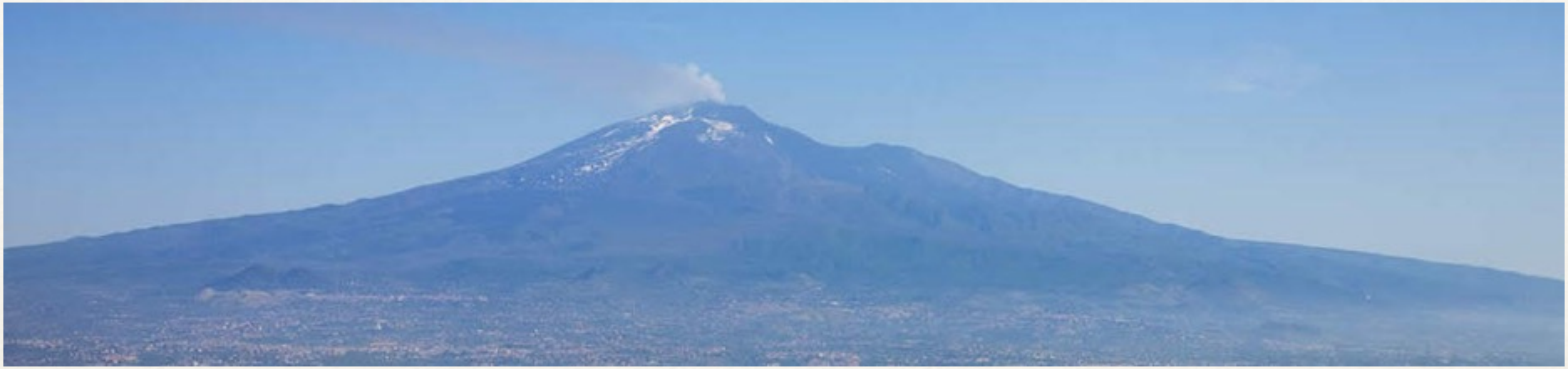

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Computation of NLO processes involving heavy quarks using Loop-Tree Duality

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Based on arXiv:1608.01584

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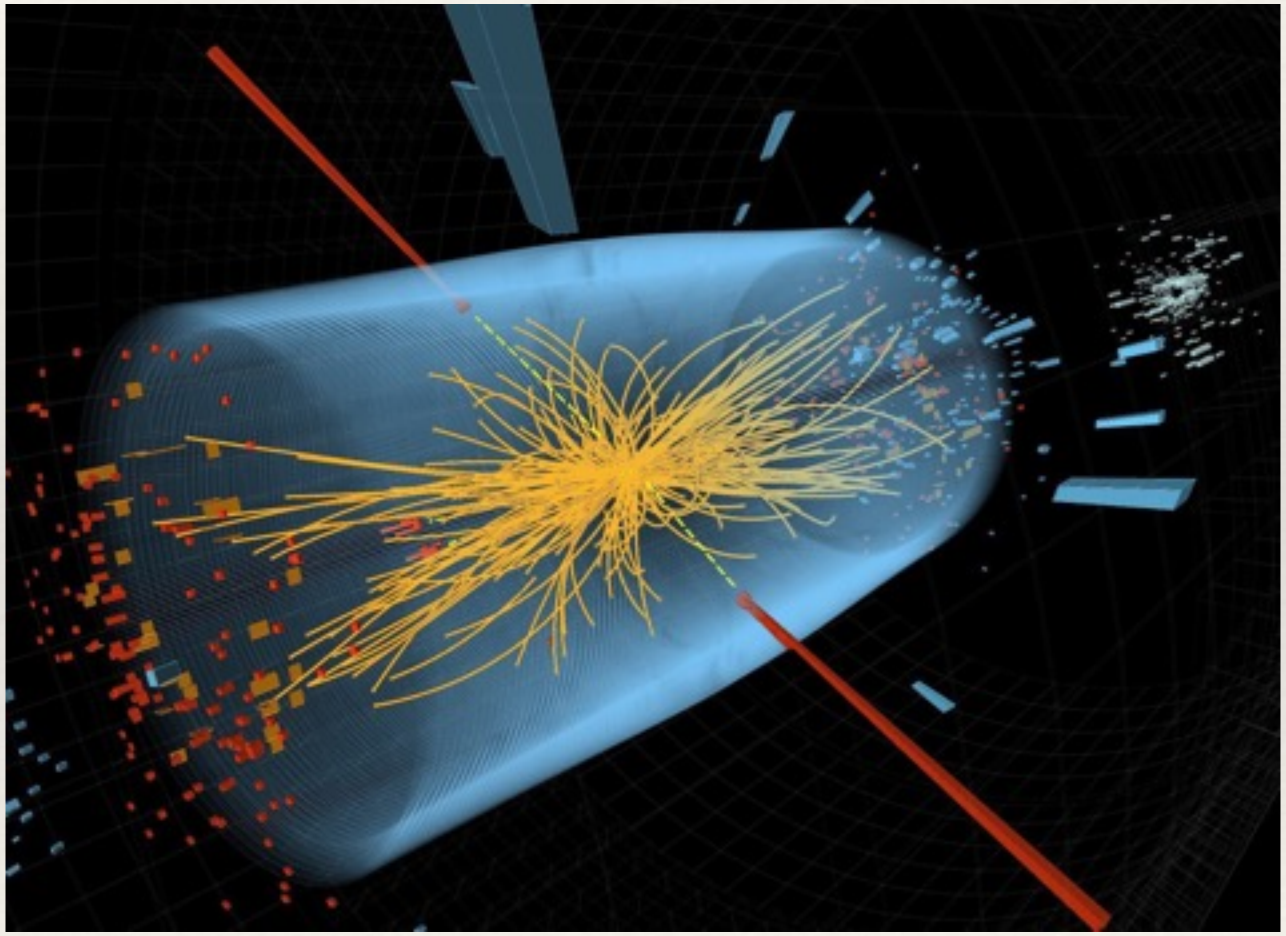
Content

Introduction

Motivation

- LHC Run II: higher energies and luminosity
  - More data & statistics
  - Higher precision measurements
- Theory has to keep up!
  - Higher orders in perturbation theories
  - Faster (numerical) computations



Introduction

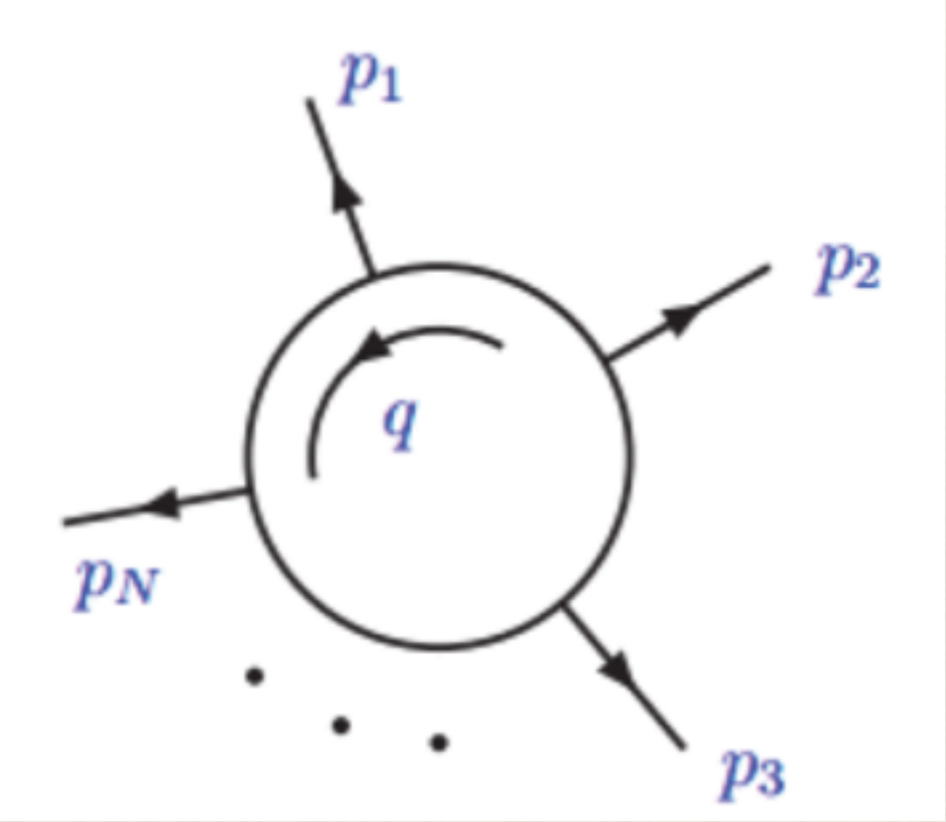
Higher order computational issues

Loop level amplitudes

Ill-defined integrals, in the UV (high-energy)

and in the IR (soft-collinear)

Singularities



Many existing methods to deal with that: DREG, FDH… and LTD

The Loop-Tree Duality

How it works

- Write the Feynman integral

$$L^{(1)}(p_1, p_2, ..., p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i)\,, \quad G_F = \frac{1}{q_i^2 - m_i^2 + \imath 0}\,, \quad q_i = \ell + p_1 + ... + p_i$$

- Apply Cauchy's residue theorem

$$L^{(1)}(p_1, p_2, ..., p_N) = -2\pi\imath \int_{\mathbf{q}} \sum \mathrm{Res}_{\{\mathrm{Im}\ q_0 < 0\}} \left[\prod_{i=1}^{N} G_F(q_i)\right]$$

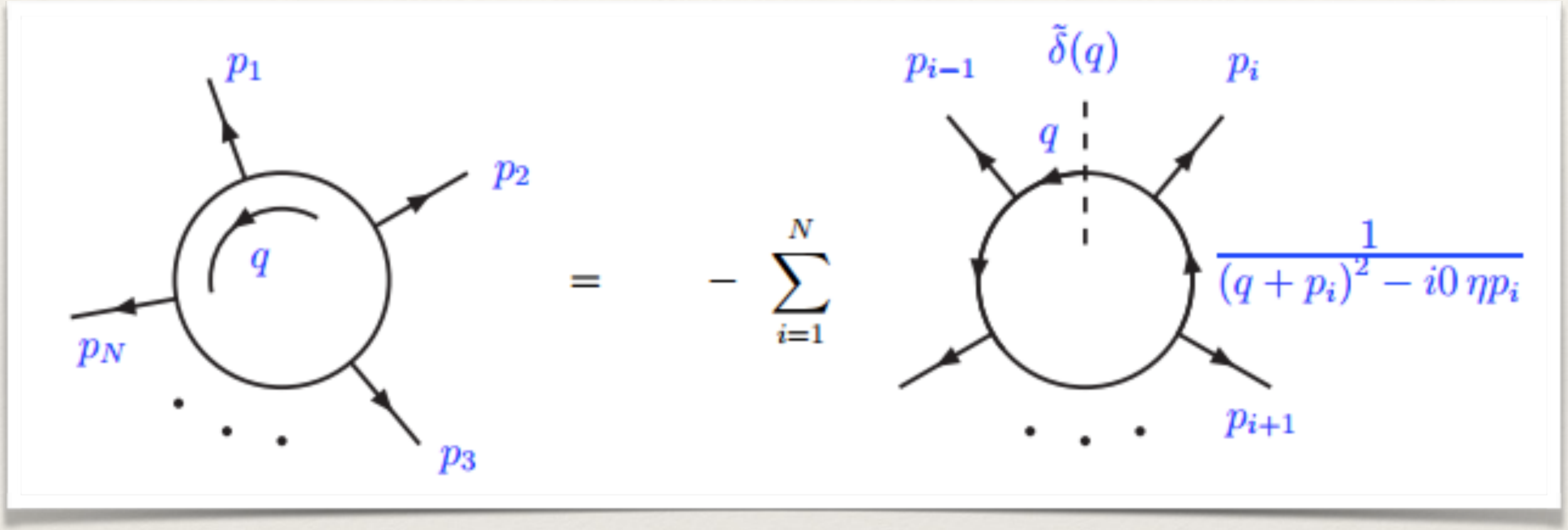
- Compute the residues, and get a sum with N contributions

$$L^{(1)}(p_1, p_2, ..., p_N) = -\int_\ell \sum_{i=1}^{N} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j\neq i}}^{N} G_D(q_i; q_j)\,, \quad G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - \imath 0\, \eta(q_j - q_i)}$$

Similar to Feynman Tree Theorem, but one gets only 1-cuts (with modified prescriptions)

The Loop-Tree Duality

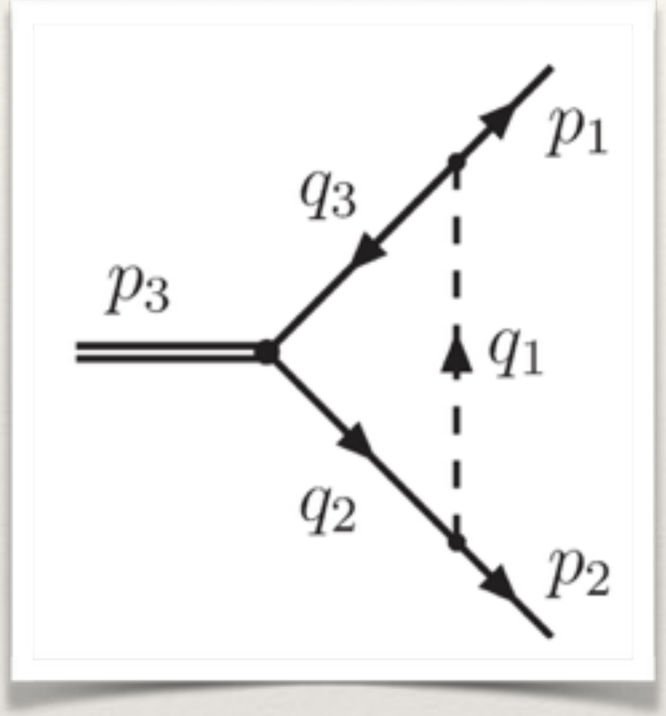
How it works



The Feynman integral becomes a sum over N 1-cuts

The Loop-Tree Duality

A basic example: the scalar three-point function



Momenta parametrisation:

$$p_1^\mu = \frac{\sqrt{s_{12}}}{2}(1,0,0,\beta)$$
$$p_2^\mu = \frac{\sqrt{s_{12}}}{2}(1,0,0,-\beta)$$
$$q_i^\mu = \frac{\sqrt{s_{12}}}{2}(\xi_{i,0},\xi_i\sin(\theta)\mathbf{e}_\perp,\xi_i\cos(\theta))$$

$$m = \frac{2M}{\sqrt{s_{12}}}$$
$$\beta = \sqrt{1-m^2}$$
$$\xi_{i,0} = \sqrt{\xi_i^2+m_i^2}$$

Three contributions:

$$L^{(1)}(p_1,p_2,-p_3) = \int_\ell \prod_{i=3}^{N} G_F(q_i) \overset{\text{LTD}}{=} \sum_{i=1}^{3} I_i$$

The Loop-Tree Duality

A basic example: the scalar three-point function

$$
\begin{aligned}
I_1 &= \frac{4}{s_{12}} \int \frac{\xi_{1,0}^{-1}\, d[\xi_{1,0}] d[v_1]}{1-(1-2v_1)^2\beta^2} \\
I_2 &= \frac{2}{s_{12}} \int \frac{\xi_{2,0}^{-1}\, \xi_2^2\, d[\xi_2] d[v_2]}{(1-\xi_{2,0}+\imath 0)(\xi_{2,0}+\beta\xi_2(1-2v_2)-m^2)} \\
I_3 &= -\frac{2}{s_{12}} \int \frac{\xi_{3,0}^{-1}\, \xi_3^2\, d[\xi_3] d[v_3]}{(1+\xi_{3,0})(\xi_{3,0}-\beta\xi_3(1-2v_3)+m^2)}
\end{aligned}
$$

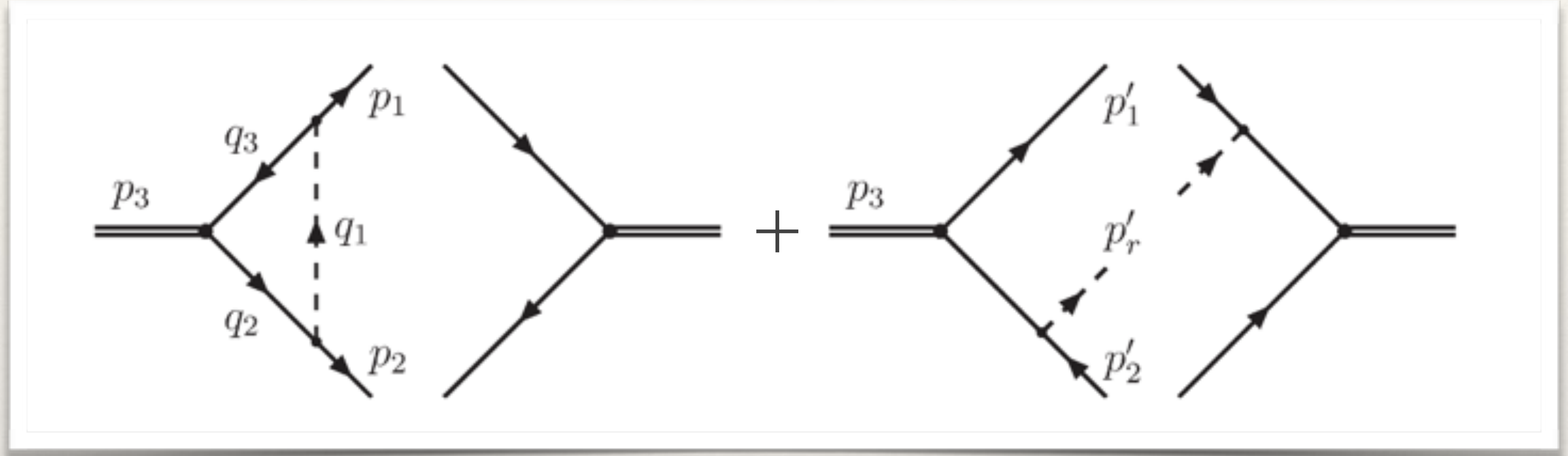
$$
\begin{aligned}
d[\xi_i] &= \frac{(4\pi)^{\epsilon-2}}{\Gamma(1-\epsilon)} \left(\frac{s_{12}}{\mu^2}\right)^{-\epsilon} \xi_i^{-2\epsilon} d\xi_i \\
d[v_i] &= v_i(1-v_i)^{-\epsilon} dv_i
\end{aligned}
$$

These integrals cannot be computed in d=4 dimensions
(Only possible in d=4-2ϵ)

One has to get rid of singularities (IR and UV)

Local cancellation of singularities

Global vs local



- Within DREG: cancellation after integration:

$$\text{Virtual} \quad \left(\frac{1}{\epsilon} + \mathcal{O}(1)\right) \quad + \quad \text{Real} \quad \left(-\frac{1}{\epsilon} + \mathcal{O}(1)\right)$$

- Within LTD: cancellation locally, <u>before integration</u>, so you can integrate in 4 dimensions

Local cancellation of singularities

Counter-term (UV)

Define suitable integrand level UV counter-terms:

- Correct integrated forms (in d dimensions) compared with DREG
- Local cancellation of UV singularities

You get the correction terms:

$$\Delta Z_2(p_1) = -g_S^2 C_F \int_\ell G_F(q_1) G_F(q_3) \left((d-2) \frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 4M^2 \left(1 - \frac{q_1 \cdot p_2}{p_1 \cdot p_2} \right) G_F(q_3) \right)$$

$$\Delta Z_M^{\mathbf{OS}}(p_1) = -g_S^2 C_F \int_\ell G_F(q_1) G_F(q_3) \left((d-2) \frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 2 \right)$$

Local cancellation of singularities

Counter-term (UV)

1. Expand the numerators and the propagators around a UV propagator

$$G_F(q_i) = \frac{1}{q_{\mathrm{UV}}^2 - \mu_{\mathrm{UV}}^2 + \imath 0} + \ldots \qquad q_{\mathrm{UV}} = \ell + k_{\mathrm{UV}}$$

2. Choose a scheme (MSbar scheme for example) and adjust subleading terms to subtract only the pole
3. Subtract the counter-term that will locally cancel the singularity, using the dual variables $(\xi_{UV},\ v_{UV})$

$$\begin{aligned}
\Delta Z_2^{\mathrm{UV}} &= -(d-2) g_S^2 C_F \int d[\xi_{\mathrm{UV}}] d[v_{\mathrm{UV}}] \frac{2\xi_{\mathrm{UV}}}{\xi_{\mathrm{UV},0}^3} \left[\left(1 + \frac{\beta \xi_{\mathrm{UV}} (1 - 2 v_{\mathrm{UV}})}{2(1+\beta^2)} \right) \right. \\
&\quad \left. \times \left(1 - \frac{3(2\beta\xi_{\mathrm{UV}}(1-2v_{\mathrm{UV}}) - m_{\mathrm{UV}}^2)}{4\xi_{\mathrm{UV},0}^2} \right) - \frac{1}{2(1+\beta^2)} \right] \\
\Delta Z_M^{\mathrm{OS,UV}} &= -g_S^2 C_F \int d[\xi_{\mathrm{UV}}] d[v_{\mathrm{UV}}] \frac{2\xi_{\mathrm{UV}}}{\xi_{\mathrm{UV},0}^3} \left[\left(d + (d-2) \frac{\beta \xi_{\mathrm{UV}} (1 - 2 v_{\mathrm{UV}})}{2(1+\beta^2)} \right) \right. \\
&\quad \left. \times \left(1 - \frac{3(2\beta\xi_{\mathrm{UV}}(1-2v_{\mathrm{UV}}) - 2d^{-1} m_{\mathrm{UV}}^2)}{4\xi_{\mathrm{UV},0}^2} \right) - \frac{d-2}{2(1+\beta^2)} \right]
\end{aligned}$$

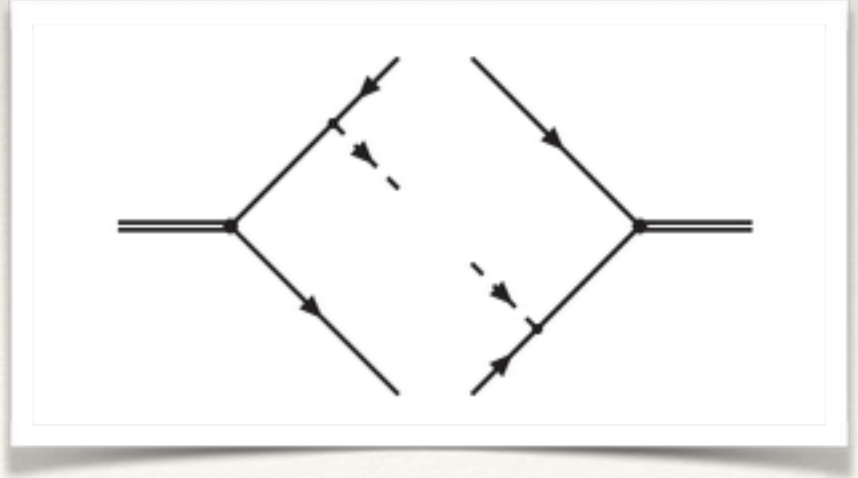
Local cancellation of singularities

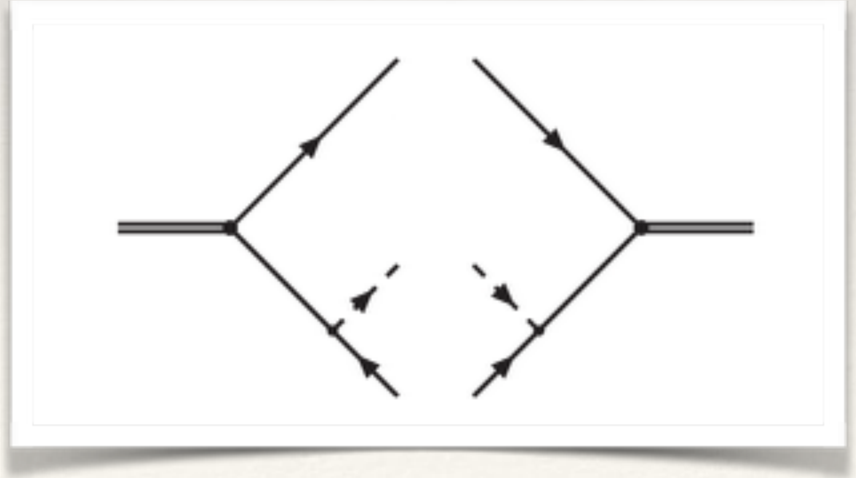
Unsubtraction algorithm (IR)

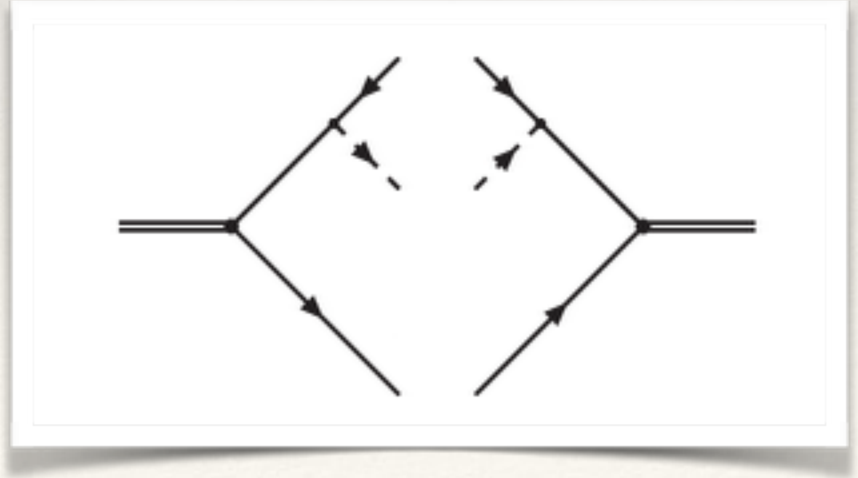
Three IR singularities:

$\xi_{1,0} = 0$		Soft
$\xi_{1,0} < 1$	$v_1 = 0$	Quasi-Collinear
$\xi_{2,0} < 1$	$v_2 = 1$	Quasi-Collinear

⇒ Cancelled by the real contributions:







Local cancellation of singularities

Phase-space partition

Before doing anything, you want to isolate each singularity

(Avoid having two soft or two quasi-collinear singularities in the same region)

$\Rightarrow$ Two regions:

- Region 1: Soft + quasi-collinear singularities

  $\longrightarrow$ First LTD contribution's singularities will be canceled

- Region 2: Quasi-collinear singularity

  $\longrightarrow$ Second LTD contribution's singularity will be canceled

But in order to do that, you need to find a mapping so you can add real and virtual amplitudes at integrand level (with the variables (ξ_i, v_i))

The real-virtual mapping

Region 1

Region 2

Using QCD factorisation properties, we define the mapping:

$$\boxed{\begin{aligned} p_r'^\mu &= q_1^\mu \\ p_1'^\mu &= (1-\alpha_1)\hat{p}_1^\mu + (1-\gamma_1)\hat{p}_2^\mu - q_1^\mu \\ p_2'^\mu &= \alpha_1\hat{p}_1^\mu + \gamma_1\hat{p}_2^\mu \end{aligned}}$$

$$\boxed{\begin{aligned} p_r'^\mu &= (1-\gamma_2)\hat{p}_1^\mu + (1-\alpha_2)\hat{p}_2^\mu - q_2^\mu \\ p_1'^\mu &= \gamma_2\hat{p}_1^\mu + \alpha_2\hat{p}_2^\mu \\ p_2'^\mu &= q_2^\mu \end{aligned}}$$

with $p_1 = \frac{1+\beta}{2}\hat{p}_1^\mu + \frac{1-\beta}{2}\hat{p}_2^\mu$ and $p_2 = \frac{1-\beta}{2}\hat{p}_1^\mu + \frac{1+\beta}{2}\hat{p}_2^\mu$

Then one has to solve:

$$\boxed{\begin{aligned} (p_1')^2 &= M^2 \\ (p_2')^2 &= M^2 \end{aligned}}$$

$$\boxed{\begin{aligned} (p_1')^2 &= M^2 \\ (p_r')^2 &= 0 \end{aligned}}$$

The real-virtual mapping

Adding real+virtual

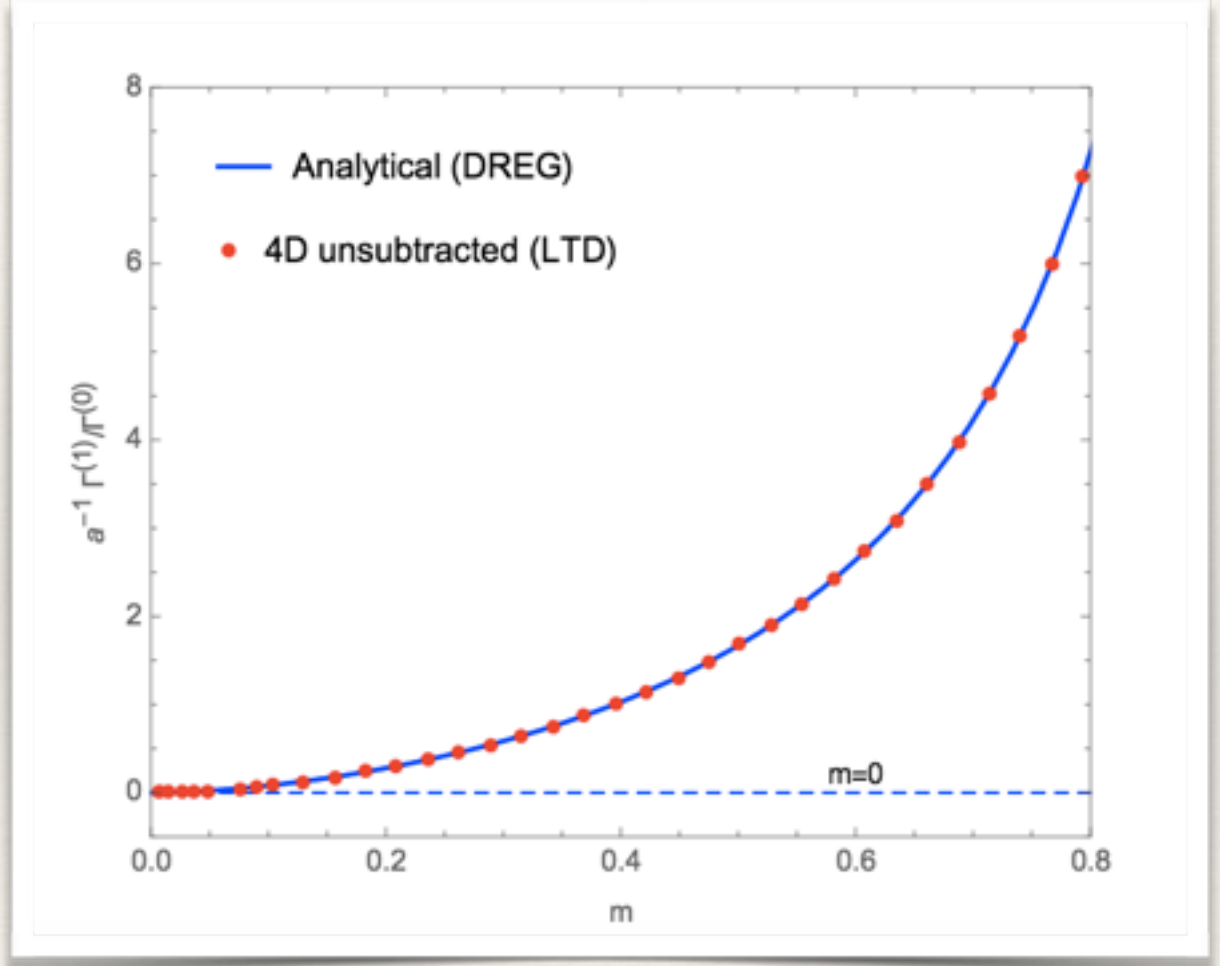
Then we get the real contribution decay rate in each region:

$$\boxed{\begin{aligned}\tilde{\Gamma}^{(1)}_{\mathrm{R},1} &= \Gamma^{(0)}\frac{2a}{\beta}\int d\xi_{1,0}dv_1\frac{\mathcal{R}_1(\xi_{1,0},v_1)\mathcal{J}_1(\xi_{1,0},v_1)(1-\xi_{1,0}(1-v_1))^2}{\xi^2_{1,0}(v_1+\alpha_1(1-2v_1))((1-v_1)(1-\xi_{1,0})-\alpha_1(1-2v_1))}\\ \tilde{\Gamma}^{(1)}_{\mathrm{R},2} &= \Gamma^{(0)}\frac{2a}{\beta}\int d\xi_2 dv_2\frac{\mathcal{R}_2(\xi_2,v_2)\mathcal{J}_2(\xi_2,v_2)(2+(1-2v_2)\xi_2-\xi_{2,0})}{(1-\xi_{2,0})(\xi_{2,0}+(1-2v_2)(1-2\alpha_2)\xi_2-m^2)}\end{aligned}}$$

And we can carefully add those to the LTD contributions and obtain something smoothly integrable in 4 dimensions!

Some results

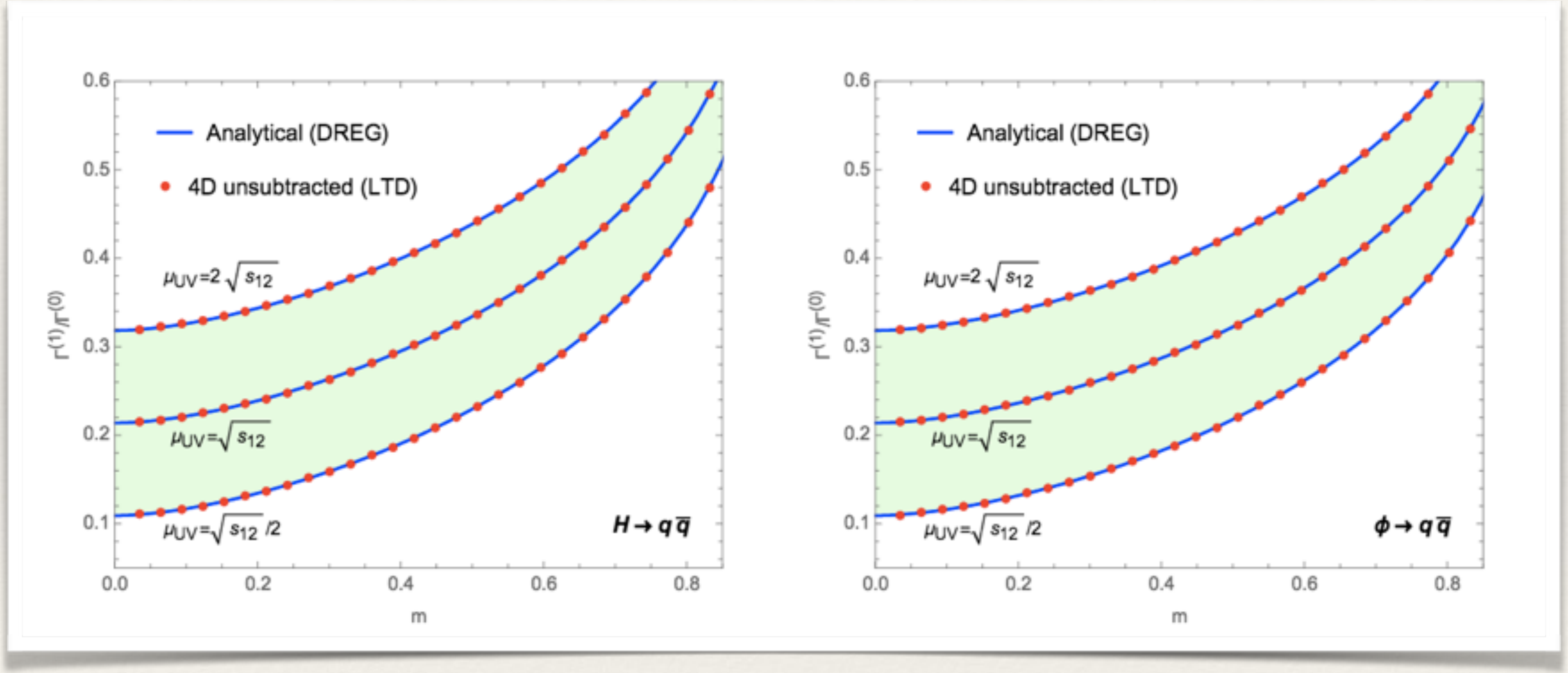
For the toy scalar case



Good agreement and expected massless limit

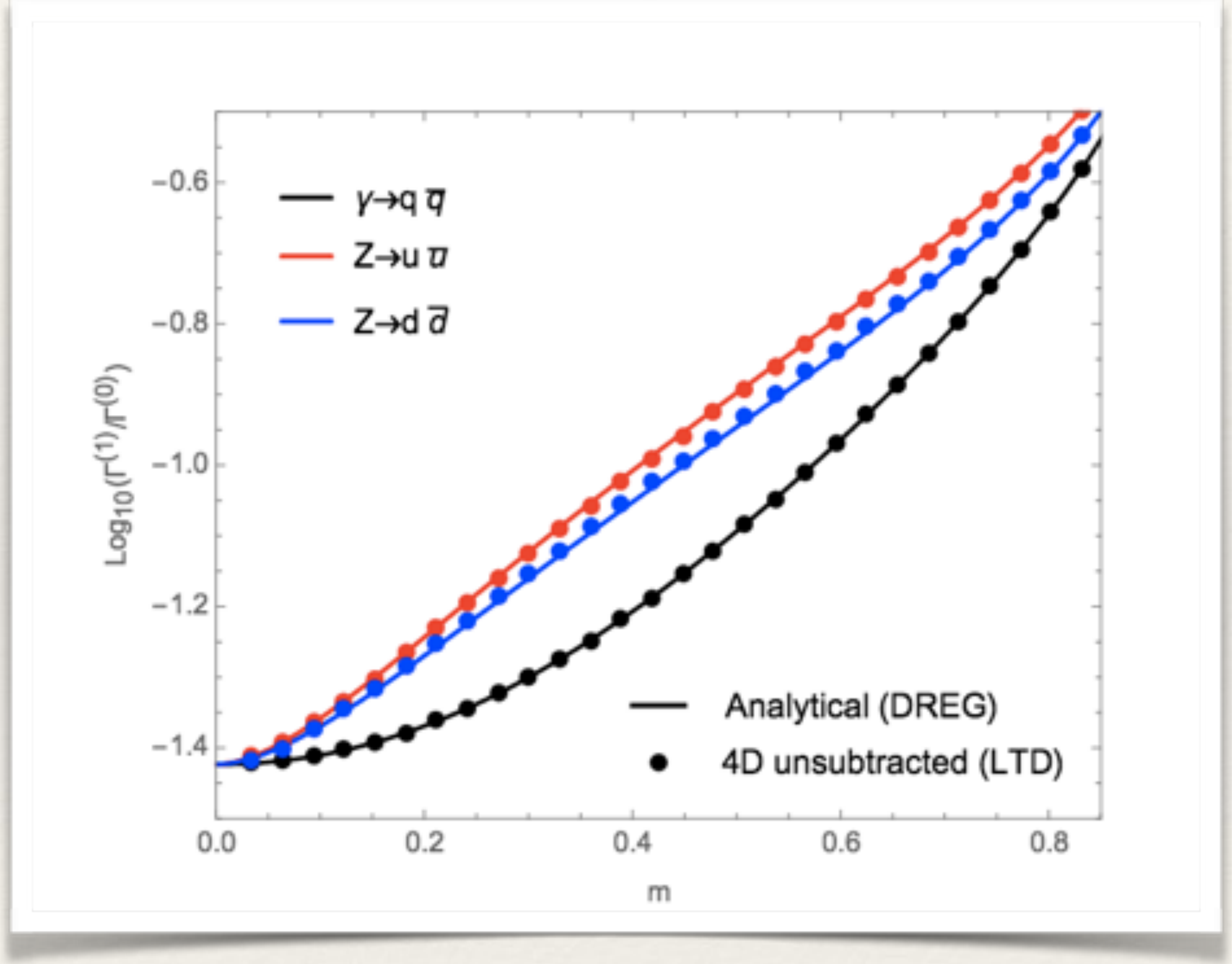
Some results

For physical processes: Higgs and pseudoscalar decay



Some results

For physical processes: Z and photon decay



Very good agreement with DREG!

Some results

Computation time

	Rank	Tensor Hexagon	Real Part	Imaginary Part	Time[s]
P20	1	SecDec	$-1.21585(12) \times 10^{-15}$		36
		LTD	$-1.21552(354) \times 10^{-15}$		6
P21	3	SecDec	$4.46117(37) \times 10^{-9}$		5498
		LTD	$4.461369(3) \times 10^{-9}$		11
P22	1	SecDec	$1.01359(23) \times 10^{-15}$	$+i\ 2.68657(26) \times 10^{-15}$	33
		LTD	$1.01345(130) \times 10^{-15}$	$+i\ 2.68633(130) \times 10^{-15}$	72
P23	2	SecDec	$2.45315(24) \times 10^{-12}$	$-i\ 2.06087(20) \times 10^{-12}$	337
		LTD	$2.45273(727) \times 10^{-12}$	$-i\ 2.06202(727) \times 10^{-12}$	75
P24	3	SecDec	$-2.07531(19) \times 10^{-6}$	$+i\ 6.97158(56) \times 10^{-7}$	14280
		LTD	$-2.07526(8) \times 10^{-6}$	$+i\ 6.97192(8) \times 10^{-7}$	85

Very effective numerically!

Conclusion and outlook

What the LTD can achieve

- Local cancellation of both UV and IR singularities
- Integration in d=4 dimensions without DREG
- Fast numerical computations: real and virtual corrections implemented simultaneously
- Works for an arbitrary number of loops

Conclusion and outlook

And what is next?

- Numerical computation of multi-leg processes at hadron colliders
- NN(…)LO
- Automatising
- All of that together

Thank you for your attention!