Could Reggeon Field theory serve as effective theory for QCD at high energies?

Collaboration with C. Contreras and G. P. Vacca

• Introduction
• Search for fixed points: first results
• Short glimpse at phenomenology
• Conclusions

JB, Contreras, Vacca, JHEP 03 (2016) 201 and hep-th/1608.08836

→ talk G.P. Vacca
**Introduction**

**Goal:**
try to connect the Regge limit of pQCD with nonperturbative strong interaction

pQCD: short transverse distances, 

\[ \alpha(0) = 1 + \omega_{BFKL} > 0, \]
\[ \alpha' \text{ very small} \]

power-like large-b behavior

u**ltraviolet**

nonperturbative: pp scattering at LHC

\[ \alpha(0) \approx 1.1, \]
\[ \alpha' \approx 0.25 \text{ GeV}^{-2} \]

exponential large-behavior

i**nfrared**

Framework: Reggeon field theory

This talk: first step, only infrared limit.

**Method:**
renormalization group, flow equations:
integrate over large momentum modes, investigate the infrared limit
Attractive idea:

use reggeon field theory and renormalization group,
construct a flow from UV scale to IR scale

\[ S = \int dyd^2x \mathcal{L}(\psi, \psi^\dagger) \]

e.g. local approximation:
\[ \mathcal{L} = (\frac{1}{2} \psi^\dagger \partial_y \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V(\psi, \psi^\dagger) \]
\[ V(\psi, \psi^\dagger) = -\mu \psi^\dagger \psi + i\lambda \psi^\dagger (\psi^\dagger + \psi) \psi 
+ g(\psi^\dagger \psi)^2 + g' \psi^\dagger (\psi^\dagger \psi + \psi^2) \psi + \cdots \]

Study the flow as function of IR cutoff \( k \) in transverse momentum,
all fields and parameters become \( k \)-dependent,
IR limit: infinite transverse momenta, infinite energies
The formalism: functional renormalization, flow equations

Reminder: Wilson approach

The standard Wilsonian action is defined by an iterative change in the UV-cutoff induced by a partial integration of quantum fluctuations:

\[ \Lambda \rightarrow \Lambda' < \Lambda \]

\[ \int [d\varphi]^\Lambda e^{-S^\Lambda[\varphi]} = \int [d\varphi]^\Lambda' e^{-S^{\Lambda'}[\varphi]} \]

Alternatively: ERG-approach (Wetterich), sequence of theories, IR cutoff (successful use in statistical mechanics and in gravity)

define a bare theory at scale \( \Lambda \).

The integration of the modes in the interval \([k, \Lambda]\) defines a \( k \)-dependent average functional.

Letting \( k \) flowing down to 0 defines a flow for the functional which leads to full theory. \( k \)-dependent effective action:

\[
e^{-\Gamma_k[\phi]} = \int [d\varphi] \mu_k e^{-S[\varphi]} + \int_x (\varphi - \phi)_x \frac{\delta \Gamma_k[\phi]}{\delta \phi_x} - \Delta S_k[\varphi - \phi]
\]
Taking a derivative with respect the RG time $t=\log (k/k_0)$ one obtains flow equation:

$$\partial_t \Gamma_k = \frac{1}{2} Tr \left[ \left( \Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right] - \frac{\dot{\mu}_k}{\mu_k}$$

$$\mathcal{R} = \text{regulator operator}$$

which is UV and IR finite
From this derive coupled differential equations for Green’s and vertex functions (see below)

A comment on the role of transverse distances and cutoff in transverse momentum:

1) pp scattering at present energies: transverse extension grows with $s$
2) growth of total cross section varies with transverse size of projectiles
BFKL in $\gamma^*\gamma^*$, $\gamma^*p$ in DIS, pp

Trend: transverse size grows with energy, intercept decreases with size
$\rightarrow$ IR cutoff in transverse momentum is physical
This talk:
only the first steps

1) Existence of a theory in the IR limit:
fixed point in the space of reggeon field theories: existence of theory
Properties of the fixed point theory

2) How to approach the fixed point

Serious approximation: local approximation

in the UV, BFKL -Pomeron is composite field, nonlocal kernels
Solve flow equations, search for fixed points

\[
\Gamma[\psi^\dagger, \psi] = \int d^2x \, d\tau \left( Z \left( \frac{1}{2} \psi^\dagger \partial^\rightarrow \psi - \alpha' \psi^\dagger \nabla^2 \psi \right) + V[\psi^\dagger, \psi] \right), \quad \alpha(0) - 1 = \mu/Z
\]

\[
V[\psi^\dagger, \psi] = -\mu \psi^\dagger \psi + i \lambda \psi^\dagger (\psi^\dagger + \psi) \psi + g (\psi^\dagger \psi)^2 + g' \psi^\dagger (\psi^\dagger \psi + \psi^2) \psi \\
+ i \lambda_5 \psi^\dagger^2 (\psi^\dagger + \psi) \psi^2 + i \lambda_5' \psi^\dagger (\psi^3 + \psi^3) \psi + \ldots
\]

After introducing a regulator: all parameters become k-dependent

\[
\Gamma_k[\psi^\dagger, \psi] = \int d^2x \, d\tau \left( Z_k \left( \frac{1}{2} \psi^\dagger \partial^\rightarrow \psi - \alpha_k' \psi^\dagger \nabla^2 \psi \right) + \psi^\dagger R_k \psi + V_k[\psi, \psi^\dagger] \right)
\]

There is freedom in choosing a regulator, for example:
Concretely: partial differential equation for potential $V(\psi, \psi^\dagger)$:

$$
\dot{V}_k[\tilde{\psi}^\dagger, \tilde{\psi}] = (-(D+2) + \zeta_k)\dot{V}_k[\tilde{\psi}^\dagger, \tilde{\psi}] + (D/2 + \eta_k/2)(\tilde{\psi} \frac{\partial \tilde{V}_k}{\partial \psi}|_t + \tilde{\psi}^\dagger \frac{\partial \tilde{V}_k}{\partial \psi^\dagger}|_t) + \frac{\dot{V}_k}{\alpha' \kappa D + 2}.
$$

$$
\dot{V}_k = N_D A_D (\eta_k, \zeta_k) \alpha'_k \kappa^{2+D} \frac{1 + \tilde{V}_k \tilde{\psi} \tilde{\psi}^\dagger}{\sqrt{1 + 2\tilde{V}_k \tilde{\psi} \tilde{\psi}^\dagger + \tilde{V}_k^2 \tilde{\psi} \tilde{\psi}^\dagger - \tilde{V}_k \tilde{\psi} \tilde{V}_k \tilde{\psi}^\dagger \tilde{\psi}^\dagger}}.
$$

Fixed point: put rhs =0

Possible ways to solve (for constant fields, approximately):

- polynomial expansion in fields around zero (beta-functions)
- polynomial expansion in fields around stationary point
- solve differential equations in the region of large fields
Results of fixed point analysis

1) Existence of a fixed point with one relevant direction

Flow in the space of parameters of the potential (couplings): reggeon mass (intercept) $\alpha(0) - 1 = \tilde{\mu}/\tilde{Z}$, triple coupling $\tilde{\lambda}$ fixed point IR attractive inside critical surface (red), repulsive along relevant direction (green)
Convergence for higher truncations (expansion around nonzero stationary point):

<table>
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<th>5</th>
<th>6</th>
<th>7</th>
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<tr>
<td>exponent $\nu$</td>
<td>0.74</td>
<td>0.75</td>
<td>0.73</td>
<td>0.73</td>
<td>0.73</td>
<td>0.73</td>
</tr>
<tr>
<td>mass $\tilde{\mu}_{eff}$</td>
<td>0.33</td>
<td>0.362</td>
<td>0.384</td>
<td>0.383</td>
<td>0.397</td>
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<tr>
<td>$i\psi_{0,\text{diag}}$</td>
<td>0.058</td>
<td>0.072</td>
<td>0.074</td>
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<td>0.074</td>
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<tr>
<td>$i\omega_0$</td>
<td>0.173</td>
<td>0.213</td>
<td>0.218</td>
<td>0.218</td>
<td>0.218</td>
<td>0.218</td>
</tr>
</tbody>
</table>

Compare with Monte Carlo result for Directed Percolation
(same universality class): $\nu = 0.73$
Shape of the effective potential (in the subspace of imaginary fields):

Extrema, location at lowest truncation:

\((\tilde{\psi}_0, \tilde{\psi}_0^\dagger) = (0, 0), \quad \left(\frac{\tilde{\mu}}{i\tilde{\lambda}}, 0\right), \quad (0, \frac{\tilde{\mu}}{i\tilde{\lambda}}), \quad \left(\frac{\tilde{\mu}}{3i\tilde{\lambda}}, \frac{\tilde{\mu}}{3i\tilde{\lambda}}\right)\).

No further structure for larger fields.
Main result of this part:

• found a candidate for fixed point (IR stable except for one relevant direction

• robust when changing truncations

• know the effective potential
First glimpse at physics

Need to find out: on which trajectory is real physics?

Look at flow of physical physical observable: Pomeron intercept \( \alpha(0) - 1 = \mu/Z \):

So far: fixed point analysis was done in terms of dimensionless variables: reggeon energy and momentum have different dimensions

\[
S = \int d^2 x d\tau \left( Z \left( \frac{1}{2} \psi^\dagger \partial^\leftrightarrow \psi - \alpha' \psi^\dagger \nabla^2 \psi \right) + V[\psi^\dagger, \psi] \right), \quad [\psi] = [\psi^\dagger] = k^{D/2}, \quad [\alpha'] = E k^{-2}.
\]

\[
\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2} \quad \tilde{\lambda}_k = \frac{\lambda_k}{Z_k^{3/2} \alpha'_k k^2} \quad k^{D/2}
\]

Evolution of physical (=dimensionful) parameters \( \mu_k, \lambda_k, \ldots \) looks quite different from dimensionless ones \( \tilde{\mu}_k, \tilde{\lambda}_k, \ldots \)
Critical subspace (red):

$\alpha(0) \to 1$

$\lambda_{triple} \to 0$

But: theory not free!

Near critical subspace (blue):

$\alpha_k(0) \to \alpha_k=0 < 1$

In the following: consider a scenario inside the critical subspace
A simple model:
single Pomeron exchange - a scaling law

\[ T_{el}(s, t) = i s \int \frac{d\omega}{2\pi} s^{i\omega} \beta_p(t) \frac{1}{Z_k(i\omega + \alpha'_k q^2) - \mu_k} \beta_p(t) \]

\[ = i s \beta_p(t) Z_k^{-1} s^{\mu_k / Z - \alpha'_k q^2} \beta_p(t). \]

For small k:

\[ T_{el}(s, t) \sim i s k^{\eta} s^{k^{(2-\zeta)}} \tilde{\mu}_f f(\ln s q^2 k^{-\zeta}) \]

\[ \eta \approx -0.331 (-0.6), \ \zeta \approx 0.172 (0.28). \]

anomalous dimensions : directed percolation

Assume: for very large energies

\[ \alpha'_k k^2 \sim \frac{1}{\ln s} \]

\[ (R^2 \sim \frac{1}{k^2} \sim R_0^2 + \alpha'_k \ln s) \]

\[ T_{el}(s, t) \sim i s (\ln s)^{-\eta/(2-\zeta)} s^{(\ln s)^{-1} \tilde{\mu}_f p} f(t(\ln s)^{2/(2-\zeta)}) \]
Triple Pomeron cross section:

\[
\frac{d\sigma}{dtdM^2} = \frac{1}{16\pi M^2} \int \frac{d\omega}{2\pi i} \int \frac{d\omega_1}{2\pi i} \int \frac{d\omega_2}{2\pi i} \left( \frac{s}{M^2} \right)^{\omega_1 + \omega_2} \left( \frac{M^2}{M_0^2} \right)^{\omega}
\]

\[
\beta(0) \frac{1}{Z_k i \omega - \mu_k} \lambda_k \frac{1}{Z_k (i \omega_1 + \alpha_k q^2) - \mu_k} \frac{1}{Z_k (i \omega_2 + \alpha_k' q^2) - \mu_k} \beta(t)^2.
\]

Additional energy dependence:

\[
\lambda_k / Z_k^3 \sim (\ln s)^{-1 + \frac{1 - 3/2\eta}{2 - \zeta}}
\]
Comparison with previous work:

2 x Gribov, Migdal
Abarbanel, Bronzan
Migdal, Polyakov, Ter-Martirosyan

Question: how could a truly asymptotic theory of Pomerons look like?
Impose obvious condition: (renormalized) intercept must be at one

RG analysis of RFT with triple coupling near $D=4$:

$$T_{el}(s, t) \sim i\sigma(\ln s)^{-\frac{\eta}{2}} F(t(\ln s)^{\frac{z}{2}})$$

$$= i\sigma(\ln s)^{-\frac{\eta}{2}} F(t(\ln s)^{\frac{2}{2}})$$

For comparison: we did not impose condition on intercept

$$T_{el}(s, t) \sim i\sigma(\ln s)^{-\frac{\eta}{2}} \tilde{\mu}_{FP} f(t(\ln s)^{\frac{2}{2}})$$

↑

Difference in intercept

Closer to real physics!
Conclusions

1) Defined the framework (ERG) for reggeon field theory

2) Have studied the IR (long distance) limit of a general class of Reggeon Field Theories:

there exists a fixed point which describes an acceptable effective theory
Desirable improvements: get away from the local approximation

3) First attempt to connect with reality:

intercept at finite energies above one, approaches zero at infinite energies,
qualitative agreement with real physics.
Takes care of finite transverse size

4) Next step:

go to the UV region (BFKL, perturbative QCD reggeon field theory),
connect with IR region.
Backup slides
Energy dependence of total cross sections varies with transverse size:

\[ \gamma \gamma \quad \sigma_{\text{tot}} \approx S^{\omega_{\text{BFKL}}} \]

calculable in pQCD

\[ \gamma p \quad \sigma_{\text{tot}} \approx (W^2)^\lambda \]

Partly calculable in pQD

\[ pp \quad \sigma_{\text{tot}} \approx S^{0.08} \]

nonperturbative

Small: strong rise ---- large: slow rise

HERA forward jets

LEP

HERA

LHC

Figure 6: The slope \( \lambda_{\text{eff}} \) of \( F_2 \) as a function of \( Q^2 \).
Question: how to continue small-x physics from pQCD to the nonperturbative region?

Regge description: 2+1-dimensional field theory
- (1) short distance region: BFKL, reggeon field theory of reggeized gluons
- (2) saturation: nonlinear BK-equation (fan diagrams)
- (∞) soft region (large distances): Regge poles (DL, Kaidalov, Tel Aviv, Durham)
Tentative interpretation: different phases:

n-1 dim. critical subspace: massless divides the n-dimensional space into two (subcritical, supercritical) half spaces

Which phase: depends upon starting point at k=0 (UV)
Possible interpretation of IR cutoff, evolution time \( \tau = \ln \frac{k}{k_0} \):

**IR-cutoff:** \( k^2 \sim 1/\text{transverse distance}^2 \sim 1/\ln s \)

\[
R^2 = R_a^2 + R_b^2 + \alpha' \ln s
\]
Possible physical scenario:

\[ \tilde{\lambda} \]

\[ \tilde{\mu} \]

\[ t \rightarrow - \infty \quad \mu = \alpha(0) - 1 \rightarrow 0, \lambda \rightarrow 0 \]

\[ t = 0 \quad \mu = \alpha(0) - 1 > 0 \]
Vertex functions, Green’s functions, physical observables: take functional derivatives w.r.t. the fields:

\[
\partial_t \Gamma_k = \frac{1}{2} G_{k;AB} \partial_t R_{k;BA}
\]

\[
\partial_t \Gamma_{k;A_1}^{(1)} = -\frac{1}{2} G_{k;AB} \Gamma_{k;A_1BC}^{(3)} G_{k;CD} \partial_t R_{k;DA}
\]

\[
\partial_t \Gamma_{k;A_1A_2}^{(2)} = \frac{1}{2} G_{k;AB} \Gamma_{k;A_1BC}^{(3)} G_{k;CD} \Gamma_{k;A_2DE}^{(3)} G_{k;EF} \partial_t R_{k;FA}
\]
\[
+ \frac{1}{2} G_{k;AB} \Gamma_{k;A_2BC}^{(3)} G_{k;AB} \Gamma_{k;A_1BC}^{(3)} G_{k;CD} \partial_t R_{k;DA}
\]
\[
- \frac{1}{2} G_{k;AB} \Gamma_{k;A_1A_2BC}^{(4)} G_{k;CD} \partial_t R_{k;DA}
\]

coupled partial differential equations
First step:

Expand the potential in powers of fields, derive beta-functions for parameters of the potential (coupling constants):

\[ 
\dot{\tilde{\mu}} = \tilde{\mu}(-2 + \zeta + \eta) + 2 N_D A_D(\eta_k, \zeta_k) \frac{\tilde{\lambda}^2}{(1 - \tilde{\mu})^2}, 
\]

\[ 
\dot{\tilde{\lambda}} = \tilde{\lambda} \left((-2 + \zeta + \frac{D}{2} + \frac{3\eta}{2}) + 2 N_D A_D(\eta_k, \zeta_k) \left( \frac{4\tilde{\lambda}^2}{(1 - \tilde{\mu})^3} + \frac{(\tilde{g} + 3\tilde{g}')}{(1 - \tilde{\mu})^2} \right) \right), 
\]

\[ 
\dot{\tilde{g}} = \tilde{g}(-2 + D + \zeta + 2\eta) + 2 N_D A_D(\eta_k, \zeta_k) \left( \frac{27\tilde{\lambda}^4}{(1 - \tilde{\mu})^4} + \frac{(16\tilde{g} + 24\tilde{g}')\tilde{\lambda}^2}{(1 - \tilde{\mu})^3} + \frac{\tilde{g}^2 + 9\tilde{g}'^2}{(1 - \tilde{\mu})^2} \right), 
\]

\[ 
\dot{\tilde{g}'} = \tilde{g}'(-2 + D + \zeta + 2\eta) + 2 N_D A_D(\eta_k, \zeta_k) \left( \frac{12\tilde{\lambda}^4}{(1 - \tilde{\mu})^4} + \frac{(4\tilde{g} + 18\tilde{g}')\tilde{\lambda}^2}{(1 - \tilde{\mu})^3} + \frac{3\tilde{g}\tilde{g}'}{(1 - \tilde{\mu})^2} \right), 
\]

Fixed points: zeroes of the beta-functions
First results: fixed points

Local reggeon field theory:

\[ \mathcal{L} = \left( \frac{1}{2} \psi^\dagger \partial_y \psi - \alpha' \psi^\dagger \nabla^2 \psi \right) + V(\psi, \psi^\dagger) \]

\[ V(\psi, \psi^\dagger) = -\mu \psi^\dagger \psi + i\lambda \psi^\dagger (\psi^\dagger + \psi) \psi \]

\[ + g(\psi^\dagger \psi)^2 + g' \psi^\dagger (\psi^\dagger^2 + \psi^2) \psi + \ldots \]

\[ \mu = \alpha(0) - 1 \]

Some history:
In early seventies: first studies of RFT with triple couplings, expansion near D=4 (\(\varepsilon\)-expansion). IR-fixed point.

In 1980: J. Cardy and R. Sugar noticed that the RFT is in the same universality class of a Markov process known as Directed Percolation (DP).
Critical exponents can then be accessed also with numerical montecarlo computations.

This attempt:
search in the full space of theories, no restriction to D=4

Gribov, Migdal; Abarbanel, Bronzan; Migdal, Polyakov, Ter-Martirosyan

Sunday, September 4, 16
Effective action with local potential:

\[ \Gamma_k = \int dy \, d^D x \left[ Z_k \left( \frac{1}{2} \psi^{\dagger} \partial_y \psi - \alpha'_k \psi^{\dagger} \nabla^2 \psi \right) + V_k(\psi, \psi^\dagger) \right] \]

Propagator of flow equations:

\[ \Gamma_k^{(2)} + \mathbb{R} = \begin{pmatrix} V_{k\psi\psi} & -iZ_k \omega + Z_k \alpha'_k q^2 + R_k + V_k \psi \psi^{\dagger} \\ iZ_k \omega + Z_k \alpha'_k q^2 + R_k + V_k \psi \psi^{\dagger} & V_{k\psi\psi^{\dagger}} \end{pmatrix} \]

Flow equation for potential:

\[ \dot{V}_k(\psi, \psi^\dagger) = \frac{1}{2} \text{tr} \left\{ \int \frac{d\omega \, d^D q}{(2\pi)^{D+1}} \left[ \left( \Gamma_k^{(2)} + \mathbb{R} \right)^{-1} \dot{\mathcal{R}}_k \right] \right\} \]

Coarse graining in momentum space:

\[ R_k(q) = Z_k \alpha'_k (k^2 - q^2) \Theta(k^2 - q^2) \]