

INTRODUCTION TO EFFECTIVE FIELD THEORIES OF QCD

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Outline

- Effective Field Theories
- QCD at Low Energies
- Towards Nuclear Structure
 - ▶ Contact Nuclear EFT
 - ▶ Few-Body Systems
 - ▶ No-Core Shell Model
 - ▶ Halo/Cluster EFT
 - ▶ Conclusions and Outlook

References:

U. van Kolck,

Effective field theory of short-range forces,

Nucl.Phys.A645:273-302,1999, **nucl-th/9808007**

P.F. Bedaque, H.-W. Hammer, and U. van Kolck,

The three-boson system with short-range interactions,

Nucl.Phys.A646:444-466,1999, **nucl-th/9811046**

I. Stetcu, B.R. Barrett, and U. van Kolck,

No-core shell model in an effective-field-theory framework,

Phys.Lett.B653:358-362,2007, **nucl-th/0609023**

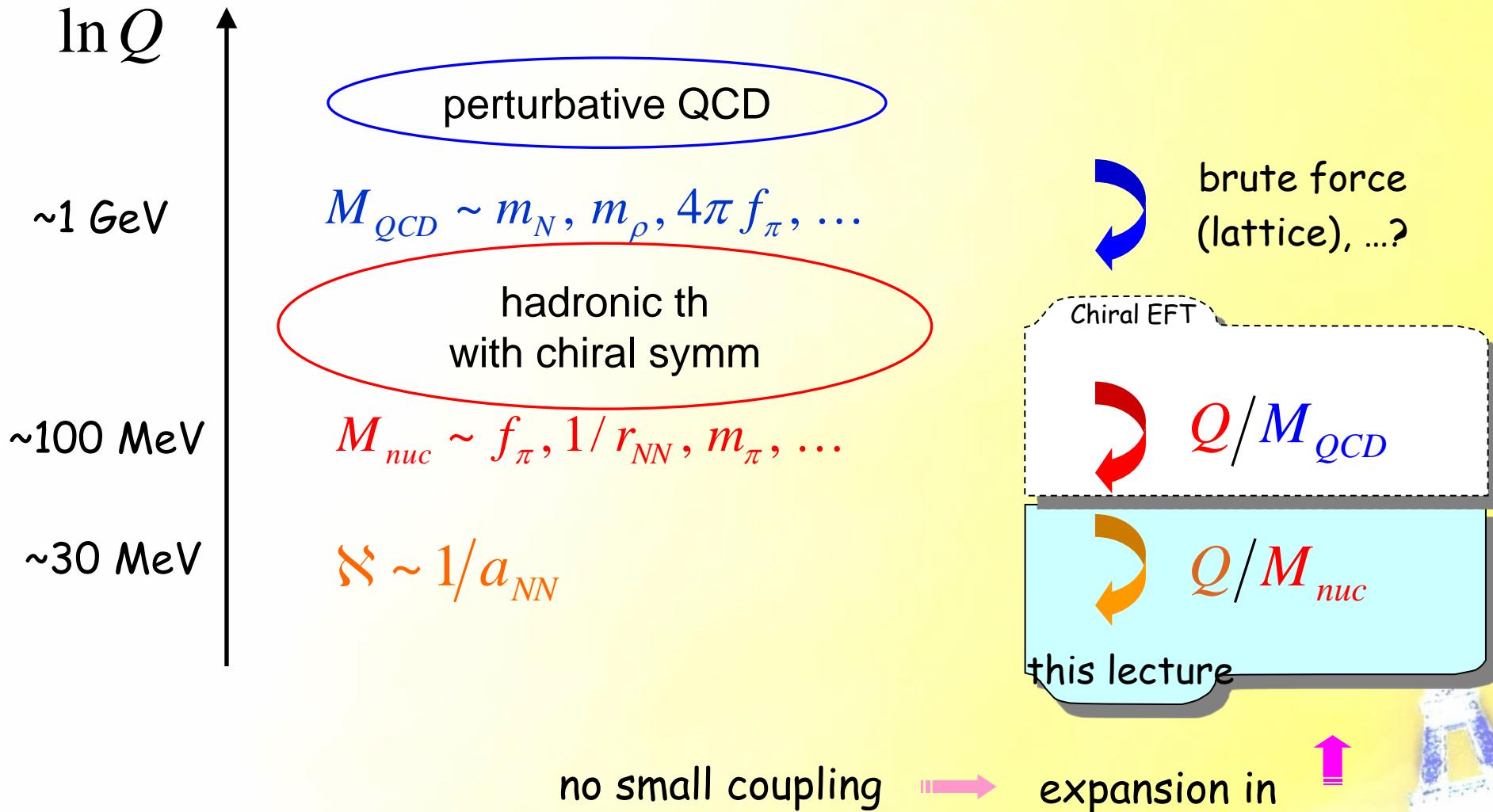
P.F. Bedaque, H.-W. Hammer, and U. van Kolck,

Narrow resonances in effective field theory,

Phys.Lett.B569:159-167,2003, **nucl-th/0304007**

Nuclear physics scales

"His scales are His pride", Book of Job



Lots of interesting nuclear physics at $E \sim 1$ MeV
instead of $E \sim 10$ MeV

within a few MeV of thresholds:

- many energy levels and resonances (cluster structures)
 - most reactions of astrophysical interest

show universal features,

i.e. to a very good approximation are independent
of details of the short-range dynamics

bonus: same techniques can be used
for dilute atomic/molecular systems

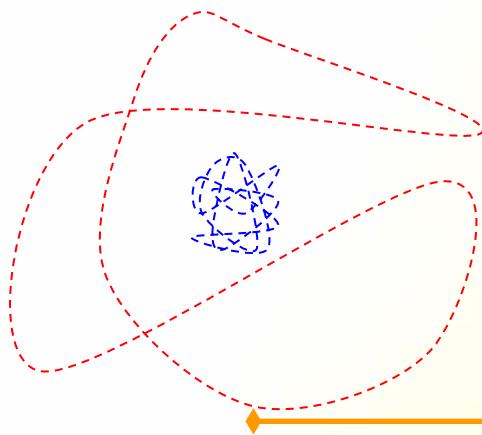
- pionful EFT an overkill at lower energies!

cf. Bethe + Peierls '35

e.g. NN s_1 channel:

(real) bound state = deuteron

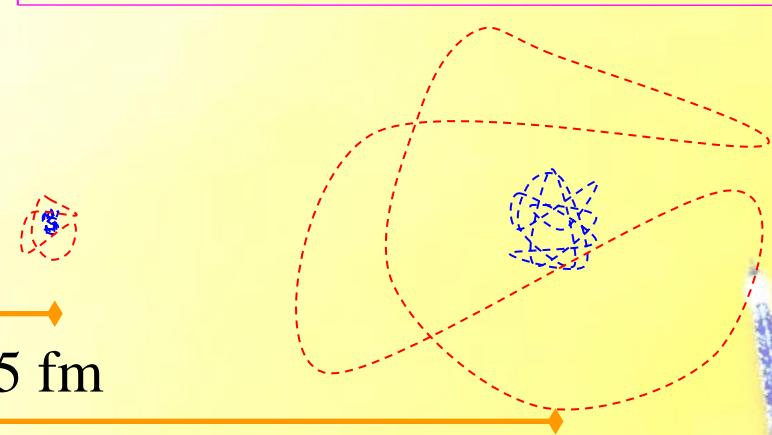
$$\mathfrak{X}_1 \sim \sqrt{m_N B_d} \cong 45 \text{ MeV} < m_\pi$$



s_0 channel:

(virtual) bound state

$$\mathfrak{X}_0 \sim \sqrt{m_N B_{d^*}} \cong 8 \text{ MeV} \ll m_\pi$$

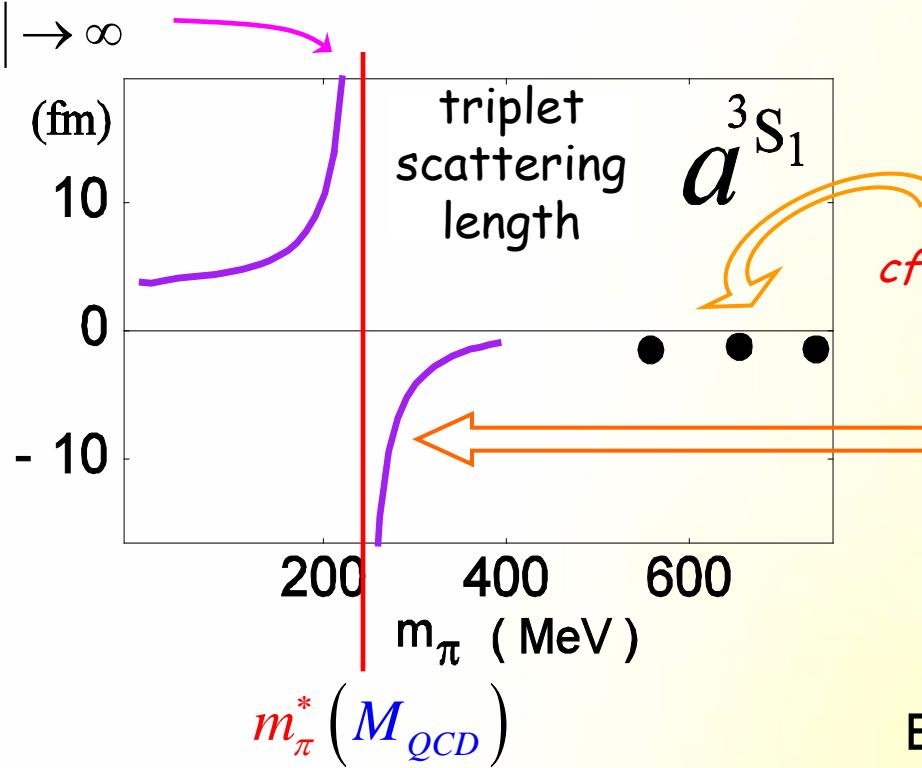


$$1/\mathfrak{X}_1 \cong 4.5 \text{ fm}$$

 multipole expansion of meson cloud:
contact interactions among local nucleon fields

Pion-mass dependence

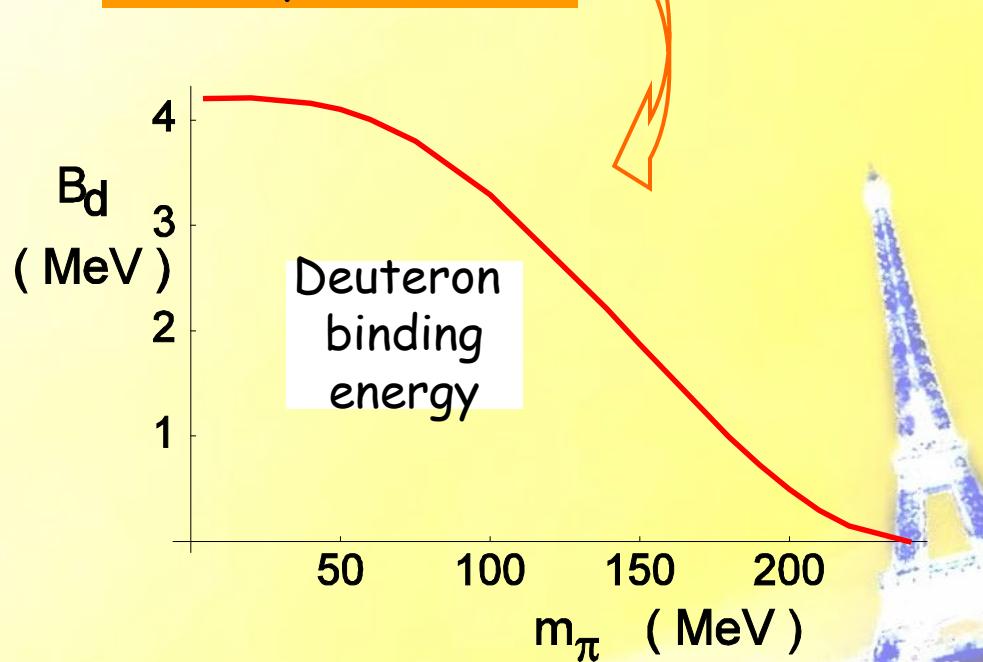
unitarity limit



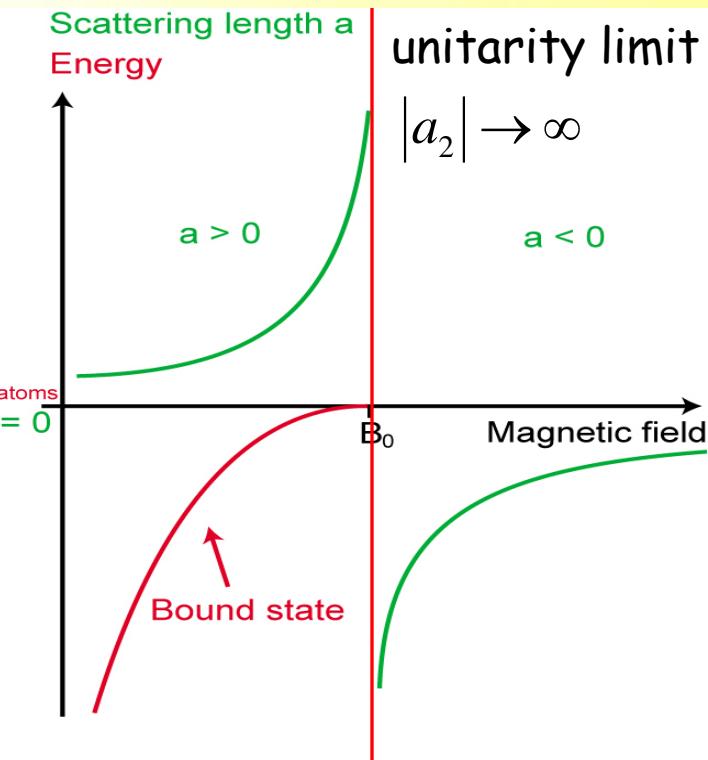
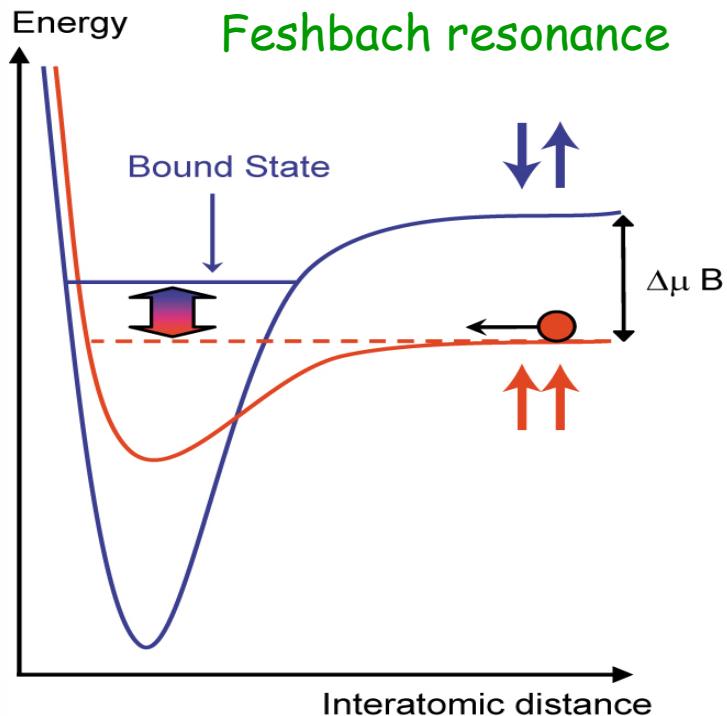
Large deuteron size because

$$m_\pi \sim m_\pi^* (M_{QCD})$$

$\propto \sim \frac{m_\pi - m_\pi^*}{m_\pi^*} M_{nuc}$



Cf. trapped fermions



quark masses analog to magnetic field:
close to critical values

$$m_\pi^{*2} = \mathcal{O}\left(\left(m_u^* + m_d^*\right) M_{QCD}\right) \simeq (200 \text{ MeV})^2$$

contact EFT can, and has been, used for atomic systems with large scatt lengths:
universality!

Example: square well $V(r) = -\frac{\alpha^2}{mR^2} \theta\left(1 - \frac{r}{R}\right)$

$$\Rightarrow T(\mathbf{k}) = -i \left[e^{-2i\mathbf{k}R} \frac{\sqrt{\alpha^2 + (\mathbf{k}R)^2} \cot \sqrt{\alpha^2 + (\mathbf{k}R)^2} + i\mathbf{k}r}{\sqrt{\alpha^2 + (\mathbf{k}R)^2} \cot \sqrt{\alpha^2 + (\mathbf{k}R)^2} - i\mathbf{k}r} - 1 \right]$$

zero-energy poles when

$$\alpha_c \equiv (2n+1)\pi/2$$

generic

$$\alpha = \mathcal{O}(1)$$

fine-tuning

$$|1 - \alpha/\alpha_c| \ll 1$$

$$a_0 = R \left(1 - \frac{\tan \alpha}{\alpha} \right)$$

$$a_0 \sim R$$

$$a_0 = -\frac{R}{\alpha_c^2} \left(1 - \frac{\alpha}{\alpha_c} \right)^{-1} \{1+\dots\} \sim \frac{1}{\mathfrak{N}}$$

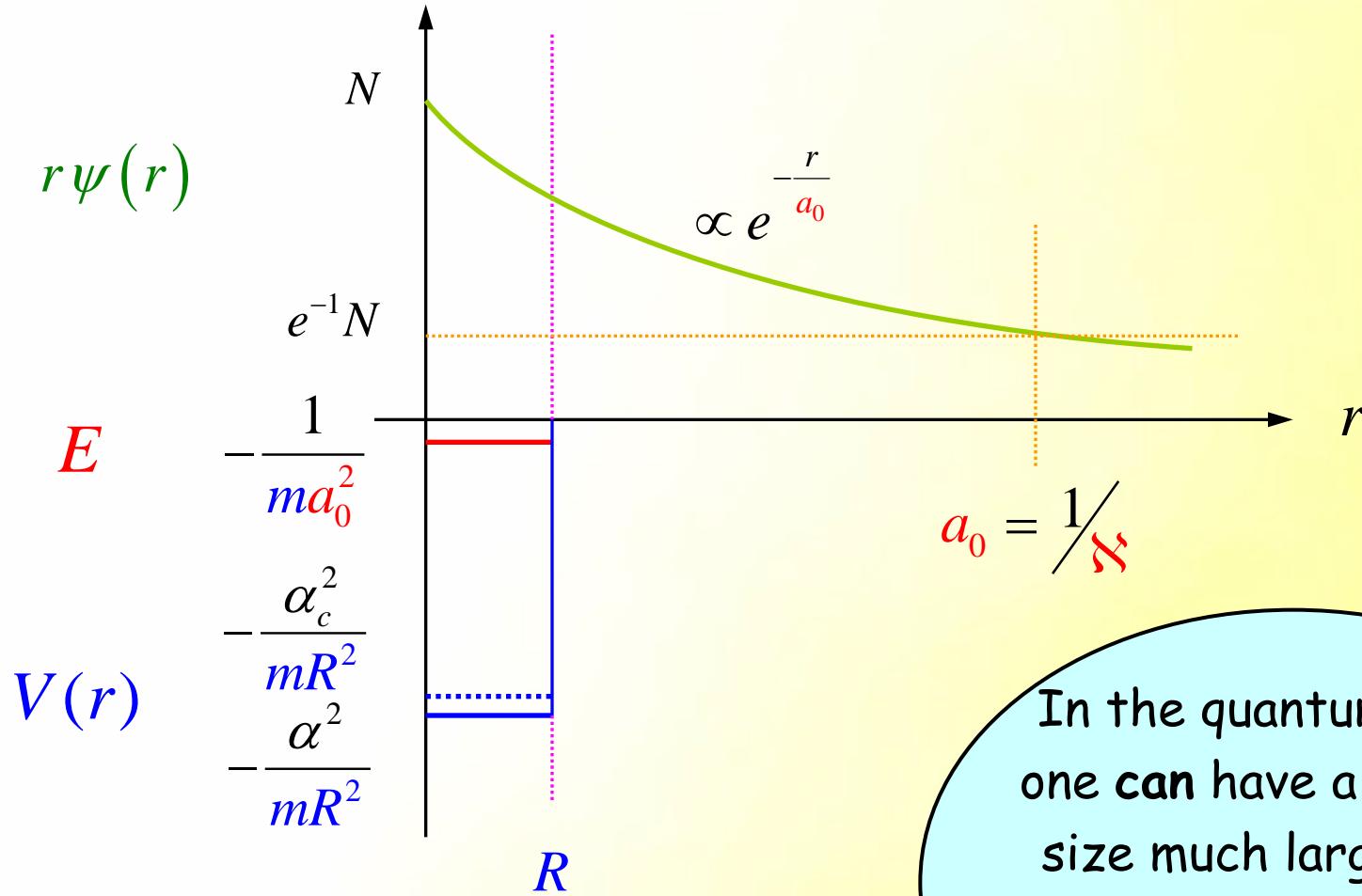
$$r_0 = R \left(1 - \frac{R}{a_0 \alpha^2} - \frac{R^2}{3a_0^2} \right)$$

$$r_0 \sim R$$

$$r_0 = R \{1+\dots\} \sim R$$

etc.

$$\mathfrak{N} \equiv \frac{|1 - \alpha/\alpha_c|}{R} \ll \frac{1}{R}$$



In the quantum world,
 one can have a b.s. with
 size much larger than
 the range of the force
provided
 there is fine-tuning

$$Q \sim \aleph \ll M_{nuc}$$

pionless EFT

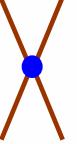
- d.o.f.: nucleons
- symmetries: Lorentz, P, T

$$\mathcal{L}_{EFT} = N^+ \left(i\partial_0 + \frac{\nabla^2}{2m_N} \right) N + C_0 N^+ N N^+ N$$

$$+ N^+ \frac{\nabla^4}{8m_N^3} N + C_2 N^+ N N^+ \nabla^2 N$$

$$+ C'_2 N^+ \vec{\nabla} N \cdot N^+ \vec{\nabla} N + \dots$$

omitting
spin, isospin

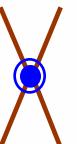

 $\sim iC_0(\Lambda)$


 $\sim C_0^2(\Lambda) \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^0 + p^0 - \frac{(\vec{l} + \vec{p})^2}{2m_N} + i\varepsilon} \quad \textcircled{1}$

$p^0 \equiv \frac{k^2}{2m_N} = -i m_N C_0^2(\Lambda) \int \frac{d^3 l}{(2\pi)^3} \frac{1}{\vec{l}^2 - k^2 - i\varepsilon} \quad \textcircled{2}$

$= -i \frac{m_N}{2\pi^2} C_0^2(\Lambda) \left\{ \int_0^\Lambda dl + k^2 \int_0^\Lambda dl \frac{1}{l + k + i\varepsilon} \frac{1}{l - k - i\varepsilon} \right\} \quad \textcircled{1} \quad \textcircled{2}$

$= -i m_N C_0^2(\Lambda) \left\{ \underbrace{\frac{1}{2\pi^2} \Lambda}_{\text{absorbed in } C_0(\Lambda)} + i \underbrace{\frac{k}{4\pi}}_{\text{non-analytic in } E} + \mathcal{O}\left(\frac{k^2}{4\pi\Lambda}\right) \right\} \equiv -i C_0^2(\Lambda) I_0(\Lambda) \quad \textcircled{1} \quad \textcircled{2}$


 $\sim i C_2(\Lambda) k^2 \quad \text{absorbed in } C_0(\Lambda) \quad \text{non-analytic in } E \quad \text{absorbed in } C_2(\Lambda)$

etc.

$$\left\{ \begin{array}{l} C_0(\Lambda) \rightarrow C_0^{(R)} \equiv C_0(\Lambda) \left\{ 1 - \frac{\textcolor{blue}{m}_N \Lambda}{2\pi^2} C_0(\Lambda) + \dots \right\} = \frac{C_0(\Lambda)}{1 + \frac{\textcolor{blue}{m}_N \Lambda}{2\pi^2} C_0(\Lambda)} \\ \\ C_2(\Lambda) \rightarrow C_2^{(R)} \equiv C_2(\Lambda) - \frac{\textcolor{blue}{m}_N}{4\pi\Lambda} C_0^2(\Lambda) + \dots \\ \\ \dots \end{array} \right.$$

Naïve dimensional analysis

$$C_0^{(R)} \equiv \frac{4\pi}{\textcolor{blue}{m}_N \textcolor{violet}{M}_0}$$

$$C_0^{(R)} \sim \frac{4\pi}{\textcolor{blue}{m}_N \textcolor{blue}{M}_{nuc}} \quad \Rightarrow \quad \textcolor{violet}{M}_0 \sim \textcolor{blue}{M}_{nuc}$$

$$C_2^{(R)} \equiv \frac{4\pi}{\textcolor{blue}{m}_N \textcolor{blue}{M}_{nuc} \textcolor{violet}{M}_2^2}$$

$$C_2^{(R)} \sim \frac{\textcolor{blue}{m}_N}{4\pi \textcolor{blue}{M}_{nuc}} C_0^{(R)2} \quad \Rightarrow \quad \textcolor{violet}{M}_2 \sim \textcolor{blue}{M}_0$$

etc.

But in this case:

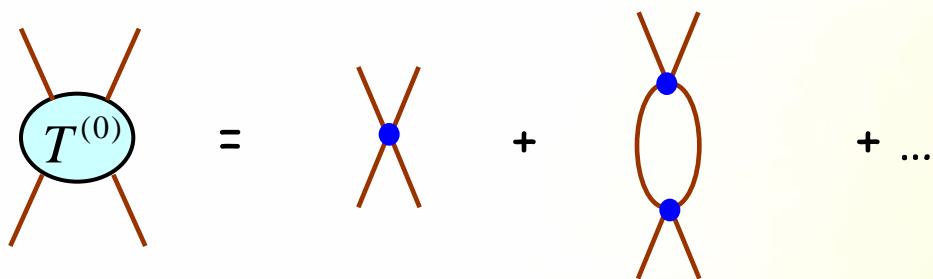
$$\rightarrow C_0^{(R)} \sim \frac{4\pi}{m_N M_0}$$

$$\rightarrow \frac{m_N Q}{4\pi} C_0^{(R)2} \sim \frac{44\pi}{m_N M_0 M_{nuc}} \frac{Q^2}{M_0} \ll 1 \text{ for } M_Q \sim M_{nuc}$$

$$\rightarrow C_2^{(R)} Q^2 \sim \frac{44\pi}{m_N m_{N_0} M_{nuc}} \frac{Q^2}{M_0} \ll 1 \text{ if } M_0 \sim M_{nuc}$$

etc.

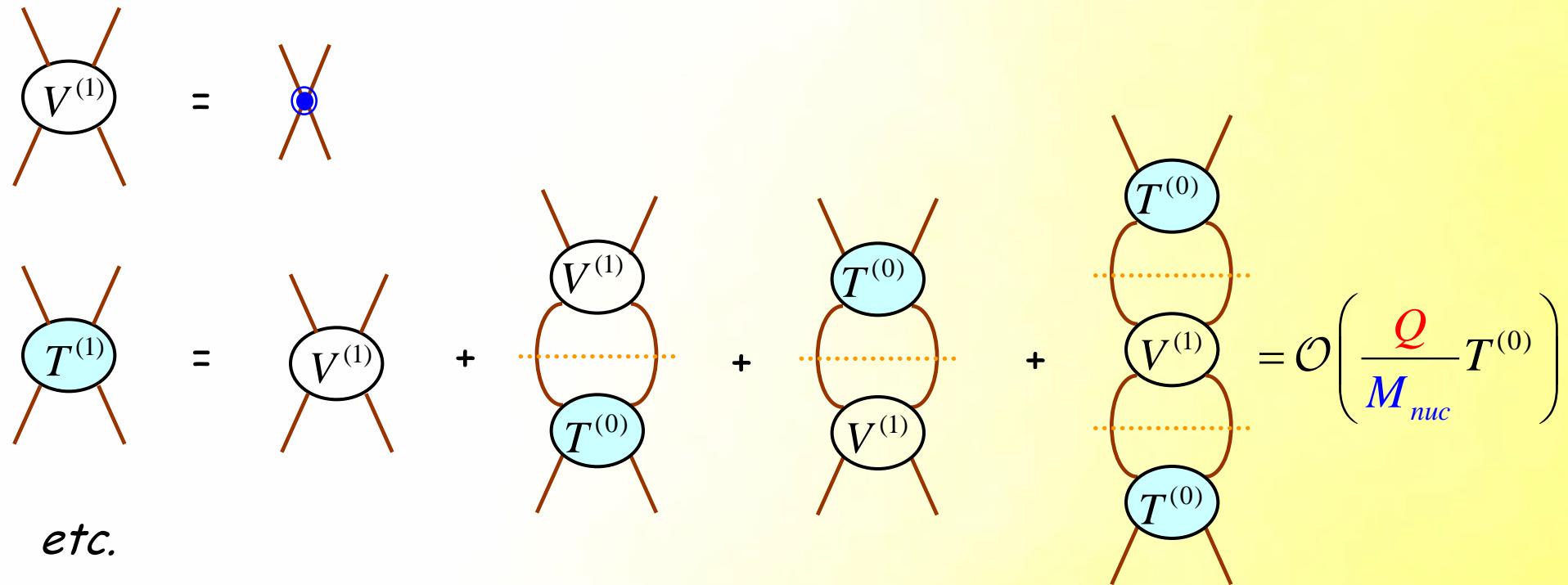
- ➡ no b.s. at $Q \lesssim M_{nuc}$, no good: just perturbation theory
need one fine-tuning: $M_0 \equiv \propto \ll M_{nuc}$
assume no other, e.g. still $M_2 \sim M_0$, etc.



$$\begin{aligned}
 &= iC_0 \left\{ 1 - C_0 I_0 + (C_0 I_0)^2 + \dots \right\} = \frac{i}{\frac{1}{m_N} + I_0} = \frac{4\pi}{m_N} \frac{i}{\underbrace{\frac{4\pi}{m_N C_0(\Lambda)} + \frac{2\Lambda}{\pi}}_{\text{red}} + ik + \mathcal{O}\left(\frac{k^2}{\Lambda}\right)} \\
 &= \frac{4\pi}{m_N} \frac{i}{\mathfrak{N} + ik} \left[1 + \mathcal{O}\left(\frac{k}{\Lambda}, \frac{k^2}{\mathfrak{N}\Lambda}\right) \right]
 \end{aligned}$$

cf. effective range expansion
s wave

$$\left\{ \begin{array}{ll} \text{scattering length} & a_0 = 1/\mathfrak{N} \\ \text{bound state} & k = i\mathfrak{N} \quad -E = \frac{\mathfrak{N}^2}{2m_N} \end{array} \right.$$



$\Rightarrow T_{NN} \sim \frac{4\pi}{m_N M_{nuc}} \left\{ \frac{M_{nuc}}{\aleph + iQ} + \left(\frac{Q}{\aleph + iQ} \right)^2 + \dots \right\}$

$\nu = -1$ $\nu = 0$

s wave

{ scattering length
 $a_0 \sim 1/\aleph$

{ effective range
 $r_0 \sim 1/M_{nuc}$

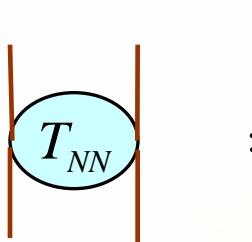
{ p, other waves

Alternative: auxiliary field

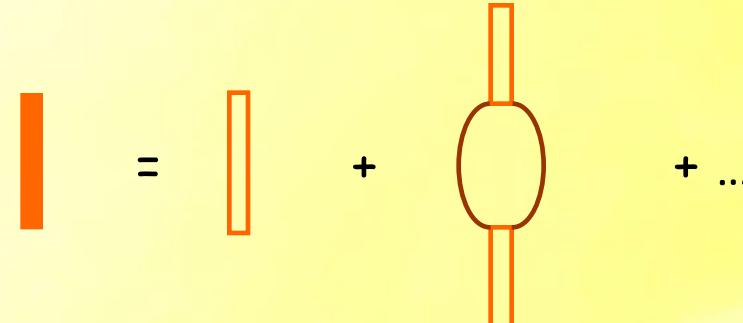
$$\mathcal{L}_{EFT} = N^+ \left(i\partial_0 + \frac{\nabla^2}{2m_N} \right) N + T^+ (-\Delta) T + \frac{g}{\sqrt{2}} [T^+ NN + N^+ N^+ T] + N^+ \frac{\nabla^4}{8m_N^3} N + \underset{n}{\text{sig}} T^+ \left(i\partial_0 + \frac{\nabla^2}{4m_N} \right) T + \dots$$

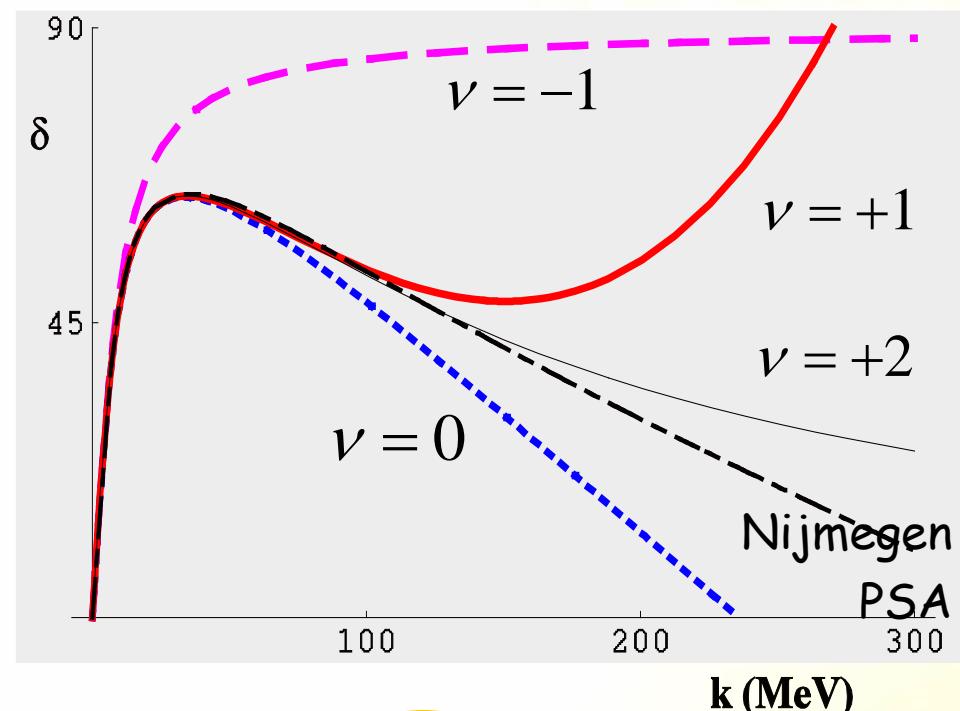
integrate out auxiliary field: same Lag as before with $C_0 = \frac{g^2}{\Delta}, \dots$

$$\Delta \sim \mathfrak{N}, \quad \frac{g^2}{4\pi} \sim \frac{1}{m_N}, \dots$$



=





fitted $a_0 = -20.0$ fm (exp)

$r_0 = 2.78$ fm (exp)

predicted

$B_{d^*} = 0.09$ MeV ($\nu = 0$)

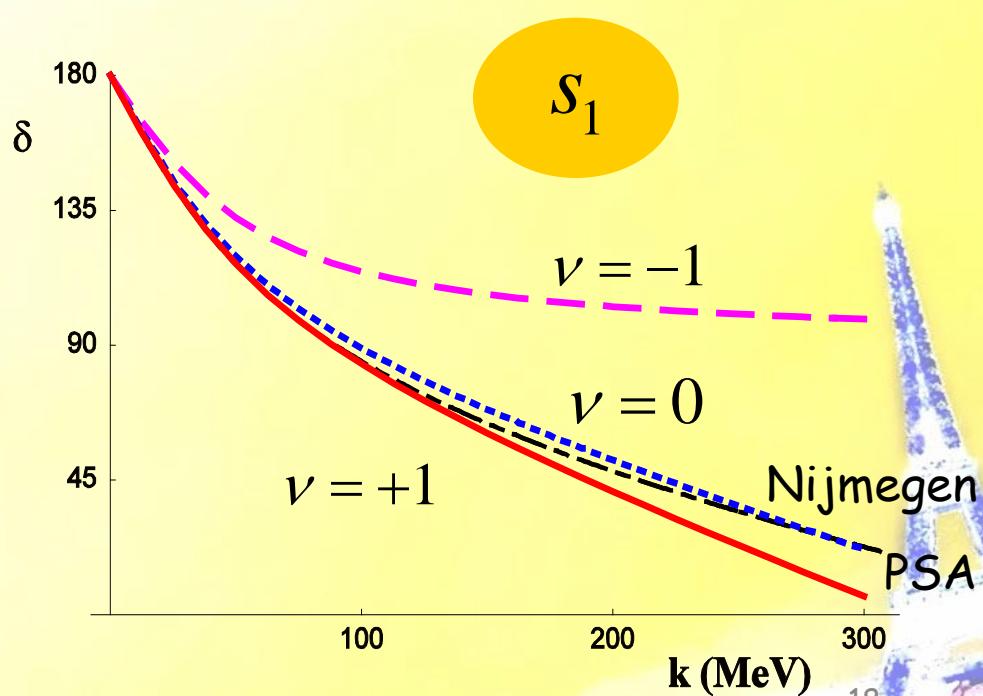
fitted $a_1 = 5.42$ fm (exp)

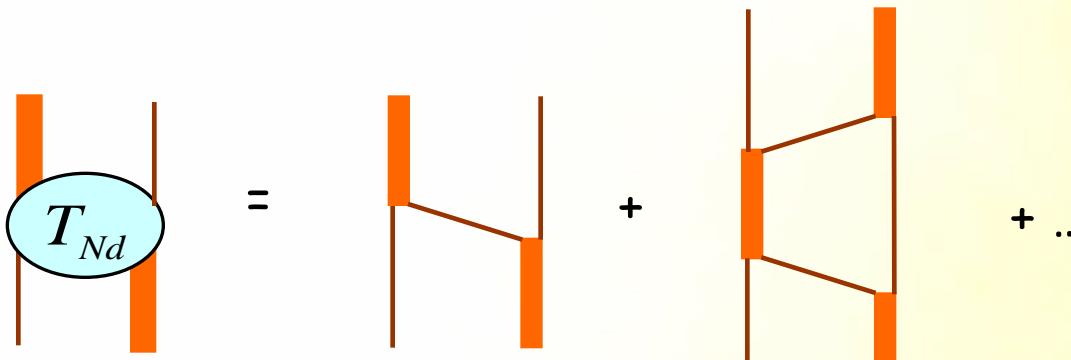
$r_1 = 1.75$ fm (exp)

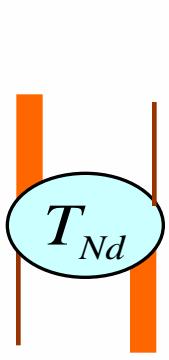
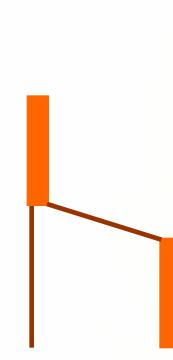
predicted

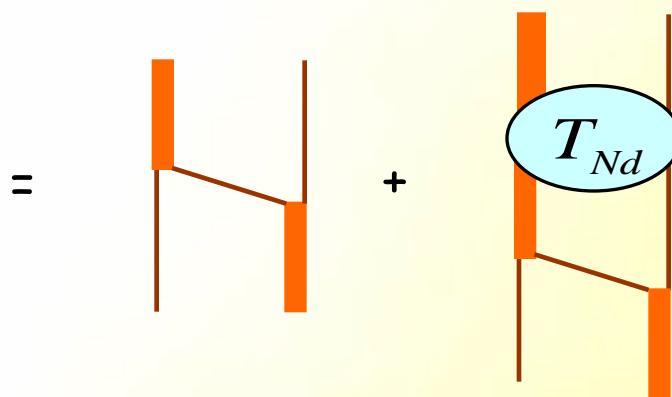
$B_d = 1.91$ MeV ($\nu = 0$)

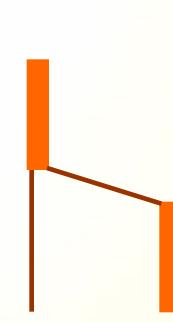
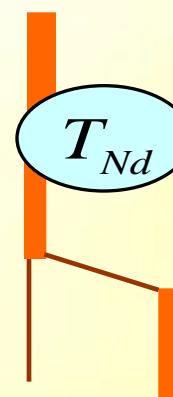
$B_d = 2.22$ MeV (exp)





T_{Nd} =  +  + ...
 $\sim \frac{g^2}{Q^2/m_N}$ $\sim \frac{Q^3}{4\pi} \left(\frac{g^2}{Q^2/m_N} \right)^2$ $\sim \frac{g^2}{Q^2/m_N} \frac{Q}{\propto}$



$=$  + 

$$T_{Nd} = K_{ONE} + \lambda \int_0^\Lambda \frac{d^3 l}{(2\pi)^3} \frac{K_{ONE} T_{Nd}}{D}$$

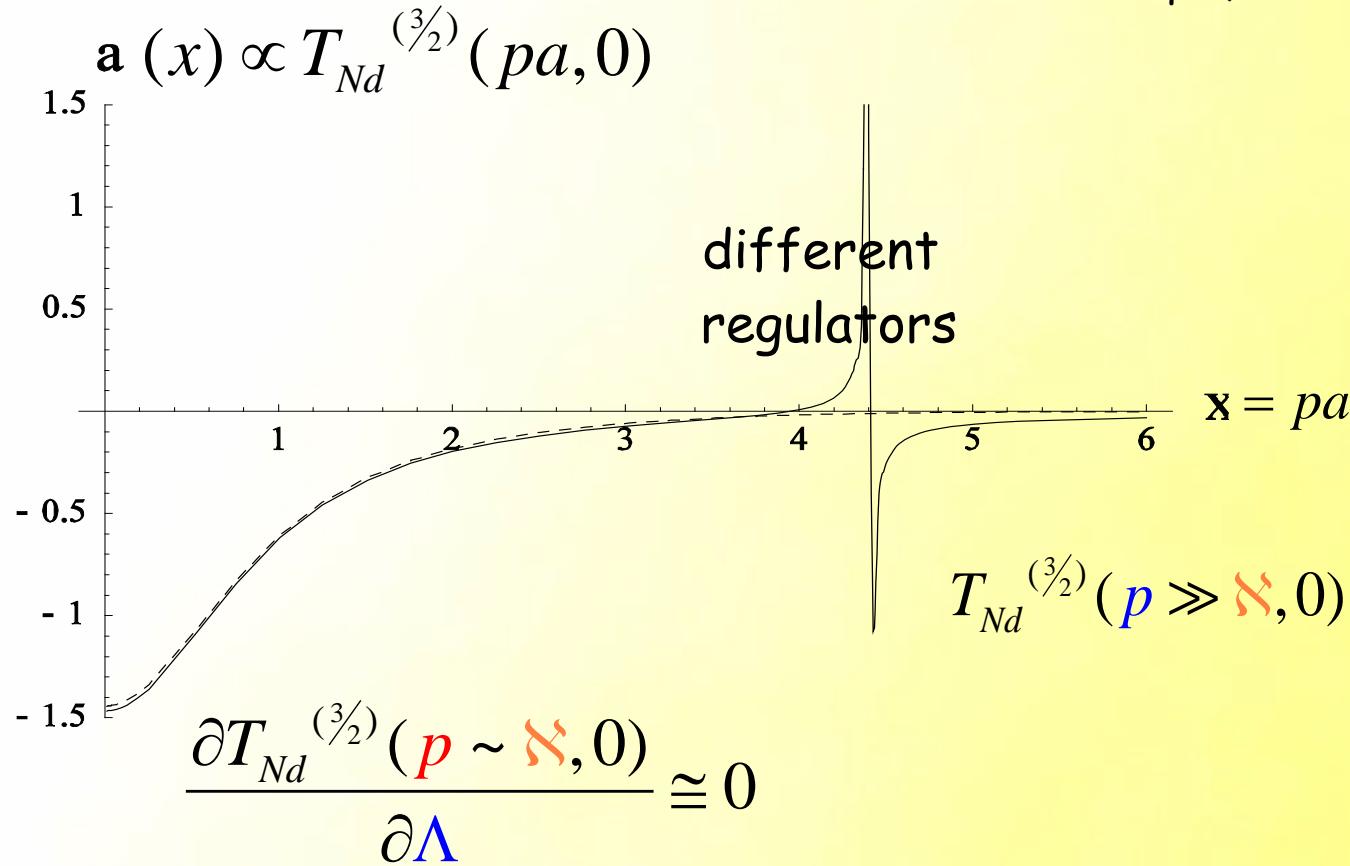
renormalization: quartet s wave

$$\lambda \leq \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

Skornyakov + Ter-Martirosian '60

Bedaque + v.K. '97

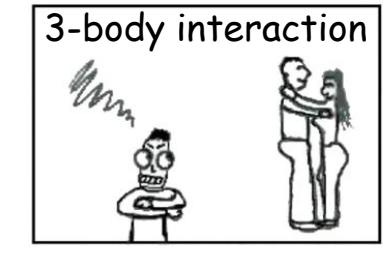
Bedaque, Hammer + v.K. '98



$$\mathcal{L}_{EFT} = \dots + D_0 N^+ N N^+ N N^+ N + \dots$$

naïve
dimensional analysis

$$D_0 \sim \left(\frac{4\pi}{m_N} \right)^2 \frac{1}{M_{nuc}^3} \quad (\nu = +1)$$



$S_{3/2}$

no three-body force up to $\nu = +3$

$$\lambda \leq \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p \gg \infty} \frac{1}{p^2}$$

$$\xrightarrow{\frac{\partial T_{Nd}}{\partial \Lambda}} \xrightarrow{p \sim \infty} 0$$

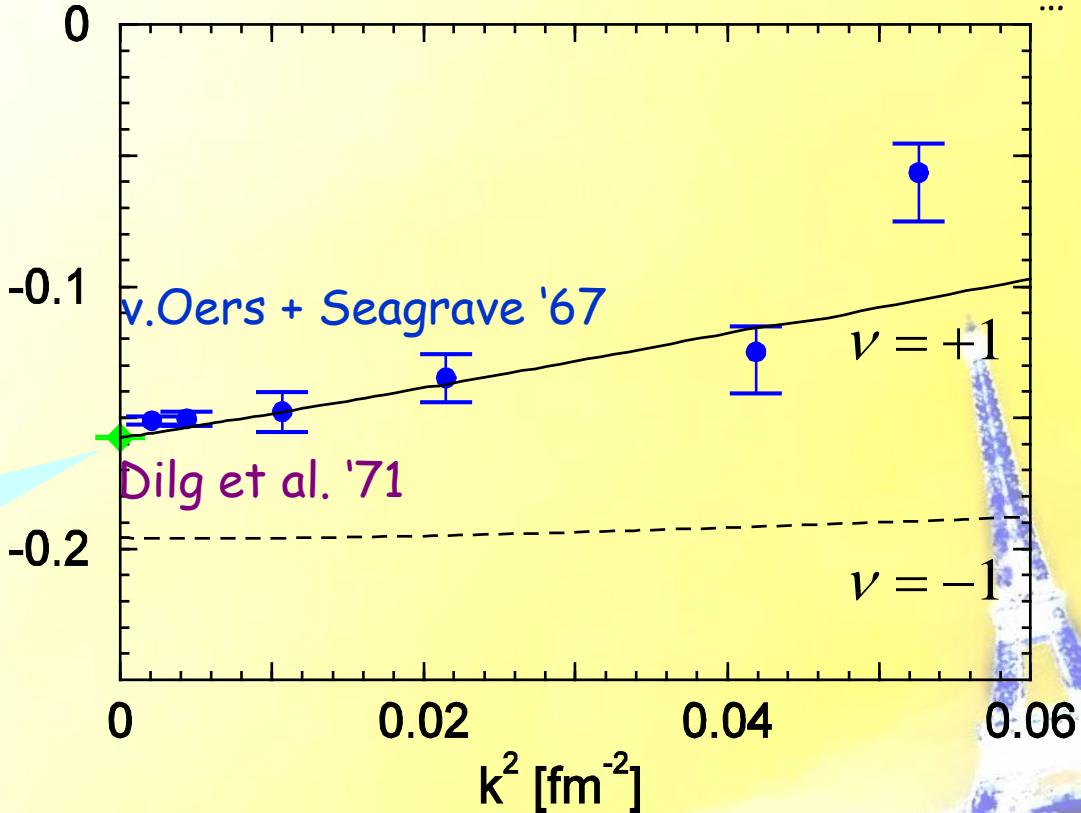
predicted

$$a_{3/2} = 6.33 \pm 0.10 \text{ fm } (\nu = +1)$$

$$a_{3/2} = 6.35 \pm 0.02 \text{ fm } (\text{exp})$$

QED-like precision!

$k \cot \delta [\text{fm}^{-1}]$



Bedaque, Hammer + v.K. '99
'00

Hammer + Mehen '01

$s_{1/2}$

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{\textcolor{blue}{p \gg \aleph}} A \cos\left(s_0 \ln \frac{p}{\Lambda} + \delta\right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{\textcolor{red}{p \sim \aleph}} \neq 0 \quad \text{unless}$$

$$D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{\aleph^2 M_{nuc}} \quad (\nu = -1)$$

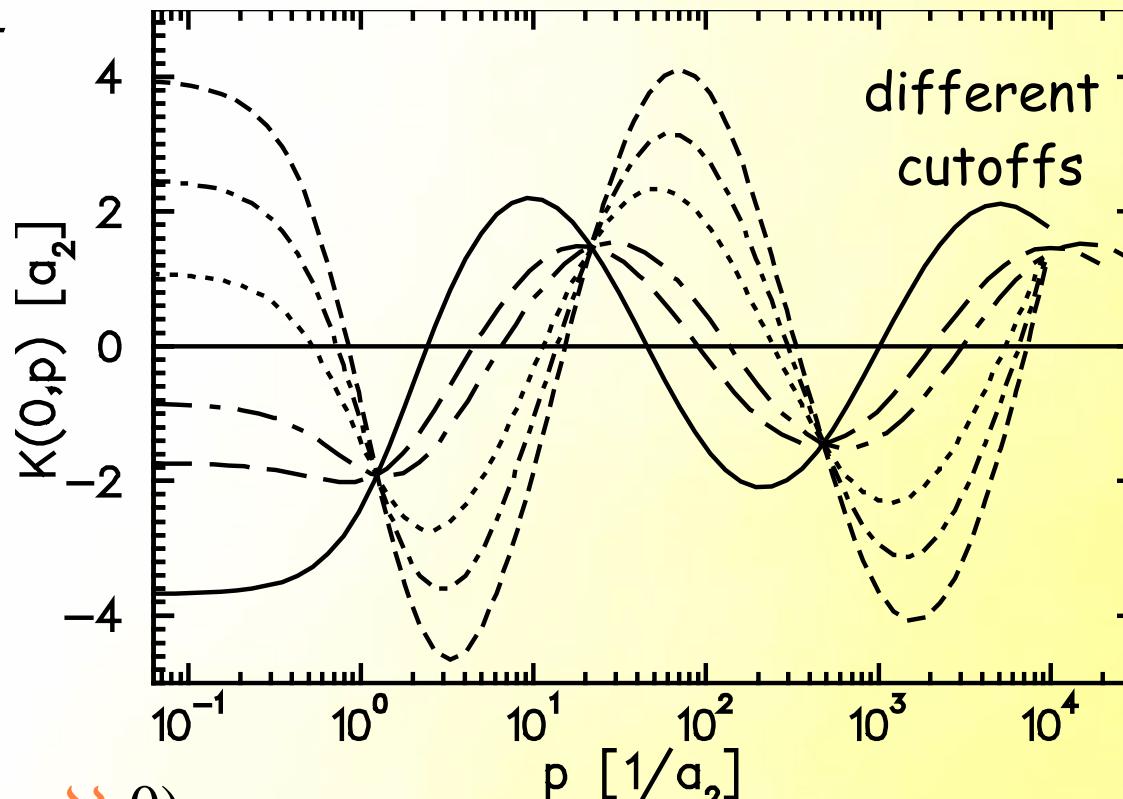
Bedaque et al. '03

renormalization: doublet s wave -I

Danilov '63

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

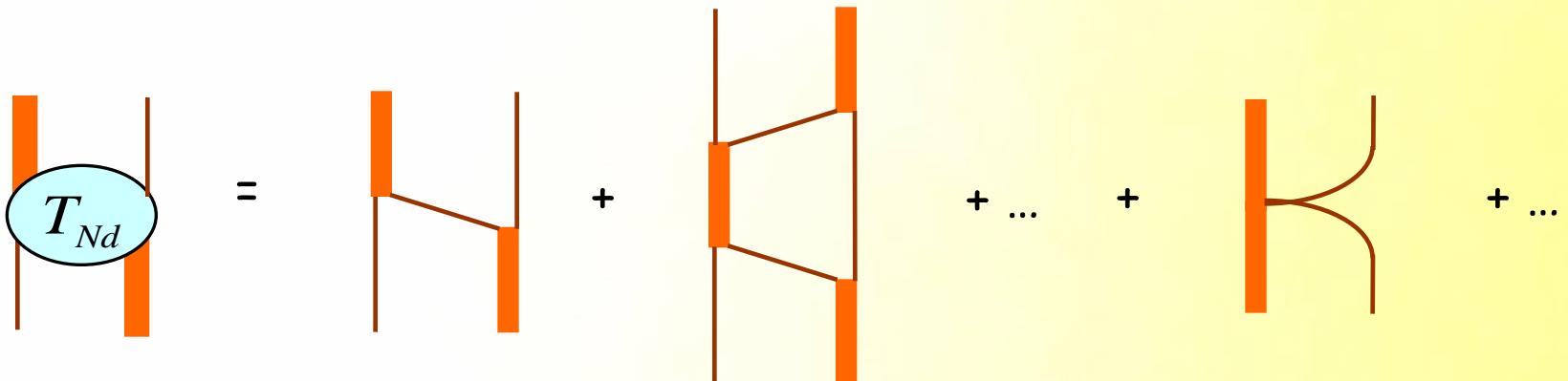
Bedaque, Hammer + v.K. '99 '00



$$\frac{\partial T_{Nd}^{(1/2)}(p \sim \aleph, 0)}{\partial \Lambda} \neq 0$$

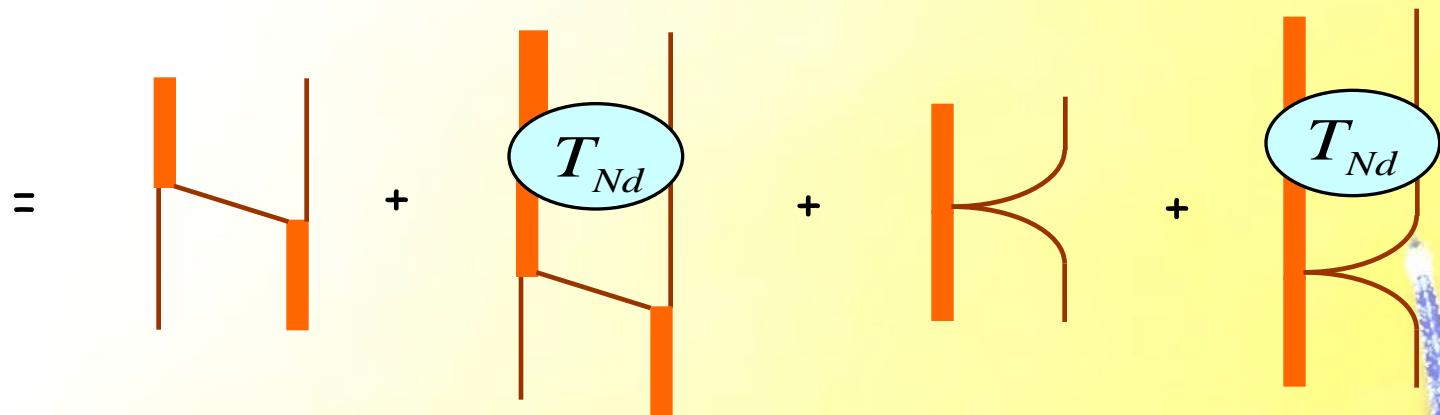
$$T_{Nd}^{(1/2)}(p \gg \aleph, 0) \propto \cos\left(s_0 \ln \frac{p}{\Lambda} + \delta\right)$$

$$s_0 \approx 1.0024$$



$$\sim \frac{g^2}{Q^2/m_N}$$

$$\sim \frac{4\pi}{\alpha^2}$$



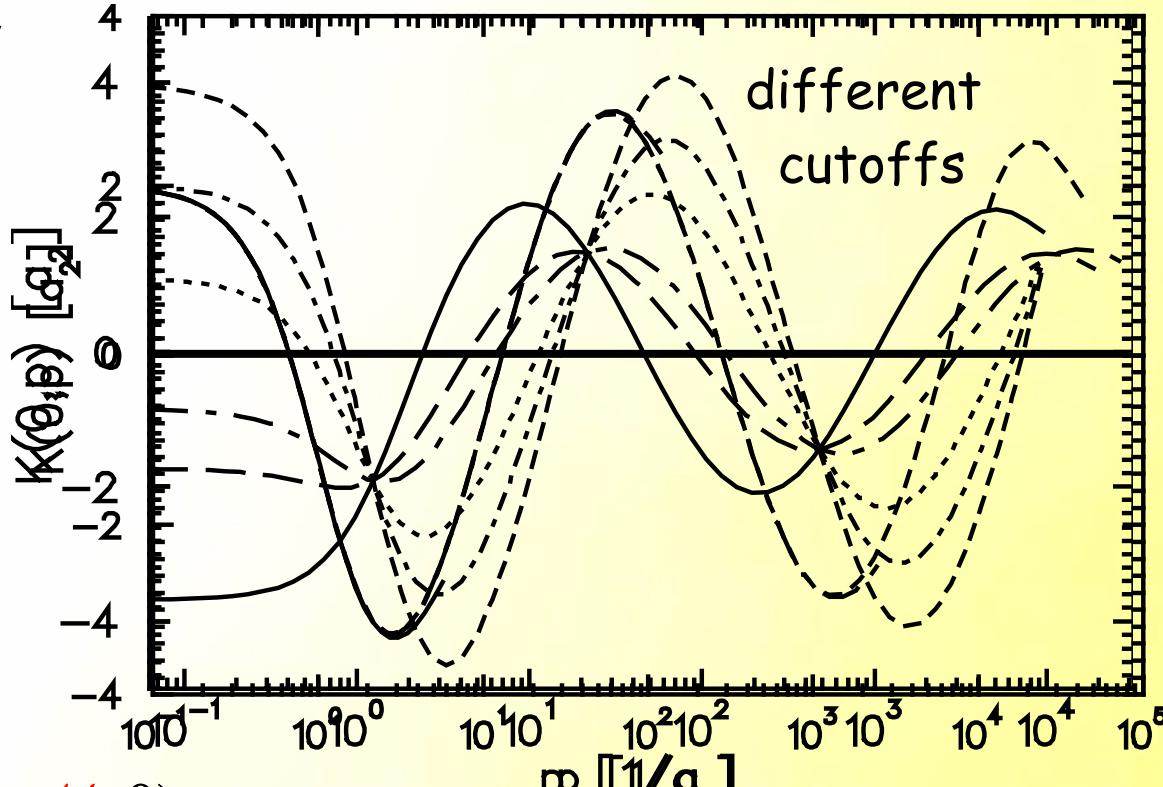
$$T_{Nd} = K_{ONE} + \lambda \int_0^\Lambda \frac{d^3 l}{(2\pi)^3} \frac{K_{ONE} T_{Nd}}{D} + K_{TBF} + \lambda \int_0^\Lambda \frac{d^3 l}{(2\pi)^3} \frac{K_{TBF} T_{Nd}}{D}$$

renormalization: doublet s wave -I

Danilov '63

Bedaque, Hammer + v.K. '99 '00

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$



$$\frac{\partial T_{Nd}^{(1/2)}(p \sim x, 0)}{\partial \Lambda} \not\equiv 0$$

$$T_{Nd}^{(1/2)}(p \gg x, 0) \propto \cos\left(s_0 \ln \frac{p}{\Lambda_*} + \delta\right)$$

$$s_0 \approx 1.0024$$

Bedaque, Hammer + v.K. '99 '00
 Hammer + Mehen '01
 Bedaque et al. '03
 ...

$s_{1/2}$

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p \gg \aleph} A \cos\left(s_0 \ln \frac{p}{\Lambda} + \delta\right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} \neq 0 \quad \text{unless}$$

$$D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{\aleph^2 M_{nuc}} \quad (\nu = -1)$$

(limit cycle!)

fitted

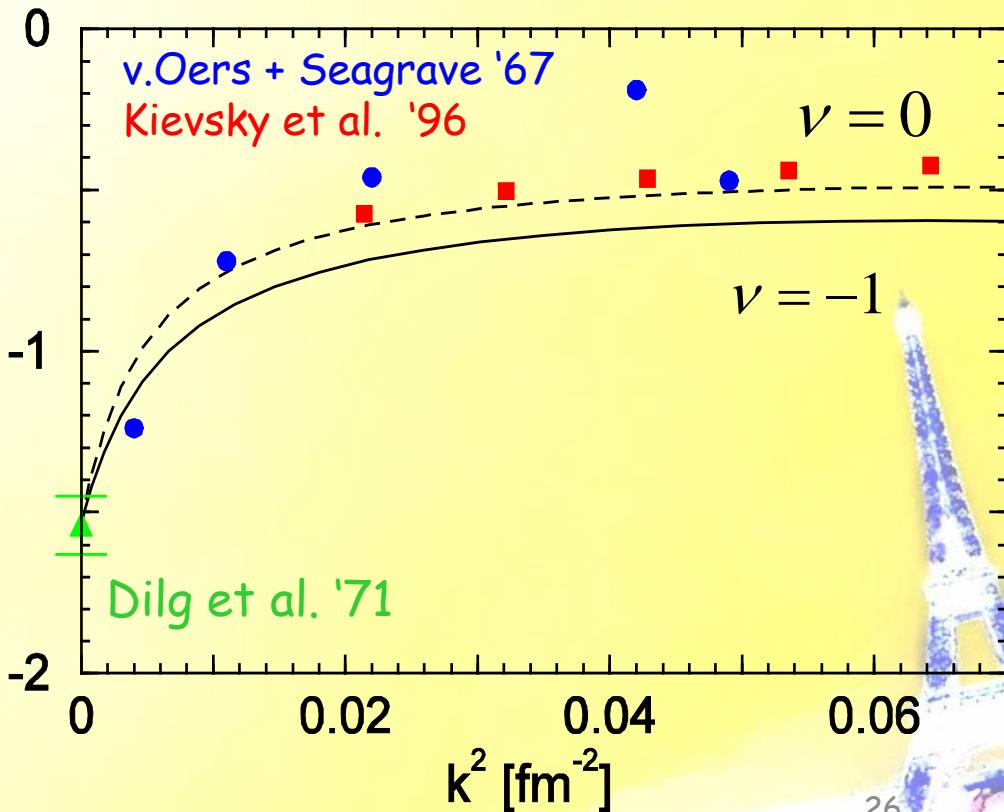
$$a_{1/2} = 0.65 \text{ fm (exp)}$$

predicted

$$B_t = 8.3 \text{ MeV } (\nu = 0)$$

$$B_t = 8.48 \text{ MeV (expt)}$$

$k \cot \delta [\text{fm}^{-1}]$

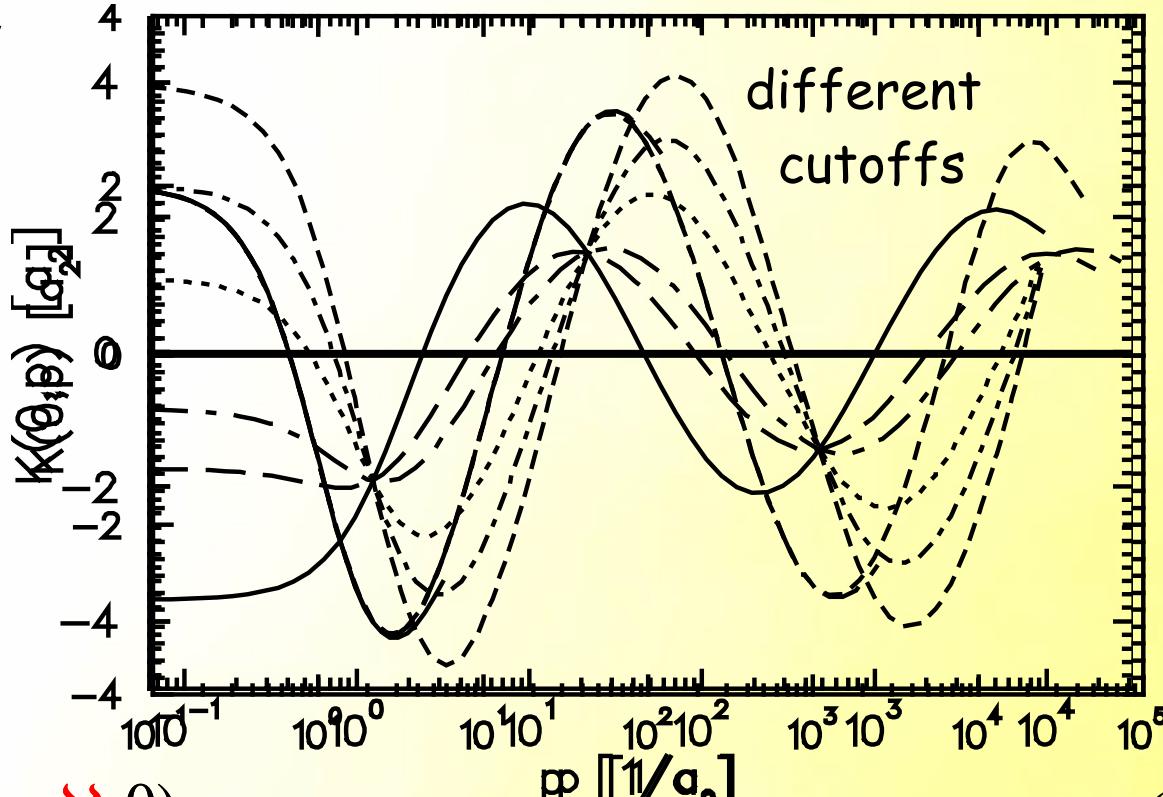


renormalization: doublet s wave -I

Danilov '63

Bedaque, Hammer + v.K. '99 '00

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$



$$\frac{\partial T_{Nd}^{(1/2)}(p \sim x, 0)}{\partial \Lambda} \not\equiv 0$$

$$T_{Nd}^{(1/2)}(p \gg x, 0) \propto \cos\left(s_0 \ln \frac{p}{\Lambda_*} + \delta\right)$$

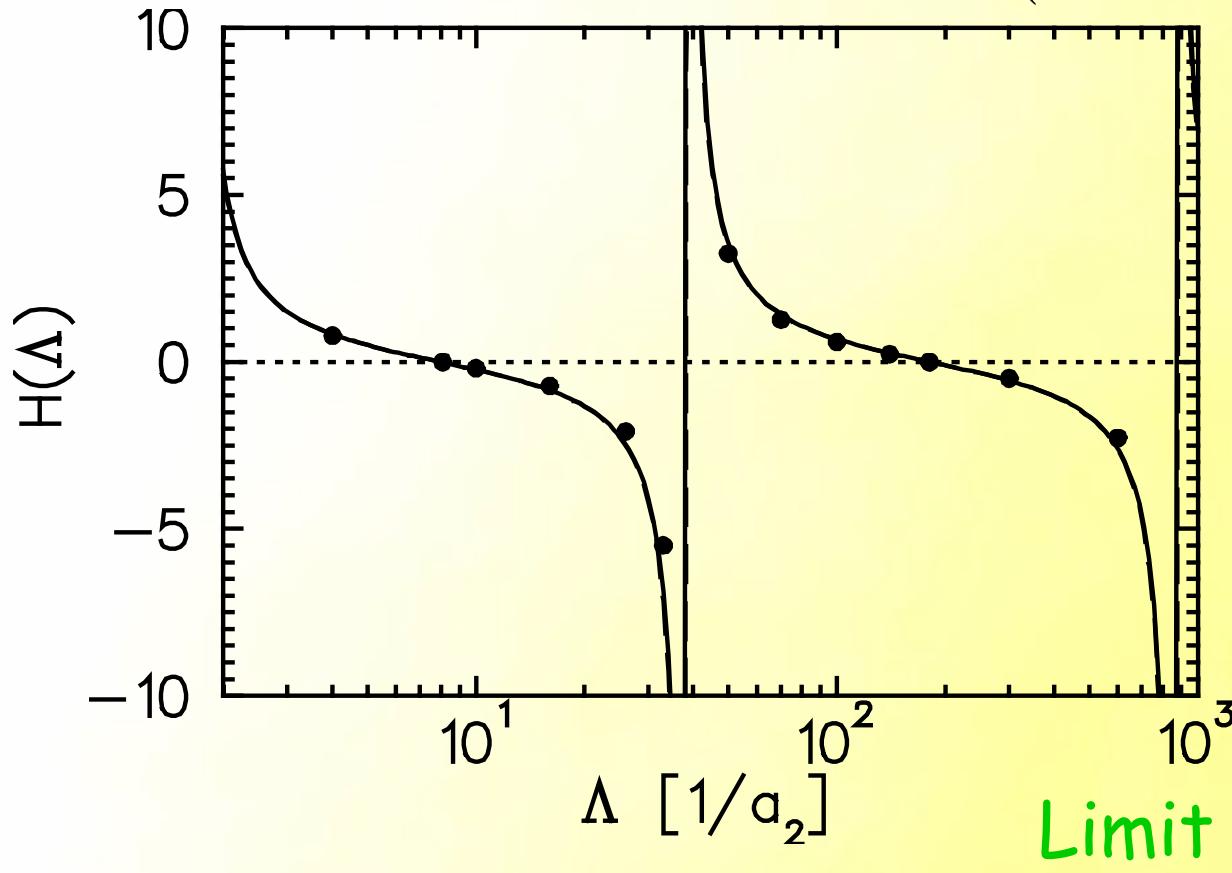
$$s_0 \approx 1.0024$$

renormalization: doublet s wave -II

Bedaque, Hammer + v.K. '99 '00

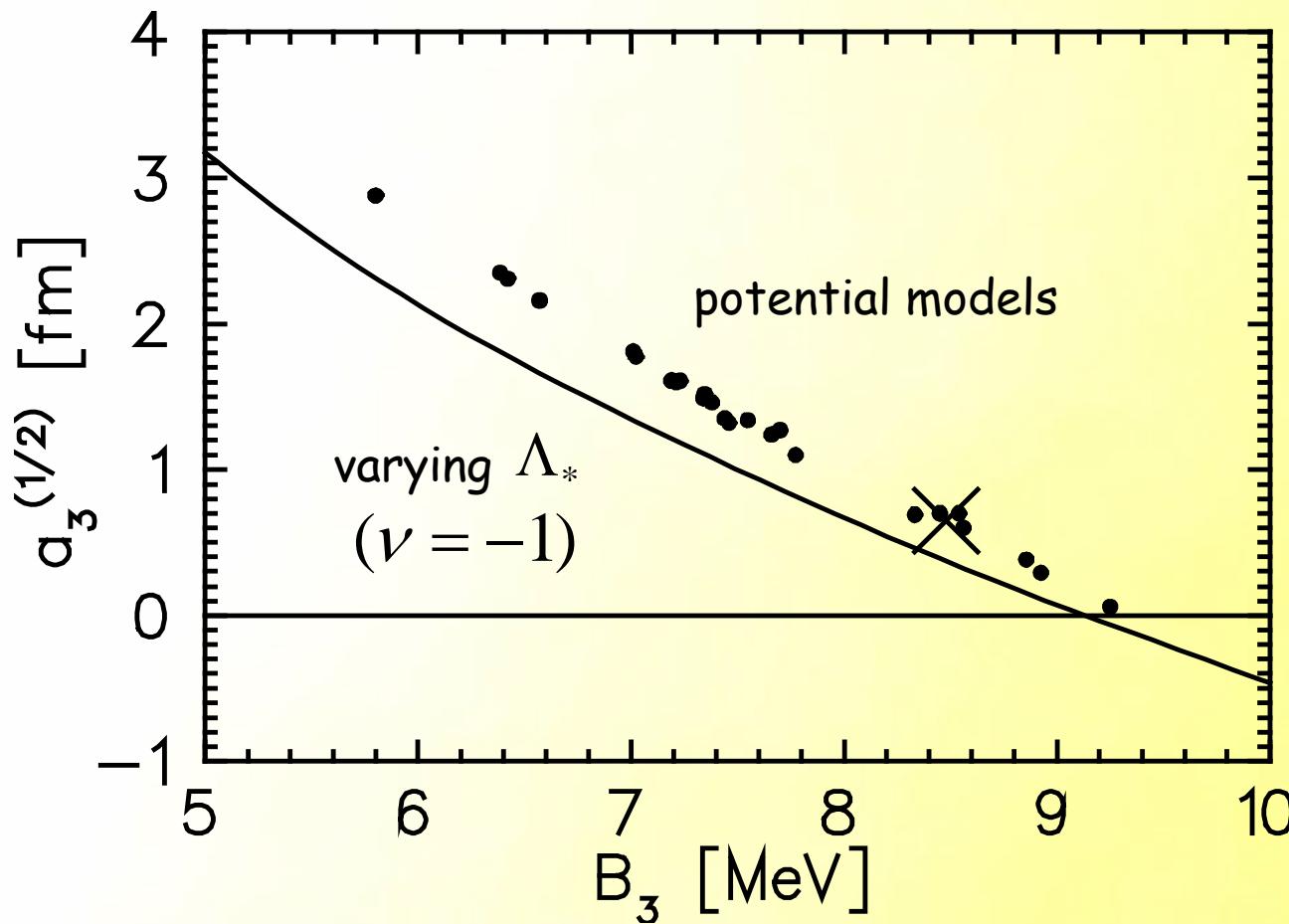
$$D_0 \sim \left(\frac{4\pi}{m_N} \right)^2 \frac{1}{8^2 M_{nuc}} H(\Lambda)$$

$$H(\Lambda) = \frac{\sin(\ln(\Lambda/\Lambda_*) + \arctan(1/s_0))}{\sin(\ln(\Lambda/\Lambda_*) - \arctan(1/s_0))}$$



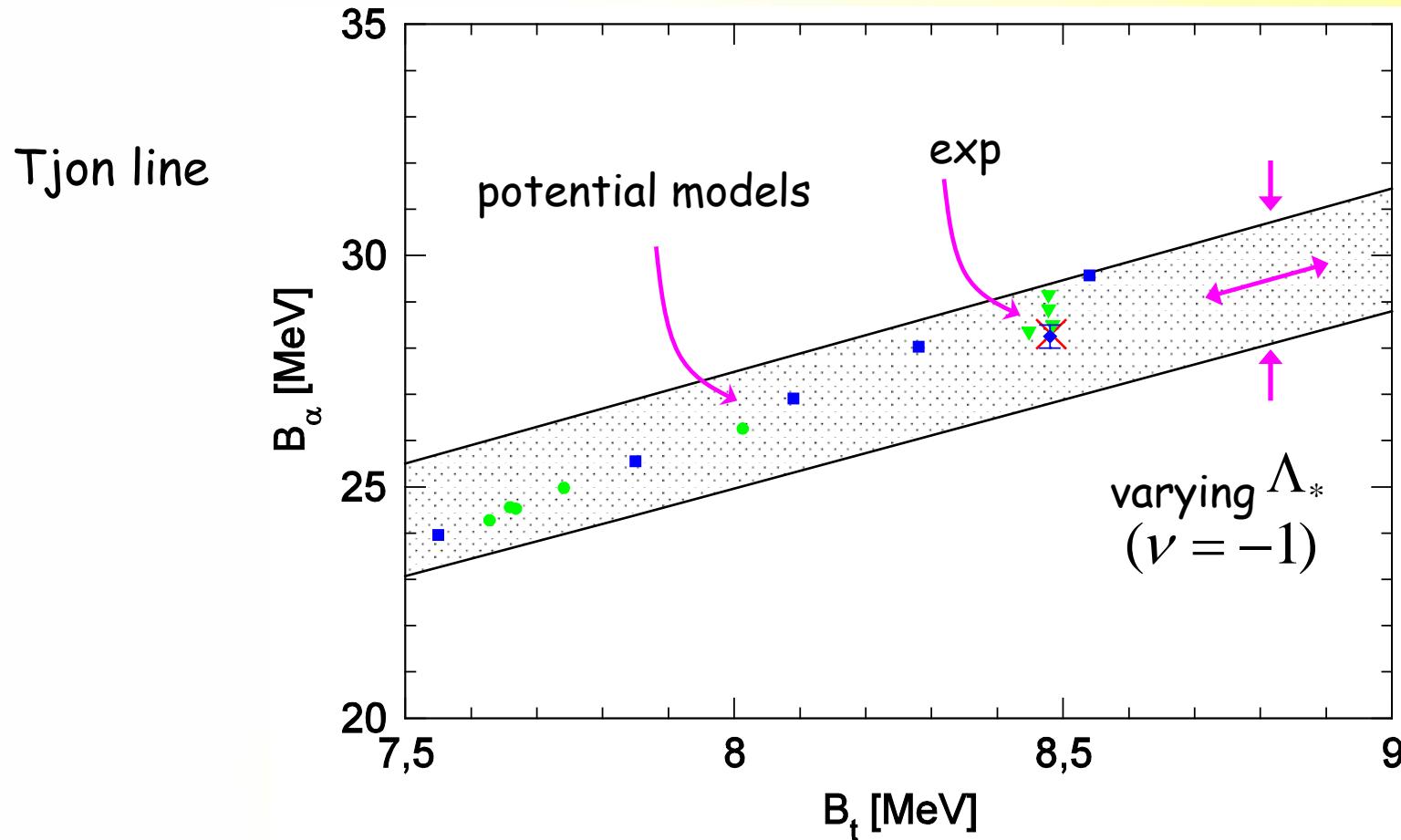
Phillips line

Bedaque, Hammer + v.K. '99 '00



+ four-body bound state can be addressed similarly
➡ no four-body force at $\nu = -1$

Hammer, Meissner + Platter '04



Summary:

Expansion parameter

$$\frac{Q}{M_{nuc}} \sim \frac{\aleph}{M_{nuc}} \sim \frac{r_0}{a_0}$$

- LO: two two-nucleon + one three-nucleon interactions
 $C_0^{(0)}, C_0^{(1)}, D_0$
- NLO: two more two-nucleon interactions
- etc.

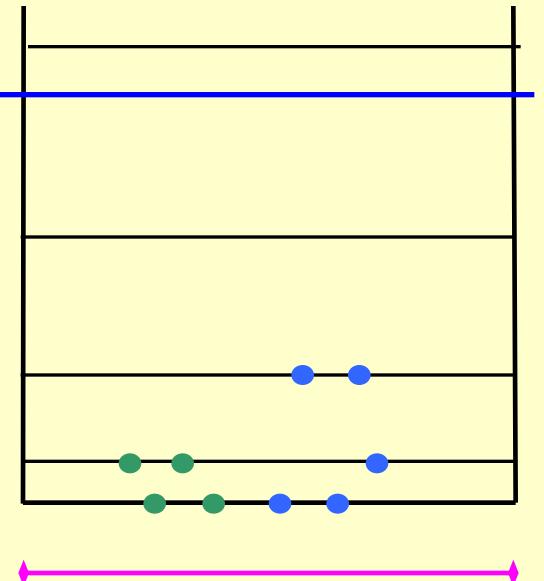
~ larger nuclei?

As A grows,
given computational power limits
number of accessible one-nucleon states

IR cutoff $\lambda \ll Q$
in addition to
UV cutoff $\Lambda \gg Q$

Finite Volume

Lattice Box



nuclear matter

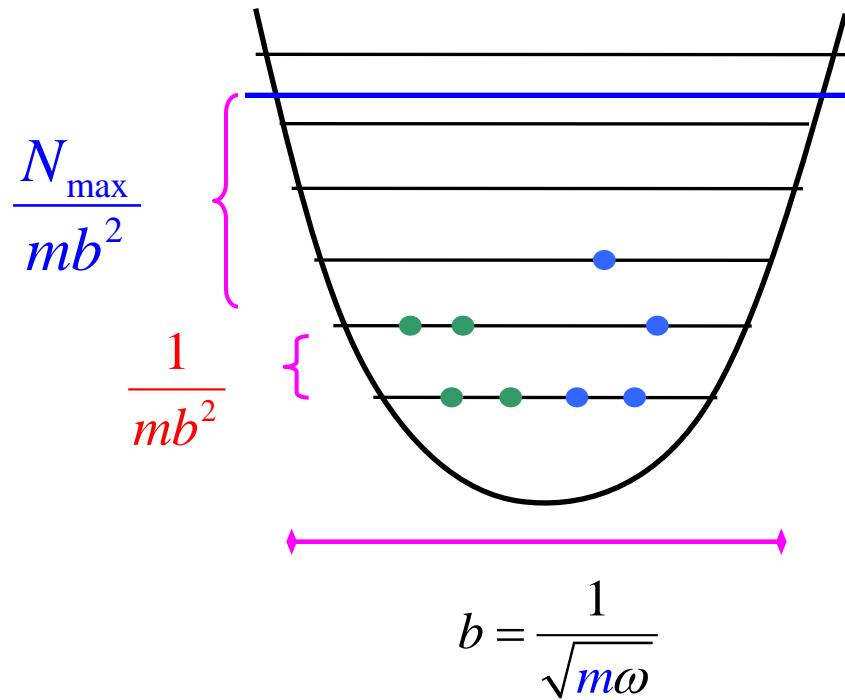
Mueller, Seki, Koonin + v.K. '99

few nucleons

Lee *et al.* '05

...

Harmonic Oscillator



finite nuclei

Stetcu, Barrett + v.K. '07
Rotureau, Stetcu, Barrett + v.K. '12

few atoms

Stetcu, Barrett + v.K. '09
Rotureau, Stetcu, Barrett + v.K. '10
Rotureau et al. '11

Two possible approaches

Lattice EFT

- Use input EFT infinite-volume potential $(0, \Lambda_0)$;
minimize regulator mismatch with $\Lambda \ll \Lambda_0$

Lee *et al* '05

...

Harmonic EFT

Barrett, Vary + Zhang '93

...

"No-Core Shell Model"

- Define EFT directly within finite volume (λ, Λ) ;
fit parameters to binding energies or to E given by

$$\sqrt{mE} \cot \delta(E) = \frac{1}{\pi L} \left[\sum_{\mathbf{n}}^{|n| < N} \frac{1}{\mathbf{n}^2 - \frac{mEL^2}{4\pi^2}} - 4\pi N \right]$$

cf. Fukuda + Newton '54

Luescher '91

$$\sqrt{mE} \cot \delta(E) = -\frac{2}{b} \frac{\Gamma\left(\frac{3}{4} - \frac{Em^2}{2}\right)}{\Gamma\left(\frac{1}{4} - \frac{Em^2}{2}\right)}$$

Busch *et al.* '98

Both being actively pursued

- many-body systems get complicated rapidly
- + (continuing) focus on simpler halo/cluster nuclei

one or more loosely-bound nucleons around one or more cores

$$\mathcal{X} \equiv \sqrt{m_N E_p} \ll \sqrt{m_N E_c} \equiv M_c \quad (\text{esp. near driplines})$$

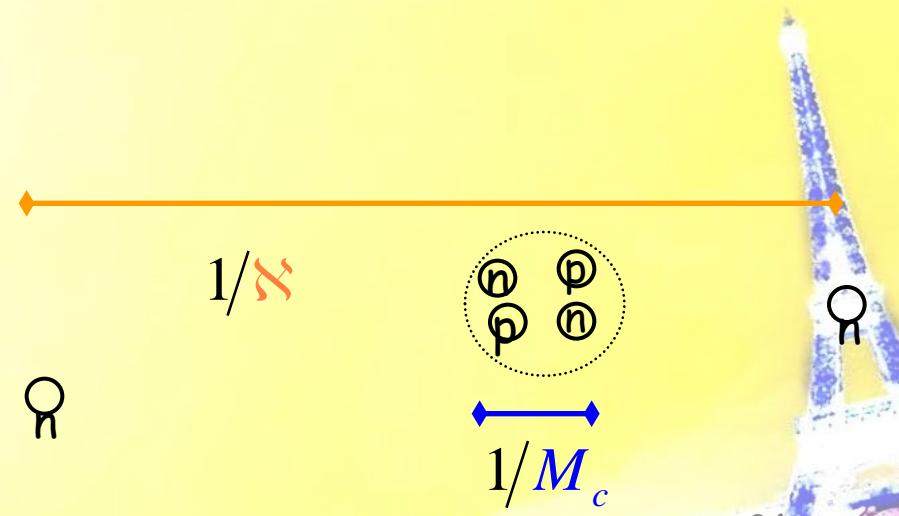
particle separation energy core excitation energy

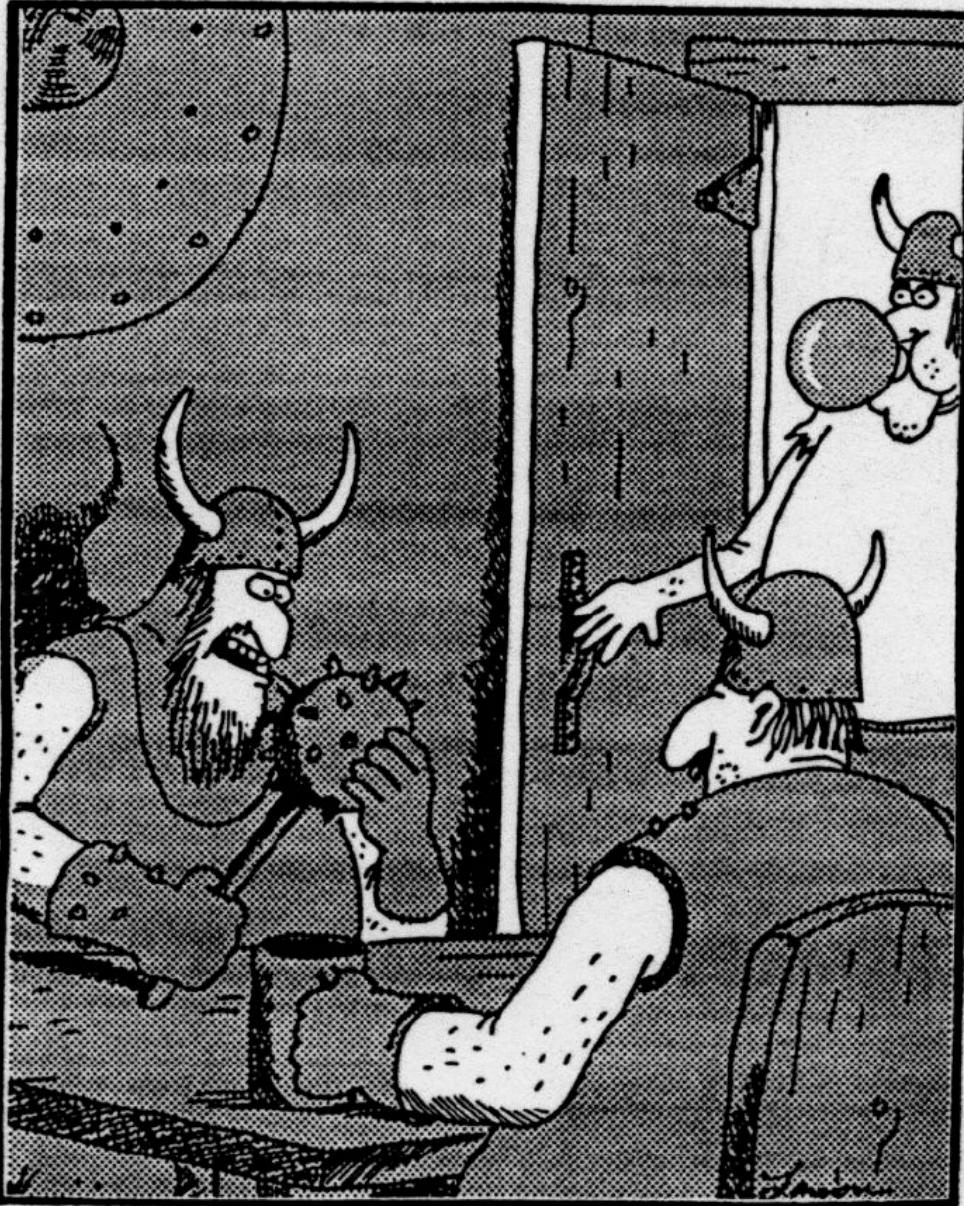
e.g.

$${}^4\text{He} \quad \left. \begin{array}{l} B_{\alpha^*} \cong 8 \text{ MeV} \\ B_\alpha \cong 28 \text{ MeV} \end{array} \right\} \quad E_\alpha = B_\alpha - B_{\alpha^*} \cong 20 \text{ MeV}$$

"⁵He" $p_{3/2}$ resonance at
 $E_n \sim 1 \text{ MeV}$

⁶He s_0 bound state at
 $E_{2n} \sim 1 \text{ MeV}$





**"You know, Bjorg, there's something about holding
a good, solid mace in your hand—you just look
for an excuse to smash something."**

$$Q \sim N \ll M_c$$

halo EFT

- degrees of freedom: nucleons, cores

- symmetries: Lorentz, ~~P, T~~

- expansion in:

$$\frac{Q}{M_c} = \begin{cases} Q/m_N, Q/m_c & \text{non-relativistic} \\ Q/m_\pi, \dots & \text{multipole} \end{cases}$$

simplest formulation: auxiliary fields for core + nucleon states

e.g.

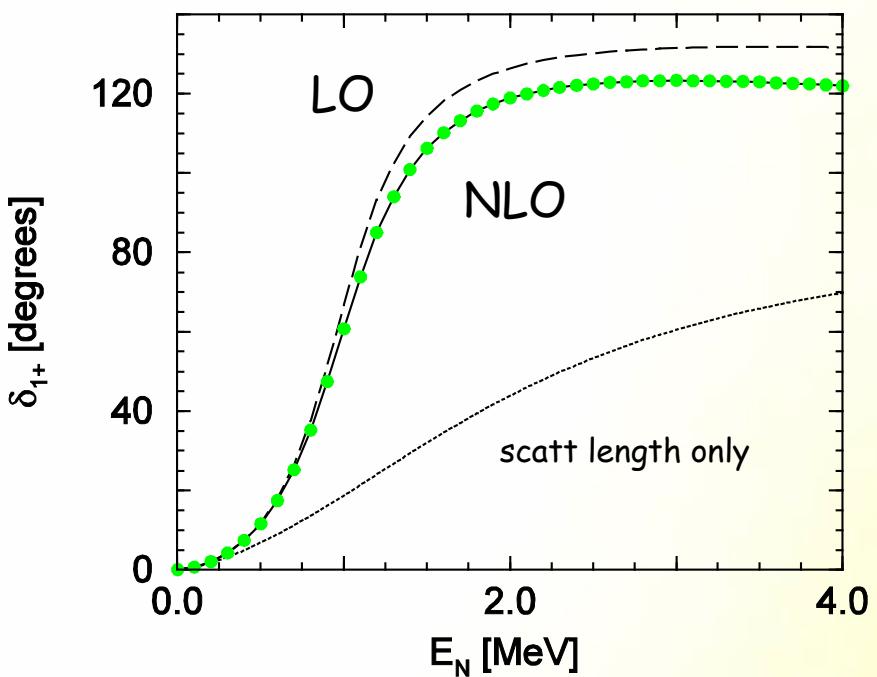
${}^4\text{He}$ \mapsto scalar field φ

$${}^4\text{He} + \text{N} \quad \begin{cases} s_{1/2} \equiv 0+ \mapsto \text{spin - 0 field } s \\ p_{1/2} \equiv 1- \mapsto \text{spin - 1/2 field } T_1 \\ p_{3/2} \equiv 1+ \mapsto \text{spin - 3/2 field } T_3 \\ \vdots \end{cases}$$

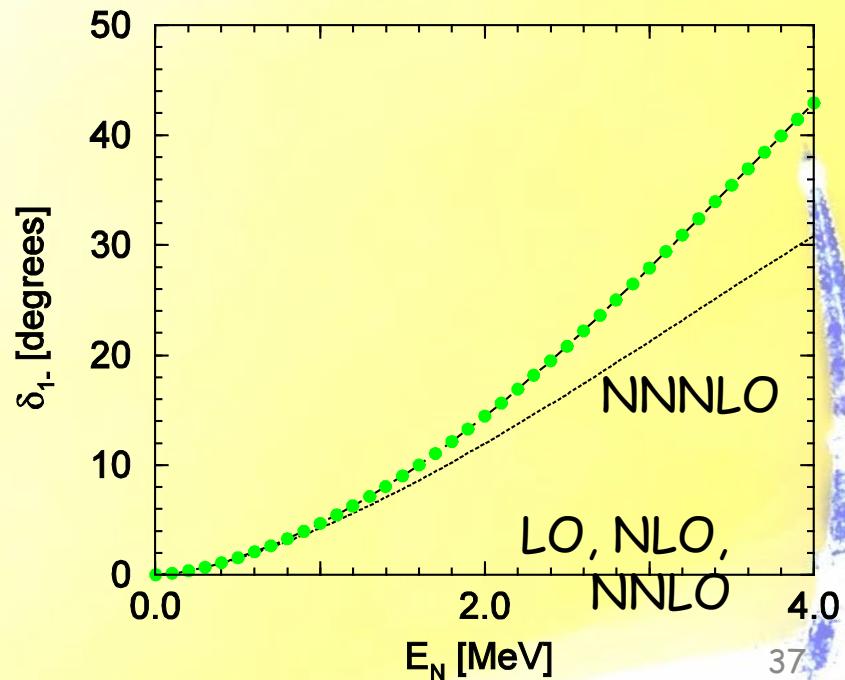
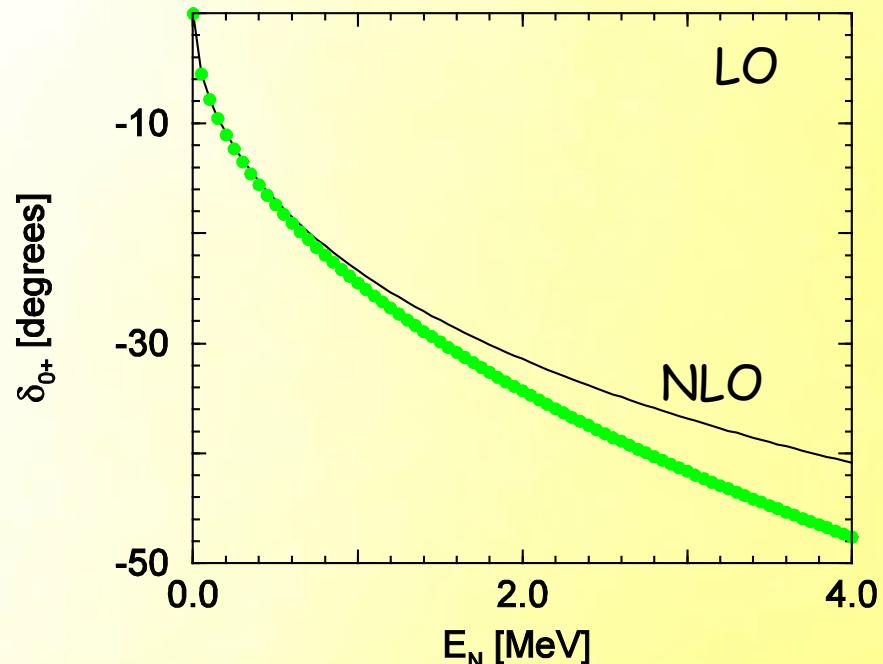
$N\alpha$

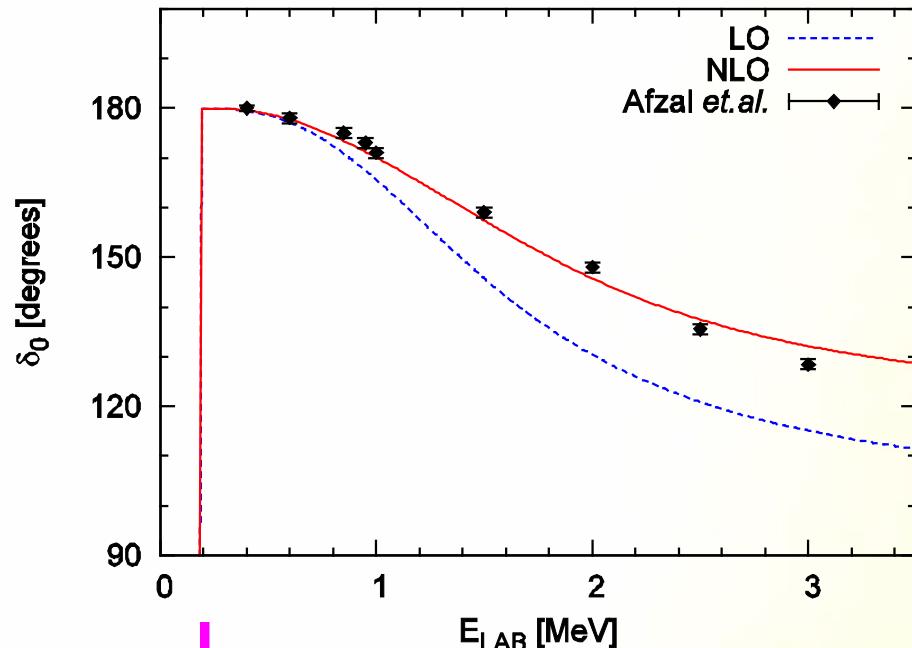
Bertulani, Hammer + v.K. '02

• PSA, Arndt et al. '73



→ $E_R \approx 0.80$ MeV
 $\Gamma(E_R) \approx 0.55$ MeV



 $\alpha\alpha$

Extra fitting parameters

$$\tilde{P}_0 = P_0 + \frac{1}{15k_C^3}$$

none

Bohr radius

$$\frac{1}{k_C} \equiv \frac{1}{Z_\alpha^2 \alpha_{em} \mu} \\ \simeq 3.6 \text{ fm}$$

$E_R = 92.07 \pm 0.03 \text{ keV}$ }
 $\Gamma(E_R) = 5.57 \pm 0.25 \text{ eV}$

fitted with a_0 and $\tilde{r}_0 = r_0 - \frac{1}{3k_C}$

More
fine-tuning!!!

	$a_0 (10^3 \text{ fm})$	$r_0 (\text{fm})$	$P_0 (\text{fm}^3)$
LO	-1.80	1.083	—
NLO	-1.92 ± 0.09	1.098 ± 0.005	-1.46 ± 0.08

fine-tuning of
1 in 1000!

$$\left| a_0^{E \& M} \right| = \mathcal{O}(1/2k_C) \simeq 1.8 \text{ fm}$$

$$\left| a_0 \right| \sim M_c^2 / \mathfrak{K}^3 \quad r_0 \sim 1/M_c \quad \tilde{r}_0 = -0.13 \text{ fm}$$

fine-tuning of
1 in 10

What next

- Coulomb interaction in higher waves:
e.g. $p + {}^4\text{He} \rightarrow {}^4\text{He} + p$ Bertulani, Higa + v.K., in progress
[cf. $p + p \rightarrow p + p$ Kong + Ravndal '99]
- three-body bound states:
e.g. 1) ${}^6\text{He} = \text{b.s.}({}^4\text{He} + n + n)$ Rotureau + v.K., '12 + in progress
2) ${}^{12}\text{C} = \text{b.s.}({}^4\text{He} + {}^4\text{He} + {}^4\text{He})$
[cf. ${}^3\text{H} = \text{b.s.}(p + n + n)$ Bedaque, Hammer + v.K. '99]
- reactions:
e.g. $p + {}^7\text{Be} \rightarrow {}^8\text{B} + \gamma$ Higa + Rupak, in progress
[cf. $p + n \rightarrow d + \gamma$ Chen et al. '00]

Conclusion

EFT the framework to describe nuclei within the SM

- ✓ is consistent with symmetries
- ✓ incorporates hadronic physics
- ✓ has controlled expansion

many successes so far, but still much to do


grow to larger nuclei!

➡ new, systematic approach to physics near $d_{r_i p}$ lines?