \mathcal{E}(x_1,\boldsymbol{b}_\perp)$$

longitudinal momentum fraction carried by the parton

transverse distance between the parton and center of proton

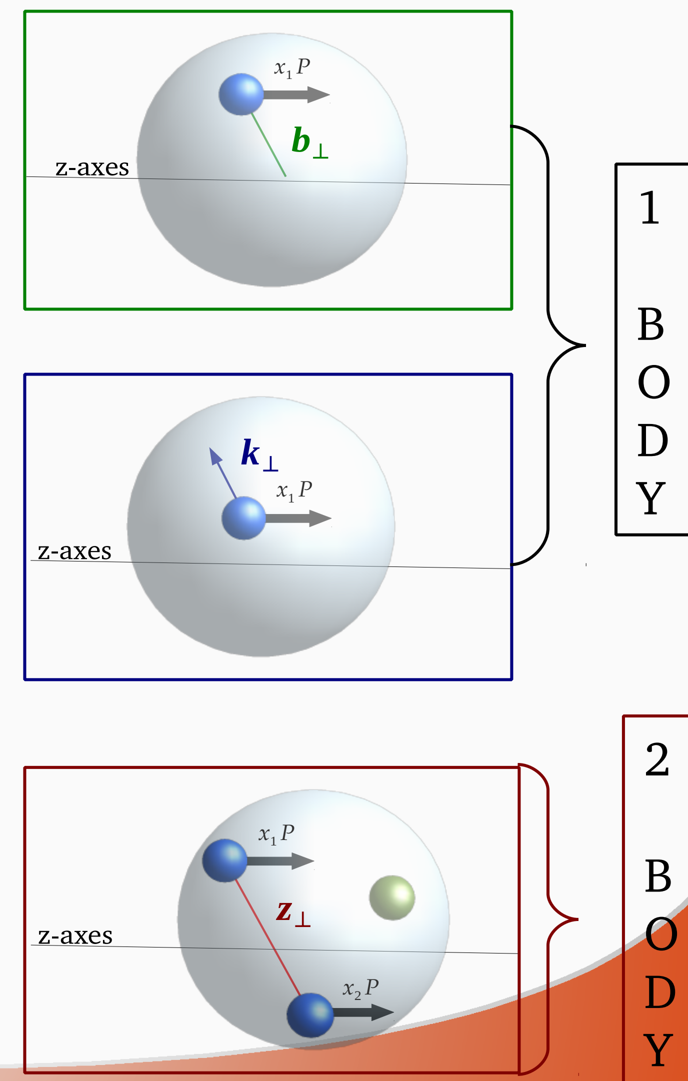
SIDIS.. *Transverse Momentum Dependent parton distribution functions*

$$f_1(x_1,\boldsymbol{k}_\perp) \quad g_{1L}(x_1,\boldsymbol{k}_\perp) \, h_1(x_1,\boldsymbol{k}_\perp) \; f_{1T}^{\perp}(x_1,\boldsymbol{k}_\perp)..$$

transverse component of the parton momentum

DPS *Double Parton Distribution Functions*

$$F_{UU}(x_1,x_2,\boldsymbol{z}_\perp) \quad F_{LL}(x_1,x_2,\boldsymbol{z}_\perp)$$



Parton correlations and dPDFs

@ LHC kinematics it is often used a factorized form of the dPDFs: $(\mathbf{x_1}, \mathbf{x_2}) - \mathbf{z}_\perp$ factorization:

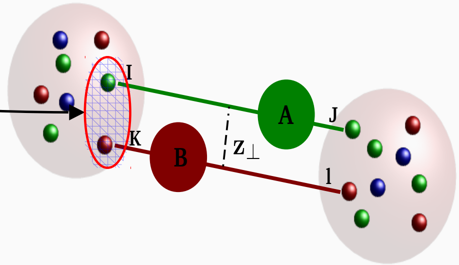
$$F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = F_{ij}(x_1, x_2, \mu) T(\vec{z}_\perp, \mu)$$

* Here and in the following: $\mu = \mu_A = \mu_B$

and $\mathbf{x_1}, \mathbf{x_2}$ factorization:

$$F_{ij}(x_1, x_2, \mu) = \underbrace{\overbrace{q_i(x_1, \mu)}^{\text{PDF}}\, q_j(x_2, \mu)}_{}\, \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n$$

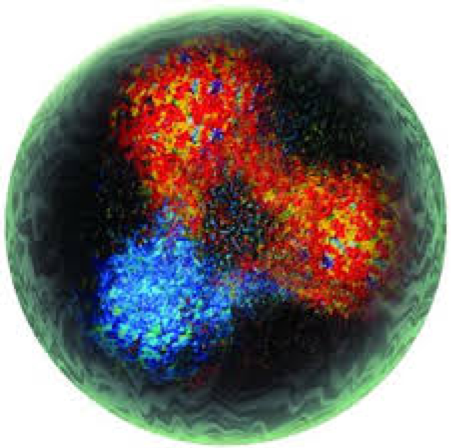
NO CORRELATION ANSATZ



In this scenario, parton correlations inside the proton are neglected!

- In principle, they are present!
- Several authors addressing this issue:
  Many published papers: Calucci and Treleani (1999), Korotkikh and Snigirev (2004), Gaunt and Stirling (2010), Diehl and Schäfer (2011), Snigirev (2011), Blok et al. (2012-2014), Schweitzer, Strikman and Weiss (2013), Gaunt and Szczurek (2015).......
- dPDFs are non-perturbative quantities ⟺ DPCs not calculated directly from QCD

DPCs in constituent quark models (CQM)



- Main features:
  - potential model
  - effective particles
  - particles are strongly bound and correlated

- CQM are a proper framework to describe DPCs, but their predictions are reliable ONLY in the valence quark region at low energy scale, while LHC data are available at small x

- At very low x, due to the large population of partons, the role of correlations may be less relevant BUT theoretical microscopic estimates are necessary

pQCD evolution of the calculated dPDFs is necessary to move towards the experimental kinematics!

i) dPDF evaluated at the initial scale of the model $\xrightarrow{\text{pQCD}}$ *ii)* dPDF evaluated at high generic scale

CQM calculations are able to reproduce the gross-feature of experimental PDFs in the valence region. CQM calculations are useful tools for the interpretation of data and for the planning of measurements of unknown quantities (e.g., TMDs in SiDIS, GPDs in DVCS...)

Similar expectations motivate the present investigation of dPDFs

The Light-Front approach

Relativity can be implemented, for a CQM, by using a Light-Front (LF) approach yielding, among other good features, the correct support. In the Relativistic Hamiltonian Dynamics (RHD) of an interacting system, introduced by Dirac (1949), one has:

$\boxed{a^{\pm} = a_0 \pm a_3}$

- Full Poincaré covariance
- fixed number of on-mass-shell particles

RHD
- Instant Form: $t_0 = 0$
  Evolution Operator: $P^0 = E$
- **Front Form (LF):**
  $x^+ = t_0 + z = 0$
  Evolution Operator: P^-

Among the 3 possibles forms of RHD we have chosen the LF one since there are several advantages. The most relevant are the following:

- 7 Kinematical generators (maximum number): i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) $\mathbf{P}^+$, $\mathbf{P}_\perp$, iii) Rotation around z.
- The LF boosts have a subgroup structure, then one gets a trivial separation of the intrinsic motion from the global one (as in the non relativistic (NR) case).
- In a peculiar construction of the Poincaré generators (Bakamjian-Thomas) it is possible to obtain a Mass equation, Schrödinger-like. A clear connection to NR.
- The IMF (Infinite Momentum Frame) description of DIS is easily included.

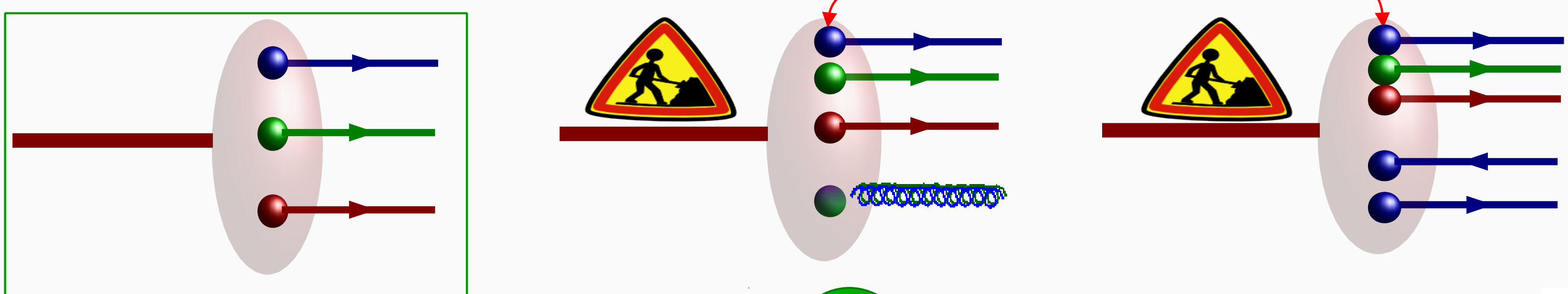
The LF approach is extensively used for hadronic studies (e.m. form factors, PDFs, GPDs, TMDs........)

A Light-Front wave function representation

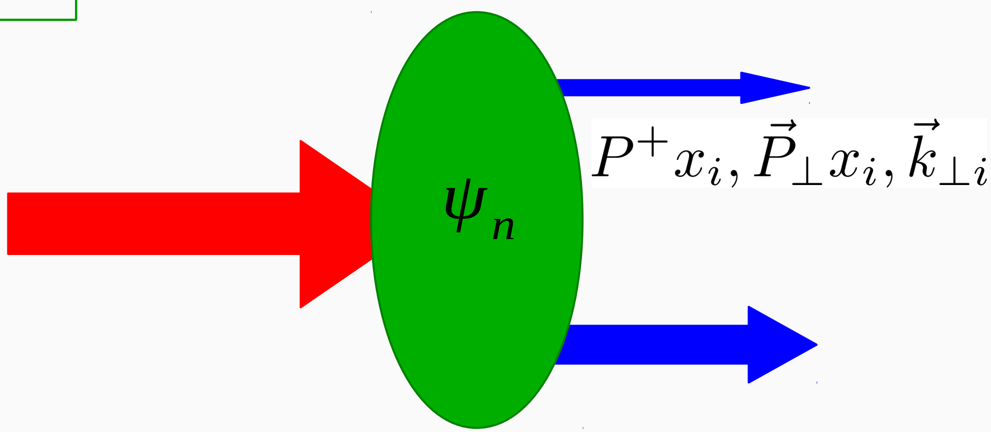
The proton wave function can be represented in the following way:

see *e.g.:* **S. J. Brodsky, H. -C. Pauli, S. S. Pinsky, Phys.Rept. 301, 299 (1998)**

$$|p, P^+ \; \vec{P}_\perp\rangle = \psi_{qqq}|qqq\rangle + \psi_{qqq\;g}|qqq\;g\rangle + \psi_{qqq\;q\bar{q}}|qqq\;q\bar{q}\rangle$$



p: $\mathrm{P}^+ \; \vec{P}_\perp$



$$\left\{ \begin{array}{l} x_i = \dfrac{k_i^+}{P^+} \\ \displaystyle\sum_i^n x_i = 1 \\ \displaystyle\sum_i^n \vec{k}_{\perp i} = 0 \end{array} \right.$$

$\psi_n^{[l]}(x_i, \vec{k}_{\perp i}, \lambda_i) \longleftrightarrow$ Invariant under LF boosts!

A Light Front wave function representation

It is possible to connect the front-form description of states and the canonical, instant-form one:

See *e.g.*: **B. D. Keister, W. N. Polyzou Adv. Nucl .Phys. 20, 225 (1991)**

$$|\vec{k}_\perp, \lambda, \tau\rangle_{[l]} \propto \sqrt{2k_0} \sum_{\lambda'} D^{1/2}_{\lambda\lambda'}(R_{il}(\vec{k})) |\vec{k}_\perp, \lambda', \tau\rangle_{[i]}$$

Melosh rotations

A relation between hadron wave functions $\psi^{[l]}_\lambda, \ \psi^{[i]}_\lambda$ can be obtained, *e.g.* for n=3:

$$\psi^{[l]}_\lambda(\beta_1, \beta_2, \beta_3) \propto \left[\frac{\omega_1\omega_2\omega_3}{M_0 x_1 x_2 x_3}\right] \sum_{\mu_1\mu_2\mu_3} D^{1/2*}_{\mu_1\lambda_1}(R_{il}(\vec{k}_1)) D^{1/2*}_{\mu_2\lambda_2}(R_{il}(\vec{k}_2)) D^{1/2*}_{\mu_3\lambda_3}(R_{il}(\vec{k}_3)) \psi^{[i]}_\lambda(\alpha_1, \alpha_2, \alpha_3)$$

$$\begin{cases} \beta_i = x_i, \vec{k}_i, \lambda_i, \tau_i \\ \alpha_i = \vec{k}_i, \mu_i, \tau_i \\ \omega_i = k_{0i} \\ M_0 = \sum_i \omega_i \end{cases}$$

dPDFs in a Light-Front approach

Extending the procedure developed in S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 649, 243 (2003) for GPDs, we obtained the following expression of the dPDF in momentum space, often called $_2$GPDs from the Light-Front description of quantum states in the intrinsic system:

$$\boxed{\vec{k}_1 + \vec{k}_2 + \vec{k}_3}$$

$$F_{ij}(x_1, x_2, k_\perp) = 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \Phi^*(\{\vec{k}_i\}, k_\perp)\Phi(\{\vec{k}_i\}, -k_\perp)$$

$$\times \quad \delta\left(x_1 - \frac{k_1^+}{M_0}\right)\delta\left(x_2 - \frac{k_2^+}{M_0}\right)$$

Conjugate to $z_\perp$

$$\boxed{M_0 = \sum_i \sqrt{\vec{k}_i^2 + m^2}}$$

GOOD SUPPORT

$$x_1 + x_2 > 1 \Rightarrow F_{ij}(x_1, x_2, k_\perp) = 0$$

$$\Phi(\{\vec{k}_i\}, \pm k_\perp) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_\perp}{2}, \vec{k}_2 \mp \frac{\vec{k}_\perp}{2}, \vec{k}_3\right)$$

Now we need a model to properly describe the hadron wave function in order to estimate the LF $_2$GPDs

$$\Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3) = D^{\dagger 1/2}(R_{il}(\vec{k}_1))D^{\dagger 1/2}(R_{il}(\vec{k}_2))D^{\dagger 1/2}(R_{il}(\vec{k}_3))\boxed{\psi^{[i]}(\vec{k}_1, \vec{k}_2, \vec{k}_3)}$$

Melosh rotation

Instant form proton w.f.

We need a CQM!

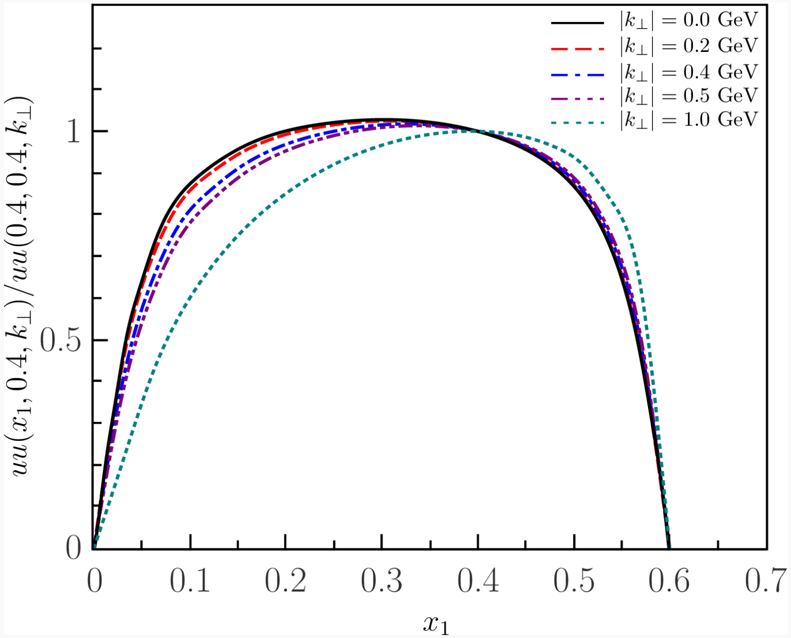
Numerical Results

M. R., S. Scopetta, M. Traini and V.Vento, JHEP 1412, 028 (2014))

Here, ratios, sensitive to correlations, are shown in order to test the factorization ansatz! For the calculation use has been made of the model of Ref.: **P. Faccioli *et al*, Nucl. Phys. A 656, 400-420 (1999)** based on the non-relativistic model of Ref.: **E. Santopinto *et al*, PLB 364 (1995)**

$$\frac{F_{uu}(x_1, 0.4, k_\perp)}{F_{uu}(0.4, 0.4, k_\perp)}$$

Factorization: $(x_1, x_2) - k_\perp$

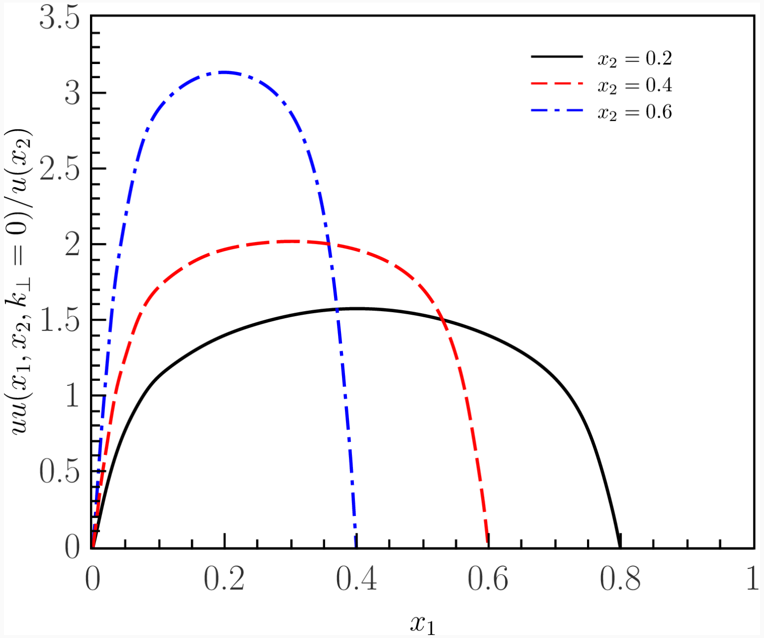


$_2$GPD

PDF

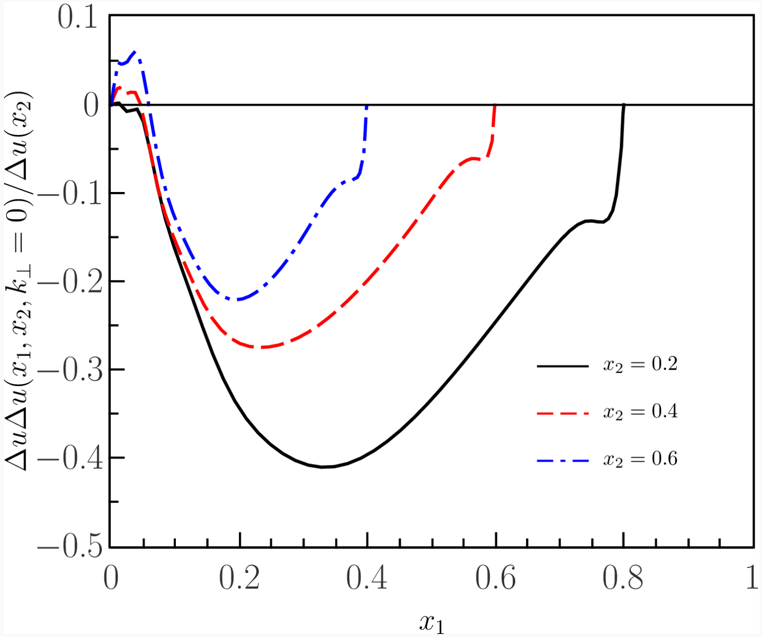
$$\frac{F_{uu}(x_1, x_2, k_\perp = 0)}{F_u(x_2)}$$

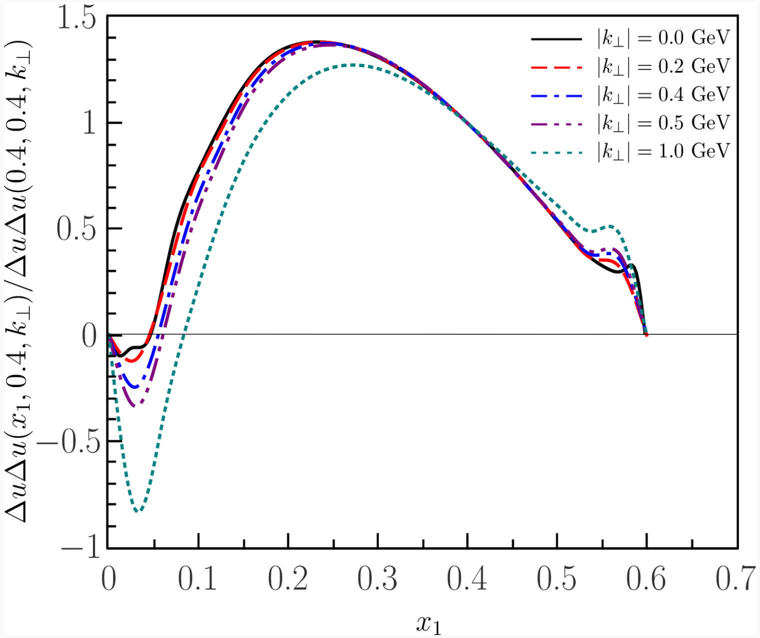
Factorization: $x_1 - x_2$



- Thanks to the good support of the calculated $_2$GPDs, the symmetry, due to the particle indistinguishability, is found! $\mathbf{uu(x_1, x_2, k_\perp = 0) = uu(x_2, x_1, k_\perp = 0)}$
- The $(x_1, x_2) - k_\perp$ and $x_1 - x_2$ factorizations are violated!

Results for spin correlations





$$u_{\uparrow(\downarrow)} u_{\uparrow(\downarrow)}(x_1, x_2, k_\perp) 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right) \Phi^*(\{\vec{k}_i\}, k_\perp) \frac{1 \pm \sigma_3(1)}{2} \frac{1 \pm \sigma_3(2)}{2} \Phi(\{\vec{k}_i\}, -k_\perp)$$

Here we have calculated: $\Delta u \Delta u(x_1, x_2, k_\perp) = \sum_{i=\uparrow,\downarrow} u_i u_i - \sum_{i \neq j = \uparrow,\downarrow} u_i u_j;$

$$\boxed{|\Delta u \Delta u| \leq uu}$$

Positivity bound

(defined in **M. Diehl et Al, JHEP 1203, 089 (2012), M. Diehl and T. Kasemets, JHEP 1305, 150 (2013))**

This particular distribution, different from zero also in an unpolarized proton, contains more information on spin correlations, which could be important at small x and large *t* (LHC) !

Also in this case, both factorizations, $x_1 - x_2$ and $(x_1, x_2) - k_\perp$ are strongly violated!

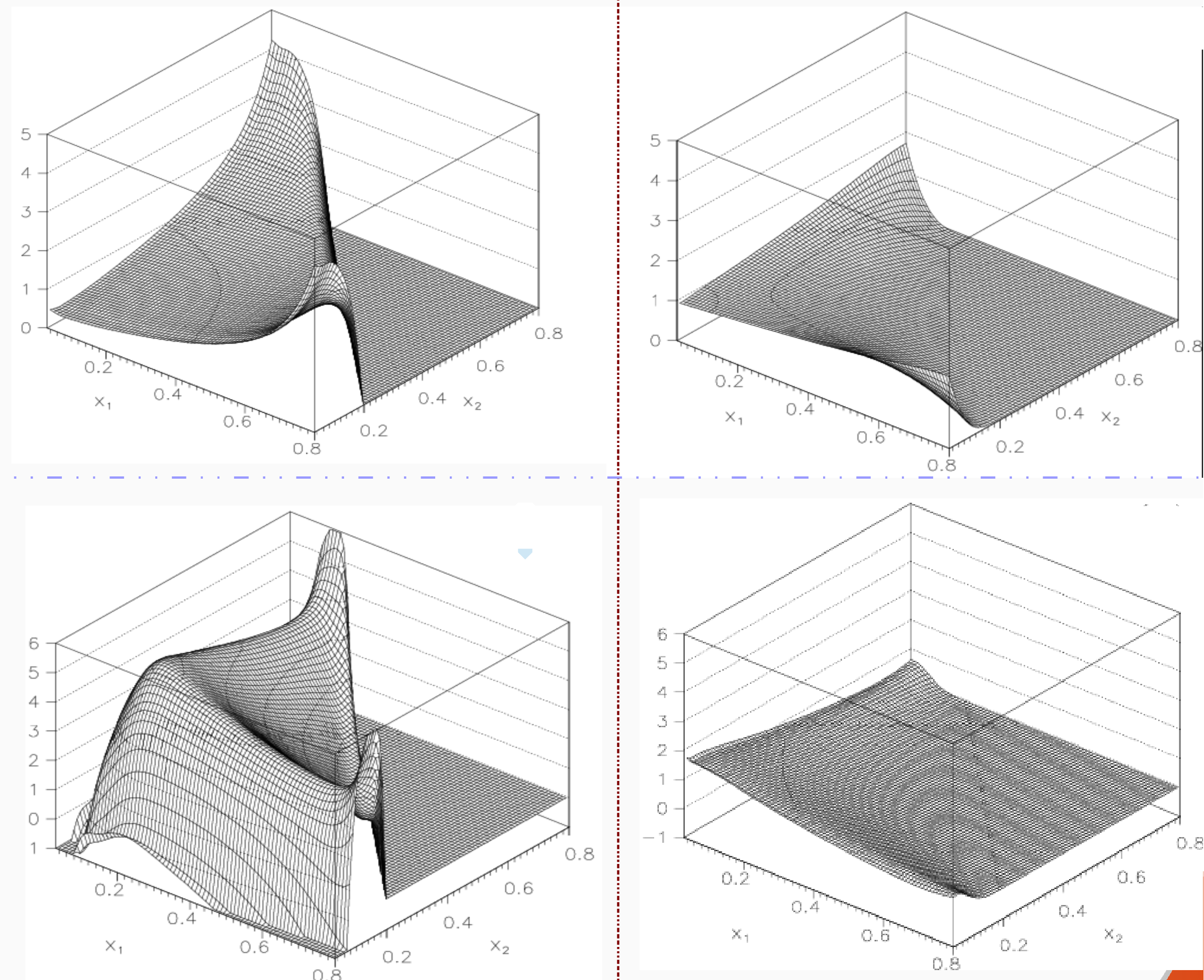
A pQCD evolution of the LF ${}_2$GPDs: the non-singlet sector

$$\mathbf{r_u(x_1, x_2; Q^2) = C_u \frac{uu(x_1, x_2; Q^2)}{u(x_1; Q^2) u(x_2; Q^2)}}$$

$$\mathbf{r_{\Delta u}(x_1, x_2; Q^2) = C_{\Delta u} \frac{\Delta u \Delta u(x_1, x_2; Q^2)}{\Delta u(x_1; Q^2) \Delta u(x_2; Q^2)}}$$

$$C_i = \frac{[\int dx F_i]^2}{\int dx_1 dx_2 F_{ii}(x_1, x_2, k_\perp = 0)}$$

$Q^2 = \mu_0^2 \simeq 0.1\ \mathrm{GeV}^2$ | $Q^2 = 10\ \mathrm{GeV}^2$

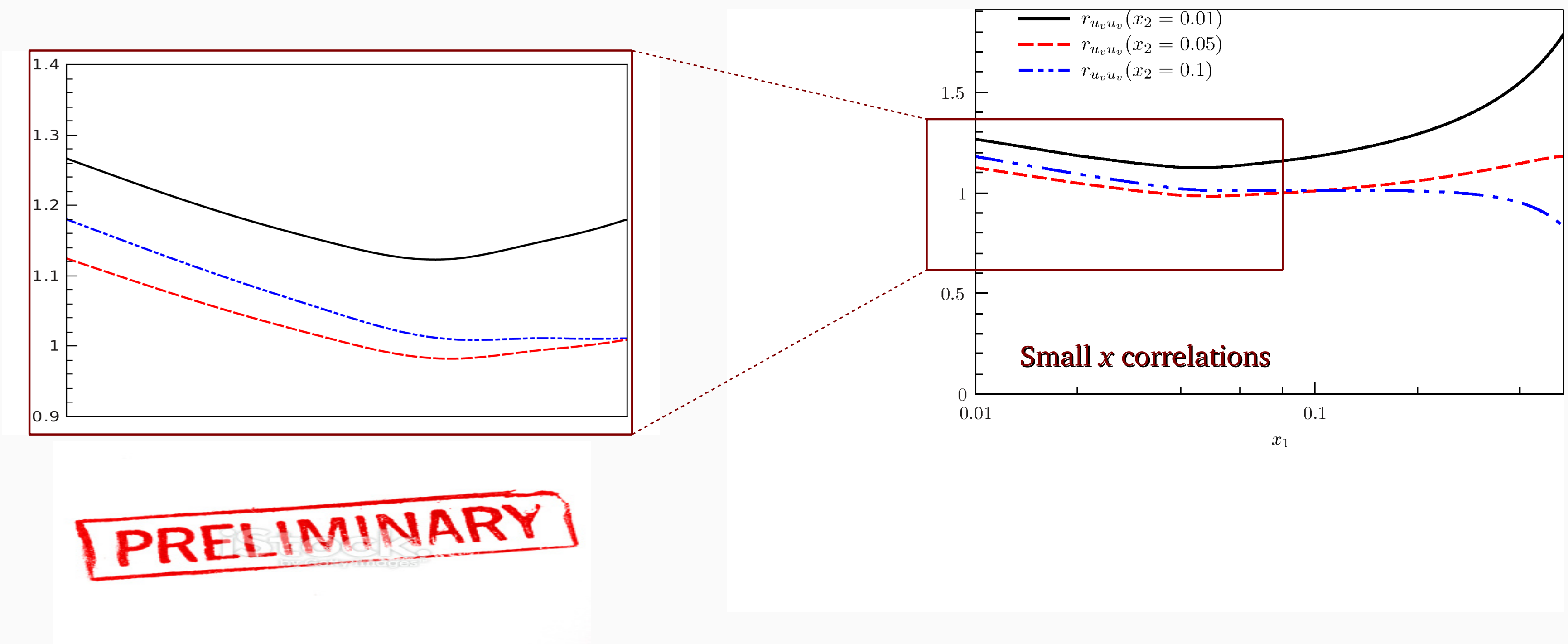


All these ratios would be 1 if there were no correlations!

A pQCD evolution of the LF $_2$GPDs: the non-singlet sector

Since DPS cross section depends on the final state and on the partonic flavours, active in the process, it is useful to analyse the following ratio in order to estimate the role of DPCs:

$$\mathrm{r}_{u_v u_v} = \frac{F_{u_v u_v}(x_1, x_2, k_\perp = 0; Q^2)}{u_v(x_1; Q^2) u_v(x_2; Q^2)}$$

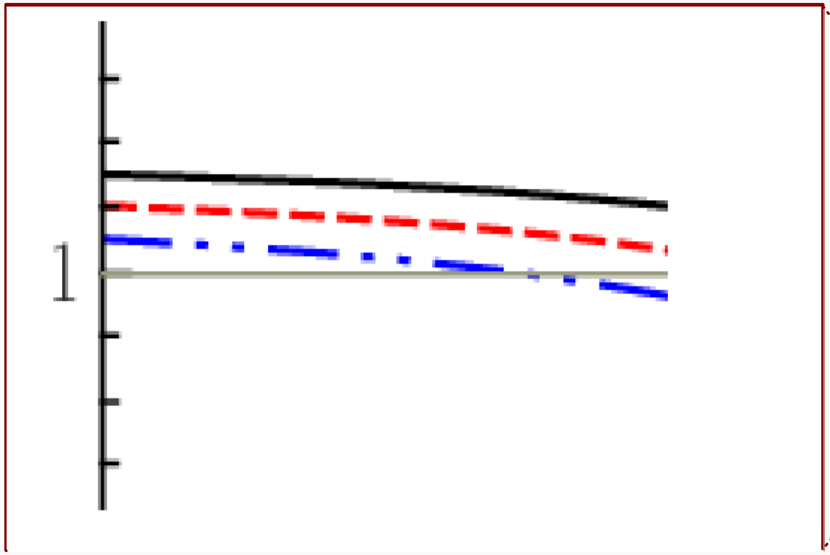


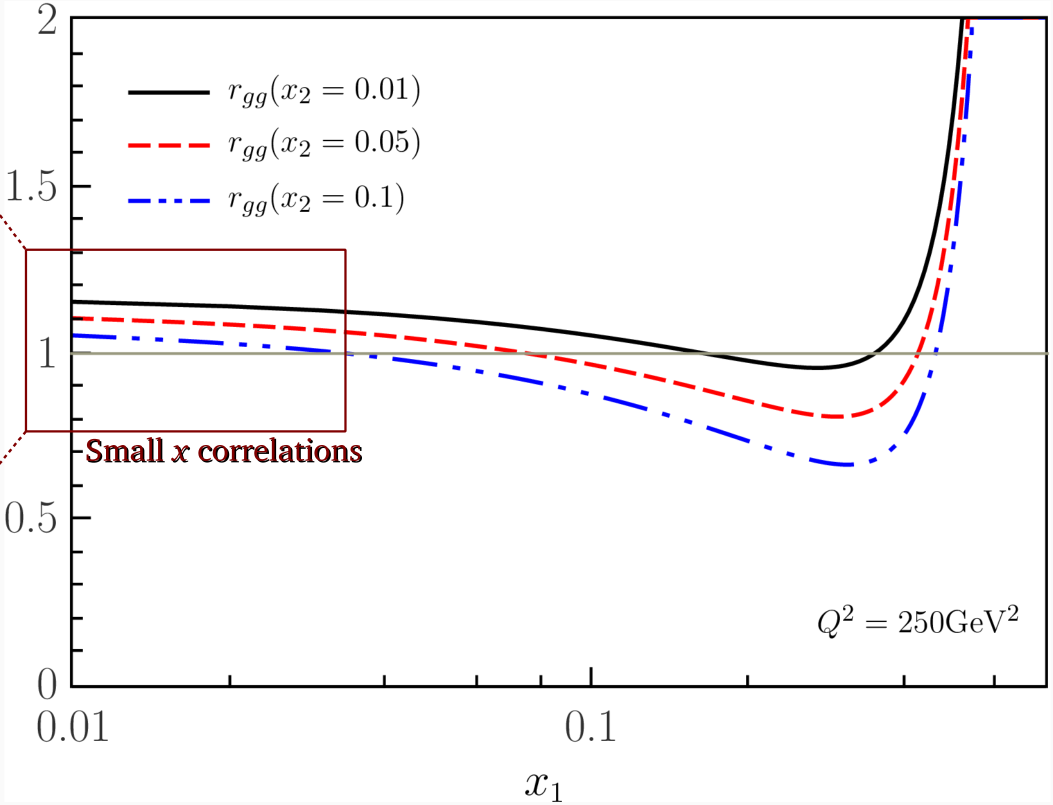
$r_{u_v u_v} \neq 1 \Longrightarrow$ **CORRELATIONS**

A pQCD evolution of the LF $_2$GPDs: perturbative sea and gluons

Since DPS cross section depends on the final state and on the partonic flavours, active in the process, it is useful to analyse the following ratio in order to estimate the role of DPCs:

$$\mathrm{r}_{gg} = \frac{F_{gg}(x_1, x_2, k_\perp = 0; Q^2)}{g(x_1; Q^2) g(x_2; Q^2)}$$





$r_{gg} \neq 1$ → **CORRELATIONS**

The Effective X-section

A fundamental tool for the comprehension of the role of DPS in hadron-hadron collisions is the so called "effective X-section": σ_{eff}

This object can be defined through the "pocket formula":

$$\sigma_{eff} = \boxed{\frac{m}{2}} \frac{\sigma_A^{pp'} \boxed{\sigma_B^{pp'}}}{\sigma_{double}^{pp}}$$

Sensitive to correlations

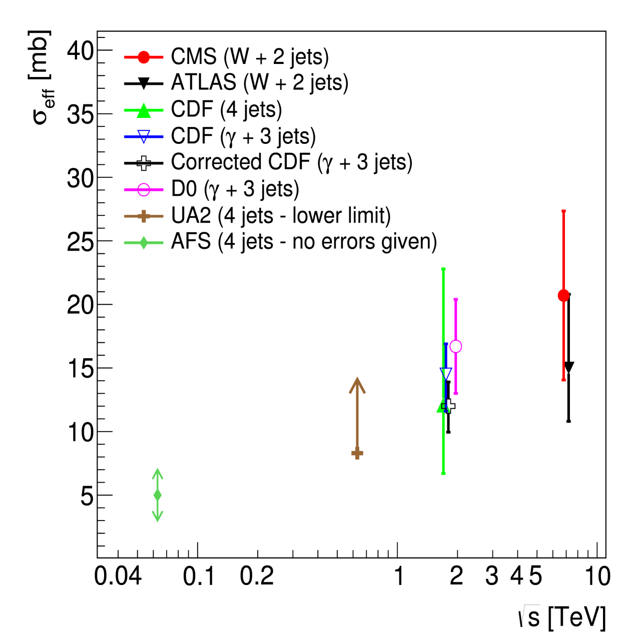
Combinatorial factor

Differential cross section for the process: pp' $\rightarrow A(B) + X$

Differential cross section for a DPS event: pp' $\rightarrow A + B + X$

....EXPERIMENTAL STATUS:

- Difficult extraction, approved analysis of for the production of same sign *WW* @LHC (RUN 2)
- the model dependent extraction of σ_{eff} from data is consistent with a "constant" , nevertheless there are large errorbars:
- different ranges in x_i accessed in different experiments!



High x for hard jets (heavy particles detected, large partonic s):

AFS $\longrightarrow$ $y \sim 0;\ x_1 \sim x_2;\ 0.2 < x_{1,2} < 0.4$

CDF $\longrightarrow$ $0.02 < x_{1,2,3,4} < 0.4$

valence region included!

The Effective X-section calculation

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

$$\boxed{\sigma_{eff} = \frac{m}{2}\frac{\sigma_A^{pp'}\ \sigma_B^{pp'}}{\sigma_{double}^{pp}}}$$

This quantity can be written in terms of PDFs and dPDFs!

In terms of PARTON DISTRIBUTIONS, $\sigma_{A(B)}^{pp'}$ and σ_{double}^{pp} can be written as follows:

$$\sigma_{A(B)}^{pp'}(x_1, x_1', \mu_1) = \sum_{\boxed{i,k}} F_i^p(x_1,\mu_1)\boxed{F_k^{p'}(x_{1'},\mu_1)}\hat{\sigma}_{ik}^{A(B)}(x_1, x_{1'}, \mu_1)$$

$$\boxed{i,k = \{q, \bar{q}, g\}}$$

Standard PDF

Proportional to colour coefficient and universal function: $C_{ij}\bar{\sigma}(x,x')$

$$\sigma_{double}^{pp}(x_1,x_1',x_2,x_2',\mu) = \frac{m}{2}\sum_{i,j,k,l}\hat{\sigma}_{ik}^A(x_1,x_1',\mu)\hat{\sigma}_{jl}^B(x_2,x_2',\mu) \times \int \frac{d\vec{k}_\perp}{(2\pi)^2} F_{ij}(x_1,x_2,k_\perp,\mu)\boxed{F_{kl}(x_1',x_2',-k_\perp,\mu)}$$

$_2$GPDs

Finally, combining the previous equations in the "pocket formula", one obtains:

Here the scale is omitted

$$\boxed{\sigma_{\mathbf{eff}}(\mathbf{x_1},\mathbf{x_1'},\mathbf{x_2},\mathbf{x_2'}) = \frac{\sum_{\mathbf{i,k,j,l}}\mathbf{F_i}(\mathbf{x_1})\mathbf{F_k}(\mathbf{x_1'})\mathbf{F_j}(\mathbf{x_2})\mathbf{F_l}(\mathbf{x_2'})\mathbf{C_{ik}}\mathbf{C_{jl}}}{\sum_{\mathbf{i,j,k,l}}\mathbf{C_{ik}}\mathbf{C_{jl}}\int \mathbf{F_{ij}}(\mathbf{x_1},\mathbf{x_2};\mathbf{k_\perp})\mathbf{F_{kl}}(\mathbf{x_1'},\mathbf{x_2'};-\mathbf{k_\perp})\frac{\mathbf{dk_\perp}}{(\mathbf{2\pi})^\mathbf{2}}}}$$

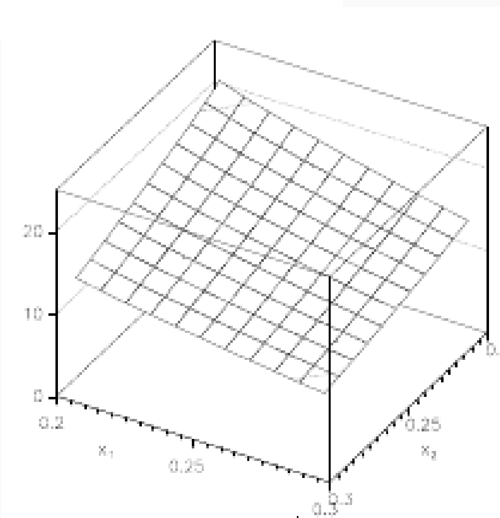
Non trivial x-dependence

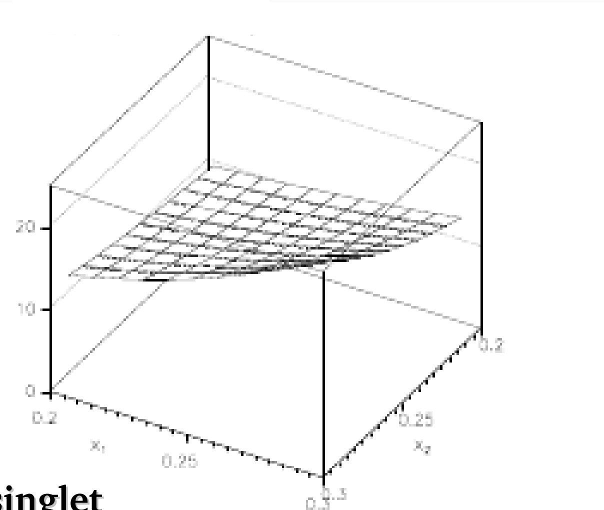
Numerical results

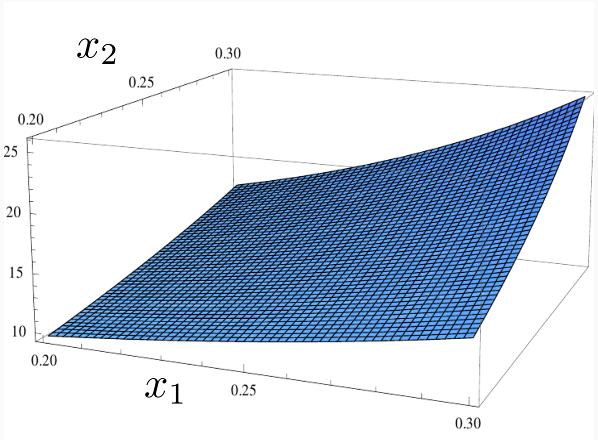
M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

Our predictions of σ_{eff} in the valence region at different energy scales:

$\sigma_{eff}(x_1, x_2, \mu_0^2)$ $\sigma_{eff}(x_1, x_2, Q^2 = 250 \text{ GeV}^2)$ $\sigma_{eff}(x_1, x_2, Q^2 = 250 \text{ GeV}^2)$



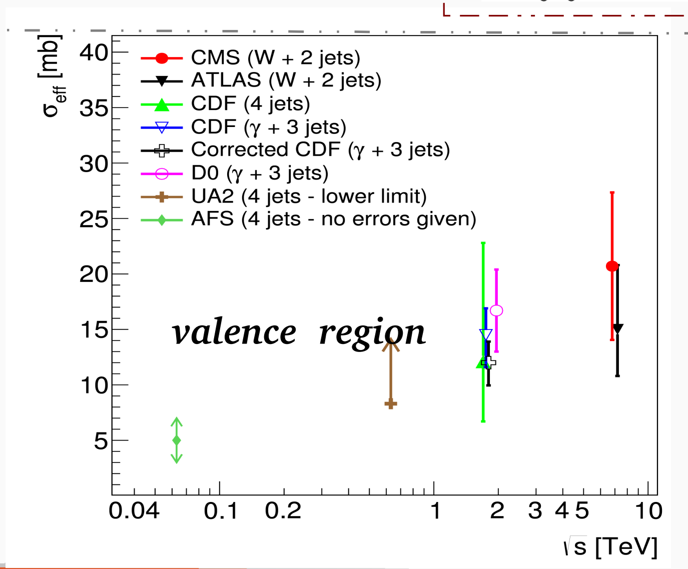


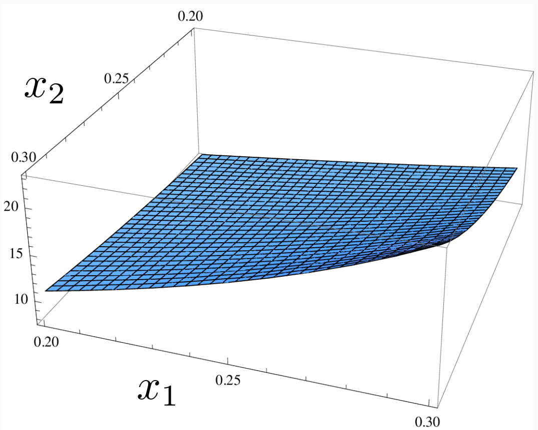


Non-singlet

$\overline{\sigma_{eff}} \sim 11$ mb

Non-singlet & gluons





PRELIMINARY

Non-singlet & sea

The old data lie in the obtained range of σ_{eff}

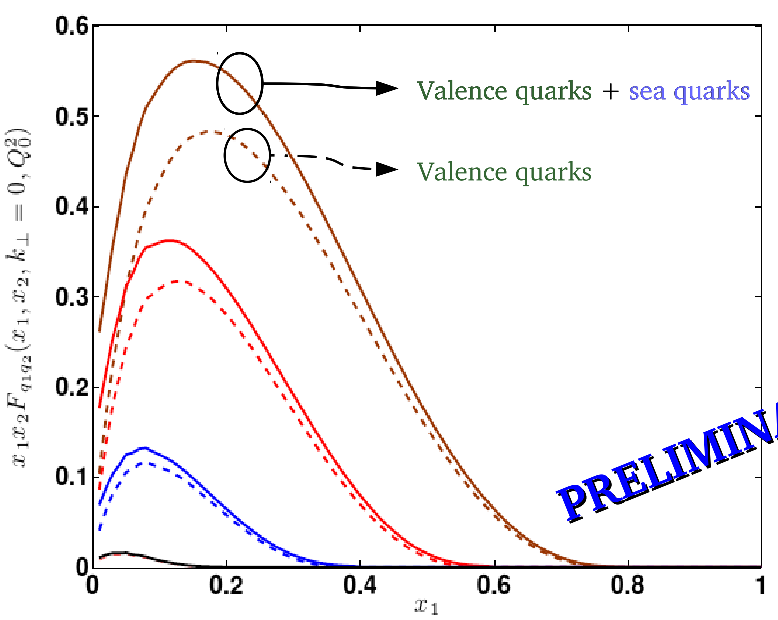
Introduction of non perturbative sea quarks at $k_\perp = 0$

From PDF analyses it is clear the necessity of including non perturbative sea quarks and gluons at the initial scale of the model. In order face this problem a simplified approach has been used:

$$F_{uu}(x_1, x_2, k_\perp = 0; Q_0^2) \sim \underbrace{F_{u_v u_v}(x_1, x_2, k_\perp = 0; Q_0^2)} + (1 - x_1 - x_2)^n \theta(1 - x_1 - x_2)$$
$$+ u_v(x_1; Q_0^2)\bar{u}(x_2; Q_0^2)$$
$$+ \bar{u}(x_1; Q_0^2)u_v(x_2; Q_0^2)$$

- Pure valence contribution obtained evolving in pQCD the model calculation of dPDF from the initial scale μ_0^2 to the scale Q_0^2

- Non perturbative sea quark contributions (effective high Fock states) $n=0.2$



PRELIMINARY!!!

PDF LO MSTW2008

$\bar{u}(x; Q_0^2)$ $u_v(x; Q_0^2)$ $Q_0^2 = 1$ GeV^2

$x_2 = 0.8$; $x_2 = 0.6$; $x_2 = 0.4$; $x_2 = 0.2$;

Conclusions

A CQM calculation of the dPDFs with a fully covariant approach:

M. R., S. Scopetta, M. Traini and V.Vento, JHEP 1412, 028 (2014)

- symmetry in the exchange of two partons in the dPDFs correctly restored
- violations of both the $(x_1, x_2) - k_\perp$ and x_1, x_2 factorizations for the polarized and unpolarized $_2$GPDs
- at very small x, the role of correlations is less important after evolution to experimental scales, spin correlations are still important;
- Evaluation of dPDF with sea quarks and gluons perturbatively generated

Calculation of the effective X-section

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2015)

- Calculation of the effective X-section at the hadronic and at high energy scales
- **x-dependent quantity obtained!** Qualitatively in agreement with data
- The x-dependence of the "effective X-section" could give information on the

3d structure of the proton!

What are we working on

- pQCD evolution of the calculated $_2$GPDs taking into account the sea contribution; PRELIMINARY
- analysis of the inhomogeneous contribution in the pQCD evolution;
- non perturbative Gluons and sea quarks (higher Fock states) to be included into the scheme.

PRELIMINARY

Direct link to LHC Physics