Predictions for diffractive ϕ meson production using AdS/QCD light-front wavefunction

Mohammad Ahmady

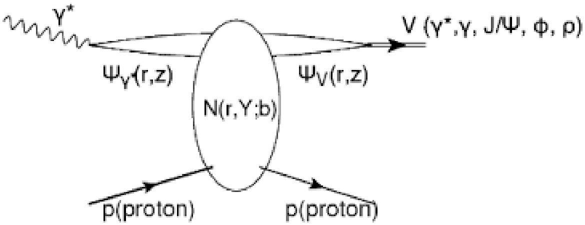
Department of Physics
Mount Allison University

September 4, 2016

Based on the joint article with Ruben Sandapen and Neetika Sharma : arXiv:1605.07665

Overview

Diffractive vector meson production



- $ep \to epV$ or $\gamma^* p \to pV$
- Sensitivity to non-perturbative physics
- Can be used to fine tune the vector meson wavefunction

Motivation

1. Diffractive ρ production had already been investigated using AdS/QCD wavefunctions resulting in excellent agreement with data: R. Sandapen and J. Forshaw, PRL 109, 081601 (2012)
2. AdS/QCD provides a light front wavefunction for vector mesons with no free parameters.
3. We have obtained updated parameters for dipole-proton cross section using HERA 2015 F2 data.
4. It would be interesting to check the predictions of AdS/QCD for diffractive ϕ production.
5. If AdS/QCD is successful in predicting diffractive vector meson production then we can be confident to use it in other contexts like $B \to (\rho, K^*)\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ decays.

The dipole model

The forward scattering amplitude for the diffractive process $\gamma^* p \to Vp$ factorizes into an overlap of photon and vector meson light-front wavefunctions and a dipole cross-section.

$$\Im\mathrm{m}\, \mathcal{A}_\lambda(s,t;Q^2) = \sum_{h,\bar{h}} \int \mathrm{d}^2\mathbf{r}\, \mathrm{d}z\, \Psi^{\gamma^*,\lambda}_{h,\bar{h}}(r,z;Q^2) \Psi^{V,\lambda}_{h,\bar{h}}(r,z)^* e^{-iz\mathrm{r}\cdot\mathbf{\Delta}} \mathcal{N}(x,\mathrm{r},\mathbf{\Delta})$$

- $t = -\mathbf{\Delta}^2$ is the momentum transfer to proton.
- $\Psi^{\gamma^*,\lambda}_{h,\bar{h}}(r,z;Q^2)$ and $\Psi^{V,\lambda}_{h,\bar{h}}(r,z)$ are the light-front wavefunctions of photon and vector meson respectively.
- h is the helicity of the quark and $\bar{h}$ is the helicity of the antiquark.
- $\lambda = L, T$ is the polarization of the photon or vector meson.
- $\mathcal{N}(x,\mathrm{r},\Delta)$ is the proton-dipole scattering amplitude.

Connection to inclusive $\gamma^* p \to X$ process

$\mathcal{N}(x, \mathrm{r}, \mathbf{\Delta})$ is a universal object.

In Deep Inelastic Scattering (DIS), one can replace the vector meson by a virtual photon in the previous equation to obtain the forward amplitude for elastic Compton scattering $\gamma^* p \to \gamma^* p$:

$$\Im\mathrm{m}\, \mathcal{A}_\lambda(s,t)|_{t=0} = s \sum_{h,\bar{h}} \int \mathrm{d}^2\mathbf{r}\, \mathrm{d}z\, |\Psi^{\gamma^*,\lambda}_{h,\bar{h}}(r,z;Q^2)|^2 \hat{\sigma}(x,r)$$

The elastic scattering of the dipole on the proton depends on the photon-proton centre-of-mass energy via the modified Bjorken variable x where

$$x = x_{\mathrm{Bj}} \left(1 + \frac{4m_f^2}{Q^2}\right) \text{ with } x_{\mathrm{Bj}} = \frac{Q^2}{s}$$

CGC dipole model parameters

The model resulting from the impact parameter being integrated in dipole-proton amplitude is known as the CGC dipole model and is given by

$$\hat{\sigma}(x,r) = \sigma_0 \mathcal{N}(x, rQ_s, 0)$$

with

$$\begin{aligned}
\mathcal{N}(x, rQ_s, 0) &= \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left[\gamma_s + \frac{\ln(2/rQ_s)}{\kappa\,\lambda\,\ln(1/x)}\right]} && \text{for} \quad rQ_s \leq 2 \\
&= 1 - \exp[-\mathcal{A}\ln^2(\mathcal{B}\, rQ_s)] && \text{for} \quad rQ_s > 2
\end{aligned}$$

where the saturation scale $Q_s = (x_0/x)^{\lambda/2}$ GeV. The coefficients $\mathcal{A}$ and $\mathcal{B}$ are determined from the condition that the $\mathcal{N}(rQ_s, x)$ and its derivative with respect to rQ_s are continuous at $rQ_s = 2$. This leads to

$$\mathcal{A} = -\frac{(\mathcal{N}_0\gamma_s)^2}{(1-\mathcal{N}_0)^2 \ln[1-\mathcal{N}_0]}, \qquad \mathcal{B} = \frac{1}{2}(1-\mathcal{N}_0)^{-\frac{(1-\mathcal{N}_0)}{\mathcal{N}_0\gamma_s}}$$

$\mathcal{N}_0 = 0.7$, $\kappa = 9.9$ fixed from LL BFKL predictions.

Fit to HERA data

1. The CGC dipole model extracted from fits to 2015 combined H1 and ZEUS data (H. Abramowicz et al. (ZEUS, H1) (2015), 1506.06042.) with $x \leq 0.01$ and $Q^2 \in [0.045, 45]$ GeV^2
2. Sensitivity to the input quark mass.
3. Previous fits had resulted $\gamma_s = 0.74$, $\sigma_0 = 27.4$ mb, $x_0 = 1.63 \times 10^{-5}$ and $\lambda = 0.216$ with $m_{u,d}, m_s = 0.14$ GeV (JHEP 11, 025 (2006))

$[m_{u,d}, m_s]$/GeV	γ_s	σ_0/mb	x_0	λ	χ^2/d.p
$[0.046, 0.357]$	0.741	26.3	1.81×10^{-5}	0.219	535/520=1.03
$[0.046, 0.14]$	0.722	24.9	1.80×10^{-5}	0.222	529/520=1.02
$[0.14, 0.14]$	0.723	25.5	1.61×10^{-5}	0.221	554/520=1.07

Photon and Meson wavefunctions

To lowest order in α_{em}, the perturbative photon wavefunctions are given by

$$\Psi^{\gamma,L}_{h,\bar{h}}(r,x;Q^2,m_f) = \sqrt{\frac{N_c}{4\pi}}\delta_{h,-\bar{h}}e\,e_f 2x(1-x)Q\frac{K_0(\epsilon r)}{2\pi}$$

$$\Psi^{\gamma,T}_{h,\bar{h}}(r,x;Q^2,m_f) = \pm\sqrt{\frac{N_c}{2\pi}}e\,e_f\left[ie^{\pm i\theta_r}(x\delta_{h\pm,\bar{h}\mp}-(1-x)\delta_{h\mp,\bar{h}\pm})\partial_r + m_f\delta_{h\pm,\bar{h}\pm}\right]\frac{K_0(\epsilon r)}{2\pi}$$

where $\epsilon^2 = x(1-x)Q^2 + m_f^2$ and $re^{i\theta_r}$ is the complex notation for the transverse separation between the quark and anti-quark.

The vector meson light-front wavefunction cannot be computed in perturbation theory but assumed to have the same spinor structure as in the photon case, together with an unknown non-perturbative wavefunction.

$$\Psi^{V,L}_{h,\bar{h}}(r,z) = \frac{1}{2}\delta_{h,-\bar{h}}\left[1+\frac{m_f^2-\nabla_r^2}{z(1-z)M_V^2}\right]\Psi_L(r,z)$$

$$\Psi^{V,T}_{h,\bar{h}}(r,z) = \pm\left[ie^{\pm i\theta_r}(z\delta_{h\pm,\bar{h}\mp}-(1-z)\delta_{h\mp,\bar{h}\pm})\partial_r + m_f\delta_{h\pm,\bar{h}\pm}\right]\frac{\Psi_T(r,z)}{z(1-z)}$$

Non-perturbative meson wavefunction

- Hadronic light-front wavefunctions based on the anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence have been proposed by Brodsky and de Téramond.
- In light-front QCD and for massless quarks, the meson wavefunction can be written as

$$\Psi(\zeta, z, \phi) = e^{iL\phi} X(z) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- $\zeta = \sqrt{z(1-z)}r$
- z to denote the light-front momentum fraction carried by the quark

Holographic Schrödinger equation

$\phi(\zeta)$ is a solution of the so-called holographic light-front Schrödinger equation:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

where M is the mass of the meson, L the orbital quantum number and $U(\zeta)$ becomes a harmonic oscillator potential in physical spacetime:

$$U(\zeta, J) = \kappa^4\zeta^2 + \kappa^2(J-1)$$

The eigenvalue and eigenfunction of the holographic Schrödinger equation

$$M^2 = 4\kappa^2\left(n + L + \frac{S}{2}\right) \Rightarrow \kappa = 0.54 \text{ GeV from Regge slope}$$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{1/2+L}\exp\left(\frac{-\kappa^2\zeta^2}{2}\right)L_n^L(x^2\zeta^2)$$

Light front Wavefunction for ϕ

For the vector mesons ρ and ϕ, we set $n = 0, L = 0$ to obtain

$$\Psi_{0,0}(z,\zeta) = \frac{\kappa}{\sqrt{\pi}} \sqrt{z(1-z)} \exp\left[-\frac{\kappa^2 \zeta^2}{2}\right]$$

Allowing for small quark masses, the wavefunction becomes

$$\Psi_{\lambda}(z,\zeta) = \mathcal{N}_{\lambda} \sqrt{z(1-z)} \exp\left[-\frac{\kappa^2 \zeta^2}{2}\right] \exp\left[-\frac{m_f^2}{2\kappa^2 z(1-z)}\right]$$

$m_{u,d} = 0.046$ GeV and $m_s = 0.357$ GeV are fixed from the y-intercepts of the Regge trajectories. Decay constant provides the first test of the wave function:

$$f_V P^+ = \langle 0|\bar{q}(0)\gamma^+ q(0)|V(P,L)\rangle$$

$$f_V = \sqrt{\frac{N_c}{\pi}} \int_0^1 \mathrm{d}z \left[1 + \frac{m_f^2 - \nabla_r^2}{z(1-z)M_V^2}\right] \Psi_L(r,z)\big|_{r=0}$$

Predictions for leptonic decay width

We can use this decay constant to predict the experimentally measured electronic decay width $\Gamma_{V\to e^+e^-}$ of the vector meson:

$$\Gamma_{V\to e^+e^-} = \frac{4\pi\alpha_{em}^2 C_V^2}{3M_V} f_V^2$$

where $C_\phi = 1/3$ for the $C_\rho = 1/\sqrt{2}$.

Meson	f_V [GeV]	$\Gamma_{e^+e^-}$ [KeV]	$\Gamma_{e^+e^-}$[KeV] (PDG)
ρ	$0.210, 0.211$	$6.355, 6.383$	7.04 ± 0.06
ϕ	$0.191, 0.205$	$0.891, 1.024$	1.251 ± 0.021

Table: Predictions for the electronic decay widths of the ρ and ϕ vector mesons using the holographic wavefunction with $m_{u,d} = 0.046, 0.14$ GeV and $m_s = 0.357, 0.14$ GeV.

Cross section for $\gamma^* p \to \rho p$

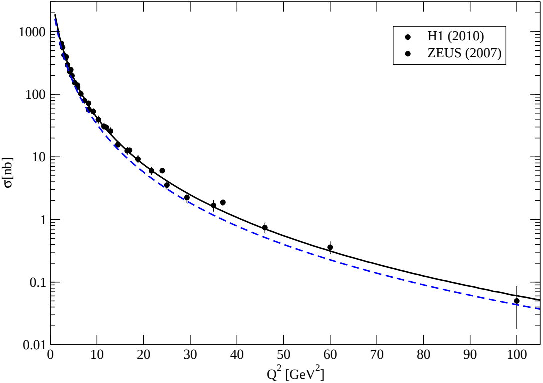


Figure: Predictions for cross section for $\gamma^* p \to \rho p$ as a function of Q^2 at $W = 75$ GeV compared to HERA data. The black solid curve is obtained using $m_{u,d} = 0.046$ GeV and the blue dashed curve is obtained using $m_{u,d} = 0.14$ GeV.

Cross section for $\gamma^* p \to \phi p$

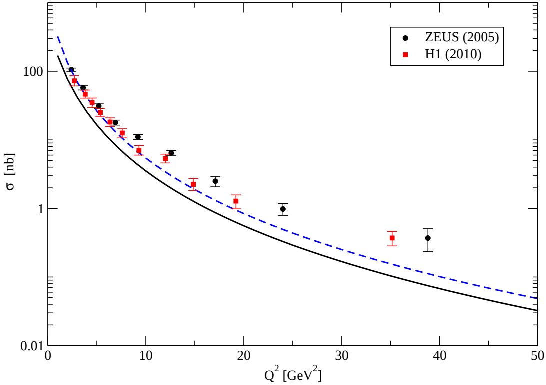


Figure: Predictions for the total cross section for ϕ production as a function of Q^2 compared to HERA data. The solid black curve is obtained using $m_s = 0.357$ GeV and the dashed blue curve is obtained using $m_s = 0.14$ GeV. The theory predictions are at $W = 90$ GeV.

σ_L/σ_T ratio in ρ production

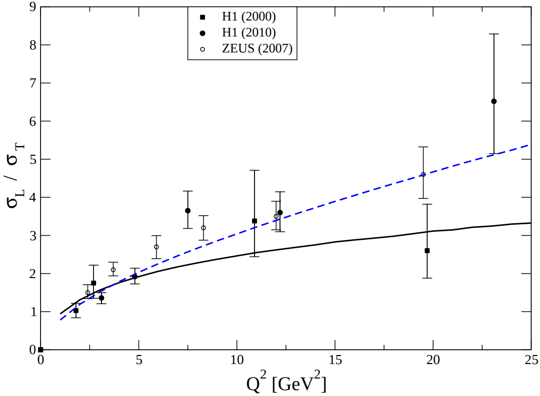


Figure: Predictions for the ratio of longitudinal to transverse cross-sections as a function of Q^2 at $W = 90$ GeV. The solid black curve is obtained using $m_{u,d} = 0.046$ GeV (holographic masses) and the dashed blue curve is obtained using $m_{u,d} = 0.14$ GeV.

σ_L/σ_T ratio in ϕ production

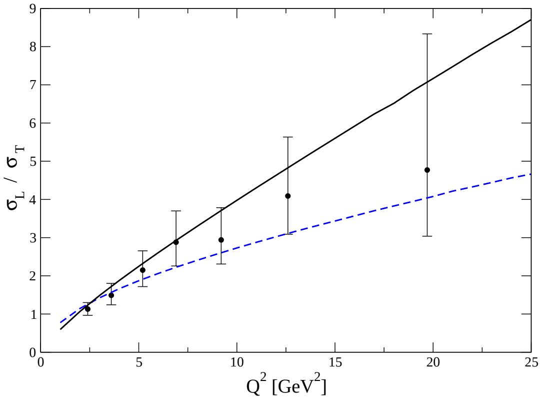


Figure: Predictions for the longitudinal to transverse cross-section ratio at $W = 90$ GeV. The solid black curves are obtained with $m_s = 0.357$ GeV and the dashed blue curves are obtained with $m_s = 0.14$ GeV.

ϕ production cross-section as a function of W VS ZEUS data

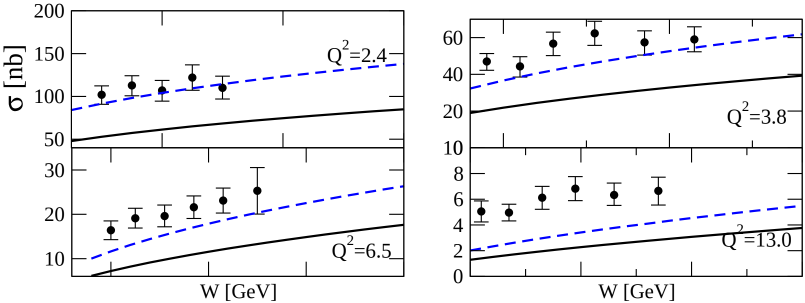


Figure: Predictions for the total ϕ production cross-section as a function of W in different Q^2 bins. The solid black curves are obtained with $m_s = 0.357$ GeV and the dashed blue curves are obtained with $m_s = 0.14$ GeV.

ϕ production cross-section as a function of W VS H1 data

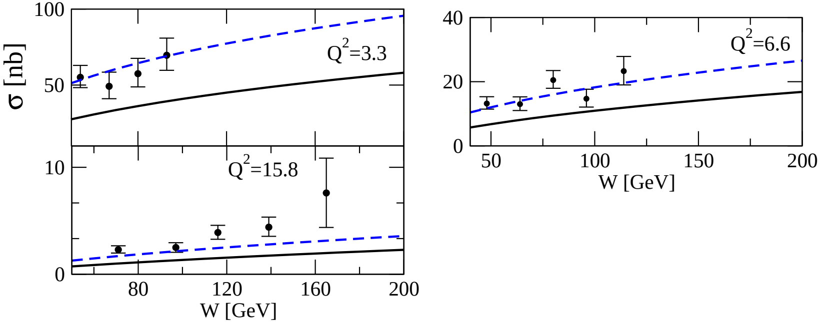


Figure: Predictions for the total ϕ production cross-section as a function of W in different Q^2 bins. The solid black curves are obtained with $m_s = 0.357$ GeV and the dashed blue curves are obtained with $m_s = 0.14$ GeV.

Conclusion

1. CGC dipole model provides a good fit to 2015 HERA combined data on F2
2. AdS/QCD predictions for diffractive ρ production cross-section is in good agreement with the data when the light quark mass is $m_{u,d} = 0.046$ as required by meson spectroscopy or even heavier around 0.14 GeV. However, the ratio σ_L/σ_T clearly prefers $m_{u,d} = 0.14$ GeV.
3. The data on ϕ production prefer a lower strange quark mass than the one required by AdS/QCD spectroscopy.
4. We are in the process of repeating our ϕ production analysis using b-CGC model.

Fixing the model parameters

Specifically, these collaborations measured the reduced cross-section

$$\sigma_r(Q^2, x, y) = F_2(Q^2, x) - \frac{y^2}{1 + (1-y)^2} F_L(Q^2, x)$$

where $y = Q^2/sx$ and $\sqrt{s}$ is the centre of mass energy of the *ep* system. The structure functions are given by

$$F_2(Q^2, x) = \frac{Q^2}{4\pi^2\alpha_{\text{em}}}(\sigma_L^{\gamma^* p}(Q^2, x) + \sigma_T^{\gamma^* p}(Q^2, x))$$

and

$$F_L(Q^2, x) = \frac{Q^2}{4\pi^2\alpha_{\text{em}}}\sigma_L^{\gamma^* p}(Q^2, x)$$

$\sigma_{L,T}^{\gamma^* p}(Q^2, x)$ as defined on previous slide.

Photon light-front wavefunction

- The photon light-front wavefunctions can be computed perturbatively in QED.
- To lowest order in α_{em}, they are given by:

$$
\begin{aligned}
\Psi^{\gamma,L}_{h,\bar{h}}(r,z;Q^2,m_f) &= \sqrt{\frac{N_c}{4\pi}}\delta_{h,-\bar{h}}e\, e_f 2z(1-z)Q\frac{K_0(\epsilon r)}{2\pi}\;, \\
\Psi^{\gamma,T}_{h,\bar{h}}(r,z;Q^2,m_f) &= \pm\sqrt{\frac{N_c}{2\pi}}e\, e_f\left[ie^{\pm i\theta_r}(z\delta_{h\pm,\bar{h}\mp}-(1-z)\delta_{h\mp,\bar{h}\pm})\partial_r\right. \\
&\quad \left.+m_f\delta_{h\pm,\bar{h}\pm}\right]\frac{K_0(\epsilon r)}{2\pi}\;,
\end{aligned}
$$

- $\epsilon^2 = z(1-z)Q^2 + m_f^2$
- $re^{i\theta_r}$ is the complex notation for of the transverse separation between the quark and anti-quark.
- As can be seen, at $Q^2 \to 0$ or $z \to 0, 1$, the photon light-front wavefunctions become sensitive to the non-vanishing quark mass m_f which prevents the modified Bessel function $K_0(\epsilon r)$ from diverging.

Spin-J string mode in AdS space

1. Making the substitutions $\zeta \to z_5$ and $L^2 - (2-J)^2 \to mR^2$ (with z_5, R and m being the fifth dimension, the radius of curvature and a mass parameter in AdS), the holographic Schrödinger equation describes the propagation of spin-J string modes in AdS space.
2. The potential is given by

$$U(z_5, J) = \frac{1}{2}\varphi''(z_5) + \frac{1}{4}\varphi'(z_5)^2 + \left(\frac{2J-3}{4z_5}\right)\varphi'(z_5)$$

   where $\varphi(z_5)$ is a dilaton field which breaks the conformal invariance of AdS space.
3. A quadratic dilaton ($\varphi(z_5) = \kappa^2 z_5^2$) profile results in a harmonic oscillator potential in physical spacetime:

$$U(\zeta, J) = \kappa^4\zeta^2 + \kappa^2(J-1)$$