Fitting the Discrete BFKL Pomeron to Low-x HERA Data

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(based on work with L.N. Lipatov)

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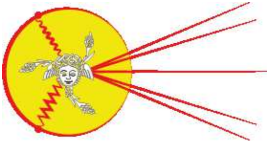
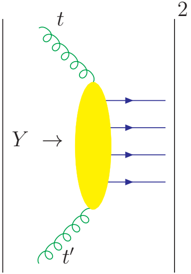




$$t = \ln(k_T^2/\Lambda^2)$$
$$t' = \ln(k_T'^2/\Lambda^2)$$
$$Y = \ln(s/k_T k_T')$$

$$\mathcal{A}(Y,t,t') = \int_{\mathcal{C}} d\omega e^{\omega Y} f_\omega(t) f_\omega^*(t'), \qquad \int dt' \, \mathcal{K}(\alpha_s, t, t') f_\omega(t') = \omega \, f_\omega(t)$$

For fixed coupling

$$f_\omega(t) \sim e^{i\nu_\omega t}$$

with ν_ω fixed for fixed ω.

For running coupling ν_ω becomes t dependent, decreasing as t increases to t_c.

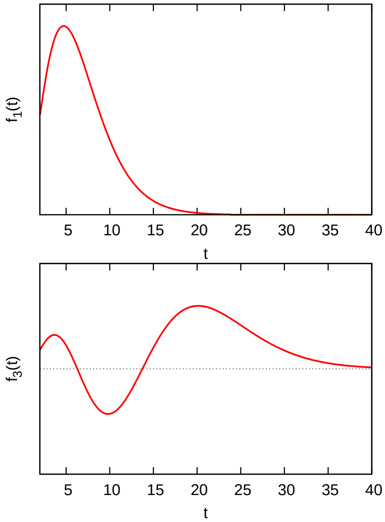
For $t > t_c$, ν_ω is imaginary $\longrightarrow$ evanescence ($f_\omega(t) \to 0$ as $t \to \infty$).

Assume that the IR (non-perturbative) properties of QCD determine the phase, η, of oscillations at some small $t = t_0$) (Lipatov 1986)
This leads to a discrete set of eigenfunctions

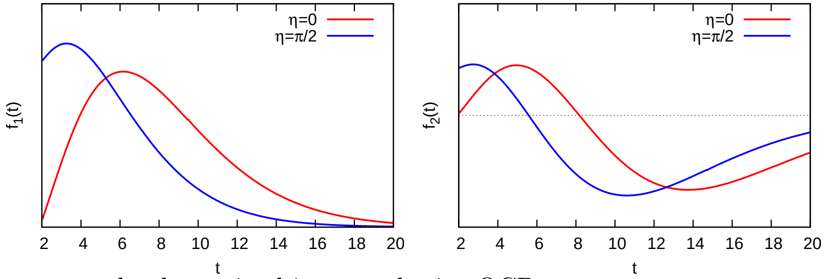
$$\mathcal{A}(Y,t,t') = \sum_n e^{\omega_n Y} f_\omega(t) f_\omega^*(t')$$

[Regge Poles]

First 4 Eigenfunctions



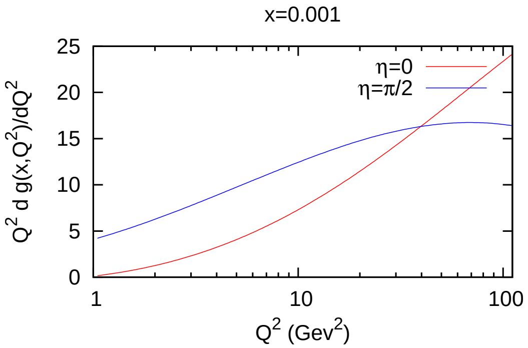
Sensitivity to Infrared phase, η



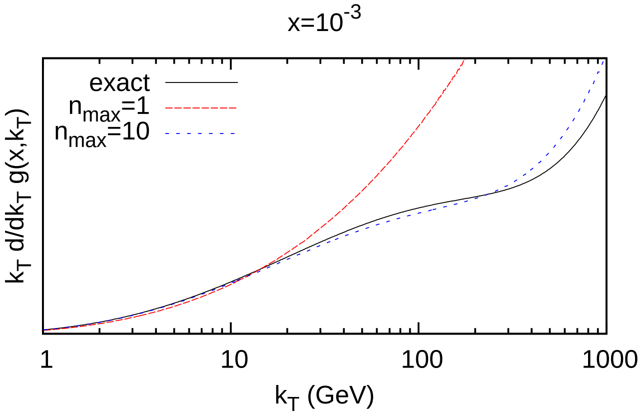
η cannot be determined in perturbative QCD.

Phases must be treated as free parameters in a fit to data.

Infrared Phase Sensitivity of Unintegrated Gluon Density

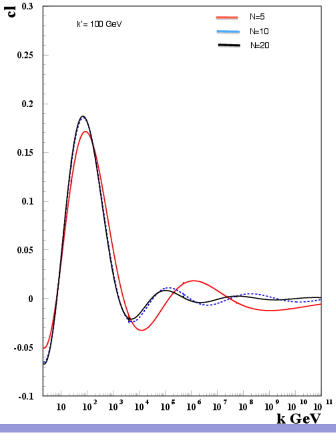


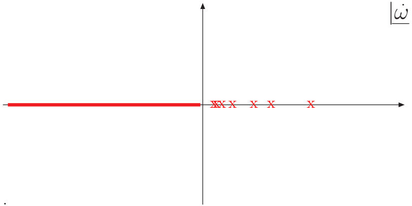
Very good convergence for the un-integrated gluon density after 10 eigenfunctions.



Completeness of Eigenfunctions

$$\sum_n f_{\omega_n}(t) f^*_{\omega_n}(t') = \delta(t - t')$$



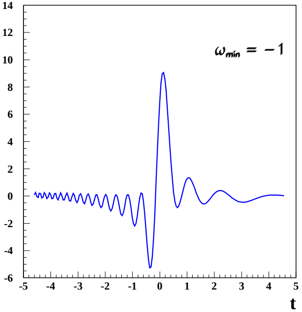


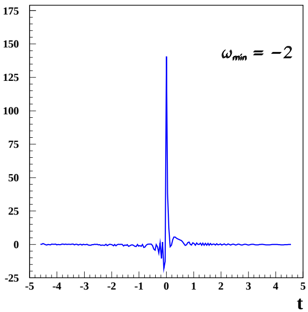
As well as a set of discrete poles for positive ω there is a cut along negative real axis.

The contribution from this cut is needed in order to reproduce the required completeness relation.

This cut contribution generates a small but non-negligible contribution to unintegrated gluon density for low-x (despite $x^{-|\omega|}$ suppression).

$$\sum_n f_{\omega_n}(t) f^*_{\omega_n}(t') + \int_{\omega_{min}}^{0} d\omega f_{-|\omega|}(t) f^*_{-|\omega|}(t') = \delta(t - t')$$





A popular parameterization of structure functions:

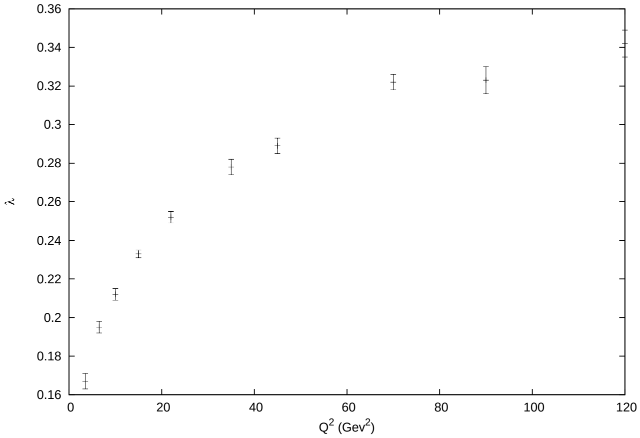
$$F_2(x, Q^2) \; = A(Q^2)x^{-\lambda(Q^2)}$$

Not motivated by either a BFKL or DGLAP analysis, but nevertheless seems to work very well (Caldwell, 2015)

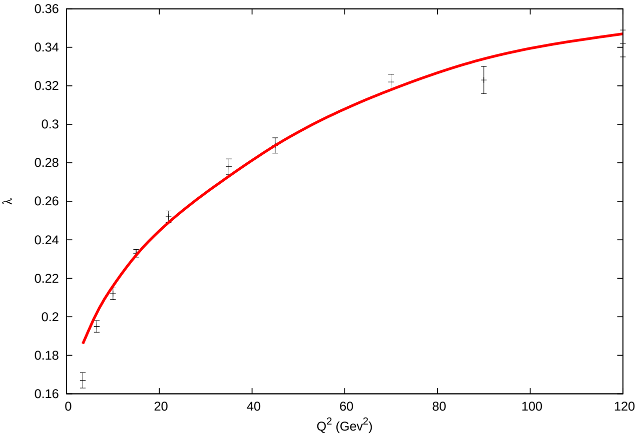
To match the discrete BFKL pomeron we need to find a fit such that

$$A(Q^2)x^{-\lambda(Q^2)} \approx \sum_n C_n f_{\omega_n}(Q^2)x^{-\omega_n}$$

Since $f_\omega(t)$ decreases for sufficiently large t, we expect that for sufficiently large Q^2, $\lambda(Q^2)$ is a decreasing function of Q^2.



Experimental data for low-x structure functions refer to Q^2 below the critical value where the leading eigenfunctions start to decay.
A fit is possible by suitable choice of the infrared phases.



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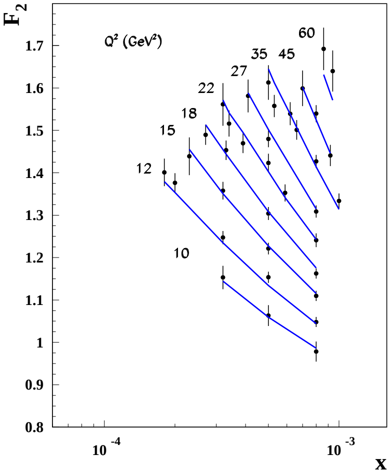
Paramterizing the Infrared Phases

3 parameter fit

$$\omega_n = \frac{A}{n+B} + C$$

(C=0 in Lipatov '86)
Best fit $A = 0.489,\ B = 1.39,\ C = 0.0014$
Invert to obtain n as a function of ω
Phase, $\eta(\omega)$ at $t = t_0$ is then given by

$$\eta(\omega) = \int_{t_0}^{t_c} \nu_\omega(t')dt' + \frac{\pi}{4} - n(\omega), \quad (\nu_\omega(t_c) = 0)$$



A global fit is only possible for data with $x \leq 10^{-3}$

Select $Q^2 > 5\,\mathrm{Gev}^2$ to avoid saturation problems.

(37 data points from HERA)

$\chi^2/\mathrm{DOF} = 0.48$

Too Good !!!!

Over-parameterized or experimental errors are correlated.

It may only be possible to get a good fit for $x \leq\sim 10^{-3}$.
BFKL is an expansion in $1/\ln|x|$.
Corrections to NLO BFKL expected to be $\sim 1/|\ln(x)|^2$.
$\sim 2\%$ for $x = 10^{-3}$ (comparable with experimental accuracy)
$\sim 5\%$ for $x = 10^{-2}$

Application of BFKL mechansm to DIS assumes that (valence) quark contribution is negligible. This may not be good to current experimental accuracy for $x > 10^{-3}$.

There are very large corrections to the photon impact factor at NLO (Chirilli & Balitsky; Bartels & Chachamis) - these need some sort of collinear resummation analogous to the NLO characteristic function (Salam, 1999)

Summary

- Eigenfunctions of discrete BFKL Pomeron are very sensitive to infrared phases.
- Rapid convergence of sum of eigenfunctions to generate unintegrated gluon density. - 10 eigenfunctions sufficient for good accuracy
- Continuum from cut along negative real axis in Mellin transform variable, ω, is needed for completeness - makes a small but signficiant contribution to unintegrated gluon density.
- For $F_2 \sim x^{-\lambda(Q^2)}$ we expect $\lambda(Q^2)$ decrease at large Q^2 - reflecting dacay of leading eigenfunctions. Data so far only gives a hint of this - perhaps this could be confirmed at LHeC !!
- Fit with $\chi^2/DOF = 0.5$ has been found for HERA DIS data with $x \leq\sim 10^{-3}$ (and $Q^2 > 5\,\mathrm{GeV}^2$)