Pomeron-Odderon interactions: a functional RG flow analysis

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Outline

- Motivations and a Reggeon Field Theory (RFT)
- Functional RG approach
- Pomeron-Odderon RFT: RG flow equations
- Critical (fixed point) solutions
- Discussion

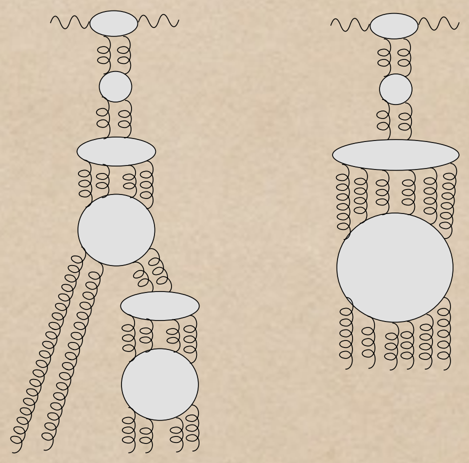
QCD in the Regge limit.

Since early perturbative BFKL analysis of QCD in terms of reggeized gluons the Pomeron has been found as a composite state ψ of 2 reggeized gluons

and later the Odderon (C,P odd), as a composite state χ of 3 reggeized gluons, solution of the BKP equation in the lowest non trivial approximation.

Simple exchanges of such objects are corrected by interactions in presence of more reggeized gluons in the t channel which are necessary to unitarize the theory.

Diagrams with reggeized gluons containing PPP and POO vertices: interactions are local in rapidity but non local in transverse space.



$$\begin{aligned} \frac{\partial N}{\partial \tau} &= KN - V_{PPP}NN + V_{POO}OO \\ \frac{\partial O}{\partial \tau} &= KO - V_{OPO}(NO + ON) \end{aligned}$$

Similar objects are found in other approaches to the Regge limit of QCD: CGC, Dipole/Wilson lines.

RFT might appear at high energies (large rapidities) and large transverse distances.

Goals

- Can QCD in the high energy limit and at large distances be described by an effective theory such as Reggeon Field Theory (RFT), with local fields and local interactions?
- Possible transition from QCD to the RFT regime:
  - BFKL physics: fundamental gluon (and quarks) organise themselves in composite fields of reggeized gluons giving as leading objects interacting Pomeron and Odderon,
    BFKL Pomeron ($J > 1$), Odderon ($J \simeq 1$) and both $\alpha' \simeq 0$
  - This should be at the "UV" boundary of RFT, below which (at larger distances) they may be considered approximately local with $J \simeq 1$ and a non zero α' and descibed by Regge poles.
- Such a transition should involve perturbative physics,
- Here we investigate some features of RFT in 2 transverse dimensions
  Is the RG flow of this theory able to tell us if RFT can be a useful description?

Strong interactions and old Regge theory

About half a century ago V.N. Gribov introduced phenomenologically the RFT.

Starting point: Sommerfeld-Watson representation of the elastic scattering amplitudes.

$$\mathcal{T}_{AB}(s,t) = \int \frac{\mathrm{d}\omega}{2\mathrm{i}} \xi(\omega) s^{1+\omega} \mathcal{F}(\omega,t). \qquad \xi(\omega) = \frac{\tau - \mathrm{e}^{-\mathrm{i}\pi\omega}}{\sin \pi\omega}$$

$$\tau = \pm 1$$

- Regge pole description in the complex $\omega = J - 1$ plane
- The leading pole: even signatured Pomeron with vacuum quantum numbers, trajectory $\alpha(t)$.
- Unitarity in the crossed (t-channel): multi pomeron states, branch-point singularities (Regge cuts)
- Analysis of experimental inclusive cross sections in the triple Regge region showed that a triple Pomeron interaction should be introduced.
- In the '70 it was conjectured that another pole with odd quantum numbers (P,C, τ) could exist, the so called Odderon with $\alpha(0)$ close to 1.
- In general one may introduce interacting vertices of any order for many reggeons (Pomeron, Odderon and subleading reggeons).

RFT with Pomeron and Odderon fields

Interactions are constrained by signature: conservation +
reggeons with different signature factors,
multi reggeon cut has discontinuity with overall sign from $-\mathrm{i}\Pi_j(\mathrm{i}\xi_j)$

Pomeron: $\xi \simeq i$ (imaginary) , Odderon: $\xi \simeq -\frac{2}{\pi\omega}$ (real)

- n Pomeron t-channel states induced by interactions gets a factor $(-1)^{n-1}$
  Therefore the pomeron self energy is negative.
  The triple Pomeron coupling by convention is chosen imaginary.
  Quartic Pomeron couplings are real.

- Odderon has negative signature:
  transition $P \to OO$ is real valued; transition $O \to OP$ is imaginary
  Quartic interactions: most coupling remain real, but
  $O \to OOO$ and $P \to P + OO$ have imaginary coupling

Local effective action for RFT

$$\Gamma[\psi^\dagger,\psi,\chi^\dagger,\chi] = \int \mathrm{d}^D x\, \mathrm{d}\tau \left(Z_P(\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial}_\tau \psi - \alpha'_P \psi^\dagger \nabla^2 \psi) + Z_O(\frac{1}{2}\chi^\dagger \overleftrightarrow{\partial}_\tau \chi - \alpha'_O \chi^\dagger \nabla^2 \chi) + V_k[\psi,\psi^\dagger,\chi,\chi^\dagger] \right)$$

- Cubic interactions

$$\begin{aligned} V_3 \,=\, & -\mu_P \psi^\dagger \psi + i\lambda \psi^\dagger(\psi+\psi^\dagger)\psi - \\ & -\mu_O \chi^\dagger \chi + i\lambda_2 \chi^\dagger(\psi+\psi^\dagger)\chi + \lambda_3(\psi^\dagger \chi^2 + {\chi^\dagger}^2 \psi) \end{aligned}$$

- Quartic interactions

$$\begin{aligned} V_4 \,=\, & \lambda_{41}(\psi\psi^\dagger)^2 + \lambda_{42}\psi\psi^\dagger(\psi^2 + {\psi^\dagger}^2) + \lambda_{43}(\chi\chi^\dagger)^2 + i\lambda_{44}\chi\chi^\dagger(\chi^2 + {\chi^\dagger}^2) \\ & + i\lambda_{45}\psi\psi^\dagger(\chi^2 + {\chi^\dagger}^2) + \lambda_{46}\psi\psi^\dagger\chi\chi^\dagger + \lambda_{47}\chi\chi^\dagger(\psi^2 + {\psi^\dagger}^2) \end{aligned}$$

- …

- States with even and odd Odderon number do not mix.
- The couplings λ_3 and similarly λ_{44} and λ_{45} play a special role: they are responsible for the change of the Odderon number

We shall study the RG flow equation for a generic potential expanded as polynomial in the weak field approximation.

We shall consider a generic D dimensional transverse space but mainly work in D=2.

Functional RG approach

We consider the flow of the IR regulated generator of 1PI functions

$$e^{-\Gamma_k[\bar\phi]} = \int D\phi\, \mu_k\, e^{-S[\phi]+\frac{\delta\Gamma_k}{\delta\bar\phi}\cdot(\phi-\bar\phi)-\Delta S_k[\phi-\bar\phi]}$$

The coarse-graining is implemented by a cutoff operator defined by

$$\Delta S_k[\phi] = \tfrac{1}{2}\phi\cdot R_k \cdot \phi$$

$R_k(p^2) > 0$ for $p^2 \ll k^2$
$R_k(p^2) \to 0$ for $p^2 \gg k^2$
$R_k(p^2) \to \infty$ for $k \to \Lambda$ $(\to \infty)$

Taking derivatives (functional and with respect to $t = \ln k/k_0$) one can obtain

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_t R_k\right] - \frac{\dot\mu_k}{\mu_k}$$

Wetterich, Morris

as well as the flow equations for the n-point 1PI functions.

Example with a scalar field: $\Gamma_k[\phi] = \int \mathrm{d}^d x \left\{ V_k(\phi) + \frac{1}{2} Z_k(\phi)(\partial\phi)^2 + \frac{1}{2} X_{1,k}(\phi)(\partial^2\phi)^2 + \frac{1}{4} X_{2,k}(\phi)(\partial\phi)^4 + \cdots \right\}$

At lowest order (LPA), choosing $R_k(p^2) = (k^2-p^2)\theta(k^2-p^2)$, the flow (dimless quantities)

of the potential is given by
$$\dot{\tilde v}_k(\phi) = -d\,\tilde v_k + \left(\frac{d}{2}-1\right)\tilde\phi\,\tilde v_k' + \frac{1}{(4\pi)^{d/2}\Gamma[1+d/2]}\,\frac{1}{1+\tilde v_k''}$$

Functional RG approach

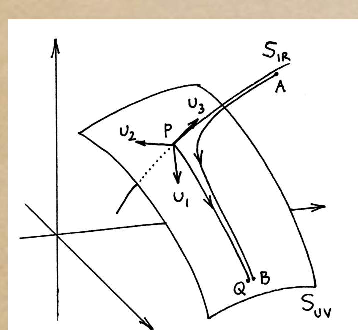
Physics requires the solution defined <u>for any value of the fields</u>
(problem of finding <u>global solutions</u> of non linear PDE, <u>infinitely many couplings</u>)

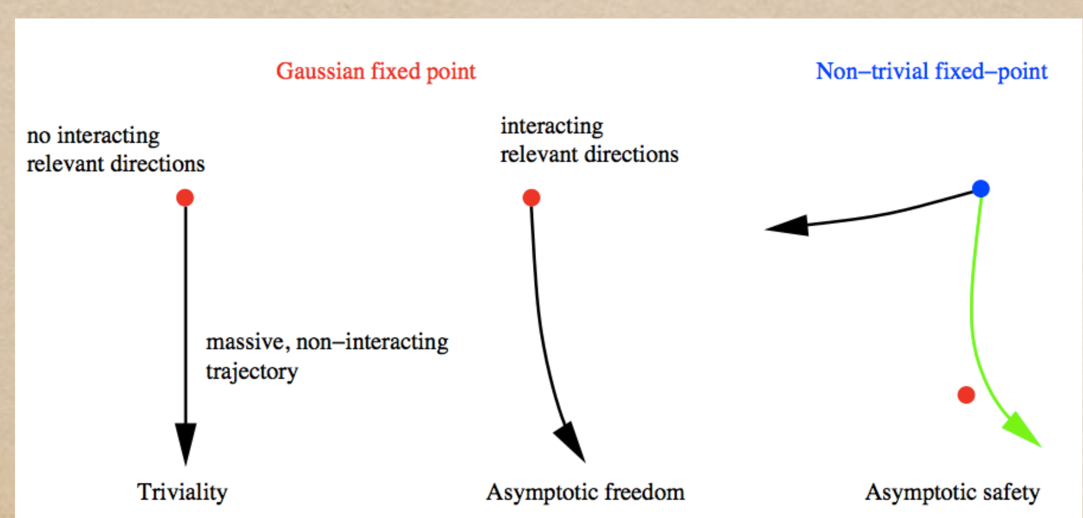
Approximation schemes (truncations): expansion in some operator basis
(e.g. polynomial around some field configuration). Care with radius of convergence!

General strategy: investigate scale invariant solutions (fixed points)
linear deformations (eigenperturbations and eigenvalues)

Universality

If the theory admit a UV non trivial fixed point with a finite number of relevant directions it may define an asymptotically safe theory.





RTF: construction of the flow equations

General strategy used here for a polynomial truncation of the potential.

$$[\Gamma^{(2)} + \mathbb{R}]^{-1} = [\Gamma^{(2)}_{free} - V_{int}]^{-1}$$
$$= G(\omega,q) + G(\omega,q)V_{int}G(\omega,q) + G(\omega,q)V_{int}G(\omega,q)V_{int}G(\omega,q) + ...$$

$$G(\omega,q) = \begin{pmatrix} G_P(\omega,q) & 0 \\ 0 & G_O(\omega,q) \end{pmatrix}$$

$$G_P(\omega,q) = \begin{pmatrix} 0 & (Z_P(-i\omega + \alpha'_P q^2) + R_P - \mu_P)^{-1} \\ (Z_P(i\omega + \alpha'_P q^2) + R_P - \mu_P)^{-1} & 0 \end{pmatrix}$$

$$G_O(\omega,q) = \begin{pmatrix} 0 & (Z_O(-i\omega + \alpha'_O q^2) + R_O - \mu_O)^{-1} \\ (Z_O(i\omega + \alpha'_O q^2) + R_O - \mu_O)^{-1} & 0 \end{pmatrix}$$

$$V_{int} = -\begin{pmatrix} V^r_{\psi\psi} & V^r_{\psi\psi^\dagger} & V^r_{\psi\chi} & V^r_{\psi\chi^\dagger} \\ V^r_{\psi^\dagger\psi} & V^r_{\psi^\dagger\psi^\dagger} & V^r_{\psi^\dagger\chi} & V^r_{\psi^\dagger\chi^\dagger} \\ V^r_{\chi\psi} & V^r_{\chi\psi^\dagger} & V^r_{\chi\chi} & V^r_{\chi\chi^\dagger} \\ V^r_{\chi^\dagger\psi} & V^r_{\chi^\dagger\psi^\dagger} & V^r_{\chi^\dagger\chi} & V^r_{\chi^\dagger\chi^\dagger} \end{pmatrix}$$

IR regulator for the coarse-graining:

$$R_P(q^2) = Z_P\alpha'_P(k^2 - q^2)\Theta(k^2 - q^2),$$
$$R_O(q^2) = Z_O\alpha'_O(k^2 - q^2)\Theta(k^2 - q^2) = rZ_O\alpha'_P(k^2 - q^2)\Theta(k^2 - q^2) \qquad r = \frac{\alpha'_O}{\alpha'_P}$$

<u>Anomalous dimensions</u>: $\eta_P = -\frac{1}{Z_P}\partial_t Z_P, \quad \eta_O = -\frac{1}{Z_O}\partial_t Z_O \qquad \zeta_P = -\frac{1}{\alpha'_P}\partial_t\alpha'_P, \quad \zeta_O = -\frac{1}{\alpha'_O}\partial_t\alpha'_O$

Dimensionless quantities: $\tilde\psi = Z_P^{1/2}k^{-D/2}\psi, \quad \tilde\chi = Z_O^{1/2}k^{-D/2}\chi \qquad \tilde V = \frac{V}{\alpha'_P k^{D+2}}$

For example:

$$\tilde\mu_P = \frac{\mu_P}{Z_P\alpha'_P k^2}, \quad \tilde\mu_O = \frac{\mu_O}{Z_O\alpha'_P k^2},$$
$$\tilde\lambda = \frac{\lambda}{Z_P^{3/2}\alpha'_P k^2}k^{D/2}, \quad \tilde\lambda_{2,3} = \frac{\lambda_{2,3}}{Z_O Z_P^{1/2}\alpha'_P k^2}k^{D/2}$$

<u>Classical scaling</u>:

$$(-(D+2) + \zeta_P)\,\tilde V + \left(\frac{D}{2} + \frac{\eta_P}{2}\right)\left(\tilde\psi\frac{\partial\tilde V}{\partial\tilde\psi} + \tilde\psi^\dagger\frac{\partial\tilde V}{\partial\tilde\psi^\dagger}\right) + \left(\frac{D}{2} + \frac{\eta_O}{2}\right)\left(\tilde\chi\frac{\partial\tilde V}{\partial\tilde\chi} + \tilde\chi^\dagger\frac{\partial\tilde V}{\partial\tilde\chi^\dagger}\right)$$

Cubic truncation: beta functions

Performing the traces, the beta functions for dimensionless quantities are:

$$\dot\mu_P = (-2+\eta_P+\zeta_P)\mu_P + 2A_P\frac{\lambda^2}{(1-\mu_P)^2} - 2A_O r\frac{\lambda_3^2}{(r-\mu_O)^2}$$

$$\dot\mu_O = (-2+\eta_O+\zeta_P)\mu_O + 2(A_P+A_O r)\frac{\lambda_2^2}{(1+r-\mu_P-\mu_O)^2}$$

$$\dot\lambda = (-2+D/2+\zeta_P+\frac{3}{2}\eta_P)\lambda + 8A_P\frac{\lambda^3}{(1-\mu_P)^3} - 4A_O r\frac{\lambda_2\lambda_3^2}{(r-\mu_O)^3}$$

$$\begin{aligned}\dot\lambda_2 &= (-2+D/2+\zeta_P+\frac{1}{2}\eta_P+\eta_O)\lambda_2\\ &+\frac{2\lambda\lambda_2^2(6A_P+5A_O r)+4\lambda_2^3(A_P+A_O r)-4\lambda_2\lambda_3^2(A_P+2A_O r)}{(1+r-\mu_P-\mu_O)^3}\\ &+\frac{2A_P\lambda\lambda_2^2(r-\mu_O)^2}{(1-\mu_P)^2(1+r-\mu_P-\mu_O)^3}-\frac{4A_O r\lambda_2\lambda_3^2(1-\mu_P)^2}{(1-\mu_O)^2(1+r-\mu_P-\mu_O)^3}\\ &+\frac{2\lambda\lambda_2^2(3A_P+A_O r)(r-\mu_O)}{(1-\mu_P)(1+r-\mu_P-\mu_O)^3}-\frac{4\lambda_2\lambda_3^2(A_P+3A_O r)(1-\mu_P)}{(r-\mu_O)(1+r-\mu_P-\mu_O)^3}\end{aligned}$$

$$\begin{aligned}\dot\lambda_3 &= (-2+D/2+\zeta_P+\frac{1}{2}\eta_P+\eta_O)\lambda_3\\ &+\frac{2\lambda_2^2\lambda_3(A_P+2A_O r)}{(r-\mu_O)(1+r-\mu_P-\mu_O)^2}+\frac{4\lambda\lambda_2\lambda_3(2A_P+A_O r)}{(1-\mu_P)(1+r-\mu_P-\mu_O)^2}\\ &+\frac{2\lambda_2^2\lambda_3 A_O r(1-\mu_P)}{(r-\mu_O)^2(1+r-\mu_P-\mu_O)^2}+\frac{4\lambda\lambda_2\lambda_3 A_P(r-\mu_O)}{(1-\mu_P)^2(1+r-\mu_P-\mu_O)^2}.\end{aligned}$$

$$\dot r = r\,(-\zeta_O+\zeta_P)$$

$$A_P = N_D A_D(\eta_P,\zeta_P),\quad A_O = N_D A_D(\eta_O,\zeta_O).$$

$$N_D = \frac{2}{\sqrt{4\pi}^D\,\Gamma(D/2)}$$

$$A_D(\eta_k,\zeta_k) = \frac{1}{D} - \frac{\eta_k+\zeta_k}{D(D+2)}$$

Similarly, one can find the anomalous dimensions (from the flow of 2-point functions):

$$\eta_P = -\frac{2A_P\lambda^2}{(1-\mu_P)^3}+\frac{2A_O r\lambda_3^2}{(r-\mu_O)^3}\qquad \eta_P+\zeta_P = -\frac{N_D\lambda^2}{D(1-\mu_P)^3}+\frac{N_D r^2\lambda_3^2}{D(r-\mu_O)^3}$$

$$\eta_O = -\frac{4(A_P+A_O\,r)\lambda_2^2}{(1+r-\mu_P-\mu_O)^3}\qquad \eta_O+\zeta_O = -\frac{4N_D\lambda_2^2}{D(1+r-\mu_P-\mu_O)^3}.$$

ϵ-expansion: $D = 4 - \epsilon$

Critical theory (fixed point): perturbative one loop results:

$$\mu_P = \frac{\epsilon}{12}, \quad \lambda^2 = \frac{8\pi^2}{3}\epsilon, \quad \eta_P = -\frac{\epsilon}{6}, \quad \zeta_P = \zeta_O = \frac{\epsilon}{12},$$

$$\mu_O = \frac{95+17\sqrt{33}}{2304}\epsilon, \quad \lambda_2^2 = \frac{23\sqrt{6}+11\sqrt{22}}{48}\epsilon, \quad \lambda_3 = 0, \quad \eta_O = -\frac{7+\sqrt{33}}{72}\epsilon, \quad r = \frac{3}{16}(\sqrt{33}-1)$$

Critical exponents: two relevant directions

$$\begin{aligned} \alpha_1 &= -2 + \frac{\epsilon}{4} \to \nu_P = \frac{1}{2} + \frac{\epsilon}{16} \\ \alpha_2 &= -2 + \frac{\epsilon}{12} \to \nu_O = \frac{1}{2} + \frac{\epsilon}{48}. \end{aligned}$$

The coupling of the changing Odderon number operator is zero!

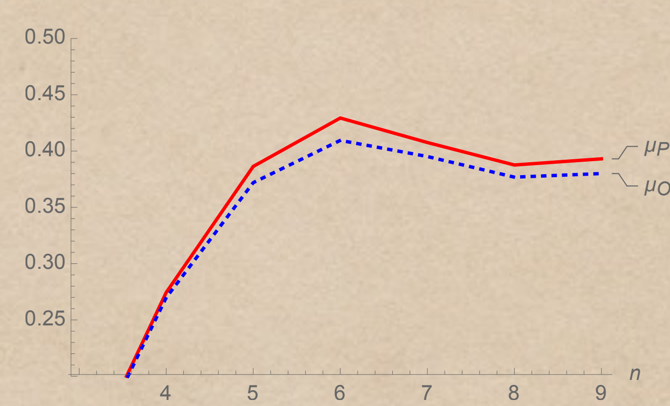
The Pomeron sector is not affected by the presence of the Odderon.

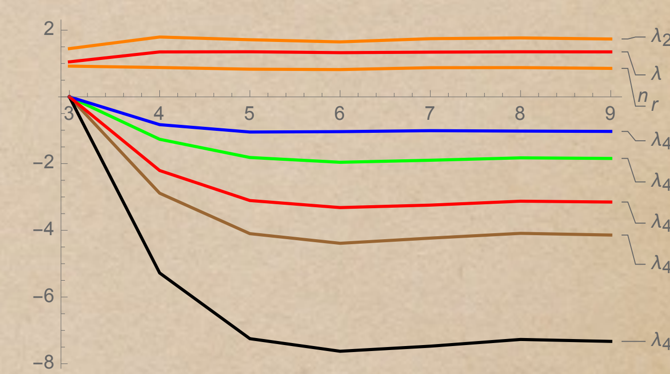
Non perturbative analysis in $D=2$

Explicit analysis at order 3,4,5 of the fixed points seems to show that the interactions changing the Odderon number are absent in the critical theory.

We perform the analysis of the fixed point up to order 9, neglecting (apart in r) the anomalous dimensions.

Couplings: fixed point values are stable at order 9.

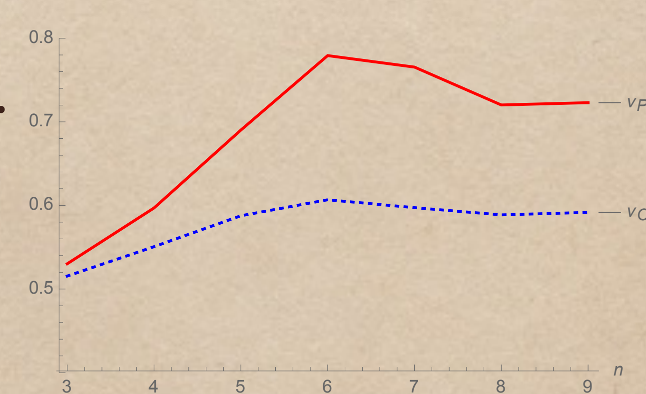


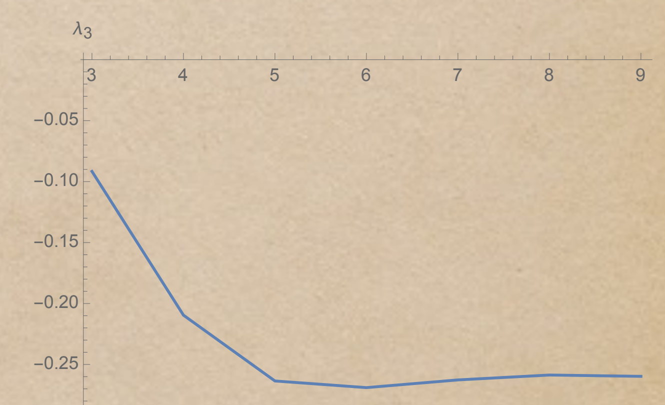


We find three relevant directions.
Critical exponents: $\nu = -1/\lambda$

$\boxed{\nu_P \simeq 0.73,\ \nu_O \simeq 0.6}$

$\boxed{\lambda_3 \simeq -0.26}$





Anomalous dimensions (cubic truncation estimate, close to ϵ-expansion result):

$$\boxed{\eta_P \simeq -0.33,\ \eta_O \simeq -0.35 \text{ and } \zeta_P = \zeta_O \simeq +0.17}$$

Discussion

- RFT with Pomeron and Odderon fields show special critical properties
  New universality class? (Relation to novel Directed Percolating systems?)
  The critical theory found is such that the Pomeron does not feel the Odderon while the Pomeron interactions renormalise the Odderon.

- Assuming RFT to be related to the QCD in its "IR" Regge limit, we do not know where QCD would places the "UV" initial conditions of the RFT flow towards its "IR".
  It could be on the critical surface (fine tuning of relevant parameters induced by QCD dynamics) or out of it, in which case such parameters can be measured in the "IR".

- The RG flow from the RFT "UV" region should be studied and we expect similar results to what has been already observed in the pure pomeron theory. Our expectations (dimensionful quantities):
  - critical theory: intercept -1 and cubic interactions —> 0 in the IR. Theory interacting (quartic couplings)
  - non critical theory: small dimensionful couplings in the "IR", "special couplings" (irrelevant) —> ~0
    intercept -1 slightly non zero.

- At phenomenological level we thus expect, assuming (to be verified) the existence of this fixed point only, in the deep "IR" a suppression of high mass diffractive processes involving the POO vertex, which could appear in the more perturbative intermediate "IR" region.
  Pure Odderon exchange processes would be instead non suppressed in the deep "iR".

Conclusions

- We have just started to study the RFT with Pomeron and Odderon fields. Interesting results which should be confirmed/extended in future work.

- If RFT is really connected to QCD, we can expect phenomenological implications in the deep "IR" limit, induced by the nature of the reggeon interactions.

- Clearly in our "IR" limit, QCD should belong, in terms of some observable, to the same universality class which we are unveiling for the RFT.

- The main question is how to connect RFT to QCD smoothly.
  We hope to investigate partially this with functional RG tools:
  the "IR" perturbative QCD region should start to overlap with
  the "UV" RFT region, with a transmutation (mainly non perturbative)
  of the dominant degrees of freedom.

Thank you!