Diffraction-2016, Acireale, Italy, 3 September 2016

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Novel approach to non-perturbative inputs for parton distributions in QCD factorization

talk based on results obtained in collaboration with
M. Greco and S.I. Troyan

Need for QCD factorization:

Description of hadronic reactions involves QCD calculations at both high and low energies. However, QCD is poorly known at low energies; the confinement problem has not been solved, so approximation methods are needed to mimic the straightforward QCD calculations at low energies. QCD factorization is the most popular approximation method.

Essence of QCD factorization:

Step 1: non-perturbative inputs are introduced through either models or fits.

Step 2: the inputs are evolved with perturbative means (evolution equations).

Non-perturbative inputs for parton distributions in hadrons are introduced through the models and fits. Alternatively, there are lattice calculations

Models:

Dmitri Diakonov, V. Petrov, P. Pobylitsa, Maxim V. Polyakov; H. Avakian, A.V. Efremov, P. Schweitzer, F. Yuan;Ivan Vitev, Leonard Gamberg, Zhongbo Kang, Hongxi Xing; Asmita Mukherjee, Sreeraj Nair, Vikash Kumar Ojha;

K. Golec-Biernat, M. Wustoff; H. Jung;
A.V. Lipatov, G.I. Lykasov, A.A. Grinyuk, N.P. Zotov;
Jon Pumplin;

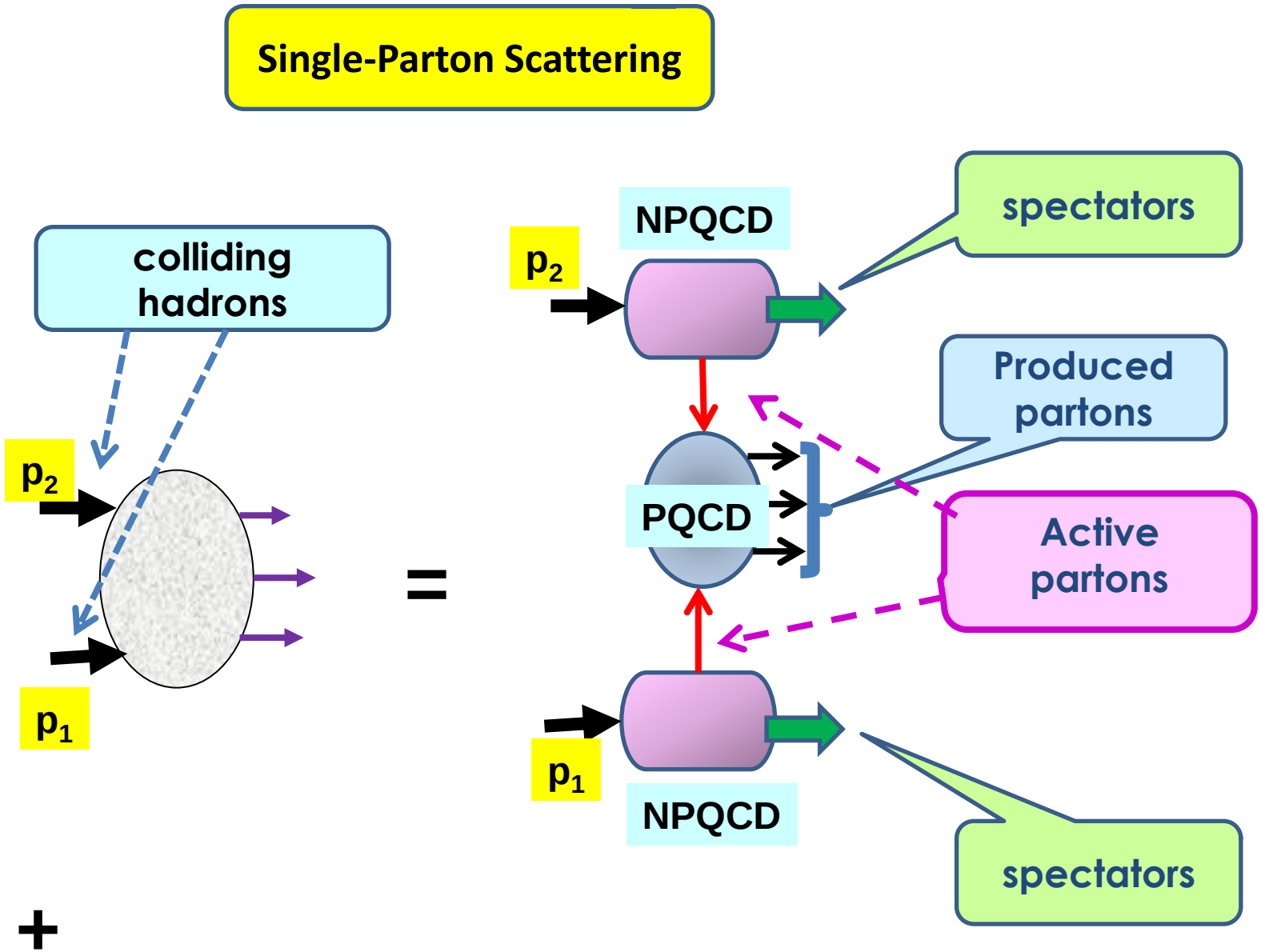
Fits:
G. Altarelli, R. Ball, S. Forte, G. Ridolfi; E. Leader,
A.V. Sidorov, D.B. Stamenov;
J. Blumlen, H. Botcher; M. Hirai

Most actively used in the context of QCD factorization

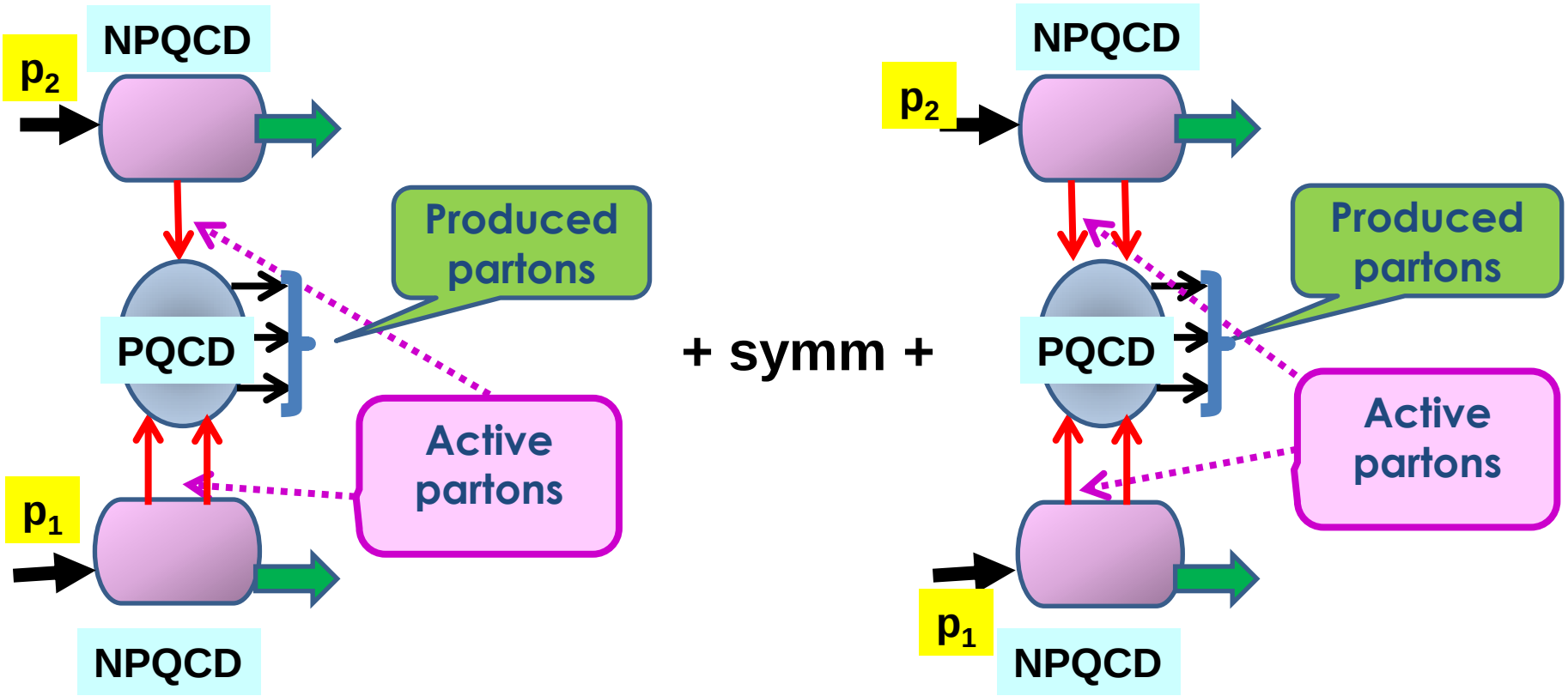
Recent Lattice Calculations:
Yan-Quing Ma, Jian-Wei Qui; Marta Constantinou

I apologize if I have overlooked some name(s) and willingly accept corrections

Scenarios of hadronic collisions at high energies



Double-Parton Scattering



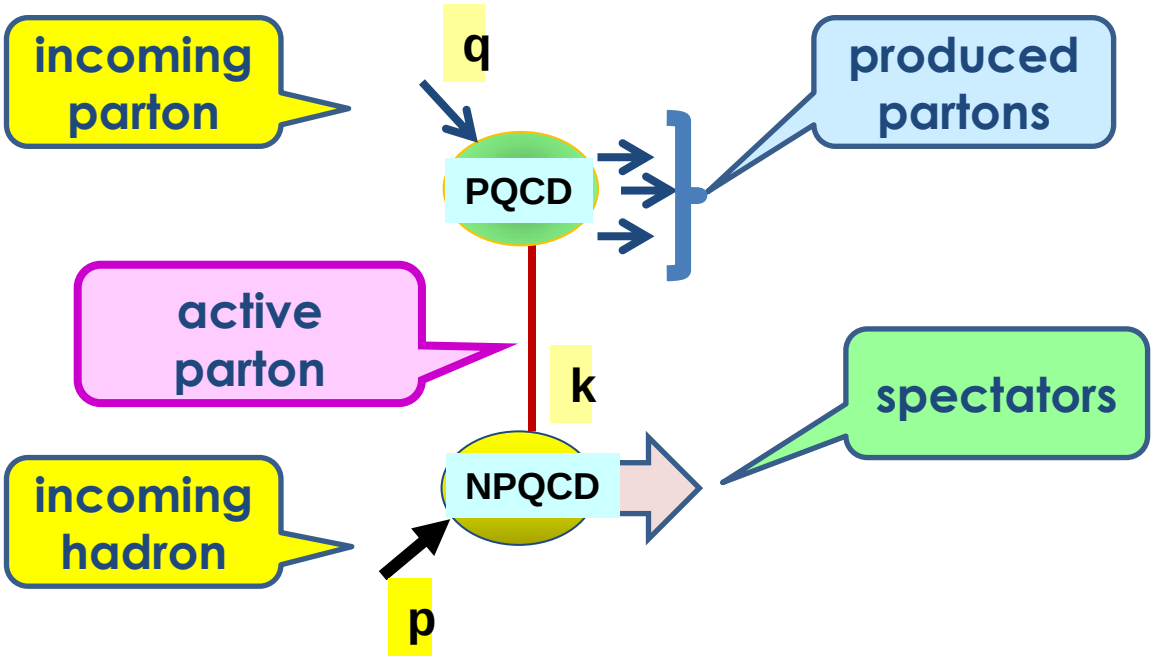
+ contributions of more complicated Multi-Parton states

As for Multi-Parton Scattering, see: Gaunt-Maciula-Szczurek,
Snigirev-Baranov-A.V. Lipatov-M.A. Malyshev

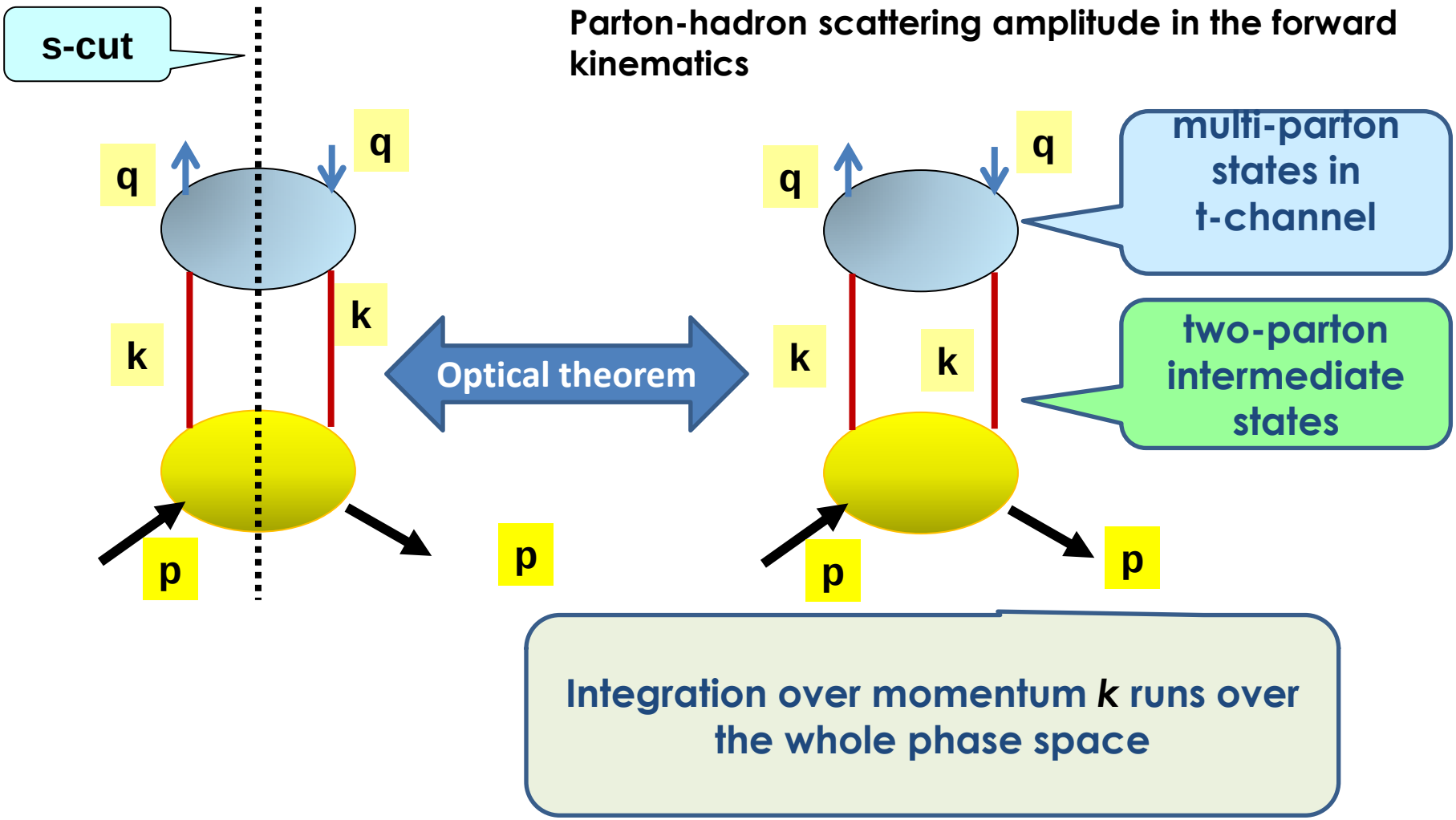
Single-Parton Scenario is much more popular than Multi-Parton one, so in the present talk I will focus on **SINGLE-PARTON SCATTERING**

Generalization to Multi-Parton Scattering is easy to do

Single-Parton Scenario for the parton-hadron scattering



Squaring the amplitude, we arrive at the parton distribution



The kinds of QCD factorization available in the literature:

Collinear Factorization

Amati-Petronzio-Veneziano, Efremov-Ginzburg-Radyushkin, Libby-Sterman, Brodsky-Lepage, Collins-Soper-Sterman

K_T- Factorization/High-Energy Factorization

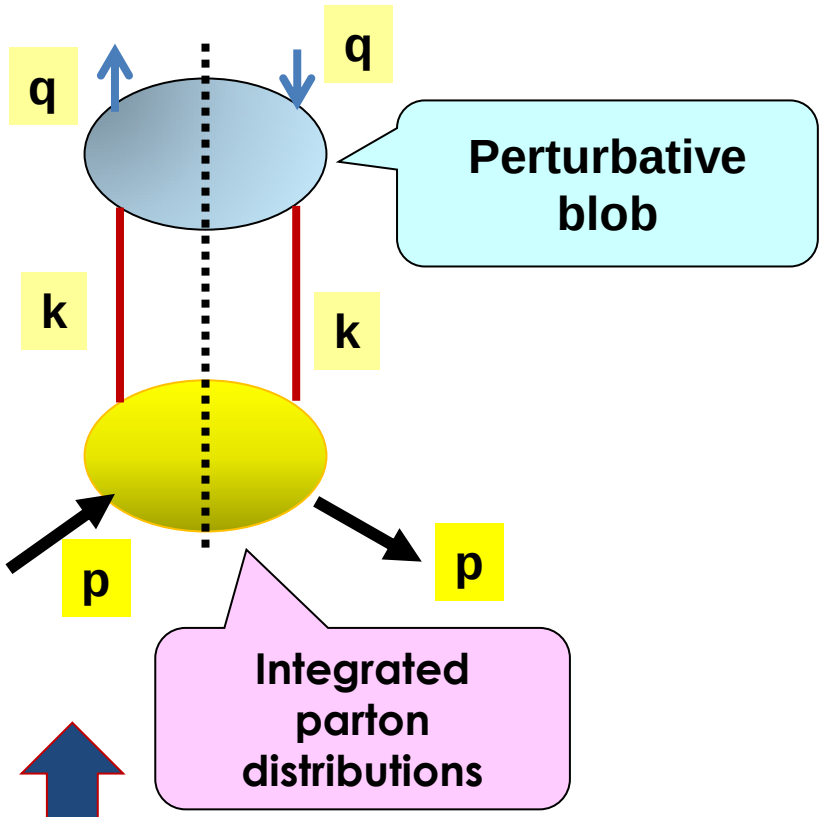
S. Catani - M. Ciafaloni – F. Hautmann;
J.C. Collins- R.K. Ellis

These two conventional forms of factorization were introduced from different considerations and are used for different perturbative approaches

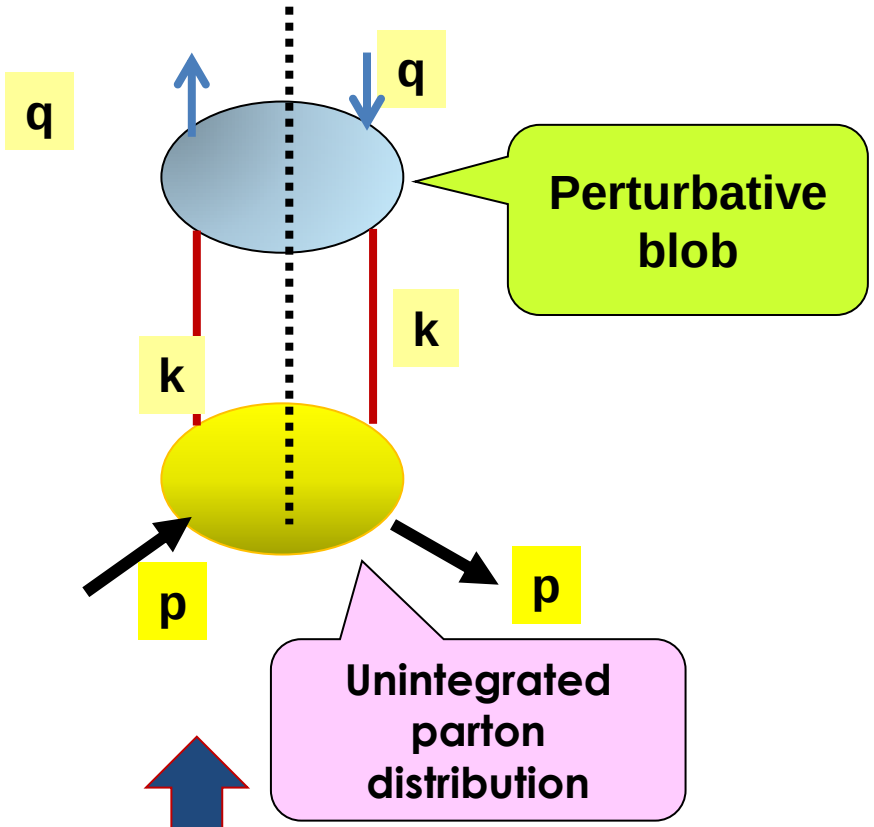
Recently we suggested a new, more general kind of factorization:
Basic Factorization
We showed how to reduce it step-by-step to K_T and Collinear Factorizations, keeping the non-perturbative inputs in a general form

Conventional illustrations of Factorizations

Collinear Factorization



K_T- factorization



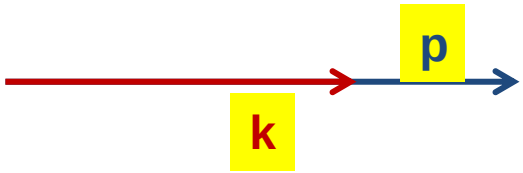
Pictures look identically but formulae differ

NB Standard Feynman diagram technique cannot be applied to these graphs

Different Factorizations imply different parameterizations of momenta of the connecting partons

Collinear Factorization

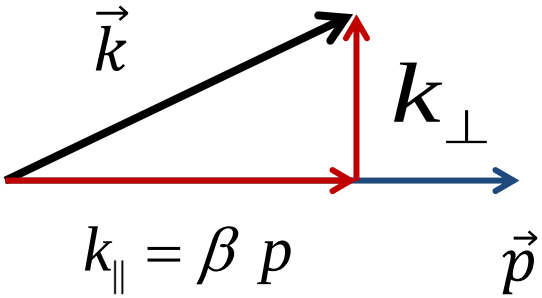
$$\vec{k} = \beta\ \vec{p} \qquad (0 < \beta < 1)$$



momentum fraction

K_T -Factorization

$$\vec{k} = \beta\ \vec{p} + \vec{k}_\perp$$



Factorization representation for parton distributions

Collinear Factorization

$$\vec{k} = \beta\, \vec{p}$$

factorization scale

$$D_{col}(x,q^2) = \int_x^1 \frac{d\beta}{\beta} D_{col}^{(pert)}(x/\beta, q^2/\mu^2)\, \varphi\,(\beta,\mu^2)$$

integrated parton distribution

Perturbative terms are calculated with evolution equations

Parton distributions found from phenomenological considerations

K_T-Factorization

$$\vec{k} = \beta\, \vec{p} + \vec{k}_\perp$$

$$D_{KT}(x,q^2) = \int \frac{d\beta}{\beta} \frac{dk_\perp^2}{k_\perp^2} D_{KT}^{(pert)}(x/\beta, q^2/k_\perp^2)\, \Phi(\beta, k_\perp^2)$$

Unintegrated parton distribution

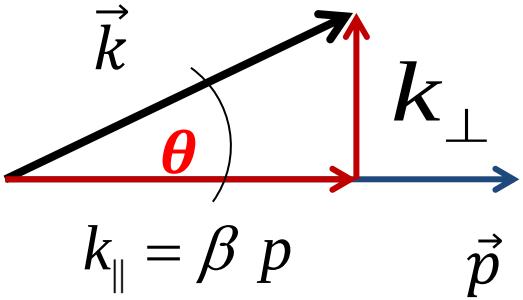
Actual situation is more involved: $k = [k_0, k_x, k_y, k_z]$
and all four components of k should be accounted for

For instance, all of them are represented by Sudakov parametrization

$$k = \alpha\, q + \beta\, p + k_\perp$$ so that

$$d^4k = d^2k_{||}\, d^2k_\perp = (s/2)\, d\alpha\, d\beta\, d^2k_\perp \approx \pi s\, d\alpha\, d\beta\, k_\perp\, dk_\perp$$

Kinematical contents of α and β



$\theta \ll 1$

$$\alpha \approx \frac{\omega}{4E}\theta^2 \qquad \beta \approx \frac{\omega}{E}$$

$\alpha << \beta$

parton energy

hadron energy

by this reason the α -dependence is often neglected compared to the β -dependence

When α -dependence is taken into account we arrive at Basic Factorization

perturbative part

non-perturbative input

new integration

$$w = 2pq$$

$$A(S_q, S_h, w, q^2) = \int \frac{d\beta}{\beta}\, dk^2{}_\perp\, d\alpha\, A^{(pert)}(S_q, w\beta, q^2, k^2) \left(\frac{k^2{}_\perp}{k^2 k^2}\right) T\,(S_h, w\alpha, k^2)$$

parton spin

hadron spin

Step-by-step reduction of Basic Factorization to other forms of factorization:

Integration over α → K_T -Factorization

↓

Integration over $k^2{}_\perp$ → Collinear Factorization

Integration over momentum k covers the whole phase space. Surely, the integration must yield a finite result. However, the integrand has singularities

HANDLING THE SINGULARITIES

singularities can be divided into Groups A and B

Group A: IR and UV singularities of the perturbative amplitude $A^{(pert)}$ originated by Feynman graphs contributions. There are IR and UV singularities. They can easily be regulated

IR singularities are regulated by k^2 and therefore $A^{(pert)}$ is IR stable as long as k^2 is not equal to zero .

UV singularities in Pert QCD are known to be absorbed by redefinitions of the couplings and masses.

Group B: When such non-singular $A^{(pert)}$ are substituted into the factorization convolution, the problem of IR and UV singularities emerges once again:

$$A(S_q, S_h, w, q^2) = \int \frac{d\beta}{\beta}\, dk^2 \quad d\alpha\, A^{(pert)}(S_q, w\beta, q^2, k^2) \left(\frac{k^2{}_\perp}{k^2 k^2}\right) T\,(S_h, w\alpha, k^2)$$

Integration over k^2 runs through the point $k^2 = 0$ and there is no reason to introduce any IR cut-off

Integration over α may yield a divergence at large $|\alpha|$

WAY OUT: input T should kill both IR and UV divergences in order to ensure IR and UV stability of the factorization convolutions

So, requirement of integrability of factorization convolutions leads to theoretical constraints on candidates for non-perturbative inputs T

IR stability: $T \sim \left(k^2\right)^{1+\eta}$ at small k^2, with $\eta > 0$

UV stability $T \sim \alpha^{-\kappa}$ at large $|\alpha|$, with $\kappa > 0$

Any model for input T for the parton-hadron scattering amplitudes in Basic Factorization must satisfy the following requirements:

(i) Input T should respect the IR and UV stability restrictions

(ii) It should have non-zero imaginary part in the s-channel in order to apply the Optical theorem

Besides, a model should ensure the step-by-step reductions of Basic Factorization to simpler forms of factorization.
In particular, the input in K_T – Factorization should have a sharp-peaked form . This ensures reducing to Collinear Factorization

These constraints can be regarded as criteria for constructing/selecting models for non-perturbative inputs

First of all, we fix the spinor part of the input for quark-hadron amplitudes

$$\hat{T} = (\hat{p} + m_h)\, T_U - (\hat{p} + m_h)\, \gamma_5 \hat{S}\, T_S$$

Hadron mass

Hadron spin

Invariant amplitude for polarized hadron

Invariant amplitude for unpolarized hadron

Such a representation obeys Conformity: When the hadron is replaced by an elementary fermion, $\hat{T}$ is replaced by the fermion density matrix

For gluon-hadron amplitudes , we choose the inputs in the following form :

$$T_{\lambda\rho} = \left(2p_\lambda p_\varrho - k_\lambda p_\varrho - pk\, g_{\lambda\rho}\right) T_U + i\, m_h \epsilon_{\lambda\varrho\tau\sigma} k_\tau S_\sigma\, T_S$$

Invariant unpolarized amplitude

Hadron spin

Invariant amplitude for polarized hadron

All such invariant amplitudes are scalars

$$T_{U,S} = T_{U,S}\left(s_1, k^2\right)$$

Invariant sub-energy

parton virtuality

$$s_1 = (p-k)^2 = w\alpha + k^2 + M^2$$

In order to fix $T_{U,S}$ we use RESONANCE MODEL

In what follows we skip the subscripts U,S

MOTIVATION FOR THE RESONANCE MODEL

After emitting the active parton(s) off the hadron, the ensemble of remaining partons becomes unstable, so it can be described in terms of resonances.

$$T = R(k^2) Z_n(s_1) \qquad Z_n(s_1) = \prod_{r=1}^{r=n} \frac{1}{(s_1 - m_r{}^2 + i\,\Gamma_r)}$$

$$s_1 = (p-k)^2 = w\alpha + k^2 + m_h{}^2$$

IR stability excludes *n=1* → *n =2,3,…*

We consider the simplest option *n=2* and rewrite Z_2 as interference of two resonances:

$$T = \tilde{R}(k^2) \left[\frac{1}{s_1 - m_1{}^2 + i\Gamma_1} - \frac{1}{s_1 - m_2{}^2 + i\Gamma_2} \right]$$

Transition from Basic factorization to K_T- factorization leads to

$$T_{KT} = R\left(k_\perp{}^2\right)\left[T_R + T_B\right] \qquad \mu_{1,2}{}^2 = \left(-m_{1,2}{}^2 + m_h{}^2\right)/\xi$$

$$T_R = \frac{1}{k_\perp{}^2/\beta - \mu_1{}^2 + i\Gamma_1} + \frac{1}{k_\perp{}^2/\beta - \mu_2{}^2 + i\Gamma_2}$$

Maximums are within the integration region

$$T_B = \frac{1}{k_\perp{}^2/\beta + \mu_1{}^2 + i\Gamma_1} + \frac{1}{k_\perp{}^2/\beta + \mu_2{}^2 + i\Gamma_2}$$

Maximums are outside the integration region, so T_B can be interpreted as background

Applying the Optical theorem, we arrive at the input for parton distributions:

$$D_{KT} = R\left(k_\perp{}^2\right)\left[D_R + D_B\right]$$

Breit-Wigner contribution

$$D_R = \frac{1}{\left(k_\perp{}^2/\beta - \mu_1{}^2\right)^2 + \Gamma_1{}^2} + \frac{1}{\left(k_\perp{}^2/\beta - \mu_2{}^2\right)^2 + \Gamma_2{}^2}$$

$$D_B = \frac{1}{\left(k_\perp{}^2/\beta + \mu_1{}^2\right)^2 + \Gamma_1{}^2} + \frac{1}{\left(k_\perp{}^2/\beta + \mu_2{}^2\right)^2 + \Gamma_2{}^2}$$

Background contribution

Specifying the factor *R*.

The only rigorous requirement on *R*: the IR stability requires that

at small $k_\perp^2$

$$R(k_\perp^2) \sim (k_\perp^2)^\eta$$

In many papers *R* is chosen
in the exponential/Gaussian form: For example

$$R(k_\perp^2) = R_1(k_\perp^2) \equiv e^{-\lambda k_\perp^2}$$

Suppressed by the IR stability requirement

K. Golec-Biernat, M. Wustoff;
Jon Pumplin

$$R(k_\perp^2) = R_2(k_\perp^2) \equiv (k_\perp^2)^\eta \, e^{-\lambda k_\perp^2}$$

H. Jung;
A.V. Lipatov, G.I. Lykasov, A.A. Grinyuk, N.P. Zotov

Agrees with the IR stability

When the exponential form is chosen $R\left(k_\perp{}^2\right) = N\left(k_\perp{}^2\right)^\eta e^{-\lambda k_\perp{}^2}$

Inputs with single resonance are allowed ⟺ Minimal Resonance Model

$$D_{KT} = R\left(k_\perp{}^2\right)\left[D_R + D_B\right]$$

with

$$D_R = \frac{\Gamma_1}{\left(k_\perp{}^2/\beta - \mu_1{}^2\right)^2 + \Gamma_1{}^2}$$

$$D_B = \frac{\Gamma_1}{\left(k_\perp{}^2/\beta + \mu_1{}^2\right)^2 + \Gamma_1{}^2}$$

Resonance contribution

Background contribution

Looks similar to the Duality scenario

Inputs in Collinear Factorization

$$D^{(col)}(w\beta,\mu^2) = \int \frac{d\,k_\perp{}^2}{k_\perp{}^2}\, D_{KT}\left(w\beta,\, k_\perp{}^2\mu^2\right)$$

$$= N\, e^{-\lambda\,\mu^2\beta}\,[1 + Y_B] \approx \widetilde{N}(1 - \lambda\,\mu^2\beta)$$

without singular factors β^{-a}

$$Y_B = \int \frac{d\,k_\perp{}^2}{k_\perp{}^2}\,\frac{\Gamma_1}{\left(k_\perp{}^2/\beta + \mu_1{}^2\right)^2 + \Gamma_1{}^2}$$

Comparison to standard fits:

$$\delta q, \delta g = N\ \ \beta^{-a}\ \ (1-\beta)^b\ \ (1+c\,\beta^d)$$

Altarelli-Ball-Forte-Ridolfi, Blumlein-Botcher, Leader-Sidorov-Stamenov, Hirai et al

Mimics resummation of $ln^n(1/x)$ and **dropped** when they are accounted for

Mimics resummation of $ln^n(1\text{-}x)$ and can be dropped when they are accounted for

At high energies the most essential contributions come from $\beta \ll 1$ and therefore $D^{(col)} \approx \widetilde{N}$

Hence, the β-dependence in the fits come from perturbative contributions

CONCLUSIONS

We obtained the most general kind of QCD factorization.
We call it Basic Factorization
Basic Factorization can be reduced first to K_T- and then to Collinear Factorizations

Imposing the requirements of IR and UV stability on the convolutions in Basic Factorization allowed us to impose general restrictions on the non-perturbative inputs for parton distributions, without specifying the inputs

Motivated by the simple observation that the ensemble of quarks and gluons in a hadron becomes unstable after the hadron emits one or several active partons and therefore it can be described through resonances, we suggested a model for non-perturbative inputs to the factorization convolutions
We call it Resonance Model. We have constructed it for Single-Parton Scattering but a generalization on Multi-Parton Scattering is easy to obtain.
This model can universally describe the inputs to parton-hadron amplitudes, parton distributions, DIS structure functions, etc., and can universally be used for the polarized and non-polarized hadrons