The growth with energy of exclusive J/Ψ and Υ photo-production cross-sections and BFKL evolution

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based on

I. Bautista, A. Ferndandez Tellez, MH. [arXiv:1607.05203] (PRD **94** 054002)

Outline

Introduction

Ingredients of our study

- NLO BFKL gluon density
- Impact factor $\gamma \to V$
- Amplitude: Real part from imaginary part

Results & Conclusions

Pomeron: effective degree of freedom which describes the rise of cross-sections with energy

BFKL Pomeron:
microscopic description in terms of quarks & gluons
$\rightarrow$ requires process with hard scale $Q^2 \gg Q_0^2 \Rightarrow \alpha_s(Q^2) \ll 1$

requires:

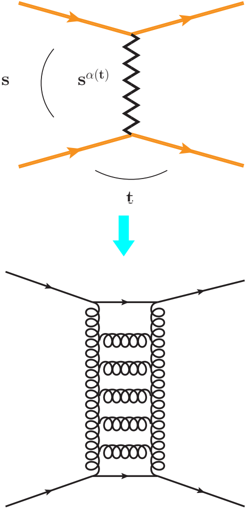
expansion of perturbative amplitudes in $1/s$

$+$ resummation of enhanced terms $\left(\alpha_s(Q^2) \ln s\right)^n \sim 1$ to all orders in α_s

➡ BFKL equation

LL: [Fadin, Kuraev, Lipatov, PLB 60 (1975) 50]
[Balitsky, Lipatov, SJNP (1978 822)]

NLL: [Fadin, Lipatov; PLB 429 (1998) 127];
[Ciafaloni, Camici; PLB 430 (1998) 349]



Phenomenology of BFKL evolution

- at LHC: first success in the description of angular decorrelation of multi-jet observables

  [Ducloué, Szymanowski, Wallon, 1312.2624], [Celiberto, Ivanov, Murdaca, Papa; 1504.08233]

  [Caporale, Chachamis, Murdaca, Sabio Vera; 1508.07711]

  ⟶ see talks by Francesco G. Celiberto

- test conformal spin $n \neq 0$ components of the BFKL kernel interesting in its own right, perturbatively stable & test of the underlying framework

- $n = 0$ component: rise of perturbative cross-sections

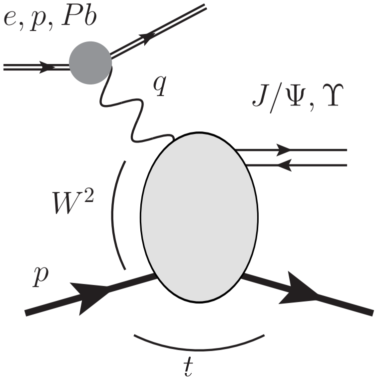
  ⟶ essentially only studied in fits to inclusive HERA DIS data

  [Kowalski, Lipatov, Ross; 1005.0355; 1205.6713]

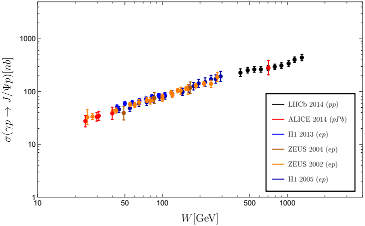
  [Hentschinski, Salas, Sabio Vera; 1209.1353, 1301.5283]

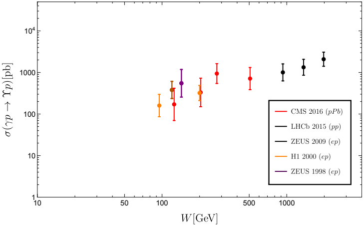
  [Levin, Potashnikova; 1307.7823]

a suitable process: excl. photo-production of J/Ψ and Υ



- measured at HERA (ep) and LHC (pp, ultra-peripheral pPb)
- charm and bottom mass provide hard scale ➡ BFKL
- rise with energy $W^2 = (p+q)^2$, *e.g.* $\sigma^{J/\Psi} \sim \left(W^2\right)^{0.335}$





Existing description of data

... work pretty well

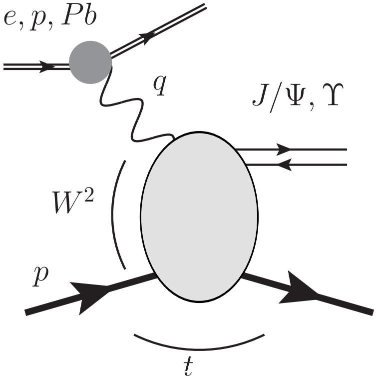
- J/Ψ: power-law fit to HERA data $\sigma \sim W^{0.67}$ [LHCb Collaboration; 1401.3288]
- collinear factorization: NLO fits [Jones, Martin, Ryskin, Teubner; 1307.7099]
- saturation models: IPsat, bCGC, rcBK

  [Armesto, Rezaeian; 1402.4831], [Goncalves, Moreira, Navarra; 1405.6977, 1408.1344]
- see also [Fiore, Jenkovszky, Libov, Machado; 1408.0530], [Cisek, Schäfer, Szczurek; 1405.2253]

BFKL special:
don't fit W -dependence, but calculate from perturbative low x evolution
don't evoke saturation (= effects beyond BFKL)

LHC: reach ultra-small x values $\simeq 4 \cdot 10^{-6}$ not constrained by HERA

Description within BFKL framework



- exclusive process: vaccuum qauntum # exchange between VM & proton
- Appropiate theoretical framework: non-forward BFKL $t \neq 0$
- kernels known up to NLO, but not explored in phenomenological studies & not sufficiently well understood how to use them

use procedure of *e.g.* DGLAP study **[Jones, Martin, Ryskin, Teubner; 1307.7099, 1312.6795]**

→ relate *exclusive* photo-production to *inclusive* gluon distribution

The framework of this BFKL study

procedure:

a) calculate diff. Xsec. at $t = 0$

→ *exclusive* scattering amplitude can be expressed through *inclusive* gluon distribution

b) parametrize t dependence $\frac{d\sigma(t)}{dt} = \frac{d\sigma(t=0)}{dt} \cdot e^{-|t|B_D(W)}$,

slope $B_D(W) = b_0 + 4\alpha' \ln \frac{W}{W_0}$ + fix parameters by (HERA) data (here: values proposed by [Jones, Martin, Ryskin, Teubner; 1307.7099, 1312.6795])

→ cross-section: $\sigma^{\gamma p \to Vp}(W) = \underbrace{\frac{1}{B_D(W)}}_{\text{phenomenological}} \underbrace{\left.\frac{d\sigma^{\gamma p \to Vp}}{dt}\right|_{t=0}}_{\text{BFKL / theory}}$

The setup: diff. Xsec. at $t = 0$

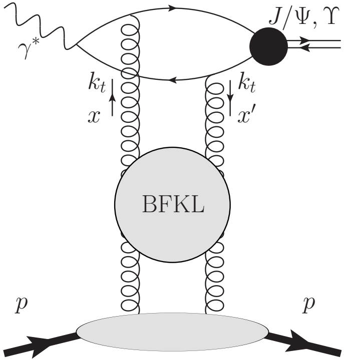
a) imaginary part of scattering amplitude:

- unintegrated gluon density from NLO BFKL fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]
- impact factor $\gamma \to J/\Psi, \Upsilon$ from light-front wave function used in dipole model studies

  [Kowalski, Motyka, Watt; hep-ph/0606272]

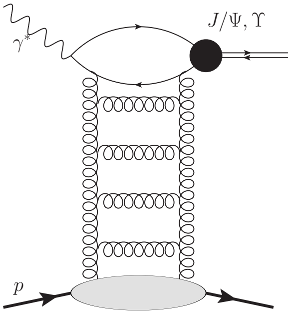
b) real part:

- $\Im \mathrm{m} \mathcal{A}(W^2, t)$ dominant, real part can be numerically large
  ➡ recover real part using dispersion relation



relate 2 pictures of the BFKL Pomeron

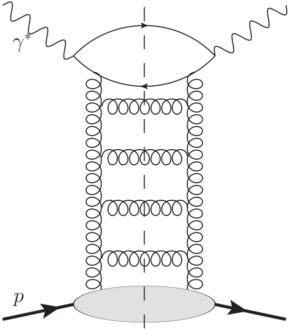
a) exclusive photo-production of vector mesons:



'uncut' Pomeron: diffractive/elastic scattering (amplitude level)

$$\mathcal{A}(s,t)$$

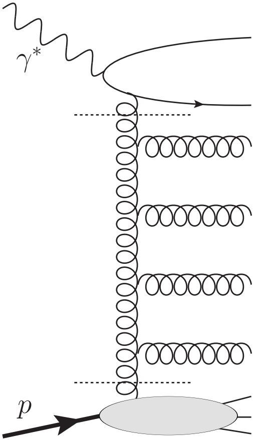
b) proton structure functions:



'cut' Pomeron: high multiplicity events (total X-sec.)

$$\sigma_{\text{tot}} = \frac{1}{s}\Im\text{m}\mathcal{A}(s,t=0)$$

The underlying NLO BFKL fit



$$F_2(x,Q^2) = \int_0^\infty d\boldsymbol{k}^2 \int_0^\infty \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \Phi_2\left(\frac{\boldsymbol{k}^2}{Q^2}\right) \mathcal{F}^{\text{DIS}}_{\text{BFKL}}(x,\boldsymbol{k}^2,\boldsymbol{q}^2)\Phi_p\left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)$$

virtual photon: quarks mass-less, $n_f = 4$ fixed

proton impact factor: $\Phi_p\left(\frac{\boldsymbol{q}^2}{Q_0^2},\delta\right) = \frac{\mathcal{C}}{\pi\Gamma(\delta)}\left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)^\delta e^{-\frac{\boldsymbol{q}^2}{Q_0^2}}$

free parameters of proton impact factor from fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

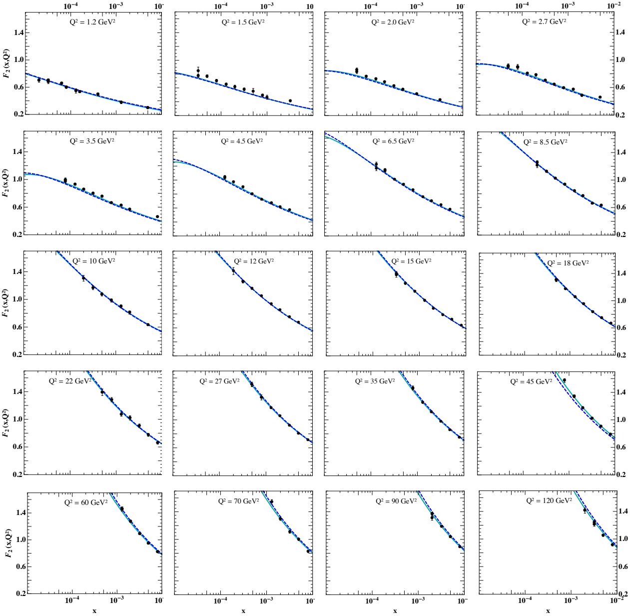
→ allows for definition of unintegrated gluon density

[Chachamis, Deak, MH, Rodrigo, Sabio Vera; 1507.05778]

$$G(x,\boldsymbol{k}^2,Q_0^2) = \int \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2}\mathcal{F}^{\text{DIS}}_{\text{BFKL}}(x,\boldsymbol{k}^2,\boldsymbol{q}^2)\Phi_p\left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)$$

	virt. photon impact factor	Q_0/GeV	δ	$\mathcal{C}$	Λ_{QCD}/ GeV
fit 1	leading order (LO)	0.28	8.4	1.50	0.21
fit 2	LO with kinematic improvements	0.28	6.5	2.35	0.21

Good description of cominbed HERA [H1 & ZEUS collab. 0911.0884]



Solve BFKL equation in conjugate (γ) Mellin space

$$G\left(x, \boldsymbol{k}^2, M\right) = \frac{1}{\boldsymbol{k}^2} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \; \hat{g}\left(x, \frac{M^2}{Q_0^2}, \frac{\overline{M}^2}{M^2}, \gamma\right) \; \left(\frac{\boldsymbol{k}^2}{Q_0^2}\right)^{\gamma}$$

re-introduce two scales: hard scale of process (M) and scale of running coupling ($\overline{M}$)

$\hat{g}$: operator in γ space!

$$\hat{g}\left(x, \frac{M^2}{Q_0^2}, \frac{\overline{M}^2}{M^2}, \gamma\right) = \frac{\mathcal{C} \cdot \Gamma(\delta - \gamma)}{\pi \Gamma(\delta)} \cdot \left(\frac{1}{x}\right)^{\chi\left(\gamma, \frac{\overline{M}^2}{M^2}\right)} \cdot$$
$$\left\{1 + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0\left(\gamma\right)}{8 N_c} \log\left(\frac{1}{x}\right) \left[- \psi\left(\delta - \gamma\right) + \log\frac{M^2}{Q_0^2} - \partial_\gamma \right] \right\},$$

resummed NLO BFKL eigenvalue with optimal scale setting ($\to$ modifies $\chi_1(\gamma)$):

$$\chi\left(\gamma, \frac{\overline{M}^2}{M^2}\right) = \bar{\alpha}_s \chi_0\left(\gamma\right) + \bar{\alpha}_s^2 \tilde{\chi}_1\left(\gamma\right) - \frac{1}{2} \bar{\alpha}_s^2 \chi_0'\left(\gamma\right) \chi_0\left(\gamma\right)$$
$$+ \chi_{RG}(\bar{\alpha}_s, \gamma, \tilde{a}, \tilde{b}) - \frac{\bar{\alpha}_s^2 \beta_0}{8 N_c} \chi_0(\gamma) \log\frac{\overline{M}^2}{M^2}.$$

Solve BFKL equation in conjugate (γ) Mellin space

$$\chi_{RG}(\bar{\alpha}_s, \gamma, a, b) = \bar{\alpha}_s(1 + a\bar{\alpha}_s)\left(\psi(\gamma) - \psi(\gamma - b\bar{\alpha}_s)\right) - \frac{\bar{\alpha}_s^2}{2}\psi''(1-\gamma) - \frac{b\bar{\alpha}_s^2 \cdot \pi^2}{\sin^2(\pi\gamma)}$$
$$+ \frac{1}{2}\sum_{m=0}^{\infty}\left(\gamma - 1 - m + b\bar{\alpha}_s - \frac{2\bar{\alpha}_s(1 + a\bar{\alpha}_s)}{1 - \gamma + m} + \sqrt{(\gamma - 1 - m + b\bar{\alpha}_s)^2 + 4\bar{\alpha}_s(1 + a\bar{\alpha}_s)}\right)$$

resums (anti-) collinear 'logs' (= γ-poles) of $\bar{\alpha}_s\chi_0(\gamma) + \bar{\alpha}_s\chi_1(\gamma) - \frac{1}{2}\bar{\alpha}_s^2\chi_0'(\gamma)\chi_0(\gamma)$
[Salam; hep-ph/9806482], [Sabio Vera; hep-ph/0505128]

optimal scale setting $\to$ γ-dependent running coupling

$$\bar{\alpha}_s\left(\overline{M} \cdot Q_0, \gamma\right) = \frac{4N_c}{\beta_0\left[\log\left(\frac{\overline{M}\cdot Q_0}{\Lambda^2}\right) + \frac{1}{2}\chi_0(\gamma) - \frac{5}{3} + 2\left(1 + \frac{2}{3}Y\right)\right]},$$

also use parametrization of running coupling in the infra-red [Webber; hep-ph/9805484]

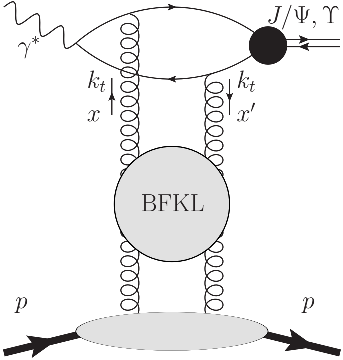
$$\alpha_s\left(\mu^2\right) = \frac{4\pi}{\beta_0 \ln\frac{\mu^2}{\Lambda^2}} + f\left(\frac{\mu^2}{\Lambda^2}\right), \quad f\left(\frac{\mu^2}{\Lambda^2}\right) = \frac{4\pi}{\beta_0}\frac{125\left(1 + 4\frac{\mu^2}{\Lambda^2}\right)}{\left(1 - \frac{\mu^2}{\Lambda^2}\right)\left(4 + \frac{\mu^2}{\Lambda^2}\right)^4},$$

The setup: diff. Xsec. at $t = 0$

a) imaginary part of scattering amplitude:

calculate diff. Xsec. at $t = 0$

- ✓ unintegrated gluon density from NLO BFKL fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]
- ✗ **impact factor** $\gamma \to J/\Psi, \Upsilon$ from light-front wave function used in dipole model studies ....
- ✗ ...



Vector mesons & dipole models ...

factorization into light-front wave function & dipole amplitude

e.g. [Kowalski, Motyka, Watt; hep-ph/0606272]

$$\Im\mathrm{m}\mathcal{A}_{T,L}^{\gamma^* p \to V p}(W, t=0) = 2\int d^2\boldsymbol{r}\int d^2\boldsymbol{b}\int_0^1 \frac{dz}{4\pi}\,(\Psi_V^*\Psi)_{T,L}\,\mathcal{N}(x,r,b)\,,$$

light-front wave function overlap

$$(\Psi_V^*\Psi)_T = \frac{\hat{e}_f e N_c}{\pi z(1-z)}\left\{m_f^2 K_0(m_f r)\phi_T(r,z) - \left[z^2 + (1-z)^2\right] m_f K_1(m_f r)\partial_r\phi_T(r,z)\right\}$$

scalar parts of VM wave function: boosted Gaussian wave-functions with Brodsky-Huang-Lepage prescription

$$\phi_T^{1s}(r,z) = \mathcal{N}_T z(1-z)\exp\left(-\frac{m_f^2\mathcal{R}_{1s}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}_{1s}^2} + \frac{m_f^2\mathcal{R}_{1s}^2}{2}\right).$$

free parameters fixed through normalization condition & leptonic decay width $\Gamma_{e^-e^+}$:

Meson	m_f/GeV	$\mathcal{N}_T$	$\mathcal{R}^2$/GeV^{-2}	M_V/GeV	$8\mathcal{R}^{-2}$/GeV^2	$\frac{1}{4}M_V^2$/GeV^2
J/ψ	$m_c = 1.27$	0.596	2.45	3.097	3.27	2.40
Υ	$m_b = 4.2$	0.481	0.57	9.460	15.38	22.42

use parameters obtained by [Armesto, Rezaeian; 1402.4831], [Goncalves, Moreira, Navarra; 1408.1344]

From wave functions to impact factors

BFKL study requires impact factor in γ space $\Phi_{V,T}(\gamma)$:

$$\Im\mathrm{m}\mathcal{A}_T^{\gamma^* p \to V p}(W, t=0) = 2\int d^2\boldsymbol{b}\int d^2\boldsymbol{r}\int_0^1 \frac{dz}{4\pi}\,(\Psi_V^*\Psi)_T(r)\;\cdot\sigma_0 N(x,r)$$

$$= \alpha_s(\overline{M}\cdot Q_0)\int\limits_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{d\gamma}{2\pi i}\int\limits_0^1\frac{dz}{4\pi}\;\hat{g}\left(x, \frac{M^2}{Q_0^2}, \frac{\overline{M}^2}{M^2}, Q_0, \gamma\right)\cdot\Phi_{V,T}(\gamma, z, M)\cdot\left(\frac{M^2}{Q_0^2}\right)^{\gamma}$$

can be derived using relation dipole $\leftrightarrow$ unintegrated gluon *e.g.* **[Kutak, Stasto; hep-ph/040811]**

$$2\int d^2\boldsymbol{b}\,\mathcal{N}(x,r,b) = \frac{4\pi}{N_c}\int\frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2}\left(1-e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\right)\alpha_s G(x,\boldsymbol{k}^2)\,.$$

yields

$$\Phi_{V,T}(\gamma,z,M) = e\hat{e}_f 8\pi^2\mathcal{N}_T\frac{\Gamma(\gamma)\Gamma(1-\gamma)}{m_f^2}\left(\frac{m_f^2\mathcal{R}^2}{8z(1-z)}\right)^2 e^{-\frac{m_f^2\mathcal{R}^2}{8z(1-z)}}e^{\frac{m_f\mathcal{R}^2}{2}}\left(\frac{8z(1-z)}{M^2\mathcal{R}^2}\right)^{\gamma}$$

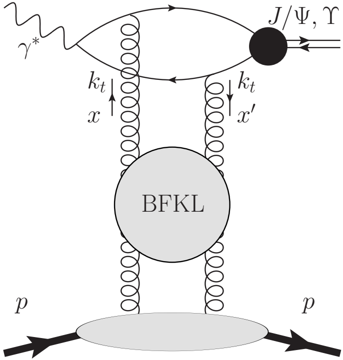
$$\left[U\left(2-\gamma,1,\frac{m_f^2\mathcal{R}^2}{8z(1-z)}\right)+[z^2+(1-z)^2]\frac{(2-\gamma)}{2}U\left(3-\gamma,2,\frac{m_f^2\mathcal{R}^2}{8z(1-z)}\right)\right],$$

[$U(a,b,z)$ hypergeometric function of the second kind or Kummer's function]

The setup: diff. Xsec. at $t = 0$

a) imaginary part of scattering amplitude:

- ✓ unintegrated gluon density from NLO BFKL fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]
- ✓ impact factor $\gamma \to J/\Psi, \Upsilon$ from light-front wave function used in dipole model studies ....



b) real part:

- ✗ $\Im\mathrm{m}\mathcal{A}(W^2, t)$ dominant, real part can be numerically large
  → recover **real part** using dispersion relation

- for $t = 0$ the (pseudo-)inclusive process fixes imaginary part of $\mathcal{A}(W^2, t = 0)$ of the exclusive process;
- Common approach: $\frac{\Re\text{e}\mathcal{A}(W^2,t)}{\Im\text{m}\mathcal{A}(W^2,t)} = \tan\frac{\lambda\pi}{2}$, with $\lambda = \frac{d\ln\mathcal{A}(W^2,t)}{d\ln W^2}$

  follows from analytic representation of scattering amplitudes of (scalar) particles in the Regge limit for positive signature
- Often: $\lambda =$ const. ➡ constant ratio of real & imaginary part

Real part from imaginary part

here: reconstruct real part through using representation in ω-Mellin space, conjugate to W^2:

$$\mathcal{A}(W^2, t) = \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} \left(i + \tan\frac{\omega\pi}{2}\right) a(\omega, t), \qquad x = \frac{M_V^2}{W^2 - m_p^2}$$

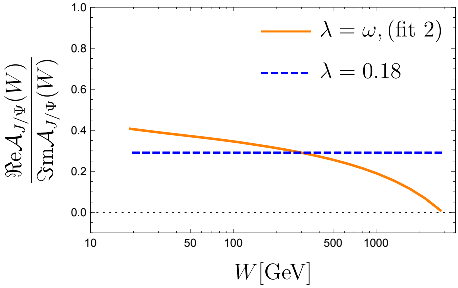
partial wave $a(\omega, t)$ can be fixed from imaginart part

$$a(\omega, 0) = \alpha_s \int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} \frac{d\gamma}{2\pi i} \left(\frac{M^2}{Q_0^2}\right)^{\gamma} \int_0^1 \frac{dz}{4\pi} \Phi_{V,T}(\gamma, z) \frac{\mathcal{C} \cdot \Gamma(\delta - \gamma)}{\pi \Gamma(\delta)} \cdot \left\{ \frac{1}{\omega - \chi\left(\gamma, \frac{\overline{M}^2}{M^2}\right)} + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma)/(8N_c)}{\left[\omega - \chi\left(\gamma, \frac{\overline{M}^2}{M^2}\right)\right]^2} \left[-\psi(\delta - \gamma) - \frac{d \ln[\Phi_{V,T}(\gamma, z)]}{d\gamma} \right] \right\}$$

yields *energy dependent* ratio of real & imaginary part

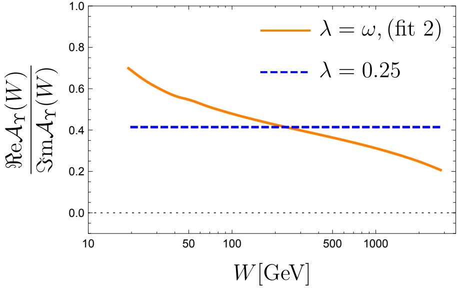
energy dependence of $r(W) = \Re\mathrm{e}\mathcal{A}/\Im\mathrm{m}\mathcal{A}$

J/Ψ:



ratio decreases with energy

Υ:



sizeable effect on overall energy dependence

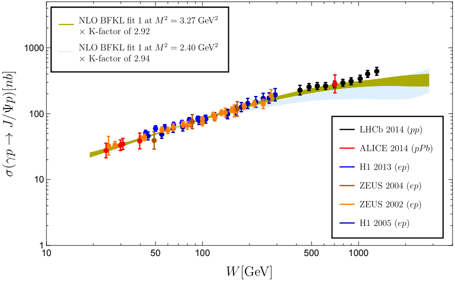
$\rightarrow$ slows down the growth

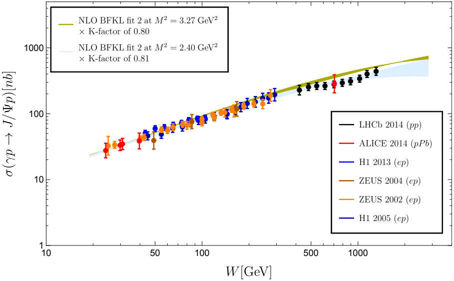
Comparison to data

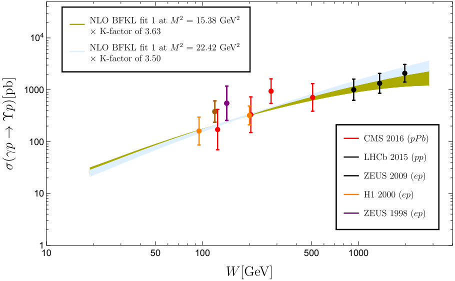
- provide results for both HERA fits (standard (fit 1) & kinematic improved (fit 2) LO impact factor)
- hard scale M^2:
  - photoproduction scale $M_{\text{pp}} = M_V/2$

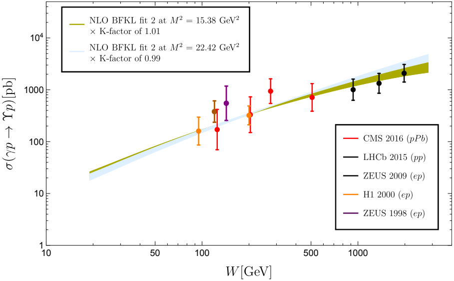
    $\left(M_{\text{pp}}^2\right)_{J/\Psi} = 2.40 \text{ GeV}^2$
    $\left(M_{\text{pp}}^2\right)_{\Upsilon} = 22.42 \text{ GeV}^2$
  - impact factor motivated: $M_{\text{if}}^2 = 8\mathcal{R}_V^{-2}$ – eliminates $(...)^\gamma$ factor & minimizes NLO running coupling correction related to impact factor

    $\left(M_{\text{if}}^2\right)_{J/\Psi} = 3.27 \text{ GeV}^2$
    $\left(M_{\text{if}}^2\right)_{J/\Psi} = 15.38 \text{ GeV}^2$
- (hard) running coupling scale $\overline{M} = M$, but vary in range $[M^2/2, M^2 \cdot 2]$ to check stability of result
- fix normalization by low energy ALICE (J/Ψ) and H1 (Υ) data point $\rightarrow$ K-factor


NLO BFKL fit 1 at $M^2 = 3.27$ GeV^2
$\times$ K-factor of 2.92
NLO BFKL fit 1 at $M^2 = 2.40$ GeV^2
$\times$ K-factor of 2.94
LHCb 2014 (pp)
ALICE 2014 (pPb)
H1 2013 (ep)
ZEUS 2004 (ep)
ZEUS 2002 (ep)
H1 2005 (ep)
$\sigma(\gamma p \to J/\Psi p)[nb]$
1
10
100
1000
10
50
100
500
1000
W[GeV]

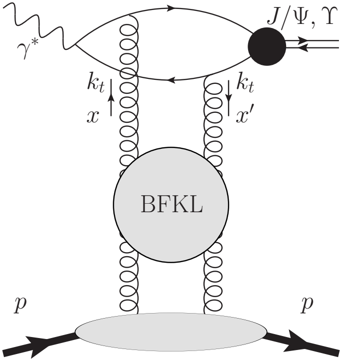

NLO BFKL fit 2 at $M^2 = 3.27$ GeV^2
$\times$ K-factor of 0.80
NLO BFKL fit 2 at $M^2 = 2.40$ GeV^2
$\times$ K-factor of 0.81
LHCb 2014 (pp)
ALICE 2014 (pPb)
H1 2013 (ep)
ZEUS 2004 (ep)
ZEUS 2002 (ep)
H1 2005 (ep)
$\sigma(\gamma p \to J/\Psi p)[nb]$
1
10
100
1000
10
50
100
500
1000
W[GeV]


NLO BFKL fit 1 at $M^2 = 15.38$ GeV^2
$\times$ K-factor of 3.63
NLO BFKL fit 1 at $M^2 = 22.42$ GeV^2
$\times$ K-factor of 3.50
CMS 2016 (pPb)
LHCb 2015 (pp)
ZEUS 2009 (ep)
H1 2000 (ep)
ZEUS 1998 (ep)
$\sigma(\gamma p \to \Upsilon p)$[pb]
1
10
100
1000
10^4
10
50
100
500
1000
W[GeV]


NLO BFKL fit 2 at $M^2 = 15.38$ GeV^2
$\times$ K-factor of 1.01
NLO BFKL fit 2 at $M^2 = 22.42$ GeV^2
$\times$ K-factor of 0.99
CMS 2016 (pPb)
LHCb 2015 (pp)
ZEUS 2009 (ep)
H1 2000 (ep)
ZEUS 1998 (ep)
$\sigma(\gamma p \to \Upsilon p)$[pb]
1
10
100
1000
10^4
10
50
100
500
1000
W[GeV]

Observations:

- K-factor: small for fit 2, sizeable for fit 1 – likely related to the impact factors in used in the HERA fit (massless, $n_f = 4$, $(\mathcal{C}_1/\mathcal{C}_2)^2 = 2.45$)
- common correction not included: GPD motivated factor to take into account $x' \neq x$; currently calculated for collinear pdf [Shuvaev, Golec-Biernat, Martin, Ryskin, hep-ph/9902410] ➡ to be calculated for k_T factorized BFKL impact factor



very good description of W-dependence

$W_{J/\Psi} > 471$ GeV & $W_\Upsilon > 669$ GeV $\equiv$ beyond region of incl. HERA fit (from $x = 4.3 \cdot 10^{-5}$ to $x = 3.5 \cdot 10^{-6}$) ➡ direct test of BFKL evolution

Caveats ....

- both BFKL HERA fit & VM photoproduction use LO impact factor
  → large corrections at NLO possible
- BFKL HERA fit for $n_f = 4$ mass-less quarks

both effects should affect the normalization, not so much W-dependence

- unintegrated gluon density can develop instability at ultra-small x:

$$G\left(x, \boldsymbol{k}^2, M\right) = \frac{1}{\boldsymbol{k}^2} \int\limits_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \; \hat{g}\left(x, \gamma\right) \; \left(\frac{\boldsymbol{k}^2}{Q_0^2}\right)^{\gamma}$$

$$\hat{g}(x,\gamma) \sim \left(\frac{1}{x}\right)^{\chi\left(\gamma, \frac{\overline{M}^2}{M^2}\right)} \cdot \left\{1 + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0\left(\gamma\right)}{8N_c} \log\left(\frac{1}{x}\right)\left[-\psi\left(\delta-\gamma\right) + \log\frac{M^2}{Q_0^2} - \partial_\gamma\right]\right\},$$

- will enter at some point region $\alpha_s^2 \ln(1/x) \sim 1$ → control of such terms will become necessary

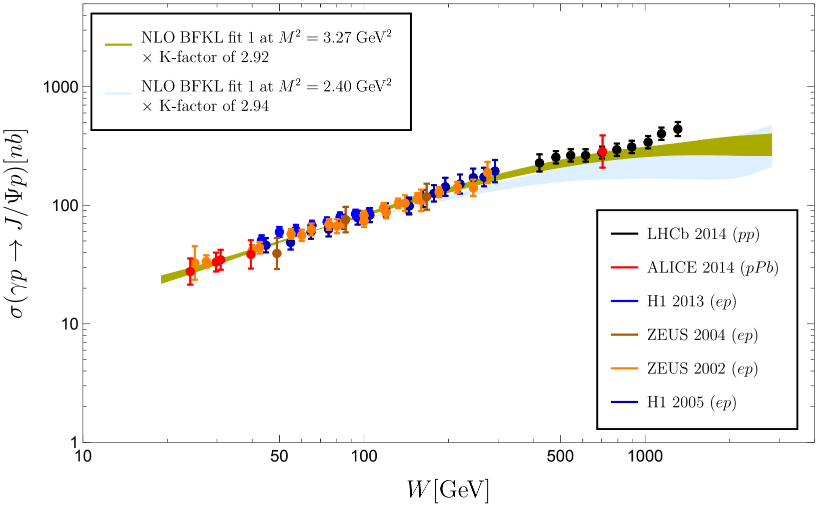
Conclusion

- maybe there is work to be left done, until we can claim that exclusive vector meson production provides a definite proof of BFKL evolution in the conformal spinn $n = 0$ sector

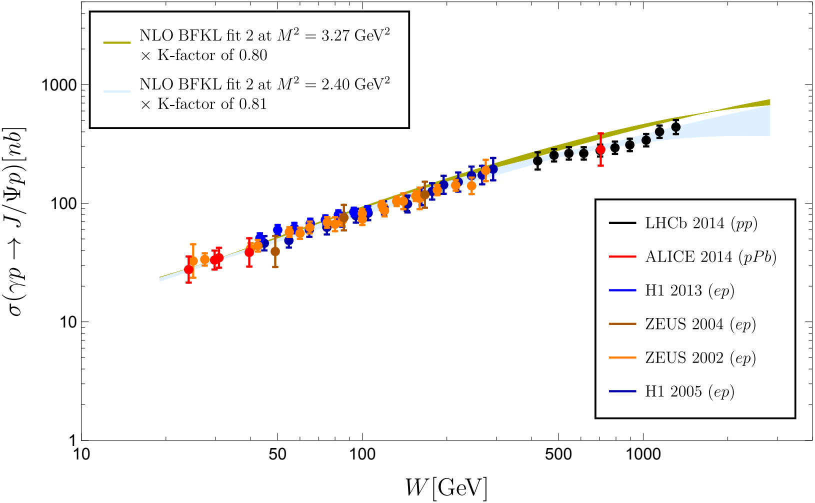
for the moment:

- most simple combination of existing NLO BFKL fit & existing VM impact factor gives amazingly good description of W-dependence of the exclusive VM photo-production cross-section, evolving the unintegrated gluon one order of magnitude lower in x, than directly constrained by HERA

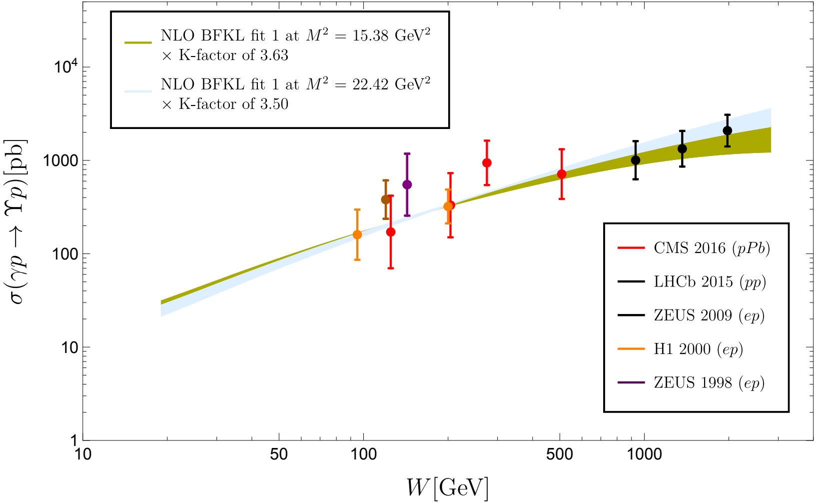
Supplementary material

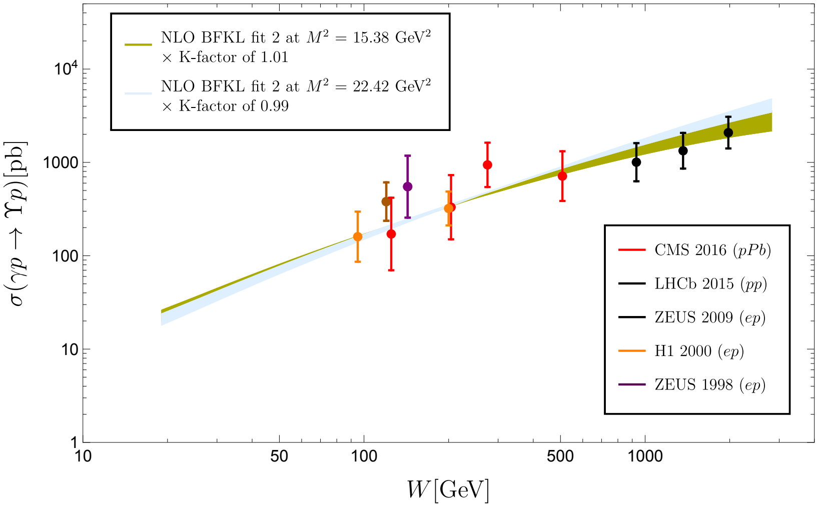


NLO BFKL fit 1 at $M^2 = 3.27$ GeV^2
$\times$ K-factor of 2.92
NLO BFKL fit 1 at $M^2 = 2.40$ GeV^2
$\times$ K-factor of 2.94
1000
100
10
1
$\sigma(\gamma p \to J/\Psi p)[nb]$
LHCb 2014 (pp)
ALICE 2014 (pPb)
H1 2013 (ep)
ZEUS 2004 (ep)
ZEUS 2002 (ep)
H1 2005 (ep)
10
50
100
500
1000
$W\,[\mathrm{GeV}]$



NLO BFKL fit 2 at $M^2 = 3.27$ GeV^2 $\times$ K-factor of 0.80
NLO BFKL fit 2 at $M^2 = 2.40$ GeV^2 $\times$ K-factor of 0.81
LHCb 2014 (pp)
ALICE 2014 (pPb)
H1 2013 (ep)
ZEUS 2004 (ep)
ZEUS 2002 (ep)
H1 2005 (ep)
$\sigma(\gamma p \to J/\Psi p)[nb]$
W [GeV]
1000
100
10
1
10
50
100
500
1000


NLO BFKL fit 1 at $M^2 = 15.38$ GeV^2 $\times$ K-factor of 3.63
NLO BFKL fit 1 at $M^2 = 22.42$ GeV^2 $\times$ K-factor of 3.50
CMS 2016 (pPb)
LHCb 2015 (pp)
ZEUS 2009 (ep)
H1 2000 (ep)
ZEUS 1998 (ep)
$\sigma(\gamma p \to \Upsilon p)$[pb]
W[GeV]
10^4
1000
100
10
1
10
50
100
500
1000


NLO BFKL fit 2 at $M^2 = 15.38$ GeV^2
$\times$ K-factor of 1.01
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CMS 2016 (pPb)
LHCb 2015 (pp)
ZEUS 2009 (ep)
H1 2000 (ep)
ZEUS 1998 (ep)
10^4
1000
100
10
1
10
50
100
500
1000
$\sigma(\gamma p \to \Upsilon p)$[pb]
W [GeV]