Computation of NLO processes involving heavy quarks using Loop-Tree Duality

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Based on arXiv:1608.01584
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Motivation

❖ LHC Run II: higher energies and luminosity
  • More data & statistics
  • Higher precision measurements

❖ Theory has to keep up!
  • Higher orders in perturbation theories
  • Faster (numerical) computations
Introduction

Higher order computational issues

Loop level amplitudes

Ill-defined integrals, in the UV (high-energy)

and in the IR (soft-collinear)

Singularities

Many existing methods to deal with that: DREG, FDH... and LTD
The Loop-Tree Duality

How it works

- Write the Feynman integral

\[ L^{(1)}(p_1, p_2, \ldots, p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i), \quad G_F = \frac{1}{q_i^2 - m_i^2 + i0}, \quad q_i = \ell + p_1 + \ldots + p_i \]

- Apply Cauchy’s residue theorem

\[ L^{(1)}(p_1, p_2, \ldots, p_N) = -2\pi i \int_q \sum_i \text{Res}\{\text{Im } q_0 < 0\} \prod_{i=1}^{N} G_F(q_i) \]

- Compute the residues, and get a sum with N contributions

\[ L^{(1)}(p_1, p_2, \ldots, p_N) = -\int_\ell \sum_{i=1}^{N} \tilde{\delta}(q_i) \prod_{j=1}^{N} G_D(q_i; q_j), \quad G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0 \eta(q_j - q_i)} \]

Similar to Feynman Tree Theorem, but one gets only 1-cuts (with modified prescriptions)
The Loop-Tree Duality

How it works

The Feynman integral becomes a sum over $N$ 1-cuts
The Loop-Tree Duality

A basic example: the scalar three-point function

Momenta parametrisation:

\[ p_1^\mu = \frac{\sqrt{s_{12}}}{2} (1, 0, 0, \beta) \]
\[ p_2^\mu = \frac{\sqrt{s_{12}}}{2} (1, 0, 0, -\beta) \]
\[ q_i^\mu = \frac{\sqrt{s_{12}}}{2} (\xi_{i,0}, \xi_i \sin(\theta)e_\perp, \xi_i \cos(\theta)) \]

\[ m = \frac{2M}{\sqrt{s_{12}}} \]
\[ \beta = \sqrt{1 - m^2} \]
\[ \xi_{i,0} = \sqrt{\xi_i^2 + m_i^2} \]

Three contributions:

\[ L^{(1)}(p_1, p_2, -p_3) = \int \prod_{i=3}^{N} G_F(q_i) = \sum_{i=1}^{3} I_i \]
The Loop-Tree Duality

A basic example: the scalar three-point function

\[
I_1 = \frac{4}{s_{12}} \int \frac{\xi_{1,0}^{-1} d[\xi_{1,0}]d[v_1]}{1 - (1 - 2v_1)^2/\beta^2}
\]

\[
I_2 = \frac{2}{s_{12}} \int \frac{\xi_{2,0}^{-1} \xi_2^2 d[\xi_2]d[v_2]}{(1 - \xi_{2,0} + \nu)(\xi_{2,0} + \beta \xi_2 (1 - 2v_2) - m^2)}
\]

\[
I_3 = -\frac{2}{s_{12}} \int \frac{\xi_{3,0}^{-1} \xi_3^2 d[\xi_3]d[v_3]}{(1 + \xi_{3,0})(\xi_{3,0} - \beta \xi_3 (1 - 2v_3) + m^2)}
\]

These integrals cannot be computed in d=4 dimensions
(Only possible in d=4-2\epsilon)

One has to get rid of singularities (IR and UV)
Local cancellation of singularities

Global vs local

✧ Within DREG: cancellation after integration:

\[
\text{Virtual } \left( \frac{1}{\epsilon} + \mathcal{O}(1) \right) + \text{Real } \left( -\frac{1}{\epsilon} + \mathcal{O}(1) \right)
\]

✧ Within LTD: cancellation locally, before integration, so you can integrate in 4 dimensions
Local cancellation of singularities

Counter-term (UV)

Define suitable integrand level UV counter-terms:

- Correct integrated forms (in d dimensions) compared with DREG
- Local cancellation of UV singularities

You get the correction terms:

\[
\Delta Z_2(p_1) = -g_S^2 C_F \int_{\ell} G_F(q_1) G_F(q_3) \left( (d - 2) \frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 4M^2 \left( 1 - \frac{q_1 \cdot p_2}{p_1 \cdot p_2} \right) G_F(q_3) \right)
\]

\[
\Delta Z_M^{OS}(p_1) = -g_S^2 C_F \int_{\ell} G_F(q_1) G_F(q_3) \left( (d - 2) \frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 2 \right)
\]
Local cancellation of singularities

**Counter-term (UV)**

1. Expand the numerators and the propagators around a UV propagator

\[ G_F(q_i) = \frac{1}{q_{UV}^2 - \mu_{UV}^2 + i0} + \ldots \quad q_{UV} = \ell + k_{UV} \]

2. Choose a scheme (MSbar scheme for example) and adjust subleading terms to subtract only the pole

3. Subtract the counter-term that will locally cancel the singularity, using the dual variables \((\xi_{UV}, \nu_{UV})\)

\[
\Delta Z_{2,UV}^{\xi} = -(d - 2)g_S^2 C_F \int d[\xi_{UV}] d[\nu_{UV}] \frac{2\xi_{UV}}{\xi_{UV,0}^3} \left[ \left( 1 + \frac{\beta_{UV}(1 - 2\nu_{UV})}{2(1 + \beta^2)} \right) \times \left( 1 - \frac{3(2\beta_{UV}(1 - 2\nu_{UV}) - m_{UV}^2)}{4\xi_{UV,0}^2} \right) \right] - \frac{1}{2(1 + \beta^2)} \\
\Delta Z_{OS,UV}^{\xi} = -g_S^2 C_F \int d[\xi_{UV}] d[\nu_{UV}] \frac{2\xi_{UV}}{\xi_{UV,0}^3} \left[ \left( d + (d - 2)\frac{\beta_{UV}(1 - 2\nu_{UV})}{2(1 + \beta^2)} \right) \times \left( 1 - \frac{3(2\beta_{UV}(1 - 2\nu_{UV}) - 2d^{-1}m_{UV}^2)}{4\xi_{UV,0}^2} \right) \right] - \frac{d - 2}{2(1 + \beta^2)}
\]
Local cancellation of singularities

Unsubtraction algorithm (IR)

Three IR singularities:

- $\xi_{1,0} = 0$  
- $\xi_{1,0} < 1$  
- $\xi_{2,0} < 1$

- $v_1 = 0$
- $v_2 = 1$

Soft  
Quasi-Collinear

Quasi-Collinear

Cancelled by the real contributions:
Local cancellation of singularities

Phase-space partition

Before doing anything, you want to isolate each singularity (Avoid having two soft or two quasi-collinear singularities in the same region)

Two regions:

• Region 1: Soft + quasi-collinear singularities
  → First LTD contribution’s singularities will be canceled

• Region 2: Quasi-collinear singularity
  → Second LTD contribution’s singularity will be canceled

But in order to do that, you need to find a mapping so you can add real and virtual amplitudes at integrand level (with the variables \((\xi_i, v_i)\))
The real-virtual mapping

**Region 1**

Using QCD factorisation properties, we define the mapping:

\[ p'_r \mu = q_1^{\mu} \]
\[ p'_1 \mu = (1 - \alpha_1)\hat{p}_1^{\mu} + (1 - \gamma_1)\hat{p}_2^{\mu} - q_1^{\mu} \]
\[ p'_2 \mu = \alpha_1\hat{p}_1^{\mu} + \gamma_1\hat{p}_2^{\mu} \]

with \( p_1 = \frac{1+\beta}{2}\hat{p}_1^{\mu} + \frac{1-\beta}{2}\hat{p}_2^{\mu} \) and \( p_2 = \frac{1-\beta}{2}\hat{p}_1^{\mu} + \frac{1+\beta}{2}\hat{p}_2^{\mu} \)

Then one has to solve:

\[ (p'_1)^2 = M^2 \]
\[ (p'_2)^2 = M^2 \]

**Region 2**

\[ p'_r \mu = (1 - \gamma_2)\hat{p}_1^{\mu} + (1 - \alpha_2)\hat{p}_2^{\mu} - q_2^{\mu} \]
\[ p'_1 \mu = \gamma_2\hat{p}_1^{\mu} + \alpha_2\hat{p}_2^{\mu} \]
\[ p'_2 \mu = q_2^{\mu} \]

\[ (p'_1)^2 = M^2 \]
\[ (p'_r)^2 = 0 \]
The real-virtual mapping

Adding real+virtual

Then we get the real contribution decay rate in each region:

\[
\tilde{\Gamma}^{(1)}_{R,1} = \Gamma^{(0)} \frac{2a}{\beta} \int d\xi_{1,0} dv_1 \frac{\mathcal{R}_1(\xi_{1,0}, v_1) \mathcal{J}_1(\xi_{1,0}, v_1)(1 - \xi_{1,0}(1 - v_1))^2}{\xi_{1,0}^2 (v_1 + \alpha_1 (1 - 2v_1))(1 - v_1)(1 - \xi_{1,0}) - \alpha_1 (1 - 2v_1))}
\]

\[
\tilde{\Gamma}^{(1)}_{R,2} = \Gamma^{(0)} \frac{2a}{\beta} \int d\xi_{2} dv_2 \frac{\mathcal{R}_2(\xi_{2}, v_2) \mathcal{J}_2(\xi_{2}, v_2)(2 + (1 - 2v_2)\xi_2 - \xi_{2,0})}{(1 - \xi_{2,0})(\xi_{2,0} + (1 - 2v_2)(1 - 2\alpha_2)\xi_2 - m^2)}
\]

And we can carefully add those to the LTD contributions and obtain something smoothly integrable in 4 dimensions!
Some results

For the toy scalar case

Good agreement and expected massless limit
Some results

For physical processes: Higgs and pseudoscalar decay
Some results

For physical processes: Z and photon decay

Very good agreement with DREG!
Some results

Computation time

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Very effective numerically!
Conclusion and outlook

What the LTD can achieve

• Local cancellation of both UV and IR singularities
• Integration in $d=4$ dimensions without DREG
• Fast numerical computations: real and virtual corrections implemented simultaneously
• Works for an arbitrary number of loops
Conclusion and outlook

And what is next?

- Numerical computation of multi-leg processes at hadron colliders
- NN(...)LO
- Automatising
- All of that together
Thank you for your attention!