Unintegrated double parton distributions

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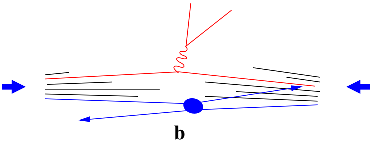
(in collaboration with Anna Staśto, to be published)

Plan

- Double parton scattering
- QCD evolution equations for double parton distributions
- Solution to the evolution equations
- Unintegrated single PDFs
- Unintegrated double PDFs
- Summary and outlook

Multiparton scattering

- Multiparton interactions (MPI) become increasingly important at the LHC.
- If no hard scale, MPI are crucial for modeling of underlying event.
- Two hard scales - double parton scattering

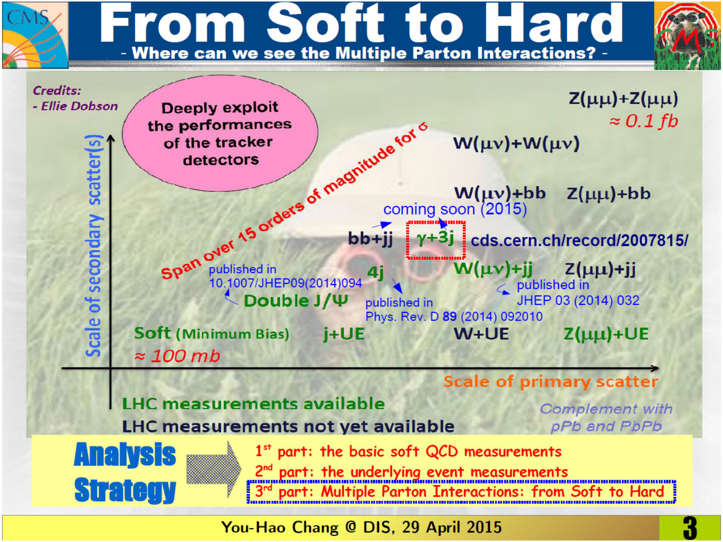


$$pp \rightarrow X_{hard} + Y_{hard} + soft$$

- Double parton distributions in addition to single PDFs.

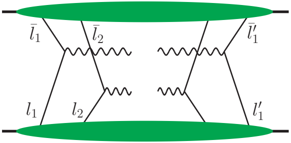
(Review by M. Diehl, D. Ostermeier, A. Schafer, JHEP 1203 (2012) 089). Thanks for some plots.

DPS at the LHC



Double parton scattering

- Two hard scatterings in one event with two hard scales, Q_1 and Q_2



- Collinear factorization cross section

$$\frac{d\sigma^{DPS}_{AB}}{dx_1\, dx_2\, dx'_1\, dx'_2} = \sum_{flav} \int d^2\mathbf{q}\, D_{f_1 f_2}(x_1, x_2, Q_1, Q_2, \mathbf{q})\, \sigma^A_{f_1 f'_1}\, \sigma^B_{f_2 f'_2}\, D_{f'_1 f'_2}(x'_1, x'_2, Q_1, Q_2, -\mathbf{q})$$

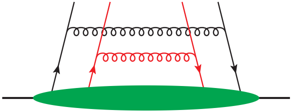
(Diehl,Gaunt,Ostermeier,Ploessl,Schaefer, JHEP 1601 (2016) 076, important steps in proof for double DY)

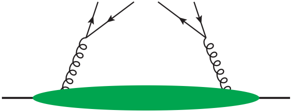
- Double parton distributions (intergrated over parton transverse momenta)

$$D_{f_1 f_2}(x_1, x_2, Q_1, Q_2, \mathbf{q}) \qquad\qquad x_1 + x_2 \le 1$$

- On shell matrix elements: $\sigma^{A,B}_{f_i f'_i}(l_i^2 = 0)$

- DGLAP type evolution equation in LLA with nonhomogeneous splitting term, which sums parton emission with $Q_0 < k_\perp < Q$.





- For equal scales, $Q_1 = Q_2 \equiv Q$, and $\mathbf{q} = \mathbf{0}$

$$\frac{\partial}{\partial \ln Q^2} D_{f_1 f_2}(x_1, x_2, Q, Q) = \frac{\alpha_s}{2\pi} \sum_{f'} \left\{ \int_{x_1}^{1-x_2} \frac{dz}{z} \mathcal{P}_{f_1 f'}\left(\frac{x_1}{z}\right) D_{f' f_2}(z, x_2, Q, Q) \right.$$

$$+ \int_{x_2}^{1-x_1} \frac{dz}{z} \mathcal{P}_{f_2 f'}\left(\frac{x_2}{z}\right) D_{f_1 f'}(x_1, z, Q, Q)$$

$$\left. + \quad \mathcal{P}_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right) D_{f'}(x_1 + x_2, Q) \right\}$$

(Konishi, Ukawa, Venziano, Snigirev, Zinovev, Shelest)

Evolution functions

- Solution with the help of parton-to-parton evolution functions $E_{ab}(x, Q, Q_0)$

$$\tilde{D}_a(n, Q) = \sum_b \tilde{E}_{ab}(n, Q, Q_0)\, \tilde{D}_b(n, Q_0) \qquad \text{(for Mellin moments)}$$

- and obey DGLAP equations with initial condition $E_{ab}(n, Q_0, Q_0) = \delta_{ab}$

$$\frac{\partial}{\partial \ln Q^2} \tilde{E}_{ab}(n, Q, Q_0) = \underbrace{\sum_{a'} \tilde{P}_{aa'}(n, Q) \tilde{E}_{a'b}(n, Q, Q_0)}_{real\ \ emission} - \tilde{E}_{ab}(n, Q, Q_0) \underbrace{\sum_{a'} \int_0^1 dz z P_{a'a}(z, Q)}_{virtual\ \ corrections}$$

- which can be transformed into integral equation

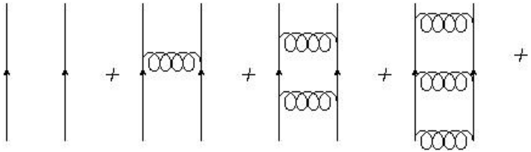
$$\boxed{\tilde{E}_{ab}(n, Q, Q_0) = T_a(Q, Q_0)\, \delta_{ab} + \int_{Q_0^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, T_a(Q, k_\perp) \sum_{a'} \tilde{P}_{aa'}(n, k_\perp)\, \tilde{E}_{a'b}(n, k_\perp, Q_0)}$$

Interpretation

- The sequence of real parton emissions interrupted by evolution with Sudakov form factor

$$T_a(Q, k_\perp) = \exp\left\{ -\int_{k_\perp^2}^{Q^2} \frac{dp_\perp^2}{p_\perp^2} \sum_{a'} \int_0^{1-\Delta} dz zP_{a'a}(z, k_\perp) \right\}$$

- Schematic partonic ladder



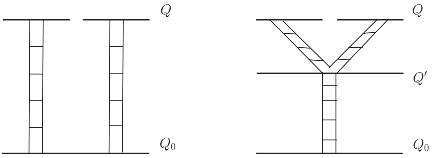
Solution to DPDF evolution equations

- The sum of homogeneous and non-homogeneous solutions:

$$\tilde{D}(n_1, n_2, Q_1, Q_2) = \tilde{E}(n_1, Q_1, Q_0)\, \tilde{D}(n_1, n_2, Q_0, Q_0)\, \tilde{E}^T(n_2, Q_2, Q_0) + \int_{Q_0^2}^{Q_{min}^2} \frac{dQ'^2}{Q'^2}\, \tilde{E}(n_1, Q_1, Q')\, \tilde{D}^{(sp)}(n_1, n_2, Q')\, \tilde{E}^T(n_2, Q_2, Q')$$

- where $\tilde{D}_{ab}(n_1, n_2, Q_0, Q_0)$ are initial conditions and

$$\tilde{D}_{ab}^{(sp)}(n_1, n_2, Q_s) = \frac{\alpha_s(Q_s)}{2\pi} \sum_f \underbrace{\tilde{D}_f(n_1 + n_2, Q_s)}_{single\ \ PDF} \int_0^1 dz z^{n_1} (1-z)^{n_2} P_{f\to ab}(z)$$



Initial conditions

(GB, Lewandowska,Serino,Snyder,Staśto, PLB 750 (2015) 559)

- Usual assumption at Q_0

$$D_{ab}(x_1, x_2) = D_a(x_1)\, D_b(x_2)\, \rho(x_1, x_2)$$

- but such DPDFs do not fulfill sum rules

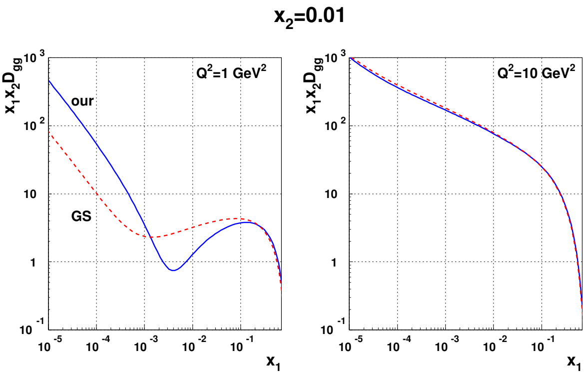
$$\begin{aligned}\sum_a \int_0^{1-x_2} dx_1 x_1\, D_{ab}(x_1, x_2) &= (1 - x_2) D_b(x_2) \\ \int_0^{1-x_2} dx_1 \left\{ D_{qb}(x_1, x_2) - D_{\bar{q}b}(x_1, x_2) \right\} &= (N_q - \delta_{qb} + \delta_{\bar{q}b})\, D_b(x_2)\end{aligned}$$

- In pure gluon case we constructed such distribution using MSTW08 parameters

$$D_{gg}(x_1, x_2) = \sum_{k=1}^{3} A_k\, (x_1\, x_2)^{\alpha_k}\, (1 - x_1 - x_2)^{\eta - \alpha_k - 1} \neq D_g(x_1) D_g(x_2)$$

- Full case leads to negative DPDFs at large x.

Numerical comparison



Why unintegrated PDFs?

- To take into account parton transverse momenta

$$D_a(x, Q) \quad \rightarrow \quad f_a(x, k_\perp, Q) \qquad\qquad \text{single PDFs}$$

$$D_{ab}(x_1, x_2, Q_1, Q_2) \quad \rightarrow \quad f_{ab}(x_1, x_2, k_{1\perp}, k_{2\perp}, Q_1, Q_2) \qquad\qquad \text{double PDFs}$$

- to be able to use them in cross sections computed with $k_\perp$ factorization

$$\sigma^{AB} \sim f_{ab} \otimes \sigma^A_{ac} \otimes \sigma^B_{bd} \otimes f_{cd}$$

where $\sigma^{A,B}_{ac}$ are off-shell matrix elements with external parton momenta $l_i^2 \neq 0$

(Kutak,Maciuła,Serino,Szczurek,van Hameren, JHEP 1604 (2016) 175, on 4-jet in DPS)

- To go beyond a simple-minded approximation

$$f_{ab}(x_1, x_2, k_{1\perp}, k_{2\perp}, Q_1, Q_2) \approx f_a(x_1, k_{1\perp}, Q_1)\, f_b(x_2, k_{2\perp}, Q_2)$$

Unintegrated single PDFs

(Kimber, Martin and Ryskin, Eur.Phys.J. C12 (2000) 655, Phys.Rev. D63 (2001) 114027)

- Convoluting $D_a(x,Q) = E_{ab}(x,Q,Q_0) \otimes D_b(x,Q_0)$, one finds

$$D_a(x,Q) = T_a(Q,Q_0) D_a(x,Q_0) + \int_{Q_0^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, f_a(x,k_\perp,Q)$$

- where f_a is unintegrated PDF for $k_\perp \geq Q_0$

$$f_a(x,k_\perp,Q) = T_a(Q,k_\perp) \sum_{a'} \int_x^{1-\Delta} \frac{dz}{z}\, P_{aa'}(z,k_\perp)\, D_{a'}\left(\frac{x}{z},k_\perp\right)$$

- f_a are computed from single PDFs, with the cutoff

$$\Delta = \frac{k_\perp}{Q} \qquad \text{DGLAP ordering :} \qquad k_\perp < Q(1-z)$$

$$\Delta = \frac{k_\perp}{k_\perp + Q} \qquad \text{CCFM angular ordering :} \quad zk_\perp < Q(1-z)$$

- For $k_\perp < Q_0$, we need a model.

Construction of unintegrated DPDFs

- Substitute the equation for evolution function

$$\tilde{E}_{ab}(n,Q,Q_0) = T_a(Q,Q_0)\,\delta_{ab} + \int_{Q_0^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, T_a(Q,k_\perp) \sum_{a'} \tilde{P}_{aa'}(n,k_\perp)\, \tilde{E}_{a'b}(n,k_\perp,Q_0)$$

- to the solution of the evolution equations for DPDFs

$$\tilde{D}(n_1,n_2,Q_1,Q_2) = \tilde{E}(n_1,Q_1,Q_0)\, \tilde{D}(n_1,n_2,Q_0,Q_0)\, \tilde{E}^T(n_2,Q_2,Q_0)$$

$$+ \int_{Q_0}^{Q_{min}} \frac{dQ'}{Q'} E(n_1,Q_1,Q')\, D^{(sp)}(n_1,n_2,Q')\, E^T(n_2,Q_2,Q')$$

- Homogeneous and non-homogeneous UDPDFs from the sum: $\tilde{D} = \tilde{D}^{(h)} + \tilde{D}^{(nh)}$

$$\tilde{f}(n_1,n_2,k_{1\perp},k_{2\perp},Q_1,Q_2) = \tilde{f}^{(h)} + \tilde{f}^{(nh)}$$

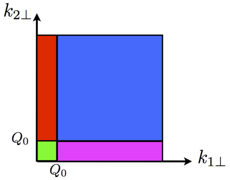
UDPDFs in four regions of transverse momenta $(k_{1\perp}, k_{2\perp})$

$$\tilde{D}^{(h)}_{a_1 a_2}(n_1, n_2, Q_1, Q_2) = T_{a_1}(Q_1, Q_0)\, T_{a_2}(Q_2, Q_0)\, \tilde{D}_{a_1 a_2}(n_1, n_2, Q_0, Q_0)$$

$$+ \int_{Q_0^2}^{Q_2^2} \frac{dk_{2\perp}^2}{k_{2\perp}^2}\, T_{a_1}(Q_1, Q_0)\, T_{a_2}(Q_2, k_{2\perp}) \sum_c \tilde{P}_{a_2 c}(n_2, k_{2\perp})\, \tilde{D}^{(h)}_{a_1 c}(n_1, n_2, Q_0, k_{2\perp})$$

$$+ \int_{Q_0^2}^{Q_1^2} \frac{dk_{1\perp}^2}{k_{1\perp}^2}\, T_{a_1}(Q_1, k_{1\perp})\, T_{a_2}(Q_2, Q_0) \sum_b \tilde{P}_{a_1 b}(n_1, k_{1\perp})\, \tilde{D}^{(h)}_{b a_2}(n_1, n_2, k_{1\perp}, Q_0)$$

$$+ \int_{Q_0^2}^{Q_1^2} \frac{dk_{1\perp}^2}{k_{1\perp}^2} \int_{Q_0^2}^{Q_2^2} \frac{dk_{2\perp}^2}{k_{2\perp}^2}\, T_{a_1}(Q_1, k_{1\perp})\, T_{a_2}(Q_2, k_{2\perp}) \sum_{b,c} \tilde{P}_{a_1 b}(n_1, k_{1\perp})\, \tilde{P}_{a_2 c}(n_2, k_{2\perp})\, \tilde{D}^{(h)}_{bc}(n_1, n_2, k_{1\perp}, k_{2\perp})$$



Homogeneous UDPDFs in x-space

- For $k_{1\perp}, k_{2\perp} > Q_0$ we have the distribution

$$f^{(h)}_{a_1 a_2}(x_1, x_2, k_{1\perp}, k_{2\perp}, Q_1, Q_2) = T_{a_1}(Q_1, k_{1\perp})\, T_{a_2}(Q_2, k_{2\perp})$$
$$\times \sum_{b,c} \int_{\frac{x_1}{1-x_2}}^{1-\Delta_1} \frac{dz_1}{z_1} \int_{\frac{x_2}{1-x_1/z_1}}^{1-\Delta_2} \frac{dz_2}{z_2} P_{a_1 b}(z_1, k_{1\perp})\, P_{a_2 c}(z_2, k_{2\perp})\, D^{(h)}_{bc}\left(\frac{x_1}{z_1}, \frac{x_2}{z_2}, k_{1\perp}, k_{2\perp}\right)$$

- where the cut-off parameters

$$\Delta_i = \frac{k_{i\perp}}{Q_i} \qquad \text{or} \qquad \Delta_i = \frac{k_{i\perp}}{k_{i\perp} + Q_i}, \qquad i = 1, 2$$

Homogeneous UDPDFs in x-space

- For $k_{1\perp} > Q_0$ and $k_{2\perp} \leq Q_0$ we have the distribution integrated over $k_{2\perp} \leq Q_0$

$$f^{(h)}_{a_1 a_2}(x_1, x_2, k_{1\perp}, Q_1, Q_2) = T_{a_1}(Q_1, k_{1\perp}) T_{a_2}(Q_2, Q_0)$$
$$\times \sum_b \int_{\frac{x_1}{1-x_2}}^{1-\Delta_1} \frac{dz_1}{z_1} P_{a_1 b}(z_1, k_{1\perp})\, D^{(h)}_{ba_2}\left(\frac{x_1}{z_1}, x_2, k_{1\perp}, Q_0\right)$$

- For $k_{1\perp} \leq Q_0$ and $k_{2\perp} > Q_0$ we have the distribution integrated over $k_{1\perp} \leq Q_0$

$$f^{(h)}_{a_1 a_2}(x_1, x_2, k_{2\perp}, Q_1, Q_2) = T_{a_1}(Q_1, Q_0)\, T_{a_2}(Q_2, k_{2\perp})$$
$$\times \sum_b \int_{\frac{x_2}{1-x_1}}^{1-\Delta_2} \frac{dz_2}{z_2} P_{a_2 b}(z_2, k_{2\perp})\, D^{(h)}_{a_1 b}\left(x_1, \frac{x_2}{z_2}, Q_0, k_{2\perp}\right)$$

- For $k_{1\perp}, k_{2\perp} \leq Q_0$ - nonperturbative region parametrized by integrated density.

Non-homogeneous UPDFs

- The same treatment in the non-homomogeneous case - plug

$$\tilde{E}_{ab}(n,Q,Q_0) = T_a(Q,Q_0)\,\delta_{ab} + \int_{Q_0^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, T_a(Q,k_\perp) \sum_{a'} \tilde{P}_{aa'}(n,k_\perp)\, \tilde{E}_{a'b}(n,k_\perp,Q_0)$$

- into the non-homogeneous solution of the evolution equations for DPDFs

$$\tilde{D}^{(nh)}(n_1,n_2,Q_1,Q_2) = \int_{Q_0^2}^{Q_{min}^2} \frac{dQ'^2}{Q'^2}\, E(n_1,Q_1,Q')\, D^{(sp)}(n_1,n_2,Q')\, E^T(n_2,Q_2,Q')$$

- Transverse momenta generated by unfolding the last step of the evolution of two gluon ladders.

- They can also be generated by unfolding the distribution $D^{(sp)}(Q')$.

- Details in the forthcoming paper.

Summary and outlook

- Starting from the integrated DPDFs, we constructed unintegrated DPDFs by unfolding transverse momentum dependence in the last step of the evolution - KMR approach.
- The construction of the homogeneous part of UPDFs is rather straightforward but the the non-homogeneous part is more subtle.
- The found homogeneous UDPDFs have nontrivial correlations between longitudinal and transverse momenta, e.g

$$\frac{x_1}{1-\Delta_1} + \frac{x_2}{1-\Delta_2} \leq 1 \qquad (\Delta_i = k_{i\perp}/Q_i)$$

- Numerics is challenging but possible.