Peculiarities of the BFKL approach in the NNLLA

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Introduction

The BFKL approach provides a general way for theoretical description of processes at high c.m.s. energy $\sqrt{s}$ and fixed (not growing with energy) momentum transfers. It is based on remarkable property of QCD – gluon Reggeization which gives a very powerful tool for description of such processes.

The Reggeization allows to express an infinite number of amplitudes through several effective vertices and gluon trajectory.

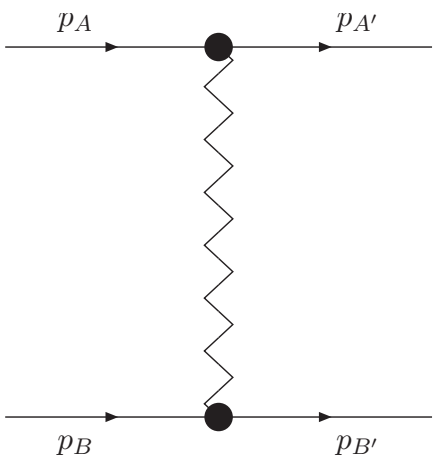
Validity of the Reggeization is proved now in all orders of perturbation theory in the coupling constant g both in the leading logarithmic approximation (LLA), where in each order of the perturbation theory only terms with the highest powers of $\ln s$ are kept, and in the next-to-leading one (NLLA), where terms with one power less are also kept.

The Reggeization provides a simple derivation of the BFKL equation both in the LLA and NLLA.

The derivation becomes more complicated.

Gluon Reggeization

For elastic scattering processes $A + B \to A' + B'$ in the Regge kinematical region: $s \simeq -u \to \infty$, t fixed (i.e. not growing with s) the Reggeization means that scattering amplitudes with the gluon quantum numbers in the t-channel can be presented as

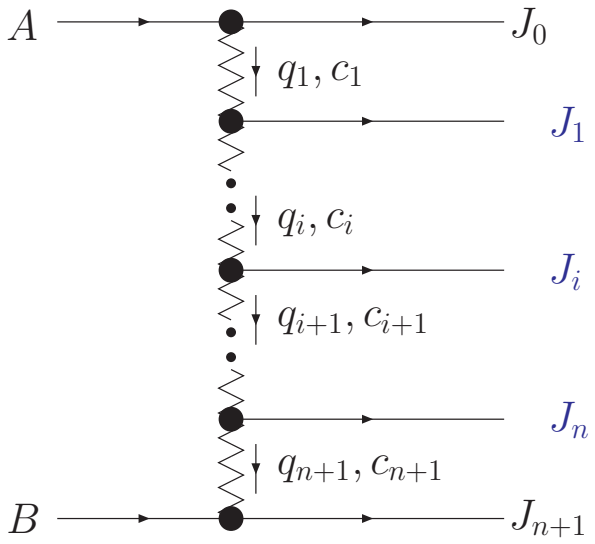


$$\mathcal{A}_{AB}^{A'B'} = \Gamma_{A'A}^{c} \left[\left(\frac{-s}{-t} \right)^{\omega(t)} - \left(\frac{s}{-t} \right)^{\omega(t)} \right] \Gamma_{B'B}^{c} \; ;$$

Gluon Reggeization

$\Gamma^c_{P'P}$–particle-particle-Reggeon (PPR) vertices or scattering vertices ("c" are colour indices); $j(t) = 1 + \omega(t)$ – Reggeon trajectory.

The Reggeization means definite form not only of elastic amplitudes, but of inelastic amplitudes in the multi-Regge kinematics (MRK) as well. It can be presented by the picture



Gluon Reggeization

and written as

$$\Re\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{J_i}(q_i,q_{i+1})\left(\frac{s_i}{s_0}\right)^{\omega(t_i)}\frac{1}{t_i}\right)$$

$$\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

Here $\gamma_{c_i c_{i+1}}^{J_i}(q_i,q_{i+1})$ – the Reggeon-Reggeon-particle (RRP) or production vertices.

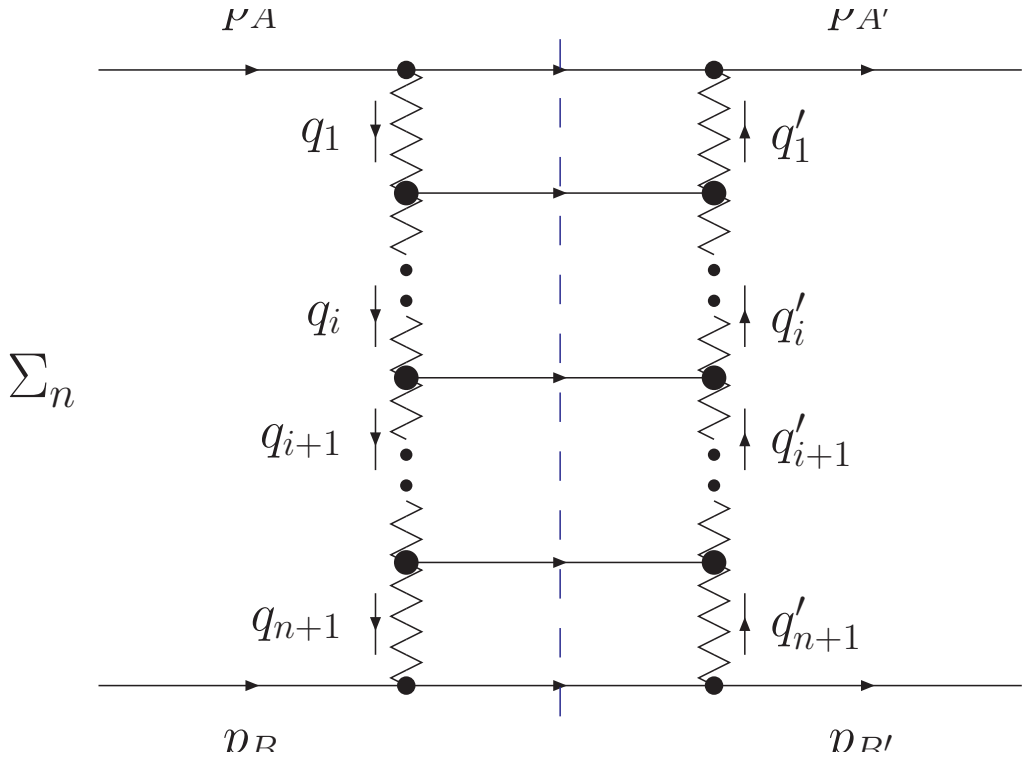
MRK is the kinematics where all particles have limited (not growing with s) transverse momenta and are combined into jets with limited invariant mass of each jet and large (growing with s) invariant masses of any pair of the jets.

The MRK gives dominant contributions to cross sections of QCD processes at high energy $\sqrt{s}$. In the LLA only a gluon can be produced. In the NLA one has to account production of $Q\bar{Q}$ and GG jets.

BFKL representation of scattering amplitudes

Amplitudes of processes with all possible quantum numbers in the t–channel are calculated using s-channel unitarity and analyticity .

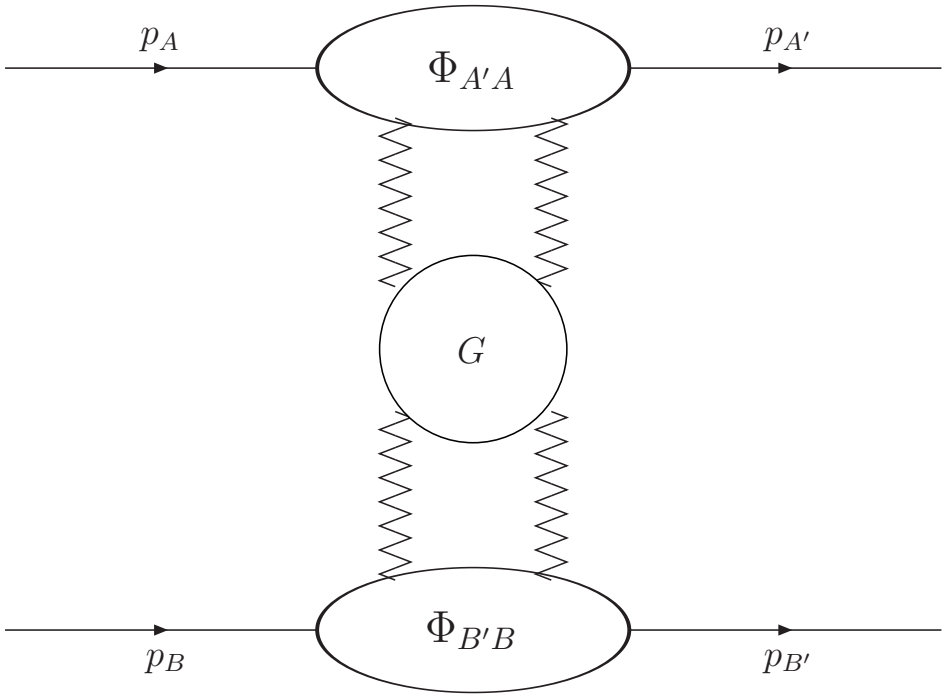
The s-channel discontinuity



BFKL representation of scattering amplitudes

The amplitudes are presented in the form :

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B}.$$



BFKL representation of scattering amplitudes

Impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describe transitions $A \to A'$ $B \to B'$,
G – Green's function for two interacting Reggeized gluons,

$$\hat{\mathcal{G}} = e^{Y\hat{\mathcal{K}}},$$

$\hat{\mathcal{K}}$ – BFKL kernel, $Y = \ln(s/s_0)$,

$$\hat{\mathcal{K}} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_r$$

$$\hat{\mathcal{K}}_r = \hat{\mathcal{K}}_G + \hat{\mathcal{K}}_{Q\bar{Q}} + \hat{\mathcal{K}}_{GG}$$

Energy dependence of scattering amplitudes is determined by the BFKL kernel.
The BFKL kernel and the impact factors are expressed in terms of the Reggeon vertices and trajectory.
The kernel is universal (process independent).

Violation of the Regge factorization

The first observation of the violation of the Regge factorization was made by
V.Del Duca, N. Glover, 2001
in the consideration of the the high-energy limit of the two-loop amplitudes for parton-parton scattering.
They consider the interference of the Born amplitude with the two-loop amplitude for gg, gq and qq elastic scattering

$$\mathcal{M}_{ij}\mathcal{M}_{ij}^{(0)} = |\mathcal{M}_{ij}^{(0)}|^2 \left(1 + \tilde{g_s}^2\, M_{ij}^{(1)} + \tilde{g_s}^4 M_{ij}^{(2)} + \mathcal{O}(\tilde{g_s}^6)\right) ,$$

with $i, j = g, q$,

$$\tilde{g_s}^2 = g_s^2 c_\Gamma \left(\frac{\mu^2}{-t}\right)^\epsilon \quad c_\Gamma = \frac{1}{(4\pi)^{2-\epsilon}} \Gamma(1+\epsilon) \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} ,$$

Violation of the Regge factorization

The interference of the tree- and two-loop amplitudes for each of the parton-parton scattering processes have been explicitly computed
C. Anastasiou, E. W. N. Glover, C. Oleari and M. E. Tejeda-Yeomans, 2001,
E. W. N. Glover, C. Oleari and M. E. Tejeda-Yeomans, 2001.
After expansion in ϵ and taking the high-energy limit, the interference between the n-loop and the tree amplitudes for ij elastic scattering has the form

$$\mathrm{Re}\,\left(\mathcal{M}^{(0)*}\mathcal{M}^{(n)}\right)_{ij} = |\mathcal{M}^{(0)}|^2_{ij\to ij}\,\tilde{g}_s^{\,2n}\,\sum_{m=0}^{n} B^{ij}_{nm}\ln^m\left(-\frac{s}{t}\right),$$

$$B^{ij}_{00} = 1\ .$$

Violation of the Regge factorization

The discrepancy appears in non-logarithmic two-loop terms. If the Reggeization would be correct in the NNLLA, they should satisfy the equation

$$B_{20}^{qg} - \frac{1}{2}\left(B_{10}^{qg}\right)^2 - \frac{1}{2}\left[B_{20}^{gg} - \frac{1}{2}\left(B_{10}^{gg}\right)^2 + B_{20}^{qq} - \frac{1}{2}\left(B_{10}^{qq}\right)^2\right] - \frac{\pi^2}{2\epsilon^2}\left(N^2 - 4\right) = 0\,.$$

Violation of the Regge factorization

The explicit calculation of the B^{ij} coefficients gives that this relation is violated by terms of $\mathcal{O}(\pi^2/\epsilon^2)$,

$$B_{20}^{qg} - \frac{1}{2}\left(B_{10}^{qg}\right)^2 - \frac{1}{2}\left[B_{20}^{gg} - \frac{1}{2}\left(B_{10}^{gg}\right)^2 + B_{20}^{qq} - \frac{1}{2}\left(B_{10}^{qq}\right)^2\right]$$

$$-\frac{\pi^2}{2\epsilon^2}\left(N^2 - 4\right) = \frac{3\pi^2}{\epsilon^2}\left(\frac{N^2+1}{N^2}\right) + \mathcal{O}(\epsilon)\,.$$

Violation of the Regge factorization

Detailed consideration of the terms responsible for the Regge factorization breaking in the case of two-loop and three-loop quark and gluon amplitudes in QCD was performed by
V. Del Duca, G. Falcioni, L, Magnea and L. Vernazza, 2014
In particular, the non-logarithmic double-pole contribution at two-loops was recovered and all non-factorizing single-logarithmic singular contributions at three loops were found using the techniques of infrared factorization.

Violation of the Regge factorization

For comparison of Regge and infrared factorizations the representation of scattering apmlitudes

$$\mathcal{M}_{\rm rs}^{[8]}\left(\frac{s}{\mu^2},\frac{t}{\mu^2},\alpha_s\right)=2\pi\alpha_s\,H_{\rm rs}^{(0),[8]}$$

$$\times\left\{C_{\rm r}\left(\frac{t}{\mu^2},\alpha_s\right)\left[A_+\left(\frac{s}{t},\alpha_s\right)+\kappa_{\rm rs}\,A_-\left(\frac{s}{t},\alpha_s\right)\right]C_{\rm s}\left(\frac{t}{\mu^2},\alpha_s\right)\right.$$

$$\left.+\,\mathcal{R}_{\rm rs}^{[8]}\left(\frac{s}{\mu^2},\frac{t}{\mu^2},\alpha_s\right)\right\},\quad \kappa_{gg}=\kappa_{qg}=0,\quad \kappa_{qq}=(4-N_c^2)/N_c^2\,,$$

was used. $H_{\rm rs}^{(0)[8]}$ represents the tree-level amplitude. A non-factorizing remainder function $\mathcal{R}_{\rm rs}$ was introduced.

Violation of the Regge factorization

The results obtained:

$$R_{qq}^{(2),0,[8]} = \left(\frac{\alpha_s}{\pi}\right)^2 \frac{\pi^2}{4\epsilon^2}\left(1 - \frac{3}{N_c^2}\right)\left(1 - \epsilon^2\zeta(2)\right),$$

$$R_{gg}^{(2),0,[8]} = -\left(\frac{\alpha_s}{\pi}\right)^2 \frac{3\pi^2}{2\epsilon^2}\left(1 - \epsilon^2\zeta(2)\right),$$

$$R_{qg}^{(2),0,[8]} = -\left(\frac{\alpha_s}{\pi}\right)^2 \frac{\pi^2}{4\epsilon^2}\left(1 - \epsilon^2\zeta(2)\right).$$

The factor $\left(1 - \epsilon^2\zeta(2)\right)$ can be absorbed in the constant c_Γ^2 by performing the expansion in terms of $\tilde{\alpha}_s = \alpha_s c_\Gamma$, instead of using α_s.

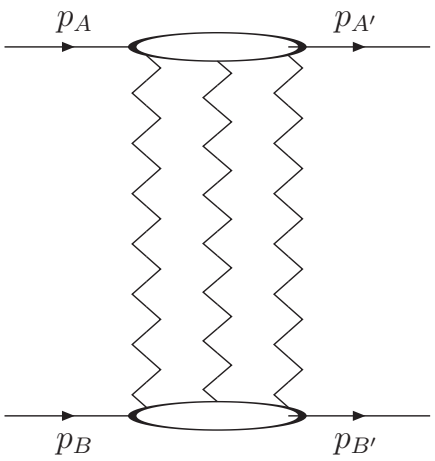
$$R_{qq}^{(3),1,[8]} = \left(\frac{\alpha_s}{\pi}\right)^3 \frac{\pi^2}{\epsilon^3} \frac{2N_c^2 - 5}{12N_c} \left(1 - \frac{3}{2}\epsilon^2\zeta(2)\right) + \mathcal{O}\left(\epsilon^0\right),$$

$$R_{gg}^{(3),1,[8]} = -\left(\frac{\alpha_s}{\pi}\right)^3 \frac{\pi^2}{\epsilon^3} \frac{2}{3} N_c \left(1 - \frac{3}{2}\epsilon^2\zeta(2)\right) + \mathcal{O}\left(\epsilon^0\right),$$

$$R_{qg}^{(3),1,[8]} = -\left(\frac{\alpha_s}{\pi}\right)^3 \frac{\pi^2}{\epsilon^3} \frac{N_c}{24} \left(1 - \frac{3}{2}\epsilon^2\zeta(2)\right) + \mathcal{O}\left(\epsilon^0\right),$$

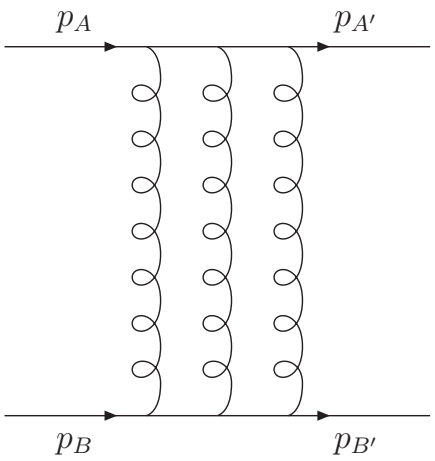
Regge cut contributions

It is well known that Regge poles generate Regge cuts.
Due to the signature conservation the cut responsible for the violation has to be 3-Reggeon one



Regge cut contributions

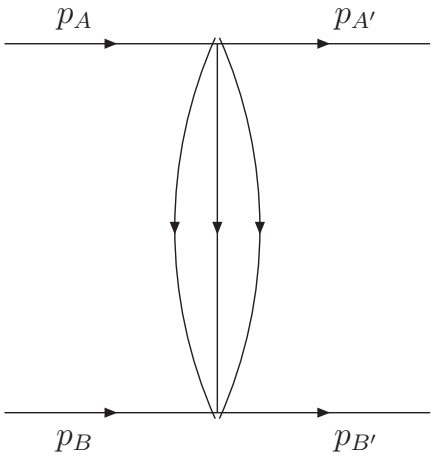
Since our Reggeon is the Reggeized gluon, the cut starts with the diagrams



$+$ all permutations of gluon lines.
These diagrams provide required colour structure, as well as π^2 factors in the non-factorized contributions.

Regge cut contributions

The π^2 factors come from integration over longitudinal momenta.
After this integration the diagrams are reduced to the diagrams



in the transverse momentum space.

Regge cut contributions

Corresponding integral

$$I_3 = \int \frac{d\vec{l}_1 d\vec{l}_2}{(2\pi)^{2(3+2\epsilon)} \vec{l}_1^{\,2} \vec{l}_2^{\,2} (\vec{q} - \vec{l}_1 - \vec{l}_2)^2} =$$

$$= 3 C_L^2 \frac{4}{\epsilon^2} \frac{1}{(\vec{q}^{\,2})^{1-2\epsilon}} \frac{\Gamma^2(1+2\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+3\epsilon)} ,$$

$$C_\Gamma = \frac{\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon}\Gamma(1+2\epsilon)} ,$$

has needed infrared behaviour, because

$$\frac{\Gamma^2(1+2\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+3\epsilon)} = 1 + O(\epsilon^3) .$$

In this order all the terms violated factorization can be attributed to the 3-reggeon cut.

Regge cut contributions

In the next order one has to take into account the gluon Reggeization and interaction.
It gives contributions of the type

$$-g^2 N_c C_\Gamma \frac{2}{\epsilon} \int \frac{d\vec{l}_1 d\vec{l}_2}{(2\pi)^{2(3+2\epsilon)} (\vec{l}_1^{\,2})^{1+2\epsilon} \vec{l}_2^{\,2} (\vec{q} - \vec{l}_1 - \vec{l}_2)^2} =$$

$$= -g^2 N_c C_\Gamma \frac{4}{3\epsilon} (\vec{q}^{\,2})^\epsilon \frac{\Gamma(1-3\epsilon)\Gamma(1+2\epsilon)\Gamma(1+3\epsilon)}{\Gamma(1-2\epsilon)\Gamma(1+\epsilon)\Gamma(1-\epsilon)\Gamma(1+4\epsilon)} \; I_3 \; ,$$

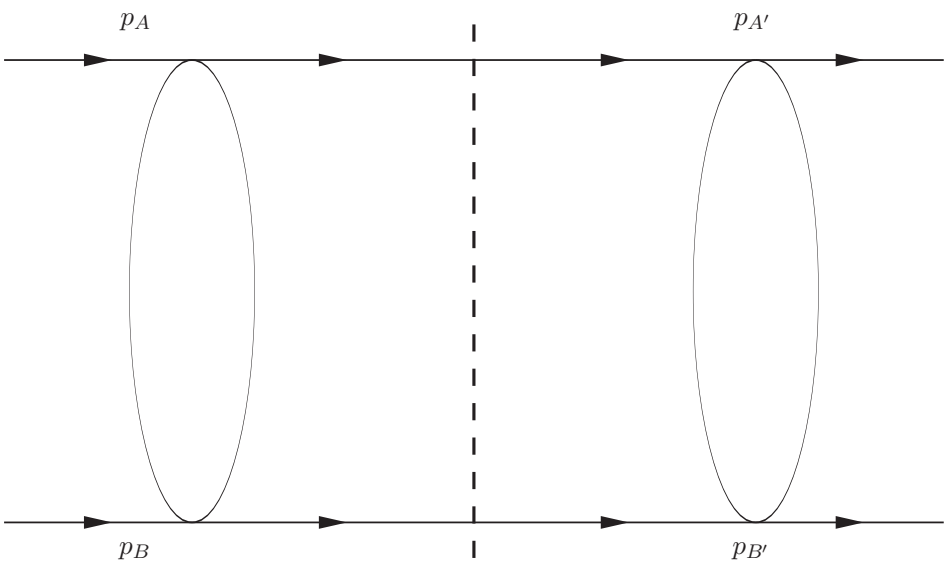
again with needed infrared behaviour, because

$$\frac{\Gamma(1-3\epsilon)\Gamma(1+2\epsilon)\Gamma(1+3\epsilon)}{\Gamma(1-2\epsilon)\Gamma(1+\epsilon)\Gamma(1-\epsilon)\Gamma(1+4\epsilon)} = 1 + O(\epsilon^3) \; .$$

Account of imaginary parts of amplitudes

Another difficulty in development of the BFKL approach in the NNLLA is necessity to account imaginary parts of amplitudes in the unitarity relations.

In the simplest two-particle intermediate state:



In imaginary parts, one ln s is lost. Products of imaginary and real parts in the unitarity relations cancel due to summation of contributions complex conjugated to each other. Due to this, imaginary parts don't play any role in the NLLA.

But they become important in the NNLLA.

Account of imaginary parts of amplitudes

It is a bad news.

Attenuating circumstance:
For the amplitudes in the unitarity relation, LLA is sufficient.

Nevertheless, account of imaginary parts leads to great changes.
Remind that for two Reggeized gluons in the t-channel in QCD, that is for tree colours, there are 6 irreducible representations:

$$\underline{1}, \underline{8_a}, \underline{8_s}, \underline{10}, \underline{\overline{10}}, \underline{27}.$$

For $N_c > 3$ there are 7 possible representations.
The representations $\underline{8_a}, \underline{10}, \underline{\overline{10}}$ are anti-symmetric
the representations $\underline{1}, \underline{8_s}, \underline{27}$ and the extra one are symmetric.

Account of imaginary parts of amplitudes

In real parts, with the NLLA accuracy, only the Reggeon channel, $\underline{8_a}$, is important.

It provides universality of the NLLA:

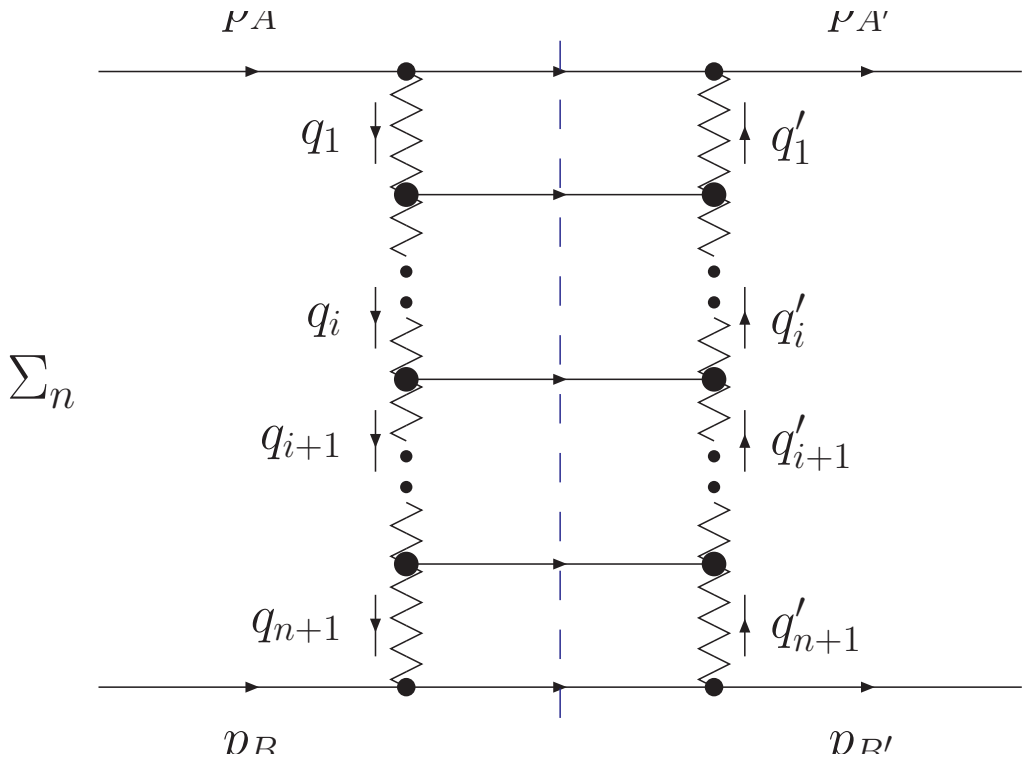
gg, qg and qq scattering can be considered in an unique way.

But account imaginary parts violate the universality.

gg scattering amplitudes can contain all the representations, while qg and qq scattering amplitudes only $\underline{1}, \underline{8_a}, \underline{8_s}$.

Account of imaginary parts of amplitudes

Consideration of many-particle states in the unitarity condition



is even more complicated problem.

Summary

- The basis of the BFKL approach is the remarkable property of QCD – gluon Reggeization
- In the LLA and in the NLLA the Reggeization provides a simple factorized form of QCD amplitudes
- This form is violated in the NNLLA by the 3-Regge cut
- It seems, that the cut is the only source of the violation
- Another NNLLA complication is the necessity of account of imaginary parts of multiparticle amplitudes in the unitarity relations