

Exclusive diffractive production of pion pairs and resonances in proton-proton collisions within tensor pomeron approach



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DIFFRACTION 2016

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Plan

- 1) Diffractive mechanism of $\pi^+\pi^-$ production (continuum, scalar and tensor resonances)
- 2) Photoproduction mechanism (ρ^0 and non-resonant (Drell-Söding) contributions)
- 3) Diffractive production of $\pi^+\pi^-\pi^+\pi^-$ via the intermediate $\sigma\sigma$ and $\rho\rho$ states

- P. Lebiedowicz, O. Nachtmann, A. Szczurek, *Exclusive central diffractive production of scalar and pseudoscalar mesons; tensorial vs. vectorial pomeron*, [arXiv:1309.3913](#), *Annals Phys.* **344** (2014) 301
- P. Lebiedowicz, O. Nachtmann, A. Szczurek, ρ^0 and Drell-Söding contributions to central exclusive production of $\pi^+\pi^-$ pairs in proton-proton collisions at high energies, [arXiv:1412.3677](#), *Phys. Rev. D* **91** (2015) 07402300
- P. Lebiedowicz, O. Nachtmann, A. Szczurek, *Central exclusive diffractive production of the $\pi^+\pi^-$ continuum, scalar and tensor resonances in pp and $p\bar{p}$ scattering within the tensor Pomeron approach*, [arXiv:1601.04537](#), *Phys. Rev. D* **93** (2016) 054015
- P. Lebiedowicz, O. Nachtmann, A. Szczurek, *Exclusive diffractive production of $\pi^+\pi^-\pi^+\pi^-$ via the intermediate $\sigma\sigma$ and $\rho\rho$ states in proton-proton collisions within tensor Pomeron approach*, [arXiv:1606.05126](#), *Phys. Rev. D* **94** (2016) 034017

The nature of soft pomeron

High-energy small-angle hadron-hadron scattering is dominated by the exchange of the soft pomeron. It is clear that it has vacuum internal quantum numbers.

What is much less clear is the spin structure of the soft pomeron.

- However, a vector pomeron is widely used in the literature [Donnachie and Landshoff]
- In [C. Ewerz, M. Maniatis, O. Nachtmann, *Annals Phys.* 342 (2014) 31] it was proposed to describe the soft pomeron as an effective rank-2 symmetric tensor exchange.

There, the all reggeon exchanges with $C = +1$ ($C = -1$) were described as effective tensor (vector) exchanges and a large number of the couplings of these objects to hadrons were determined from experimental data.

- In [C. Ewerz, P. Lebiedowicz, O. Nachtmann, A. Szczurek, *Helicity in proton-proton elastic scattering and the spin structure of the pomeron*, arXiv:1606.08067] we have confronted three hypotheses for the soft pomeron – tensor, vector, and scalar – with experimental data on polarised high-energy pp elastic scattering from the STAR collaboration.

pp elastic scattering

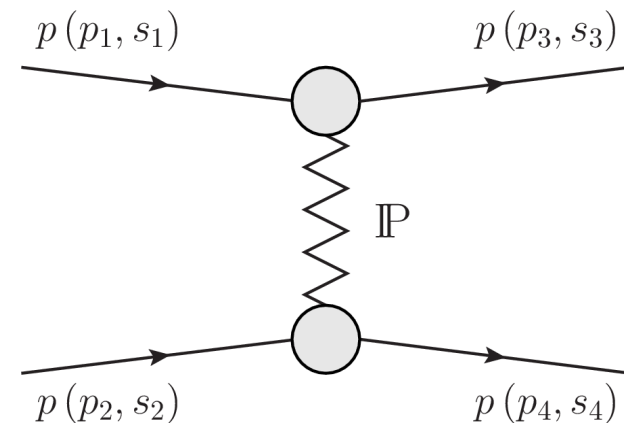
There are 16 helicity amplitudes defined as the \mathcal{T} -matrix elements

$$\langle p(p_3, s_3), p(p_4, s_4) | \mathcal{T} | p(p_1, s_1), p(p_2, s_2) \rangle \equiv \langle 2s_3, 2s_4 | \mathcal{T} | 2s_1, 2s_2 \rangle$$

$$s_j \in \{1/2, -1/2\}, \quad j = 1, \dots, 4.$$

and only five are independent

$$\begin{aligned} \phi_1(s, t) &= \langle ++ | \mathcal{T} | ++ \rangle, \\ \phi_2(s, t) &= \langle ++ | \mathcal{T} | -- \rangle, \\ \phi_3(s, t) &= \langle +- | \mathcal{T} | +- \rangle, \\ \phi_4(s, t) &= \langle +- | \mathcal{T} | -+ \rangle, \\ \phi_5(s, t) &= \langle ++ | \mathcal{T} | +- \rangle. \end{aligned}$$



The amplitudes with no helicity flip are ϕ_1 and ϕ_3 , with single flip ϕ_5 , and with double flip ϕ_2 and ϕ_4 .

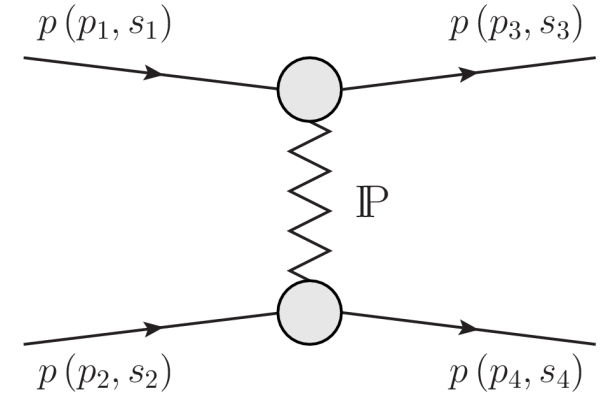
The total cross section for unpolarised protons is

$$\begin{aligned} \sigma_{tot}(pp) &= \frac{1}{s(s - 4m_p^2)} \frac{1}{4} \sum_{s_1, s_2} \text{Im} \langle 2s_3, 2s_4 | \mathcal{T} | 2s_1, 2s_2 \rangle |_{t=0} \\ &= \frac{1}{2\sqrt{s(s - 4m_p^2)}} \text{Im} [\phi_1(s, 0) + \phi_3(s, 0)] \end{aligned}$$

Tensor and scalar pomeron ansatz

Tensor pomeron

$$i < 2s_3, 2s_4 | \mathcal{T} | 2s_1, 2s_2 > = i < p(p_3, s_3) | J_{T\mu\nu}(0) | p(p_1, s_1) > \\ \times i \Delta^{(\mathbb{P}_T)\mu\nu, \kappa\lambda}(s, t) \\ \times i < p(p_4, s_4) | J_{T\kappa\lambda}(0) | p(p_2, s_2) >$$



$$< p(p', s') | J_{T\mu\nu}(0) | p(p, s) > = \bar{u}(p', s') \Gamma_{\mu\nu}^{((\mathbb{P}_T)pp)}(p', p) u(p, s)$$

see C. Ewerz, M. Maniatis, O. Nachtmann, *Annals Phys.* 342 (2014) 31

$$i \Delta_{\mu\nu, \kappa\lambda}^{(\mathbb{P}_T)}(s, t) = \frac{1}{4s} \left(g_{\mu\kappa} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\kappa} - \frac{1}{2} g_{\mu\nu} g_{\kappa\lambda} \right) (-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$i \Gamma_{\mu\nu}^{(\mathbb{P}_T)pp)}(p', p) = i \Gamma_{\mu\nu}^{(\mathbb{P}_T \bar{p} \bar{p})}(p', p) = -i 3 \beta_{\mathbb{P}NN} F_1((p' - p)^2) \left\{ \frac{1}{2} [\gamma_\mu(p' + p)_\nu + \gamma_\nu(p' + p)_\mu] - \frac{1}{4} g_{\mu\nu} (\not{p}' + \not{p}) \right\}$$

$$\beta_{\mathbb{P}NN} = 1.87 \text{ GeV}^{-1}. \quad F_1(t) = \frac{4m_p^2 - 2.79t}{(4m_p^2 - t)(1 - t/m_D^2)^2}, \quad m_D^2 = 0.71 \text{ GeV}^2$$

$$\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}} t,$$

$$\alpha_{\mathbb{P}}(0) = 1 + \epsilon_{\mathbb{P}},$$

$$\epsilon_{\mathbb{P}} = 0.0808,$$

$$\alpha'_{\mathbb{P}} = 0.25 \text{ GeV}^{-2}$$

Scalar pomeron

$$i \Delta^{(\mathbb{P}_S)}(s, t) = \frac{s}{2m_p^2 M_0^2} (-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$i \Gamma_{\mu\nu}^{(\mathbb{P}_S)pp)}(p', p) = i \Gamma_{\mu\nu}^{(\mathbb{P}_S \bar{p} \bar{p})}(p', p) = -i 3 \beta_{\mathbb{P}NN} M_0 F_1((p' - p)^2)$$

Vector pomeron ansatz

The corresponding $\mathbb{P}_V pp$ vertex and \mathbb{P}_V propagator are

$$i\Gamma_{\mu}^{(\mathbb{P}_V pp)}(p', p) = -i 3\beta_{\mathbb{P}NN} F_1[(p' - p)^2] \gamma_{\mu}$$

$$i\Delta_{\mu\nu}^{(\mathbb{P}_V)}(s, t) = \frac{1}{M_0^2} g_{\mu\nu} (-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

where $M_0 \equiv 1$ GeV is introduced for dimensional reasons.

A charge-conjugation transformation C gives

$$\langle \bar{p}(p_3, s_3) | J_{V\mu}(0) | \bar{p}(p_1, s_1) \rangle = - \langle p(p_3, s_3) | J_{V\mu}(0) | p(p_1, s_1) \rangle .$$

Thus, we get for a vector pomeron

$$\langle \bar{p}(p_3, s_3), p(p_4, s_4) | \mathcal{T} | \bar{p}(p_1, s_1), p(p_2, s_2) \rangle = - \langle p(p_3, s_3), p(p_4, s_4) | \mathcal{T} | p(p_1, s_1), p(p_2, s_2) \rangle$$

$$\sigma_{tot}(\bar{p}p) = -\sigma_{tot}(pp) .$$

Clearly, this result does not make sense for a non-vanishing cross section and would contradict the rules of QFT.

Comparison with experiment

In order to discriminate between the tensor and scalar pomeron cases we turn to data from the STAR experiment at RHIC [[L. Adamczyk et al. \(STAR Collaboration\), Phys. Lett. B719 \(2013\) 62](#)].

There, a measurement of the ratio of single-flip to non-flip amplitudes was performed. The relevant quantity is

$$r_5(s, t) = \frac{2m_p \phi_5(s, t)}{\sqrt{-t} \operatorname{Im} [\phi_1(s, t) + \phi_3(s, t)]} .$$

We find ([arXiv:1606.08067](#)) for the tensor and scalar pomeron cases

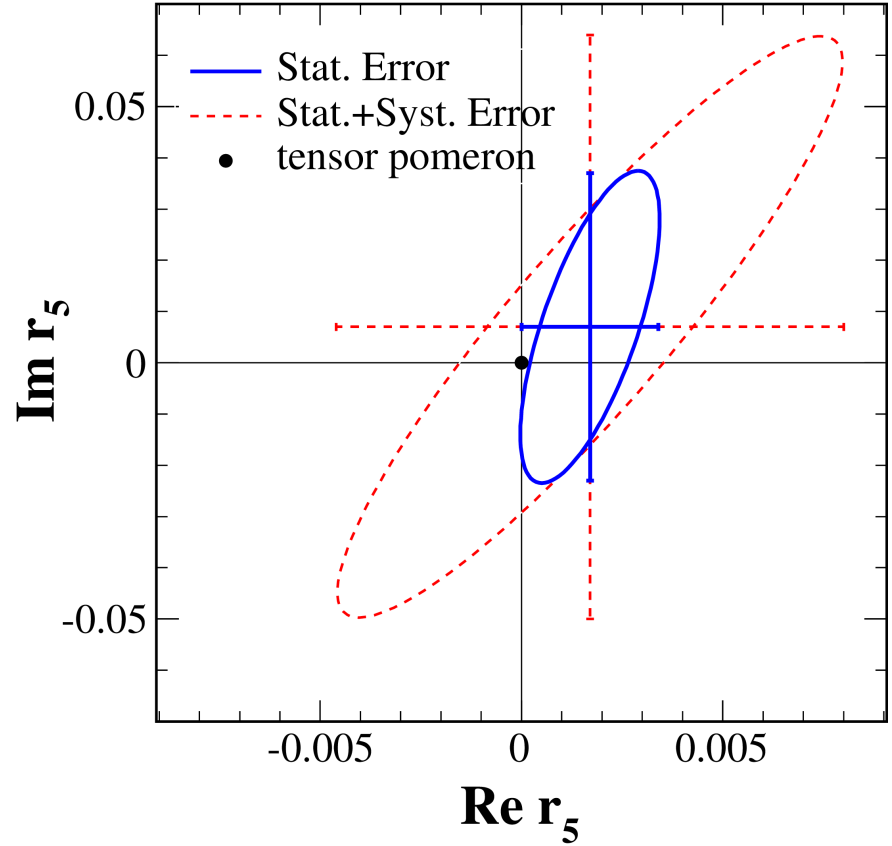
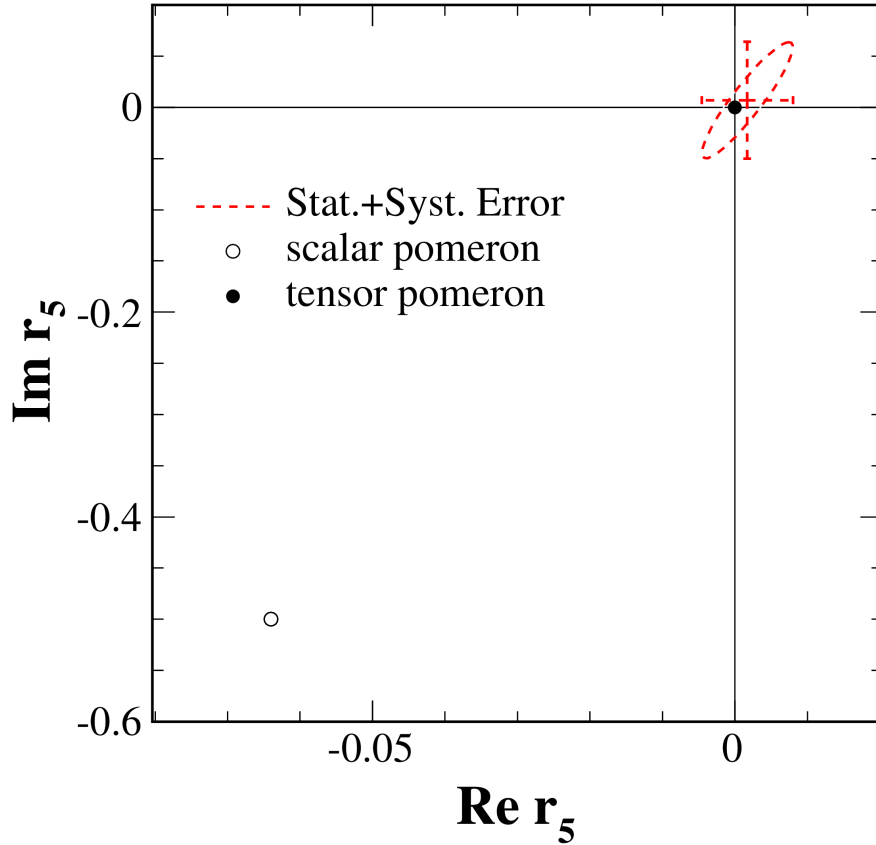
$$r_5^T(s, t) = -\frac{m_p^2}{s} \left[i + \tan \left(\frac{\pi}{2} (\alpha(t) - 1) \right) \right] ,$$

$$r_5^S(s, t) = -\frac{1}{2} \left[i + \tan \left(\frac{\pi}{2} (\alpha(t) - 1) \right) \right] .$$

The measurement of r_5 is done for $0.003 \leq |t| \leq 0.035 \text{ GeV}^2$ and no t -dependence of r_5 is observed in this range. Therefore, we can approximately set $t = 0$ and obtain with $\sqrt{s} = 200 \text{ GeV}$

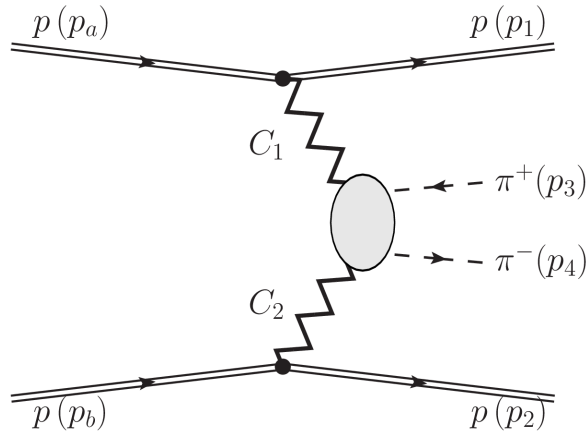
$$r_5^T(s, 0) = (-0.28 - i2.20) \times 10^{-5},$$

$$r_5^S(s, 0) = -0.064 - i0.500.$$



Clearly, the tensor-pomeron result is compatible with the experiment whereas the scalar-pomeron result is far outside the experimental error ellipse.

Dipion continuum production



$$(C_1, C_2) = (1, 1) : \quad (\mathbb{P} + f_{2\mathbb{R}}, \mathbb{P} + f_{2\mathbb{R}})$$

$$(C_1, C_2) = (-1, -1) : \quad (\rho_{\mathbb{R}} + \gamma, \rho_{\mathbb{R}} + \gamma)$$

$$(C_1, C_2) = (1, -1) : \quad (\mathbb{P} + f_{2\mathbb{R}}, \rho_{\mathbb{R}} + \gamma)$$

$$(C_1, C_2) = (-1, 1) : \quad (\rho_{\mathbb{R}} + \gamma, \mathbb{P} + f_{2\mathbb{R}})$$

Ewerz-Maniatis-Nachtmann model: Regge-type model respecting the rules of QFT to describe high-energy soft reactions

$C = +1$ exchanges (\mathbb{P} , $f_{2\mathbb{R}}$, $a_{2\mathbb{R}}$) are represented as rank-2 tensor

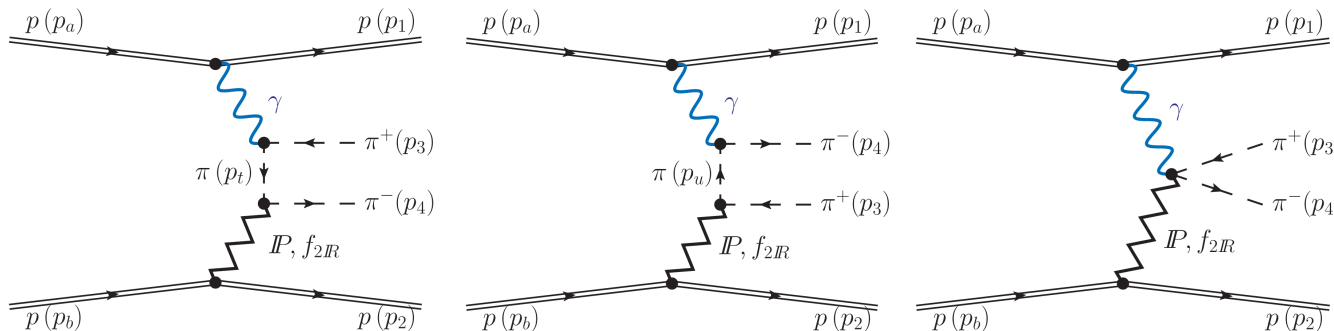
$C = -1$ exchanges (odderon (?), $\omega_{\mathbb{R}}$, $\rho_{\mathbb{R}}$) represented as vector

Exchange object	C	G
\mathbb{P}	1	1
$f_{2\mathbb{R}}$	1	1
$a_{2\mathbb{R}}$	1	-1
γ	-1	
\mathbb{O}	-1	-1
$\omega_{\mathbb{R}}$	-1	-1
$\rho_{\mathbb{R}}$	-1	1

G parity invariance forbids the vertices:

$$a_{2\mathbb{R}}\pi\pi, \omega_{\mathbb{R}}\pi\pi, \mathbb{O}\pi\pi$$

for the cases involving the photon exchange one also has to take into account the diagrams involving the contact terms



The inclusion of these diagrams is a **gauge invariant version of the Drell-Söding mechanism**.

Diffractive dipion continuum production

The full amplitude of dipion production is a sum of continuum and resonances amplitudes:

$$\mathcal{M}_{pp \rightarrow pp\pi^+\pi^-} = \mathcal{M}_{pp \rightarrow pp\pi^+\pi^-}^{\pi\pi\text{-continuum}} + \mathcal{M}_{pp \rightarrow pp\pi^+\pi^-}^{\pi\pi\text{-resonances}}$$

$$\mathcal{M}_{pp \rightarrow pp\pi^+\pi^-}^{\pi\pi\text{-continuum}} = \mathcal{M}^{(IP \bar{P} \rightarrow \pi^+\pi^-)} + \mathcal{M}^{(IP f_{2R} \rightarrow \pi^+\pi^-)} + \mathcal{M}^{(f_{2R} \bar{P} \rightarrow \pi^+\pi^-)} + \mathcal{M}^{(f_{2R} f_{2R} \rightarrow \pi^+\pi^-)}$$

The $IP \bar{P}$ - exchange amplitude can be written as

$$\mathcal{M}^{(IP \bar{P} \rightarrow \pi^+\pi^-)} = \mathcal{M}_{\lambda_a \lambda_b \rightarrow \lambda_1 \lambda_2 \pi^+\pi^-}^{(\hat{t})} + \mathcal{M}_{\lambda_a \lambda_b \rightarrow \lambda_1 \lambda_2 \pi^+\pi^-}^{(\hat{u})}$$

$$\mathcal{M}_{\lambda_a \lambda_b \rightarrow \lambda_1 \lambda_2 \pi^+\pi^-}^{(\hat{t})} =$$

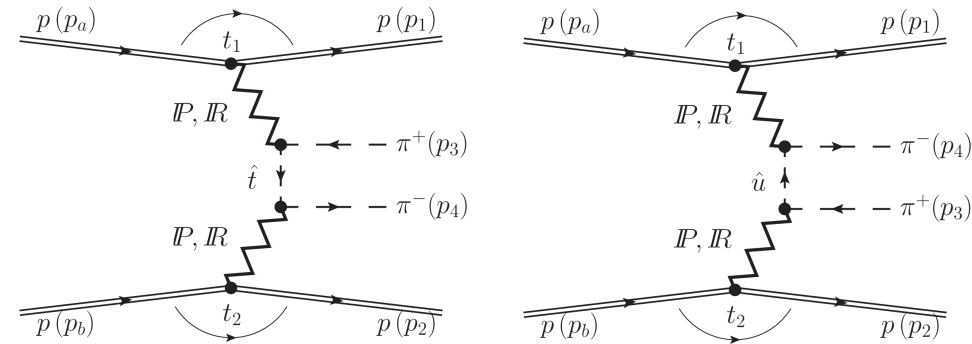
$$\begin{aligned} & (-i) \bar{u}(p_1, \lambda_1) i\Gamma_{\mu_1 \nu_1}^{(IP pp)}(p_1, p_a) u(p_a, \lambda_a) i\Delta^{(IP)}_{\mu_1 \nu_1, \alpha_1 \beta_1}(s_{13}, t_1) i\Gamma_{\alpha_1 \beta_1}^{(IP \pi\pi)}(p_t, -p_3) \\ & \times i\Delta^{(\pi)}(p_t) i\Gamma_{\alpha_2 \beta_2}^{(IP \pi\pi)}(p_4, p_t) i\Delta^{(IP)}_{\alpha_2 \beta_2, \mu_2 \nu_2}(s_{24}, t_2) \bar{u}(p_2, \lambda_2) i\Gamma_{\mu_2 \nu_2}^{(IP pp)}(p_2, p_b) u(p_b, \lambda_b) \end{aligned}$$

in terms of effective tensor pomeron propagator, proton and pion vertex functions

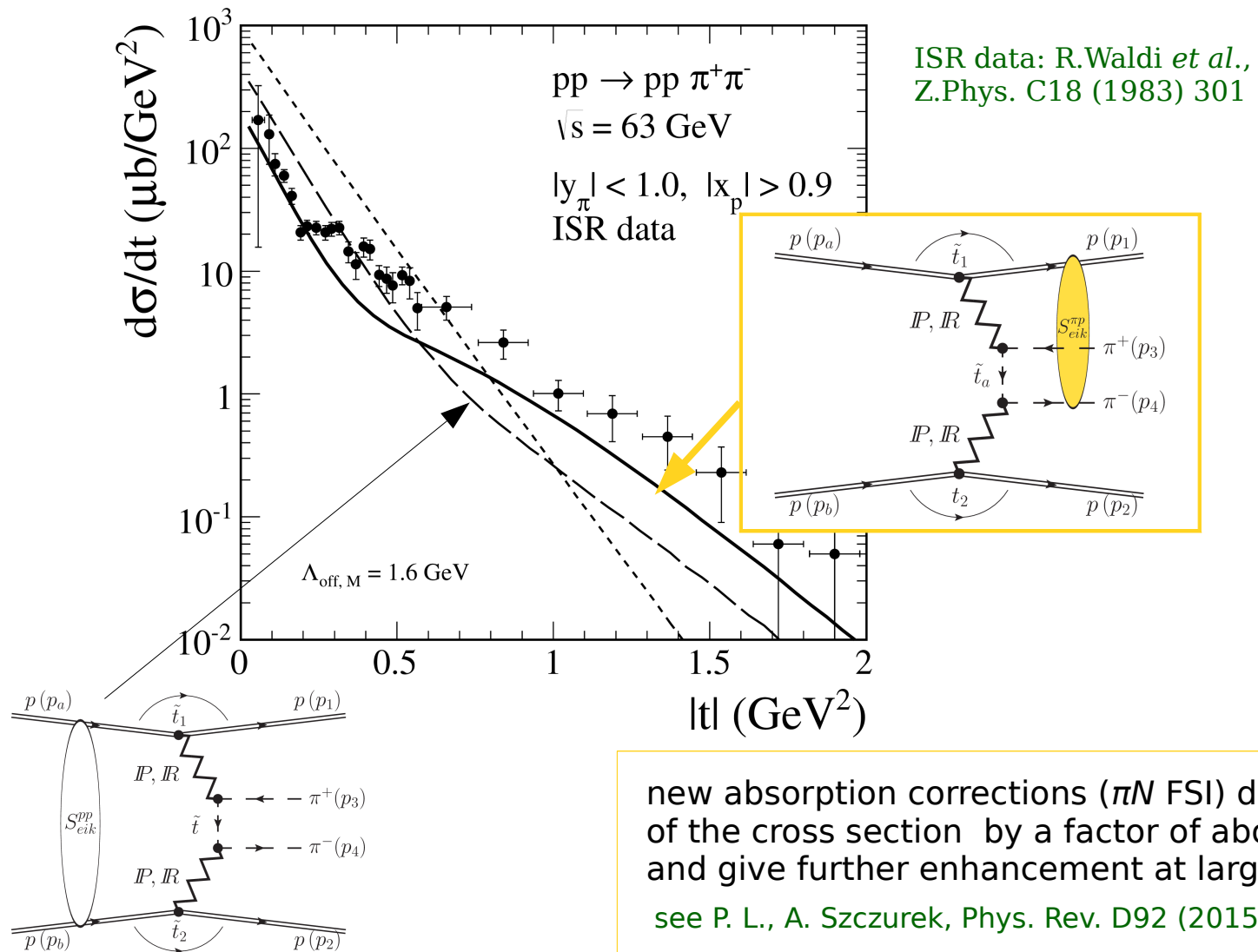
see C. Ewerz, M. Maniatis, O. Nachtmann, *Annals Phys.* 342 (2014) 31

$$i\Gamma_{\mu\nu}^{(IP \pi\pi)}(k', k) = -i2\beta_{IP\pi\pi} F_M((k' - k)^2) \left[(k' + k)_\mu (k' + k)_\nu - \frac{1}{4} g_{\mu\nu} (k' + k)^2 \right]$$

$$\beta_{IP\pi\pi} = 1.76 \text{ GeV}^{-1}, \quad F_M(t) = \frac{1}{1-t/\Lambda_0^2}, \quad \Lambda_0^2 = 0.5 \text{ GeV}^2$$



Absorption corrections, $\pi\pi$ continuum term



$$\mathcal{M}_{pp \rightarrow pp \pi^+ \pi^-} = \mathcal{M}_{pp \rightarrow pp \pi^+ \pi^-}^{\text{Born}} + \mathcal{M}_{pp \rightarrow pp \pi^+ \pi^-}^{pp\text{-rescattering}} + \mathcal{M}_{pp \rightarrow pp \pi^+ \pi^-}^{\pi p\text{-rescattering}}$$

$$\mathcal{M}_{pp \rightarrow pp \pi^+ \pi^-}^{pp\text{-rescattering}}(s, \vec{p}_{1\perp}, \vec{p}_{2\perp}) = \frac{i}{8\pi^2 s} \int d^2 \vec{k}_\perp \mathcal{M}_{pp \rightarrow pp \pi^+ \pi^-}^{\text{Born}}(s, \vec{p}_{1\perp} - \vec{k}_\perp, \vec{p}_{2\perp} + \vec{k}_\perp) \mathcal{M}_{pp \rightarrow pp}^{P\text{-exch.}}(s, -\vec{k}_\perp^2)$$

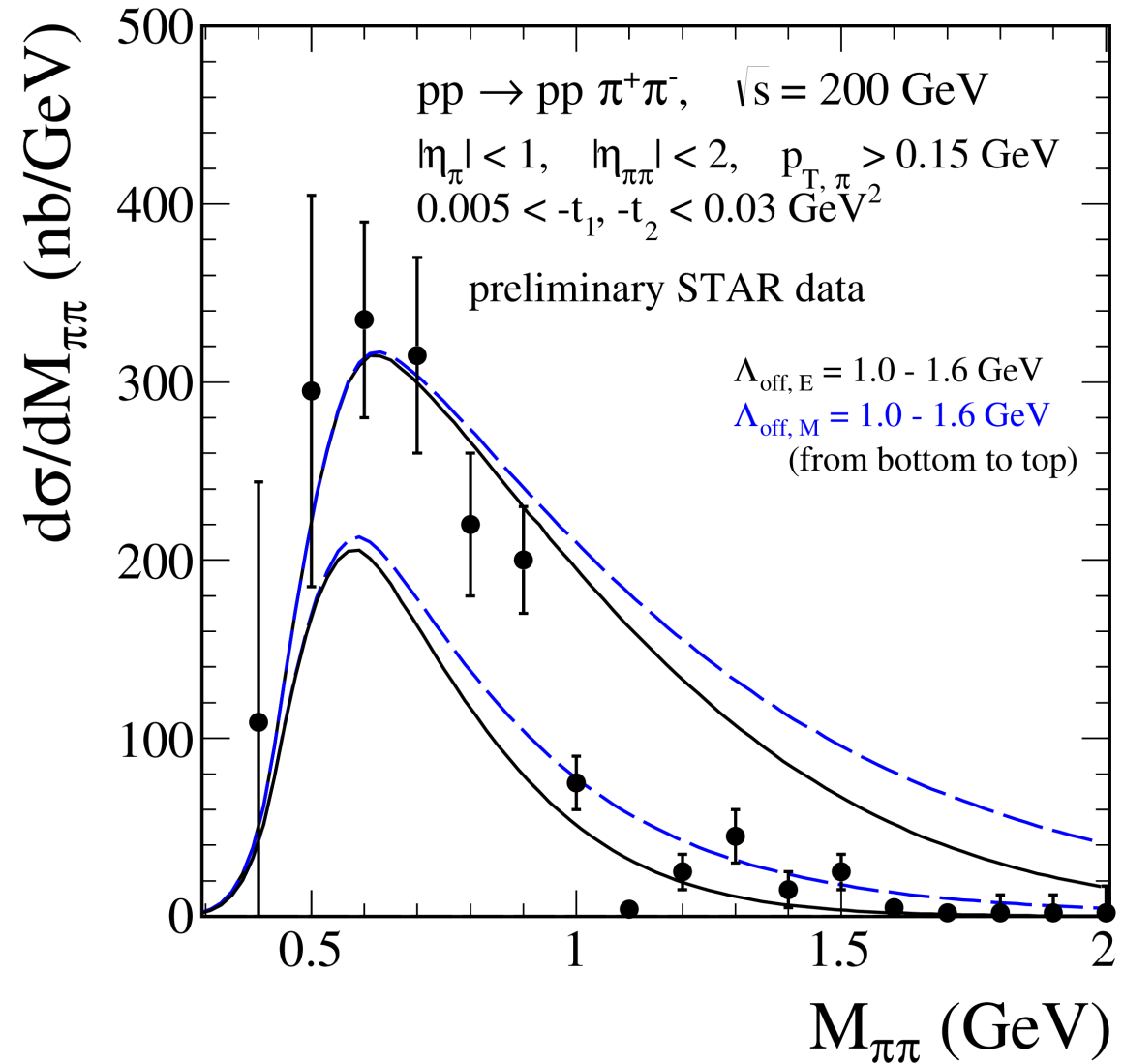
Off-shell pion form factor, $\pi\pi$ continuum term

preliminary STAR data: L. Adamczyk et al., Int.J.Mod.Phys. A29 no. 28, (2014) 1446010

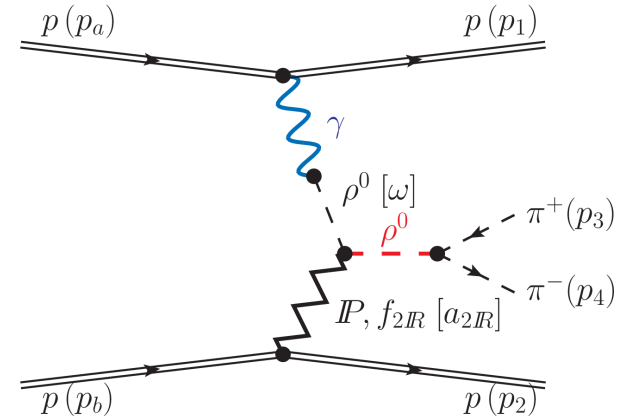
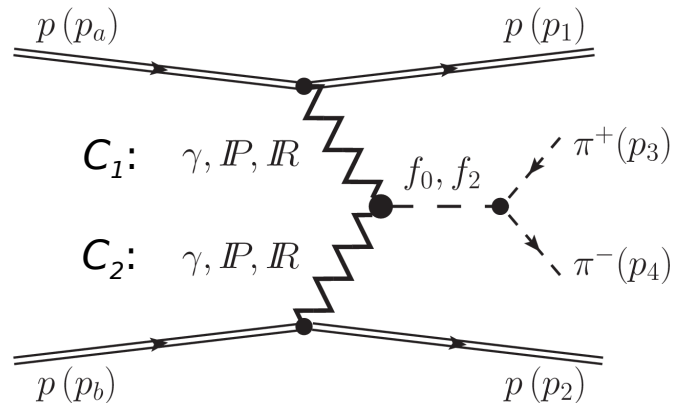
off-shell effects of the intermediate pions can be described by the form factors

$$F_\pi(\hat{t}) = \exp\left(\frac{\hat{t} - m_\pi^2}{\Lambda_{off,E}^2}\right)$$

$$F_\pi(\hat{t}) = \frac{\Lambda_{off,M}^2 - m_\pi^2}{\Lambda_{off,M}^2 - \hat{t}}$$



Dipion resonant production



In general, many exchanges are possible in the dipion resonance production process.

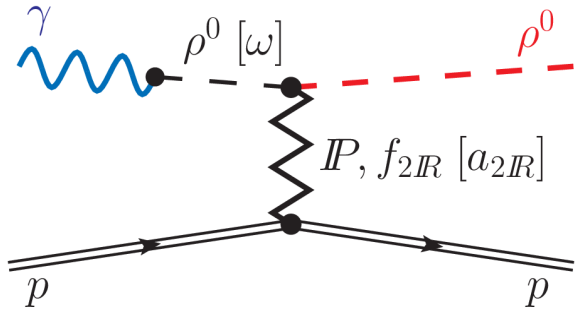
$I^G J^{PC}$, resonances	(C_1, C_2) production modes
$0^+ 0^{++}$, $f_0(500)$, $f_0(980)$, $f_0(1500)$, $f_0(1370)$, $f_0(1710)$ $0^+ 2^{++}$, $f_2(1270)$, $f_2'(1525)$, $f_2(1950)$ $0^+ 4^{++}$, $f_4(2050)$	$\left\{ (\mathbb{P} + f_{2\mathbb{R}}, \mathbb{P} + f_{2\mathbb{R}}), (a_{2\mathbb{R}}, a_{2\mathbb{R}}), \right.$ $\left. (\odot + \omega_{\mathbb{R}} + \gamma, \odot + \omega_{\mathbb{R}} + \gamma), (\rho_{\mathbb{R}}, \rho_{\mathbb{R}}), \right.$ $\left. (\gamma, \rho_{\mathbb{R}}), (\rho_{\mathbb{R}}, \gamma) \right\}$
$1^+ 1^{--}$, $\rho(770)$, $\rho(1450)$, $\rho(1700)$ $1^+ 3^{--}$, $\rho_3(1690)$	$\left\{ (\gamma + \rho_{\mathbb{R}}, \mathbb{P} + f_{2\mathbb{R}}), (\mathbb{P} + f_{2\mathbb{R}}, \gamma + \rho_{\mathbb{R}}), \right.$ $\left. (\odot + \omega_{\mathbb{R}}, a_{2\mathbb{R}}), (a_{2\mathbb{R}}, \odot + \omega_{\mathbb{R}}) \right\}$

At high energies, we shall concentrate on the dominant (C_1, C_2) contributions:

$(\mathbb{P} + f_{2\mathbb{R}}, \mathbb{P} + f_{2\mathbb{R}})$ for purely diffractive mechanism;

$(\gamma, \mathbb{P} + f_{2\mathbb{R}}), (\mathbb{P} + f_{2\mathbb{R}}, \gamma)$ for photoproduction mechanism.

Photoproduction of ρ^0 meson

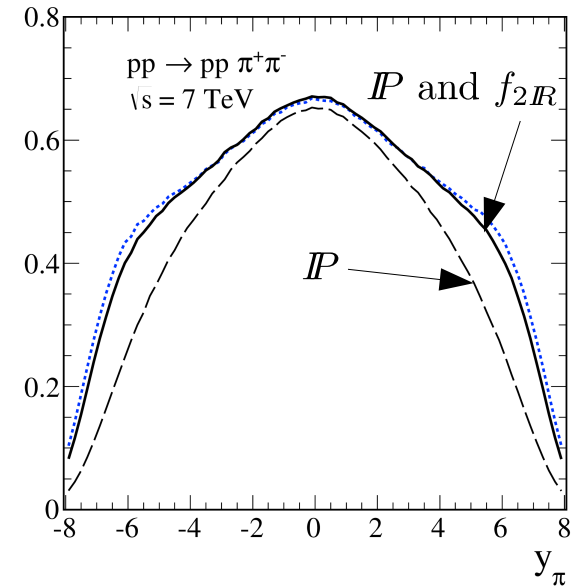
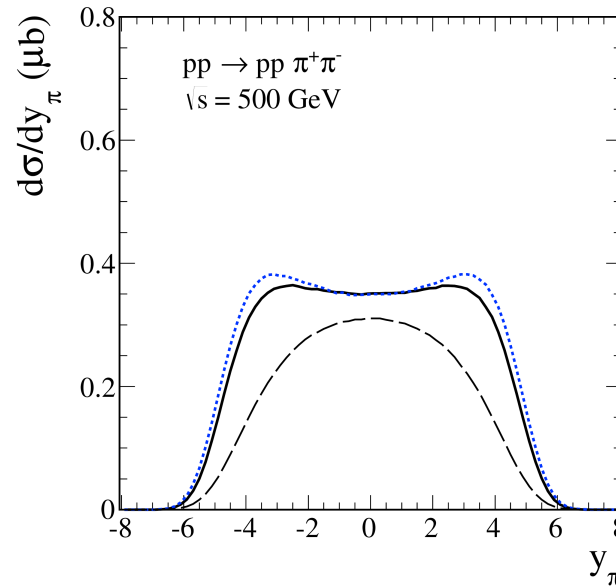
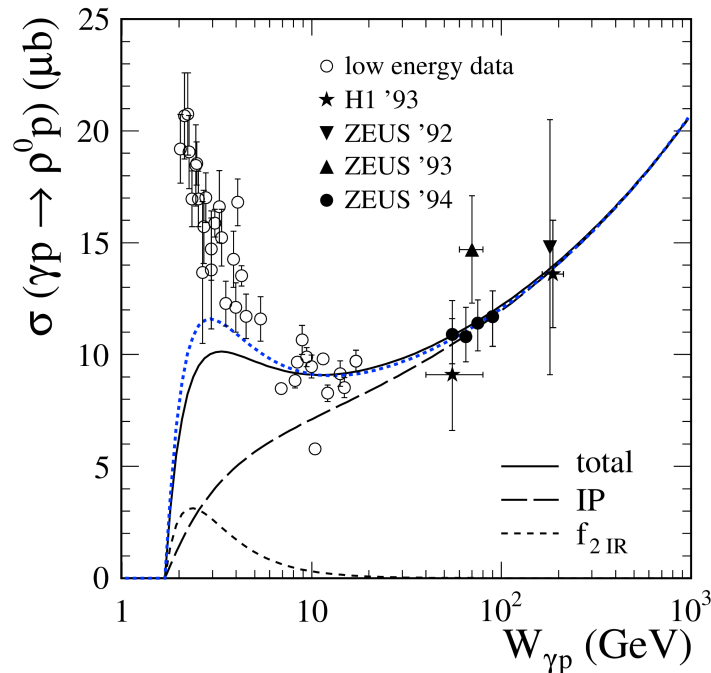


$$\mathcal{M}_{\lambda_\gamma \lambda_b \rightarrow \lambda_\rho \lambda_2}(s, t) \cong ie \frac{m_\rho^2}{\gamma_\rho} \Delta_T^{(\rho)}(0) (\epsilon^{(\rho)\mu})^* \epsilon^{(\gamma)\nu} V_{\mu\nu\kappa\lambda}(s, t, q, p_\rho) \times 2(p_2 + p_b)^\kappa (p_2 + p_b)^\lambda \delta_{\lambda_2 \lambda_b} F_1(t) F_M(t)$$

alternatively, $F_1(t) F_M(t) \rightarrow$ factorised form $F_{\rho p}^{(P/R)}(t) = \exp\left(\frac{B_{\rho p}^{(P/R)} t}{2}\right)$
(see the blue dotted line)

$$V_{\mu\nu\kappa\lambda}(s, t, q, p_\rho) = \frac{1}{4s} \left\{ 2\Gamma_{\mu\nu\kappa\lambda}^{(0)}(p_\rho, -q) \left[3\beta_{IPNN} a_{IP\rho\rho} (-is\alpha'_{IP})^{\alpha_{IP}(t)-1} + M_0^{-1} g_{f_{2R}pp} a_{f_{2R}\rho\rho} (-is\alpha'_{R+})^{\alpha_{R+}(t)-1} \right] \right. \\ \left. - \Gamma_{\mu\nu\kappa\lambda}^{(2)}(p_\rho, -q) \left[3\beta_{IPNN} b_{IP\rho\rho} (-is\alpha'_{IP})^{\alpha_{IP}(t)-1} + M_0^{-1} g_{f_{2R}pp} b_{f_{2R}\rho\rho} (-is\alpha'_{R+})^{\alpha_{R+}(t)-1} \right] \right\}$$

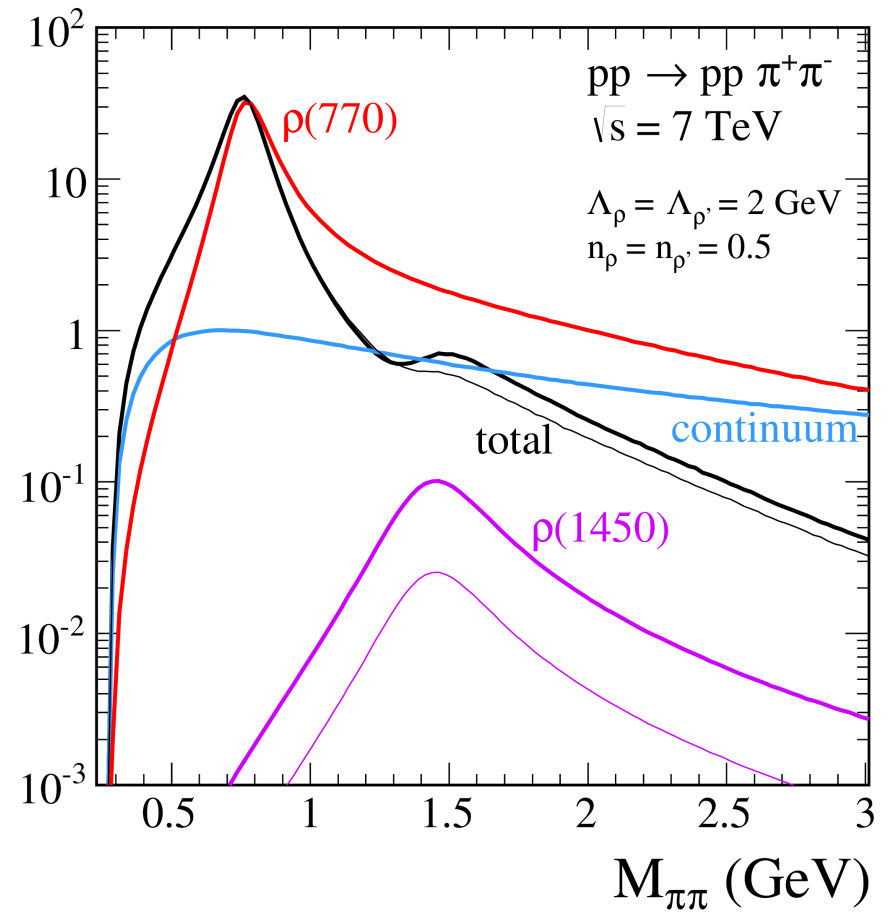
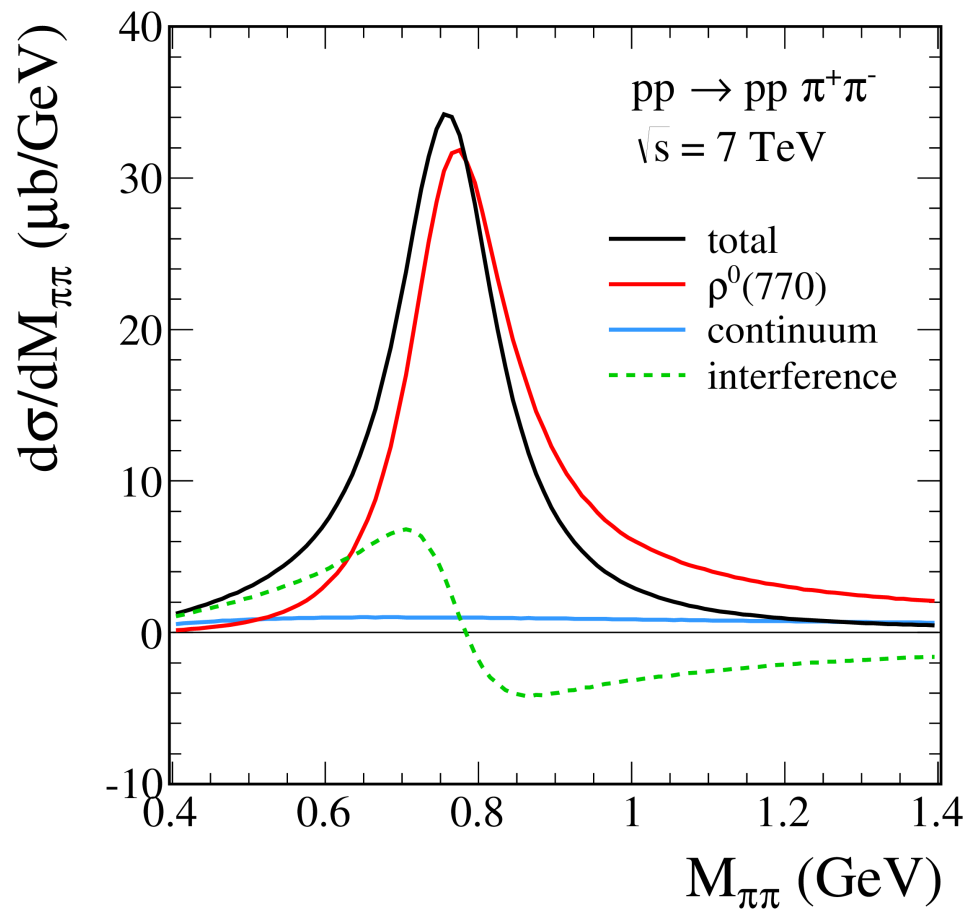
tensorial functions: *C. Ewerz, M. Maniatis and O. Nachtmann, Ann. Phys. 342 (2014) 31*



The coupling constants IP/IR - ρ - ρ have been estimated from parametrization of total cross sections for πp scattering assuming $\sigma_{tot}(\rho^0(\lambda_\rho = \pm 1), p) = \frac{1}{2} [\sigma_{tot}(\pi^+, p) + \sigma_{tot}(\pi^-, p)]$

ρ^0 and $\pi^+\pi^-$ continuum

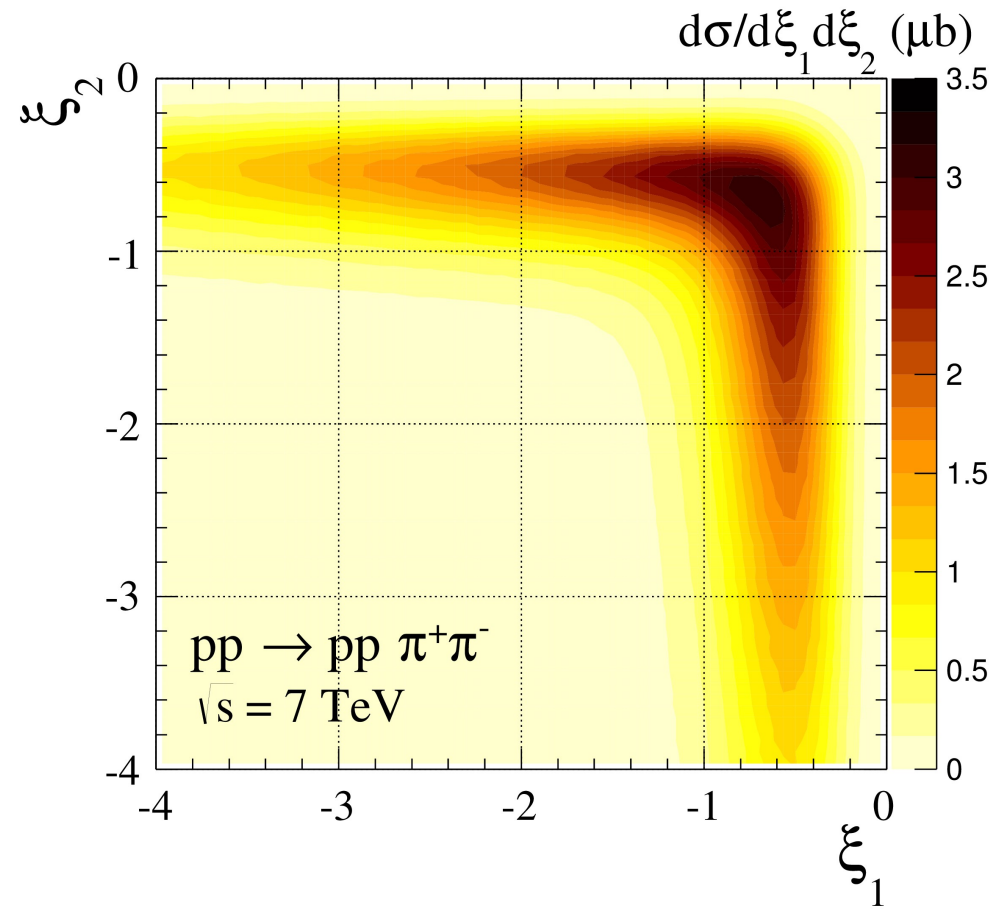
The **non-resonant (Drell-Söding)** contribution interfere with **resonant $\rho(770)$** contribution
→ skewing of ρ^0 line shape.



ρ^0 and $\pi^+\pi^-$ continuum

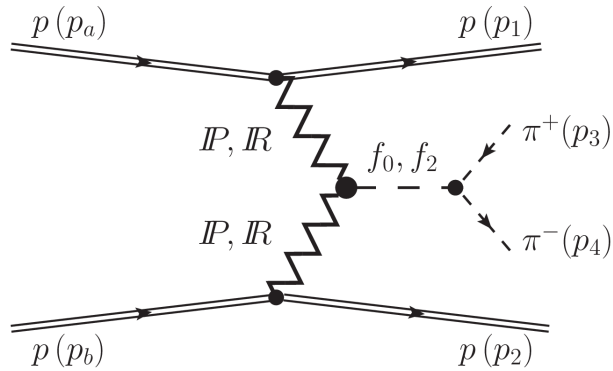
$$\xi_1 = \log_{10}(p_{1\perp}/1 \text{ GeV})$$

$$\xi_1 = -1 \text{ means } p_{1\perp} = 0.1 \text{ GeV}$$



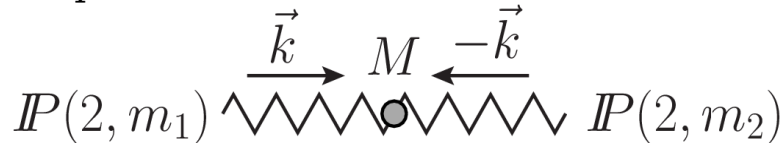
We expect the photon induced processes to be most important when at least one of the protons is undergoing only a very small $|t|$.

Pomeron-pomeron-meson couplings



M must have isospin and G parity $I^G = 0^+$ and charge conjugation $C = +1$

In table we list the values of J and P of mesons which can be produced in our fictitious reaction:



For each value of l, S, J , and P we can construct a covariant Lagrangian density \mathcal{L}' coupling (the field operator for the meson M to the pomeron fields) and the vertex corresponding to the l and S .

The values, for orbital angular momentum l , total spin S , total angular momentum J , and parity P , possible in the annihilation of two “spin 2 pomeron particles”. We have $S \in \{0, 1, 2, 3, 4\}$, $P = (-1)^l$, $|l - S| \leq J \leq l + S$, and Bose symmetry requires $l - S$ to be even.

l	S	J	P
0	0	0	+
	2	2	
	4	4	
1	1	0, 1, 2	-
	3	2, 3, 4	
2	0	2	+
	2	0, 1, 2, 3, 4	
	4	2, 3, 4, 5, 6	
3	1	2, 3, 4	-
	3	0, 1, 2, 3, 4, 5, 6	
4	0	4	+
	2	2, 3, 4, 5, 6	
	4	0, 1, 2, 3, 4, 5, 6, 7, 8	
5	1	4, 5, 6	-
	3	2, 3, 4, 5, 6, 7, 8	
6	0	6	+
	2	4, 5, 6, 7, 8	
	4	2, 3, 4, 5, 6, 7, 8, 9, 10	

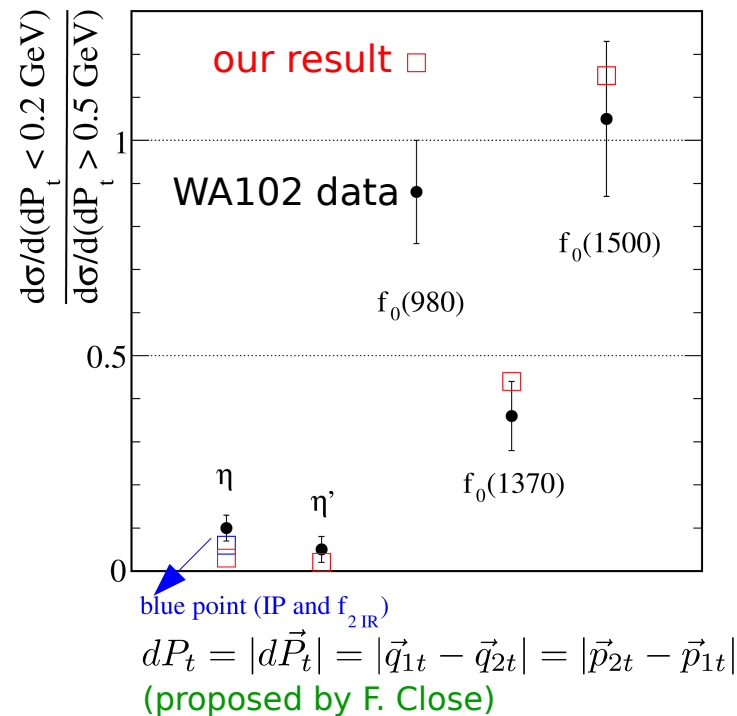
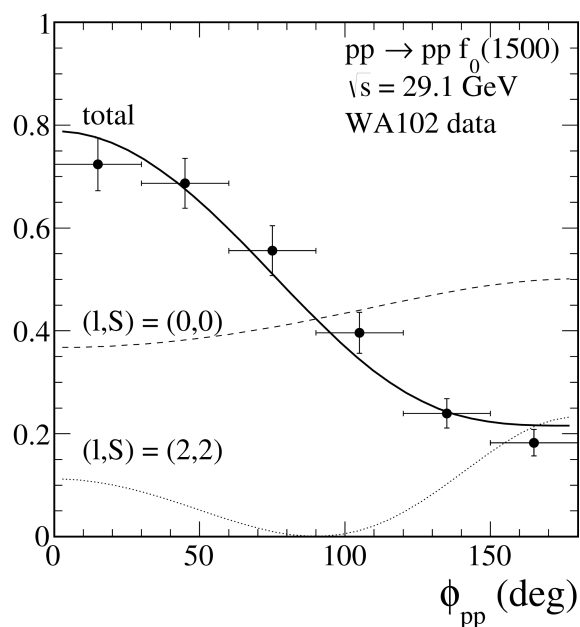
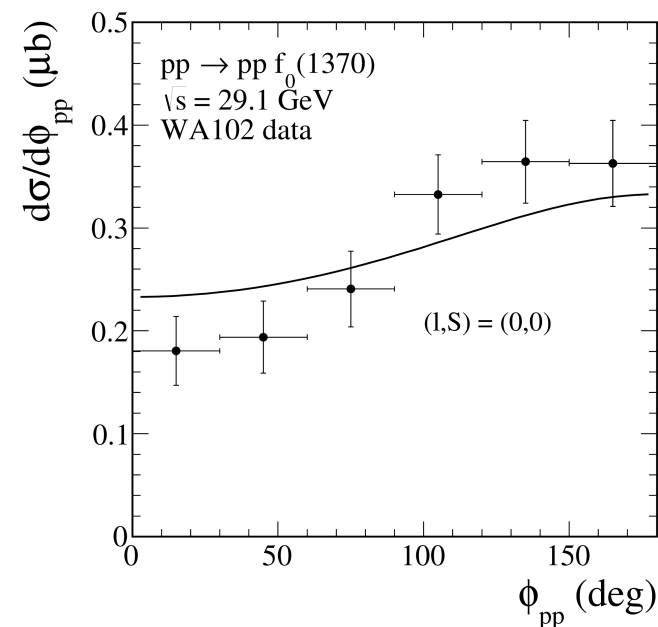
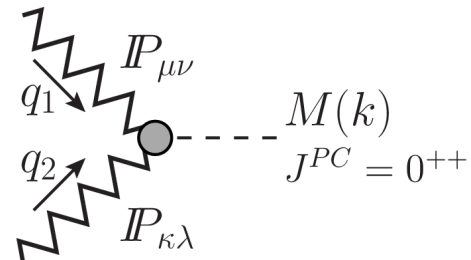
The lowest (l, S) term for a scalar meson $J^{PC} = 0^{++}$ is $(0, 0)$ while for a tensor meson $J^{PC} = 2^{++}$ is $(0, 2)$.

Scalar mesons

For a scalar mesons the “bare” tensorial IP - IP - M vertices corresponding to $(l, S) = (0, 0)$ and $(2, 2)$ terms are

$$i\Gamma_{\mu\nu, \kappa\lambda}^{(IP \rightarrow M)} = i g'_{IPM} M_0 \left(g_{\mu\kappa} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\kappa} - \frac{1}{2} g_{\mu\nu} g_{\kappa\lambda} \right)$$

$$i\Gamma_{\mu\nu, \kappa\lambda}^{(IP \rightarrow M)}(q_1, q_2) = \frac{i g''_{IPM}}{2M_0} [q_{1\kappa} q_{2\mu} g_{\nu\lambda} + q_{1\kappa} q_{2\nu} g_{\mu\lambda} + q_{1\lambda} q_{2\mu} g_{\nu\kappa} + q_{1\lambda} q_{2\nu} g_{\mu\kappa} - 2(q_1 \cdot q_2)(g_{\mu\kappa} g_{\nu\lambda} + g_{\nu\kappa} g_{\mu\lambda})]$$



Our results and WA102 data have been normalized to the mean value of the total cross section given by [A. Kirk, Phys. Lett. B489 \(2000\) 29](#).

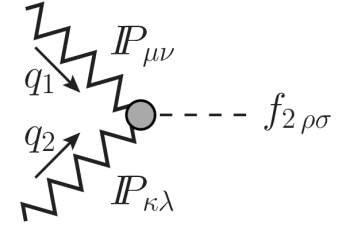
- $f_0(1370)$ peaks as $\phi_{pp} \rightarrow \pi$ whereas the $f_0(980)$, $f_0(1500)$, $f_0(1710)$ peak at $\phi_{pp} \rightarrow 0$
- $f_0(1500)$ and $f_0(1710)$ which could have a large 'gluonic component' have a large value of dP_t ratio

In most cases of scalar mesons one has to add coherently amplitudes for two lowest (l, S) couplings.

$f_2(1270)$ meson

The amplitude for the process $pp \rightarrow pp (f_2 \rightarrow \pi^+ \pi^-)$ via $\mathbb{P}\mathbb{P}$ fusion:

$$\begin{aligned} \mathcal{M}_{\lambda_a \lambda_b \rightarrow \lambda_1 \lambda_2 \pi^+ \pi^-}^{(\mathbb{P}\mathbb{P} \rightarrow f_2 \rightarrow \pi^+ \pi^-)} = & (-i) \bar{u}(p_1, \lambda_1) i\Gamma_{\mu_1 \nu_1}^{(\mathbb{P}pp)}(p_1, p_a) u(p_a, \lambda_a) i\Delta^{(\mathbb{P}) \mu_1 \nu_1, \alpha_1 \beta_1}(s_1, t_1) \\ & \times i\Gamma_{\alpha_1 \beta_1, \alpha_2 \beta_2, \rho\sigma}^{(\mathbb{P}\mathbb{P}f_2)}(q_1, q_2) i\Delta^{(f_2) \rho\sigma, \alpha\beta}(p_{34}) i\Gamma_{\alpha\beta}^{(f_2 \pi\pi)}(p_3, p_4) \\ & \times i\Delta^{(\mathbb{P}) \alpha_2 \beta_2, \mu_2 \nu_2}(s_2, t_2) \bar{u}(p_2, \lambda_2) i\Gamma_{\mu_2 \nu_2}^{(\mathbb{P}pp)}(p_2, p_b) u(p_b, \lambda_b), \end{aligned}$$



$$i\Gamma_{\mu\nu, \kappa\lambda, \rho\sigma}^{(\mathbb{P}\mathbb{P}f_2)}(q_1, q_2) = \left(i\Gamma_{\mu\nu, \kappa\lambda, \rho\sigma}^{(\mathbb{P}\mathbb{P}f_2)(1)}|_{bare} + \sum_{j=2}^7 i\Gamma_{\mu\nu, \kappa\lambda, \rho\sigma}^{(\mathbb{P}\mathbb{P}f_2)(j)}(q_1, q_2)|_{bare} \right) \tilde{F}^{(\mathbb{P}\mathbb{P}f_2)}(q_1^2, q_2^2, p_{34}^2).$$

Here $p_{34} = q_1 + q_2$ and the form factor $\tilde{F}^{(\mathbb{P}\mathbb{P}f_2)} = F_M(q_1^2)F_M(q_2^2)F^{(\mathbb{P}\mathbb{P}f_2)}(p_{34}^2)$.

$$i\Delta_{\mu\nu, \kappa\lambda}^{(f_2)}(p_{34}) = \frac{i}{p_{34}^2 - m_{f_2}^2 + im_{f_2}\Gamma_{f_2}} \left[\frac{1}{2}(\hat{g}_{\mu\kappa}\hat{g}_{\nu\lambda} + \hat{g}_{\mu\lambda}\hat{g}_{\nu\kappa}) - \frac{1}{3}\hat{g}_{\mu\nu}\hat{g}_{\kappa\lambda} \right],$$

where $\hat{g}_{\mu\nu} = -g_{\mu\nu} + p_{34\mu}p_{34\nu}/p_{34}^2$ and $\Delta_{\nu\mu, \kappa\lambda}^{(f_2)}(p_{34}) = \Delta_{\mu\nu, \lambda\kappa}^{(f_2)}(p_{34}) = \Delta_{\kappa\lambda, \mu\nu}^{(f_2)}(p_{34})$, $g^{\kappa\lambda}\Delta_{\mu\nu, \kappa\lambda}^{(f_2)}(p_{34}) = 0$.

$$i\Gamma_{\mu\nu}^{(f_2 \pi\pi)}(p_3, p_4) = -i \frac{g_{f_2 \pi\pi}}{2M_0} \left[(p_3 - p_4)_\mu (p_3 - p_4)_\nu - \frac{1}{4}g_{\mu\nu}(p_3 - p_4)^2 \right] F^{(f_2 \pi\pi)}(p_{34}^2),$$

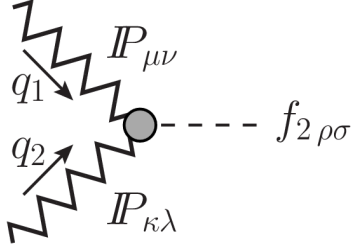
where $g_{f_2 \pi\pi} = 9.26$ was obtained from the corresponding partial decay width.

We assume that $F^{(f_2 \pi\pi)}(p_{34}^2) = F^{(\mathbb{P}\mathbb{P}f_2)}(p_{34}^2) = \exp\left(\frac{-(p_{34}^2 - m_{f_2}^2)^2}{\Lambda_{f_2}^4}\right)$, $\Lambda_{f_2} = 1 \text{ GeV}$.

IP - IP - f_2 couplings

In order to write the corresponding formulae of vertices in a compact and convenient form we find it useful to define the tensor

$$R_{\mu\nu\kappa\lambda} = \frac{1}{2}g_{\mu\kappa}g_{\nu\lambda} + \frac{1}{2}g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{4}g_{\mu\nu}g_{\kappa\lambda}$$



$$i\Gamma_{\mu\nu,\kappa\lambda,\rho\sigma}^{(IPf_2)(1)} = 2i g_{IPf_2}^{(1)} M_0 R_{\mu\nu\mu_1\nu_1} R_{\kappa\lambda\alpha_1\lambda_1} R_{\rho\sigma\rho_1\sigma_1} g^{\nu_1\alpha_1} g^{\lambda_1\rho_1} g^{\sigma_1\mu_1}$$

$$i\Gamma_{\mu\nu,\kappa\lambda,\rho\sigma}^{(IPf_2)(2)}(q_1, q_2) = -\frac{2i}{M_0} g_{IPf_2}^{(2)} \left((q_1 \cdot q_2) R_{\mu\nu\rho_1\alpha} R_{\kappa\lambda\sigma_1}{}^\alpha - q_{1\rho_1} q_2^{\mu_1} R_{\mu\nu\mu_1\alpha} R_{\kappa\lambda\sigma_1}{}^\alpha \right. \\ \left. - q_1^{\mu_1} q_{2\sigma_1} R_{\mu\nu\rho_1\alpha} R_{\kappa\lambda\mu_1}{}^\alpha + q_{1\rho_1} q_{2\sigma_1} R_{\mu\nu\kappa\lambda} \right) R_{\rho\sigma}{}^{\rho_1\sigma_1}$$

$$i\Gamma_{\mu\nu,\kappa\lambda,\rho\sigma}^{(IPf_2)(3)}(q_1, q_2) = -\frac{2i}{M_0} g_{IPf_2}^{(3)} \left((q_1 \cdot q_2) R_{\mu\nu\rho_1\alpha} R_{\kappa\lambda\sigma_1}{}^\alpha + q_{1\rho_1} q_2^{\mu_1} R_{\mu\nu\mu_1\alpha} R_{\kappa\lambda\sigma_1}{}^\alpha \right. \\ \left. + q_1^{\mu_1} q_{2\sigma_1} R_{\mu\nu\rho_1\alpha} R_{\kappa\lambda\mu_1}{}^\alpha + q_{1\rho_1} q_{2\sigma_1} R_{\mu\nu\kappa\lambda} \right) R_{\rho\sigma}{}^{\rho_1\sigma_1}$$

$$i\Gamma_{\mu\nu,\kappa\lambda,\rho\sigma}^{(IPf_2)(4)}(q_1, q_2) = -\frac{i}{M_0} g_{IPf_2}^{(4)} \left(q_1^{\alpha_1} q_2^{\mu_1} R_{\mu\nu\mu_1\nu_1} R_{\kappa\lambda\alpha_1\lambda_1} + q_2^{\alpha_1} q_1^{\mu_1} R_{\mu\nu\alpha_1\lambda_1} R_{\kappa\lambda\mu_1\nu_1} \right) R^{\nu_1\lambda_1}{}_{\rho\sigma}$$

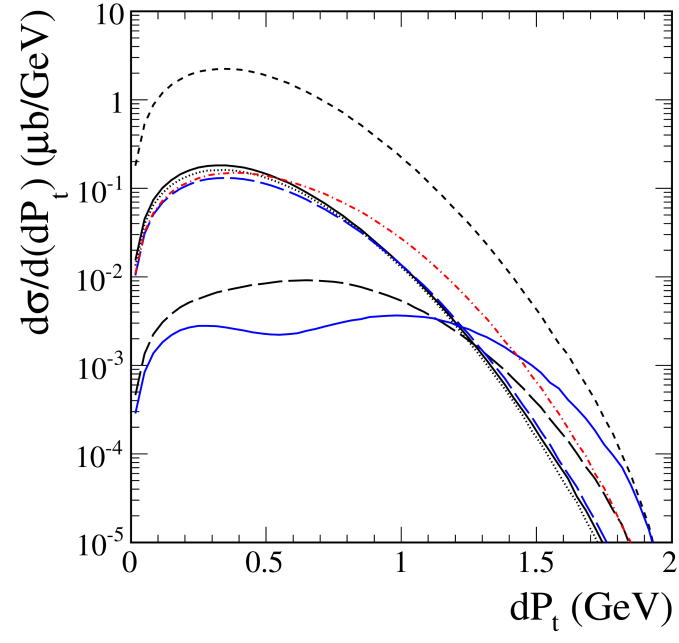
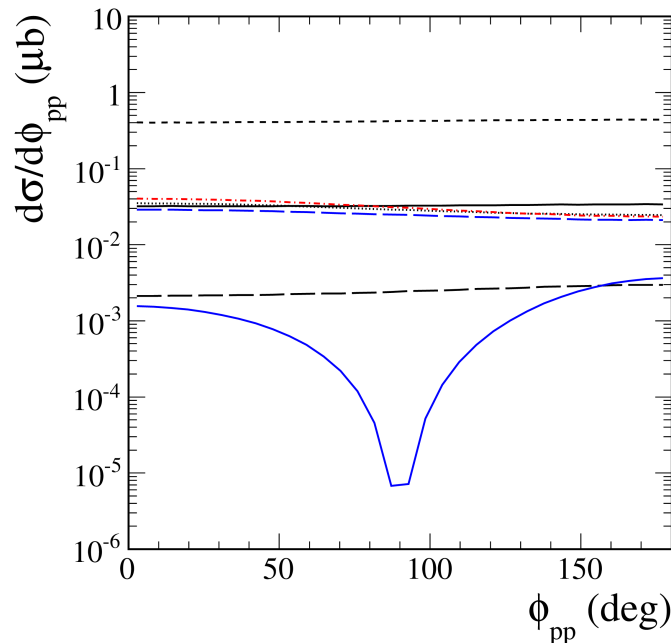
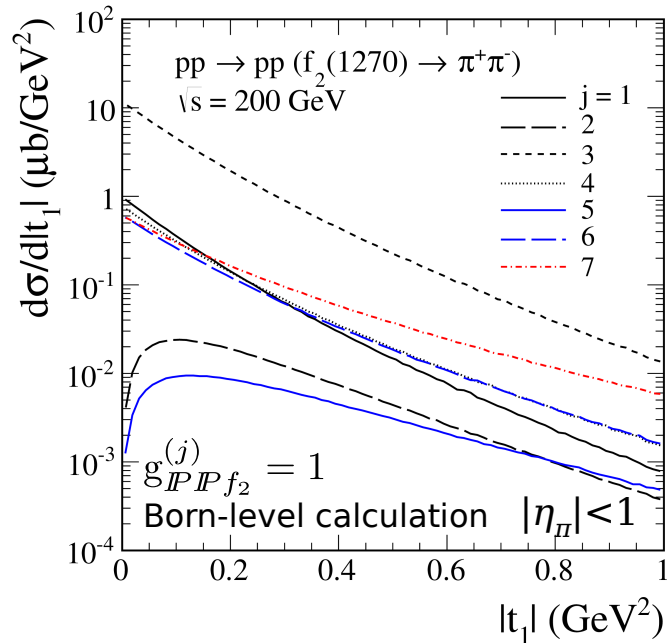
$$i\Gamma_{\mu\nu,\kappa\lambda,\rho\sigma}^{(IPf_2)(5)}(q_1, q_2) = -\frac{2i}{M_0^3} g_{IPf_2}^{(5)} \left(q_1^{\mu_1} q_2^{\nu_1} R_{\mu\nu\nu_1\alpha} R_{\kappa\lambda\mu_1}{}^\alpha + q_1^{\nu_1} q_2^{\mu_1} R_{\mu\nu\mu_1\alpha} R_{\kappa\lambda\nu_1}{}^\alpha \right. \\ \left. - 2(q_1 \cdot q_2) R_{\mu\nu\kappa\lambda} \right) q_{1\alpha_1} q_{2\lambda_1} R^{\alpha_1\lambda_1}{}_{\rho\sigma}$$

$$i\Gamma_{\mu\nu,\kappa\lambda,\rho\sigma}^{(IPf_2)(6)}(q_1, q_2) = \frac{i}{M_0^3} g_{IPf_2}^{(6)} \left(q_1^{\alpha_1} q_1^{\lambda_1} q_2^{\mu_1} q_{2\rho_1} R_{\mu\nu\mu_1\nu_1} R_{\kappa\lambda\alpha_1\lambda_1} \right. \\ \left. + q_2^{\alpha_1} q_2^{\lambda_1} q_1^{\mu_1} q_{1\rho_1} R_{\mu\nu\alpha_1\lambda_1} R_{\kappa\lambda\mu_1\nu_1} \right) R^{\nu_1\rho_1}{}_{\rho\sigma}$$

$$i\Gamma_{\mu\nu,\kappa\lambda,\rho\sigma}^{(IPf_2)(7)}(q_1, q_2) = -\frac{2i}{M_0^5} g_{IPf_2}^{(7)} q_1^{\rho_1} q_1^{\alpha_1} q_1^{\lambda_1} q_2^{\sigma_1} q_2^{\mu_1} q_2^{\nu_1} R_{\mu\nu\mu_1\nu_1} R_{\kappa\lambda\alpha_1\lambda_1} R_{\rho\sigma\rho_1\sigma_1}$$

We can associate the couplings $j = 1, \dots, 7$ with the following (l, S) values:
 $(0, 2), (2, 0) - (2, 2), (2, 0) + (2, 2), (2, 4), (4, 2), (4, 4), (6, 4)$, respectively.

Choice of $IP-IP-f_2(1270)$ coupling



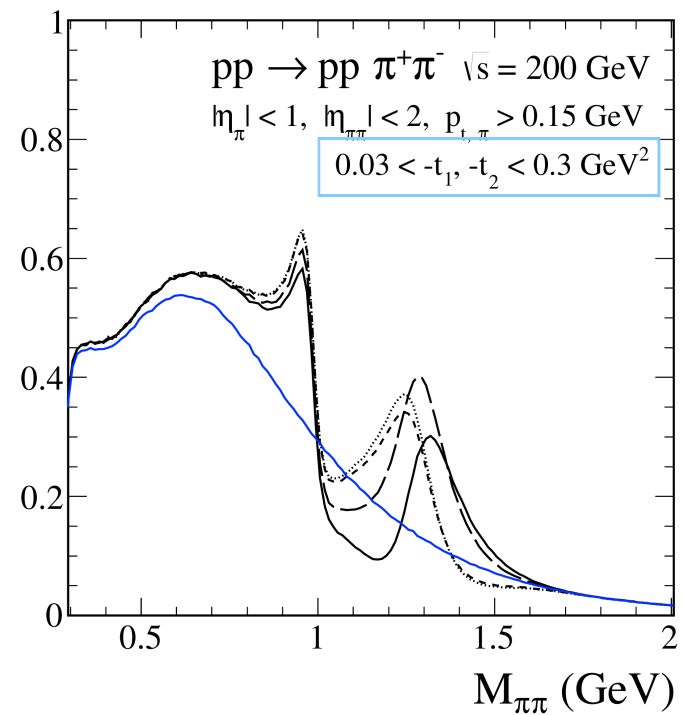
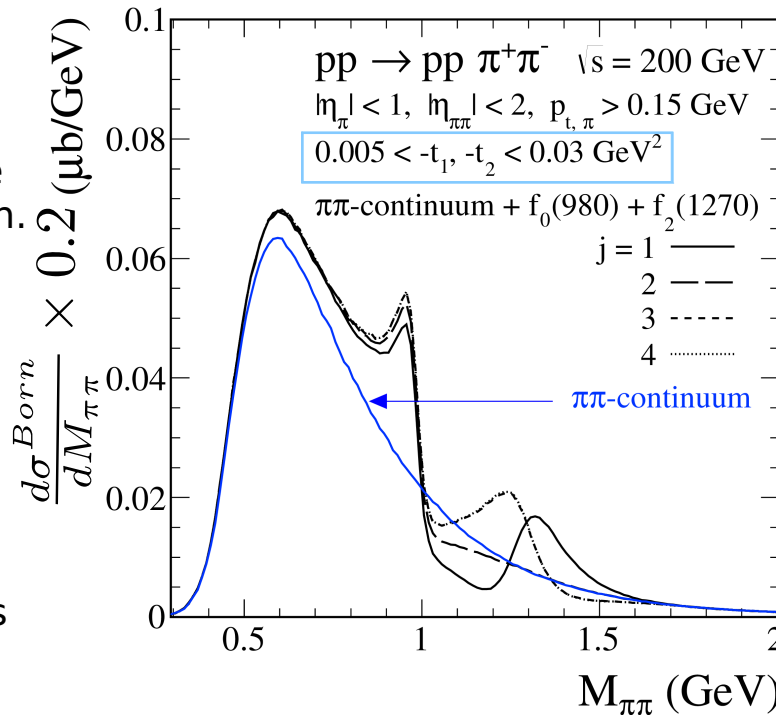
$j = 2$ coupling is in agreement with experimental observations (WA102, COMPASS, ISR)

$\rightarrow f_2(1270)$ peaks at $\phi_{pp} \sim 180^\circ$ and is most prominently observed at large $|t|$ and is suppressed as $dP_t \rightarrow 0$ (undisputed $q\bar{q}$ state)

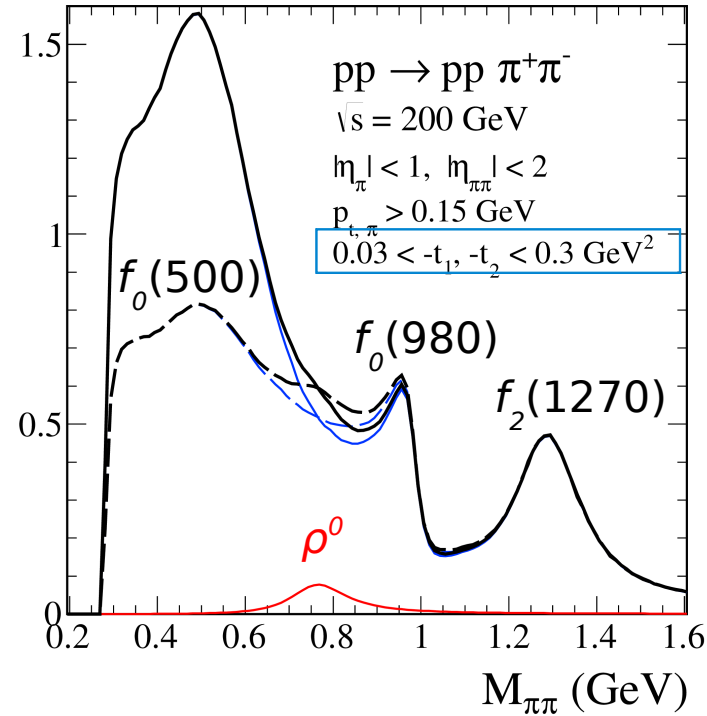
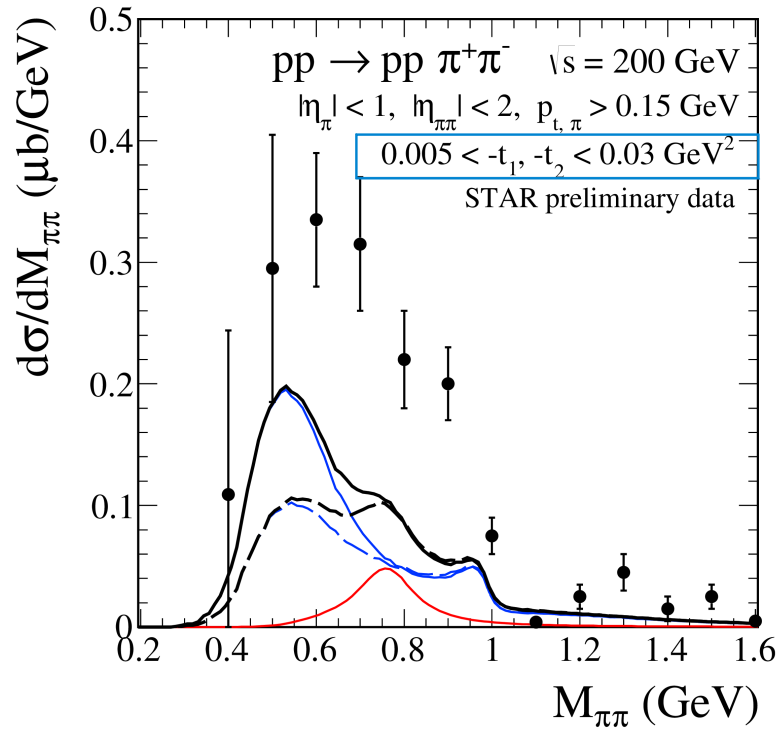
Different couplings generate different interference pattern.

The relative contribution of the resonant $f_2(1270)$ and dipion continuum strongly depends on the cut on $|t|$

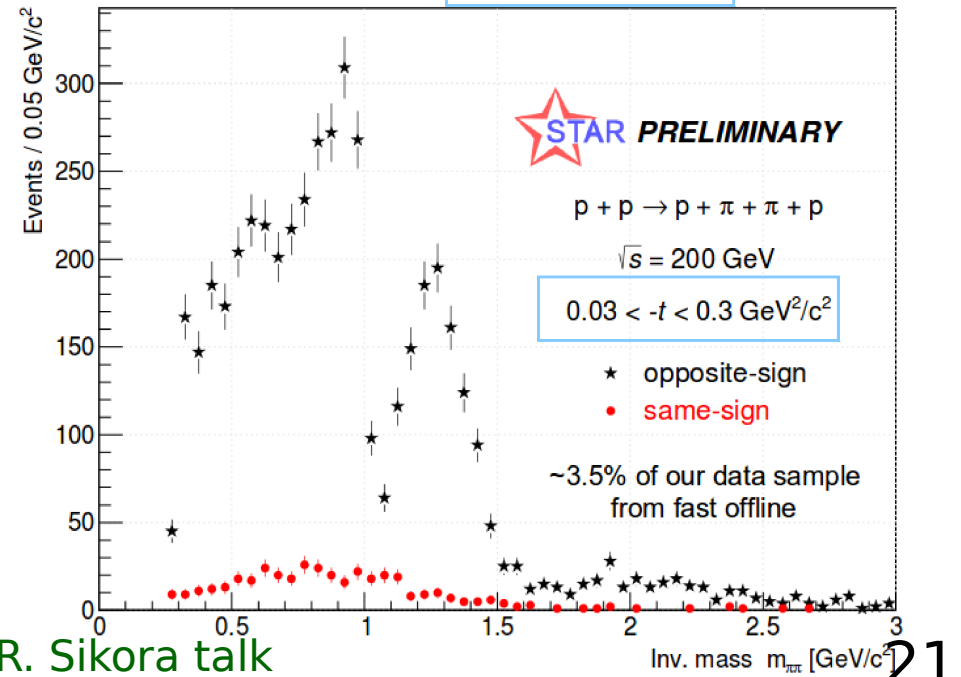
\rightarrow this may explain some controversial observation made by the ISR exp. groups (AFS, ABCDHW)



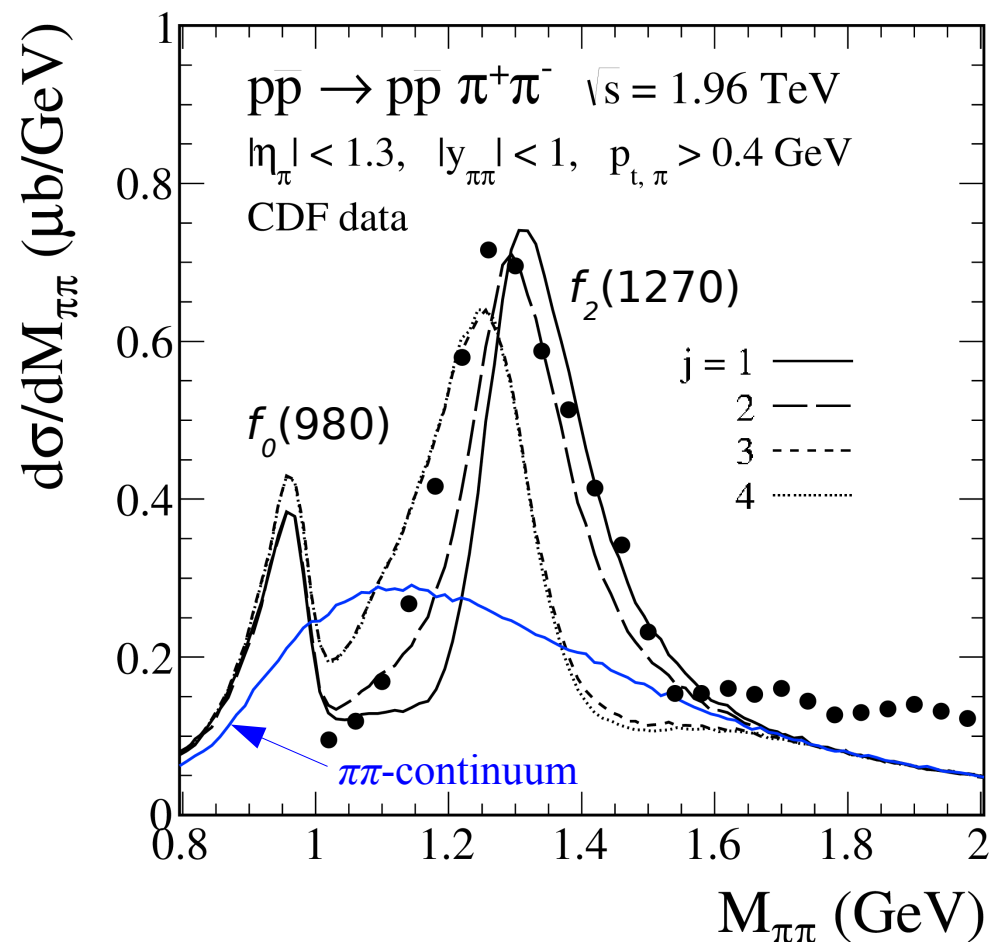
Comparison with STAR preliminary data



- Blue lines (diffractive term), red line (ρ^0 term), black lines (complete result)
- In calculation of f_2 term only one of the $IP-IP-f_2$ couplings ($j=2$) was taken
- At $M_{\pi\pi} < 1$ GeV also other processes may be important
 → $\pi\pi$ FSI effect ($f_0(500)$ meson)
- Absorption effects were included effectively:
 $\langle S^2 \rangle \simeq 0.2$ for the diffractive contribution
 $\langle S^2 \rangle \simeq 0.9$ for the photon-IP/IR contribution



Comparison with CDF data



CDF data: T. A. Aaltonen et al., (CDF Collaboration), Phys.Rev. D91 (2015) 091101.

Events with two oppositely charged particles, assumed to be pions, and no other particles detected in $|\eta| < 5.9$.

(no proton tagging → rapidity gap method)

[see M. Albrow talk](#)

The visible structure attributed to f_0 and $f_2(1270)$ mesons which interfere with the continuum.

We assume that the peak in the region 1.2 - 1.4 GeV corresponds mainly to the $f_2(1270)$ resonance.

We have adjusted the $j = 1, \dots, 4$ couplings to get the same cross section in the region 1.0 - 1.4 GeV.

There may also be a contribution from $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$.

For CDF conditions, the f_2 -to-background ratio is about a factor of 2.

We take the monopole form for off-shell pion form factors with $\Lambda_{\text{off},M} = 0.7 \text{ GeV}$.

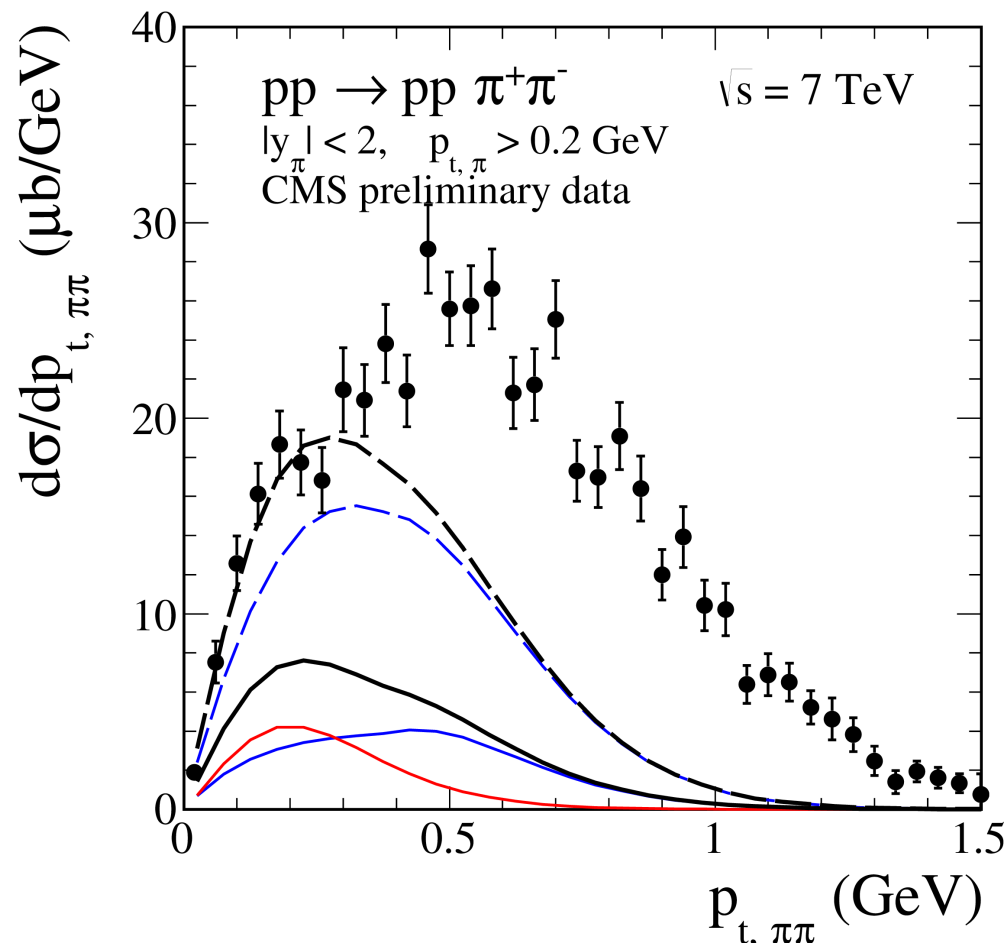
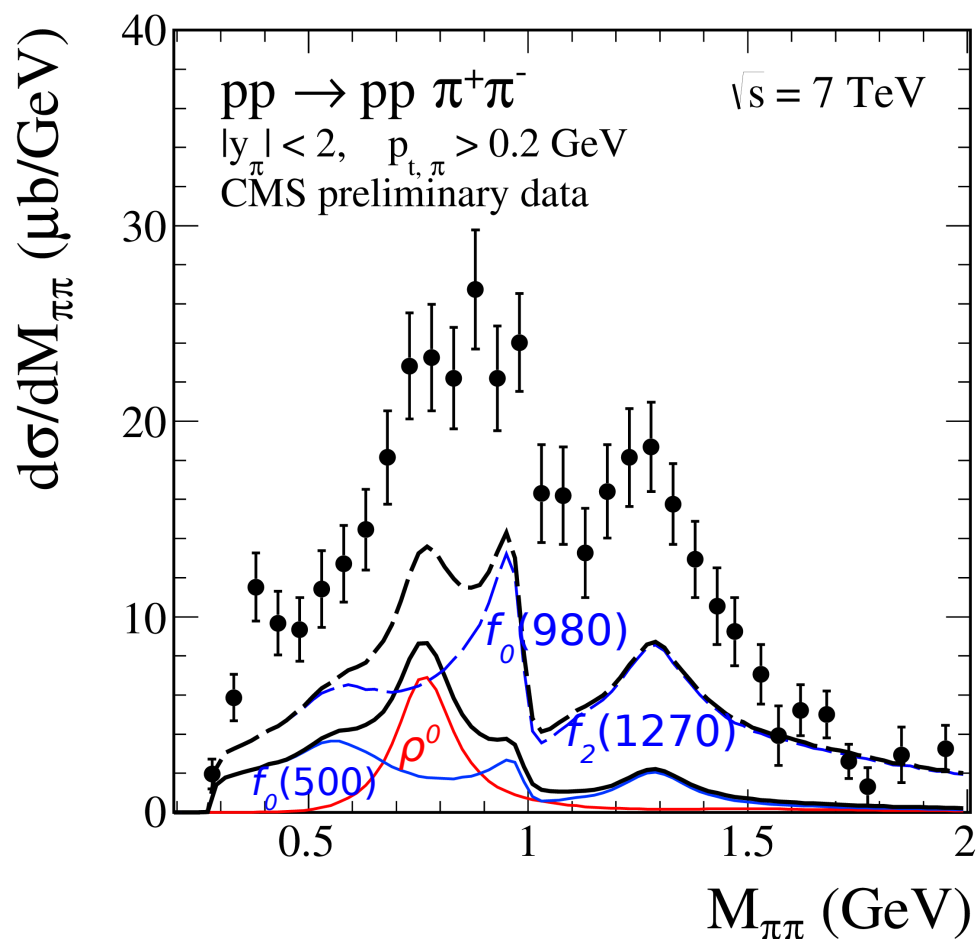
Absorption effects were included effectively:

$$\frac{d\sigma^{\text{Born}}}{dM_{\pi\pi}} \times \langle S^2 \rangle$$

$$\langle S^2 \rangle \simeq 0.1$$

ratio of full (absorbed)-to-Born cross section

Comparison with CMS preliminary data



In diff. continuum term: (solid blue line) $\Lambda_{\text{off},M} = 0.7 \text{ GeV}$ (the same couplings as for CDF predictions)
 (dashed blue line) $\Lambda_{\text{off},M} = 1.2 \text{ GeV}$, with enhanced $f_0(980)$ and f_2 couplings

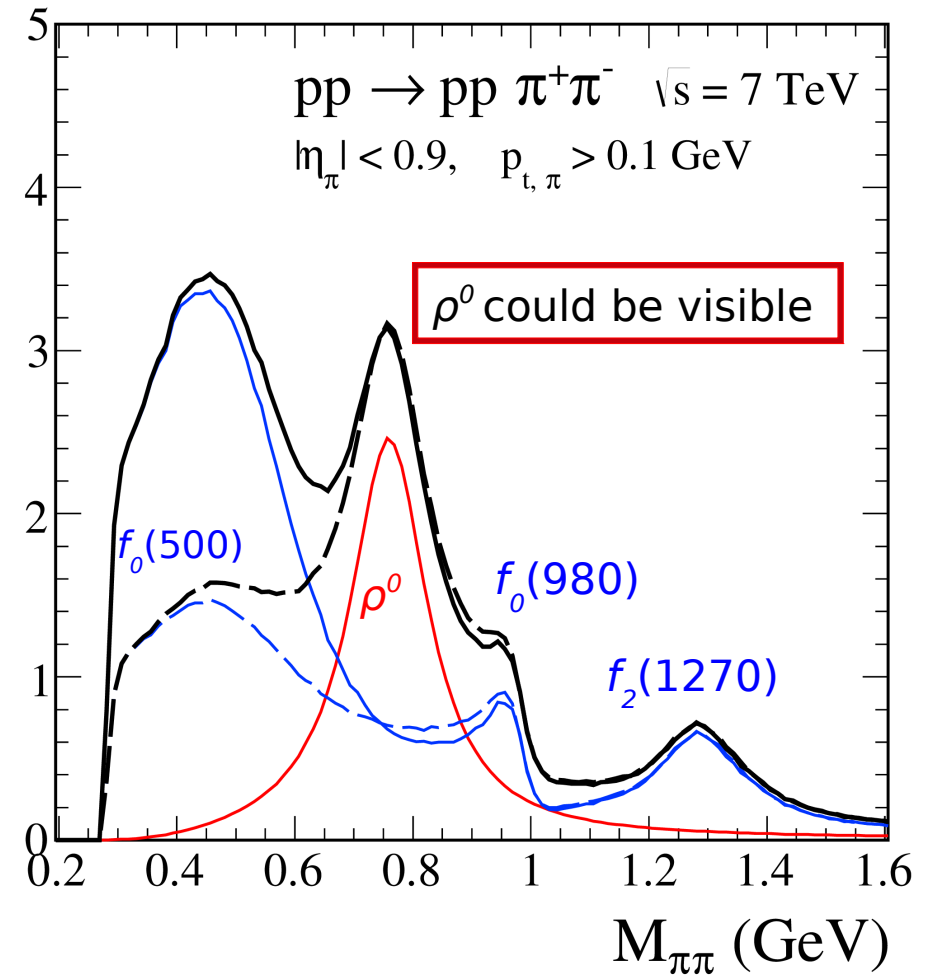
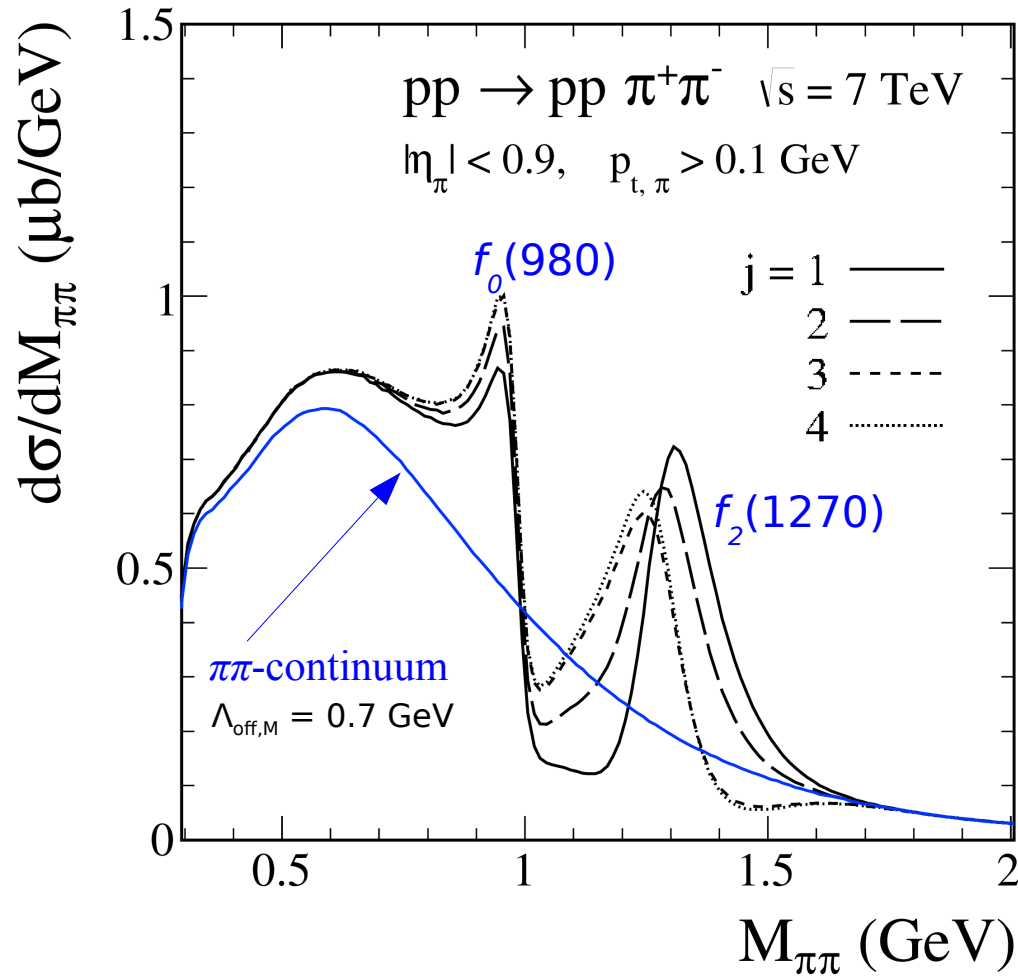
Our model results are much below the CMS preliminary data ([CMS-PAS-FSQ-12-004](#)) which could be due to a contamination of non-exclusive processes (one or both protons undergoing dissociation).

$\langle S^2 \rangle \simeq 0.1$ for the diffractive contribution
 $\langle S^2 \rangle \simeq 0.9$ for the photon-IP/IR contribution

ρ^0 could be visible

[see M. Khakzad talk](#)

Predictions for ALICE

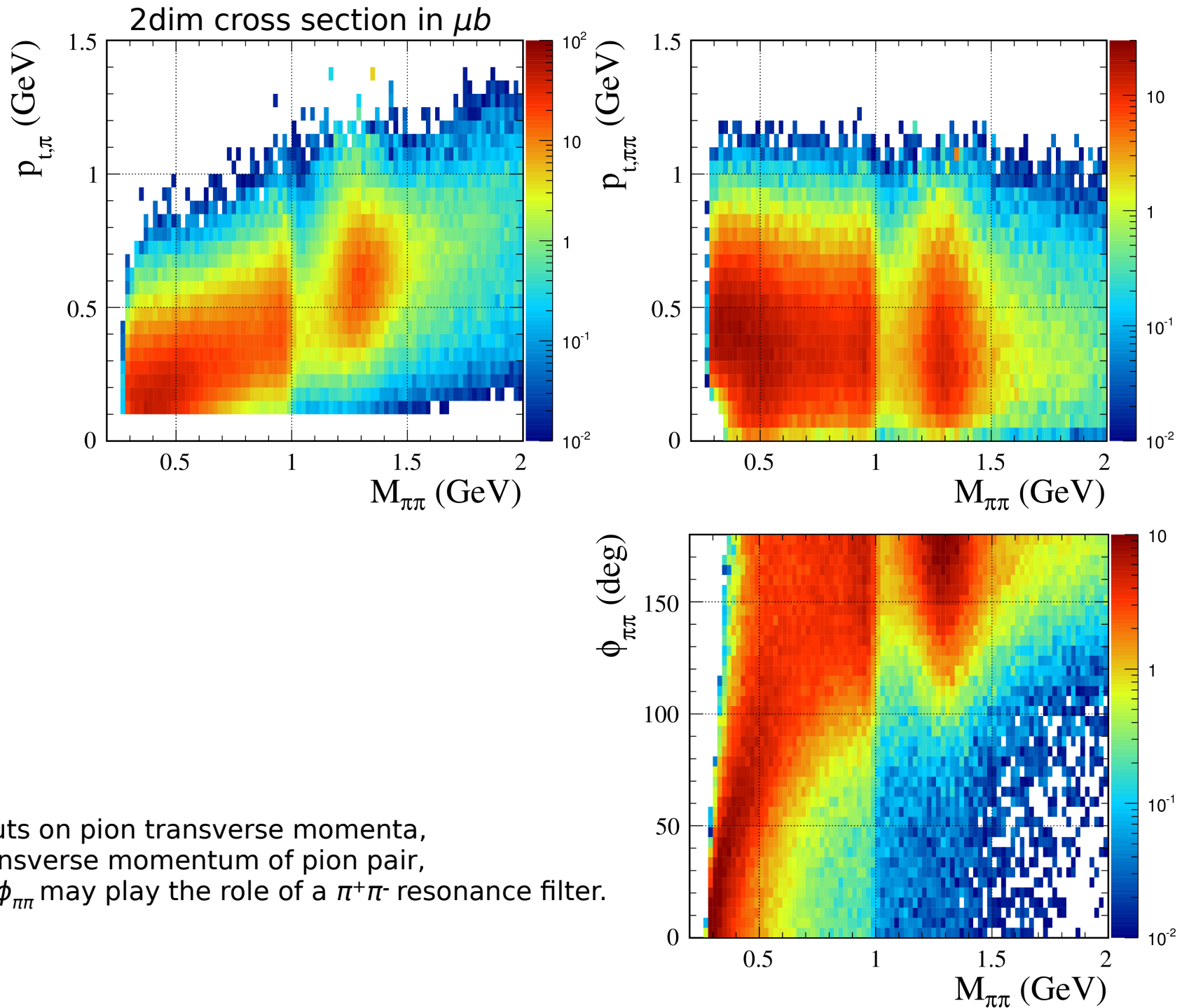


Different IP - IP - f_2 couplings generate different interference pattern.

$\langle S^2 \rangle \simeq 0.1$ for the diffractive contribution

$\langle S^2 \rangle \simeq 0.9$ for the photon- IP /IR contribution

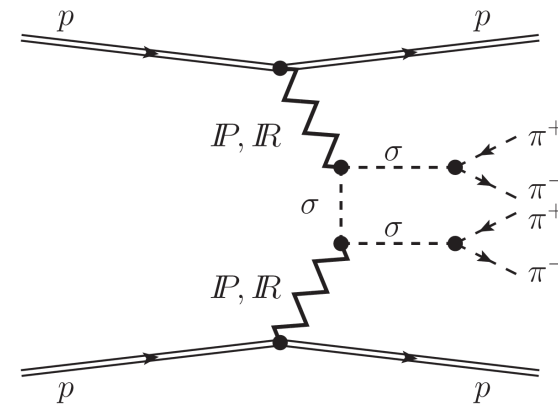
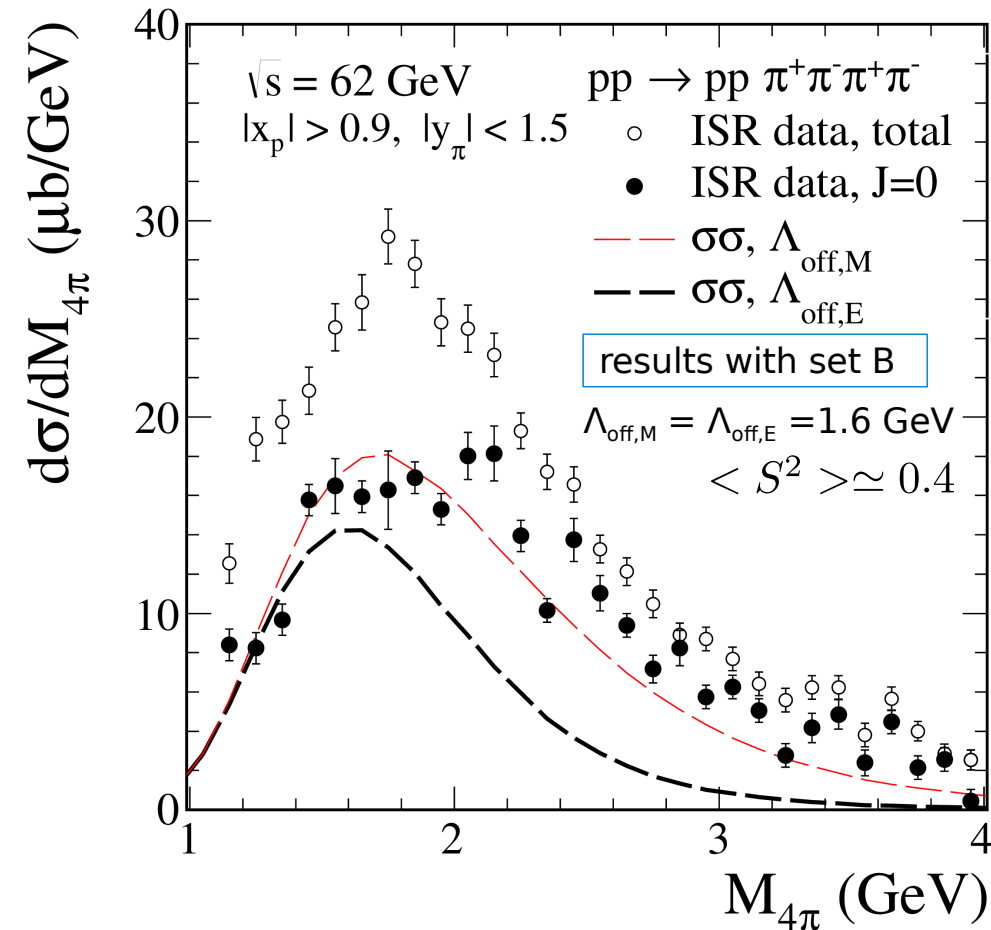
Predictions for ATLAS+ALFA



Diffractive production of $\pi^+\pi^-\pi^+\pi^-$ in pp collisions

$$\sigma_{2 \rightarrow 6} = \int_{2m_\pi}^{\max\{m_{X_3}\}} \int_{2m_\pi}^{\max\{m_{X_4}\}} \sigma_{2 \rightarrow 4}(\dots, m_{X_3}, m_{X_4}) f_M(m_{X_3}) f_M(m_{X_4}) dm_{X_3} dm_{X_4}$$

with the spectral functions of meson $f_M(m_{X_i}) = A_N \left(1 - \frac{4m_\pi^2}{m_{X_i}^2}\right)^{n/2} \frac{\frac{2}{\pi} m_M^2 \Gamma_{M,tot}}{(m_{X_i}^2 - m_M^2)^2 + m_M^2 \Gamma_{M,tot}^2}$



as for dipion continuum

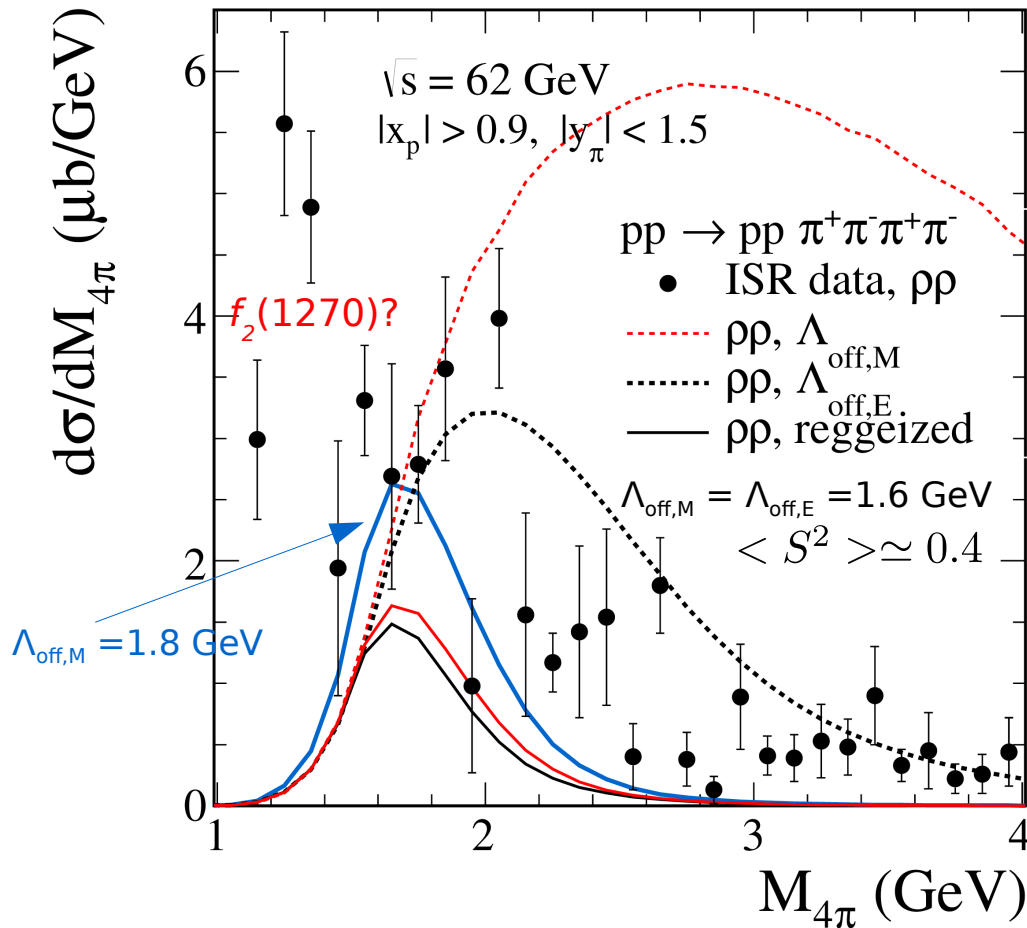
set A : $\beta_{P\sigma\sigma} = 2\beta_{P\pi\pi}$, $g_{f_{2R}\sigma\sigma} = g_{f_{2R}\pi\pi}$

set B : $\beta_{P\sigma\sigma} = 4\beta_{P\pi\pi}$, $g_{f_{2R}\sigma\sigma} = 2g_{f_{2R}\pi\pi}$
 enhanced coupling constants

The 4π ISR data contains a large $\rho^0\pi^+\pi^-$ component with an enhancement in the $J=2$ term interpreted by ABCDHW Collaboration as a $f_2(1720)$ state.

ISR data: A. Breakstone *et al.* (ABCDHW Collaboration), Z. Phys. C58 (1993) 251

4 π production ($\rho\rho$ contribution)

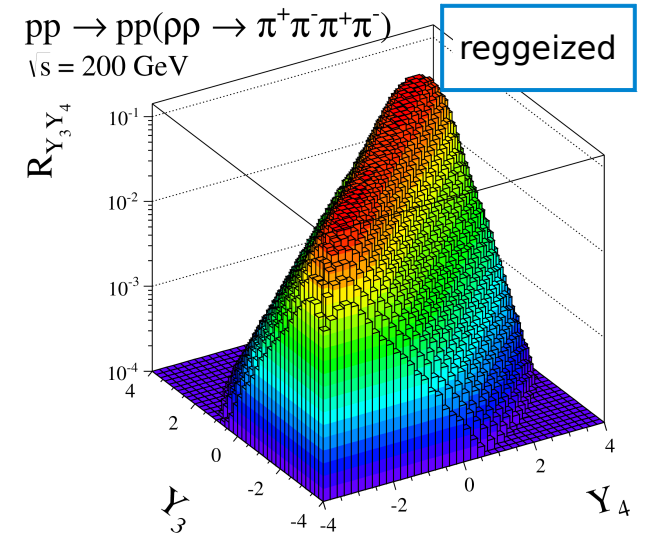
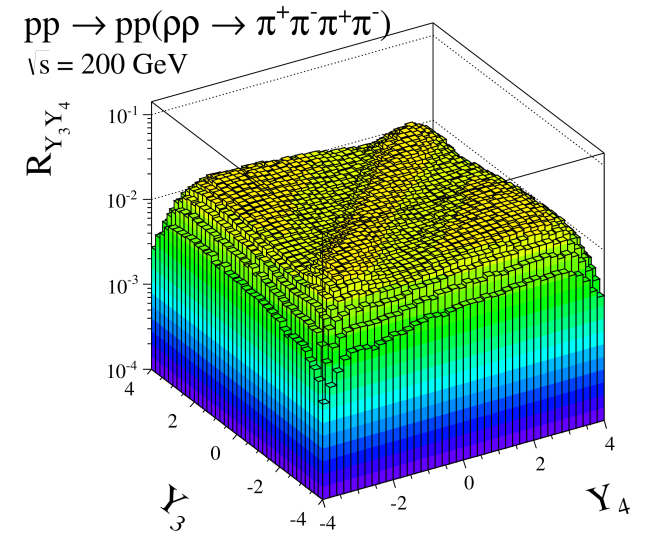


- reggeization effect

$$\Delta_{\rho_1 \rho_2}^{(\rho)}(p) \rightarrow \Delta_{\rho_1 \rho_2}^{(\rho)}(p) \left(\frac{s_{34}}{s_0} \right)^{\alpha_\rho(p^2) - 1}, \quad s_0 = 4m_\rho^2$$

becomes crucial when the separation in rapidity between two ρ mesons increases $|Y_3 - Y_4| > 0$

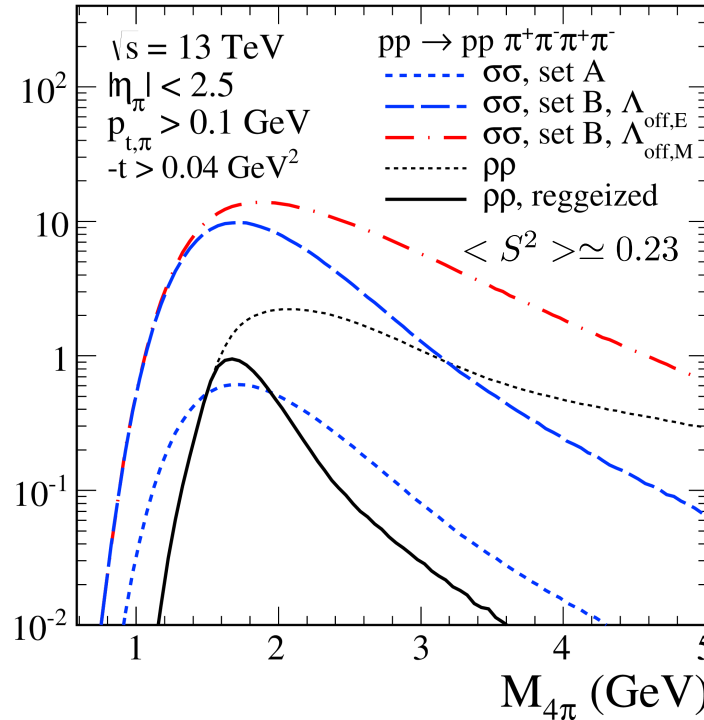
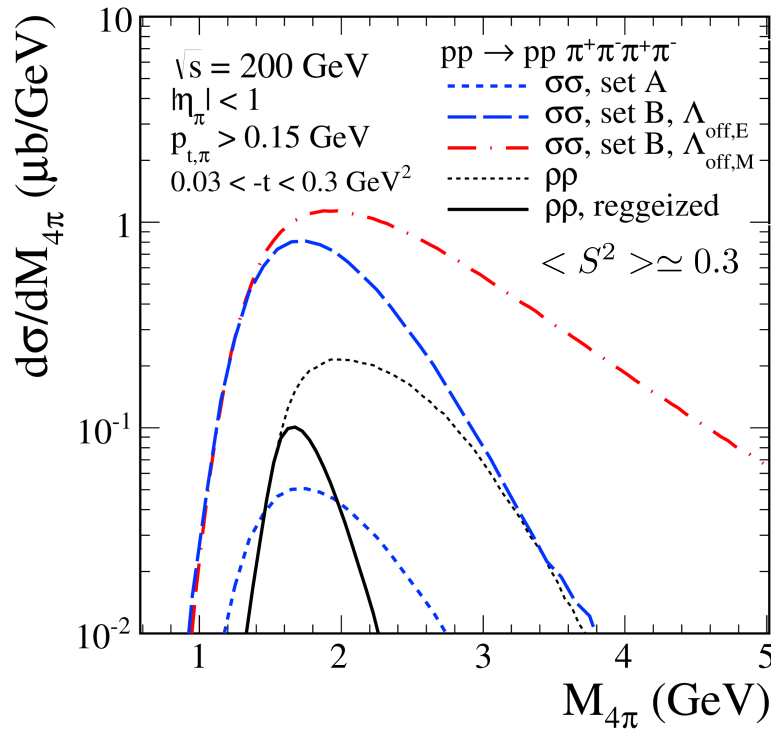
$$R_{Y_3 Y_4} = \frac{d^2 \sigma}{dY_3 dY_4} / \int dY_3 dY_4 \frac{d^2 \sigma}{dY_3 dY_4}$$



see also discussion in

L.A. Harland-Lang, V.A. Khoze, M.G. Ryskin, arXiv:1312.4553, Eur. Phys. J. C74 (2014) 2848

First predictions for RHIC and LHC (ATLAS+ALFA)



Absorption effects due to pp -rescattering were included.

Here $\Lambda_{\text{off,E}} = 1.6 \text{ GeV}$.

\sqrt{s} , TeV	Cuts	“Born level” cross sections in μb	
		$\sigma\sigma$ (set B)	$\rho\rho$
0.2	$ \eta_\pi < 1, p_{t,\pi} > 0.15 \text{ GeV}, 0.03 < -t < 0.3 \text{ GeV}^2$	2.94	0.88 (0.17)
7	$ \eta_\pi < 0.9, p_{t,\pi} > 0.1 \text{ GeV}$	10.40	2.79 (0.53)
7	$ y_\pi < 2, p_{t,\pi} > 0.2 \text{ GeV}$	34.88	17.94 (2.20)
13	$ \eta_\pi < 1, p_{t,\pi} > 0.1 \text{ GeV}$	16.18	3.56 (0.72)
13	$ \eta_\pi < 2.5, p_{t,\pi} > 0.1 \text{ GeV}$	120.06	45.58 (6.21)
13	$ \eta_\pi < 2.5, p_{t,\pi} > 0.1 \text{ GeV}, -t > 0.04 \text{ GeV}^2$	47.52	18.08 (2.44)

Table: Born cross sections in μb . The $\sigma\sigma$ contribution was calculated with the enhanced (set B) couplings while the $\rho\rho$ contribution without and with (in the parentheses) inclusion of ρ meson reggeization.

Predicted cross section can be obtained by multiplying the Born cross section by the gap survival factor: 0.3 (STAR), 0.21 (7 TeV), 0.19 (13 TeV), 0.23 (13 TeV, with cuts on $|t|$).

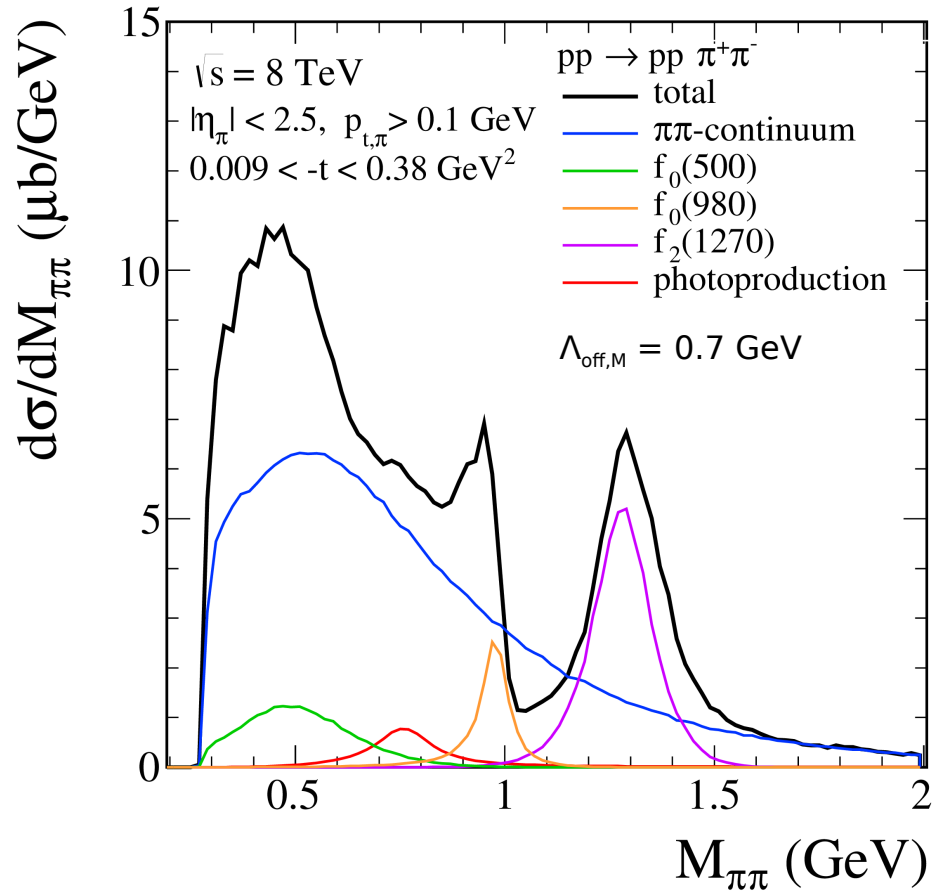
Conclusions

- Studying the ratio r_5 of single-helicity-flip to non-flip amplitudes in polarised high-energy pp elastic scattering we found that the STAR data are consistent with a tensor pomeron while they clearly exclude a scalar pomeron. We have further argued that a vector pomeron assumption is in contradiction to the rules of QFT. We therefore conclude that the tensor pomeron is the only viable option (for soft pomeron).
- The tensor-pomeron model (Ewerz-Maniatis-Nachtmann) was applied to many $pp \rightarrow pp \text{ meson}(s)$ reactions. The amplitudes are formulated in terms of effective vertices and propagators respecting the standard crossing and charge conjugation relations of QFT.
- We have given a consistence treatment of the $\pi^+\pi^-$ continuum and resonance production in proton-(anti)proton collisions. The $pp \rightarrow pp\pi^+\pi^-$ process is an attractive for different exp.: COMPASS, STAR, CDF, ALICE, CMS, ATLAS, LHCb.
- We include $f_0(500)$, $f_0(980)$, $f_2(1270)$ and ρ^0 contributions which interfere with the continuum. By assuming dominance of one of the IP - IP - f_2 couplings ($j=2$) we can get only a rough description of the recent CDF and preliminary STAR data. The model parameters have been adjusted to HERA and CDF data and then used for the predictions for STAR, ALICE, and CMS experiments.

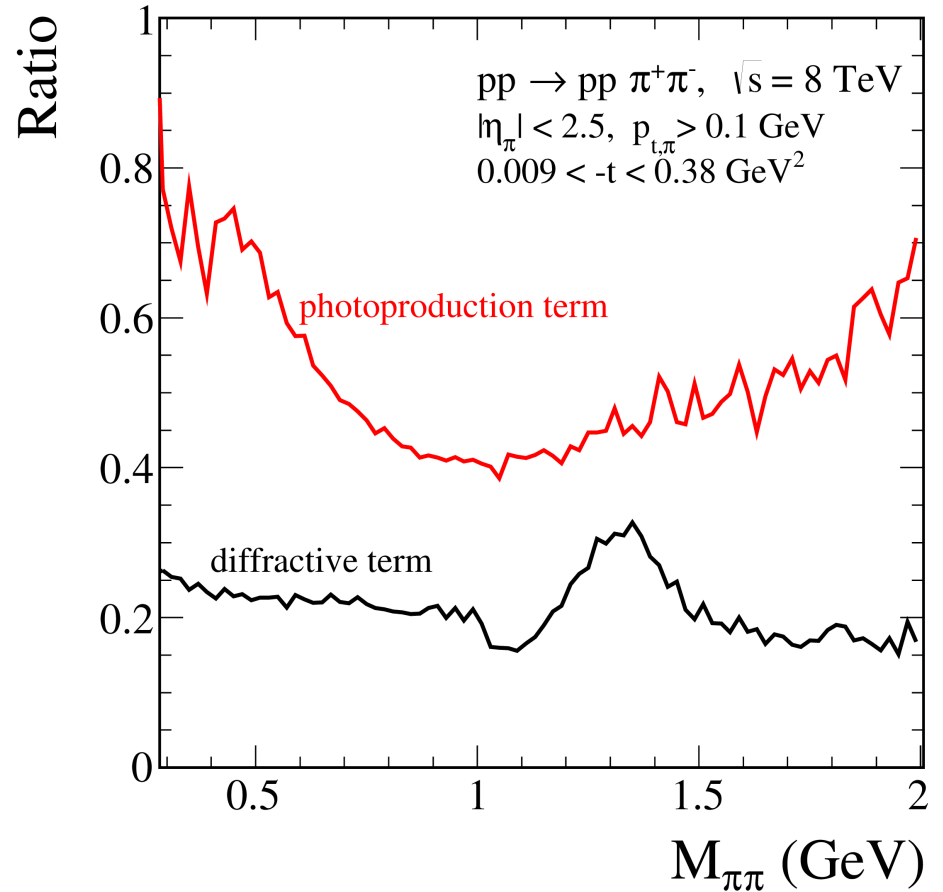
Disagreement with the preliminary CMS data could be due to a large dissociation contribution. Purely exclusive data expected from CMS+TOTEM and ATLAS+ALFA will allow us to draw definite conclusions.
- The distribution in dipion invariant mass shows a rich pattern of structure that depends on the cuts used in a particular experiment. We find that the relative contribution of the $f_2(1270)$ and $\pi\pi$ -continuum strongly depends on the cut on $|t|$ which may explain some controversial observation made by the ISR groups.
- We have estimated first predictions of the cross sections for the process $pp \rightarrow pp \pi^+\pi^-\pi^+\pi^-$ via intermediate $\sigma\sigma$ and $\rho\rho$ states. We compared our results with the ISR data. A measurable cross section of few μb was obtained including the exp. cuts relevant for LHC experiments.

Extra slides

Predictions for ATLAS+ALFA



Ratio of full (absorbed)-to-Born cross section



Here we take the absorption corrections due to pp -rescattering only.
 The absorption effect depends on the t region in which is considered.

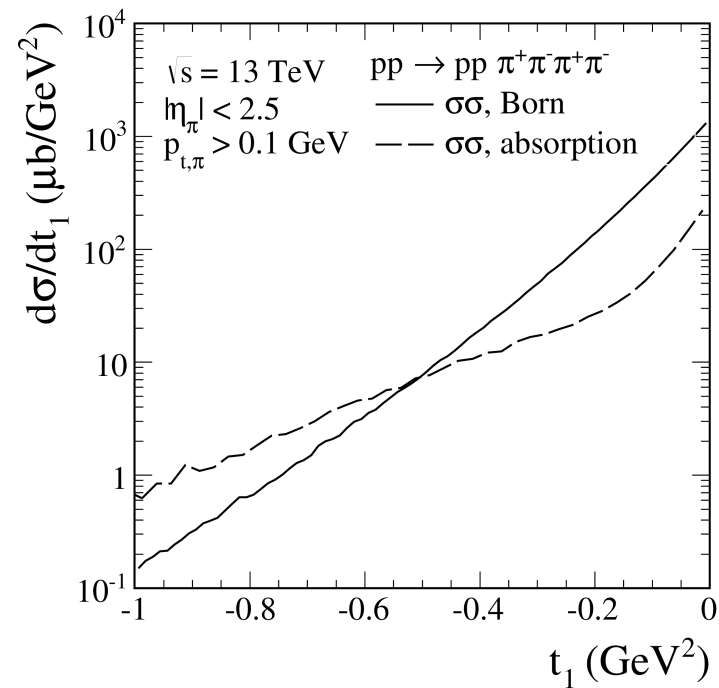
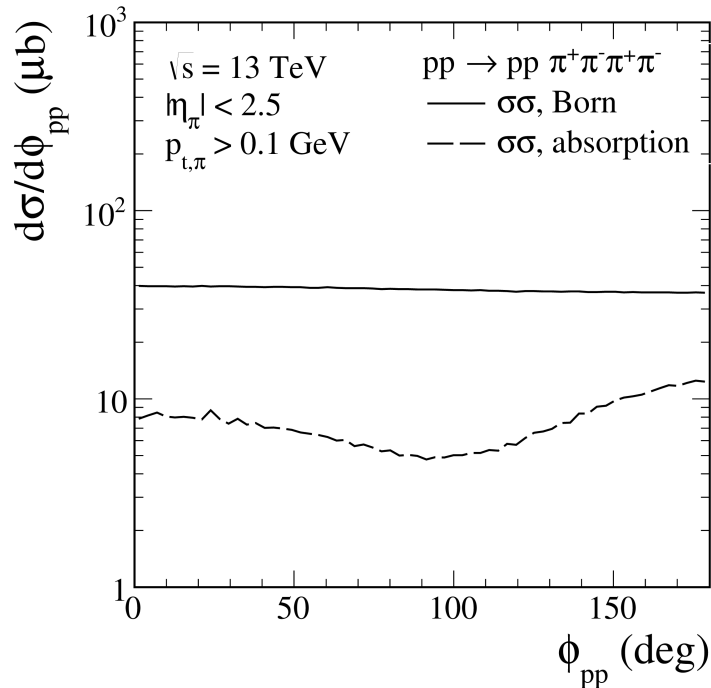
Cross sections: at 8 TeV (7.09 – 14.11) μb ,
 at 13 TeV (7.35 – 14.57) μb ,
 for two off-shell pion form factor parameters $\Lambda_{\text{off},M} = (0.7 - 1.0) \text{ GeV}$, respectively.

Cross sections (in μb) for $pp \rightarrow pp \pi^+ \pi^- \pi^+ \pi^-$

\sqrt{s} , TeV	Cuts	“Born level” cross sections in μb	
		$\sigma\sigma$ (set B)	$\rho\rho$
0.2	$ \eta_\pi < 1, p_{t,\pi} > 0.15 \text{ GeV}, 0.03 < -t < 0.3 \text{ GeV}^2$	2.94	0.88 (0.17)
7	$ \eta_\pi < 0.9, p_{t,\pi} > 0.1 \text{ GeV}$	10.40	2.79 (0.53)
7	$ y_\pi < 2, p_{t,\pi} > 0.2 \text{ GeV}$	34.88	17.94 (2.20)
13	$ \eta_\pi < 1, p_{t,\pi} > 0.1 \text{ GeV}$	16.18	3.56 (0.72)
13	$ \eta_\pi < 2.5, p_{t,\pi} > 0.1 \text{ GeV}$	120.06	45.58 (6.21)
13	$ \eta_\pi < 2.5, p_{t,\pi} > 0.1 \text{ GeV}, -t > 0.04 \text{ GeV}^2$	47.52	18.08 (2.44)

The $\sigma\sigma$ contribution was calculated using the enhanced coupling constants (set B) while the $\rho\rho$ contribution without and with (in the parentheses) the inclusion of ρ meson reggeization. Here the exponential off-shell meson form factor with $\Lambda_{\text{off,E}} = 1.6 \text{ GeV}$ was used.

The full cross section can be obtained by multiplying the Born cross section by the corresponding gap survival factor: 0.3 (STAR), 0.21 (7 TeV), 0.19 (13 TeV), 0.23 (13 TeV, with cuts on $|t|$).



Related works:

- P. Lebiedowicz, R. Pasechnik, A. Szczurek, *Measurement of exclusive production of χ_{c0} scalar meson in proton-(anti)proton collisions via $\chi_{c0} \rightarrow \pi^+\pi^-$ decay*, [arXiv:1103.5642](#), *Phys. Lett. B* **701** (2011) 434
- R. Staszewski, P. Lebiedowicz, M. Trzebiński, J. Chwastowski, A. Szczurek, *Exclusive $\pi^+\pi^-$ Production at the LHC with Forward Proton Tagging*, [arXiv: 1104.3568](#), *Acta Phys. Polon. B* **42** (2011) 1861
- L.A. Harland-Lang, V.A. Khoze, M.G. Ryskin, *Modelling exclusive meson pair production at hadron colliders*, [arXiv:1312.4553](#), *Eur. Phys. J. C* **74** (2014) 2848
- A. Bolz, C. Ewerz, M. Maniatis, O. Nachtmann, M. Sauter, A. Schöning, *Photoproduction of $\pi^+\pi^-$ pairs in a model with tensor-pomeron and vector-odderon exchange*, [arXiv:1409.8483](#), *JHEP* **1501** (2015) 151
- P. Lebiedowicz, A. Szczurek, *Revised model of absorption corrections for the $pp \rightarrow pp \pi^+\pi^-$ process*, [arXiv:1504.07560](#), *Phys. Rev. D* **92** (2015) 054001
- R. Fiore, L. Jenkovszky, R. Schicker, *Resonance production in Pomeron-Pomeron collisions at the LHC*, [arXiv:1512.04977](#), *Eur. Phys. J. C* **76** (2016) 38

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