Inclusive three- and four-jet production in multi-Regge kinematics at the LHC

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based on

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# Outline

1. **Introduction**
   - Motivation

2. **Three-jet production**
   - A new way to probe BFKL
   - Three-jet phenomenology: hadronic level analysis

3. **Four-jet production**
   - Four-jet BFKL azimuthal profile
   - Four-jet phenomenology: hadronic level predictions

4. **Conclusions & Outlook**
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4 Conclusions & Outlook
How could we further and deeply probe BFKL?

1. Study a less inclusive two-body final state...

Di-hadron production

- inclusive production of a pair of charged light hadrons well separated in rapidity
- hadrons can be detected at the LHC at much smaller values of the transverse momentum than jets
- possibility to constrain not only the PDFs, but also the FFs!

2. Study three- and four-body final state processes...

Multi-jet production

- demand the tagging of one or/and two further jets in more central regions of the detectors with a relative separation in rapidity from each other
- definition of new, suitable BFKL observables...
- ...in order to further investigate the azimuthal distribution of the final state
How could we further and deeply probe BFKL?

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- in order to further investigate the azimuthal distribution of the final state
Looking for new observables...

! We would like to study observables for which the $k_T$ (any $k_T$ along the BFKL ladder) enters the game...

- ...to probe not only the general properties of the BFKL ladder, but also “to peek into the interior”...
- ...by studying azimuthal decorrelations where the $k_T$ of extra particles introduces a new dependence...

...multi-jet production!

[R. Maciula, A. Szczurek (2014, 2015)]
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...multi-jet production!

[R. Maciula, A. Szczurek (2014, 2015)]
Three- and four-jet production

\[ p_1 \]

\[ x_1 \]

\[ k_A, \theta_A, Y_A \]

\[ k_J, \theta_J, y_J \]

\[ x_2 \]

\[ k_B, \theta_B, Y_B \]

\[ p_2 \]

\[ p_1 \]

\[ x_1 \]

\[ k_A, \theta_A, Y_A \]

\[ k_J, \theta_J, y_J \]

\[ x_2 \]

\[ k_B, \theta_B, Y_B \]

\[ k_1, \theta_1, y_1 \]

\[ k_2, \theta_2, y_2 \]

\[ p_2 \]

[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]
[F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016)]
[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]
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An event with three tagged jets

\[ Y_B < y_J < Y_A \]
The three-jet partonic cross section

Starting point: differential partonic cross-section (no PDFs)

\[
\frac{d^3\hat{\sigma}^{3\text{-jet}}}{dk_J d\theta_J dy_J} = \frac{\tilde{\alpha}_s}{\pi k_J} \int d^2\vec{p}_A \int d^2\vec{p}_B \delta^{(2)} \left( \vec{p}_A + \vec{k}_J - \vec{p}_B \right) \times \varphi \left( \vec{k}_A, \vec{p}_A, Y_A - y_J \right) \varphi \left( \vec{p}_B, \vec{k}_B, y_J - Y_B \right)
\]

- **Multi-Regge kinematics** rapidity ordering: $Y_B < y_J < Y_A$
- $k_J$ lie above the experimental resolution scale
- $\varphi$ is the LO BFKL gluon Green function
- $\tilde{\alpha}_s = \alpha_s N_c / \pi$
A three-jet primitive lego-plot
Generalized azimuthal correlations - partonic level

**Prescription:** integrate over all angles after using the projections on the two azimuthal angle differences indicated below...

→ ...to define:

\[
\int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos (M (\theta_A - \theta_J - \pi)) \cos (N (\theta_J - \theta_B - \pi)) \frac{d^3\hat{\sigma}^{3\text{-jet}}}{dk_J d\theta_J dy_J}
\]

\[
= \bar{\alpha}_s \sum_{L=0}^{N} \binom{N}{L} (k_j^2)^{L-1/2} \int_0^{\infty} dp^2 \left( p^2 \right)^{N-L} \int_0^{2\pi} d\theta (\frac{-1}{}^{M+N} \cos (M\theta) \cos ((N - L)\theta)) \frac{\sqrt{p^2 + k_j^2 + 2\sqrt{p^2k_j^2 \cos \theta}}}{N}
\]

\[
\times \phi_M (k_A^2, p^2, Y_A - y_J) \phi_N \left( p^2 + k_j^2 + 2\sqrt{p^2k_j^2 \cos \theta}, k_B^2, y_J - Y_B \right)
\]

Main observables: **generalized azimuthal correlation momenta**

\[
\mathcal{R}_{MN}^{PQ} = \frac{\mathcal{C}_{MN}}{\mathcal{C}_{PR}} = \frac{\langle \cos (M (\theta_A - \theta_J - \pi)) \cos (N (\theta_J - \theta_B - \pi)) \rangle}{\langle \cos (P (\theta_A - \theta_J - \pi)) \cos (Q (\theta_J - \theta_B - \pi)) \rangle}
\]

Remove the contribution from the zero conformal spin

→ drastically reduce the dependence on collinear configurations

study \( \mathcal{R}_{PQ}^{MN} \) with integer \( M, N, P, Q > 0 \)
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4. Conclusions & Outlook
Next step: hadronic level predictions

- Introduce PDFs and running of the strong coupling:

\[
\frac{d\sigma^{3-\text{jet}}}{dk_A dY_A d\theta_A dk_B dY_B d\theta_B dk_J dy_J d\theta_J} =
\frac{8\pi^3 C_F \bar{\alpha}_s(\mu_R)^3}{N_C^3} \frac{x_{J_A} x_{J_B}}{k_A k_B k_J} \int d^2\vec{p}_A \int d^2\vec{p}_B \delta^{(2)}(\vec{p}_A + \vec{k}_J - \vec{p}_B)
\times \left( \frac{N_C}{C_F} f_g(x_{J_A}, \mu_F) + \sum_{r=q,\bar{q}} f_r(x_{J_A}, \mu_F) \right)
\times \left( \frac{N_C}{C_F} f_g(x_{J_B}, \mu_F) + \sum_{s=q,\bar{q}} f_s(x_{J_B}, \mu_F) \right)
\times \varphi\left(\vec{k}_A, \vec{p}_A, Y_A - y_J\right) \varphi\left(\vec{p}_B, \vec{k}_B, y_J - Y_B\right)
\]

- Match the LHC kinematical cuts (integrate \(d\sigma^{3-\text{jet}}\) on \(k_T\) and rapidities):

- \(1.\) \(k_A \geq 35\) GeV; \(k_B \geq 35\) GeV; symmetric cuts
- \(2.\) \(k_A \geq 35\) GeV; \(k_B \geq 50\) GeV; asymmetric cuts
- \(Y = Y_A - Y_B\) fixed;
- \(y_J = (Y_A + Y_B)/2\)
- \(\sqrt{s} = 7, 13\) TeV
Three-jet phenomenology: hadronic level analysis

\[ R_{23}^{12} \text{ vs } Y = Y_A - Y_B \text{ for three different } k_J \text{ bins } \]

\[ k_{A}^{\text{min}} = 35 \text{ GeV}, \quad k_{B}^{\text{min}} = 35 \text{ GeV}, \quad k_{A}^{\text{max}} = k_{B}^{\text{max}} = 60 \text{ GeV} \text{ (symmetric)} \]

\[ \sqrt{s} = 13 \text{ TeV}; \quad k_{B}^{\text{min}} = 35 \text{ GeV} \]

Y is the rapidity difference between the most forward/backward jet; 
\[ y_J = \frac{Y_A + Y_B}{2} \]

[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]
$R_{12}^{23}$ vs $Y = Y_A - Y_B$ for three different $k_J$ bins

$k_A^{\text{min}} = 35 \text{ GeV, } k_B^{\text{min}} = 50 \text{ GeV, } k_A^{\text{max}} = k_B^{\text{max}} = 60 \text{ GeV (asymmetric)}$

$\sqrt{s} = 13 \text{ TeV; } k_B^{\text{min}} = 50 \text{ GeV}$

$Y$ is the rapidity difference between the most forward/backward jet; $y_J = \frac{Y_A + Y_B}{2}$. 

[F. Caporale, F. G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]
$R_{33}^{12}$ vs $Y$ at 13 TeV - NLLA preliminary results

$k_{A}^{\text{min}} = 35$ GeV, $k_{B}^{\text{min}} = 50$ GeV, $k_{A}^{\text{max}} = k_{B}^{\text{max}} = 60$ GeV (asymmetric)

$\sqrt{s} = 13$ TeV; $k_{B}^{\text{min}} = 50$ GeV; $k_{J} \in \text{\bullet bin-1, \bigcirc bin-2, \bigotimes bin-3}$

$Y$ is the rapidity difference between the most forward/backward jet; $y_{J} = \frac{Y_{A} + Y_{B}}{2}$. 

[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (in progress)]
Three-jet phenomenology: hadronic level analysis

$R_{33}^{12}$ vs $Y$ at 13 and 7 TeV - NLLA preliminary results

$\sqrt{s} = 13$ TeV; $k_B^{\text{min}} = 50$ GeV; $k_j \in \text{bin-1, bin-2, bin-3}$

$\sqrt{s} = 7$ TeV; $k_B^{\text{min}} = 50$ GeV; $k_j \in \text{bin-1, bin-2, bin-3}$

[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (in progress)]
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A four-jet primitive lego-plot
Generalized azimuthal correlations - partonic level

As for the three-jet case...

Prescription: integrate over all angles after using the projections on the three azimuthal angle differences indicated below...

→ ...to define:

$$C_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi))$$

$$\frac{d^6 \hat{\sigma}_{4\text{-jet}}(\vec{k}_A, \vec{k}_B, Y_A - Y_B)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}$$

Main observables: generalized azimuthal correlation momenta

$$R_{PQR}^{MNL} = \frac{C_{MNL}}{C_{PRQ}} = \frac{\langle \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \rangle}{\langle \cos(P(\vartheta_A - \vartheta_1 - \pi)) \cos(Q(\vartheta_1 - \vartheta_2 - \pi)) \cos(R(\vartheta_2 - \vartheta_B - \pi)) \rangle}$$
Partonic prediction of $C_{MNL}$ vs $k_{1,2}$

$C_{121}$

$k_A = 40$ GeV; $k_B = 50$ GeV

$C_{212}$

$k_A = 40$ GeV; $k_B = 50$ GeV

$C_{121}$

$k_A = 30$ GeV; $k_B = 60$ GeV

$C_{212}$

$k_A = 30$ GeV; $k_B = 60$ GeV

[F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016)]
Partonic prediction of $R_{PQR}^{MNL}$ vs $k_{1,2}$

$R_{122}^{111}$

$k_A = 40$ GeV; $k_B = 50$ GeV

$R_{211}^{222}$

$k_A = 30$ GeV; $k_B = 60$ GeV
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Next step: hadronic level predictions

- Introduce PDFs and running of the strong coupling

- Use realistic LHC kinematical cuts:
  1. $k_A^{min} = 35 \text{ GeV}, k_A^{max} = 60 \text{ GeV}$
     $k_B^{min} = 45 \text{ GeV}, k_B^{max} = 60 \text{ GeV}$
     $k_1^{min} = 20 \text{ GeV}, k_1^{max} = 35 \text{ GeV}$
     $k_2^{min} = 60 \text{ GeV}, k_2^{max} = 90 \text{ GeV}$
  2. $k_A^{min} = 35 \text{ GeV}, k_A^{max} = 60 \text{ GeV}$
     $k_B^{min} = 45 \text{ GeV}, k_B^{max} = 60 \text{ GeV}$
     $k_1^{min} = 25 \text{ GeV}, k_1^{max} = 50 \text{ GeV}$
     $k_2^{min} = 60 \text{ GeV}, k_2^{max} = 90 \text{ GeV}$

- $Y = Y_A - Y_B$ fixed;
  $Y_A - y_1 = y_1 - y_2 = y_2 - Y_B = Y/3$

- $\sqrt{s} = 7, 13 \text{ TeV}$
Four-jet phenomenology: hadronic level predictions

$R_{221}^{122}$ at $\sqrt{s} = 7$ TeV vs $Y = Y_A - Y_B$ for two $k_1$ bins

- $\sqrt{s} = 7$ TeV
- $k_{A\text{min}} = 35$ GeV
- $k_{B\text{min}} = 45$ GeV
- $k_{2\text{min}} = 60$ GeV
- $k_{2\text{max}} = 90$ GeV

$Y$ is the rapidity difference between the most forward/backward jet;

$$Y_A - y_1 = y_1 - y_2 = y_2 - Y_B = Y / 3.$$
$R_{221}^{122}$ at $\sqrt{s} = 13$ TeV vs $Y = Y_A - Y_B$ for two $k_1$ bins

$\sqrt{s} = 13$ TeV; \( k_A^{\text{min}} = 35 \text{ GeV} \); \( k_B^{\text{min}} = 45 \text{ GeV} \); \( k_2^{\text{min}} = 60 \text{ GeV} \); \( k_2^{\text{max}} = 90 \text{ GeV} \)

Y is the rapidity difference between the most forward/backward jet;

\[
Y_A - y_1 = y_1 - y_2 = y_2 - Y_B = Y / 3.
\]

[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]
$R_{221}^{122}$ and $R_{112}^{221}$ vs $Y = Y_A - Y_B$ and $\sqrt{s}$ for two $k_1$ bins

[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]
Conclusions...

- Study of processes with **three** and **four** tagged jets to propose new, more exclusive, BFKL observables: **generalized azimuthal correlation momenta**
- Ratios of correlation functions used to minimize the influence of higher order corrections
- Comparison with experimental data suggested and needed

...Outlook

- Three- and four-jets in the NLLA accuracy: improved kernel(s), scale optimization
  
  [F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (in progress)]

- Dependence on rapidity bins (asymmetric configurations for the central jet(s))
  
  [F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (in progress)]

- Comparison with analyses where the four-jet predictions stem from two independent gluon ladders
  
  [R. Maciula, A. Szczurek (2014, 2015)]

- Probe BFKL through other processes...
  
  - Di-hadron production in the full NLLA
  
  [F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (in progress)]
  
  - Heavy quark pair production
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  [F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (in progress)]
Thanks for your attention!!
BACKUP slides
Motivation

So far, search for BFKL effects had these general drawbacks:

- too low $\sqrt{s}$ or rapidity intervals among tagged particles in the final state
- too inclusive observables, other approaches can fit them

Advent of LHC:

- higher energies $\leftrightarrow$ larger rapidity gaps
- unique opportunity to test pQCD in the high-energy limit
- disentangle applicability region of energy-log resummation (BFKL approach)

$[V.S. \text{ Fadin}, E.A. \text{ Kuraev}, L.N. \text{ Lipatov} (1975, 1976, 1977)]$
$[Y.Y. \text{ Balitskii}, L.N. \text{ Lipatov} (1978)]$

Last years:

hadroproduction of two jets featuring high transverse momenta and well separated in rapidity, so called Mueller–Navelet jets...

- possibility to define infrared-safe observables...
- ...and constrain the PDFs...
- ...theory vs experiment

$[B. \text{ Ducloué}, L. \text{ Szymanowski}, S. \text{ Wallon} (2014)]$
$[F. \text{ Caporale}, D.Yu. \text{ Ivanov}, B. \text{ Murdaca}, A. \text{ Papa} (2014)]$
...large jet transverse momenta: $\vec{k}_{J,1}^2 \sim \vec{k}_{J,2}^2 \gg \Lambda_{QCD}^2$

...large rapidity gap between jets (high energies) $\Rightarrow \Delta y = \ln \frac{x_{J,1}x_{J,2}s}{|k_{J,1}||k_{J,2}|}$
Partonic prediction of $R_{22}^{21}$ for $k_J = 30, 45, 70$ GeV

$Y_A - Y_B$ is fixed to 10; $y_J$ varies between 0.5 and 9.5.
$R^{23}_{12}$ vs $Y = Y_A - Y_B$, $\sqrt{s}$ and $k_B^{\text{min}}$ for three $k_J$ bins

$\sqrt{s} = 7$ TeV; $k_B^{\text{min}} = 35$ GeV

$\sqrt{s} = 7$ TeV; $k_B^{\text{min}} = 50$ GeV

$\sqrt{s} = 13$ TeV; $k_B^{\text{min}} = 35$ GeV

$\sqrt{s} = 13$ TeV; $k_B^{\text{min}} = 50$ GeV

[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]
\( R_{33}^{12} \) vs \( Y \) at 7 TeV - NLLA preliminary results

\( k_{A}^{\text{min}} = 35 \) GeV, \( k_{B}^{\text{min}} = 50 \) GeV, \( k_{A}^{\text{max}} = k_{B}^{\text{max}} = 60 \) GeV (asymmetric)

\[ \sqrt{s} = 7 \text{ TeV}; \quad k_{B}^{\text{min}} = 50 \) GeV; \quad k_{j} \in \text{ bin--1, bin--2, bin--3} \]

\[ Y \text{ is the rapidity difference between the most forward/backward jet; } y_{J} = \frac{Y_{A} + Y_{B}}{2}. \]
$R_{23}^{22}$ vs $Y$ at 13 and 7 TeV - NLLA preliminary results

[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (in progress)]
Four-jets: generalized azimuthal coefficients - partonic level

\[ C_{MNL} = \left( \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \right) \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \]

\[ \frac{d^6 \sigma^{4\text{-jet}} \left( \vec{k}_A, \vec{k}_B, Y_A - Y_B \right)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2} \]

\[ = \frac{2\pi^2 \tilde{\alpha}_s (\mu_R)^2}{k_1 k_2} \left( -1 \right)^{M+N+L} (\tilde{\Omega}_{M,N,L} + \tilde{\Omega}_{M,N,-L} + \tilde{\Omega}_{M,-N,L} + \tilde{\Omega}_{M,-N,-L} + \tilde{\Omega}_{-M,N,L} + \tilde{\Omega}_{-M,N,-L} + \tilde{\Omega}_{-M,-N,L} + \tilde{\Omega}_{-M,-N,-L}) \]

with \( \tilde{\Omega}_{m,n,l} = \int_0^{+\infty} dp_A p_A \int_0^{+\infty} dp_B p_B \int_0^{2\pi} d\phi_A \int_0^{2\pi} d\phi_B \]

\[ \frac{e^{-im\phi_A} e^{il\phi_B} (p_A e^{i\phi_A} + k_1)^n (p_B e^{-i\phi_B} - k_2)^n}{\sqrt{(p_A^2 + k_1^2 + 2p_A k_1 \cos \phi_A)^n} \sqrt{(p_B^2 + k_2^2 - 2p_B k_2 \cos \phi_B)^n}} \]

\[ \varphi_m \left( |\vec{k}_A|, |\vec{p}_A|, Y_A - y_1 \right) \varphi_l \left( |\vec{p}_B|, |\vec{k}_B|, y_2 - Y_B \right) \]

\[ \varphi_n \left( \sqrt{p_A^2 + k_1^2 + 2p_A k_1 \cos \phi_A}, \sqrt{p_B^2 + k_2^2 - 2p_B k_2 \cos \phi_B}, y_1 - y_2 \right) \]
Four-jets: generalized azimuthal coefficients - partonic level

\[ C_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \cos(M(\vartheta_A - \vartheta_1 - \pi)) \]

\[ \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \]

\[ \frac{d^6\sigma^{4\text{-jet}}(\vec{k}_A, \vec{k}_B, Y_A - Y_B)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2} \]

Main observables: \textbf{generalized azimuthal correlation momenta}

\[ R_{PQR}^{MNL} = \frac{C_{MNL}}{C_{PRQ}} = \frac{\langle \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \rangle}{\langle \cos(P(\vartheta_A - \vartheta_1 - \pi)) \cos(Q(\vartheta_1 - \vartheta_2 - \pi)) \cos(R(\vartheta_2 - \vartheta_B - \pi)) \rangle} \]
$\mathcal{R}_{111}^{111}$ and $\mathcal{R}_{112}^{111}$ vs $Y = Y_A - Y_B$ and $\sqrt{s}$ for two $k_1$ bins.
$R_{211}^{112}$ and $R_{111}^{212}$ vs $Y = Y_A - Y_B$ and $\sqrt{s}$ for two $k_1$ bins

$\sqrt{s} = 7$ TeV; $k_A^{\text{min}} = 35$ GeV; $k_B^{\text{min}} = 45$ GeV; $k_2^{\text{min}} = 60$ GeV; $k_2^{\text{max}} = 90$ GeV

$\sqrt{s} = 13$ TeV; $k_A^{\text{min}} = 35$ GeV; $k_B^{\text{min}} = 45$ GeV; $k_2^{\text{min}} = 60$ GeV; $k_2^{\text{max}} = 90$ GeV

[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]