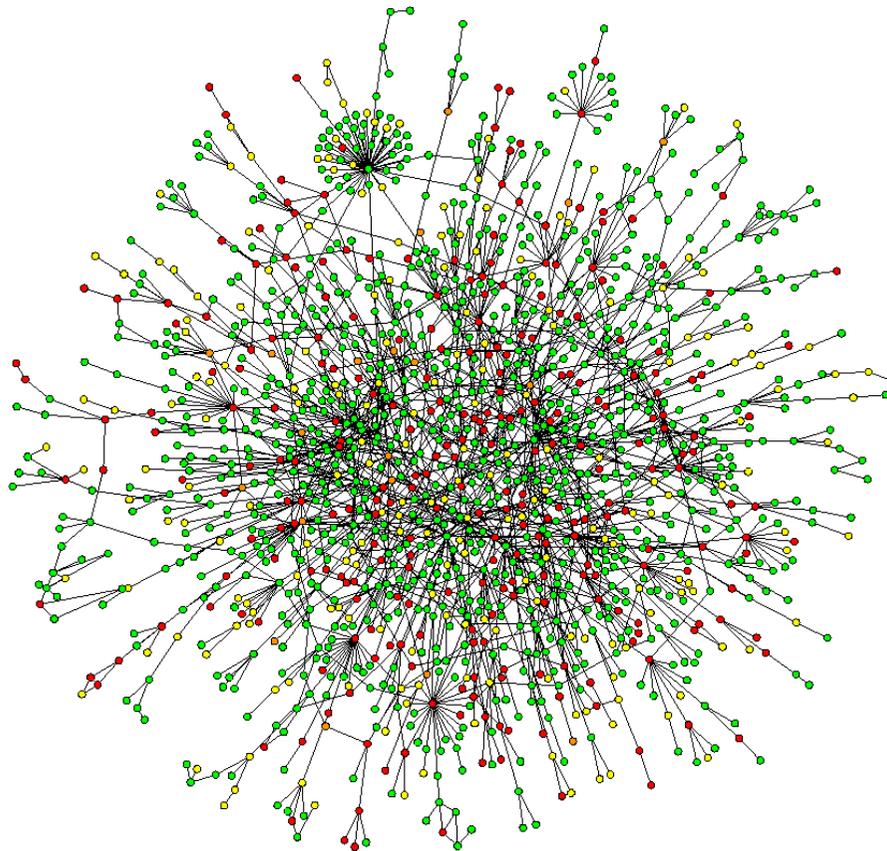


La Fisica delle Reti Complesse

Sebino Stramaglia

Dip. Fisica, Univ. Bari

I.N.F.N.





Cerca: “Complex Networks”

Risultati **1 - 10** su circa **2.080.000** per **complex networks**. (**0,36** secondi)

Cosa hanno in comune Al Qaeda,
gli attori di Hollywood e le
proteine ?

Risposta: Si organizzano
allo stessa maniera

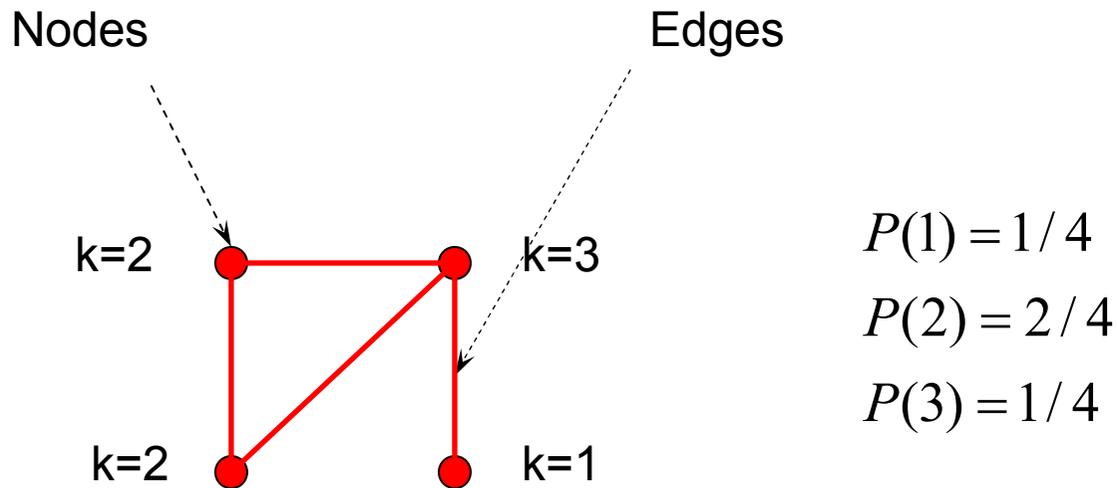
Sono sistemi complessi: formati da un
grande numero di elementi che
interagiscono

**Su grande scala, le proprietà dei
singoli elementi non sono
importanti, ma sono fondamentali
le relazioni tra essi.**

Rete complessa: modello geometrico (fatto di punti e
linee) per rappresentare le relazioni tra gli elementi di
un sistema complesso

Networks: Definizioni

1) **Network** è un insieme di nodi (oggetti) collegati da segmenti (Edges).



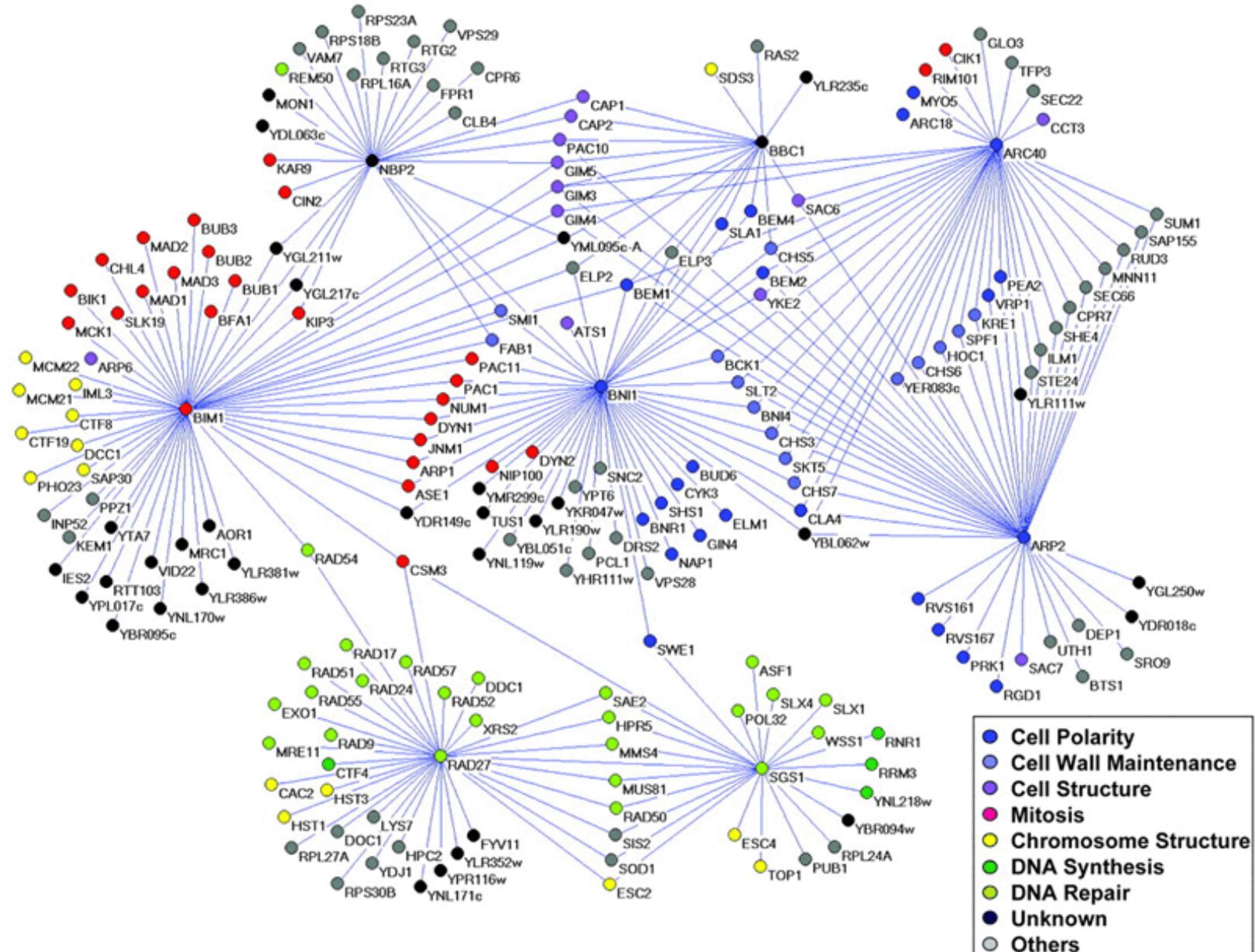
2) **Degree** (k), o grado, di un nodo è il numero di edges cui è collegato

3) **Degree Distribution** $P(k)$ è la probabilità (frazione) che un nodo abbia grado k

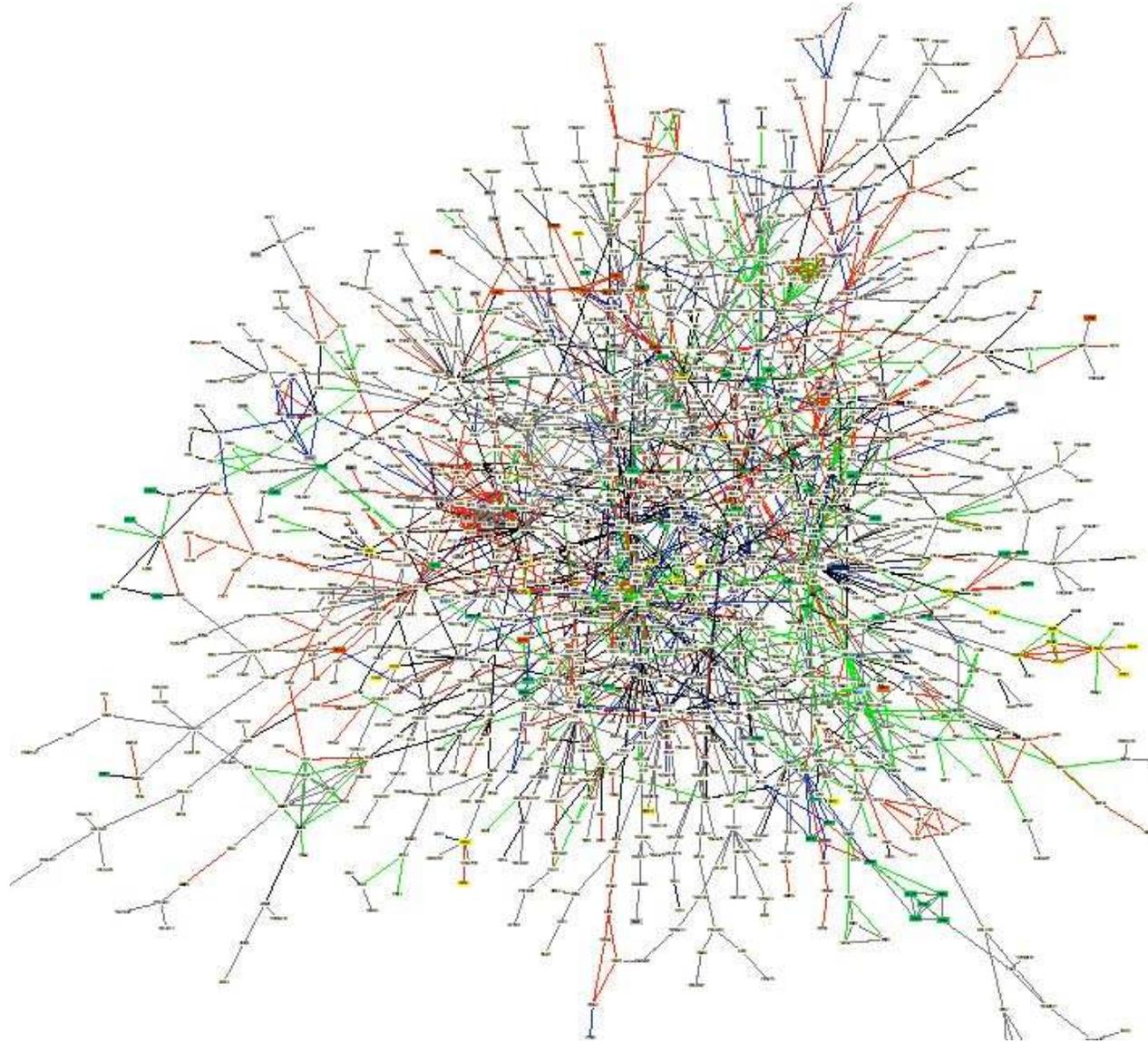
Cervello: nodi (neuroni) linee (sinapsi)



Genetic interaction network



Yeast protein-protein interaction network

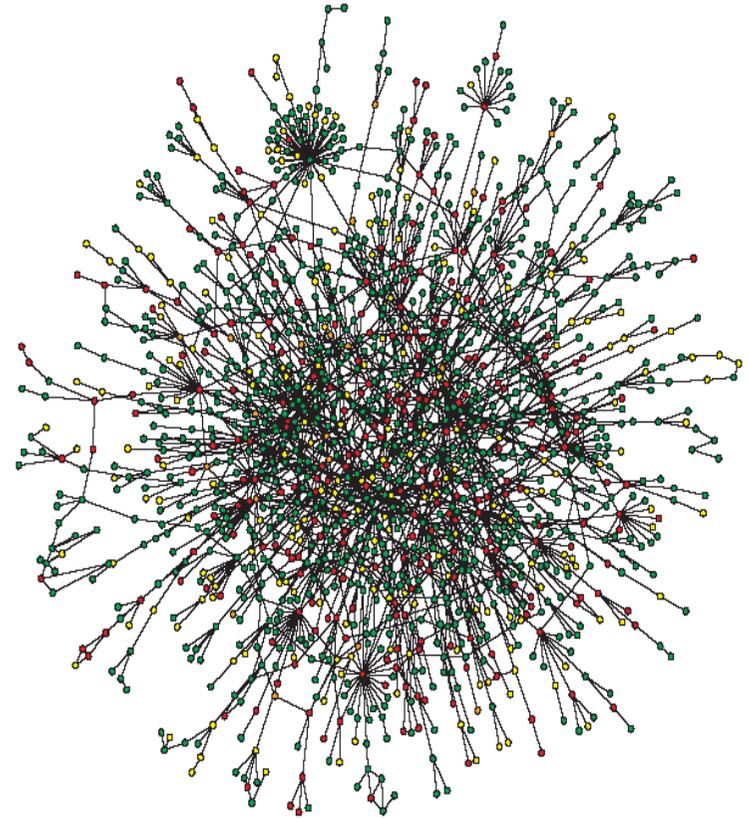


Altre reti biologiche

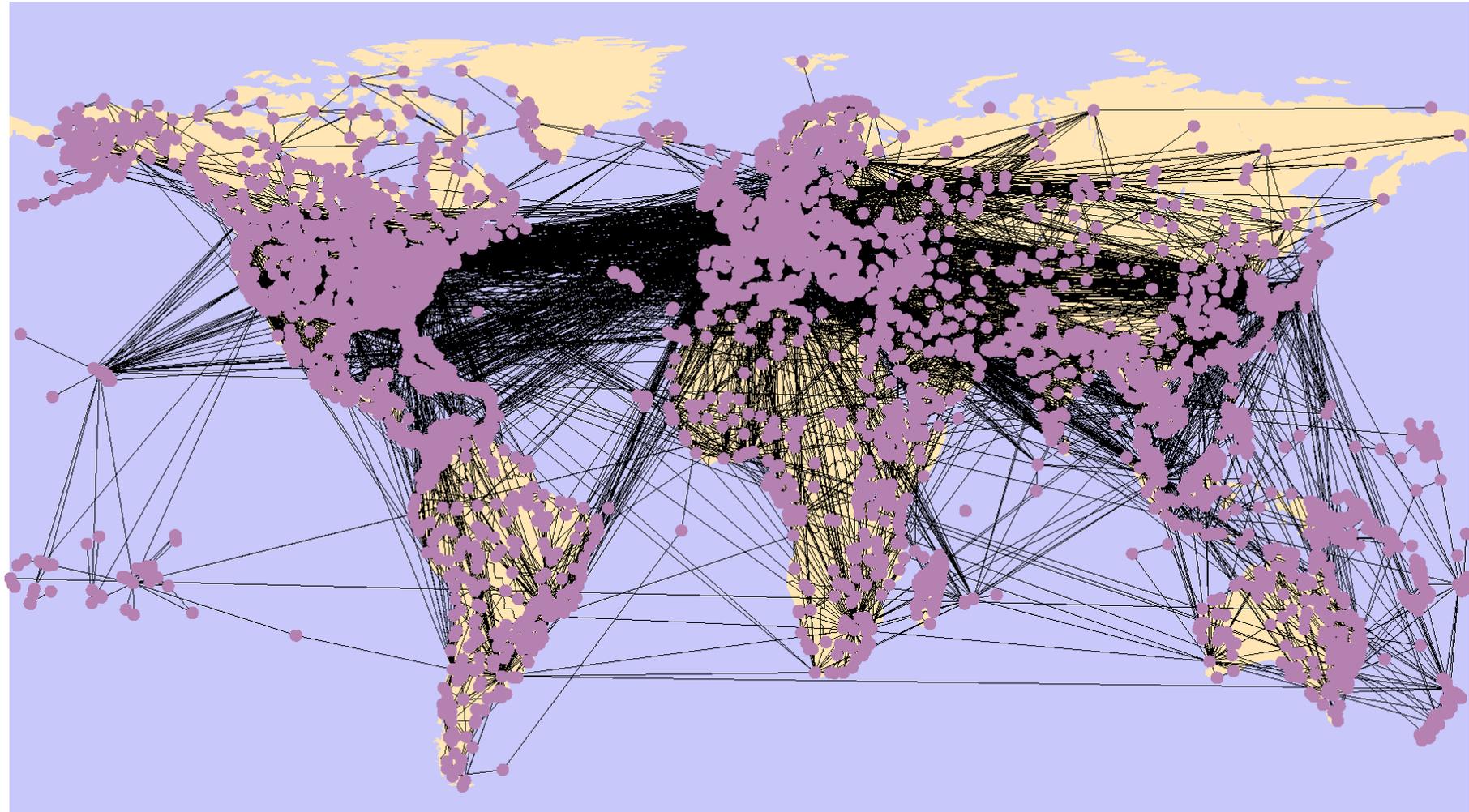
Reti di proteine: proteine connesse da interazioni fisiche

Reti di metaboliti: connessioni tra prodotti e substrati che partecipano in una reazione

Gene regulatory networks: due geni sono connessi se l'espressione di un gene modula l'espressione di un altro, sia attivandola o inibendola



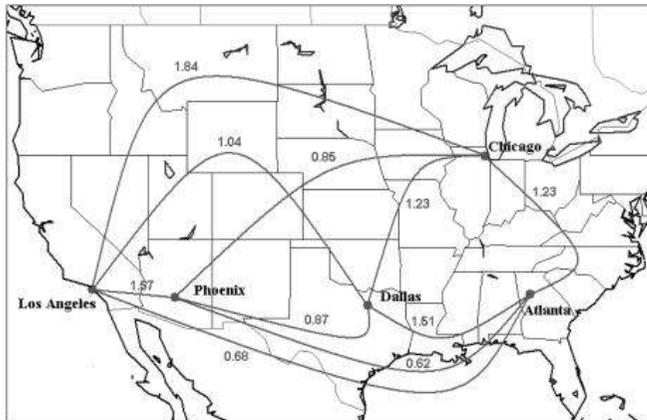
System Biology



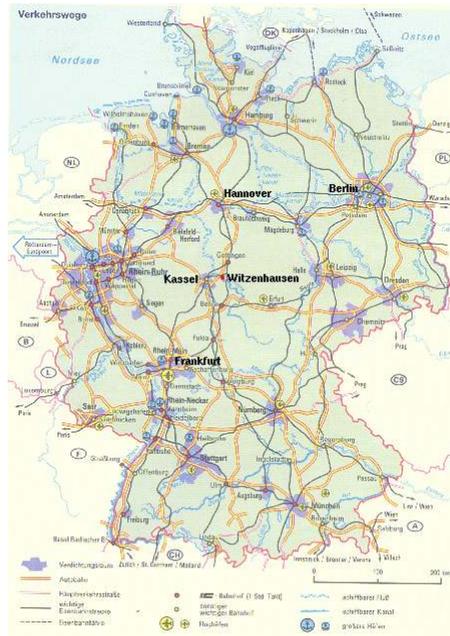
World-wide airport
network

Transportation Networks

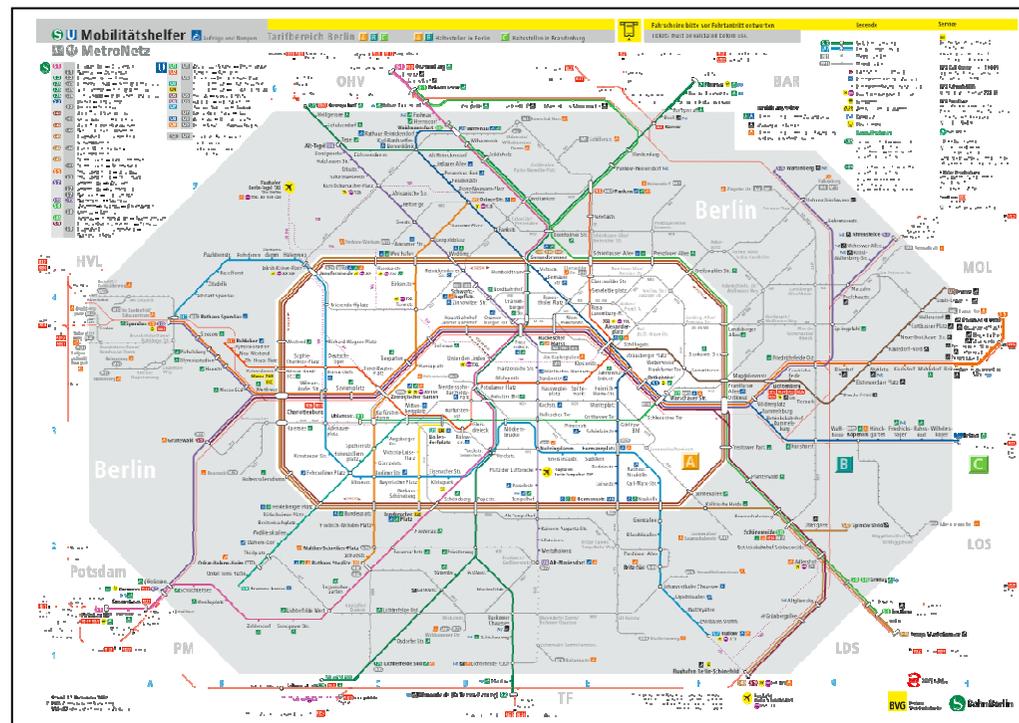
Airport Networks



Road Maps



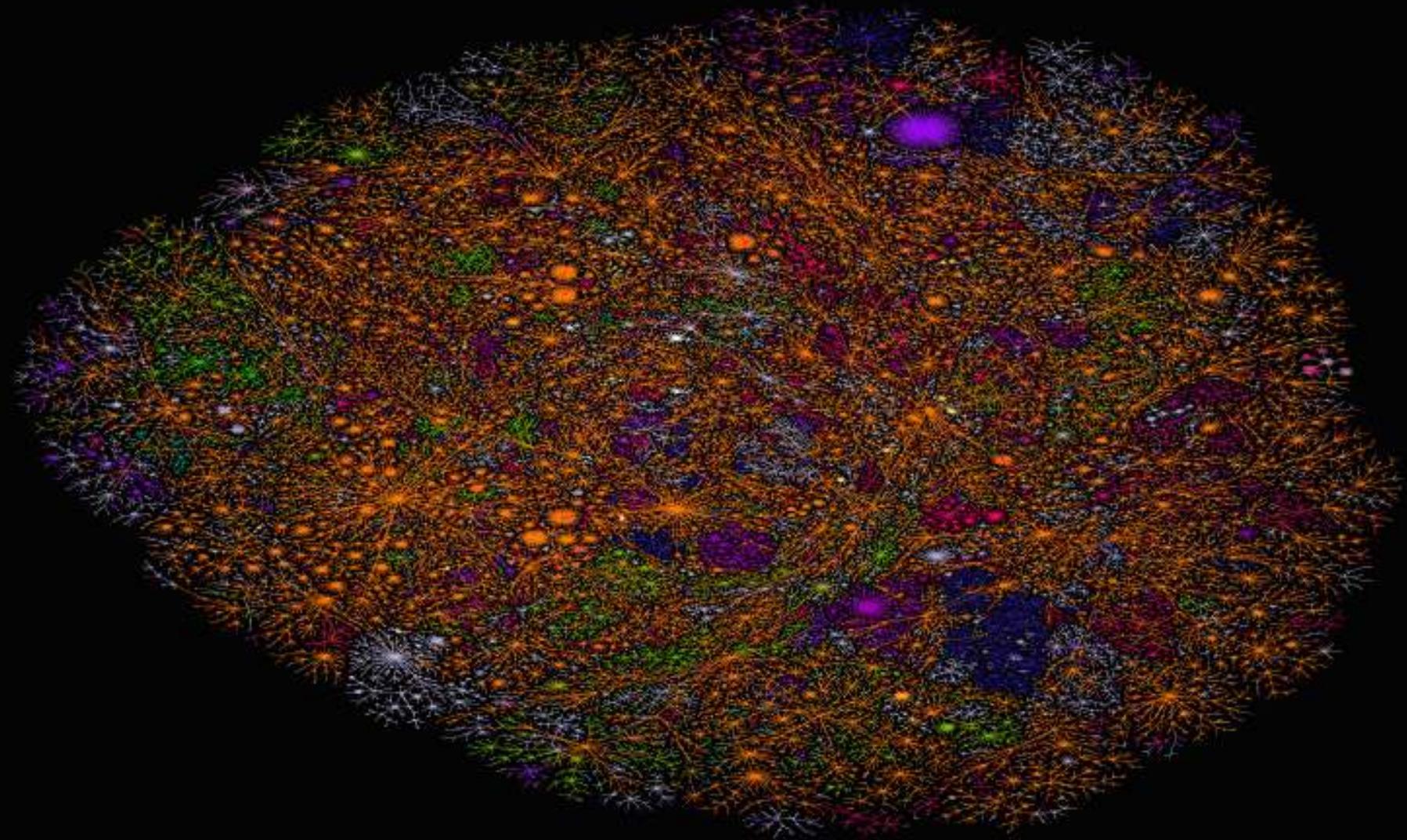
Local Transportation



Power Grid Network



The Internet: 2002



Graph by Iain Stewart and Bill Stanger. Nodes by name, edges by degree. Nodes

Copyright © L3 LARITA and Network Team, Inc., 2002

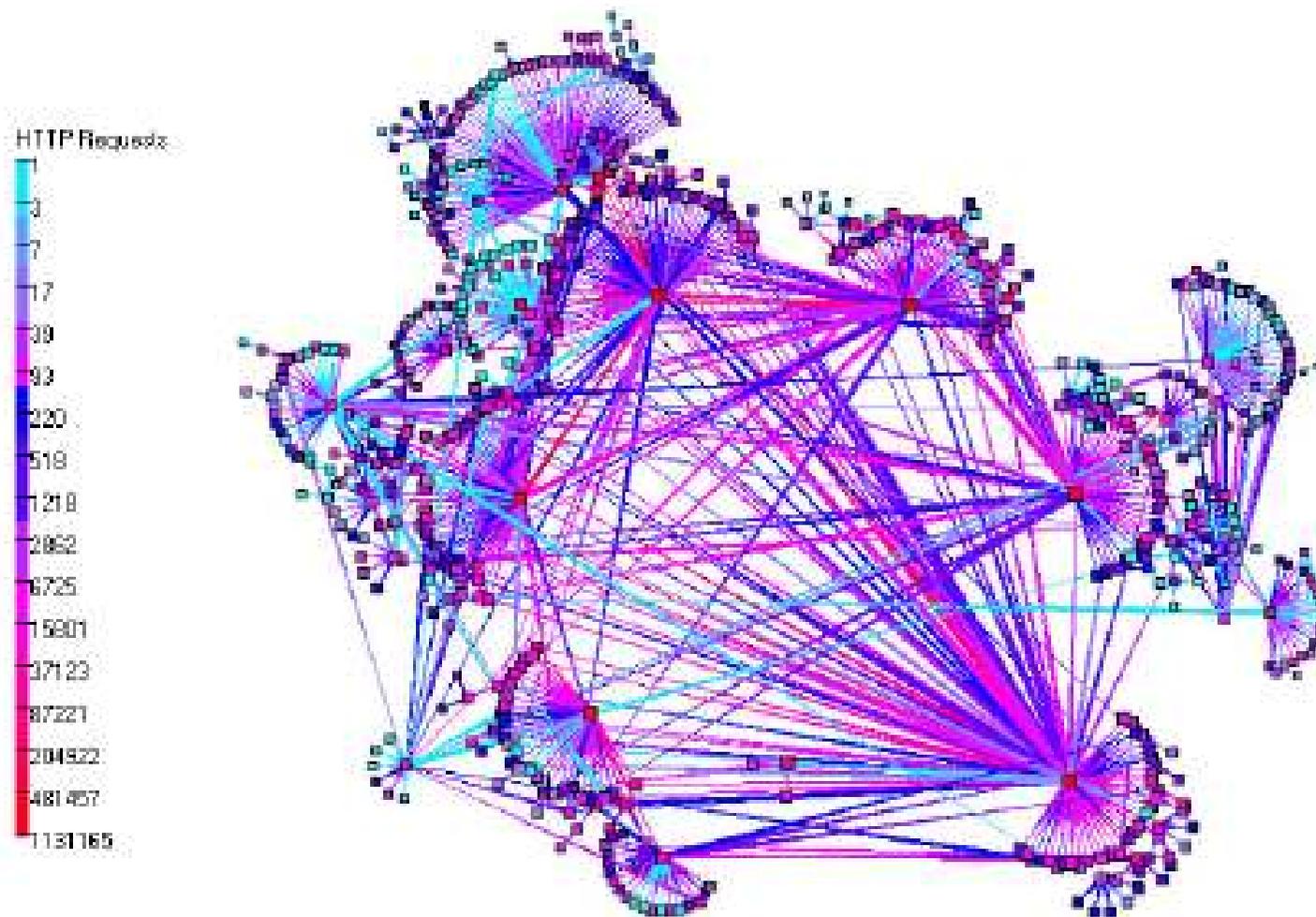
- North America
- Central America
- South America
- Africa
- South Africa
- Europe
- Germany
- France
- Netherlands
- United Kingdom
- Asia
- Japan
- Pacific Islands
- Australia
- New Zealand
- USA
- UK
- CA
- RU
- IT
- IN
- Other

This graph of the Internet was created by loading the Internet path database of connected computers in Germany, New Zealand, and the USA into a database and then using the 111 000 domains listed in the public Internet register of each to identify and color-code individual nodes. The data was collected in January 2002.

Published by
PERIOGRAPH
www.perio-graph.com

Nodes about the size of the Internet domain they represent (with the exception of the USA, UK, CA, RU, IT, IN, and Other). Nodes are colored by country or region. Nodes are colored by country or region. Nodes are colored by country or region.

Reti di informazione: World Wide Web



Hierarchical topology of the international Web cache

(Bradley Huffaker, <http://www.caida.org/tools/visualization/plankton>)



Quetelet (1796-1874)

Quando consideriamo una massa costituita da un grande numero di individui, le dinamiche sociali, più che dalla volontà individuale, dipendono da sollecitazioni collettive della rete sociale cui l'individuo appartiene.

Come le molecole di un gas!

THE PHYSICS OF SOCIETY

“Whatever concerns the human species, considered en masse, belongs to the domain of physical facts; the greater the number of individuals, the more the individual will is submerged beneath the series of general facts which depend on the general causes according to which the society exists and is conserved.” --Quetelet



Laplace



Quetelet



Buckle



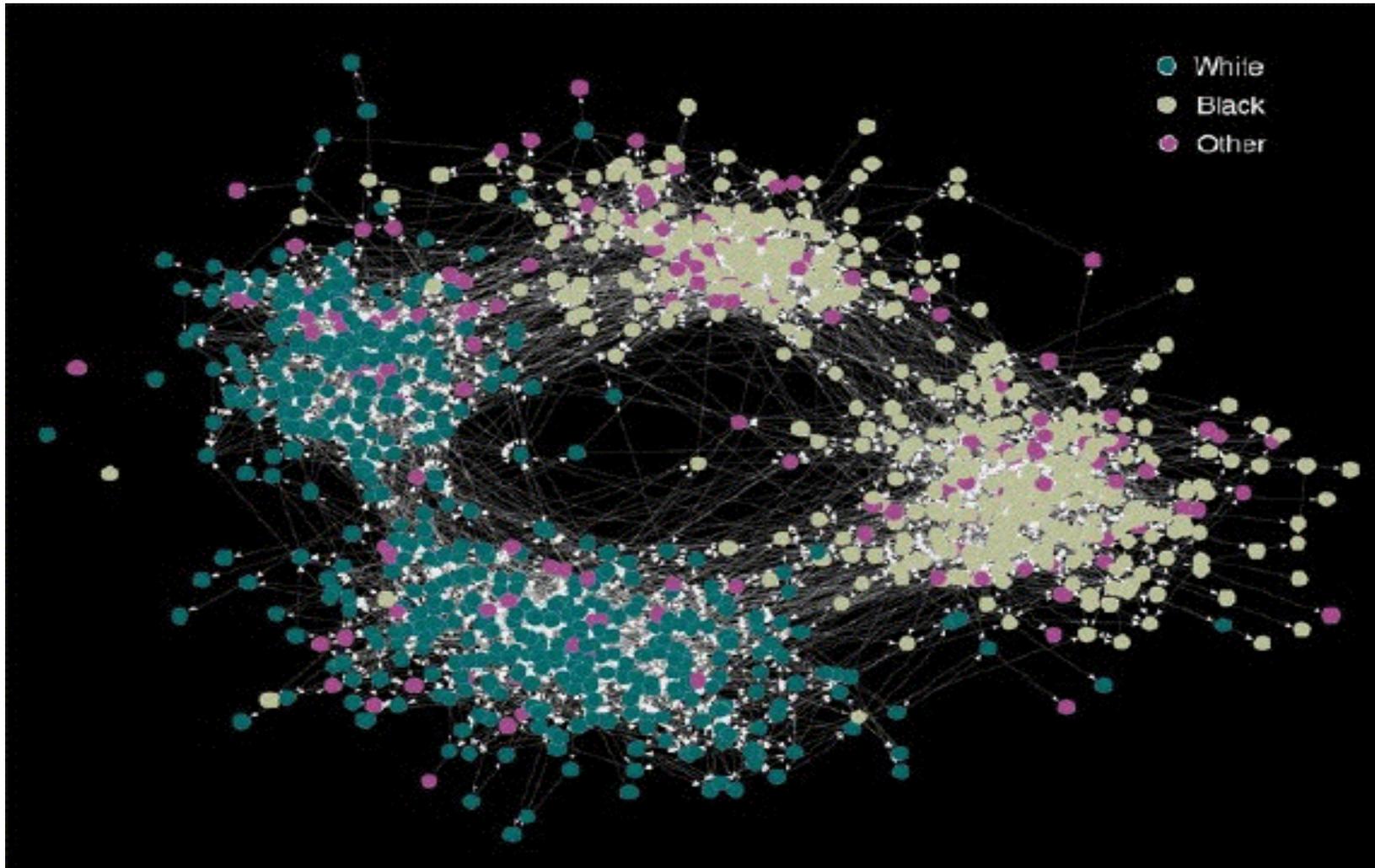
Maxwell



Boltzmann

“The molecules are like so many individuals, having the most various states of motion, and the properties of gases only remain unaltered because the number of these molecules which on the average have a given state of motion is constant.” --Boltzmann

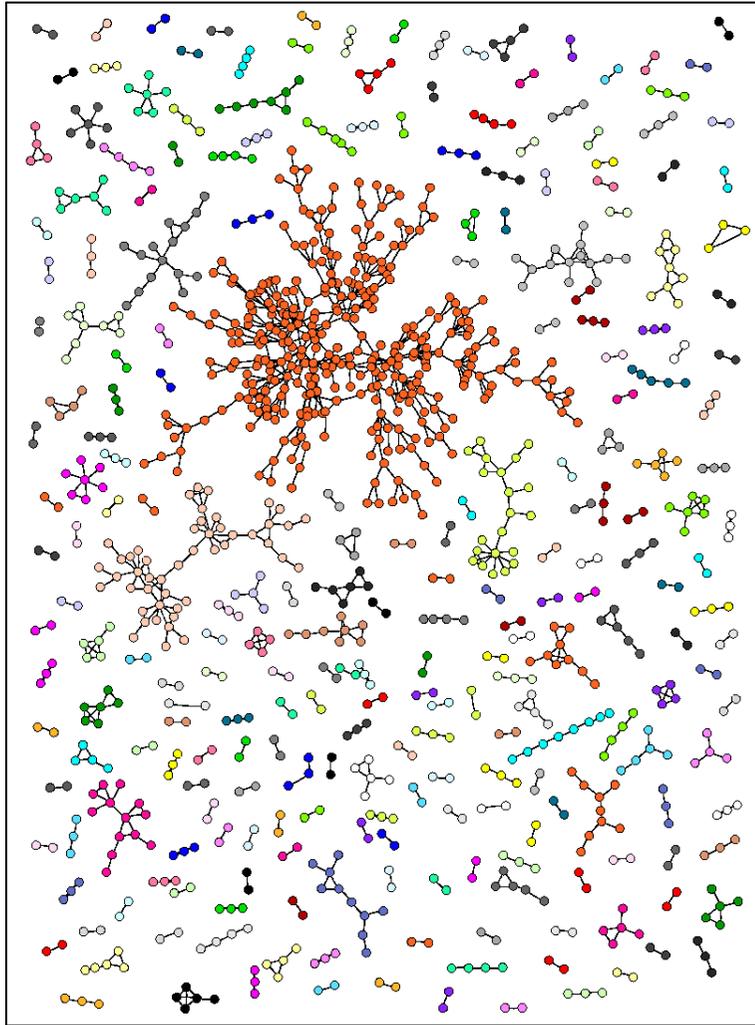
Reti di amicizie



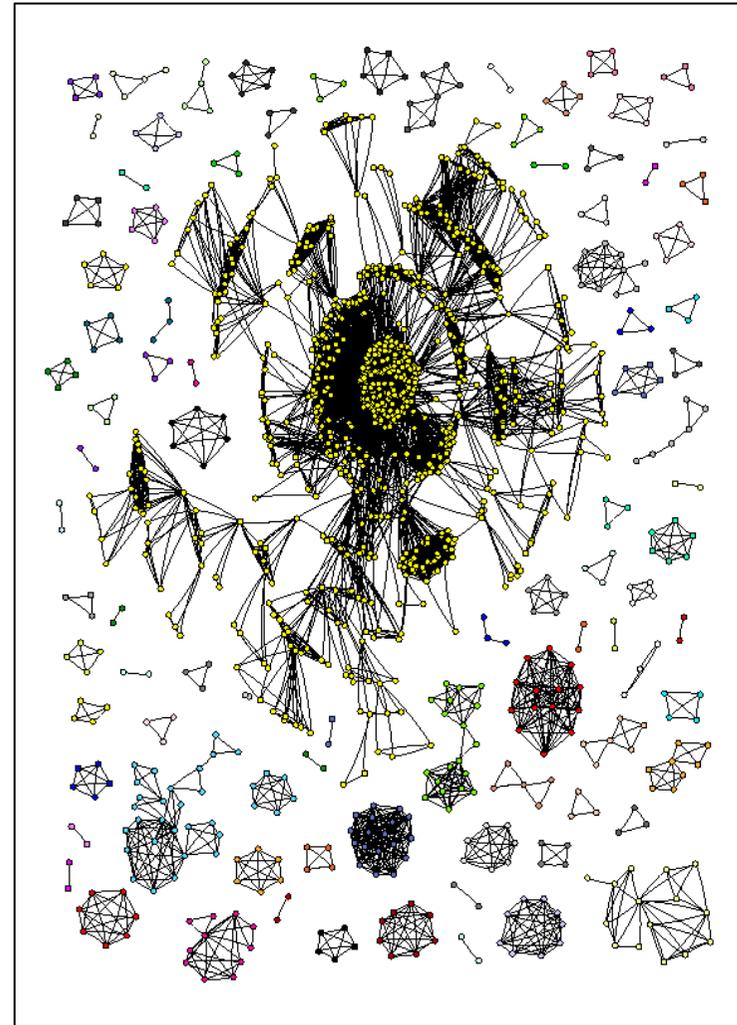


Rete dei terroristi dell'undici settembre

Collaborazioni Scientifiche



Collaborations in *Econometrica*

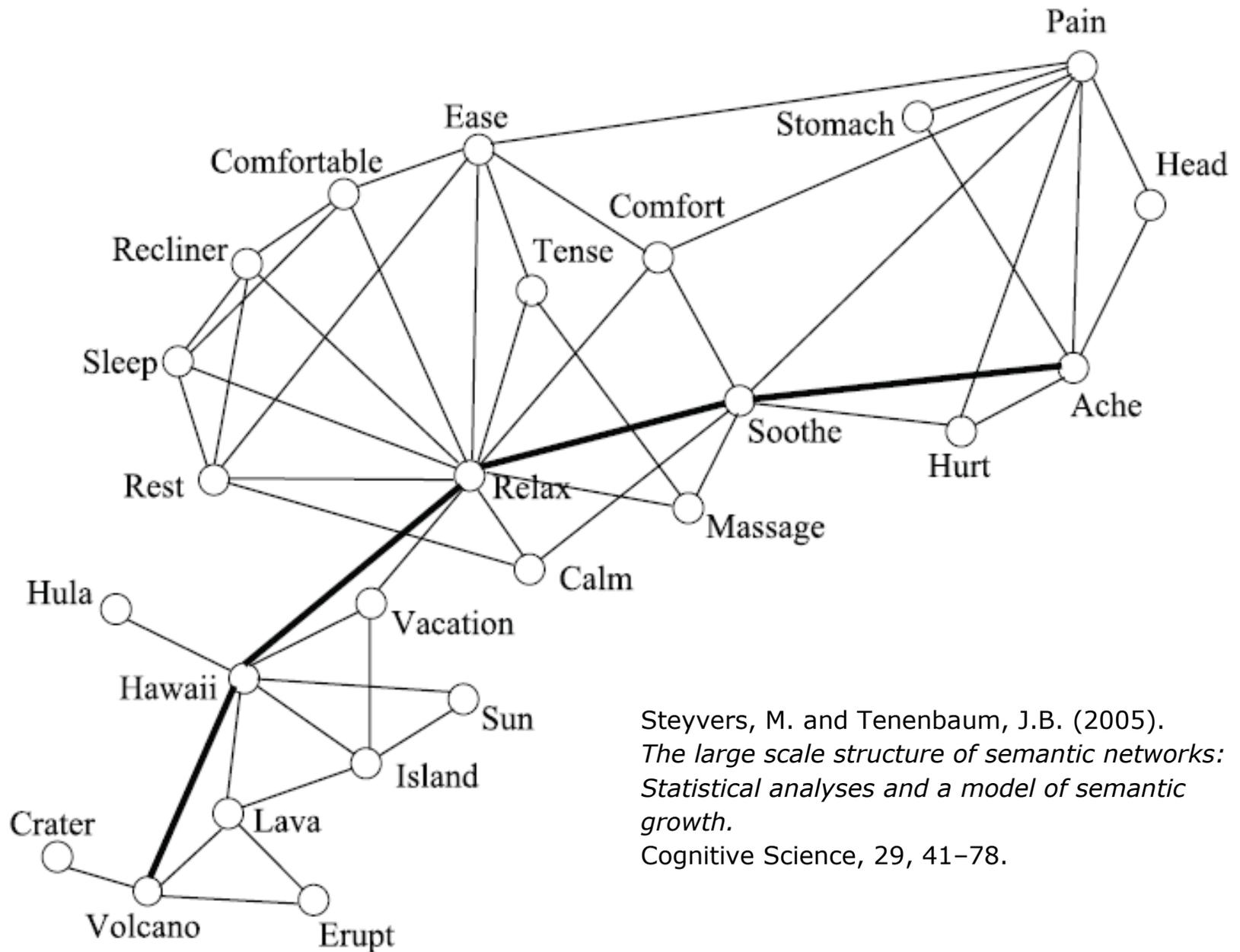


Collaborations in the
Astronomical Journal

facebook



Rete semantica: associazioni tra parole



Steyvers, M. and Tenenbaum, J.B. (2005).
*The large scale structure of semantic networks:
Statistical analyses and a model of semantic
growth.*
Cognitive Science, 29, 41–78.

RETE	NODI	COLLEGAMENTI
<i>Metabolismo cellulare</i>	<i>Molecole coinvolte nella trasformazione di cibo in energia</i>	<i>Partecipare alla stessa reazione biochimica</i>
Hollywood	Attori	Partecipare allo stesso film
<i>Internet</i>	<i>Router</i>	<i>Le connessioni fisiche</i>
Collaborazioni scientifiche	Scienziati	Aver scritto articoli in collaborazione
<i>Relazioni sessuali</i>	<i>Esseri umani</i>	<i>Rapporti sessuali</i>
Relazioni sociali	Esseri umani	amicizia
<i>Distribuzione en. elettrica</i>	<i>Impianti di produzione & consumo</i>	<i>Linee di trasmissione</i>
Traffico aereo	aeroporti	Voli di linea & commerciali
<i>Relazioni economiche</i>	<i>Consiglio di Amministrazione (CDA) aziende</i>	<i>Avere un membro del CDA in comune</i>
World Wide Web	Pagine web	Link tra le pagine

Fisici statistici hanno sviluppato tecniche di analisi di queste reti complesse

- Le reti complesse mostrano proprietà **universali**
 - Queste proprietà sono **emergenti**
(nascono spontaneamente dall'iterazione di semplici regole per la creazione delle connessioni)

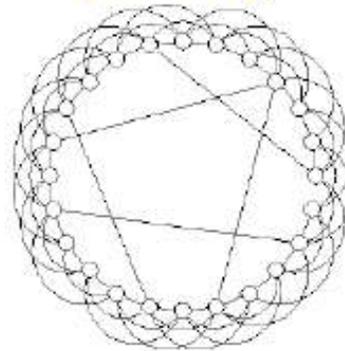
Structural properties of real-world networks

The small world effect



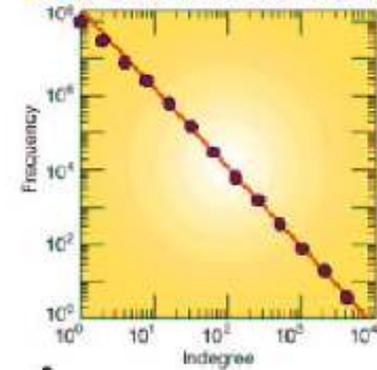
Diameter small compared to system size.

Clustering



Two of your friends are likely to be friends with one another.

Degree distributions



World-Wide Web

Degree distributions are often heavy tailed.

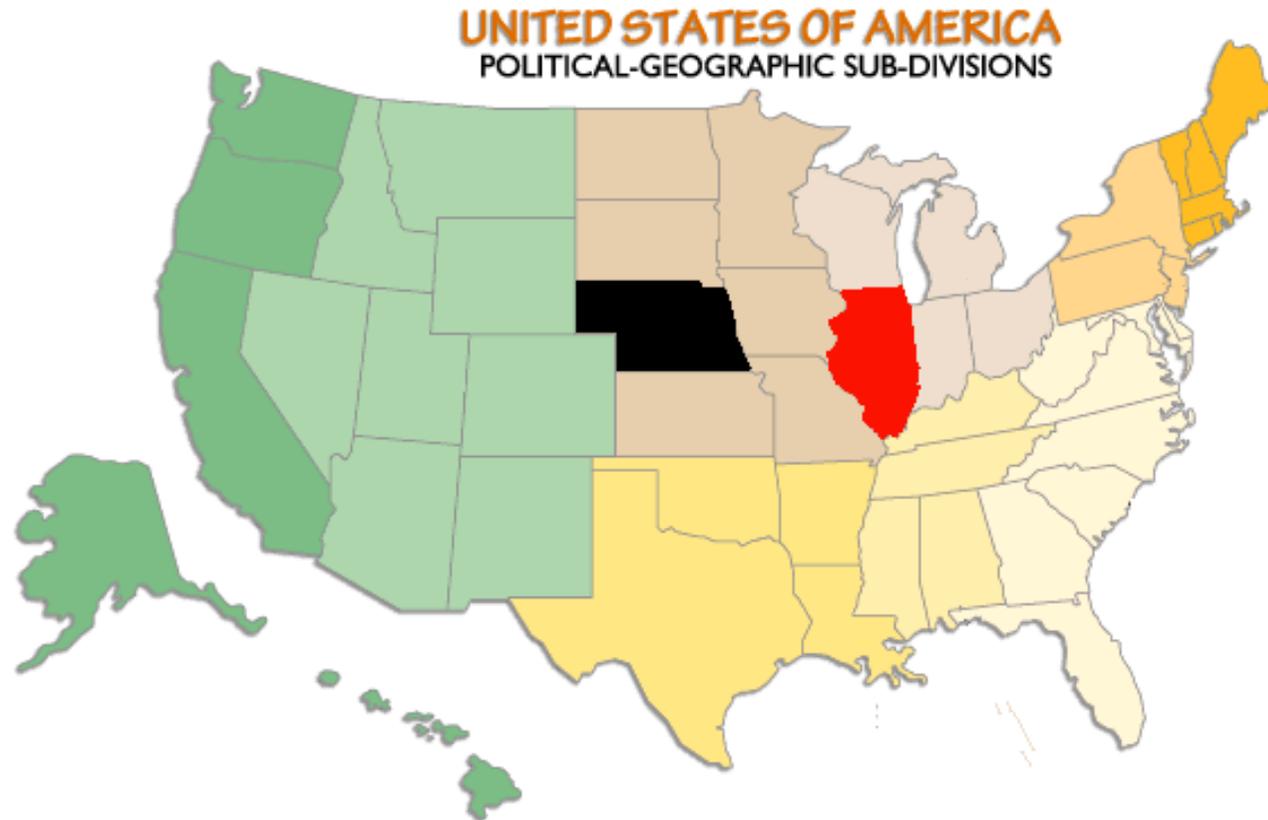
Domande che ci si può porre

- Questi networks possiedono una simmetria?
- Questi networks sono la creazione di oggetti intelligenti o le loro proprietà sono “emergenti”?
- Qual’è l’origine della loro struttura complessa?
 - Trovare semplici regole che, iterate, portano alla formazione di strutture complesse

Domande che ci si può porre

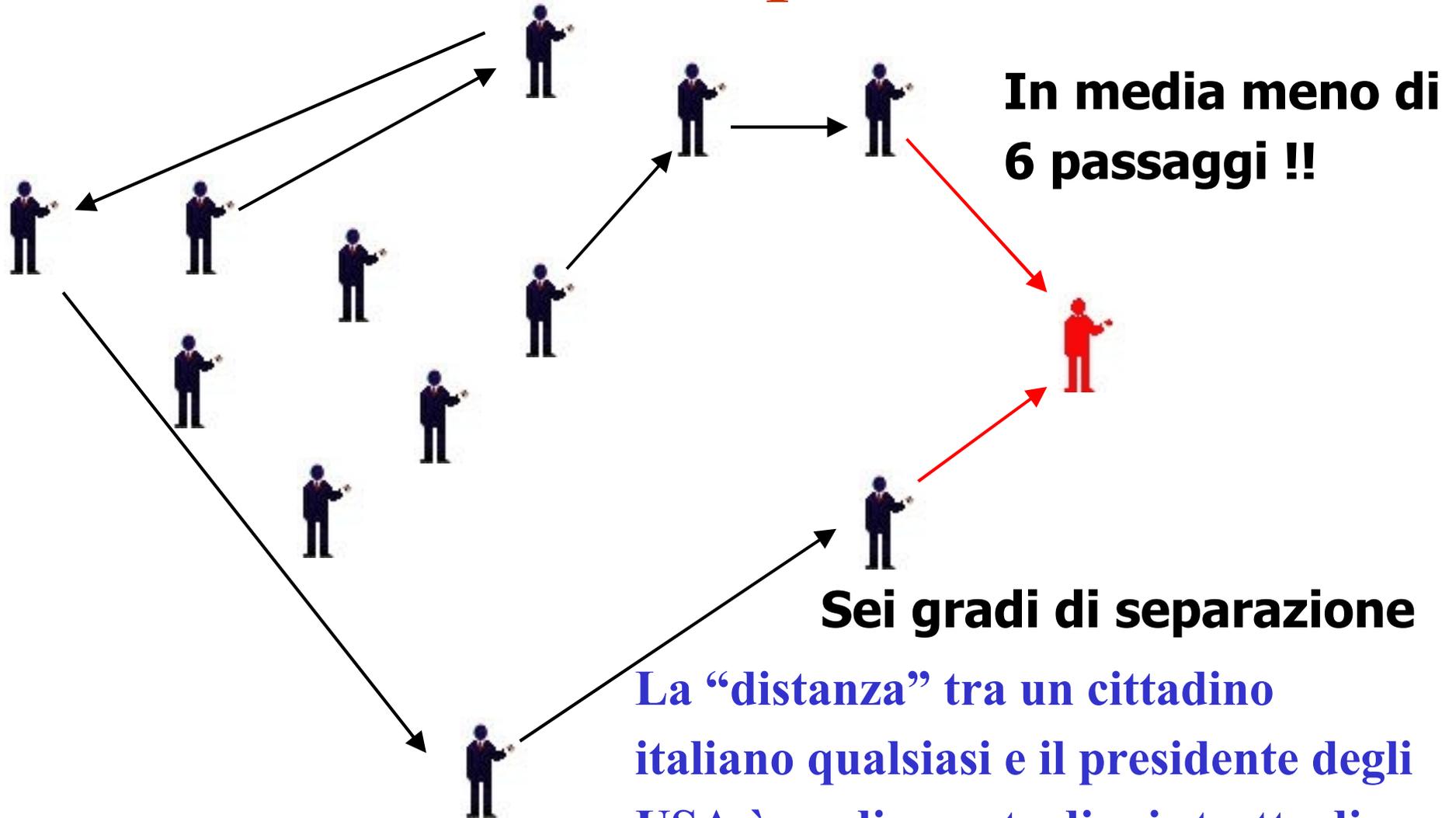
- Analizzando le proprietà del network possiamo valutare lo “stato di salute” del sistema descritto dal network? O far previsioni sul suo comportamento futuro?
- Questi network sono robusti rispetto a un “guasto” in qualche nodo?
- Questi networks favoriscono il flusso di informazioni tra i nodi?
- Possiamo costruire questi network? (engineering complex systems).

Effetto small world: Milgram (1967)



E' possibile consegnare un messaggio ad un agente di cambio a Chicago partendo da persone prese a caso nel Nebraska ?

Il mondo è piccolo !



La “distanza” tra un cittadino italiano qualsiasi e il presidente degli USA è mediamente di sei strette di mano.

Le reti reali hanno anche un'altra proprietà (aggregazione)

A è amico di B

A è amico di C

→ è probabile che B e C siano amici

(proprietà di aggregazione o clustering)

Clustering → Comunità nella rete

Clustering: misura la densità di triangoli nel network

$$C^{(1)} = \frac{\sum_i \text{triangles centered at node } i}{\sum_i \text{triples centered at node } i}$$

I networks reali sono “small worlds”

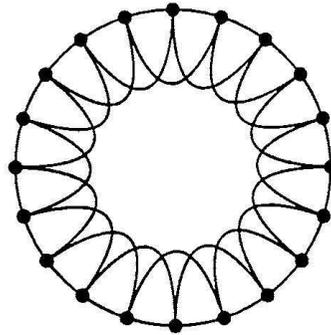
	L	C
Movie actors	3.65	0.79
Power grid	18.7	0.080
C.elegans	2.65	0.28

L = grado medio di separazione; C = coefficiente di clustering

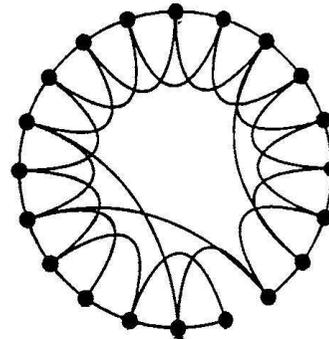
Modello di Watts-Strogatz (1998)

Come può diventare piccolo il mondo?

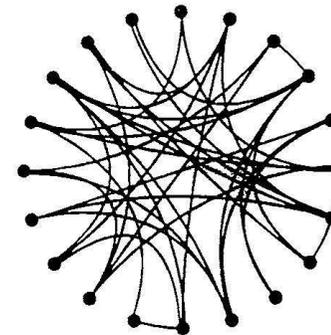
Regular



Small-World



Random

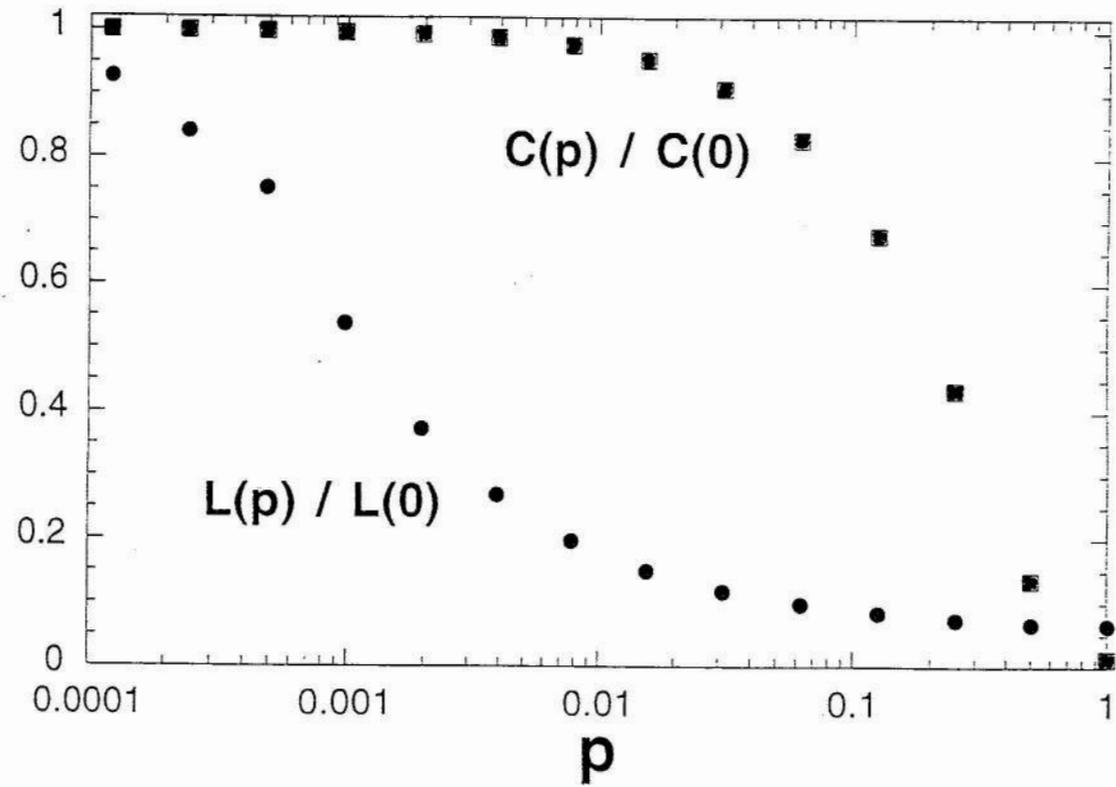


$p = 0$

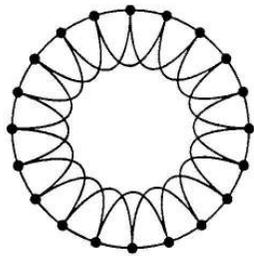


$p = 1$

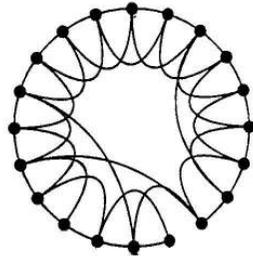
Increasing randomness



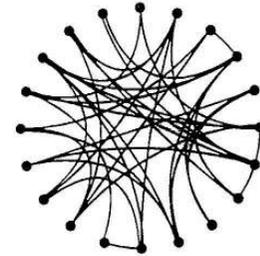
Regular



Small-World

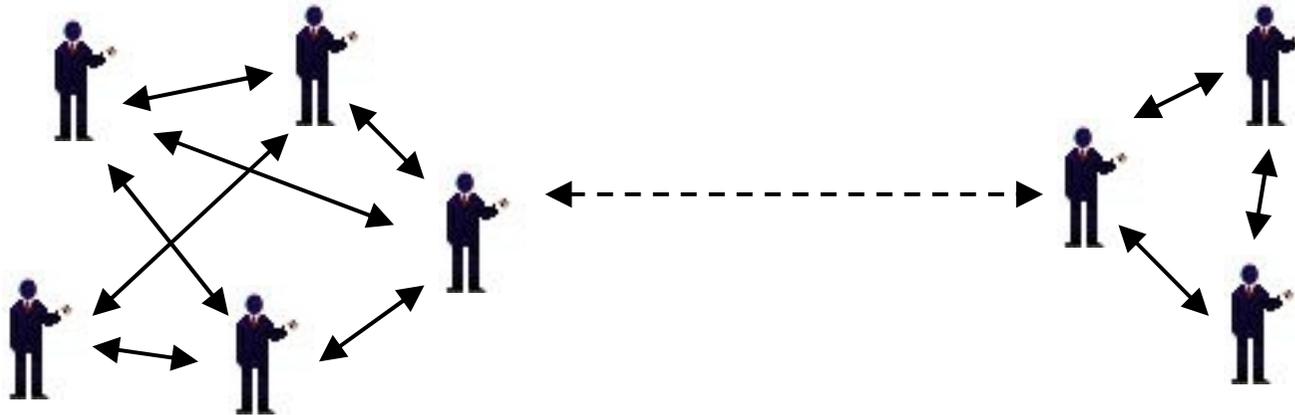


Random



La struttura delle reti sociali

Mark Granovetter (1973): le scorciatoie sono i “legami deboli”

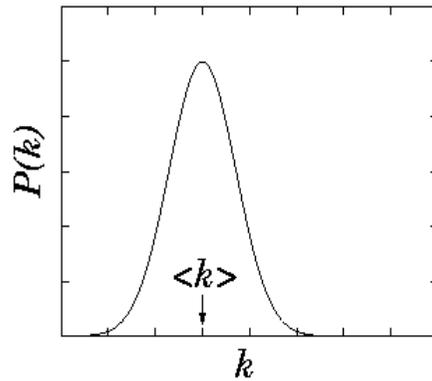


“The Strength of Weak Ties” (1973): in politica ed in marketing i legami deboli permettono di raggiungere comunità non accessibili con i legami forti.

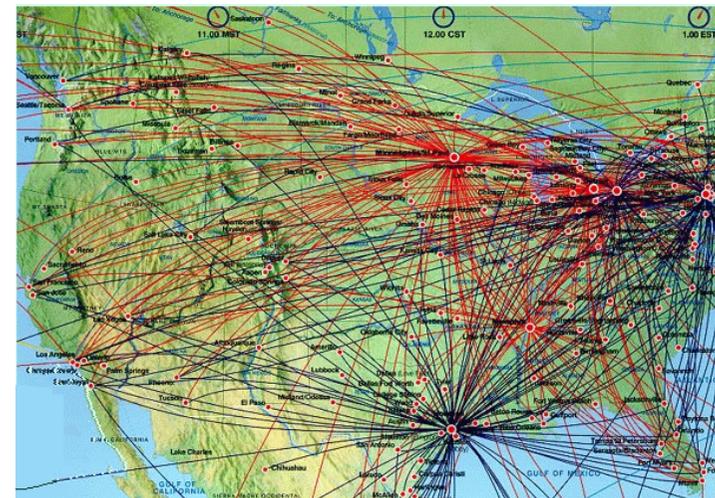
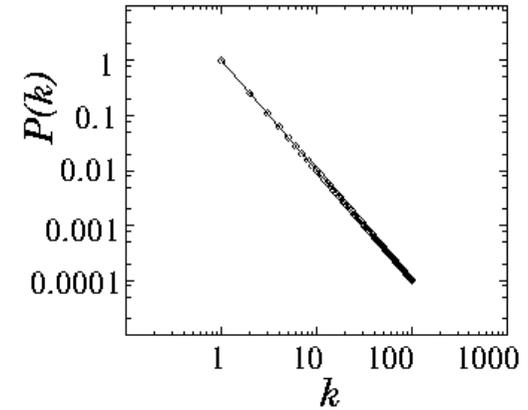
Movimenti “Grassroots”, Mobilità sociale, Trovare Lavoro

Simmetria

Network egualitario

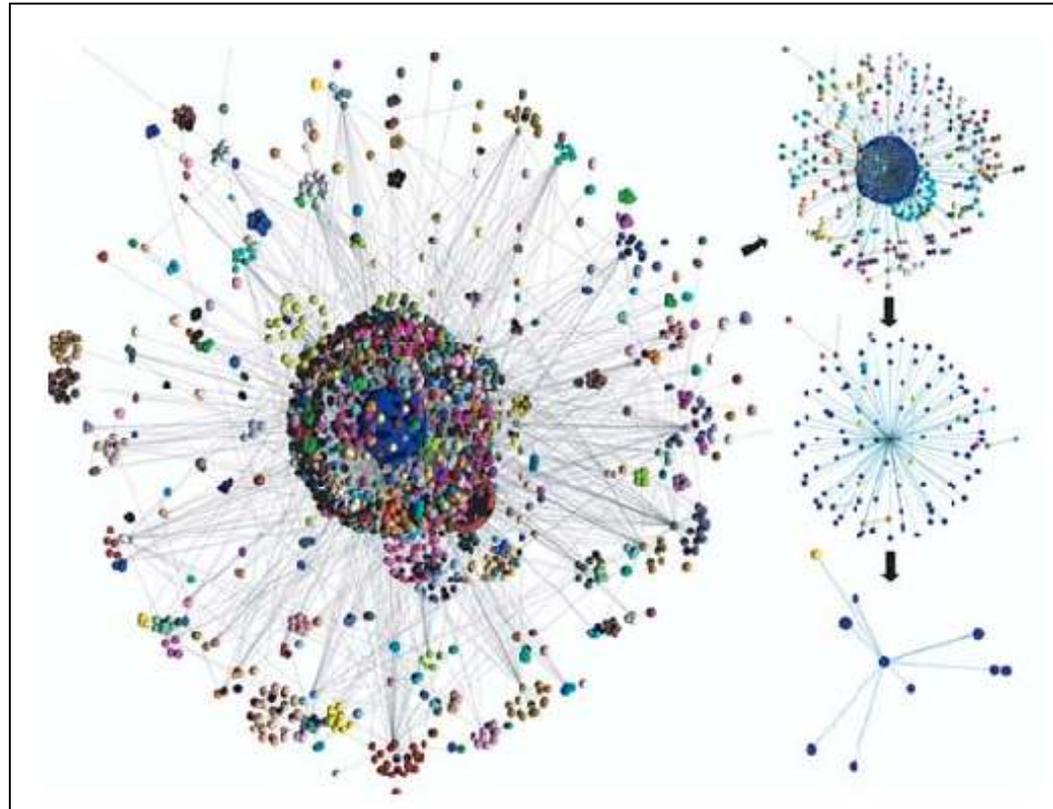


Scale-free (gerarchico):
presenza di hub



Auto-similarità delle reti scale-free

$$P(k) \sim k^{-\gamma}$$

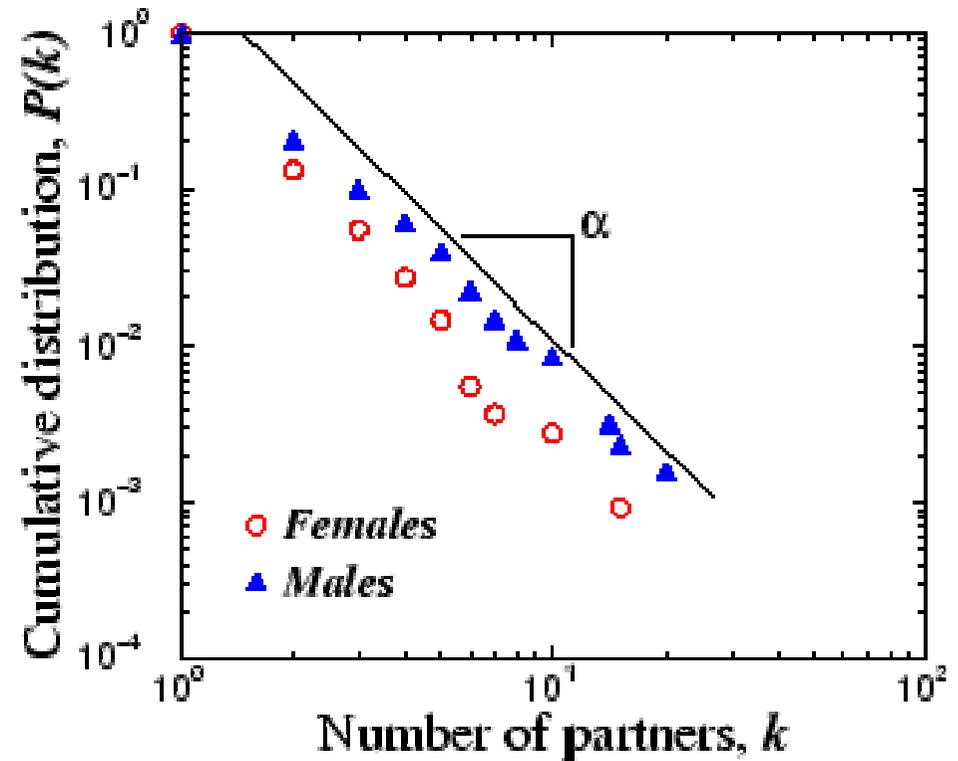


WWW (in)	Internet	Actor	Citation index	Sex Web	Cellular network	Phone call network	linguistics
$\gamma = 2.1$	$\gamma = 2.5$	$\gamma = 2.3$	$\gamma = 3$	$\gamma = 3.5$	$\gamma = 2.1$	$\gamma = 2.1$	$\gamma = 2.8$

Swedish sex-web

Nodes: people (Females; Males)

Links: sexual relationships

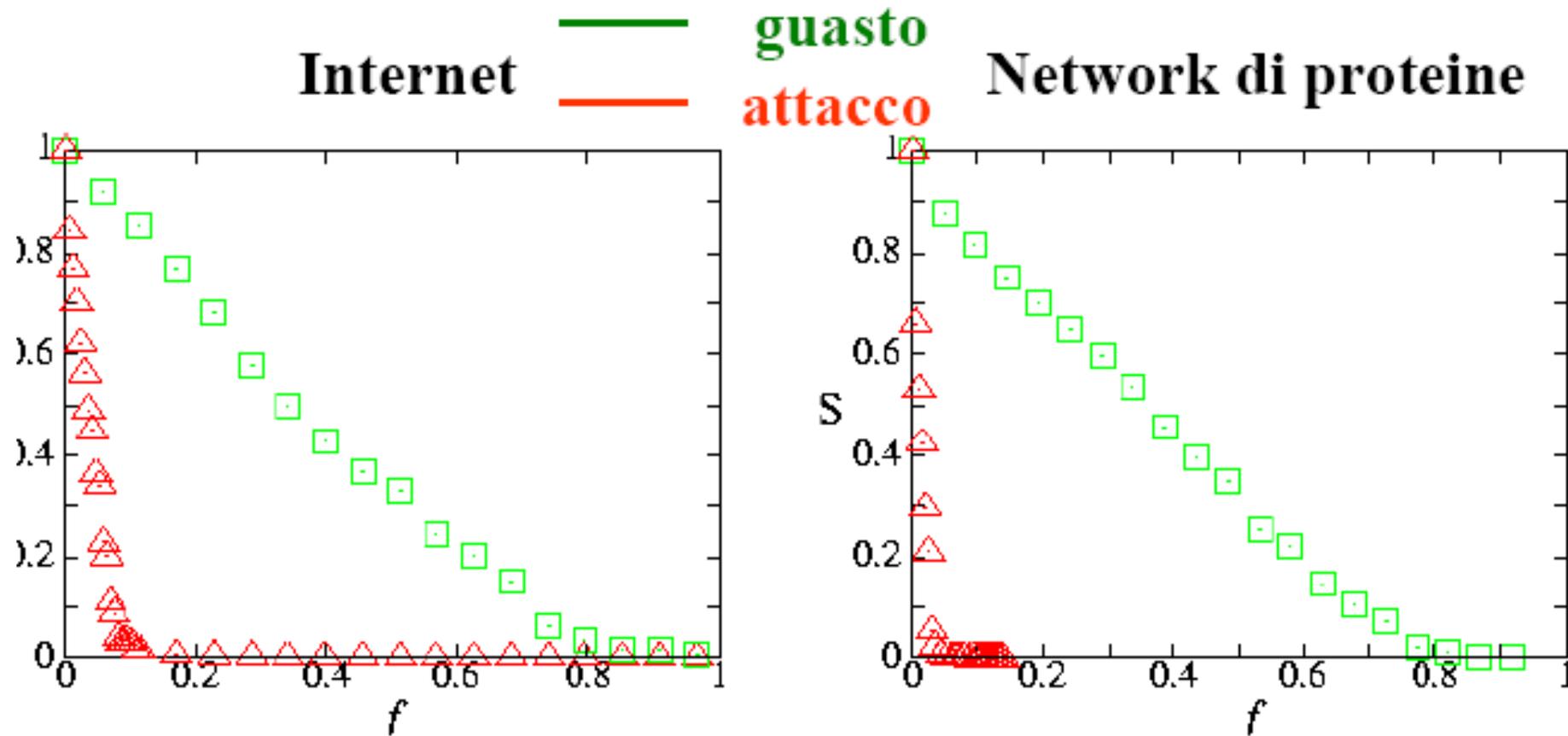


4781 Swedes; 18-74;
59% response rate.

Liljeros et al. *Nature* 2001

Robustezza rispetto a guasti casuali

- Andamento della percentuale di nodi connessi in funzione della frazione di nodi guasti (o attaccati)



Sulle reti complesse le informazioni si propagano molto rapidamente

Questo vale anche per i virus informatici o le malattie

immunizzare gli hub come strategia di prevenzione di malattie infettive (o virus informatici)



Qual è il meccanismo che da origine ai
network scale-free?

PREFERENTIAL ATTACHMENT

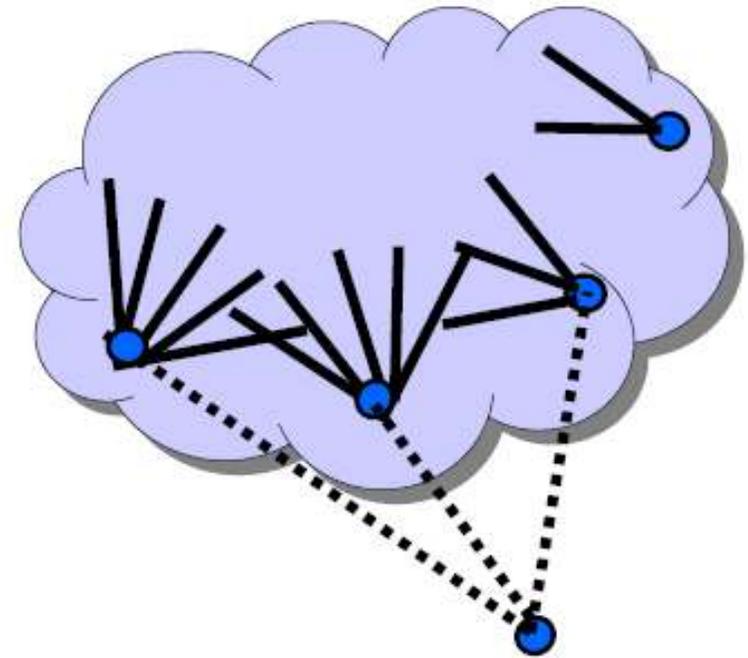
(Albert & Barabasi, 1999)



Effetto Matteo:

“così a chi ha sarà dato
e sarà nell’abbondanza”.
(vangelo Matteo, 25:29)

I ricchi diventano sempre più ricchi!

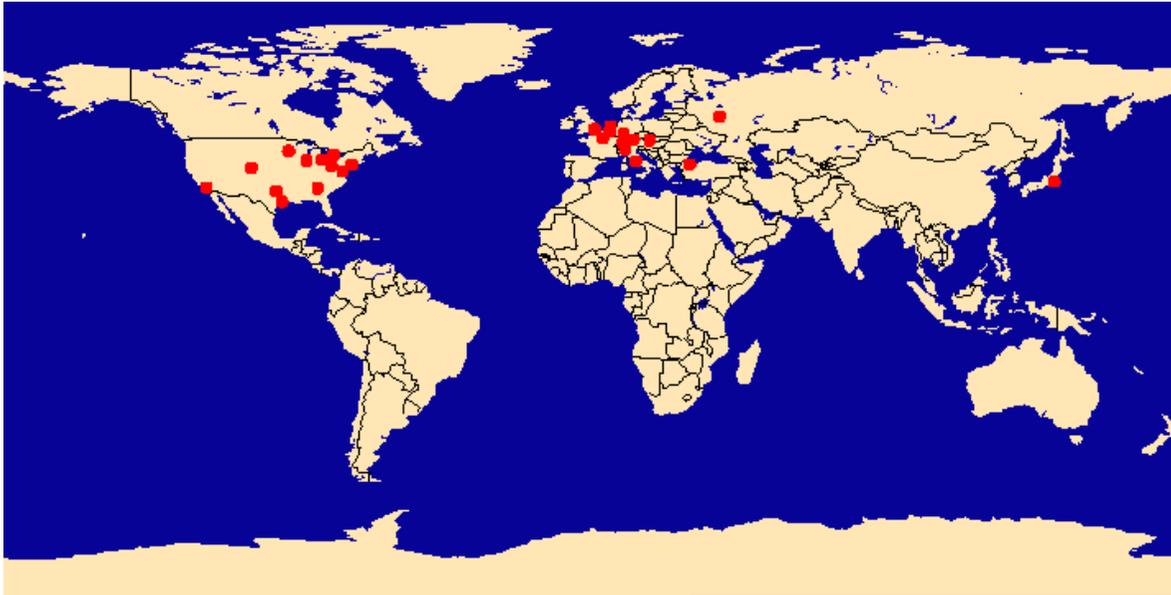


Regola universale 80/20

Importanza di un nodo

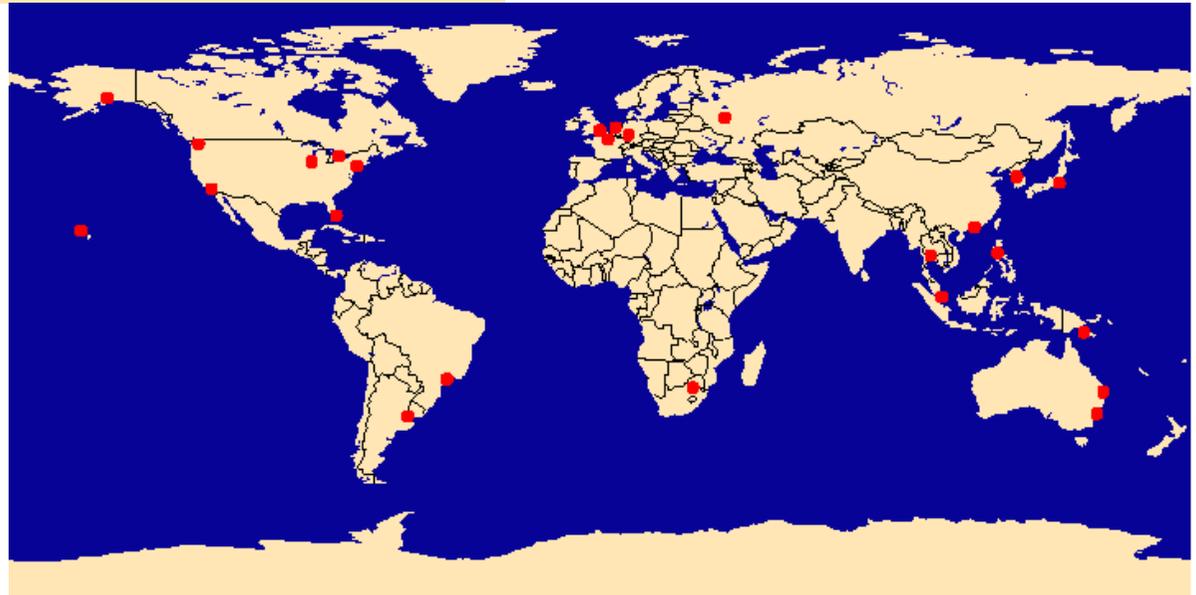
- Il numero di connessioni non è sufficiente a quantificare l'importanza di un nodo
- Centralità di un nodo = frazione di cammini di lunghezza minima che passano per quel nodo

Quali sono gli aeroporti più importanti della rete mondiale?



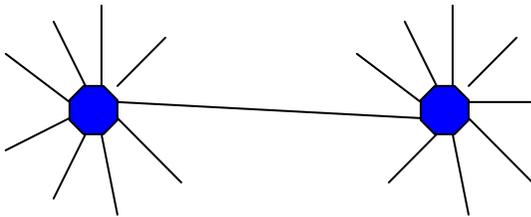
Aeroporti più **connessi**

Aeroporti più **centrali**

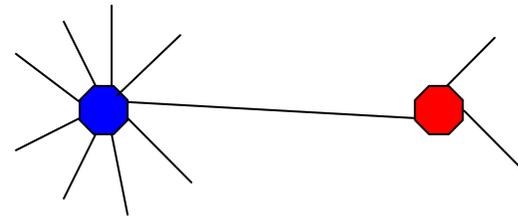


Assortatività

Assortative networks



Disassortative networks



- Le reti reali mostrano uno o l'altro di questi due comportamenti

- reti “**↑**” mostrano comportamenti “**↑** simile”

Reti sociali

Reti tecnologiche e biologiche

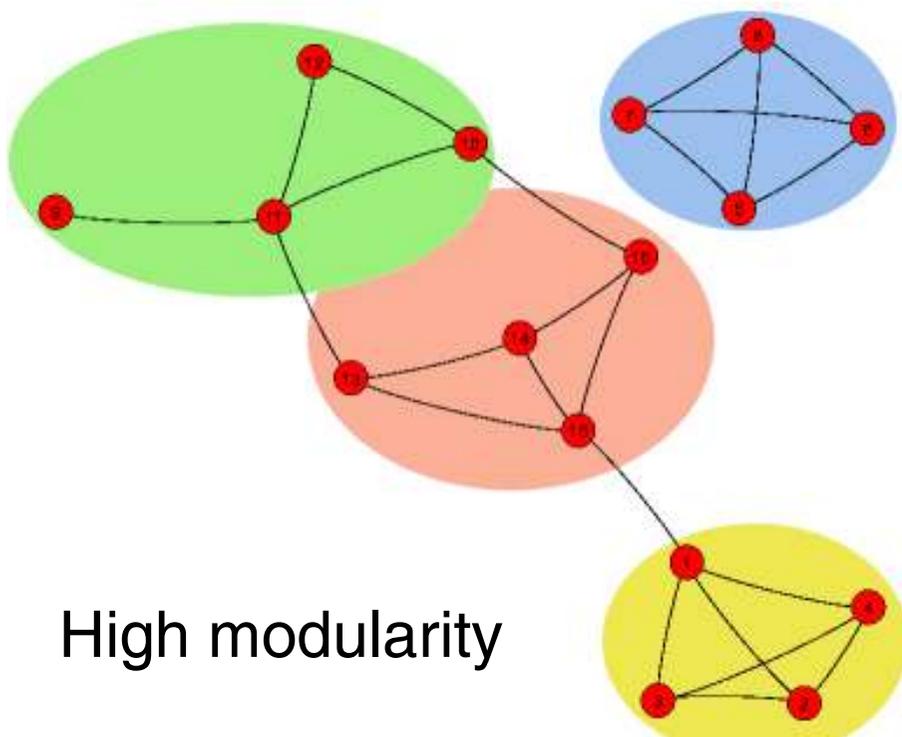
Fenomeno del Rich-Club

Assortatività di network reali

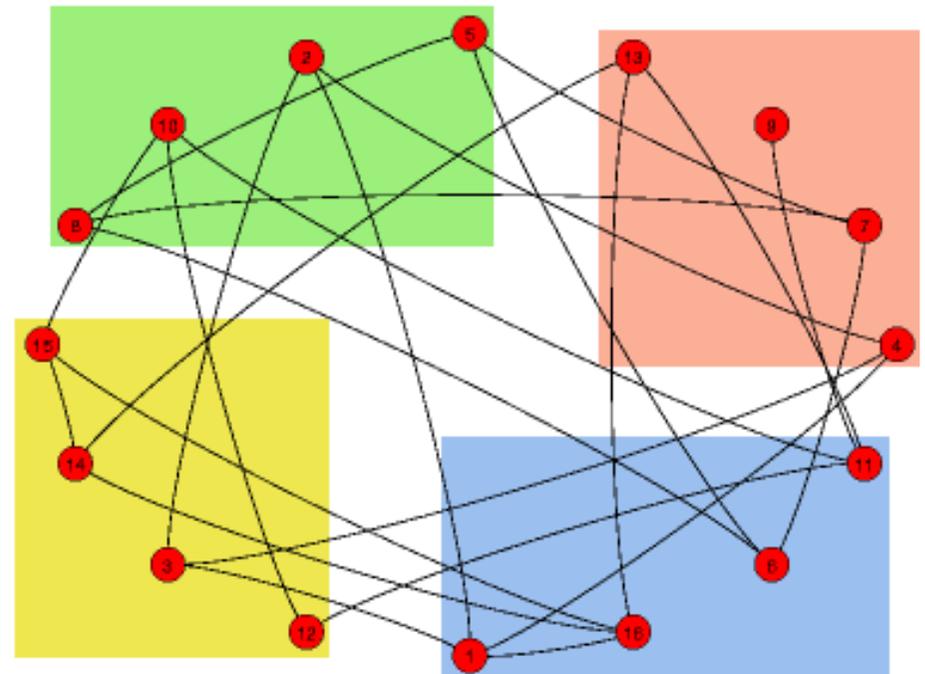
	network	type	size n	assortativity r
social	physics coauthorship	undirected	52 909	0.363
	biology coauthorship	undirected	1 520 251	0.127
	mathematics coauthorship	undirected	253 339	0.120
	film actor collaborations	undirected	449 913	0.208
	company directors	undirected	7 673	0.276
	email address books	directed	16 881	0.092
technol.	Internet	undirected	10 697	-0.189
	World-Wide Web	directed	269 504	-0.067
	software dependencies	directed	3 162	-0.016
biological	protein interactions	undirected	2 115	-0.156
	metabolic network	undirected	765	-0.240
	neural network	directed	307	-0.226
	marine food web	directed	134	-0.263
	freshwater food web	directed	92	-0.326

M.E.J Newman and M. Girvan, *Mixing Patterns and Community Structure in Networks* (2002).

Modularità

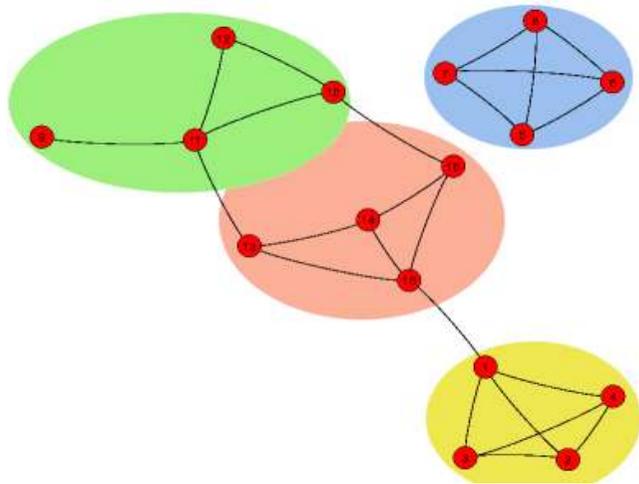


High modularity

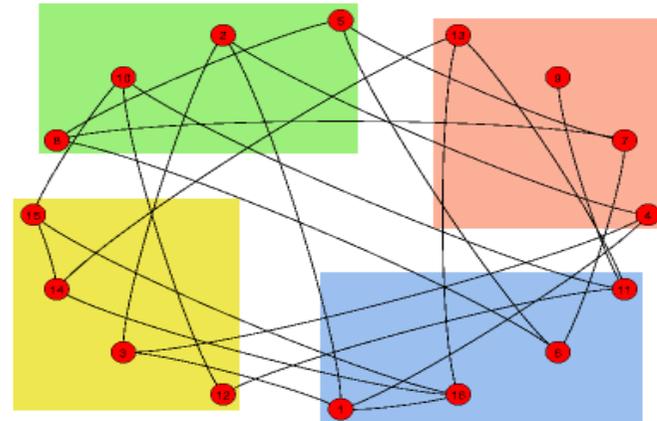


Low modularity

Misura quantitativa della *modularità*



f_s : frazione di links
all'interno del modulo s



F_s : frazione attesa di
links all'interno del
modulo s , per una
partizione random dei
nodi

Modularity of a partition:

$$M = \sum_s (f_s - F_s)$$

Matrice di modularità

- Modularity of network defined as

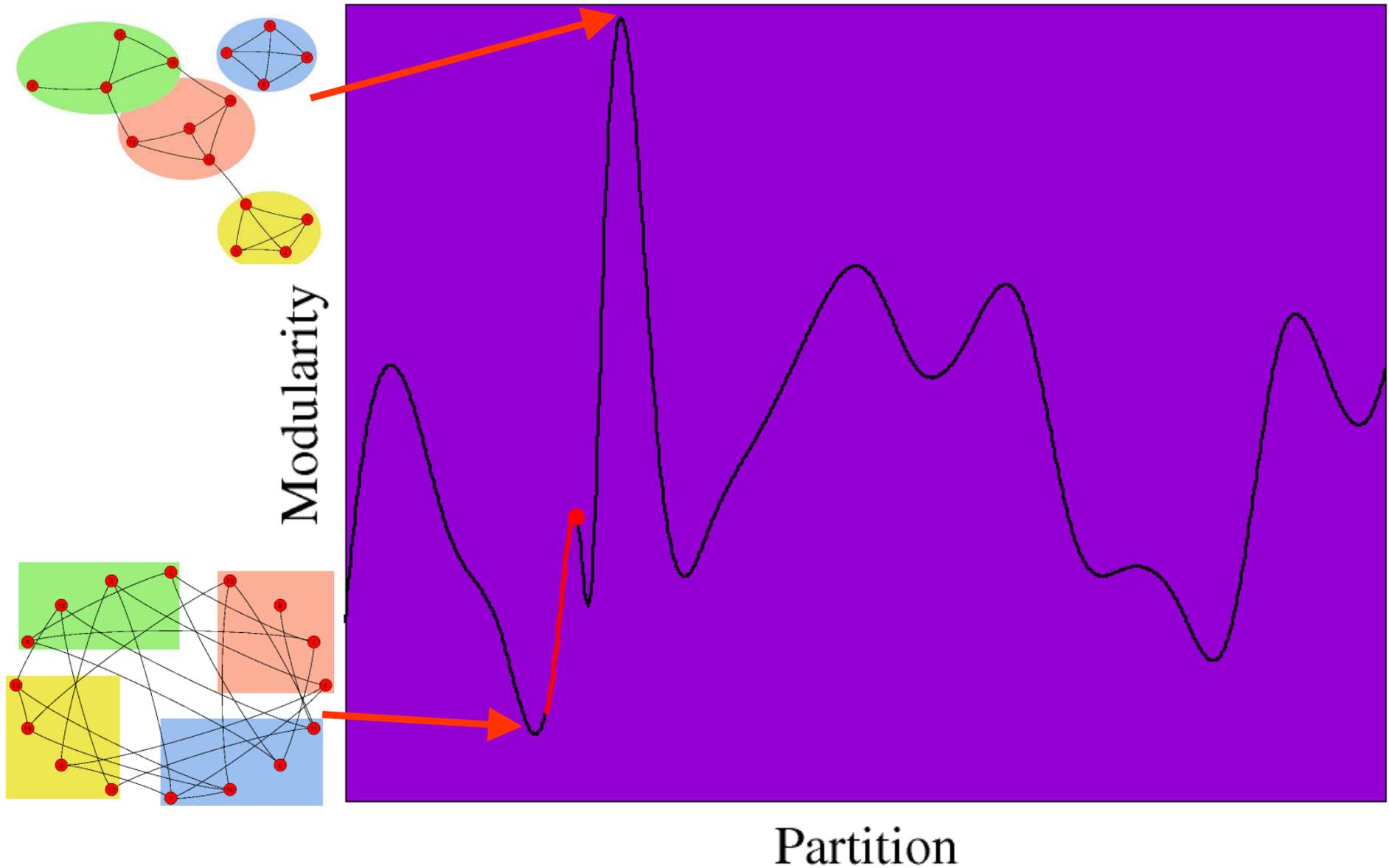
$$Q = \frac{1}{4m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) s_i s_j = \frac{1}{4m} \mathbf{s}^T \mathbf{B} \mathbf{s}$$

A = adjacency matrix, m = total number of edges

- **B** is defined as the *modularity matrix*

$$B_{ij} = A_{ij} - \frac{k_i k_j}{2m}$$

Algoritmi di ricerca della migliore partizione

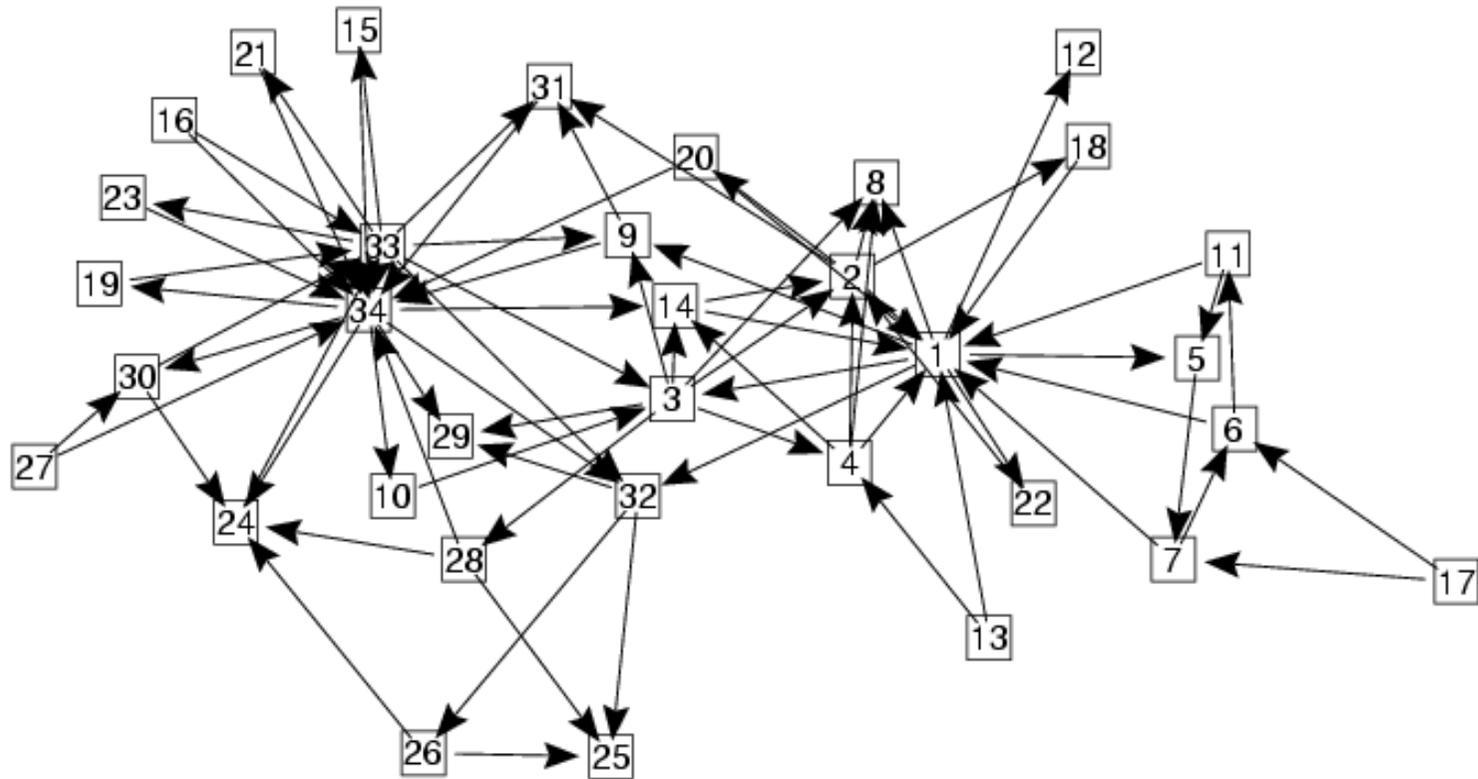


Inferenza di networks

Sebino Stramaglia



Se non c'è accesso alle connessioni
fisiche tra i nodi?
Inferenza del network dai dati !



Dati temporalmente correlati: Un sistema dinamico in ogni nodo

- Applicazioni più importanti
- Neuroscienze (fMRI, EEG, MEG, ECoG)
- System biology (gene regulatory networks)

Paradigma generale (Kolaczyk 2009)

- 1) Scelta della misura dell'accoppiamento
(bivariata o multivariata)
- 2) Test di significatività (p-value)
- 3) Controllo del false discovery rate

Misure dell'accoppiamento:

Correlazioni

Mutua informazione

Sincronizzazione di fase

Sincronizzazione generalizzata

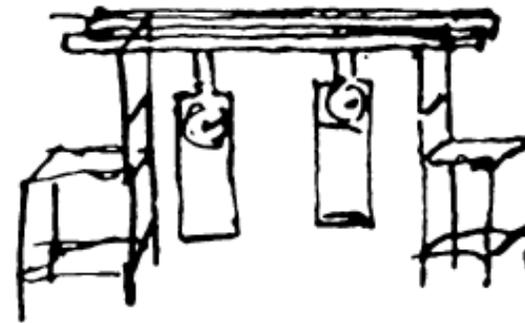
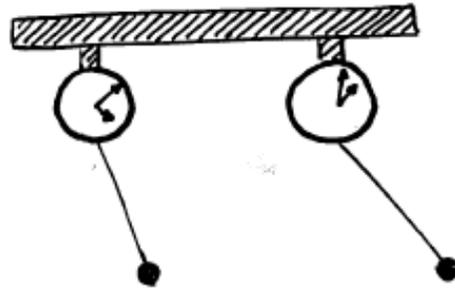
Coerenza spettrale

Entropia di trasferimento

Causalità alla Granger

La sincronizzazione è l'adattamento del ritmo di un sistema di oscillatori non-lineari dovuta alla loro debole interazione.

Esempio: gli orologi a pendolo (Huygens, 1665)



I due **orologi** (=oscillatori), separati, hanno **velocità** di avanzamento leggermente diverse.

Connessi allo stesso **supporto** (=debole interazione), gli orologi avanzano a **velocità** perfettamente identica (=adattamento del ritmo).

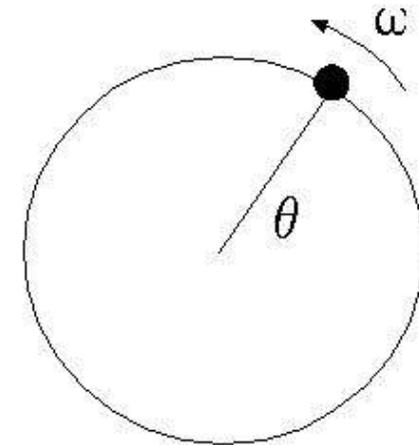
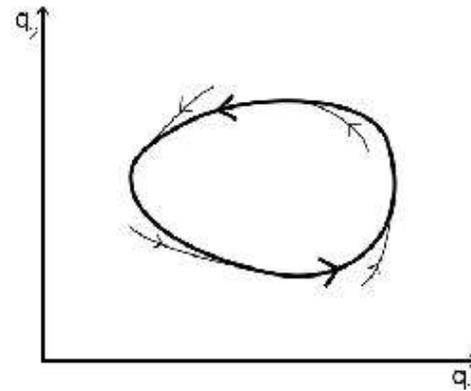
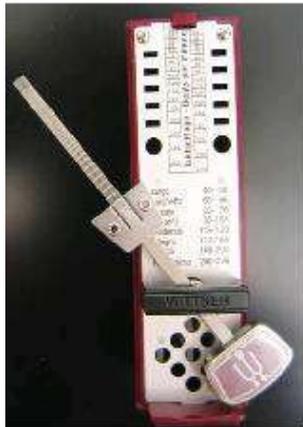


CHRISTIANI
HUGENII
ZVLIHEMII, CONST. F.
HOROLOGIVM
OSCILLATORIVM
SIVE
DE MOTV PENDVLORVM
AD HOROLOGIA APTATO
DEMONSTRATIONES
GEOMETRICÆ.



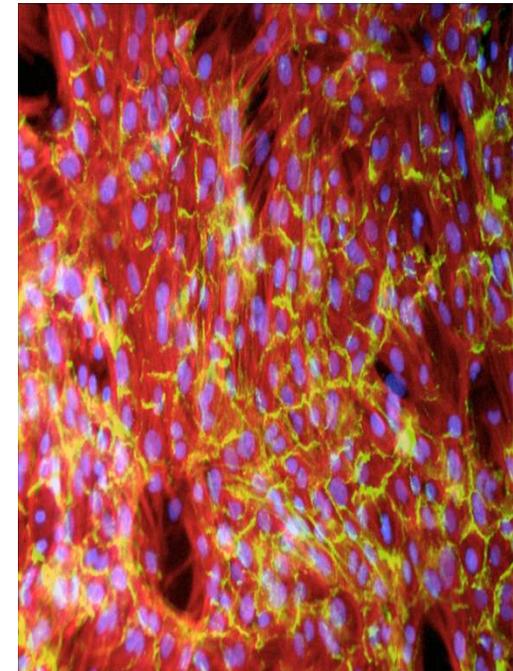
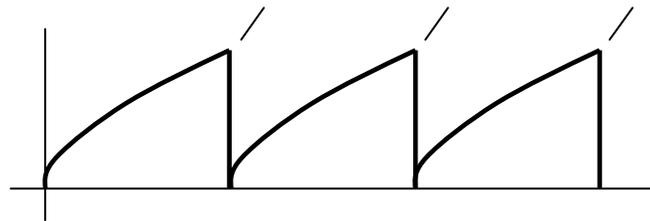
PARISIIS,
Apud E. MUGNETY, Regis & Illustrissimi Archiepiscopi Typographum
via Catharæ, ad insignem trami Regiam.
MDCLXXIII
CVM PRIVILEGIO REGIS.

La dinamica di un sistema, anche non perfettamente periodico, può essere vista come la manifestazione di un punto che si muove su un ciclo limite.



Battiti cardiaci

- Qualche migliaia di cellule sino-atriali
- Ognuna oscilla ad una sua frequenza caratteristica
- Peskin's model: quando una cellula emette un impulso, tutti le altre cellule hanno un piccolo salto nel voltaggio
- Questa debole interazione fa sincronizzare le cellule, che “sparano” sincrone dando origine al battito cardiaco



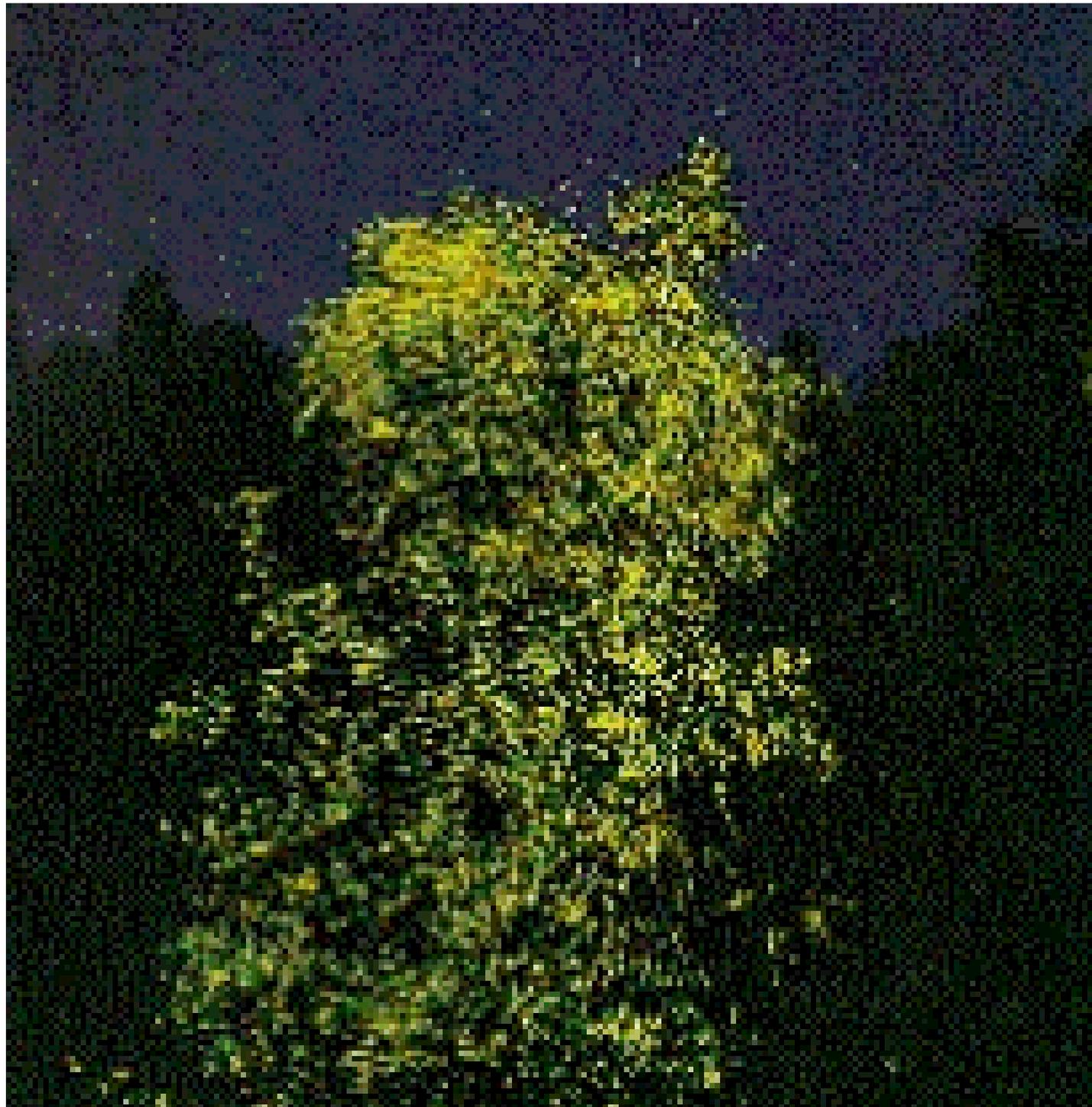
Sincronizzazione nel cervello

- I neuroni attivi nella banda alfa presi individualmente lancerebbero segnali elettrici con frequenze tutte diverse tra loro.
- Presi assieme formano un vero e proprio orologio che batte con la frequenza di circa 10 pulsazioni al secondo

Flash sincronizzati delle lucciole

- Scoperte in Malesia nell'800: colonie di migliaia di lucciole lampeggianti
- Flash usati nel corteggiamento rituale in gruppo
- Dopo un breve periodo il lampeggiamento diviene sincrono





Spiegazioni

- Accidentale ?
- Effetto ottico (sbattere delle ciglia dell'osservatore) ?
- Colpi di vento?
- **In realta':**
- Le lucciole hanno meccanismi neurali che vengono stimolati dai flash

Comportamenti simili negli esseri umani

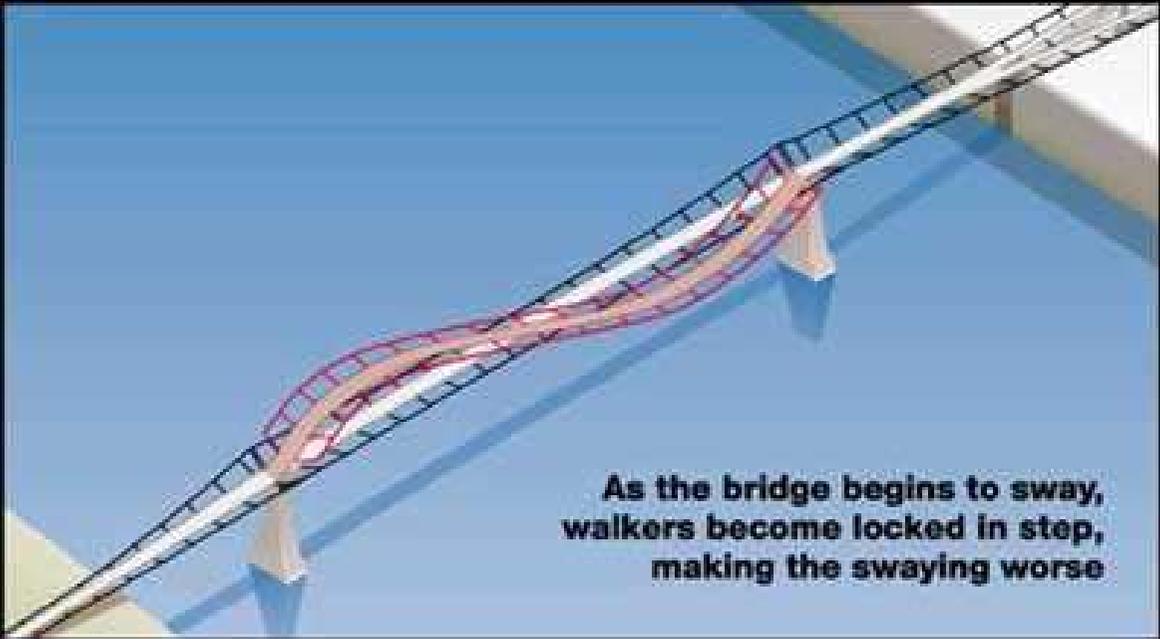
- Chiudete e aprite gli occhi e battete un dito sul tavolo
- Quasi subito i due movimenti diventano sincroni
- Applausi
- Compagne di collegio

Dark side of the moon



Millennium bridge

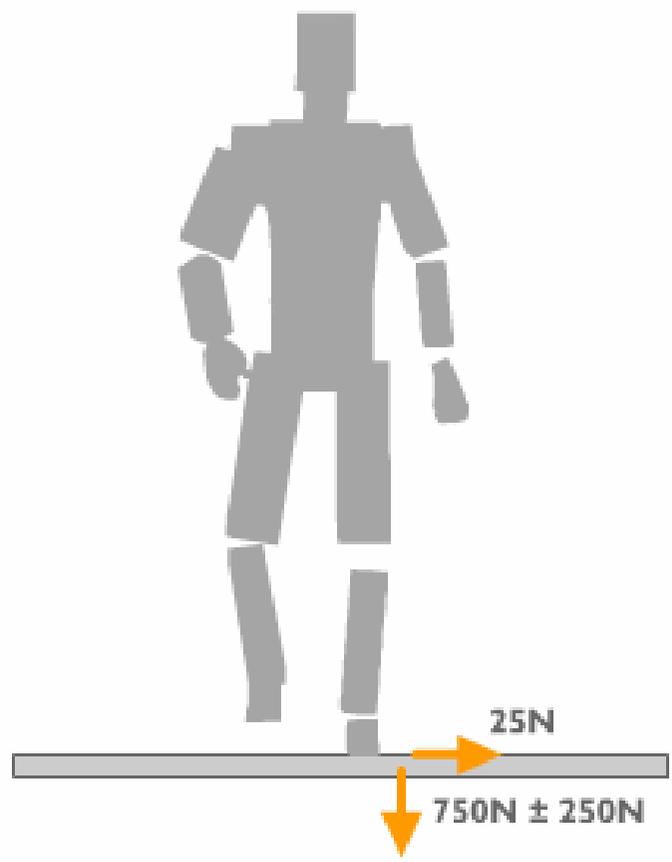




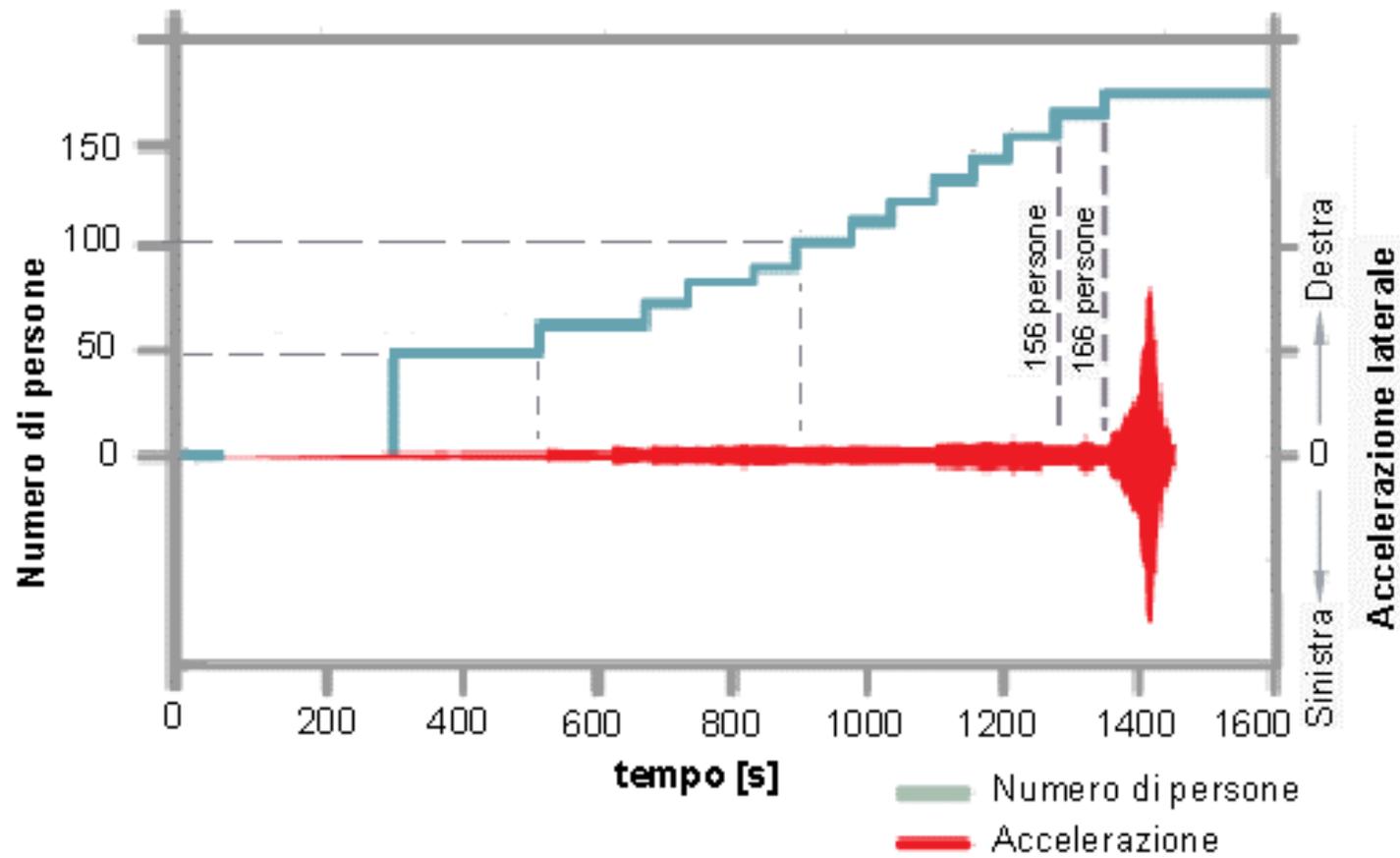
As the bridge begins to sway, walkers become locked in step, making the swaying worse

Front view





Transizione di fase



Onde α nel cervello

EEG rithms:

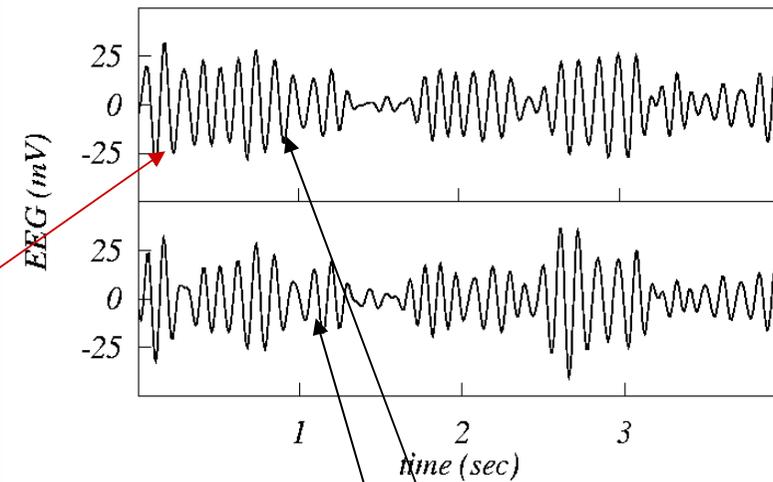
δ (0.5-3.5 Hz): deep sleep

θ (3.5-7.5 Hz): sleep

α (7.5-12.5 Hz): wake, relax

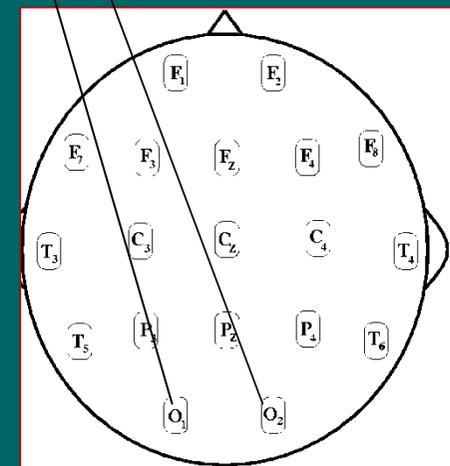
β (12.5-30 Hz): tension

γ (30-60 Hz): Cellular level exp.

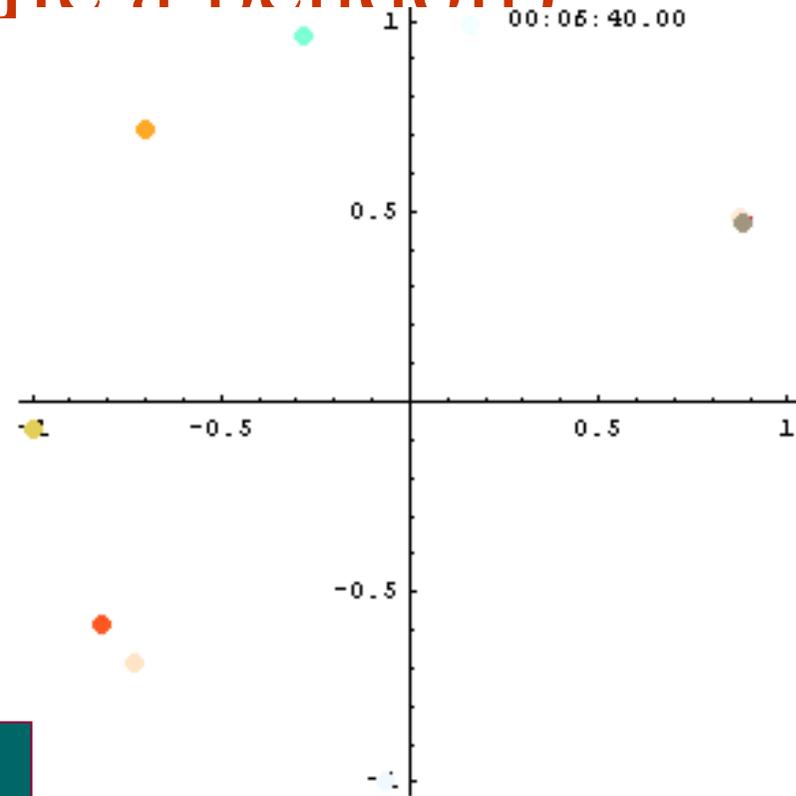


Periodicity is produced by spontaneous synchronization of microscopic signals

Synchronization between signals from different electrodes can be noted,

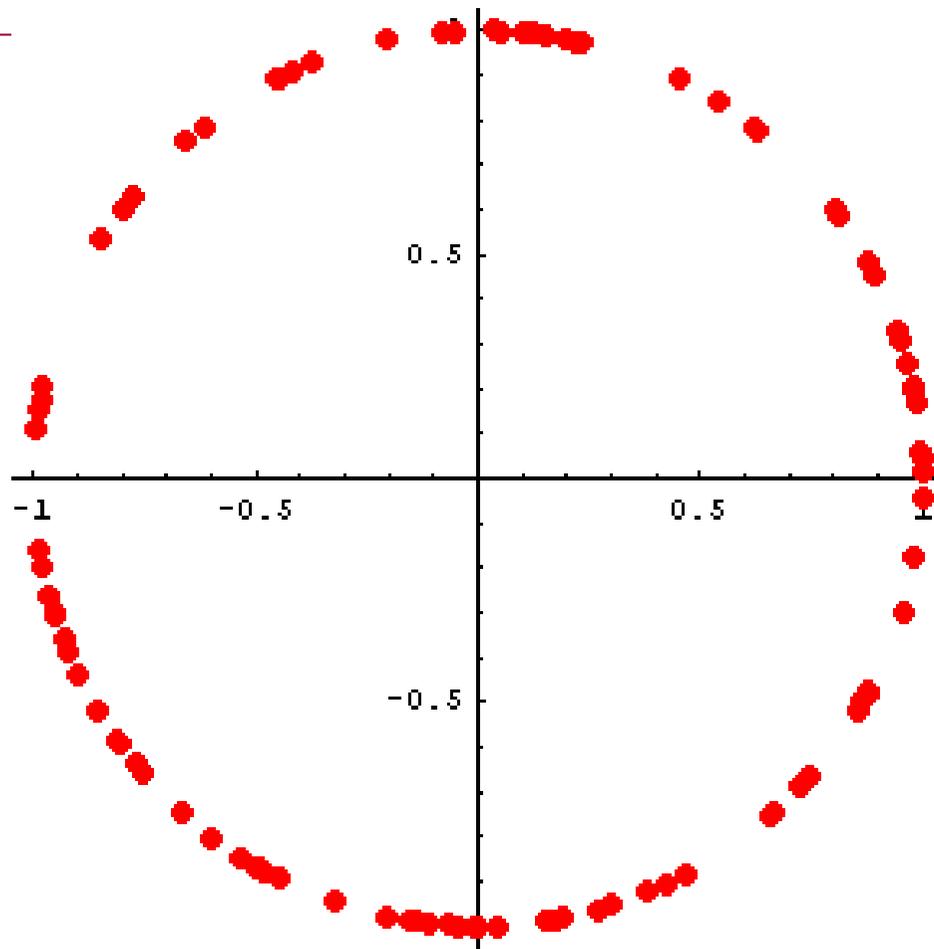


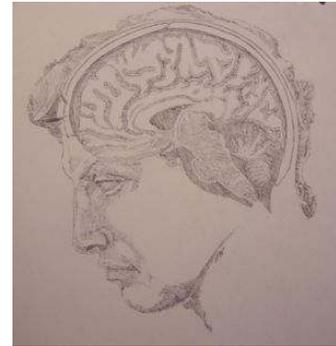
Modello matematico: Eq. di oscillatori (simile a pendoli)



Oscillatori: punti che si muovono su di una circonferenza con velocita' diverse:

(N grande, interazione non lineare)





- Il cervello mostra una specializzazione locale
- Compiti complessi richiedono cooperazione tra diverse aree del cervello
- La sincronizzazione è un meccanismo chiave per l'integrazione funzionale
- La sincronizzazione porta alla formazione di reti funzionali con struttura spaziale e temporale

Binding problem

- Come comunicano tra loro diverse zone del cervello in presenza (o in assenza) di determinati stimoli?

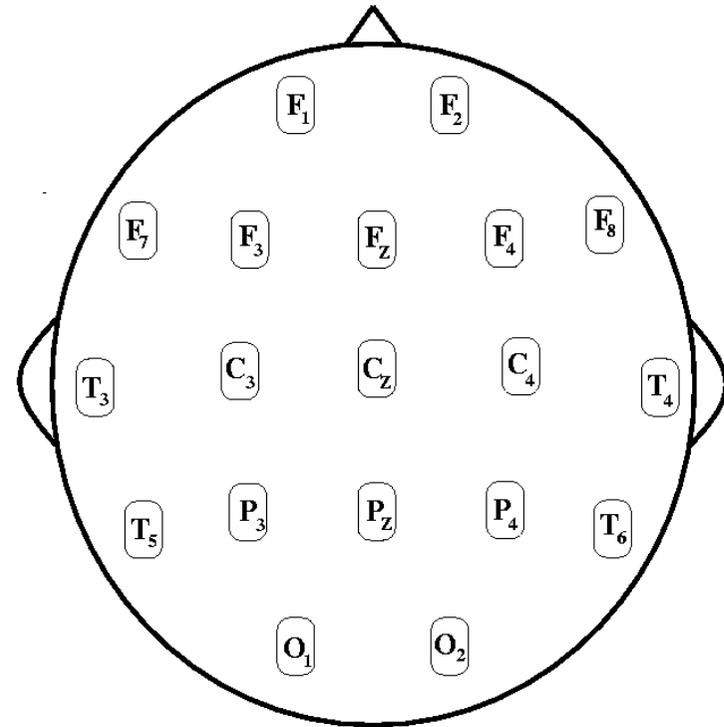


Eemicrania



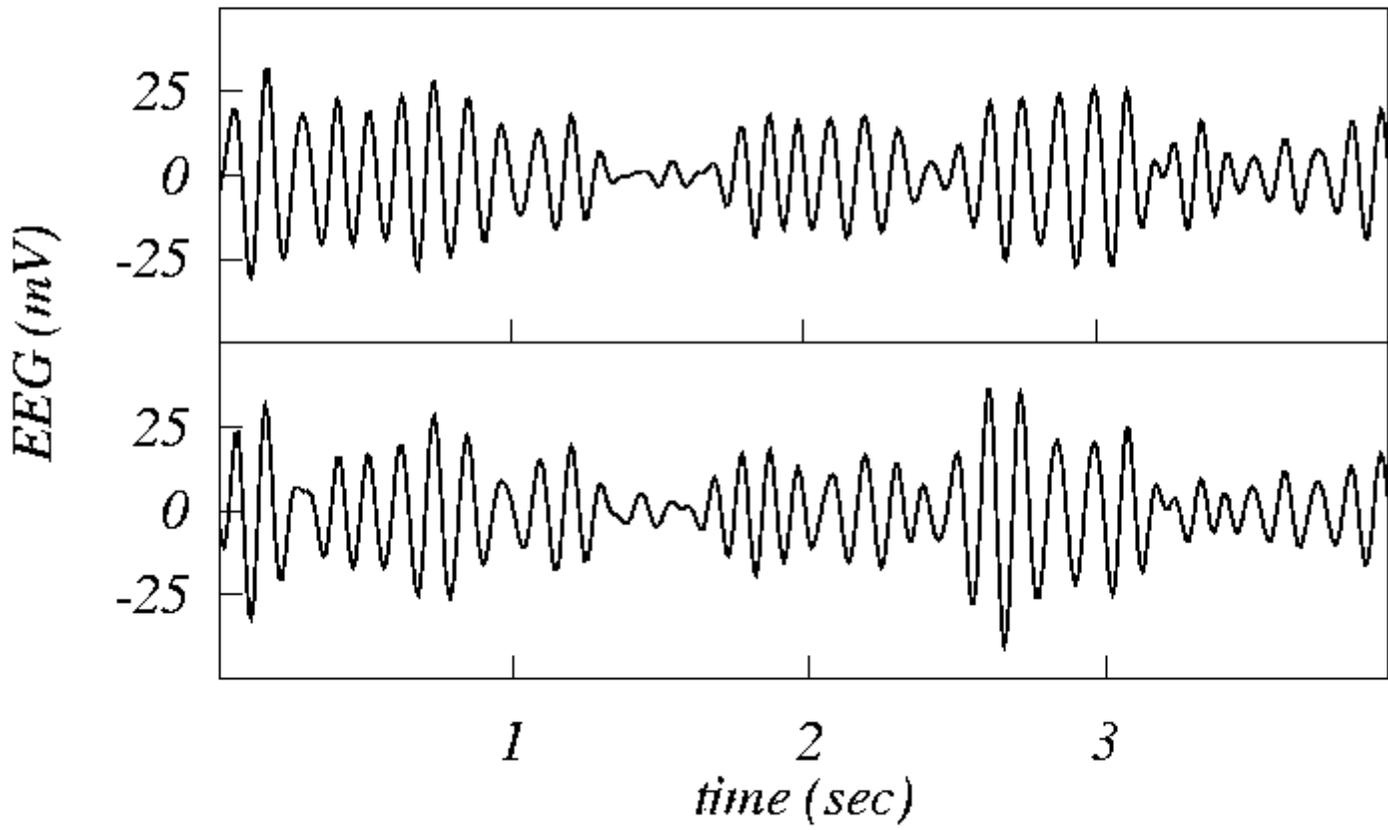
- Patologia molto complicata e diffusa
- Disordine del sistema nervoso centrale.
- Stress, sforzi fisici, cambiamenti di tempo, forti emozioni, luci intermittenti,
- Nessun modello valido sperimentale
- In cosa differisce la risposta a questi stimoli tra emicranici e persone sane?

- Theta (4-7 Hz)
- Alpha(8-12.5 Hz)
- Beta (14-30 Hz)
- Gamma (>30 Hz)



- EEG spontaneo
- Flash: 3-6-9-12-15-18-21-24-27 Hz

-
-



Steady-State Visual Evoked Potentials and Phase Synchronization in Migraine Patients

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M. Pellicoro,^{1,2,3} C. Pierro,¹ and S. Stramaglia^{1,2,3}

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⁴*Department of Neurological and Psychiatric Sciences, University of Bari, Italy*

⁵*Center for Polymer Studies and Department of Physics, Boston University, Boston, Massachusetts, USA*

⁶*Institute of Solid State Physics, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria*

⁷*D.E.T.O., University of Bari, Italy*

(Received 8 October 2003; revised manuscript received 5 April 2004; published 15 July 2004)

We investigate phase synchronization in EEG recordings from migraine patients. We use the analytic signal technique, based on the Hilbert transform, and find that migraine brains are characterized by enhanced alpha band phase synchronization in the presence of visual stimuli. Our findings show that migraine patients have an overactive regulatory mechanism that renders them more sensitive to external stimuli.

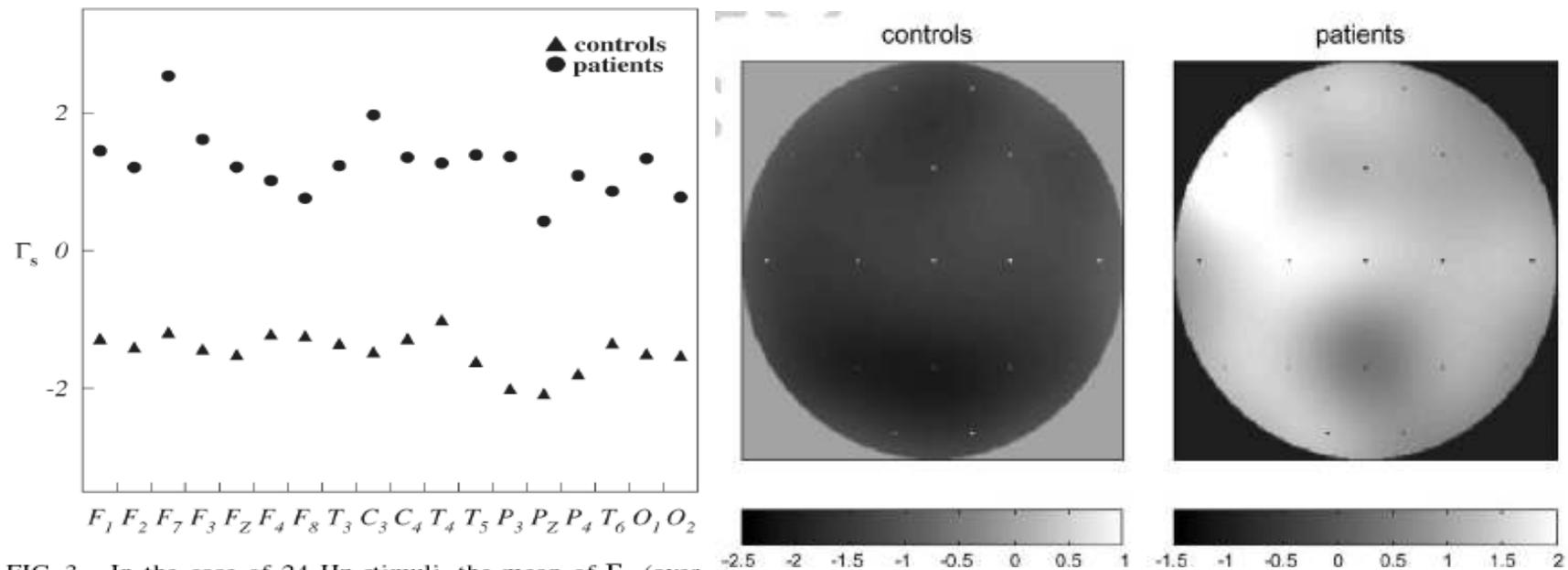


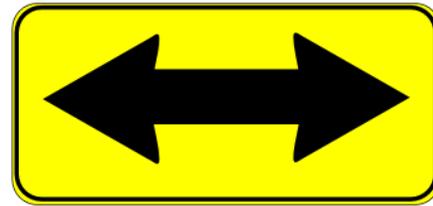
FIG. 3. In the case of 24 Hz stimuli, the mean of Γ_s (over patients and over controls) is represented for all the 18 electrodes. On the average, phase synchronization increases for patients and decreases for controls.

Relazioni di causa-effetto tra i sottosistemi

In molte situazioni siamo interessati non solo a rivelare stati sincronizzati, ma anche a mettere in evidenza relazioni di causa-effetto (drive-response) tra i sottosistemi di un sistema complesso.

Esempio: sync vs causality

Sync



Causality



Esempi di applicazione

- Neuroscienze
- Epilessia
- Epidemiologia sperimentale
- Ecologia e climatologia
- Macroeconomia
- Sistemi caotici spazio-temporali
- Reo-caos
- Genomica
-

Necessità di dare rigore statistico alla nozione di causa !

Adamo ed Eva



Esempio: Osserviamo una statua della regina Vittoria

- Perché la statua é qui?
- Causa materiale: la pietra con cui la statua é costruita
- Causa formale: la regina Vittoria
- Causa efficiente: lo scultore che ha fatto la statua
- Causa finale: volontà del popolo di esprimere gratitudine alla regina Vittoria

Causalit  in fisica

In fisica l'applicazione della causalit  materiale e di quella formale non crea problemi di natura filosofica.

L'applicazione della causalit  efficiente, in fisica, ha un importante corollario: la causa deve precedere l'effetto (in modo invariante, cio  in tutti i sistemi di riferimento inerziali).

Esperimenti come EPR sulla non-localit  solo apparentemente mettono in crisi la causalit  efficiente, perch    stato dimostrato che effetti tipo EPR non possono essere utilizzati per mandare segnali con velocit  maggiore di c .

Causalit  finale in Fisica? Ha senso porsi questa domanda? Esempio: principio antropico.

GALTON'S PLOT DI DATI CORRELATI (1888)

PROCEEDINGS

OF THE

ROYAL SOCIETY OF LONDON.

December 23, 1888.

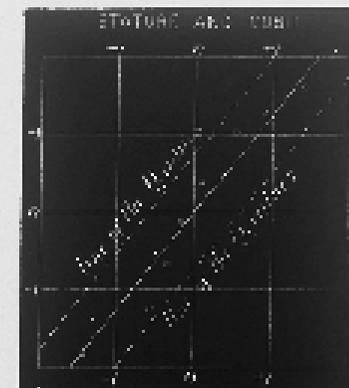
Professor G. G. STICKERS, D.C.L., President, in the Chair.

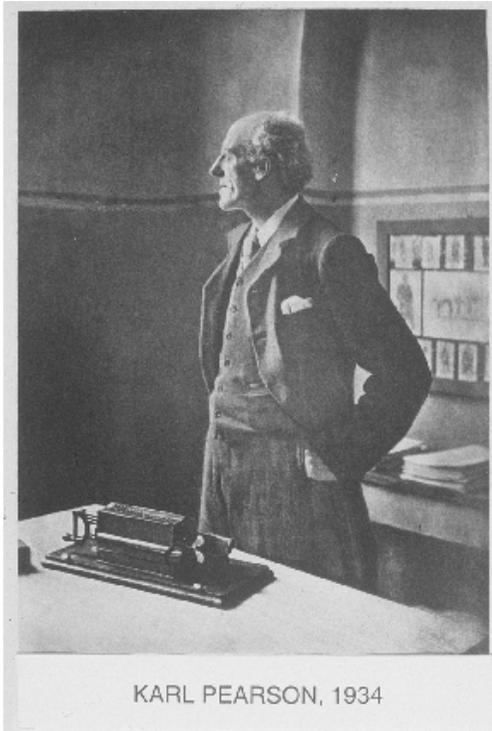
The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read —

1. "Correlations and their Measurement, chiefly from Anthropometric Data." By FRANCIS GALTON, F.R.S. Received December 5, 1888.

"Correlation or correlation of structure" is a phrase much used in biology, and used here in that branch of it which refers to heredity, and the idea is even more frequently present than the phrase; but I am not aware of any previous attempt to define it clearly, to trace its mode of action in detail, or to show how to measure its degree.





KARL PEARSON, 1934

"I felt like a buccaneer of Drake's days -... I interpreted that sentence of Galton to mean that there was a category broader than causation, namely correlation, of which causation was only the limit, and that this new conception of correlation brought psychology, anthropology, medicine, and sociology in large parts into the field of mathematical treatment."

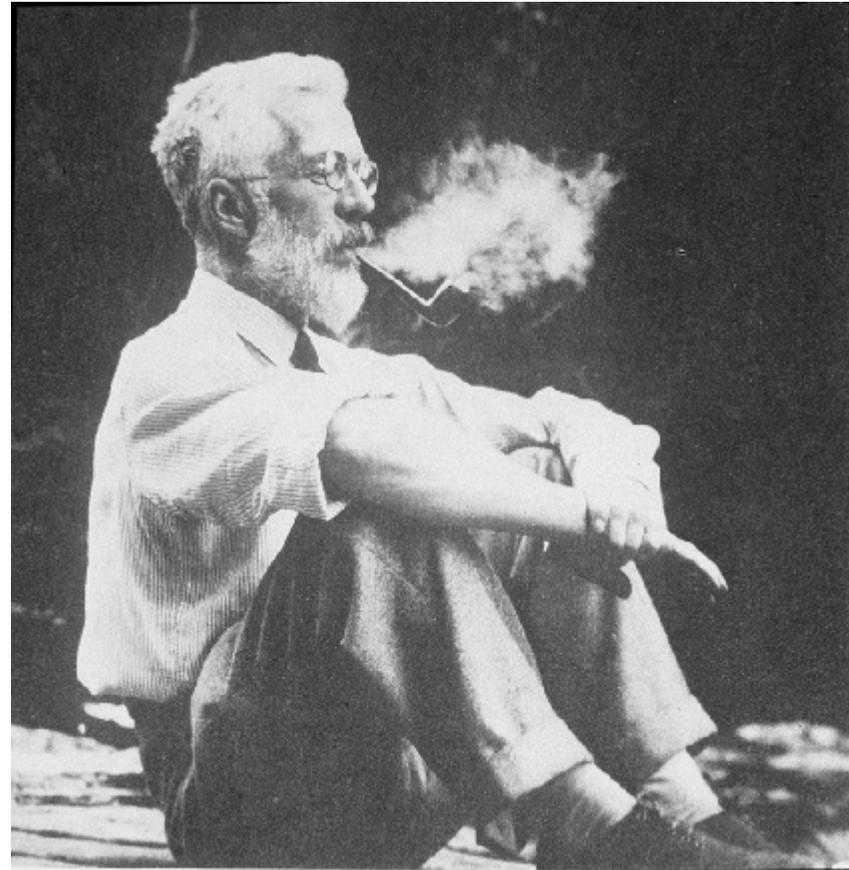
B_1 occurs n_{11} , B_2 occurs n_{12} times, and so on. We thus obtain a general distribution of B's for each class of A that we can form, and were we to go through the whole population, N, of A's in this manner we should obtain a table of the following kind:—

TYPE OF A OBSERVED

	A_1	A_2	A_3	A_p	Total
B_1	n_{11}	n_{21}	n_{31}	n_{p1}	$n_{.1}$
B_2	n_{12}	n_{22}	n_{32}	n_{p2}	$n_{.2}$
B_3	n_{13}	n_{23}	n_{33}	n_{p3}	$n_{.3}$
...
...
B_c	n_{1c}	n_{2c}	n_{3c}	n_{pc}	$n_{.c}$
...
...
Total	$n_{.1}$	$n_{.2}$	$n_{.3}$	$n_{.p}$	N

Pearson rigetta categoricamente il bisogno del concetto indipendente di causa, oltre al concetto di correlazione.

l'esperimento randomizzato



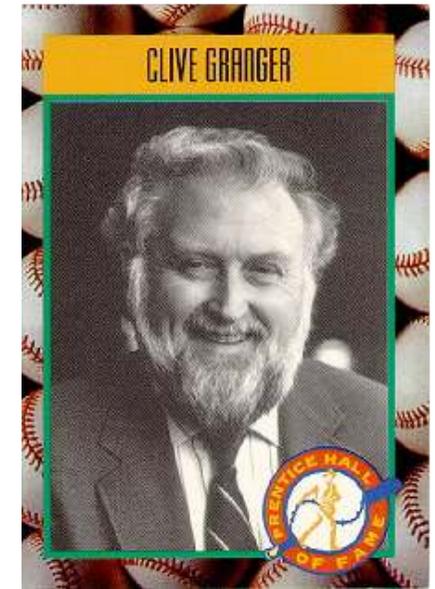
Fisher's classic example is that of the tea-drinking lady:

“A lady declares that by tasting a cup of tea made with milk she can discriminate whether the milk or the tea infusion was first added to the cup. ... Our experiment consists in mixing eight cups of tea, four in one way and four in the other, and presenting them to the subject in random order. ... Her task is to divide the cups into two sets of 4, agreeing, if possible, with the treatments received. ... The element in the experimental procedure which contains the essential safeguard is that the two modifications of the test beverage are to be prepared “in random order.” This is in fact the only point in the experimental procedure in which the laws of chance, which are to be in exclusive control of our frequency distribution, have been explicitly introduced. ... it may be said that the simple precaution of randomisation will suffice to guarantee the validity of the test of significance, by which the result of the experiment is to be judged.”

L'esperimento randomizzato permetterebbe di stabilire che il fumo è causa del cancro ai polmoni e non viceversa.

- Quando non è possibile fare l'esperimento randomizzato (come nel caso del fumo) è ancora possibile inferire relazioni causali?

Causalit : definizione operativa di Granger



Clive Granger (premio Nobel 2003 per l'economia)

X   causa di Y se la conoscenza di X permette di fare previsioni pi  accurate sui valori futuri di Y

Osserviamo che questa definizione ha senso solo per fenomeni irreversibili.

$$P(x | X) = P(x | X, Y)$$

Transfer entropy (Schreiber 2000)

Misura la violazione della generalized Markov property:

$$T(Y \rightarrow X) = \int P(x, X, Y) \log \left(\frac{P(x | X, Y)}{P(x | X)} \right) dx dX dY$$

T misura l'informazione che fluisce da una serie temporale all'altra

T è collegata ma non è equivalente all'accoppiamento

Transfer entropy and regression

The minimizer of the risk functional

$$R[f] = \int dX dx (x - f(X))^2 p(X, x)$$

represents the best estimate of x , given X , and corresponds to the regression function

$$f^*(X) = \int dx p(x|X)x.$$

Transfer entropy and regression

The best estimate of x , given X and Y , is now:

$$g^*(X, Y) = \int dx p(x|X, Y)x.$$

If $p(x|X, Y) = p(x|X)$, then

$$f^*(X) = g^*(X, Y)$$

and the knowledge of Y does not improve the prediction of x .

Transfer entropy

$$S_X = -\int dx dX p(x, X) \log[p(x | X)]$$

$$S_{XY} = -\int dx dX dY p(x, X, Y) \log[p(x | X, Y)]$$

Regression

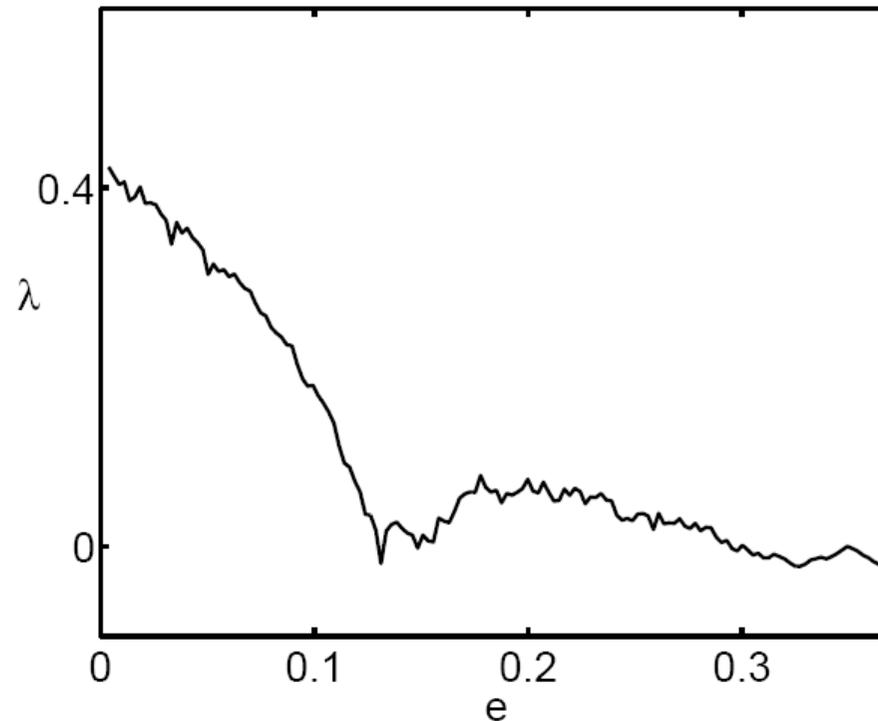
$$E_X = \int dx dX p(x, X) \left(x - \int dx' p(x' | X) x' \right)^2$$

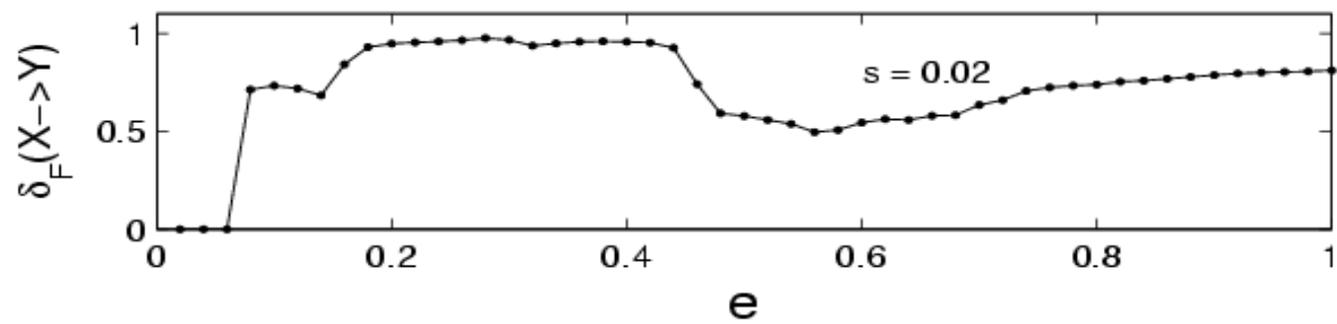
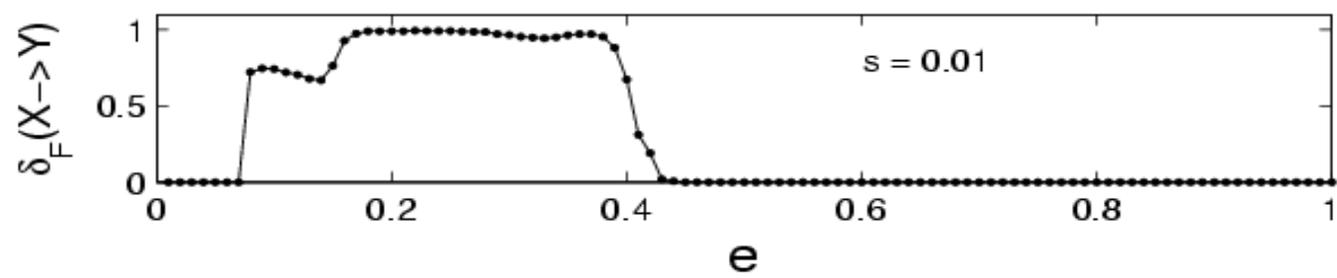
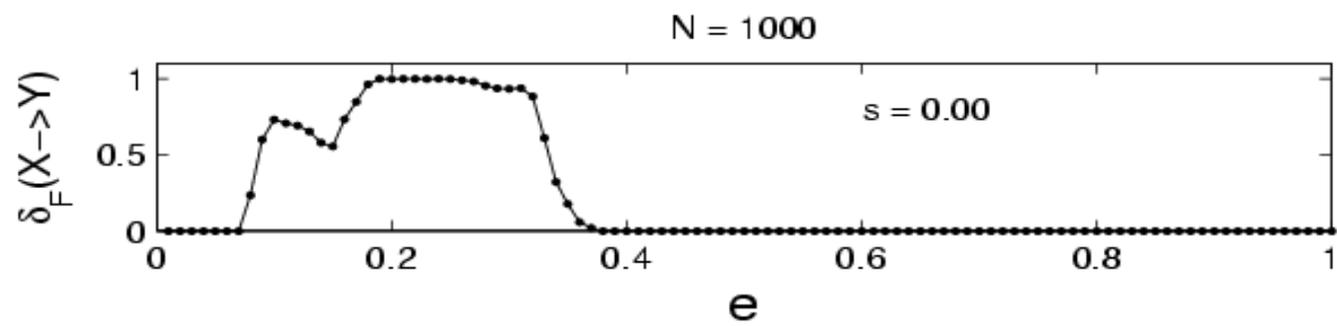
$$E_{XY} = \int dx dX dY p(x, X, Y) \left(x - \int dx' p(x' | X, Y) x' \right)^2$$

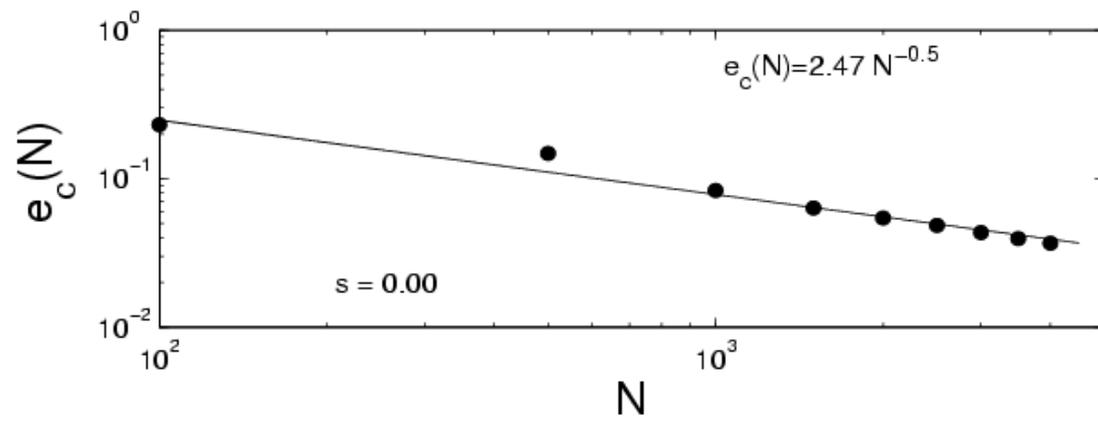
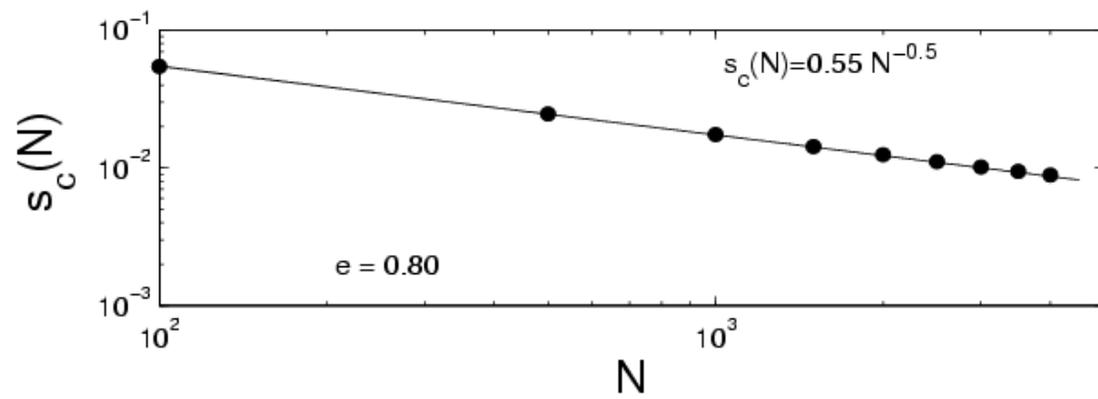
Esempi di applicazione del nostro metodo

Pair of noisy logistic maps:

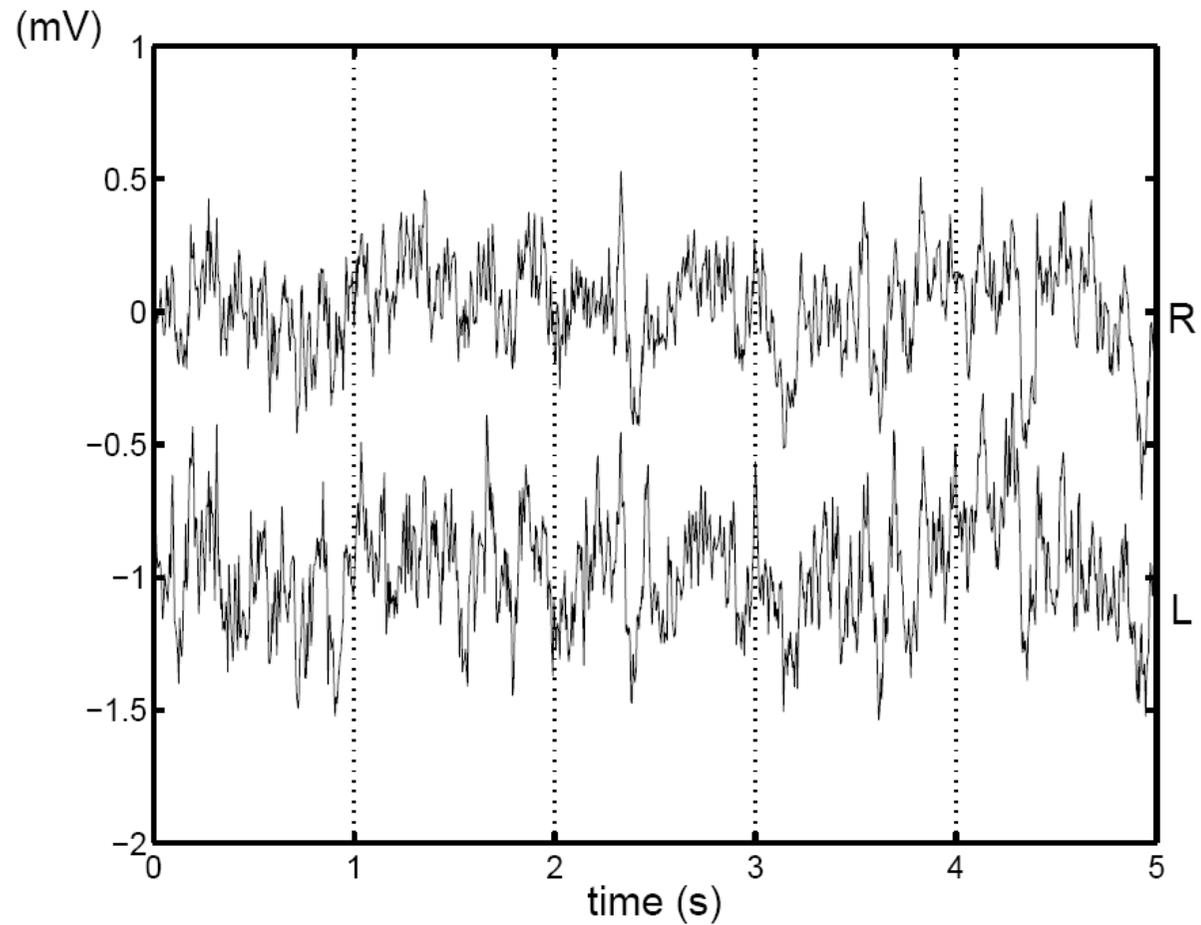
$$\begin{aligned}x_{n+1} &= a(1 - x_n^2) + s\eta_{n+1}, \\y_{n+1} &= (1 - e)a(1 - y_n^2) + ea(1 - x_n^2) + s\xi_{n+1};\end{aligned}$$



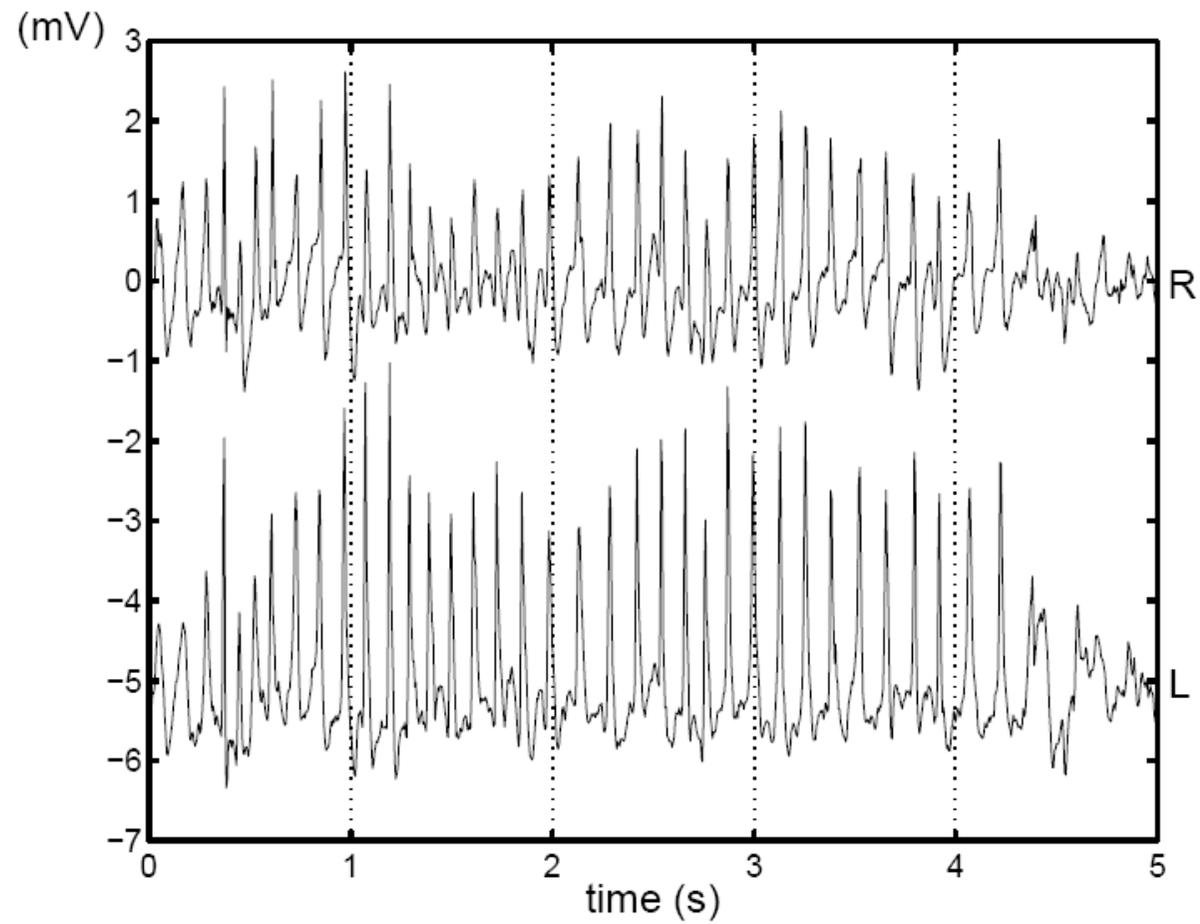


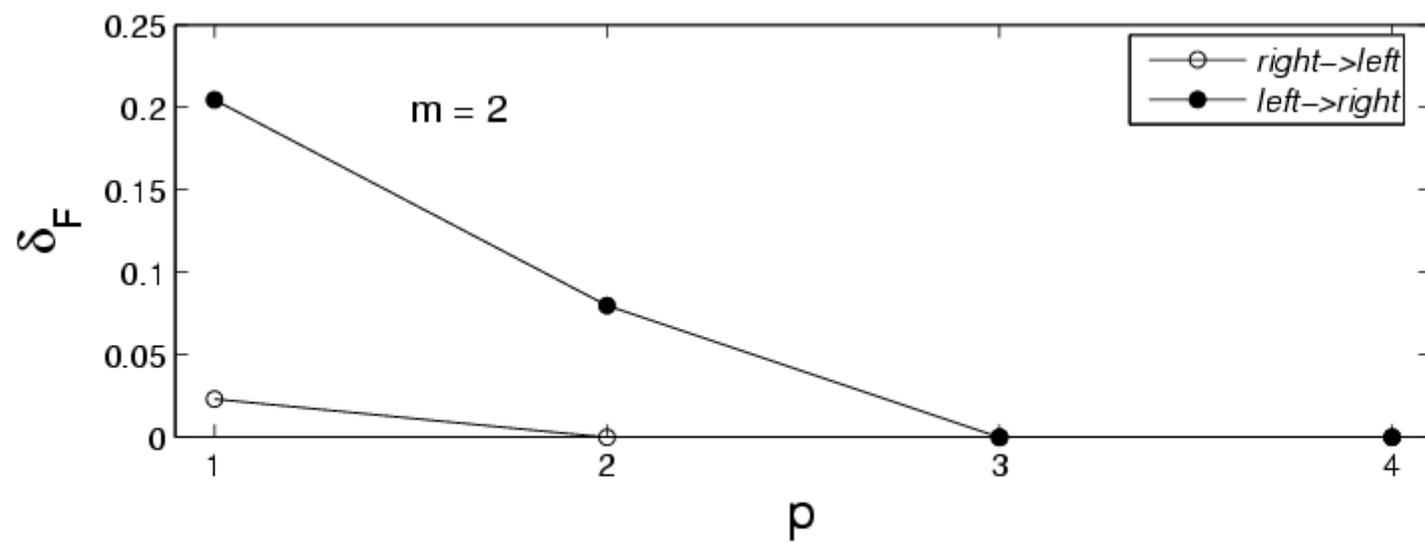
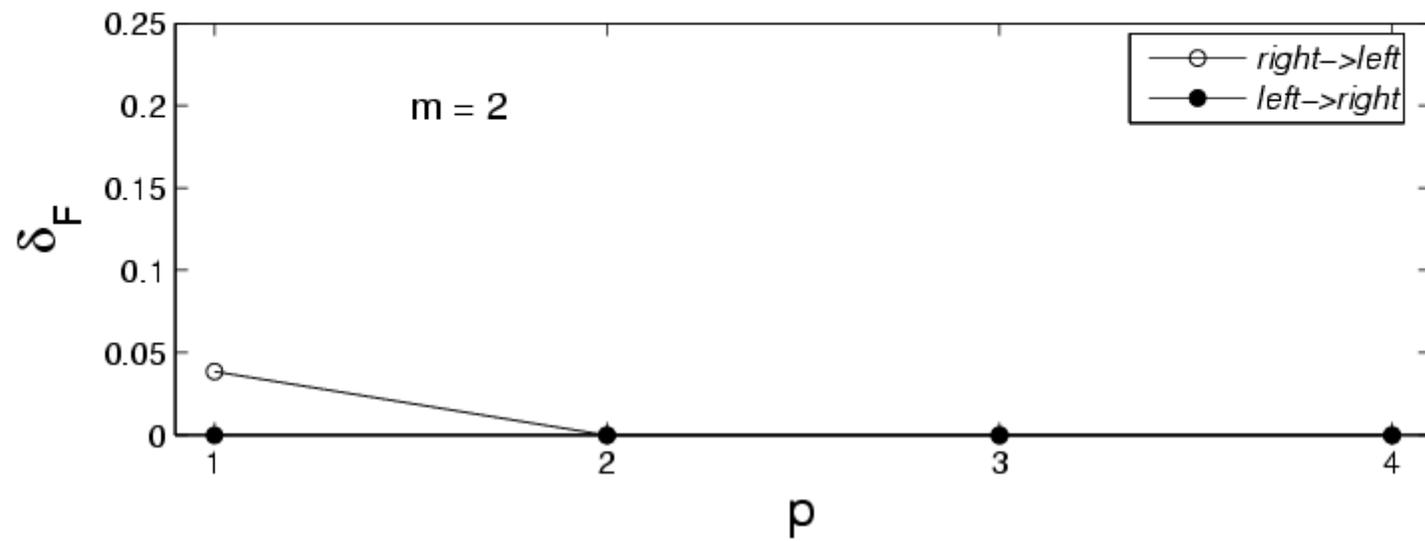


Applicazione: EEG ratto

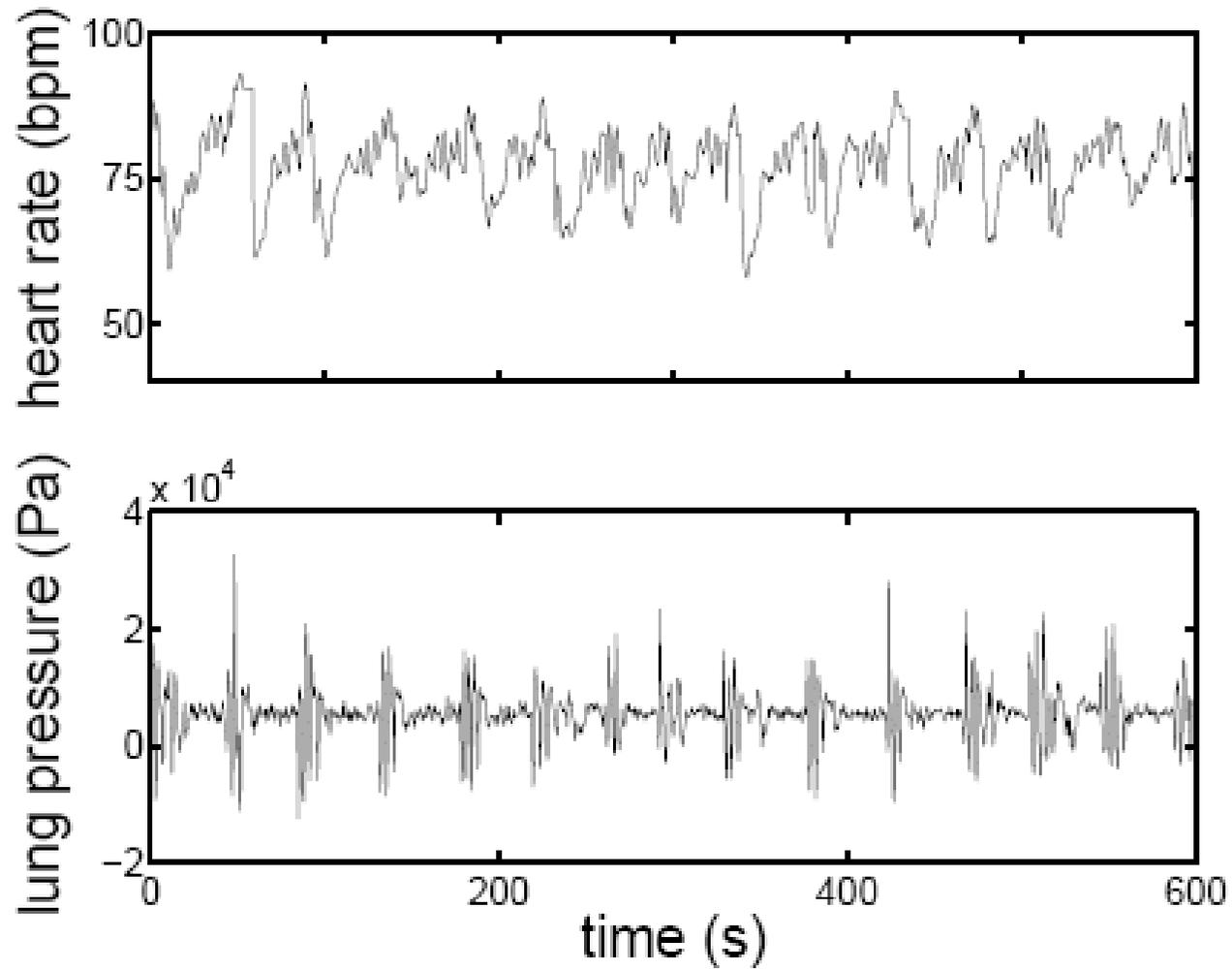


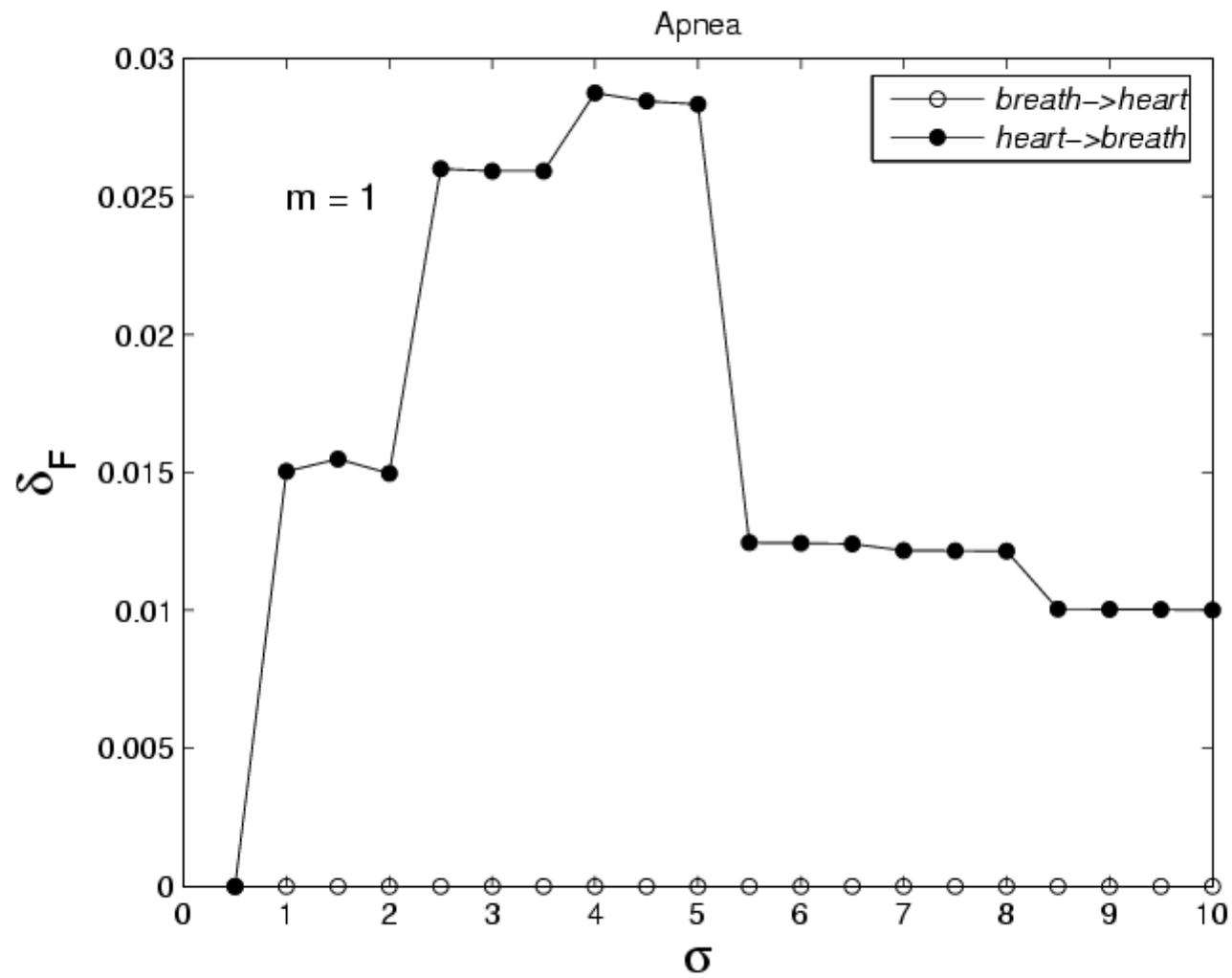
EEG ratto: dopo la lesione





Applicazione: apnea nel sonno



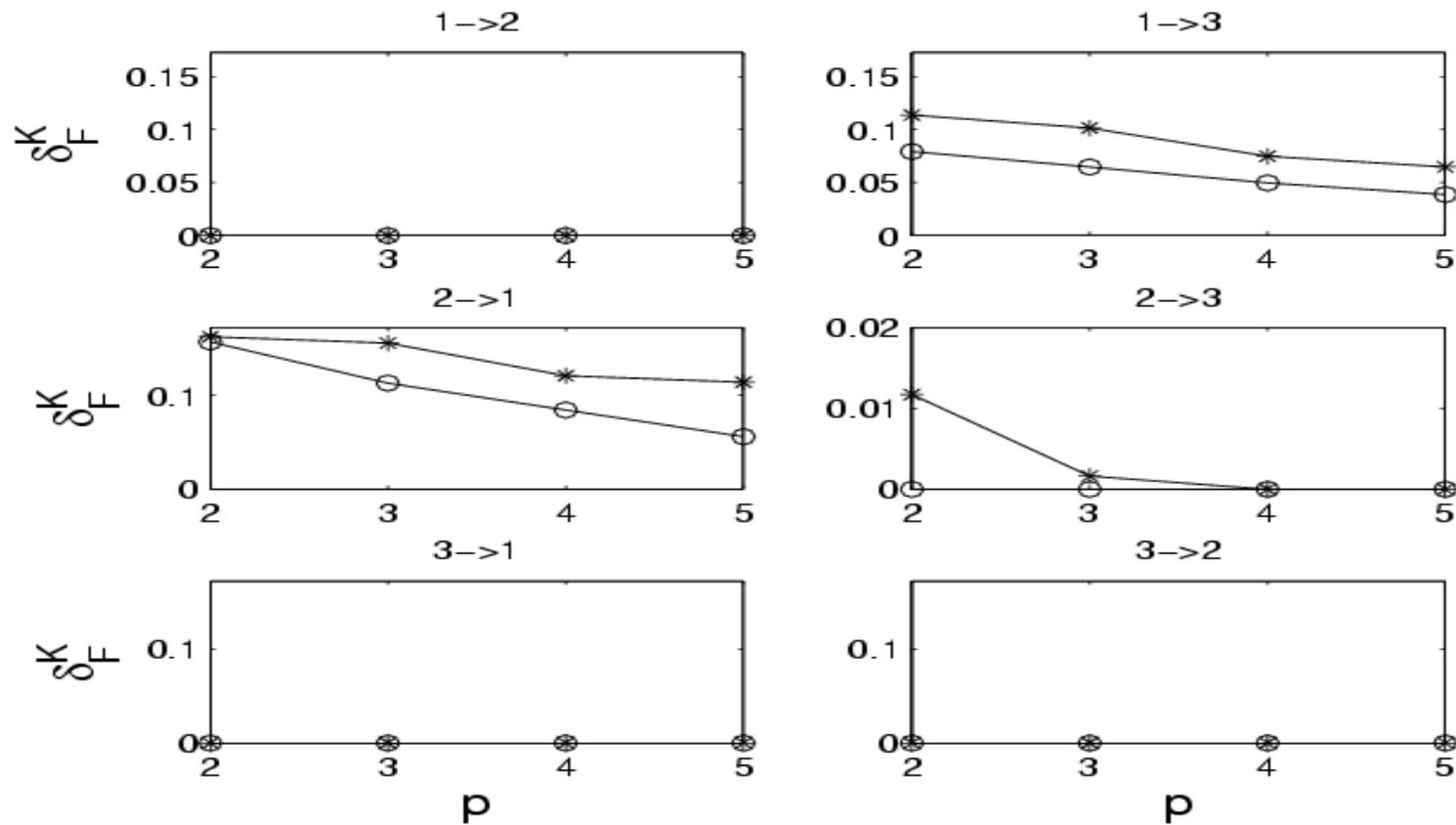


Multivariate Granger causality and indirect relationships

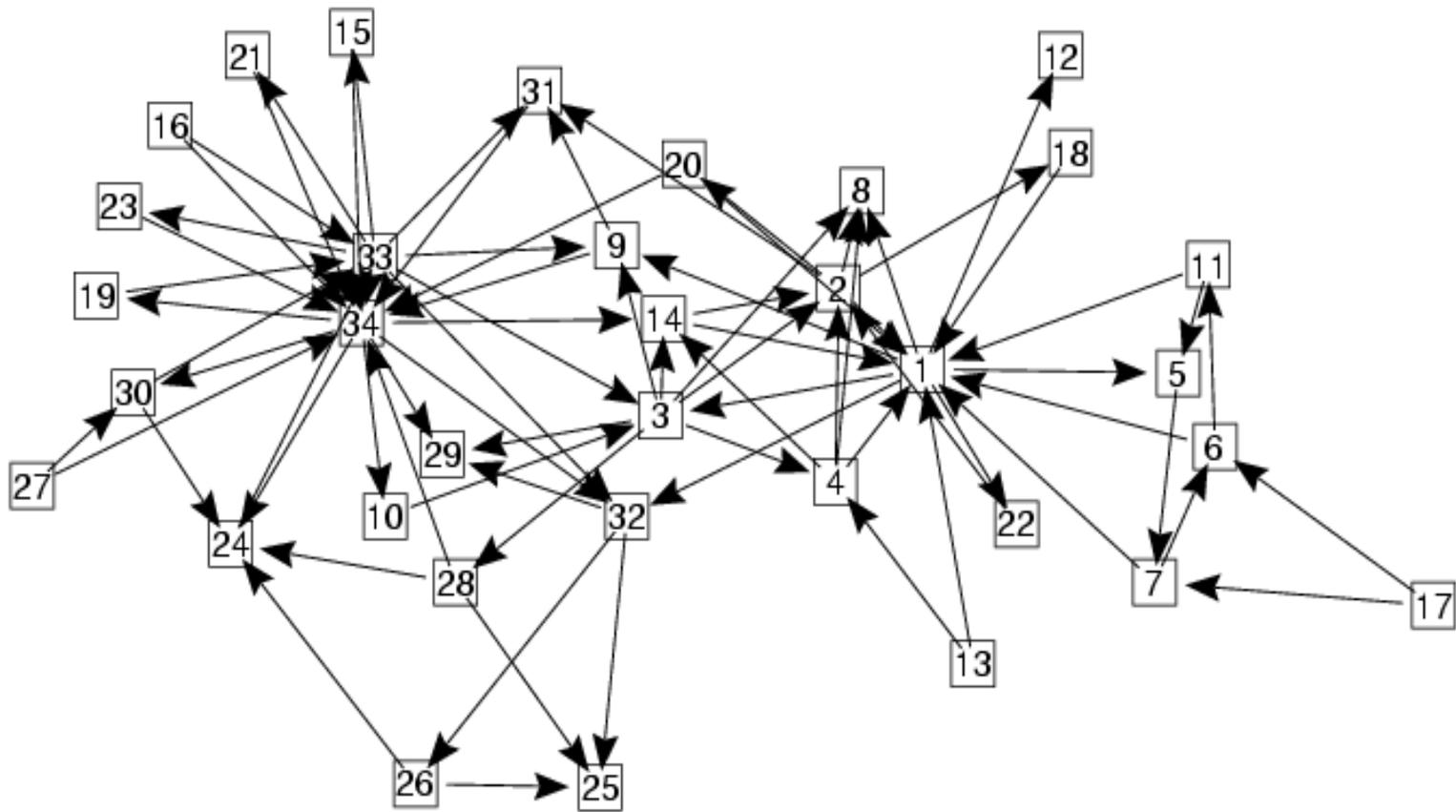
Three logistic maps: $2 \rightarrow 1 \rightarrow 3$

$$\begin{aligned}x_{1,t} &= 0.8 (1 - ax_{1,t-1}^2) + 0.2 (1 - ax_{2,t-1}^2) + s\tau_{1,t}, \\x_{2,t} &= 1 - ax_{2,t-1}^2 + s\tau_{2,t}, \\x_{3,t} &= 0.8 (1 - ax_{3,t-1}^2) + 0.2 (1 - ax_{1,t-1}^2) + s\tau_{3,t};\end{aligned}$$

Three logistic maps: $2 \rightarrow 1 \rightarrow 3$



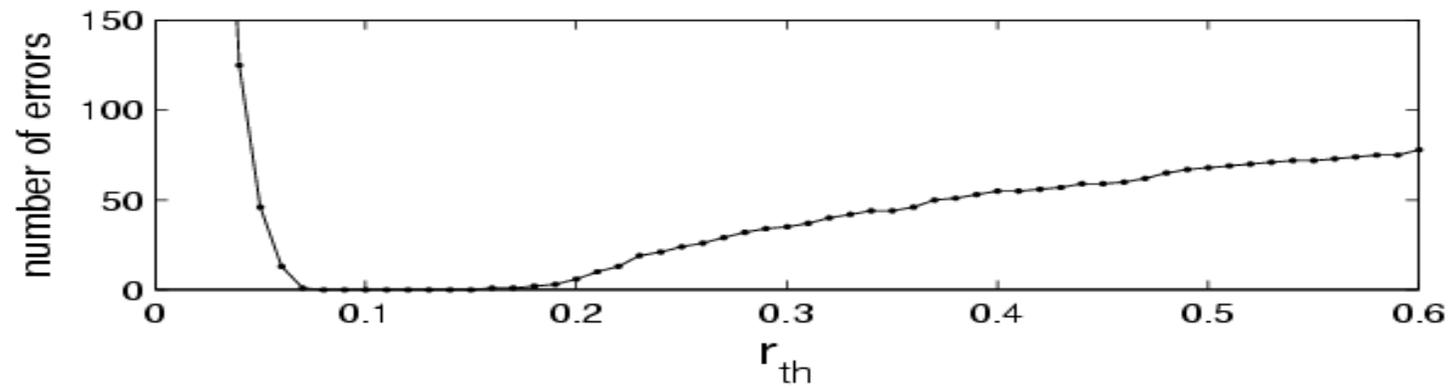
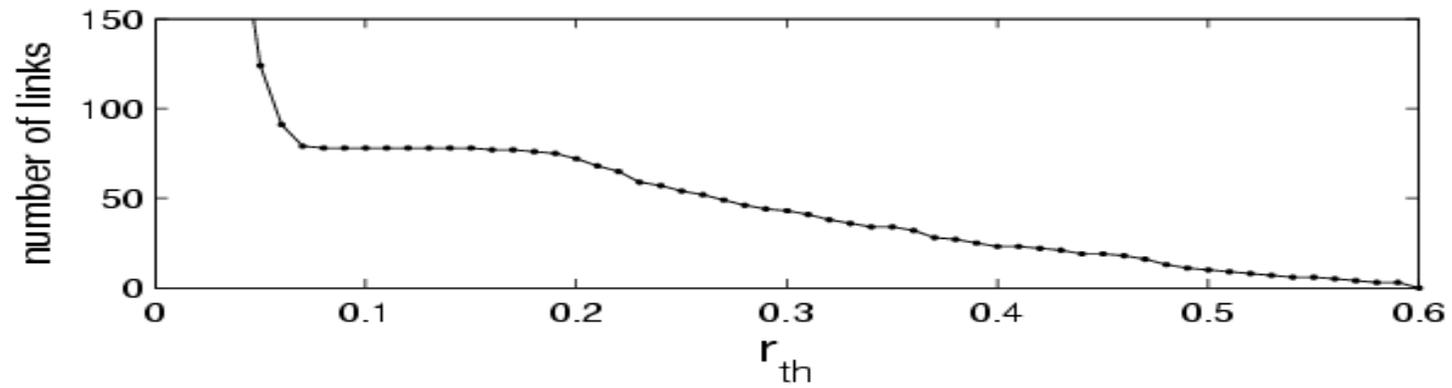
Dynamical network



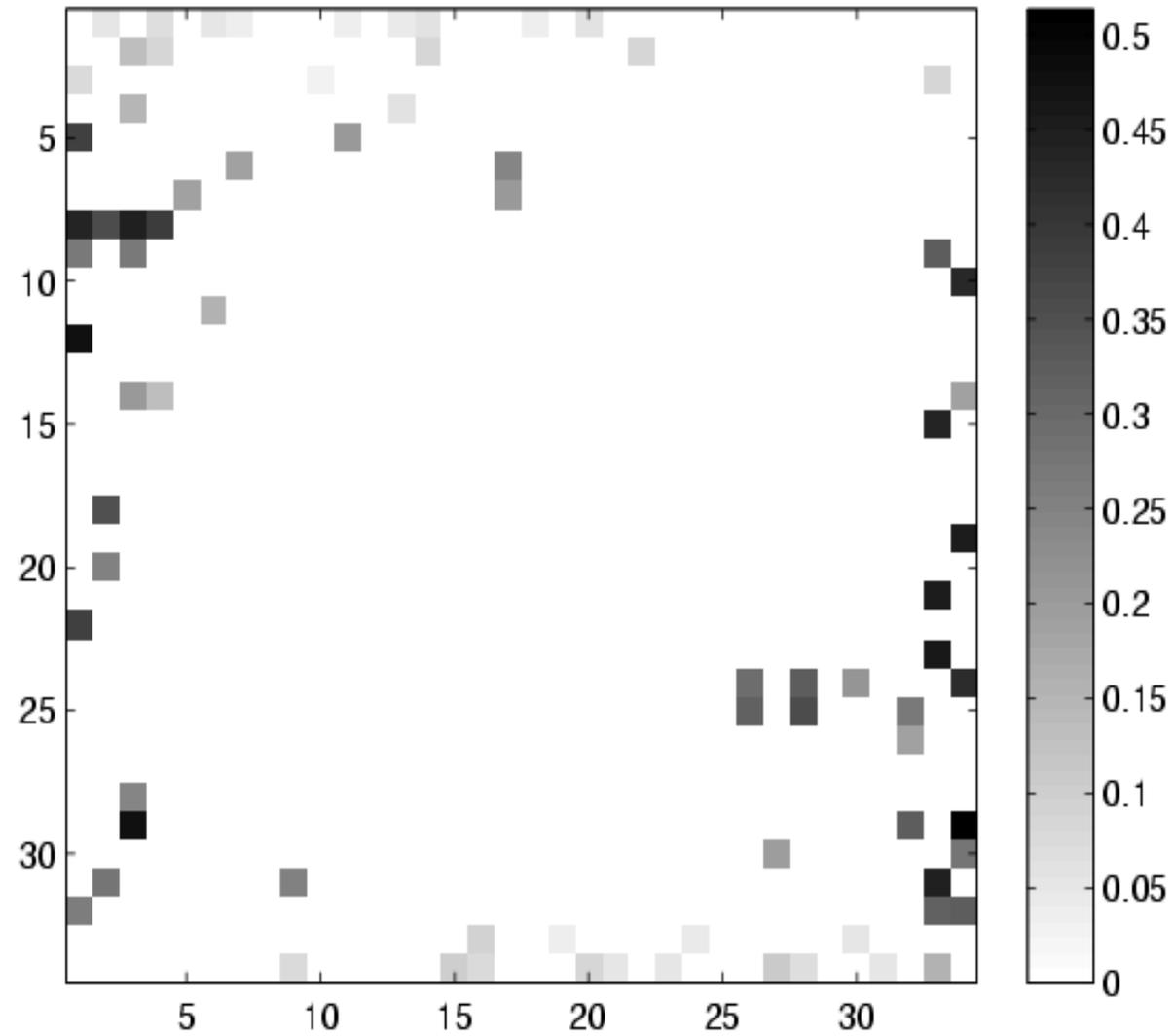
Coupled Chaotic maps

$$x_{i,t} = \left(1 - \sum_{j=1}^n c_{ij} \right) (1 - ax_{i,t-1}^2) + \sum_{j=1}^n c_{ij} (1 - ax_{j,t-1}^2) + s\tau_{i,t},$$

Results



Correlation vs coupling

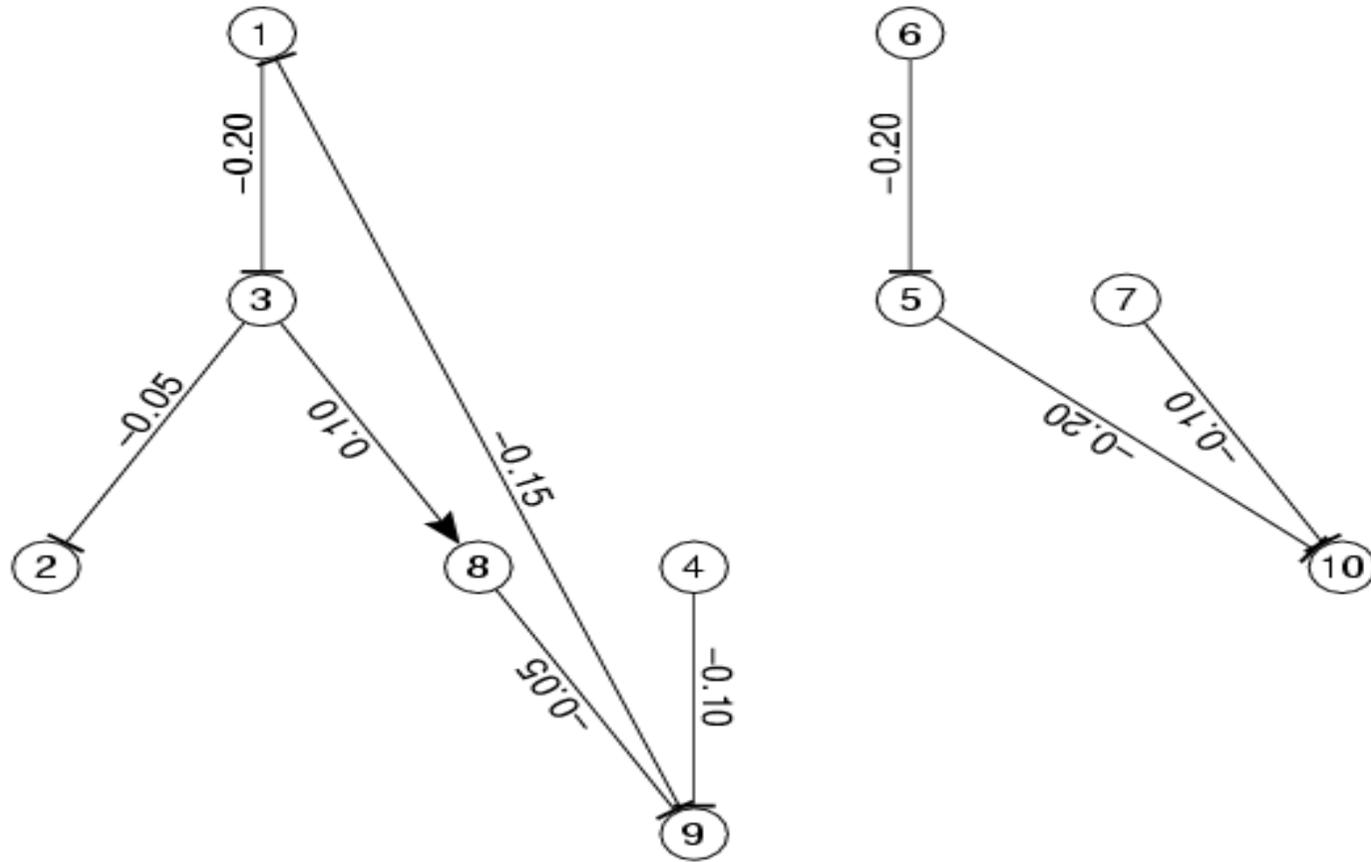


Genetic regulatory network

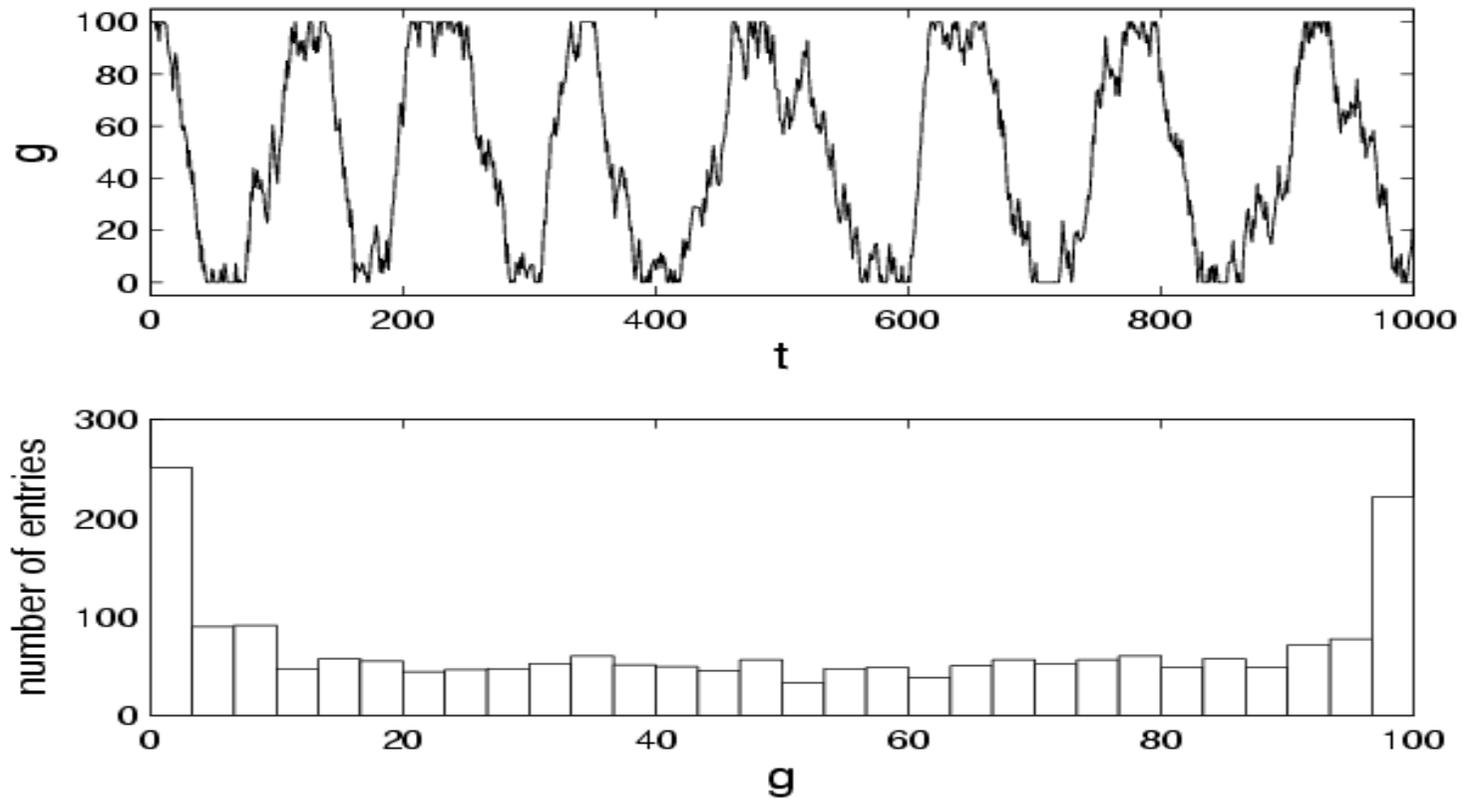
$$\mathbf{g}_{t+1} = \mathbf{g}_t + A (\mathbf{g}_t - T\mathbf{I}) + \Sigma, \quad (1)$$

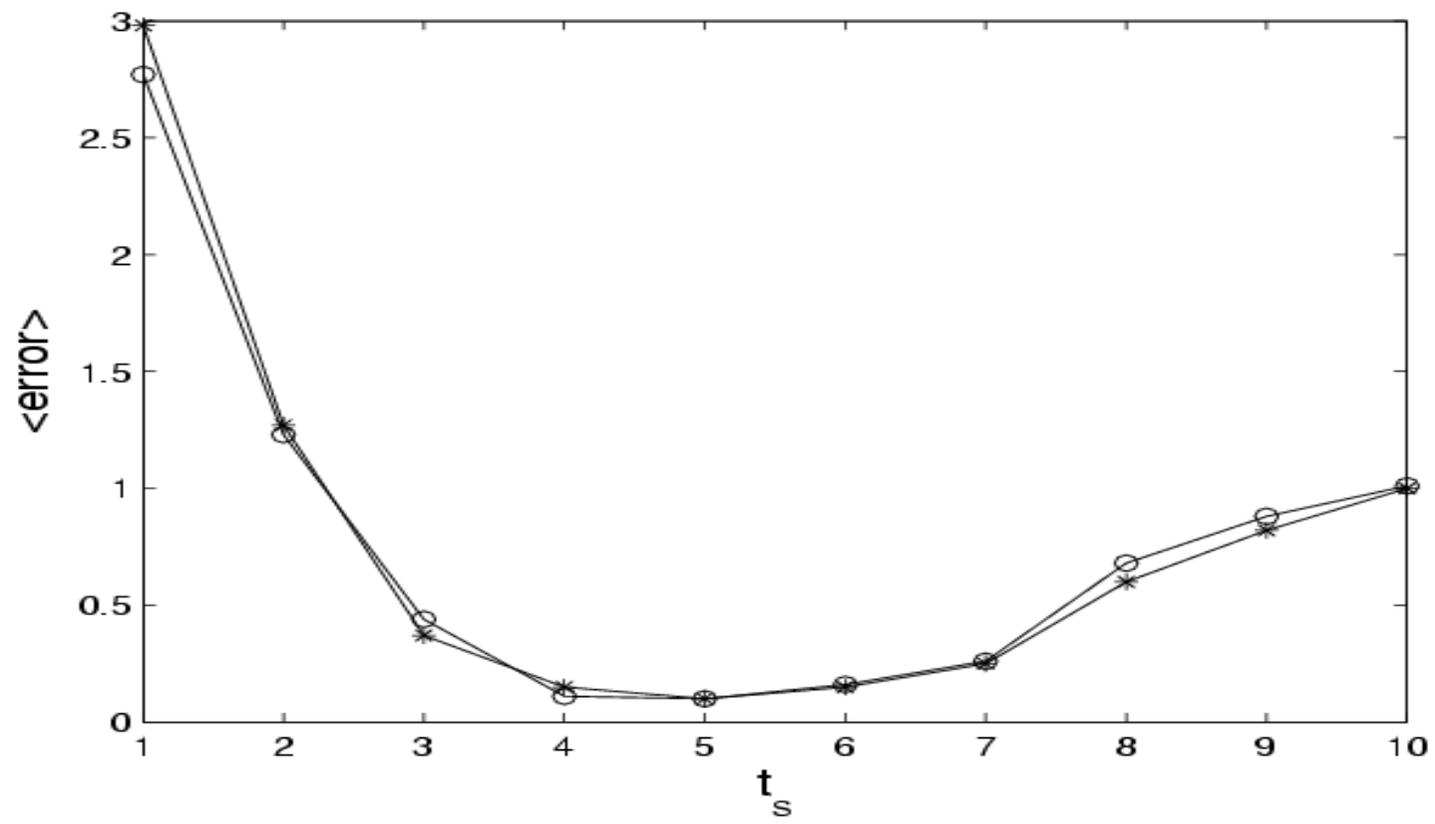
where A is a connectivity strength matrix corresponding to the network, $T = 50$, \mathbf{I} is the identity matrix and Σ is a vector of random variables uniform in $[-10, 10]$. The values of \mathbf{g} are restricted by floor and ceiling function to range in $[0, 100]$: this constraint provides the nonlinear character of the model.

Genetic regulatory network



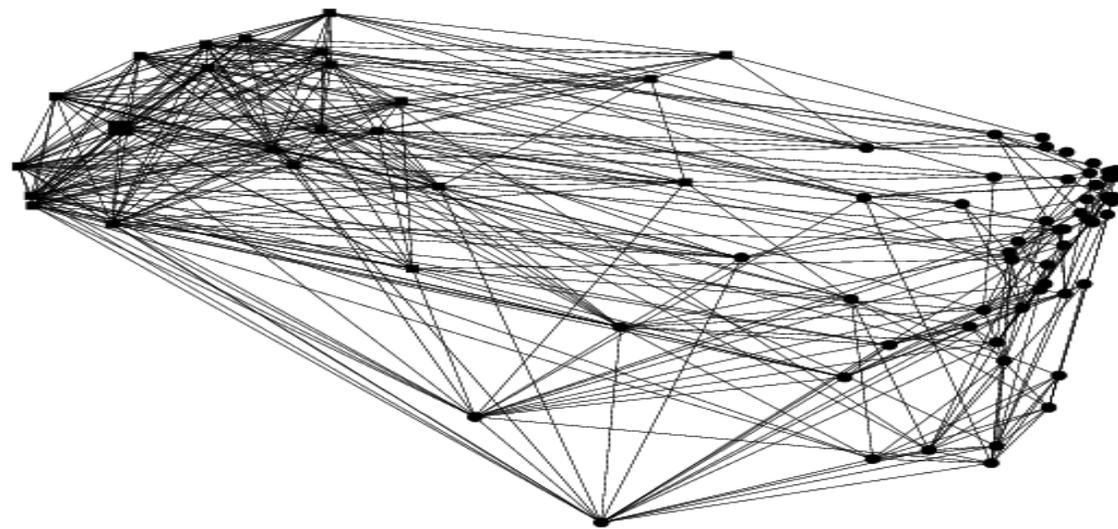
Typical evolution of the expression of a gene





HeLa cell cycle

- 94 genes and 48 time points (an hour interval separating two measures).
- Analysis of static correlations: 800 pairs of genes are significantly correlated
- The analysis of the undirected network shows two modules (23 and 62 genes resp.), corresponding to well known transcriptional factors (NFkB, p53-STAT3)



Granger causality

- 19 causalities acting on the time scale of the sampling rate are significant

TPD52L	→	TNF-a	(0.6166)
TSP1	→	Bcl-XL	(0.5935)
TSP1	→	c-myc	(0.5905)
IRF-2	→	BRCA1	(0.5669)
c-myc	→	FGFR4	(0.5653)
R2	→	c-myc	(0.5642)
PKIG	→	TSP1	(0.5475)
IAP	→	NFkB	(0.5457)
NFkB	→	B99	(0.5416)
Bcl-XL	→	OCT4	(0.5354)
OCT4	→	VCAM-1	(0.5329)
VCAM-1	→	TPD52L	(0.5315)
Killer/DR5	→	c-myc	(0.5313)
A20	→	Bcl-XL	(0.5284)
Cyclin E1	→	E2F-1	(0.5249)
PKIG	→	ICAM-1	(0.5243)
IkappaBa	→	NFkB	(0.5156)
TPD52L	→	MASPIN	(0.5128)
FGF7	→	MCP-1	(0.5107)