Impact factor for exclusive diffractive dijet production with NLO accuracy

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Diffraction 2016

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JHEP 409 (2014) 026

[1606.00419] (To be published in JHEP)
Probing QCD in the Regge limit and towards saturation

What kind of observable?

- Perturbation theory should apply: a hard scale $Q^2$ is required.

- One needs semihard kinematics: $s \gg p_T^2 \gg \Lambda_{QCD}^2$, where all the typical transverse scales $p_T$ are of the same order.

- Saturation is reached when $Q^2 \sim Q_s^2 = \left(\frac{A}{x}\right)^{\frac{1}{3}}$: the smaller $x \sim \frac{Q_s^2}{s}$ is and the heavier the target ion, the easier saturation is reached.

- Typical processes: DIS, Mueller-Navelet double jets, high $p_T$ central jets, ultraperipheral events at the LHC...
Precision tests of BFKL dynamics

- The BFKL kernel is known at NLL accuracy, resumming $\alpha_s(\alpha_s \log s)^n$ corrections (Lipatov, Fadin; Camici, Ciafaloni)
- Very few impact factors are known at NLO accuracy
  - $\gamma^* \rightarrow \gamma^*$ (Bartels, Colferai, Gieseke, Kyrielis, Qiao; Balitsky, Chirilli)
  - Forward jet production (Bartels, Colferai, Vacca; Caporale, Ivanov, Murdaca, Papa, Perri; Chachamis, Hentschinski, Madrigal, Sabio Vera)
  - Inclusive production of a pair of hadrons separated by a large interval of rapidity (Ivanov, Papa)
  - $\gamma^*_L \rightarrow \rho_L$ in the forward limit (Ivanov, Kostsky, Papa)
Rapidity gap events at HERA

Experiments at HERA: about 10% of scattering events reveal a rapidity gap

DIS events

DDIS events
Rapidity gap events at HERA

Experiments at HERA: about 10% of events reveal a rapidity gap

ZEUS, 1993

H1, 1994
Diffractive DIS

Theoretical approaches for DDIS using pQCD

- **Collinear factorization** approach
  - Relies on QCD factorization theorem, using a hard scale such as the *virtuality* $Q^2$ of the incoming photon
  - One needs to introduce a diffractive distribution function for partons *within a pomeron*

- **$k_T$ factorization** approach for two exchanged gluons
  - low-$x$ QCD approach:
    - $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron is described as a two-gluon color-singlet state
Theoretical approaches for DDIS using pQCD

Collinear factorization approach

Direct

Resolved
Theoretical approaches for DDIS using pQCD

$k_T$-factorization approach: two gluon exchange

Bartels, Ivanov, Jung, Lotter, Wüsthoff
Braun and Ivanov developed a similar model in collinear factorization
Theoretical approaches for DDIS using pQCD

Confrontation of the two approaches with HERA data

ZEUS collaboration, 2015
Assumptions

- Regge limit: \( s \gg Q^2 \gg \Lambda_{QCD} \)

- **No approximation** for the outgoing gluon, contrary to e.g.:
  - Collinear approximation [Wüsthoff, 1995]
  - Soft approximation [Bartels, Jung, Wüsthoff, 1999]

- Lightcone coordinates \((p^+, p^-, \vec{p})\) and lightcone gauge \(n_2 \cdot A = 0\)

- Transverse dimensional regularization \(d = 2 + 2\varepsilon\), longitudinal cutoff \(p_g^+ < \alpha p_\gamma^+\)

- **Shockwave** (Wilson lines) approach [Balitsky, 1995]
The shockwave approach

One decomposes the gluon field $A$ into an internal field and an external field:

$$A^\mu = A^\mu + b^\mu$$

The internal one contains the gluons with rapidity $p_g^+ > e^\eta p_\gamma^+$ and the external one contains the gluons with rapidity $p_g^+ < e^\eta p_\gamma^+$. One writes:

$$b^\mu_\eta (z) = \delta (z^+) B_\eta (\vec{z}) n^\mu_2$$

Intuitively, large boost $\Lambda$ along the + direction:

- $b^+ (x^+, x^-, \vec{x}) \rightarrow \frac{1}{\Lambda} b^+ \left( \Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x} \right)$
- $b^- (x^+, x^-, \vec{x}) \rightarrow \Lambda b^- \left( \Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x} \right)$
- $b^i (x^+, x^-, \vec{x}) \rightarrow b^i \left( \Lambda x^+, \frac{1}{\Lambda} x^-, \vec{x} \right)$
Propagator in the shockwave field

\[ G(z_2, z_0) = - \int d^4 z_1 \theta(z_2^+) \delta(z_1^+) \theta(-z_0^+) G(z_2 - z_1) \gamma^+ G(z_1 - z_0) \ U_1 \]

\[ G(q, p) = (2\pi) \theta(p^+) \int d^D q_1 \delta(q + q_1 - p) \ G(q) \gamma^+ \tilde{U}_{\vec{q}_1} G(p) \]

Wilson lines :

\[ U_i = U_{\vec{z}_i} = U(\vec{z}_i, \eta) = P \exp \left[ ig \int_{-\infty}^{+\infty} b^-_\eta(z_i^+, \vec{z}_i) dz_i^+ \right] \]

\[ U_i = 1 + ig \int_{-\infty}^{+\infty} b^-_\eta(z_i^+, \vec{z}_i) dz_i^+ + (ig)^2 \int_{-\infty}^{+\infty} b^-_\eta(z_i^+, \vec{z}_i) b^-_\eta(z_j^+, \vec{z}_j) \theta(z_{ji}^+) dz_i^+ dz_j^+ \]

\[ \cdots \]

\[ = \quad + \quad + \quad + \cdots \]
Evolution equation for a color dipole

\[ U_{12} = \frac{1}{N_c} \text{Tr} \left( U_1 U_2^\dagger \right) - 1 \]

Involving Wilson lines

\[ \vec{z}_1 \rightarrow \eta + \Delta \eta \]
\[ \vec{z}_2 \leftarrow \eta + \Delta \eta \]
Balitsky’s hierarchy of equations

\[ \frac{\partial U_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\tilde{z}_3 \frac{\tilde{z}_{12}^2}{\tilde{z}_{13}^2 \tilde{z}_{23}^2} [U_{13} + U_{32} - U_{12} - U_{13}U_{32}] \]

\[ \frac{\partial U_{13}U_{32}}{\partial \eta} = \ldots \]

B-JIMWLK equation

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]
The BK equation

Mean field approximation, or 't Hooft limit $N_c \to \infty$ in Balitsky's equation

\[ \frac{\partial U_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z} \frac{\vec{z}_{12}^2}{\vec{z}_{13}^2 \vec{z}_{23}^2} [U_{13} + U_{32} - U_{12} - U_{13}U_{32}] \]

\[ \Rightarrow \text{BK equation [Balitsky, 1995] [Kovchegov, 1999]} \]

Evolves a dipole into a double dipole

Non-linear term: saturation
Leading Order

\[ \mathcal{A}_0 = \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_0^\alpha (\vec{p}_1, \vec{p}_2) \delta (\vec{p}_{q1} + \vec{p}_{q2}) \tilde{U}_{12} \]

\[ p_{ij} = p_i - p_j \]
First kind of virtual corrections

\[ \mathcal{A}_{V_1} \propto \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_{V_1}^{\alpha} (\vec{p}_1, \vec{p}_2) \delta (\vec{p}_{q1} + \vec{p}_{\bar{q}2}) \left( \frac{N_c^2 - 1}{N_c} \right) \tilde{U}_{12} \]
Second kind of virtual corrections

\[ A_{V2} \propto \varepsilon \alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \Phi_{V2}^\alpha (\vec{p}_1, \vec{p}_2, \vec{p}_3) \delta (\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_3) \]

\[
\left[ \delta (\vec{p}_3) \left( \frac{N_c^2 - 1}{N_c} \right) \tilde{U}_{12} + N_c \left( \tilde{U}_{13} \tilde{U}_{32} + \tilde{U}_{13} + \tilde{U}_{32} - \tilde{U}_{12} \right) \right]
\]
First kind of real corrections

\[ \mathcal{A}_{R1} = \varepsilon_{\alpha} N_c \int d^d \vec{p}_1 d^d \vec{p}_2 \Phi_{R1}^{\alpha} (\vec{p}_1, \vec{p}_2) \delta (\vec{p}_{q1} + \vec{p}_{\bar{q}2} + \vec{p}_g) \left( \frac{N_c^2 - 1}{N_c} \right) \tilde{U}_{12} \]
Second kind of real corrections

\[ A_{R2} = \varepsilon_\alpha N_c \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \Phi^\alpha_{R2} (\vec{p}_1, \vec{p}_2, \vec{p}_3) \delta (\vec{p}_{q1} + \vec{p}_{\bar{q}2} + \vec{p}_g3) \]

\[ \left[ \left( \frac{N_{c}^2 - 1}{N_{c}} \right) \tilde{U}_{12} \delta (\vec{p}_3) + N_{c} \left( \tilde{U}_{13} \tilde{U}_{32} + \tilde{U}_{13} + \tilde{U}_{32} - \tilde{U}_{12} \right) \right] \]
Divergences

- **UV divergence** \( \vec{p}_g^2 \rightarrow +\infty \)
  \[ \Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^* \]

- **Soft divergence** \( p_g \rightarrow 0 \)
  \[ \Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^*, \Phi_{R1} \Phi_{R1}^* \]

- **Collinear divergence** \( p_g \propto p_q \) or \( p_{\bar{q}} \)
  \[ \Phi_{R1} \Phi_{R1}^* \]

- **Soft and collinear divergence** \( p_g = \frac{p_g^+}{p_q^+} p_q \) or \( \frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}} \), \( p_g^+ \rightarrow 0 \)
  \[ \Phi_{R1} \Phi_{R1}^* \]

- **Rapidity divergence** \( p_g^+ \rightarrow 0 \)
  \[ \Phi_{V2} \Phi_0^* + \Phi_0 \Phi_{V2}^* \]
Rapidity divergence

Double dipole virtual correction $\Phi_{V2}$

Balitsky’s evolution of the LO term: $\Phi_0 \otimes K_{BK}$
Production of dijets in DDIS

Conclusion and applications

Rapidity divergence

B-JIMWLK equation

\[
\frac{\partial \tilde{U}_{12}}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta \left( \vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2 \right) \left( \tilde{U}_{13} \tilde{U}_{32} + \tilde{U}_{13} + \tilde{U}_{32} - \tilde{U}_{12} \right)
\]

\[
\left[ \frac{d}{2} \frac{\Gamma (1 - \frac{d}{2})}{\Gamma (d - 1)} \right] \left( \frac{\delta \left( \vec{k}_2 - \vec{p}_2 \right)}{\left( \vec{k}_1 - \vec{p}_1 \right)^2} \right)^{1-\frac{d}{2}} + \frac{\delta \left( \vec{k}_1 - \vec{p}_1 \right)}{\left( \vec{k}_2 - \vec{p}_2 \right)^2} \right)^{1-\frac{d}{2}}
\]

\( \eta \) is the rapidity divide

\[
\tilde{U}_{12}^\alpha \Phi_0 \rightarrow \Phi_0 \tilde{U}_{12}^\eta + \log \left( \frac{e^{\eta}}{\alpha} \right) K_{BK} \Phi_0 \left( \tilde{U}_{13} \tilde{U}_{32} + \tilde{U}_{13} + \tilde{U}_{32} - \tilde{U}_{12} \right)
\]
Rapidity divergence

**Virtual contribution**

\[(\Phi_{V2}^\mu)_{\text{div}} \propto \Phi_0^\mu \left\{ 4 \ln \left( \frac{x \bar{x}}{\alpha^2} \right) \left[ \frac{1}{\varepsilon} + \ln \left( \frac{\vec{p}_3^2}{\mu^2} \right) \right] - \frac{6}{\varepsilon} \right\}\]

**BK contribution**

\[(\Phi_{BK}^\mu)_{\text{div}} \propto \Phi_0^\mu \left\{ 4 \ln \left( \frac{\alpha^2}{e^{2\eta}} \right) \left[ \frac{1}{\varepsilon} + \ln \left( \frac{\vec{p}_3^2}{\mu^2} \right) \right] \right\}\]

**Sum**

\[(\Phi_{V2}^\prime \mu)_{\text{div}} \propto \Phi_0^\mu \left\{ 4 \ln \left( \frac{x \bar{x}}{e^{2\eta}} \right) \left[ \frac{1}{\varepsilon} + \ln \left( \frac{\vec{p}_3^2}{\mu^2} \right) \right] - \frac{6}{\varepsilon} \right\}\]
Rapidity divergence

Convolution

\[(\Phi'_\mu V_2 \otimes UU) = \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \left\{ 4 \ln \left( \frac{x \bar{x}}{e^{2\eta}} \right) \left[ \frac{1}{\varepsilon} + \ln \left( \frac{\vec{p}_3^2}{\mu^2} \right) \right] - \frac{6}{\varepsilon} \right\} \times \delta(\vec{p}_q_1 + \vec{p}_q_2 - \vec{p}_3) \left[ \tilde{U}_{13} + \tilde{U}_{32} - \tilde{U}_{12} - \tilde{U}_{13} \tilde{U}_{32} \right] \Phi^\mu_0 (\vec{p}_1, \vec{p}_2) \]

Rq :
- \( \Phi_0 (\vec{p}_1, \vec{p}_2) \) only depends on one of the \( t \)-channel momenta.
- The double-dipole operators cancels when \( \vec{z}_3 = \vec{z}_1 \) or \( \vec{z}_3 = \vec{z}_2 \).

This permits one to show that the convolution cancels the remaining \( \frac{1}{\varepsilon} \) divergence.

Then \( \tilde{U}_{12}^\alpha \Phi_0 + \Phi_V \) is finite
Divergences

- **UV divergence** $\vec{p}_g^2 \to +\infty$
  \[ \Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^* \]

- **Soft divergence** $p_g \to 0$
  \[ \Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^*, \Phi_{R1} \Phi_{R1}^* \]

- **Collinear divergence** $p_g \propto p_q$ or $p_{\bar{q}}$
  \[ \Phi_{R1} \Phi_{R1}^* \]

- **Soft and collinear divergence** $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$
  \[ \Phi_{R1} \Phi_{R1}^* \]

- **Rapidity divergence**
Constructing a finite cross section

From partons to jets
Jet cone algorithm

We define a cone width for each pair of particles with momenta $p_i$ and $p_k$, rapidity difference $\Delta Y_{ik}$ and relative azimuthal angle $\Delta \phi_{ik}$

$$(\Delta Y_{ik})^2 + (\Delta \phi_{ik})^2 = R_{ik}^2$$

If $R_{ik}^2 < R^2$, then the two particles together define a single jet of momentum $p_i + p_k$.

Applying this in the small $R^2$ limit cancels our soft and collinear divergence.
Remaining divergence

- UV divergence $\vec{p}_g^2 \to +\infty$

  $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$

- Soft divergence $p_g \to 0$

  $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

  $\Phi_{R1}\Phi_{R1}^*$
Remaining divergence

Soft real emission

\[(\Phi_{R1} \Phi_{R1}^*)_{\text{soft}} \propto (\Phi_0 \Phi_0^*) \int_{\text{outside the cones}} \left| \frac{p_q^\mu}{(p_q \cdot p_g)} - \frac{p_{\bar{q}}^\mu}{(p_{\bar{q}} \cdot p_g)} \right|^2 \frac{d p_g^+}{p_g^+ (2\pi)^d} \]

Collinear real emission

\[(\Phi_{R1} \Phi_{R1}^*)_{\text{col}} \propto (\Phi_0 \Phi_0^*) (N_q + N_{\bar{q}}) \]

Where \(N\) is the number of jets in the quark or the antiquark

\[N_k = \frac{(4\pi)^{\frac{d}{2}}}{\Gamma(2 - \frac{d}{2})} \int_{p_{\text{jet}}^+}^{p_k^+} dp_g^+ dp_k^+ \int_{\alpha p_\gamma^+}^{p_{\gamma}^+} \frac{d^d \vec{p}_g d^d \vec{p}_k}{2 p_g^+ 2 p_k^+} \int_{\text{in cone } k} \frac{d^d \vec{p}_k}{(2\pi)^d \mu^{d-2}} \frac{\text{Tr} (\hat{p}_k \gamma^\mu \hat{p}_{\text{jet}} \gamma^\nu) d_{\mu\nu}(p_g)}{2 p_{\text{jet}}^+ (p_k^- + p_g^- - p_{\text{jet}}^-)^2} \]

Those two contributions cancel exactly the virtual divergences (both UV and soft)
Cancellation of divergences

Total divergence

\( (d\sigma_1)_{\text{div}} = \alpha_s \frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}} \left( \frac{N_c^2 - 1}{2N_c} \right) (S_V + S_V^* + S_R + N'_{\text{jet1}} + N'_{\text{jet2}}) d\sigma_0 \)

Virtual contribution

\[
S_V = \left[ 2 \ln \left( \frac{x_j x_j}{\alpha^2} \right) - 3 \right] \left[ \ln \left( \frac{x_j x_j \mu^2}{(x_j \vec{p}_j - x_j \vec{p}_j')^2} \right) - \frac{1}{\varepsilon} \right] \\
+ 2i\pi \ln \left( \frac{x_j x_j}{\alpha^2} \right) + \ln^2 \left( \frac{x_j x_j}{\alpha^2} \right) - \frac{\pi^2}{3} + 6
\]

Real contribution

\[
S_R + N'_{\text{jet1}} + N'_{\text{jet2}} = 2 \left[ \ln \left( \frac{(x_j \vec{p}_j - x_j \vec{p}_j')^4}{x_j^2 x_j^2 R^4 \vec{p}_j^2 \vec{p}_j'^2} \right) \ln \left( \frac{4E^2}{x_j x_j (p_{\gamma})^2} \right) + 2I(R, E) \right] \\
+ 2 \ln \left( \frac{x_j x_j}{\alpha^2} \right) \left( \frac{1}{\varepsilon} - \ln \left( \frac{x_j x_j \mu^2}{(x_j \vec{p}_j - x_j \vec{p}_j')^2} \right) \right) - \ln^2 \left( \frac{x_j x_j}{\alpha^2} \right) \\
+ \frac{3}{2} \ln \left( \frac{16\mu^4}{R^4 \vec{p}_j^2 \vec{p}_j'^2} \right) - \ln \left( \frac{x_j}{x_j} \right) \ln \left( \frac{x_j \vec{p}_j^2}{x_j \vec{p}_j'^2} \right) - \frac{3}{\varepsilon} - \frac{2\pi^2}{3} + 7
\]
Cancellation of divergences

Total divergence

\[
div = S_V + S_V^* + S_R + N_{\text{jet}1} + N_{\text{jet}2}
\]

\[
= 4 \left[ \frac{1}{2} \ln \left( \frac{(x_j \vec{p}_j - x_j \vec{p}_j^*)^4}{x_j^2 x_j^2 R^4 \vec{p}_j^2 \vec{p}_j^*^2} \right) \ln \left( \frac{4E^2}{x_j x_j (p_\gamma^+)^2} \right) + \frac{3}{2} \right] \\
+ I(R, E) + \ln(8) - \frac{1}{2} \ln \left( \frac{x_j}{x_j} \right) \ln \left( \frac{x_j \vec{p}_j^*}{x_j \vec{p}_j^2} \right) + \frac{13 - \pi^2}{2}
\]

Our cross section is thus finite
Conclusions about exclusive dijet production

- We computed the amplitude for the production of an open $q\bar{q}$ pair in DDIS.

- Using this result, we constructed a finite expression for the cross section for the exclusive production of dijets.

- The remaining part can be expressed as a finite integral, so it can be used straightforwardly for phenomenology.

- Any model can be used for the matrix elements of the Wilson line operators (GBW, AAMQS if the target is a proton or an ion).

- The target can also be perturbative, involving any impact factor...
Phenomenological applications: exclusive dijet production at NLO accuracy

- **HERA data** for exclusive dijet production in diffractive DIS can be fitted with our results.

- We can also give predictions for the same process in a future **electron-ion or electron-proton collider** (EIC, LHeC...)

- For $Q^2 = 0$ we can give predictions for **ultraperipheral collisions at the LHC**
Phenomenological applications: exclusive trijet production at LO accuracy

- **HERA data** for exclusive trijet production in diffractive DIS can be fitted with our results.
- We can also give predictions for the same process in a future **electron-ion collider**.
- For $Q^2 = 0$ we can give predictions for **ultraperipheral collisions at the LHC**.

Amplitude for diffractive trijet production
**General amplitude**

- **Most general kinematics**
  - The hard scale can be $Q^2$, $t$, $M_X^2$ or $m^2$ in the (future) massive extension of our computation.
  - The target can be either a proton or an ion, or another impact factor.
  - One can study ultraperipheral collision by tagging the particle which emitted the photon, in the limit $Q^2 \to 0$. 

The general amplitude

![Diagram](attachment:image.png)
One can adapt our general amplitude to obtain the NLO expression for (non diffractive) DIS.

Such a result would have to be compared with Balitsky and Chirilli’s result, and with an ingoing study by Beuf.
Diffractive production of a $\rho$ meson

- By forcing the quark and antiquark to be collinear and using the right Fierz projection, one can study $\rho$ production.
- Generalization of previous results of Ivanov, Kotsky, Papa to the non-forward case.
- This would give a better understanding of the formal transition between BFKL and BK.

$$\Phi_{BK} \otimes \mathcal{K}_{BK} \otimes \Phi'_{BK} = (\Phi_{BFKL} \otimes \mathcal{O})(\mathcal{O}^{-1} \otimes \mathcal{K}_{BFKL} \mathcal{O})(\mathcal{O}^{-1} \otimes \Phi'_{BFKL})$$
With an added mass

- Open charm production (straightforward)
- Heavy quarkonium production (in the Color Evaporation formalism)

\[ Q^2 \]

\[ J/\psi \]

rapidity gap

Amplitude for diffractive production of a charmonium