The 3-D nucleon structure in momentum space

 \boldsymbol{s}_q

b

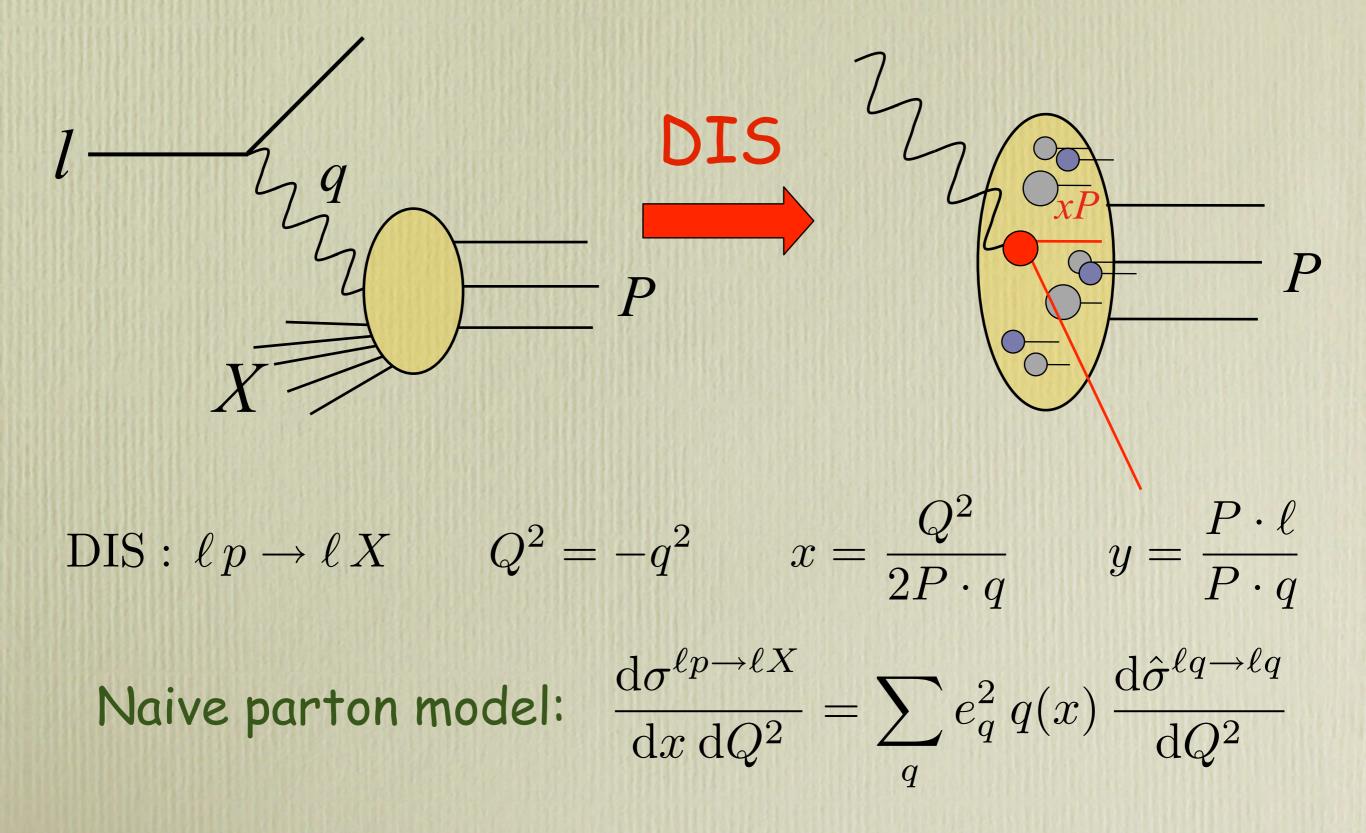
 \boldsymbol{S}

com © 2014

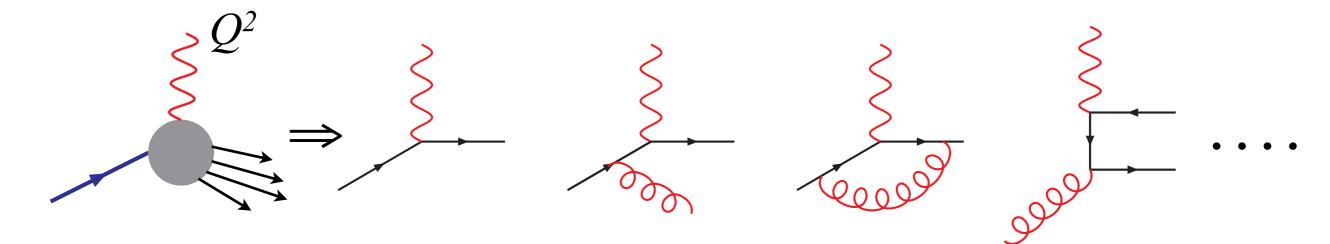
Diffraction 2016, Acireale, Sept. 4, 2016

Mauro Anselmino - Torino University & INFN

usual (successful) way of exploring the proton structure (collinear parton model)

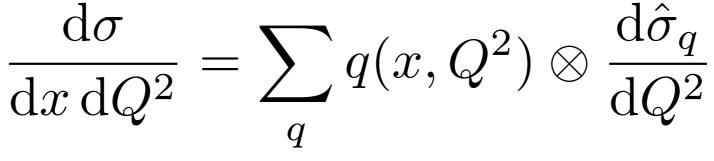


QCD interactions induce a well known Q² dependence



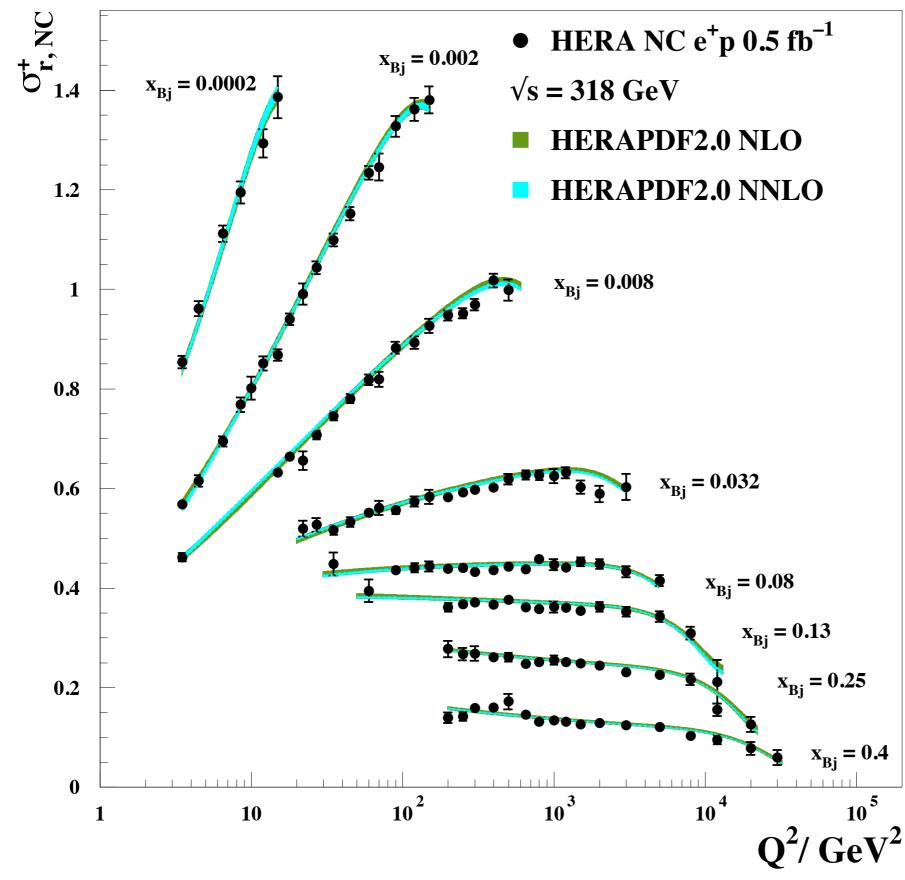
DIS – pQCD :
$$q(x) \Rightarrow \underline{q(x, Q^2)}_{PDFs}$$

factorization:



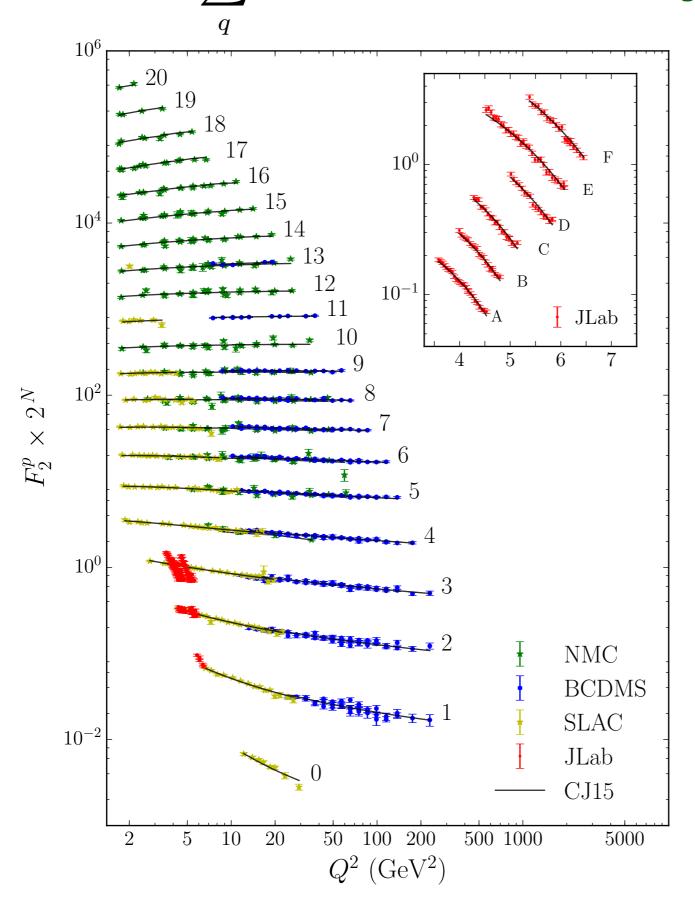
universality: same $q(x,Q^2)$ measured in DIS can be used in other processes

H1 and ZEUS



$$\sigma_{r,\text{NC}}^{\pm} = \frac{\mathrm{d}^2 \sigma_{\text{NC}}^{e^{\pm}p}}{\mathrm{d}x_{\text{Bj}} \mathrm{d}Q^2} \cdot \frac{Q^4 x_{\text{Bj}}}{2\pi \alpha^2 Y}$$
$$Y_{\pm} = 1 \pm (1 - y)^2$$

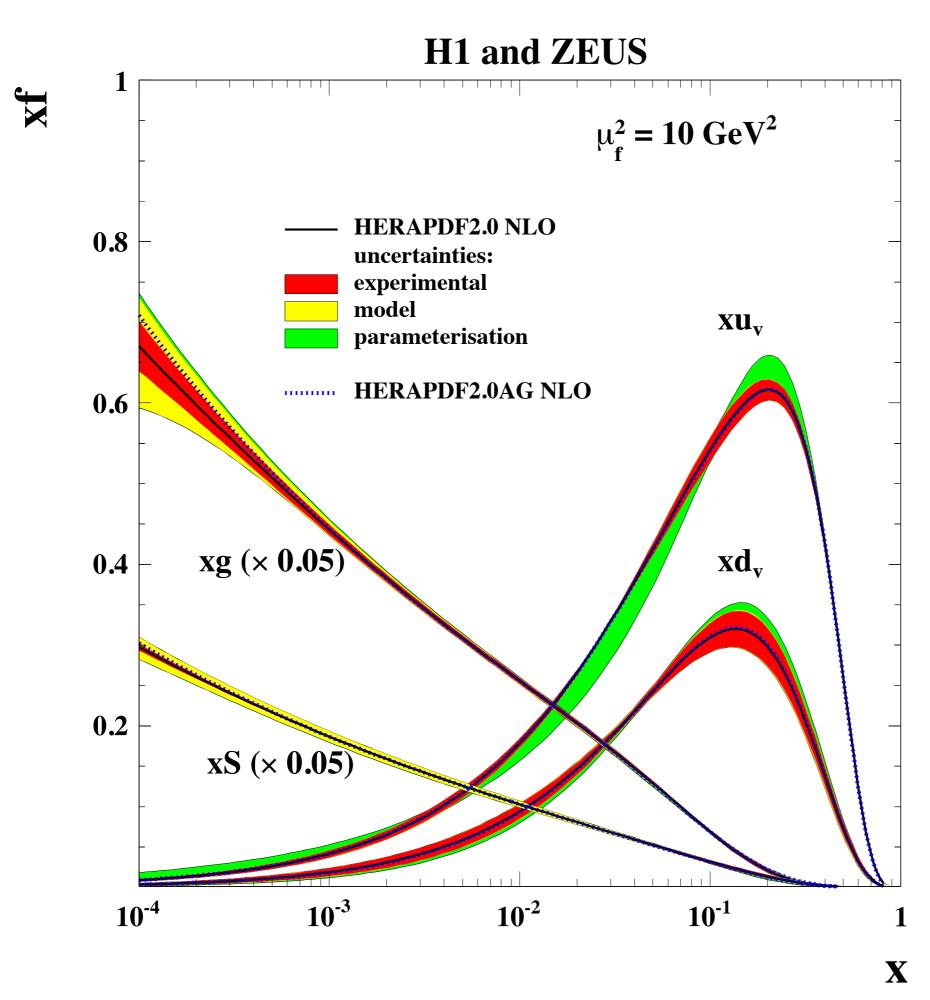
Eur. Phys. J. C75 (2015) 580 $F_2 = \sum x q(x, Q^2)$ from M. Pennington, arXiv:1604.01441



N	X	
0	0.85	
1	0.74	
2	0.65	
3	0.55	
4	0.45	
5	0.34	
6	0.28	
7	0.23	
8	0.18	
9	0.14	
10	0.11	
11	0.10	
12	0.09	
13	0.07	
14	0.05	
15	0.04	
16	0,026	
17	0,018	
18	0,013	
19	0,008	
20	0,005	

JLab insert

Ι	0	Ν
A	38°	0
В	41°	1
С	45°	2
D	55°	3
E	60°	4
F	70°	5



unpolarized distribution $xf_a(x,Q^2)$

H. Abramowicz et al., Eur. Phys. J. C75 (2015) 580

> PDFs are very useful, but do we really know the partonic nucleon structure?

despite 50 years of studies the nucleon is still a very mysterious object, and the most abundant piece of matter in the visible Universe

 $10^{-15} \,\mathrm{m}$

6

())

 $\leq 10^{-19} \,\mathrm{m}$

parton intrinsic motion spin-k_ correlations? orbiting quarks? spatial distribution? nucleon mass?

 $10^{-14} \,\mathrm{m}$

 $10^{-10} \,\mathrm{m}$

what would we like to know ? how ?

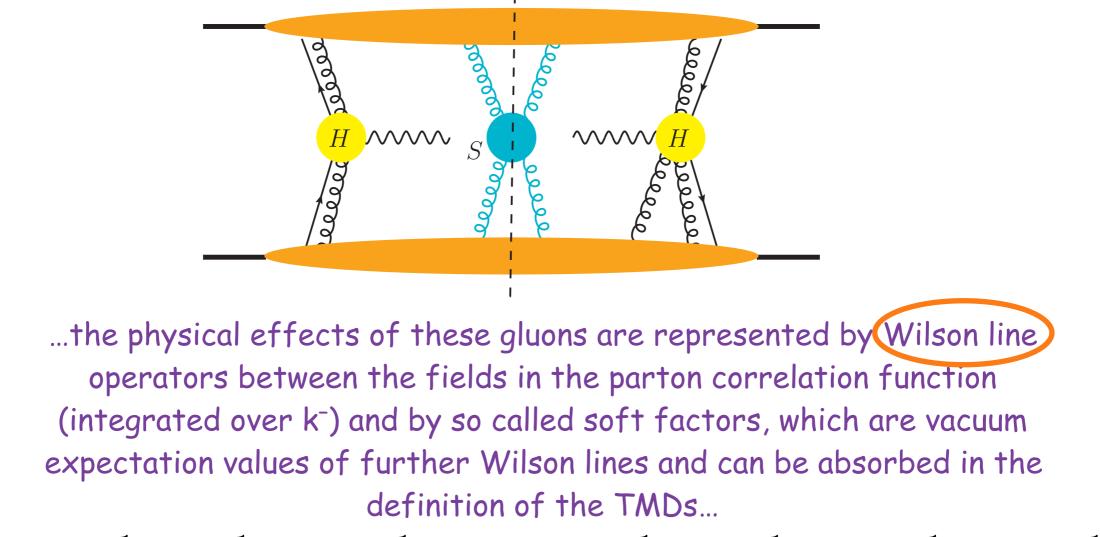
$$\begin{split} H(k,P,\Delta) &= (2\pi)^{-4} \int d^4 z \; e^{izk} & \text{two-quark correlation} \\ &\times \left\langle p(P+\frac{1}{2}\Delta) | \bar{q}(-\frac{1}{2}z) \Gamma q(\frac{1}{2}z) | p(P-\frac{1}{2}\Delta) \right\rangle & \text{function} \end{split}$$

light-cone variables

s
$$v = (v^+, v^-, v)$$
 $v^{\pm} = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$
 $x = \frac{k^+}{P^+}$ $2\xi = -\frac{\Delta^+}{P^+}$

 $\Delta=0~$ inclusive processes, cross sections $\Delta\neq 0~~{\rm exclusive~processes}, {\rm amplitudes}$

actually, things are not so simple... (example of D-Y process)

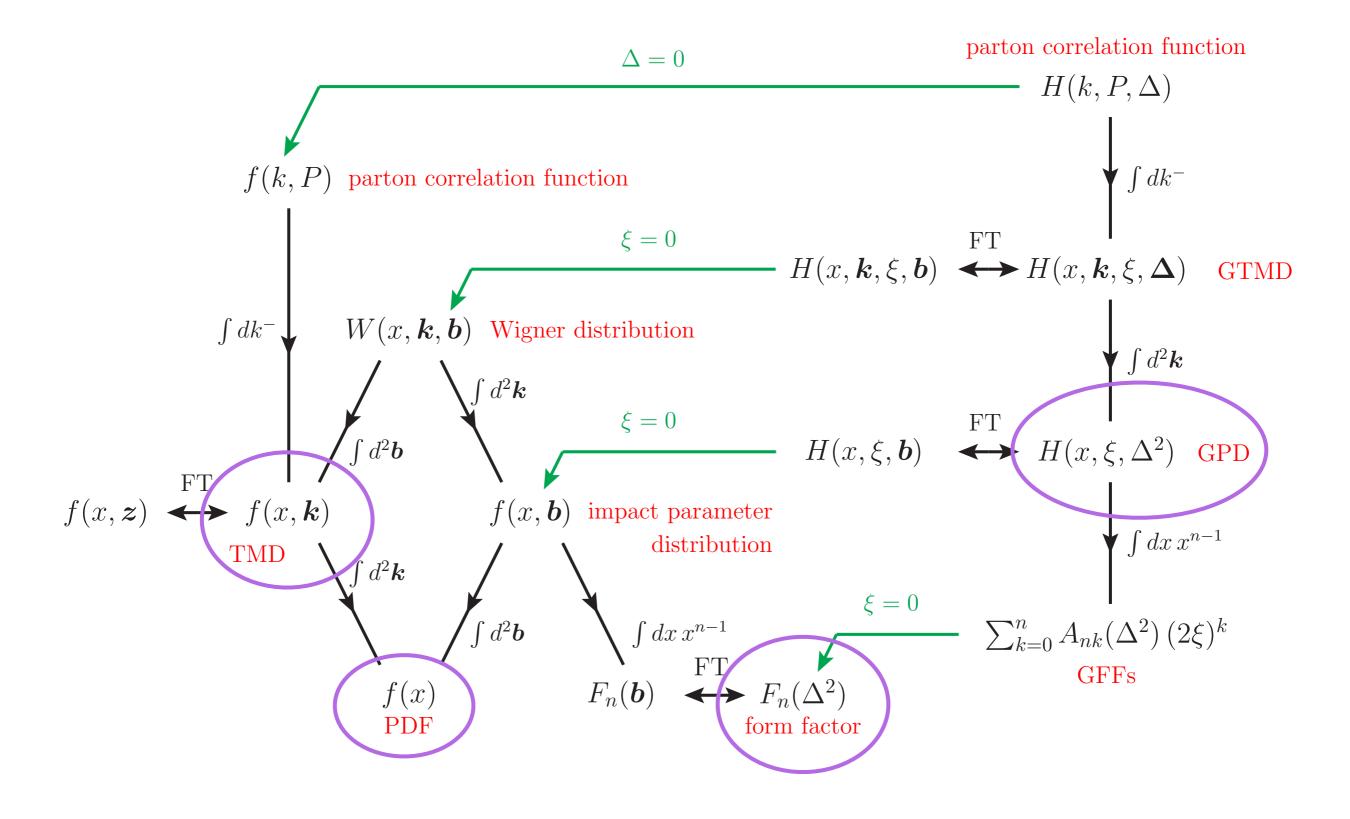


$$\langle p(P+\frac{1}{2}\Delta)|\bar{q}(-\frac{1}{2}z)\Gamma q(\frac{1}{2}z)|p(P-\frac{1}{2}\Delta)\rangle \rightarrow \langle p(P+\frac{1}{2}\Delta)|\bar{q}(-\frac{1}{2}z)\operatorname{IW}q(\frac{1}{2}z)|p(P-\frac{1}{2}\Delta)\rangle$$

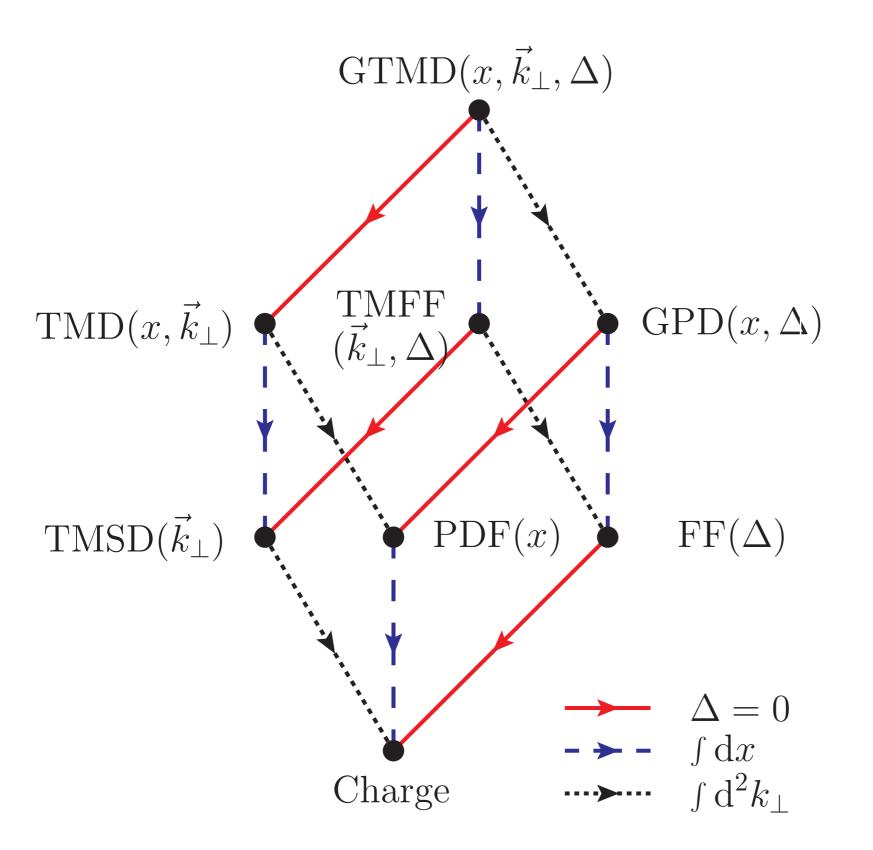
the Wilson lines are path-ordered exponential of the gauge field and turn the operator product into a gauge invariant operator, but induce some process dependence

> M. Diehl, arXiv:1512.01328 J. Collins, Cambridge University Press (2011)

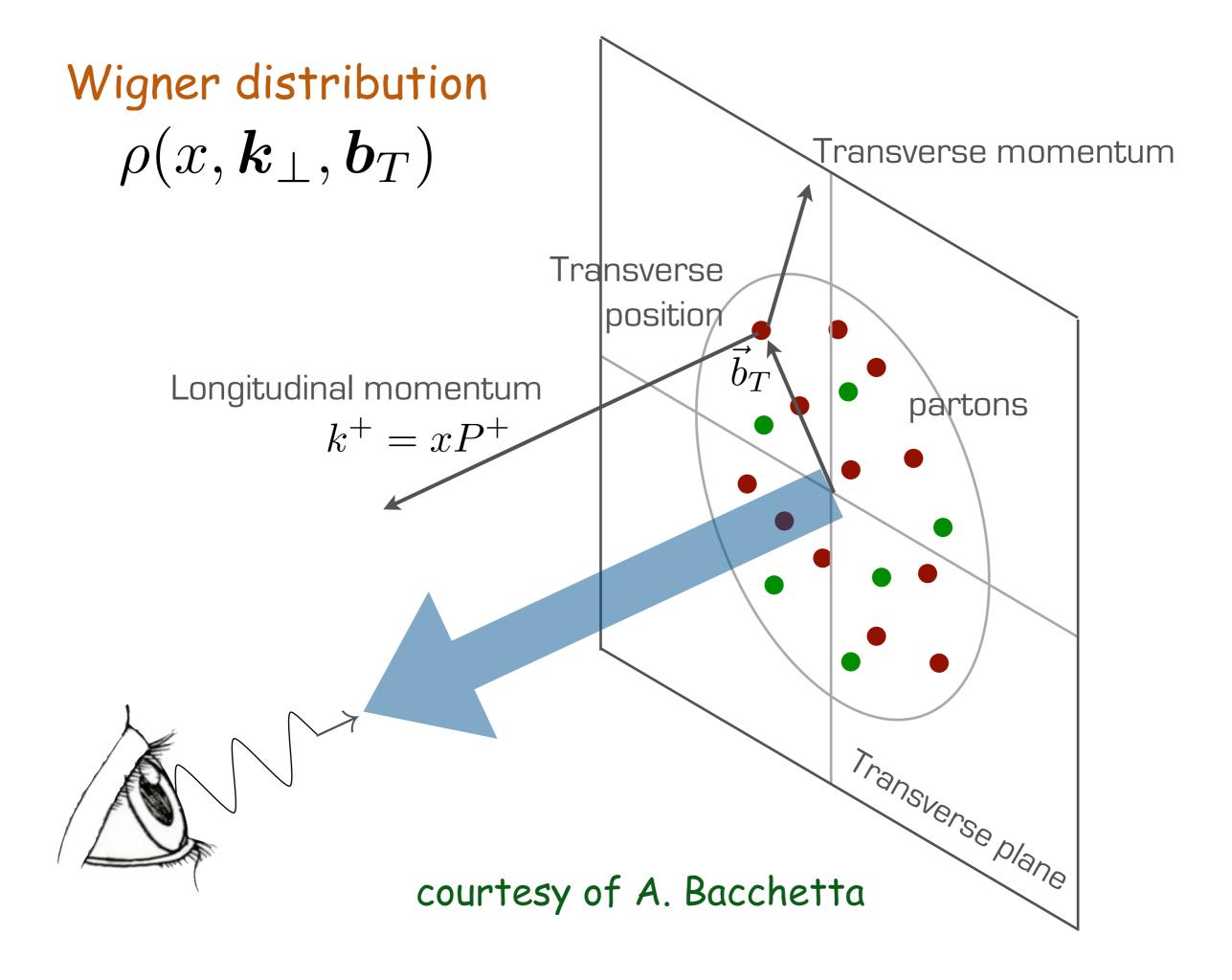
The nucleon landscape Markus Diehl, arXiv:1512.01328



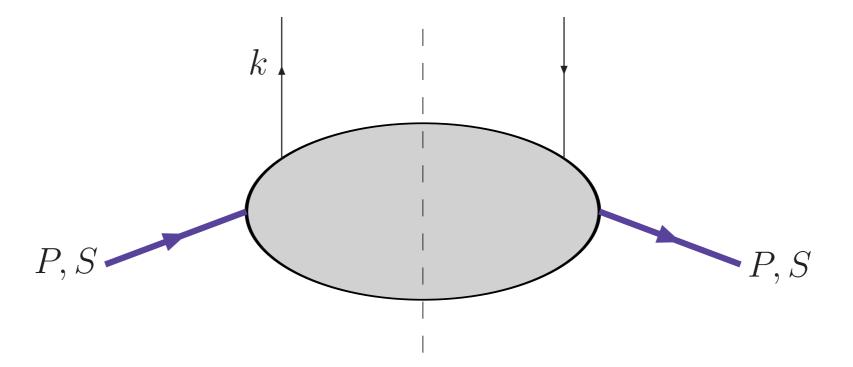
Burkardt, Pasquini, arXiv:1510.02567



special issue of EPJA dedicated to the 3D nucleon structure, EPJA 52, (2016) 164 (15 contributions, Editors M.A., P. Rossi. M. Guidal)



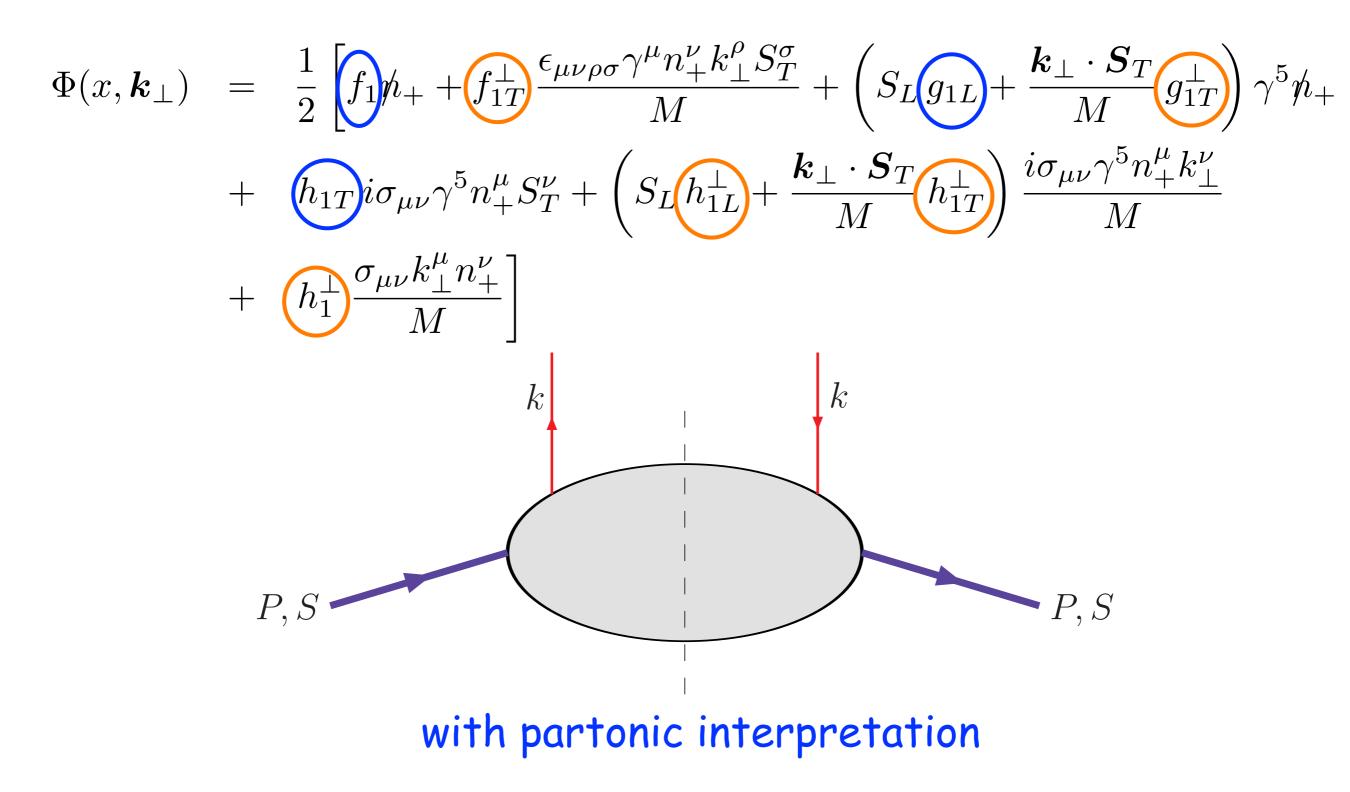
TMD formalism - The nucleon correlator, in collinear configuration: 3 distribution functions



 $\Phi_{ij}(k;P,S) = \sum_{X} \int \frac{\mathrm{d}^{3} P_{X}}{(2\pi)^{3} 2E_{X}} (2\pi)^{4} \delta^{4}(P-k-P_{X}) \langle PS|\overline{\Psi}_{j}(0)|X\rangle \langle X|\Psi_{i}(0)|PS\rangle$ $= \int \mathrm{d}^{4} \xi \, e^{ik \cdot \xi} \langle PS|\overline{\Psi}_{j}(0)\Psi_{i}(\xi)|PS\rangle$

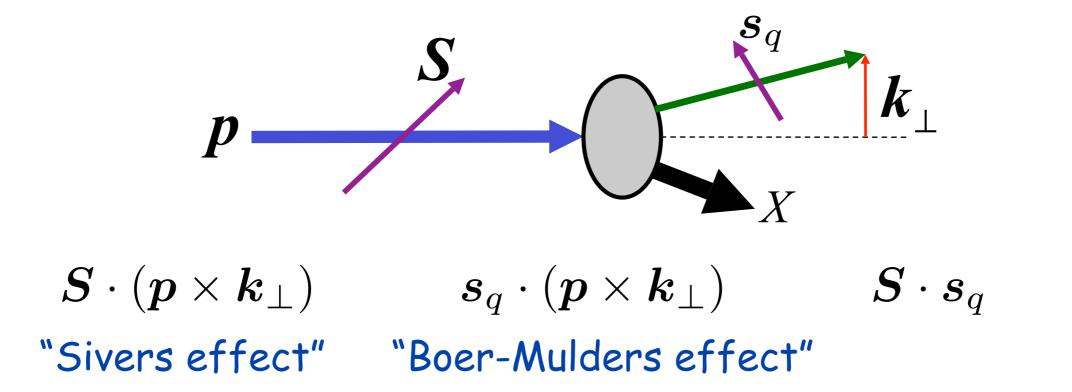
$$\Phi(x,S) = \frac{1}{2} \left[\underbrace{f_1(x)}_{\mathbf{q}} \not{n}_+ + S_L \underbrace{g_{1L}(x)}_{\Delta \mathbf{q}} \gamma^5 \not{n}_+ + \underbrace{h_{1T}}_{\Delta_{\mathsf{T}} \mathbf{q}} i\sigma_{\mu\nu} \gamma^5 n_+^{\mu} S_T^{\nu} \right]$$

TMD-PDFs: the leading-twist correlator, with intrinsic k_{\perp} , contains 8 independent functions

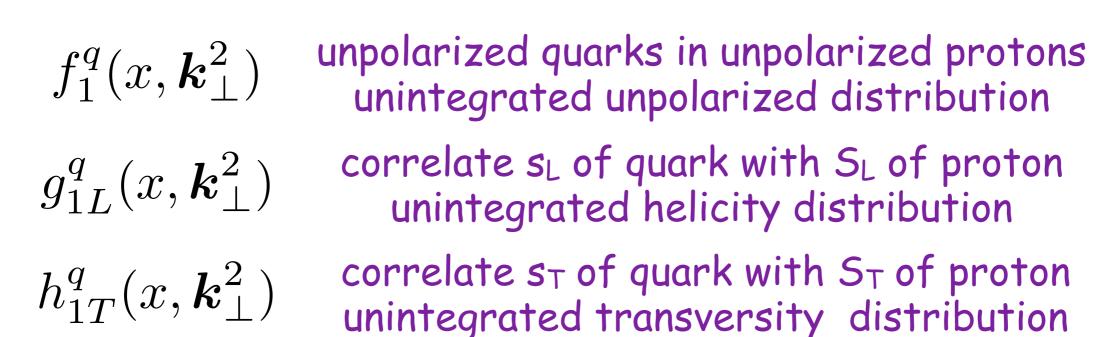


TMDs in simple parton model TMDs = Transverse Momentum Dependent Parton Distribution Functions (TMD-PDF) or Transverse Momentum Dependent Fragmentation Functions (TMD-FF)

TMD-PDFs give the number density of partons, with their intrinsic motion and spin, inside a fast moving proton, with its spin.



there are 8 independent TMD-PDFs



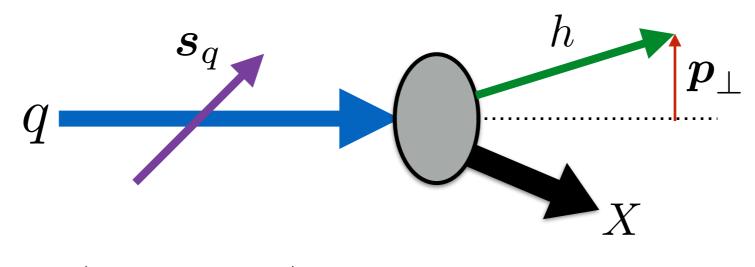
only these survive in the collinear limit

 $f_{1T}^{\perp q}(x, k_{\perp}^{2})$

correlate k_{\perp} of quark with S_{\top} of proton (Sivers) $h_1^{\perp q}(x, k_\perp^2)$ correlate k_\perp and s_T of quark (Boer-Mulders)

$$\begin{array}{ll} g_{1T}^{\perp q}(x, \boldsymbol{k}_{\perp}^2) & h_{1L}^{\perp q}(x, \boldsymbol{k}_{\perp}^2) & h_{1T}^{\perp q}(x, \boldsymbol{k}_{\perp}^2) \\ & \text{different double-spin correlations} \end{array}$$

TMD-FFs give the number density of hadrons, with their momentum, originated in the fragmentation of a fast moving parton, with its spin.



 $oldsymbol{s}_q \cdot (oldsymbol{p}_q imes oldsymbol{p}_\perp)$ "Collins effect"

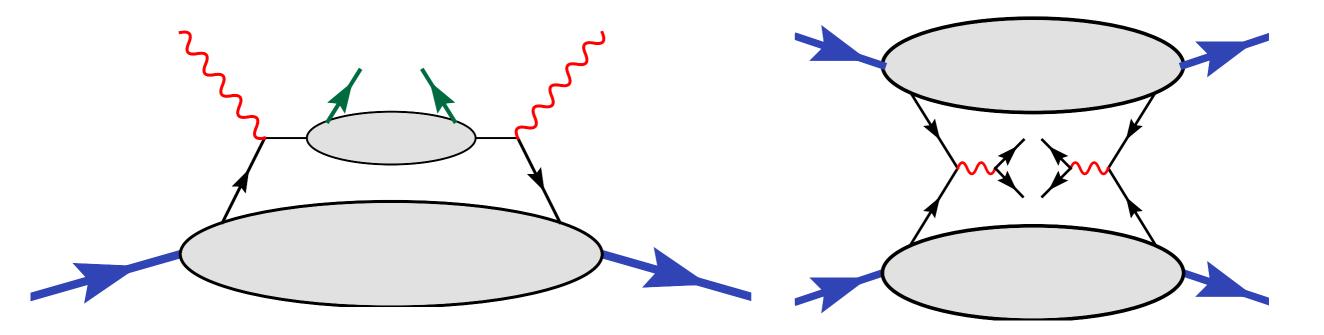
there are 2 independent TMD-FFs for spinless hadrons

 $D_1^q(z, \pmb{p}_\perp^2)$ unpolarized hadrons in unpolarized quarks unintegrated fragmentation function

 $H_1^{\perp q}(z, {m p}_{\perp}^2)$ correlate ${m p}_{\perp}$ of hadron with ${m s}_{m T}$ of quark (Collins)

how to "measure" TMDs?

needs processes which relate physical observables to parton intrinsic motion



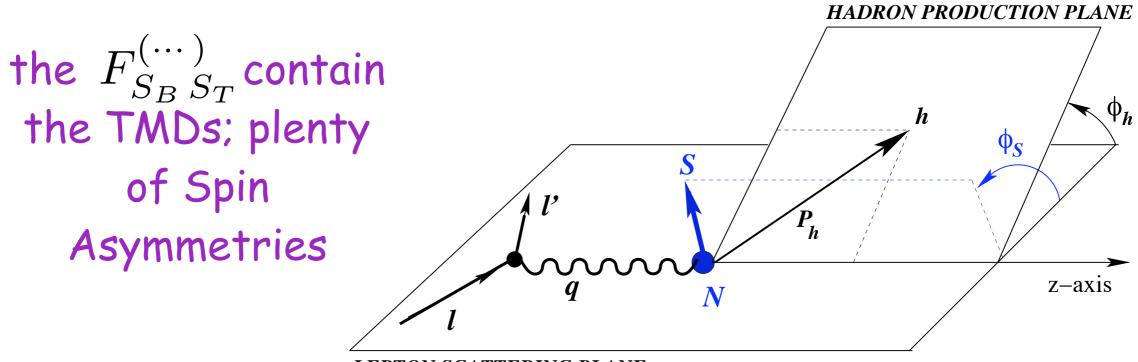
SIDISDrell-Yan processes $\ell N \to \ell h X$ $p N \to \ell^+ \ell^- X$

a similar diagram for $e^+e^- \rightarrow h_1 h_2 X$ and, possibly, for $p N \rightarrow h X$

TMDs in SIDIS Q^2 Q^2 hh $\mathrm{d}^{6}\sigma \equiv \frac{\mathrm{d}^{6}\sigma^{\ell p^{\uparrow} \to \ell h X}}{\mathrm{d}x_{B} \,\mathrm{d}Q^{2} \,\mathrm{d}z_{h} \,\mathrm{d}^{2}\boldsymbol{P}_{T} \,\mathrm{d}\phi_{S}}$ ${m P}_T\simeq {m p}_\perp+z{m k}_\perp$ p, S p, STMD factorization holds at large Q^2 , and $P_T \approx k_{\perp} \approx \Lambda_{\rm QCD}$ Two scales: $P_T \ll Q^2$ TMD-PDFs hard scattering TMD-FFs $=\sum (f_q(x, \boldsymbol{k}_{\perp}; Q^2)) \otimes d\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_{\perp}; Q^2)) \otimes D_q^h(z, \boldsymbol{p}_{\perp}; Q^2)$ $\mathrm{d}\sigma^{\ell p \to \ell h X}$

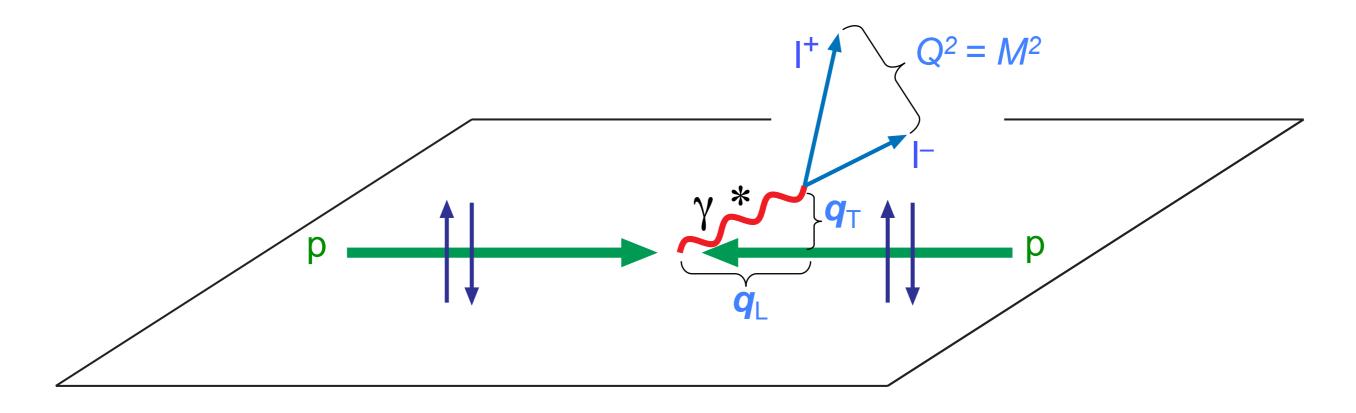
(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz...)

$$\begin{split} \frac{\mathrm{d}\sigma}{\mathrm{d}\phi} &= F_{UU} + \cos(2\phi) \, F_{UU}^{\cos(2\phi)} + \frac{1}{Q} \, \cos\phi \, F_{UU}^{\cos\phi} + \lambda \frac{1}{Q} \, \sin\phi \, F_{LU}^{\sin\phi} \\ &+ S_L \left\{ \sin(2\phi) \, F_{UL}^{\sin(2\phi)} + \frac{1}{Q} \, \sin\phi \, F_{UL}^{\sin\phi} + \lambda \left[F_{LL} + \frac{1}{Q} \, \cos\phi \, F_{LL}^{\cos\phi} \right] \right\} \\ &+ S_T \left\{ \frac{\sin(\phi - \phi_S) \, F_{UT}^{\sin(\phi - \phi_S)} + \sin(\phi + \phi_S) \, F_{UT}^{\sin(\phi + \phi_S)} + \sin(3\phi - \phi_S) \, F_{UT}^{\sin(3\phi - \phi_S)} \\ &+ \frac{1}{Q} \left[\sin(2\phi - \phi_S) \, F_{UT}^{\sin(2\phi - \phi_S)} + \sin\phi_S \, F_{UT}^{\sin\phi_S} \right] \\ &+ \lambda \left[\cos(\phi - \phi_S) \, F_{LT}^{\cos(\phi - \phi_S)} + \frac{1}{Q} \left(\cos\phi_S \, F_{LT}^{\cos\phi_S} + \cos(2\phi - \phi_S) \, F_{LT}^{\cos(2\phi - \phi_S)} \right) \right] \right\} \end{split}$$



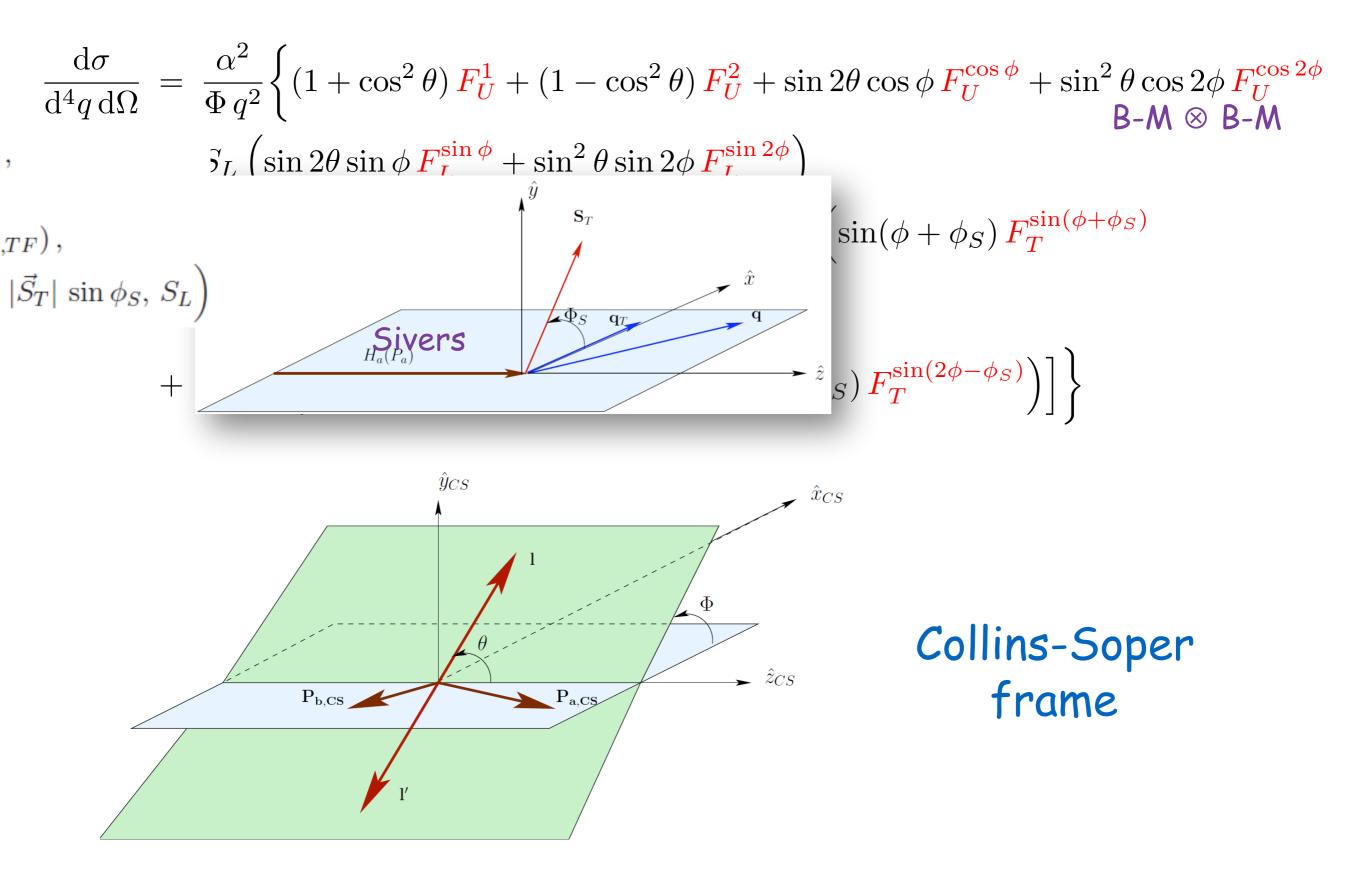
LEPTON SCATTERING PLANE

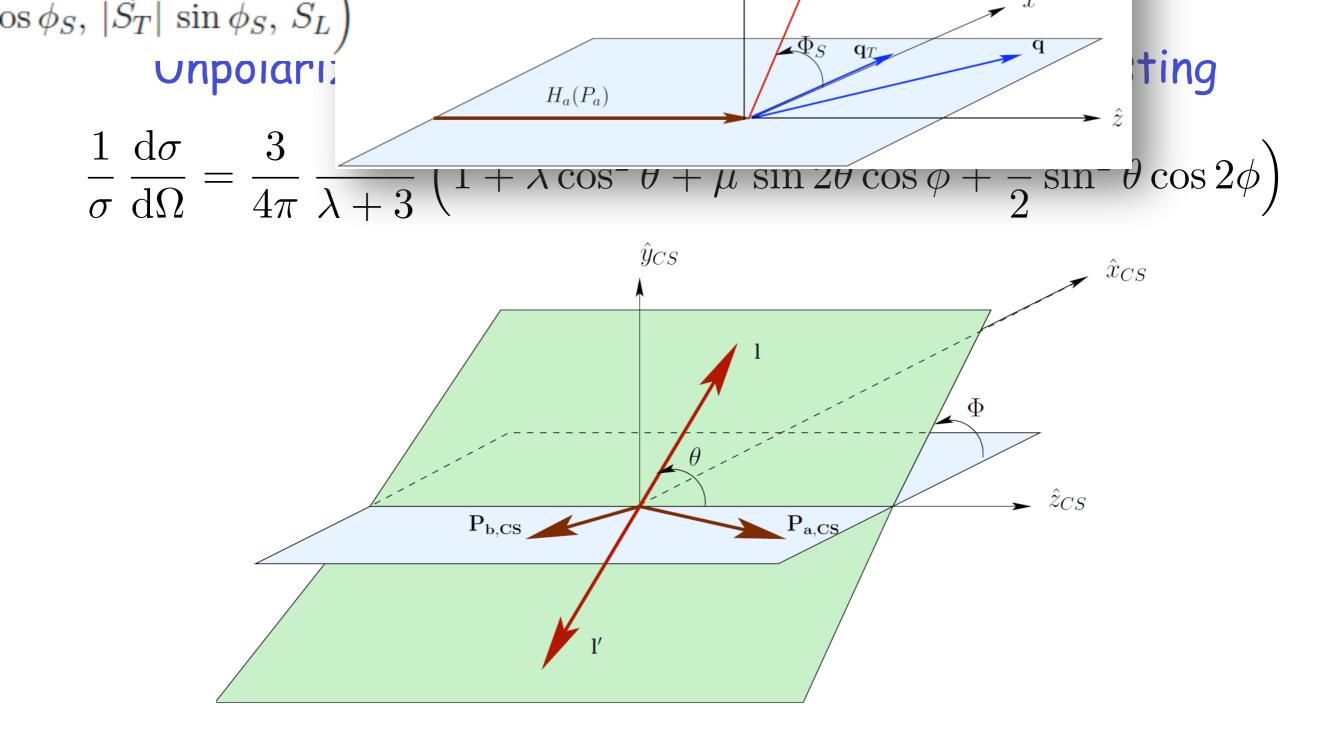
TMDs in Drell-Yan processes COMPASS, RHIC, Fermilab, NICA, AFTER...



factorization holds, two scales, M², and $q_{T} \ll M$ $d\sigma^{D-Y} = \sum_{a} f_{q}(x_{1}, \mathbf{k}_{\perp 1}; Q^{2}) \otimes f_{\bar{q}}(x_{2}, \mathbf{k}_{\perp 2}; Q^{2}) d\hat{\sigma}^{q\bar{q} \rightarrow \ell^{+}\ell^{-}}$ direct product of TMDs, no fragmentation process

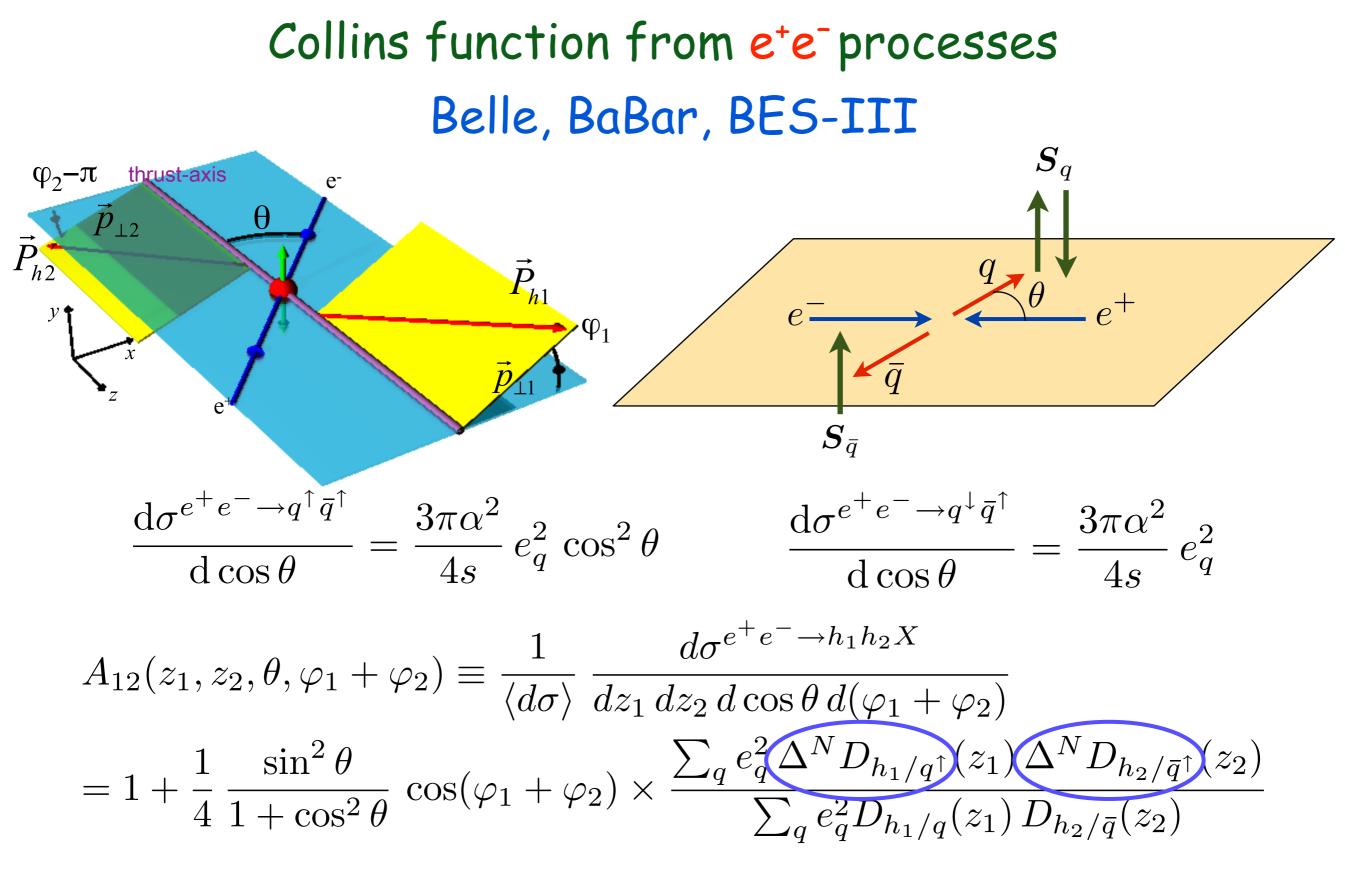
Case of one polarized nucleon only





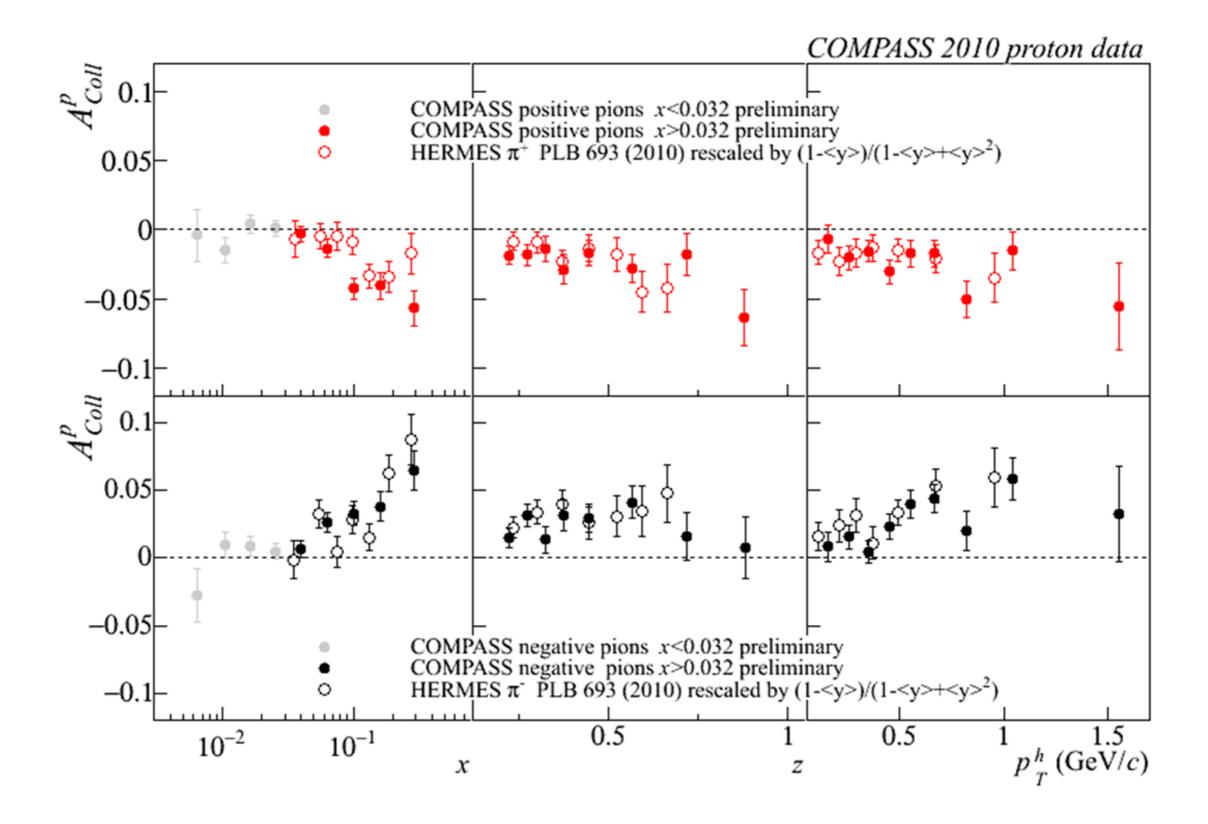
Collins-Soper frame

naive collinear parton model: $\lambda = 1$ $\mu = \nu = 0$



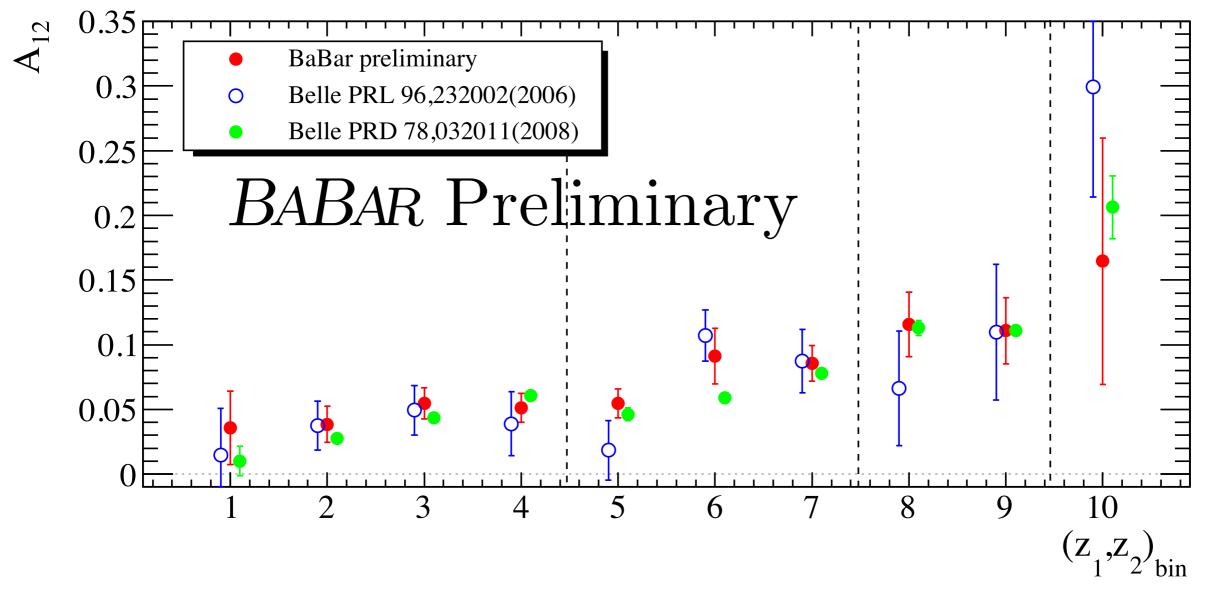
another similar asymmetry can be measured, A₀

Experimental results: clear evidence for Sivers and Collins effects from SIDIS data (HERMES, COMPASS, JLab)

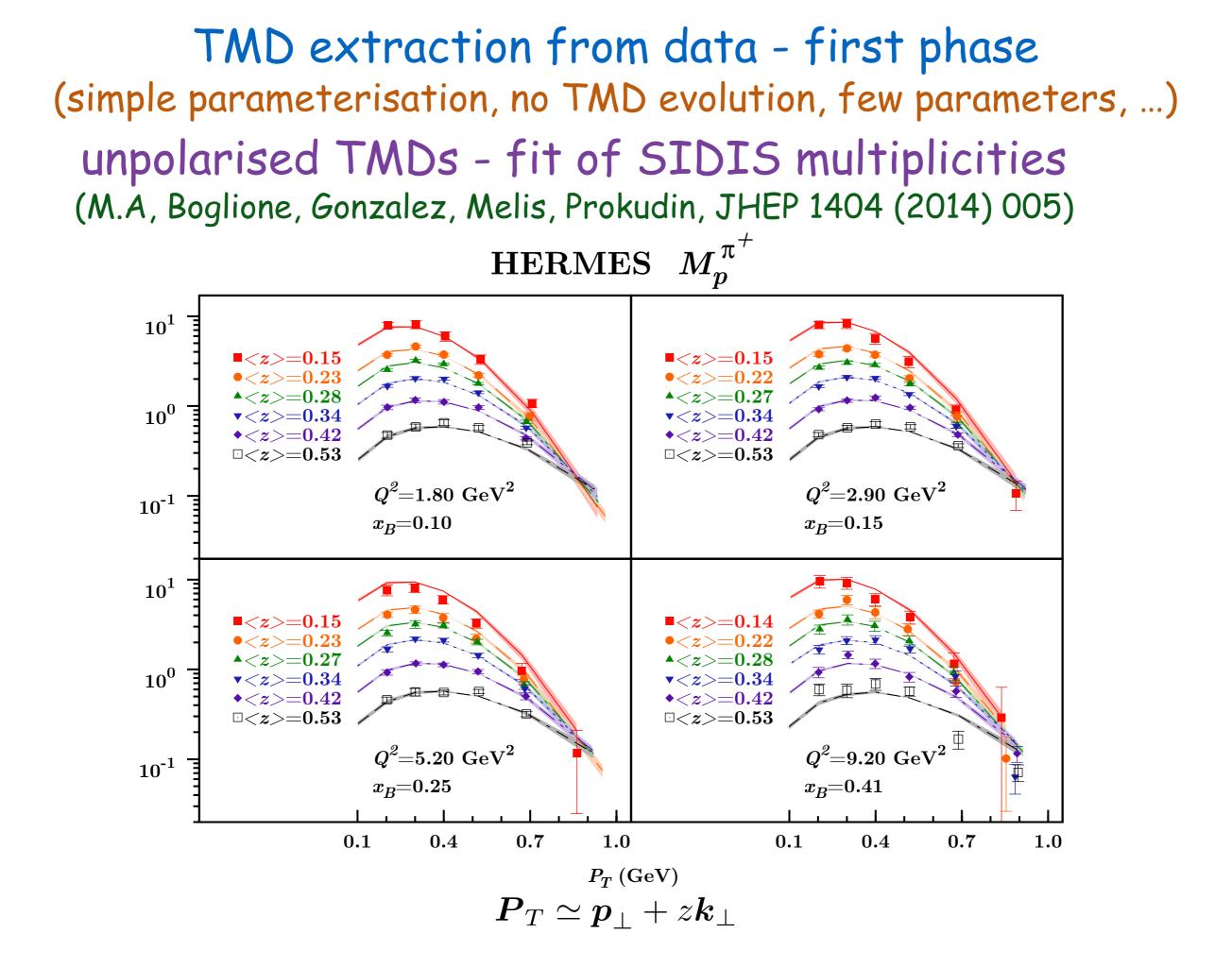


independent evidence for Collins effect from e⁺e⁻ data at Belle, BaBar and BES-III

$$A_{12}(z_1, z_2) \sim \Delta^N D_{h_1/q^{\uparrow}}(z_1) \otimes \Delta^N D_{h_2/\bar{q}^{\uparrow}}(z_2)$$



I. Garzia, arXiv:1201.4678



simple gaussian distribution works well

$$\frac{d^2 n^h(x_{\scriptscriptstyle B}, Q^2, z_h, P_T)}{dz_h dP_T^2} = \frac{1}{2P_T} M_n^h(x_{\scriptscriptstyle B}, Q^2, z_h, P_T) = \frac{\pi \sum_q e_q^2 f_{q/p}(x_{\scriptscriptstyle B}) D_{h/q}(z_h)}{\sum_q e_q^2 f_{q/p}(x_{\scriptscriptstyle B})} \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

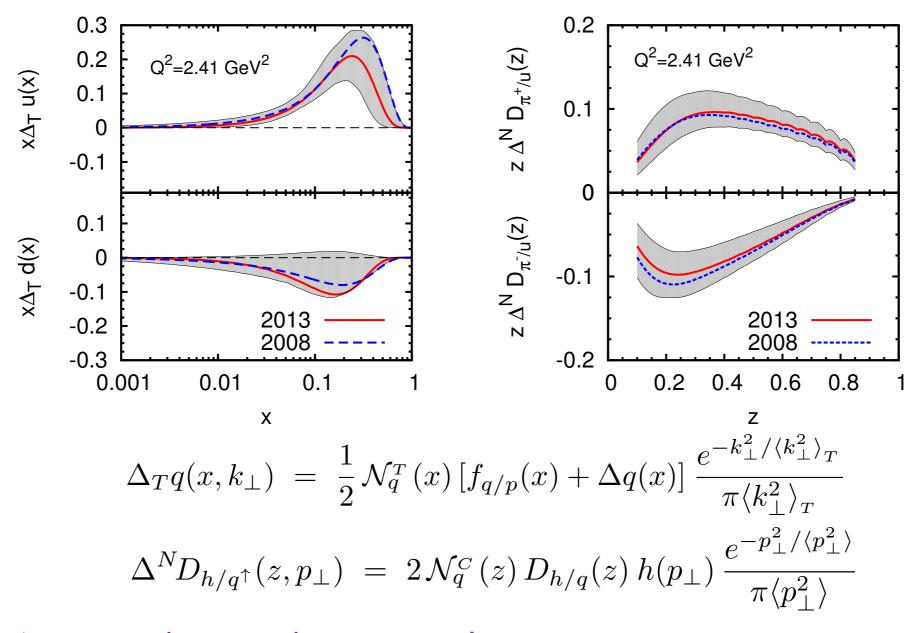
$$f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$$\langle k_\perp^2 \rangle = 0.57 \qquad \langle p_\perp^2 \rangle = 0.12$$

a similar analysis performed by Signori, Bacchetta, Radici, Schnell, JHEP 1311 (2013) 194; it also assumes gaussian behaviour

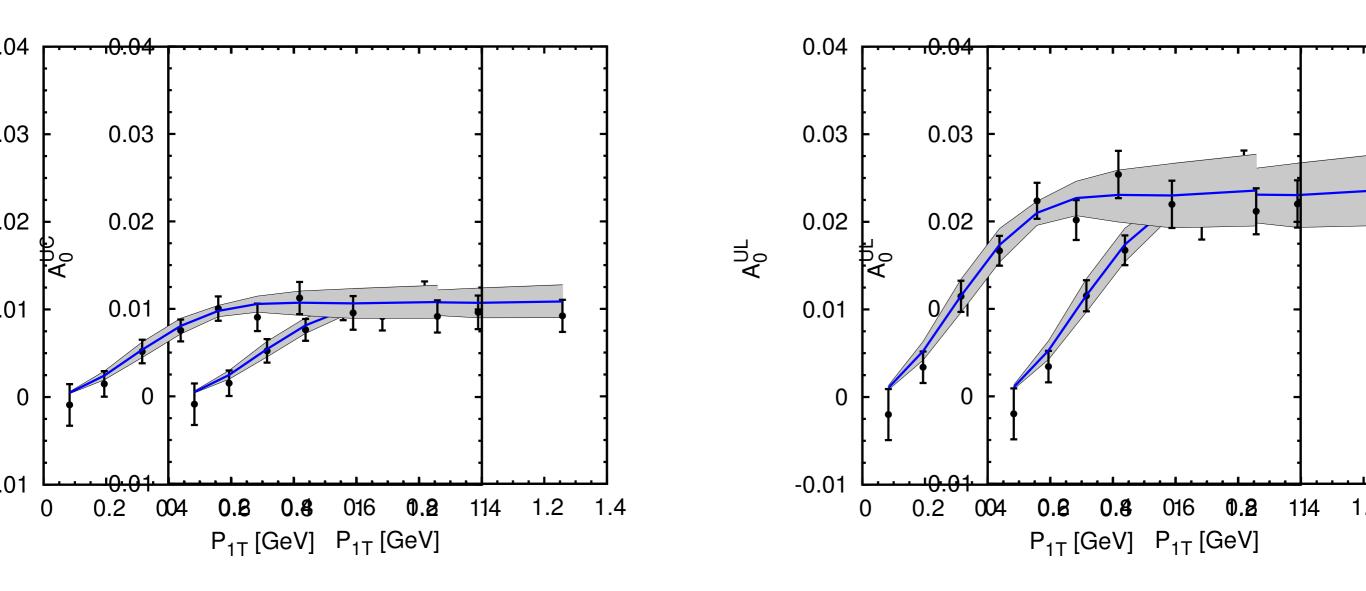
TMD extraction: transversity and Collins functions - first phase M. A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, PRD 87 (2013) 094019



SIDIS and e+e- data, simple parameterization, no TMD evolution, agreement with extraction using di-hadron FF

(recent papers by Bacchetta, Courtoy, Guagnelli, Radici, JHEP 1505 (2015) 123; Kang, Prokudin, Sun, Yuan, Phys. Rev. D91 (2015) 071501; arXiv:1505.05589)

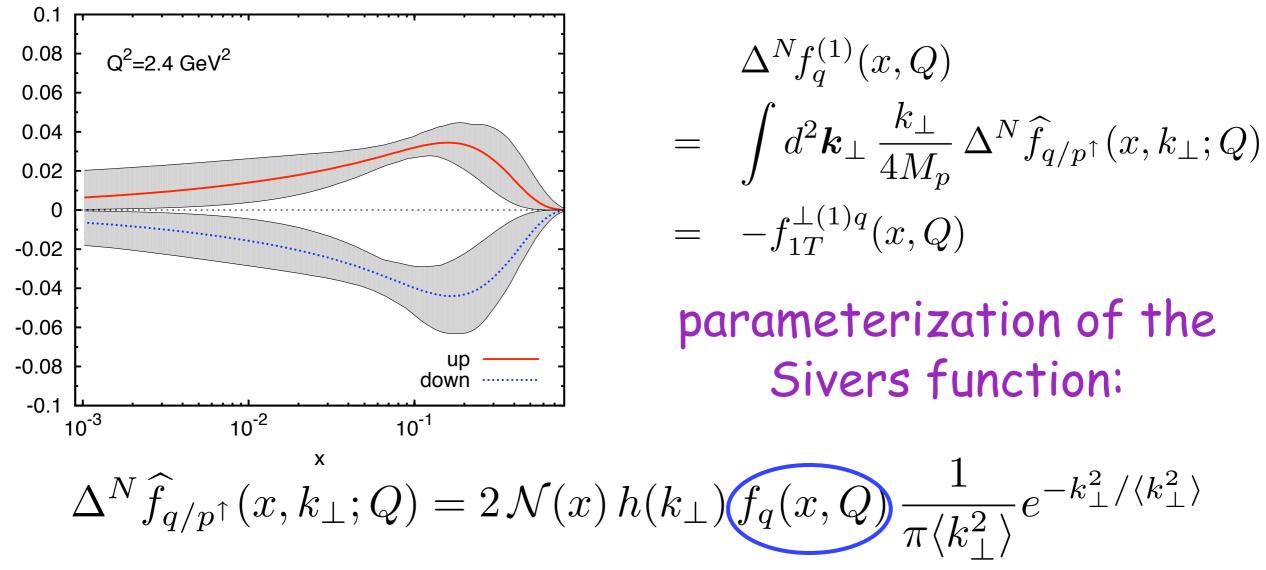
recent BaBar data on the p_{\perp} dependence of the Collins function (first direct measurement)



gaussian p_⊥ dependence of Collins functions (M.A., Boglione, D'Alesio, Gonzalez, Melis, Murgia, Prokudin, in preparation)

extraction of u and d Sivers functions - first phase M.A, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin (in agreement with several other groups)

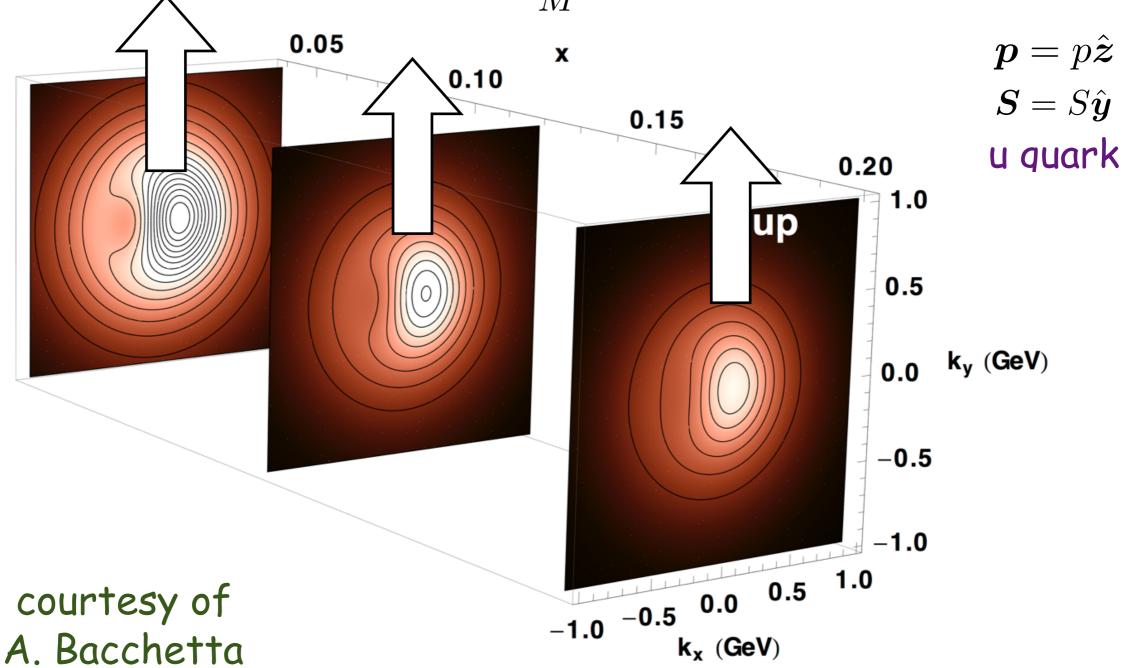
 $x \Delta^N f_q^{(1)}(x,Q)$

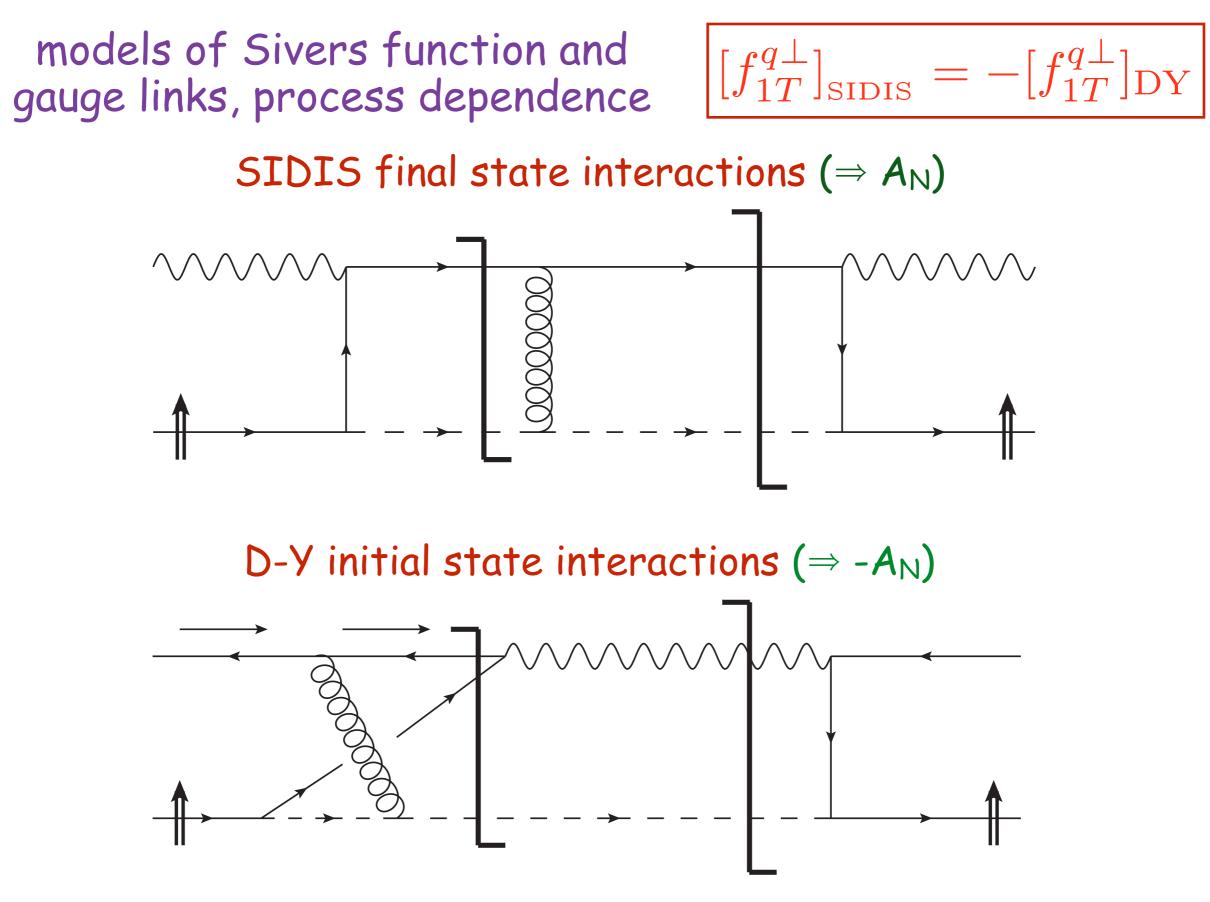


Q² evolution only taken into account in the collinear part (usual PDF)

Sivers effects induces distortions in the parton distribution

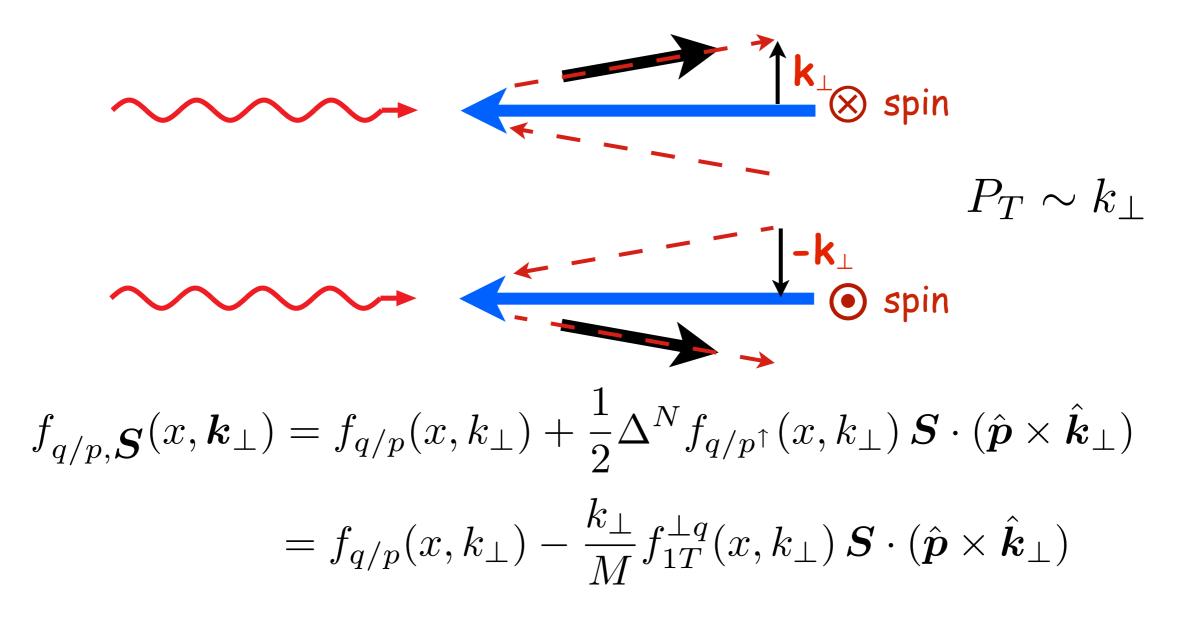
$$f_{q/p,\mathbf{S}}(x,\mathbf{k}_{\perp}) = f_{q/p}(x,k_{\perp}) + \frac{1}{2}\Delta^{N}f_{q/p^{\uparrow}}(x,k_{\perp})\,\mathbf{S}\cdot(\hat{\mathbf{p}}\times\hat{\mathbf{k}}_{\perp})$$
$$= f_{q/p}(x,k_{\perp}) - \frac{k_{\perp}}{M}f_{1T}^{\perp q}(x,k_{\perp})\,\mathbf{S}\cdot(\hat{\mathbf{p}}\times\hat{\mathbf{k}}_{\perp})$$





Brodsky, Hwang, Schmidt, PL B530 (2002) 99; NP B642 (2002) 344 Brodsky, Hwang, Kovchegov, Schmidt, Sievert, PR D88 (2013) 014032

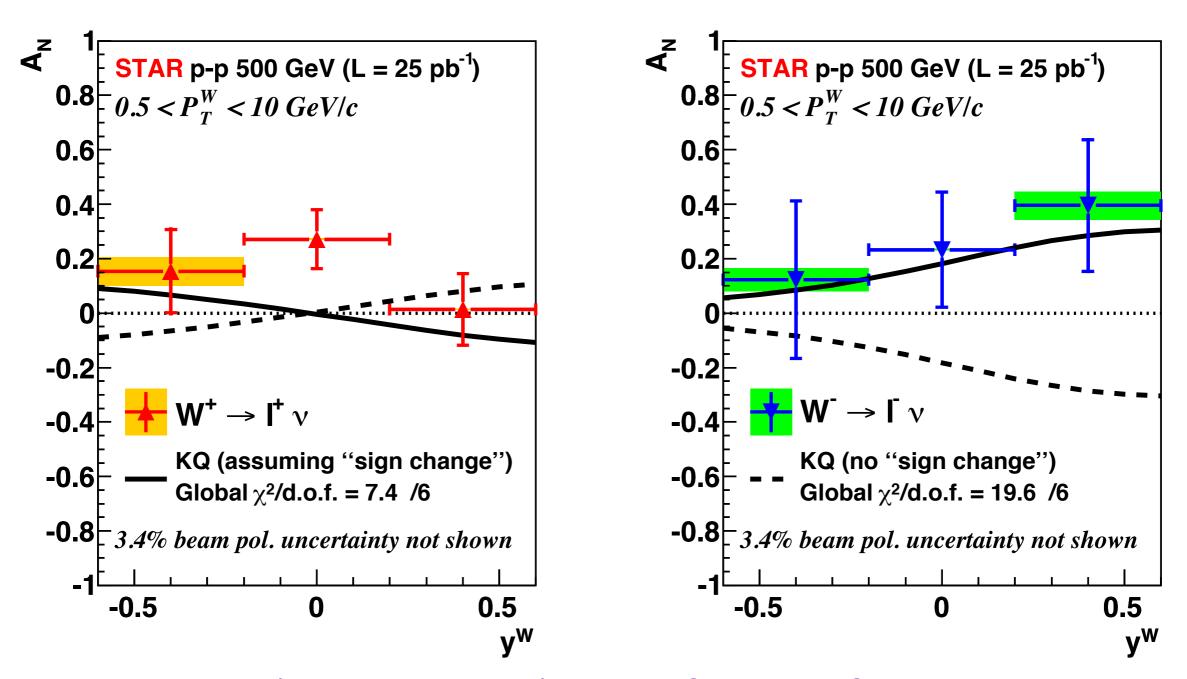
but the the Sivers effect has a simple physical picture...



left-right spin asymmetry for the process $\gamma^*q
ightarrow q$

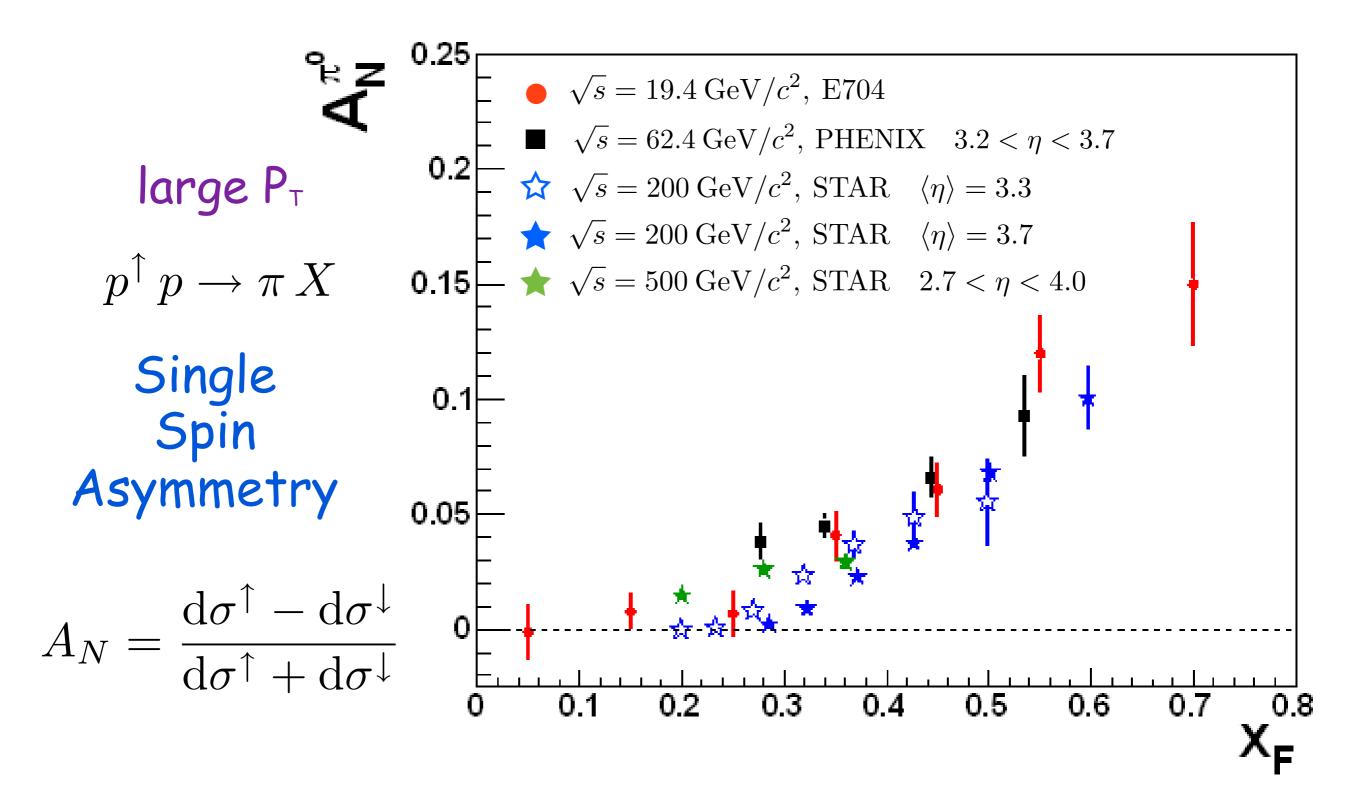
the spin- \mathbf{k}_{\perp} correlation is an intrinsic property of the nucleon; it should be related to the parton orbital motion

First results from RHIC, $p^{\uparrow}p \to W^{\pm}X$ STAR Collaboration, PRL 116 (2016) 132301



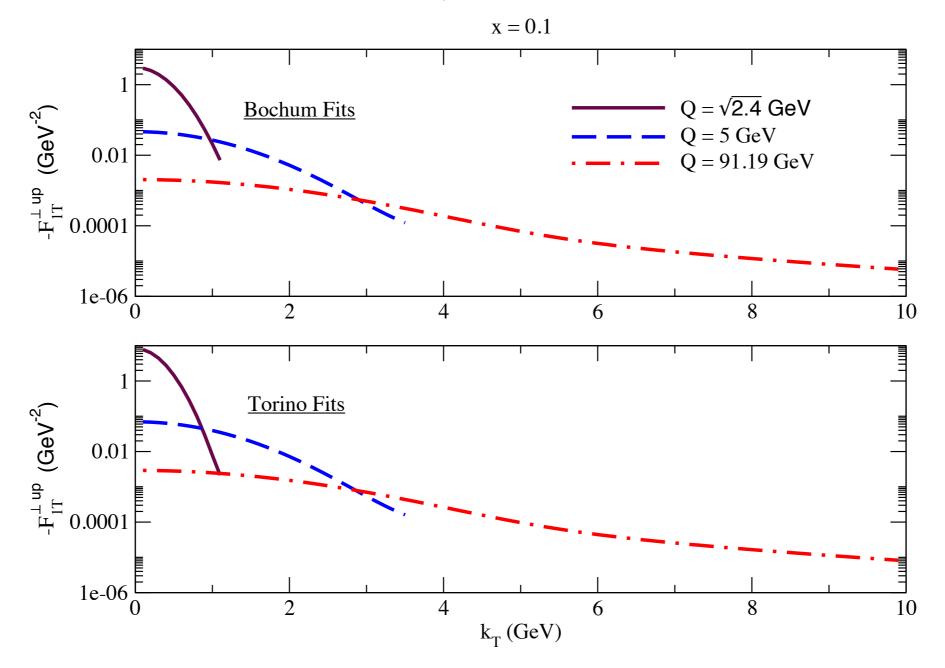
some hints at sign change of Sivers function..... (new results from COMPASS expected soon)

other experimental evidence of the Sivers and Collins effects



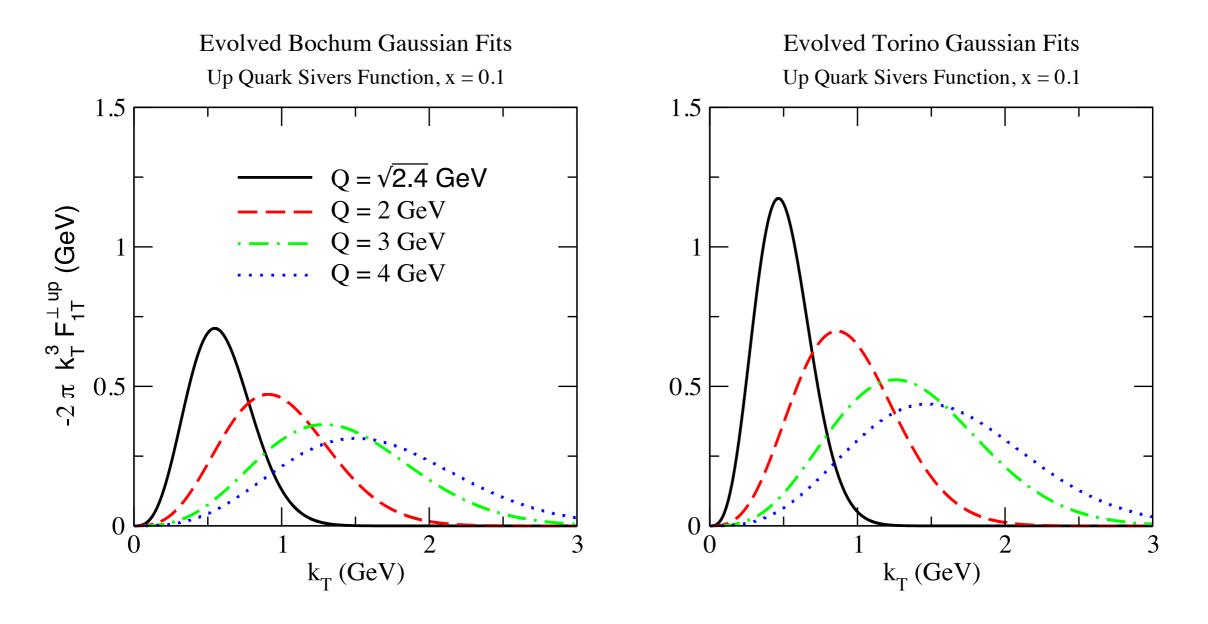
TMD phenomenology - phase 2 how does gluon emission affect the transverse motion? a few selected results

TMD evolution of up quark Sivers function



Aybat, Collins, Qiu, Rogers, Phys. Rev. D85 (2012) 034043

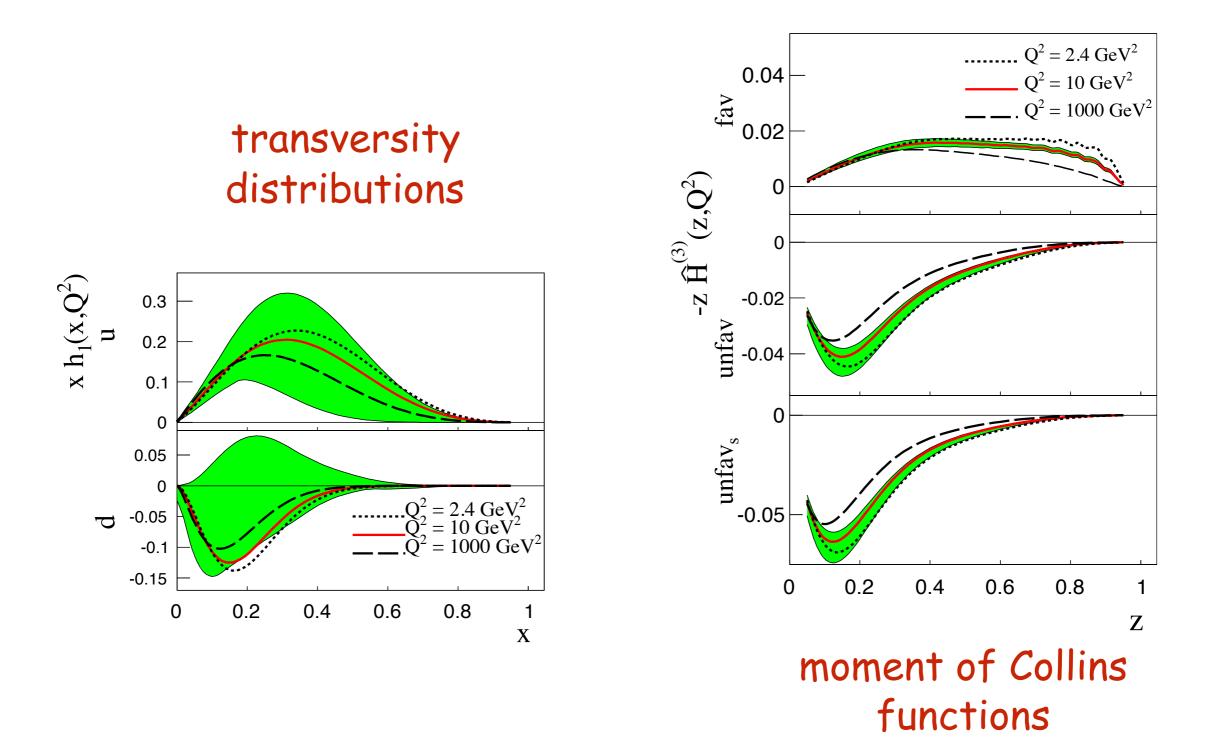
TMD evolution of up quark Sivers function

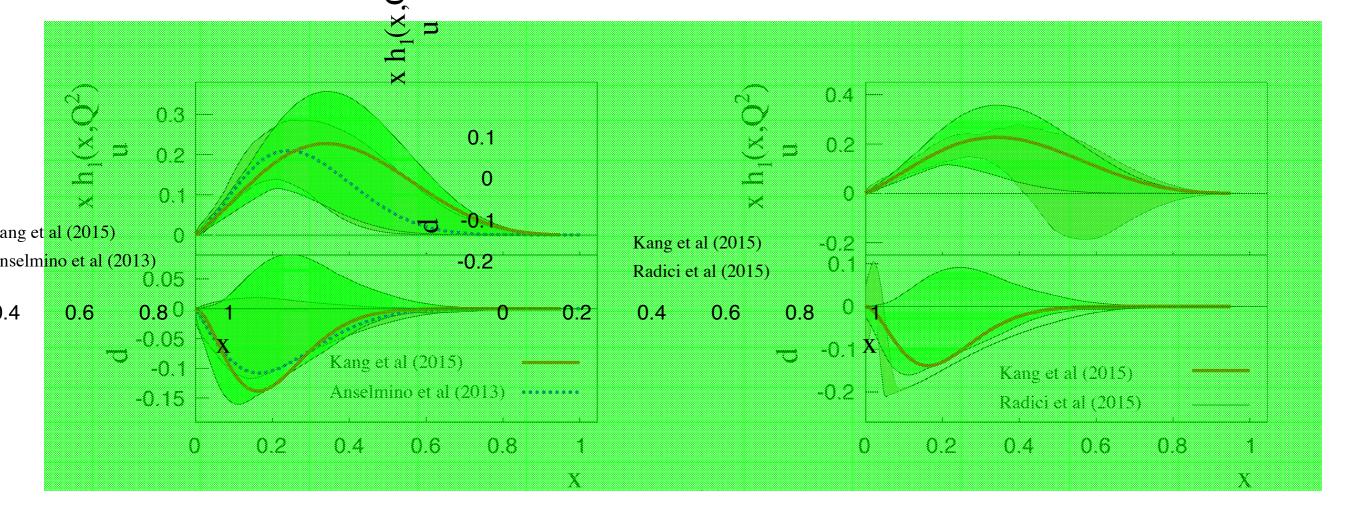


Aybat, Collins, Qiu, Rogers, Phys.Rev. D85 (2012) 034043

TMD evolution of Sivers function studied also by Echevarria, Idilbi, Kang, Vitev, Phys. Rev. D89 (2014) 074013

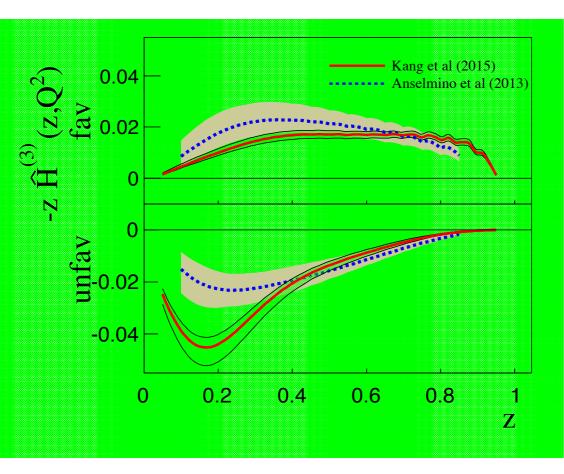
Extraction of transversity and Collins functions with TMD evolution (Kang, Prokudin, Sun, Yuan, arXiv:1505.05589)





comparison with phase 1 extraction, $Q^2 = 2.4 \text{ GeV}^2$

(Kang, Prokudin, Sun, Yuan, arXiv:1505.05589)



Conclusions

The 3D nucleon structure is mysterious and fascinating. Many experimental results show the necessity to go beyond the simple collinear partonic picture and give new information. Crucial task is interpreting data and building a consistent 3D description of the nucleon.

Sivers and Collins effects are well established, with many transverse spin asymmetries resulting from them. Sivers function, TMDs and orbital angular momentum? QCD analysis of TMDs and GPDs sound and well developed.

Combined data from SIDIS, Drell-Yan, e+e-, with theoretical modelling, should lead to a true 3D imaging of the proton

Waiting for JLab 12, new COMPASS results, and, crucially, for an **EIC dedicated facility**

Thank you!