INTRODUCTION TO EFFECTIVE FIELD THEORIES OF QCD

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Supported in part by
CNRS, Université Paris Sud, and US DOE

Outline

- Effective Field Theories
- QCD at Low Energies
  - QCD and Chiral Symmetry
  - Chiral Nuclear EFT
  - Renormalization of Pion Exchange
  - Summary
- Towards Nuclear Structure

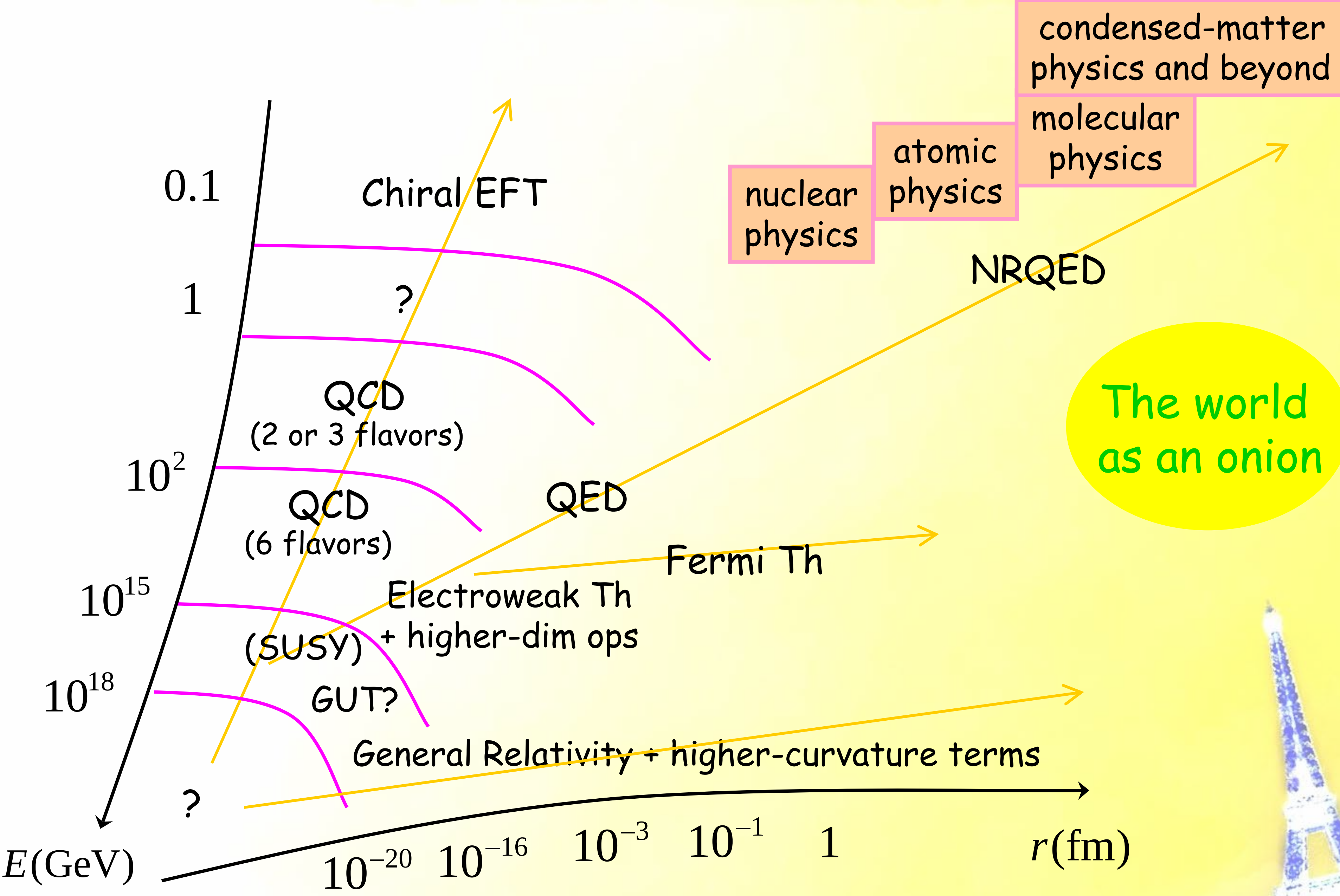
References:

S. Weinberg,
Phenomenological Lagrangians,
Physica A96:327,1979

S. Weinberg,
Effective chiral Lagrangians for nucleon-pion interactions and nuclear forces,
Nucl.Phys.B363:3-18,1991

S.R. Beane, P.F. Bedaque, L. Childress, A. Kryjevski, J. McGuire, and U. van Kolck,
Singular potentials and limit cycles,
Phys.Rev.A64:042103,2001, **quant-ph/0010073**

A. Nogga, R.G.E. Timmermans, and U. van Kolck,
Renormalization of one-pion exchange and power counting,
Phys.Rev.C72:054006,2005, **nucl-th/0506005**


condensed-matter physics and beyond
molecular physics
atomic physics
nuclear physics
NRQED
0.1
Chiral EFT
1
?
QCD
(2 or 3 flavors)
10^2
QCD
(6 flavors)
QED
Fermi Th
10^{15}
Electroweak Th
+ higher-dim ops
(SUSY)
10^{18}
GUT?
General Relativity + higher-curvature terms
?
The world as an onion
E(GeV)
10^{-20}
10^{-16}
10^{-3}
10^{-1}
1
r(fm)

EFT at a few GeV= underlying theory for nuclear physics

d.o.f.s leptons: $l_f = \begin{pmatrix} l^+ \\ \nu \end{pmatrix}_f$ quarks: $q = \begin{pmatrix} u \\ d \end{pmatrix}$ photon: A_μ gluons: G_μ^a

symmetries: $SO(3,1)$ global, $U_{em}(1)$ gauge, $SU_c(3)$ gauge

$$\mathcal{L}_{und} = \sum_{f=1}^{3} \bar{l}_f \left(i\partial\!\!\!/ + eQ_l A\!\!\!/ - m_f \right) l_f - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$+ \bar{q} \left(i\partial\!\!\!/ + eQ_q A\!\!\!/ + g_s G\!\!\!/ \right) q - \frac{1}{2} \mathrm{Tr}\left[G_{\mu\nu} G^{\mu\nu} \right]$$
$$- \frac{1}{2} (m_u + m_d) \bar{q} q - \frac{1}{2} (m_u - m_d) \bar{q} \tau_3 q$$

QED + QCD

$$Q_l = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$Q_q = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} = \frac{1 + 3\tau_3}{6}$$

$$+ \frac{m_u m_d}{m_u + m_d} \bar{\theta}\, \bar{q} i \gamma_5 q + \ldots$$

higher-dimension interactions: suppressed by larger masses — e.g. $G_F \propto 1 / M_{W,Z}^2$

unnaturally small T violation (strong CP problem) — $\bar{\theta} \lesssim 10^{-9}$

Focus on strong-interacting sector: four parameters

1) $m_u = m_d = 0,\ e = 0,\ \bar{\theta} = 0$ "chiral limit"

single, *dimensionless* parameter

$$\int d^4x\, \mathcal{L}_{QCD} = \int d^4x \left\{ \bar{q}\left(i\partial\!\!\!/ + g_s G\!\!\!/\right) q - \frac{1}{2}\mathrm{Tr}\, G^{\mu\nu} G_{\mu\nu} \right\}$$

invariant under scale transformations

$$\begin{cases} x \to \lambda^{-1} x \\ q \to \lambda^{3/2} q \\ G \to \lambda G \end{cases}$$

but in

$$Z = \int DG \int D\bar{q} \int Dq \, \exp\left(i \int d^4x\, \mathcal{L}_{QCD} \right)$$

scale invariance

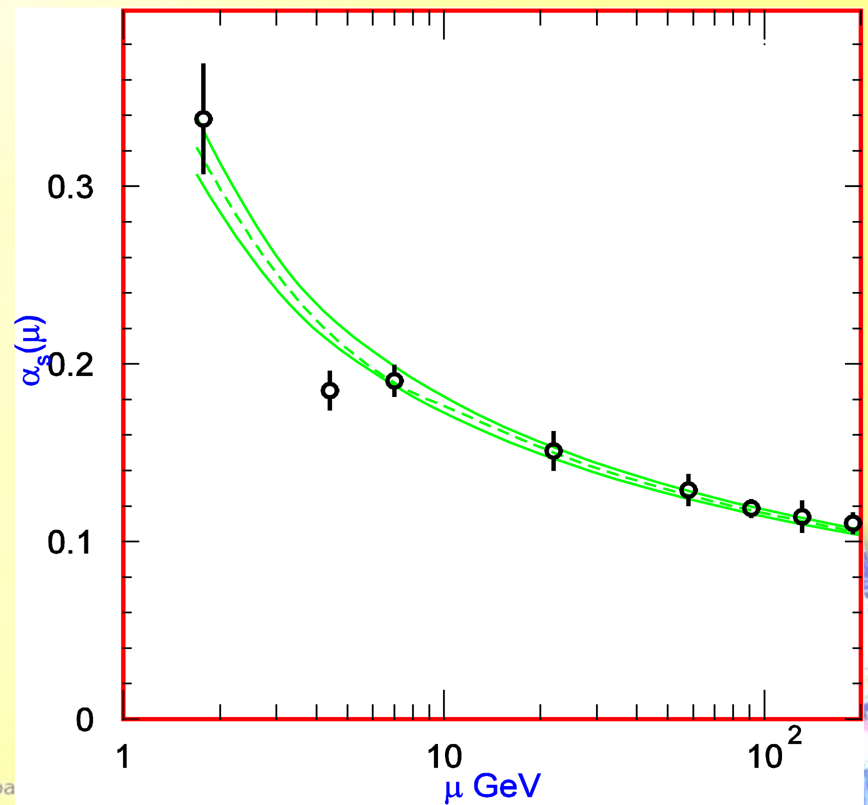
"anomalously broken"

by *dimensionful* regulator

⇨ coupling runs

$$\alpha_s\left(Q \sim 1\,\mathrm{GeV}\right) \sim 1$$

("dimensional transmutation")



Non-perturbative physics at $Q \sim 1\,\mathrm{GeV}$

Assumption 1: <u>confinement</u>

only colorless states (“hadrons”) are asymptotic

Observation: (almost) all hadron masses $\gtrsim 1\,\mathrm{GeV}$

Assumption 2: <u>naturalness</u>

masses are determined by characteristic scale

$\Rightarrow$ $M_{QCD} \sim 1\,\mathrm{GeV}$

Observation: pion mass $m_\pi \simeq 140\,\mathrm{MeV} \ll M_{QCD}$

breakdown of naturalness? NO!

“spontaneous breaking” of chiral symmetry

Why is the pion special?

$$\mathcal{L}_{QCD} = \bar{q}_L \left(i\not\partial + g_s \not G\right) q_L + \bar{q}_R \left(i\not\partial + g_s \not G\right) q_R - \frac{1}{2}\mathrm{Tr}\, G^{\mu\nu} G_{\mu\nu}$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \frac{1-\gamma_5}{2} q \qquad \frac{1+\gamma_5}{2} q$$

invariant under chiral symmetry

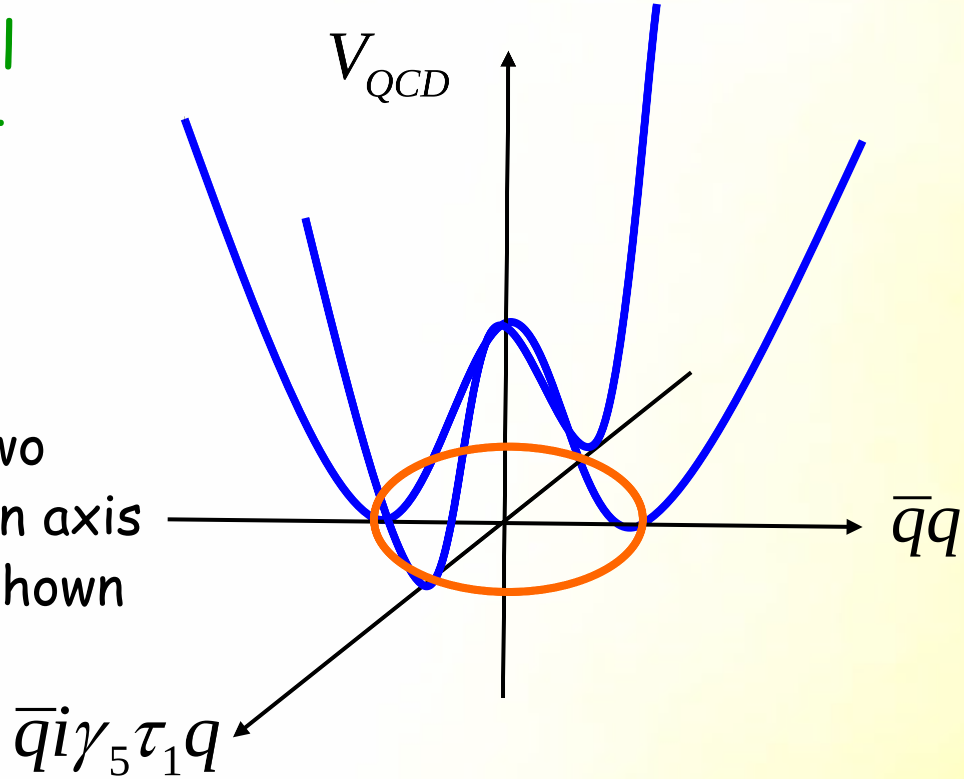
$$q_{L(R)} \to \exp\left(i\boldsymbol{\alpha}_{L(R)} \cdot \boldsymbol{\tau}\right) q_{L(R)} \qquad SU(2)_L \times SU(2)_R \sim SO(4)$$

broken by vacuum down to isospin

$$q \to \exp\left(i\boldsymbol{\alpha} \cdot \boldsymbol{\tau}\right) q \qquad SU(2)_{L+R} \sim SO(3)$$

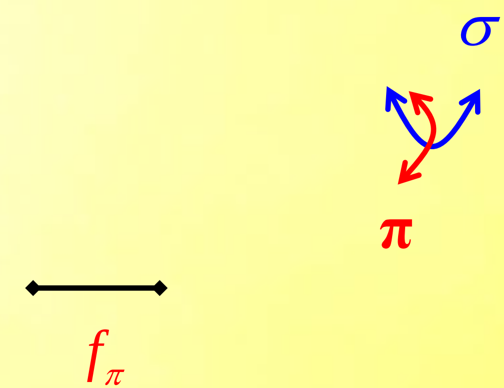
$$m_\sigma \gg m_\pi$$
$$m_{N_-} \gg m_{N_+}$$

Chiral Limit



two isospin axis not shown

chiral circle



pion decay constant (in chiral limit)

$$\mathcal{L}_{EFT} = \text{piece invariant under } \boldsymbol{\pi} \to \boldsymbol{\pi} + \boldsymbol{\varepsilon} \quad [\text{function of } \partial_\mu \boldsymbol{\pi} \text{ on chiral circle}]$$

$$\underbrace{\left(1 - \frac{\boldsymbol{\pi}^2}{4 f_\pi^2} + \ldots\right) \partial_\mu \boldsymbol{\pi}}$$

2) $m_u \neq 0 \neq m_d$, $e = 0$, $\bar{\theta} = 0$

$$\mathcal{L}_{QCD} = \bar{q}\left(i\not{\partial} + g_s \not{G}\right)q - \frac{1}{2}\mathrm{Tr}\, G^{\mu\nu} G_{\mu\nu}$$

v.K. '93

$$+\frac{1}{2}(m_u + m_d)\,\underbrace{\bar{q}q} + \frac{1}{2}(m_u - m_d)\,\underbrace{\bar{q}\tau_3 q} + \ldots$$

4th component of $SO(4)$ vector

$$S = \left(\bar{q}i\gamma_5 \boldsymbol{\tau} q,\, \bar{q}q\right)$$

3rd component of $SO(4)$ vector

$$P = \left(\bar{q}\boldsymbol{\tau} q,\, \bar{q}i\gamma_5 q\right)$$

break

$SO(4) \quad \to SO(3)$

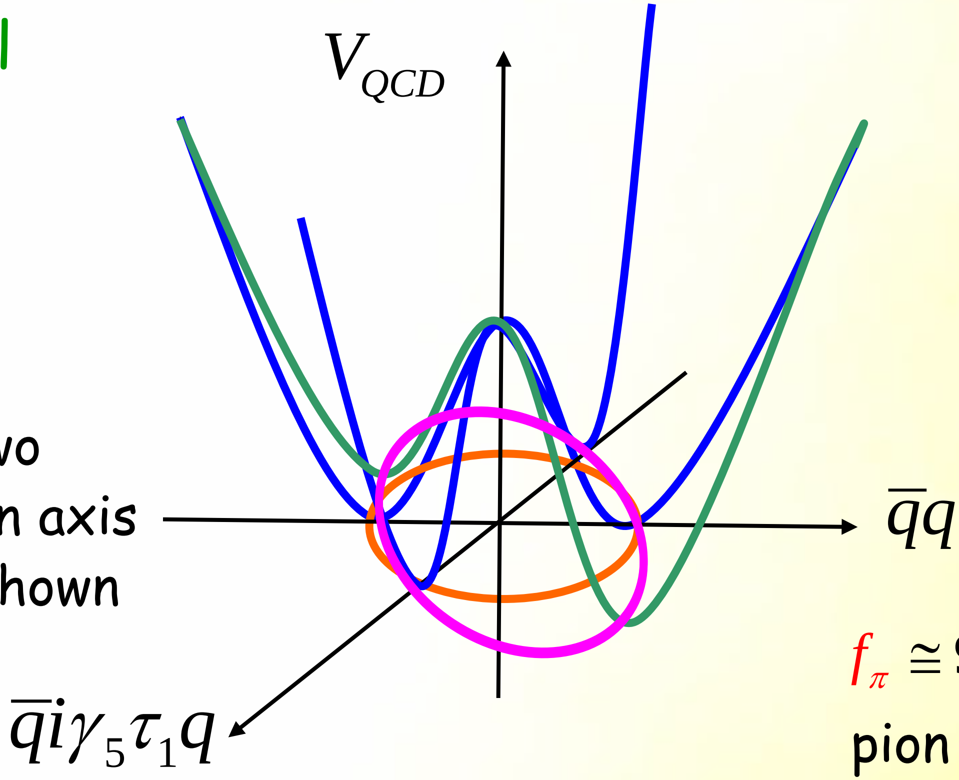
(explicit chiral-symmetry breaking)

$\to U(1)$

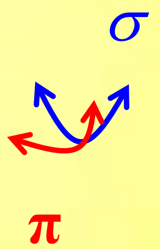
(isospin violation)

Chiral Limit

two isospin axis not shown



slightly-tilted chiral circle



$f_\pi \cong 92$ MeV

pion decay constant

$\mathcal{L}_{EFT}$ = piece invariant under $\boldsymbol{\pi} \rightarrow \boldsymbol{\pi} + \boldsymbol{\varepsilon}$ [function of $\partial_\mu \boldsymbol{\pi}$] $\quad \propto Q$

+ piece in $\bar{q}q$ direction [function of $\boldsymbol{\pi}$ explicitly] $\quad \propto (m_u + m_d)$

+ isospin breaking $\quad \propto (m_u - m_d)$

CHIRAL SYMMETRY ⇨ WEAK PION INTERACTIONS

3) $e \neq 0, \bar{\theta} = 0$

Two types of interactions:

- "soft" photons – explicit d.o.f. in the EFT

$$D_\mu = \partial_\mu - ieQ_q A_\mu$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- "hard" photons – "integrated out" of EFT

v.K. '93

$$\mathcal{L}_{und} = \ldots - e^2 \bar{q} Q_q \gamma_\mu q \, D^{\mu\nu}(\partial^2) \, \bar{q} Q_q \gamma_\nu q + \ldots$$

34 comp of antisymmetric tensor

$$F_\mu = \begin{pmatrix} \varepsilon_{ijk} \bar{q} i \gamma_\mu \gamma_5 \tau_k q & \bar{q} i \gamma_\mu \tau_j q \\ -\bar{q} i \gamma_\mu \tau_i q & 0 \end{pmatrix}$$

breaks $SO(4)$ (and $SO(3)$ in particular) $\rightarrow U(1)$

⇨ $\mathcal{L}_{EFT} =$ soft photons $\propto e$

\+ further isospin breaking $\propto \alpha / 4\pi$

4) $\bar{\theta} \neq 0$

$$\mathcal{L}_{und} = \ldots + \frac{m_u m_d}{m_u + m_d} \bar{\theta}\, \underbrace{\bar{q} i \gamma_5 q} + \ldots$$

4th component of $SO(4)$ vector $P = \left(\bar{q}\boldsymbol{\tau} q, \bar{q} i \gamma_5 q\right)$

T violation linked to isospin violation: in EFT, combination is

$$-\frac{1}{2}\left(m_u - m_d\right) P_3 + \frac{m_u m_d}{m_u + m_d} \bar{\theta}\, P_4$$

Hockings, Mereghetti + v.K., '10

5) continue with higher-order operators,
e.g. T-violating quark EDM and color-EDM
P-violating four-quark operators

De Vries, Mereghetti, Timmermans + v.K., '12

...

Kaplan + Savage '96

Zhu, Maekawa, Holstein, Musolf + v.K. '02

Nuclear physics scales

"His scales are His pride", Book of Job

$\ln Q$

perturbative QCD

~1 GeV $\quad M_{QCD} \sim m_N, m_\rho, 4\pi f_\pi, \ldots$

brute force (lattice), ...?

hadronic th
with chiral symm

~100 MeV $\quad M_{nuc} \sim f_\pi, 1/r_{NN}, m_\pi, \ldots$

~30 MeV $\quad \aleph \sim 1/a_{NN}$

this lecture

Q/M_{QCD}

Q/M_{nuc}

next lecture

no small coupling → expansion in ↑

Nuclear EFT

pionful EFT

$$Q \sim m_\pi \ll M_{QCD}$$

- d.o.f.s: nucleons, pions, deltas $\quad (m_\Delta - m_N \sim 2m_\pi)$

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad \boldsymbol{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} = \begin{pmatrix} (\pi^+ + \pi^-)/\sqrt{2} \\ -i(\pi^+ - \pi^-)/\sqrt{2} \\ \pi^0 \end{pmatrix} \qquad \Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \\ \Delta^{-} \end{pmatrix}$$

- symmetries: Lorentz, $\not{P}$, $\not{T}$, $\not{\text{chiral}}$

Weinberg '68
Callan, Coleman, Wess + Zumino '69

Non-linear realization of chiral symmetry

chiral invariants

(chiral) covariant derivatives

pion $\quad \boldsymbol{D}_\mu \equiv \left(\dfrac{\partial_\mu \boldsymbol{\pi}}{f_\pi}\right)\left(1 - \dfrac{\boldsymbol{\pi}^2}{4f_\pi^2} + \ldots\right)$

fermions $\quad \mathcal{D}_\mu \equiv \left(\partial_\mu - \dfrac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{E}_\mu\right)$

$$\boldsymbol{E}_\mu \equiv \frac{\boldsymbol{\pi}}{f_\pi} \times \boldsymbol{D}_\mu$$

$+ S_4$'s, P_3's, F_{34}'s

$$m_\pi^2 = \mathcal{O}\left((m_u + m_d) M_{QCD}\right)$$

$$\Rightarrow m_u + m_d = \mathcal{O}\left(\frac{m_\pi^2}{M_{QCD}}\right)$$

Schematically,

$$\mathcal{L}_{EFT} = \sum_{\{n,p,f\}} c_{\{n,p,f\}} \left(\frac{\mathbf{D}, \mathcal{D}, m_\Delta - m_N}{M_{QCD}} \right)^n \left(\frac{m_\pi^2}{M_{QCD}^2} \frac{\boldsymbol{\pi}^2}{f_\pi^2} \right)^{p/2} \left(\frac{\psi^+ \psi}{f_\pi^2 M_{QCD}} \right)^{f/2} f_\pi^2 M_{QCD}^2$$

$= \mathcal{O}(1)$ isospin conserving

$= \mathcal{O}\left(\varepsilon, \frac{\alpha}{4\pi} \right)$ isospin breaking

(NDA: naïve dimensional analysis)

calculated from QCD: lattice, …
fitted to data

$$= \sum_{\Delta=0}^{\infty} \mathcal{L}^{(\Delta)} \qquad \Delta \equiv n + p + \frac{f}{2} - 2 \equiv d + \frac{f}{2} - 2 \geq 0$$

"chiral index"

chiral symmetry

$$\mathcal{L}^{(0)} = \frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 \left(1-\frac{\boldsymbol{\pi}^2}{2f_\pi^2}+\ldots\right) - \frac{1}{2} m_\pi^2 \boldsymbol{\pi}^2 \left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}+\ldots\right)$$

$$+ N^+ \left[i\partial_0 - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_0\boldsymbol{\pi}) + \ldots \right] N + \frac{g_A}{2f_\pi} N^+\boldsymbol{\tau}\vec{\sigma}N \cdot\cdot(\vec{\nabla}\boldsymbol{\pi})\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}+\ldots\right)$$

$$+ \Delta^+\left[i\partial_0 - (m_\Delta - m_N) + \ldots\right]\Delta + \ldots + \frac{h_A}{2f_\pi}\left(N^+\mathbf{T}\vec{S}\Delta + \text{H.c.}\right)\cdot\cdot(\vec{\nabla}\boldsymbol{\pi})\,(1+\ldots)$$

$$- C_S\left(N^+N\right)^2 - C_T\left(N^+\vec{\sigma}N\right)^2$$

$$\mathcal{L}^{(1)} = N^+\left[\frac{1}{2m_N}\left(\vec{\nabla} + \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\vec{\nabla}\boldsymbol{\pi}) + \ldots\right)^2 + \frac{1}{2}(m_p - m_n)\left(\tau_3 - \frac{1}{2f_\pi^2}\pi_3\boldsymbol{\pi}\cdot\boldsymbol{\tau} + \ldots\right)\right]N$$

$$+ \frac{1}{f_\pi^2}N^+\left[b_2(\partial_0\boldsymbol{\pi})^2 - b_3(\vec{\nabla}\boldsymbol{\pi})^2 - 2b_1 m_\pi^2\boldsymbol{\pi}^2 + ib_4\varepsilon_{ijk}\varepsilon_{abc}\sigma_k\tau_c(\partial_i\pi_b)(\partial_j\pi_c)\right]N + \ldots$$

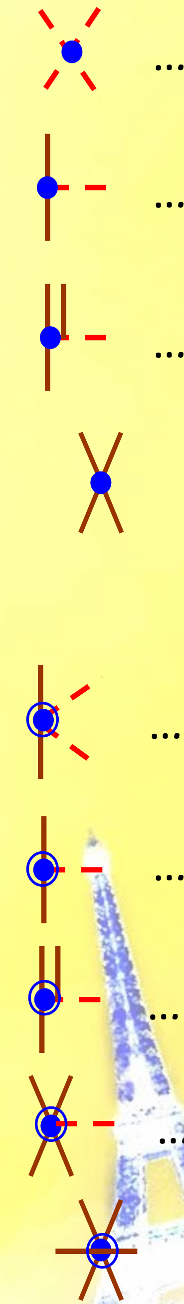
$$- \frac{g_A}{4m_N f_\pi}\left[iN^+\boldsymbol{\tau}\vec{\sigma}\cdot\vec{\nabla}N + \text{H.c.}\right]\cdot(\partial_0\boldsymbol{\pi})\,(1+\ldots)$$

$$- \frac{h_A}{4m_N f_\pi}\left[iN^+\mathbf{T}\vec{S}\cdot\vec{\nabla}N + \text{H.c.}\right]\cdot(\partial_0\boldsymbol{\pi})\,(1+\ldots)$$

$$+ \frac{d}{f_\pi}N^+N\; N^+\boldsymbol{\tau}\vec{\sigma}N\cdot\cdot(\vec{\nabla}\boldsymbol{\pi})\,(1+\ldots)$$

$$- E\left(N^+N\right)^3$$

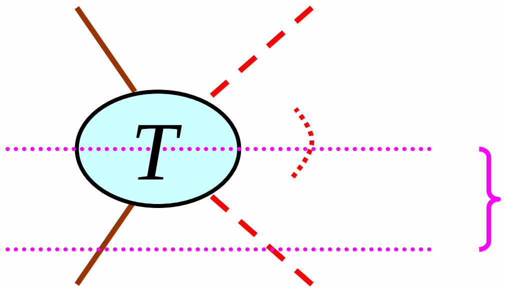
$$\mathcal{L}^{(2)} = \ldots$$



Form of pion interactions <u>determined</u> by chiral symmetry

Weinberg '79
Gasser + Leutwyler '84
...
Gasser, Sainio + Svarc '87
Jenkins + Manohar '91
...

A= 0, 1: chiral perturbation theory



$$\frac{1}{\Delta E} \sim \frac{1}{Q}$$

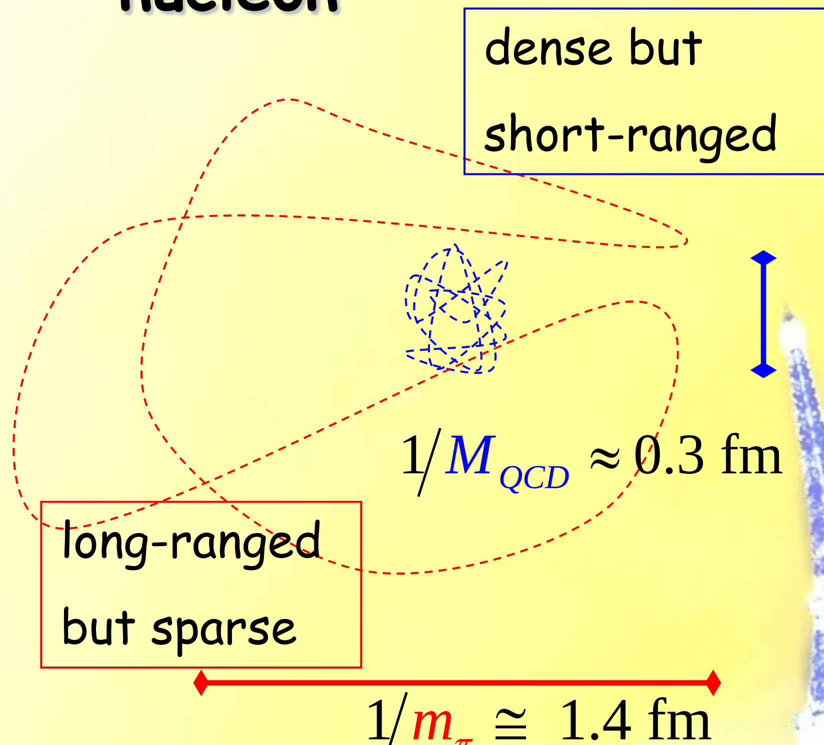
$$\sim \sum_{\nu} c_{\nu} \left(\frac{Q}{M_{QCD}} \right)^{\nu} F_{\nu}\left(\frac{Q}{m_{\pi}} \right)$$

$$\nu = 2 - A + 2L + \sum_i V_i \Delta_i \geq \nu_{\min} = 2 - A$$

loops

vertices of type *i*

nucleon

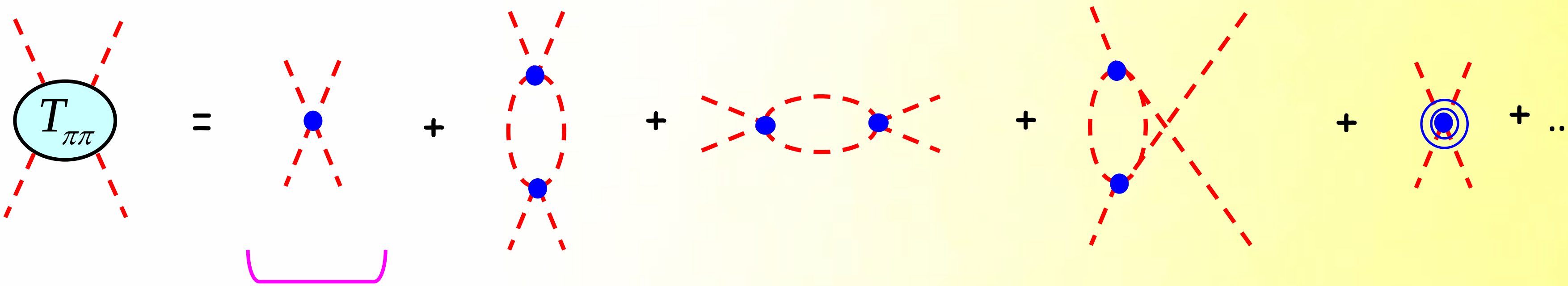


expansion in

$$\frac{Q}{M_{QCD}} \sim \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_{\rho}, \ldots & \text{multipole} \\ Q/4\pi f_{\pi} & \text{pion loop} \end{cases}$$

Analogous to NRQED…

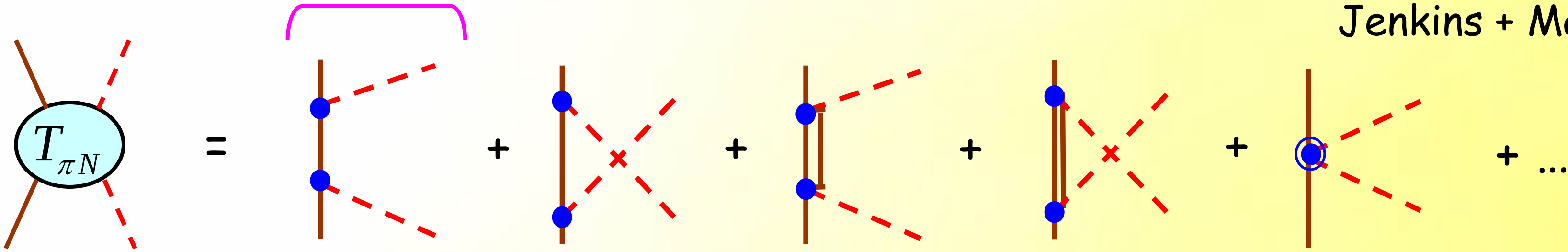
Weinberg '79
Gasser + Leutwyler '84
…



current algebra

Weinberg '66
…

Gasser, Sainio + Svarc '87
Bernard, Kaiser + Meissner '90
Jenkins + Manohar '91
…



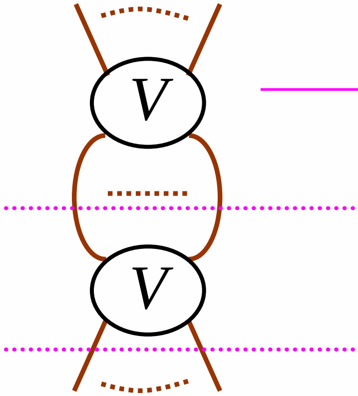
etc.

N.B. For $\left|E-(m_\Delta - m_N)\right| \lesssim \mathcal{O}\left(\frac{Q^3}{M_{QCD}^2}\right)$ a resummation is necessary

Phillips + Pascalutsa '02
Long + v.K. , '08

A ≥ 2: resummed chiral perturbation theory

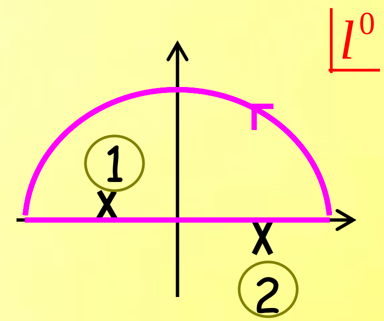
Weinberg '90, '91



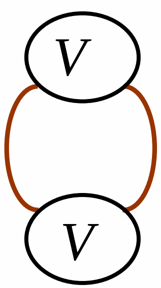
A-nucleon irreducible

$$\frac{1}{\Delta E} \sim \frac{m_N}{Q^2}$$ infrared enhancement!

A-nucleon reducible



e.g.



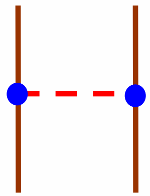
$$E = \frac{k^2}{m_N}$$

$$\simeq i\int \frac{d^4 l}{(2\pi)^4} V \frac{1}{l^0 + k^2/m_N - l^2/m_N - i\varepsilon}\ \frac{1}{-l^0 + k^2/m_N - l^2/m_N - i\varepsilon} V$$

(1) (2)

$$= \int \frac{d^3 l}{(2\pi)^3} V \frac{m_N}{l^2 - k^2} V + \ldots \qquad \sim \mathcal{O}\left(\frac{m_N Q}{4\pi} V^2\right)$$

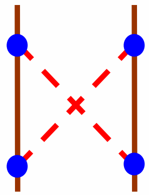
instead of $\frac{Q^2}{(4\pi)^2}$



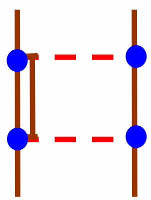
$$\sim i\left(\frac{g_A}{2f_\pi}\right)^2\left(\frac{\vec q^{\,2}}{\vec q^{\,2}+m_\pi^2}\right)\frac{\left(S_{12}(\hat q)+\vec\sigma_1\cdot\vec\sigma_2\right)}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\sim\frac{1}{f_\pi^2}$$

tensor force

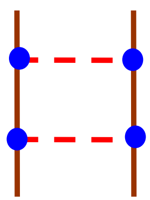
$$S_{12}(\hat q)=3\vec\sigma_1\cdot\hat q\,\vec\sigma_2\cdot\hat q-\vec\sigma_1\cdot\vec\sigma_2$$



$$\sim\frac{1}{f_\pi^4}\frac{Q^3}{(4\pi)^2}\frac{1}{Q}\frac{1}{Q}\frac{Q^2}{Q^2}\frac{Q^2}{Q}\sim\frac{1}{f_\pi^2}\underbrace{\frac{Q^2}{(4\pi f_\pi)^2}}_{=\mathcal{O}\left(\frac{Q^2}{M_{QCD}^2}\right)}$$



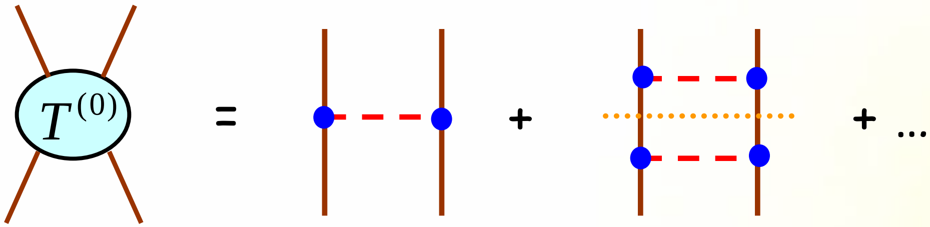
$$\sim\frac{1}{f_\pi^4}\frac{Q^3}{(4\pi)^2}\frac{1}{m_\Delta-m_N}\frac{1}{Q}\frac{Q^2}{Q^2}\frac{Q^2}{Q}\sim\frac{1}{f_\pi^2}\overbrace{\frac{Q^2}{(4\pi f_\pi)^2}}\underbrace{\frac{Q}{m_\Delta-m_N}}_{=\mathcal{O}(1)}$$



$$\sim\frac{1}{f_\pi^4}\frac{Q^3}{4\pi}\frac{m_N}{Q^2}\frac{Q^2}{Q^2}\frac{Q^2}{Q^2}\sim\frac{1}{f_\pi^2}\frac{m_N}{4\pi f_\pi}\frac{Q}{f_\pi}\sim\frac{1}{f_\pi^2}\frac{Q}{M_{NN}}$$

$$\equiv 1/M_{NN}$$

$$=\mathcal{O}(1)\ \text{for}\ Q\sim M_{NN}$$



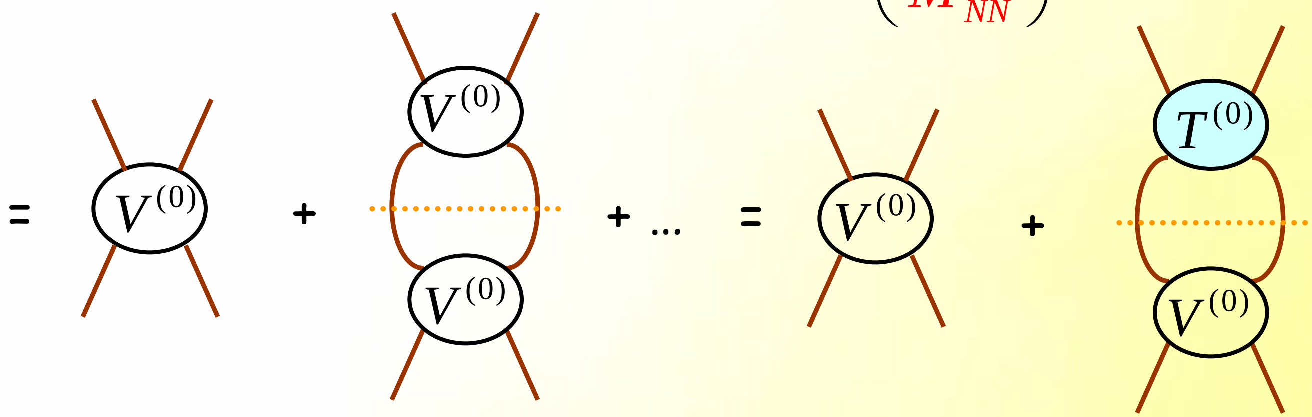
bound state at

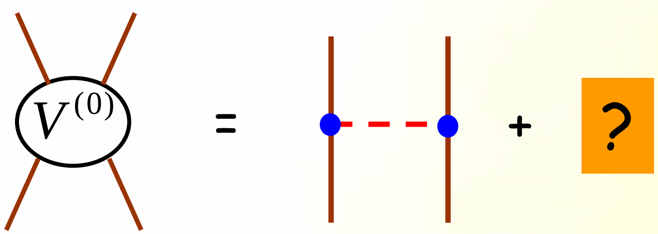
$$Q \sim M_{NN} \qquad -E \sim \frac{Q^2}{m_N} \sim \frac{M_{NN}^2}{M_{QCD}}$$

$$\sim \frac{1}{f_\pi^2}\left\{1+\mathcal{O}\left(\frac{Q}{M_{NN}}\right)+\ldots\right\} \sim \frac{1}{f_\pi^2}\frac{1}{1-\mathcal{O}\left(\frac{Q}{M_{NN}}\right)}$$

$$M_{nuc} = M_{NN} \sim \frac{4\pi f_\pi}{m_N} f_\pi \approx f_\pi$$

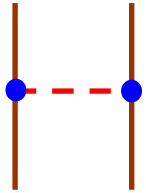
Nuclear scale arises naturally from chiral symmetry





Is 1PE all there is in leading order?
That is, are observables cutoff independent with 1PE alone?

Issue: relative importance of pion exchange and short-range interactions

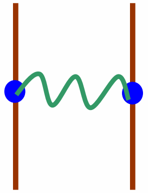


$$\sim i\left(\frac{g_A}{2f_\pi}\right)^2\left(\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\pi^2}\right)\frac{\left(S_{12}(\hat{q})+\vec{\sigma}_1\cdot\vec{\sigma}_2\right)}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\sim\frac{4\pi}{m_N M_{NN}}$$

$$\begin{cases} V(r)=\left(\dfrac{g_A}{2f_\pi}\right)^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(-\delta^{(3)}(\vec{r})+\dfrac{m_\pi^2}{4\pi r}e^{-m_\pi r}\right) & S=0 \\ V(r)=\left(\dfrac{g_A}{2f_\pi}\right)^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left\{\dfrac{1}{3}\left(\delta^{(3)}(r)-\dfrac{m_\pi^2}{4\pi r}e^{-m_\pi r}\right)+\dfrac{m_\pi^2}{4\pi r}\left(\dfrac{1}{(m_\pi r)^2}+\dfrac{1}{m_\pi r}+\dfrac{1}{3}\right)e^{-m_\pi r}\left\langle S_{12}(\hat{r})\right\rangle\right\} & S=1 \end{cases}$$

$\langle S_{12}\rangle$	$j-1$	j	$j+1$
$j-1$	$-2\frac{j-1}{2j+1}$	0	$6\frac{\sqrt{j(j+1)}}{2j+1}$
j	0	2	0
$j+1$	$6\frac{\sqrt{j(j+1)}}{2j+1}$	0	$-2\frac{j+2}{2j+1}$

much more singular --and complicated!-- than

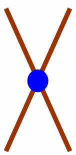


$$\sim\frac{i\,e^2}{\left(\vec{p}-\vec{p}\,'\right)^2-i\varepsilon}\sim\frac{4\pi\alpha}{Q^2}\quad\to V(r)=\frac{\alpha}{r}$$

Weinberg '90, '91
Ordonez + v.K. '92
Ordonez, Ray + v.K. '96
...
Entem + Machleidt '03...
Epelbaum, Gloeckle + Meissner '04
...

Assume contact interactions are driven by heavier dofs, and scale with M_{QCD} according to naïve dimensional analysis

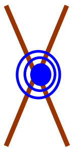
(W power counting)



$$\sim \underbrace{C_0^{(0)}}_{\equiv \frac{4\pi}{m_N M^{(0)}}} \frac{(\vec{\sigma}_1 \cdot \vec{\sigma}_2 + 1)}{4} - \underbrace{C_0^{(1)}}_{\equiv \frac{4\pi}{m_N M^{(1)}}} \frac{(\vec{\sigma}_1 \cdot \vec{\sigma}_2 - 3)}{4}$$

$$\begin{cases} V(r) = \dfrac{4\pi}{m_N M^{(0)}} \delta^{(3)}(\vec{r}) & S = 0 \\ V(r) = \dfrac{4\pi}{m_N M^{(1)}} \delta^{(3)}(\vec{r}) & S = 1 \end{cases}$$

$M^{(i)} \sim M_{NN}$ ⇨ $C_0^{(i)}$ in LO



$$\sim \frac{4\pi}{m_N M_{NN}} \frac{Q^2}{M_{QCD}^2}$$ ⇨ in NNLO

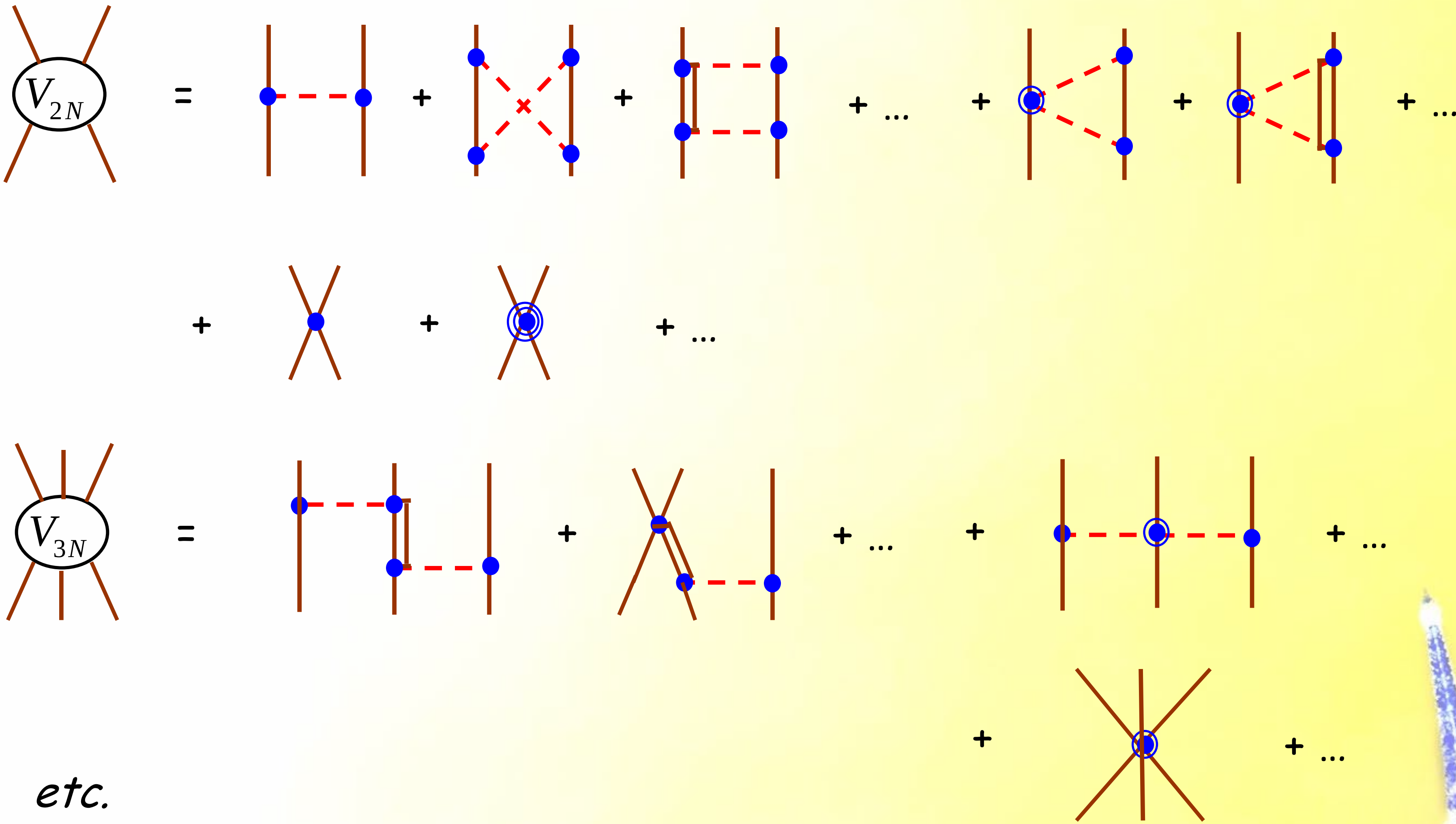
(NLO terms, linear in Q/M_{QCD}, break *P*, *T*)

etc.

Ordonez + v.K. '92

v.K. '94

...

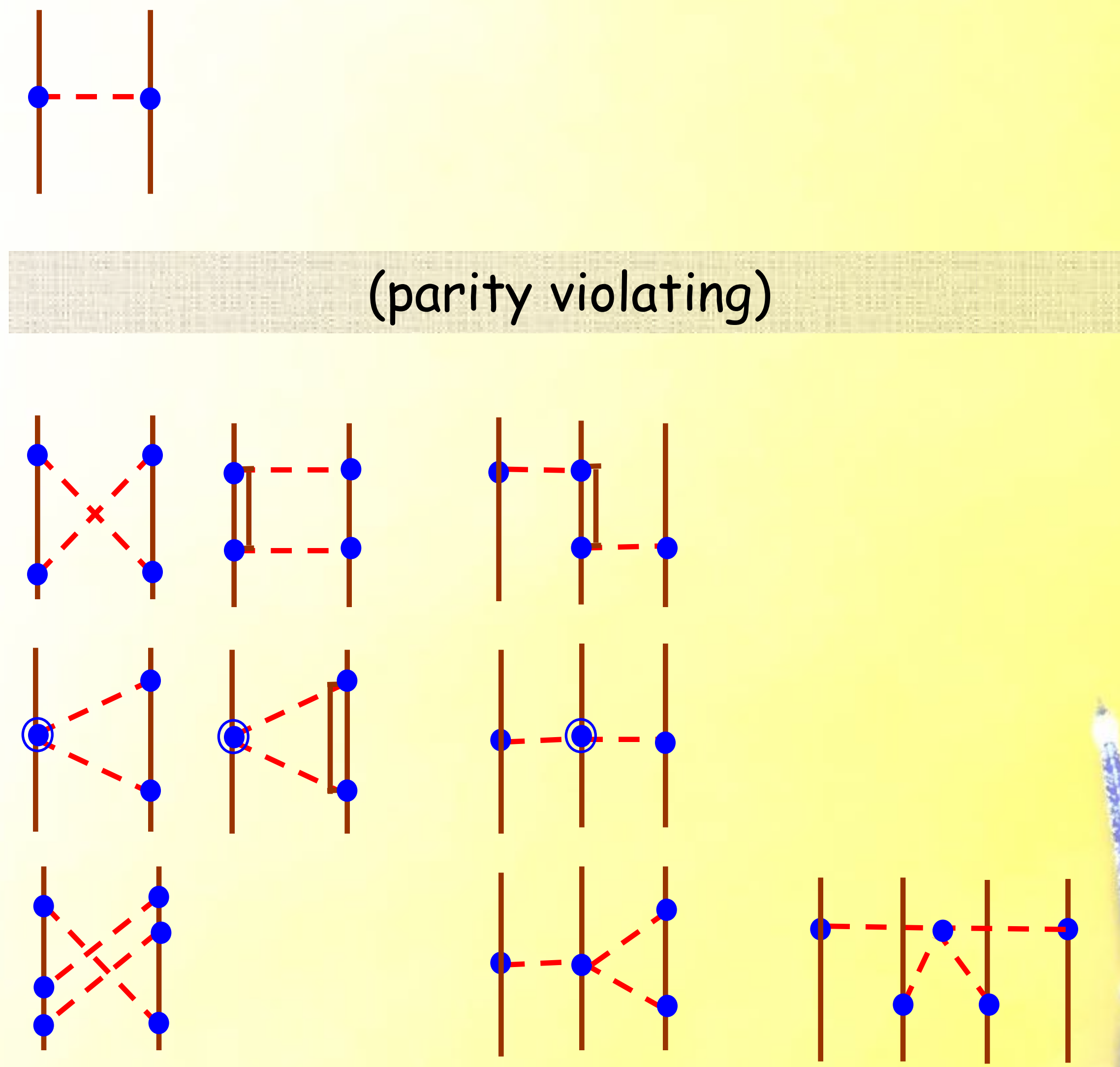


etc.

more nucleons

higher powers of Q

		2-body	3-body	4-body	...
LO	$\mathcal{O}\left(\frac{1}{f_\pi^2}\right)$				
in German NLO	$\mathcal{O}\left(\frac{1}{f_\pi^2}\frac{Q}{M_{QCD}}\right)$	(parity violating)			
NNLO	$\mathcal{O}\left(\frac{1}{f_\pi^2}\frac{Q^2}{M_{QCD}^2}\right)$				
NNNLO	$\mathcal{O}\left(\frac{1}{f_\pi^2}\frac{Q^3}{M_{QCD}^3}\right)$				
NNNNLO	$\mathcal{O}\left(\frac{1}{f_\pi^2}\frac{Q^4}{M_{QCD}^4}\right)$				
ETC.					



Hierarchies

many-body forces

$$V_{2N} \gg V_{3N} \gg V_{4N} \gg \ldots$$

A canon emerges!

Weinberg '90, '91

isospin-breaking forces

Similar explanation for

$$V_{IS} \gg V_{IV} \gg V_{CSB}$$

v.K. '93

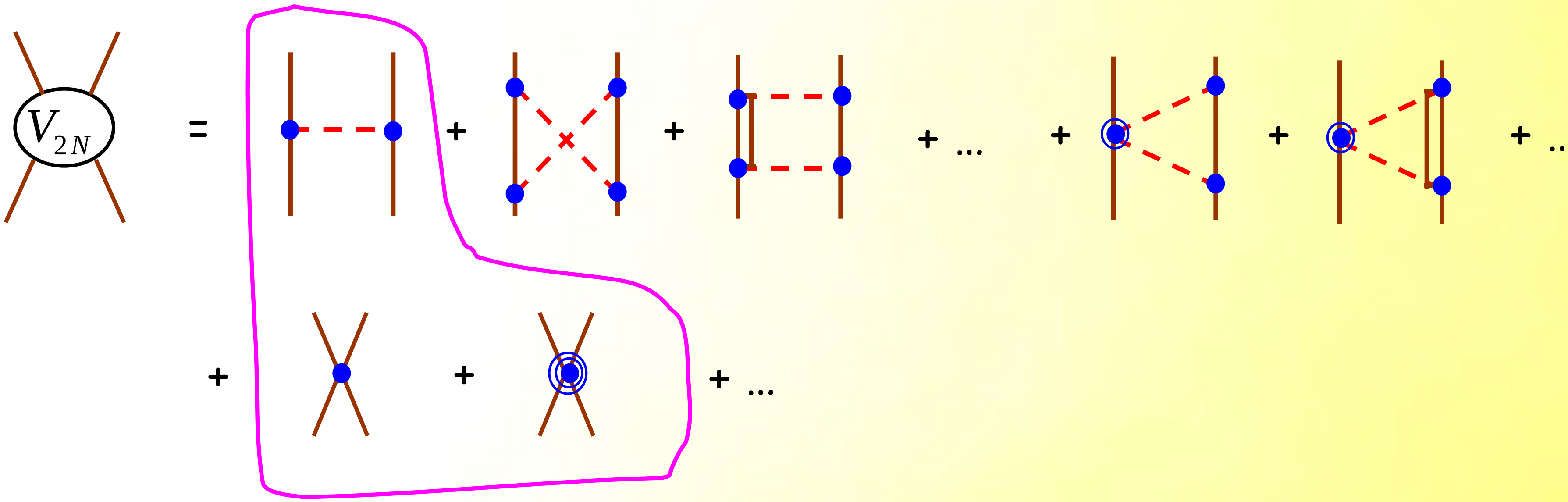
$$J_{1N} \gg J_{2N} \gg J_{3N} \gg \ldots$$

Rho '92

external currents

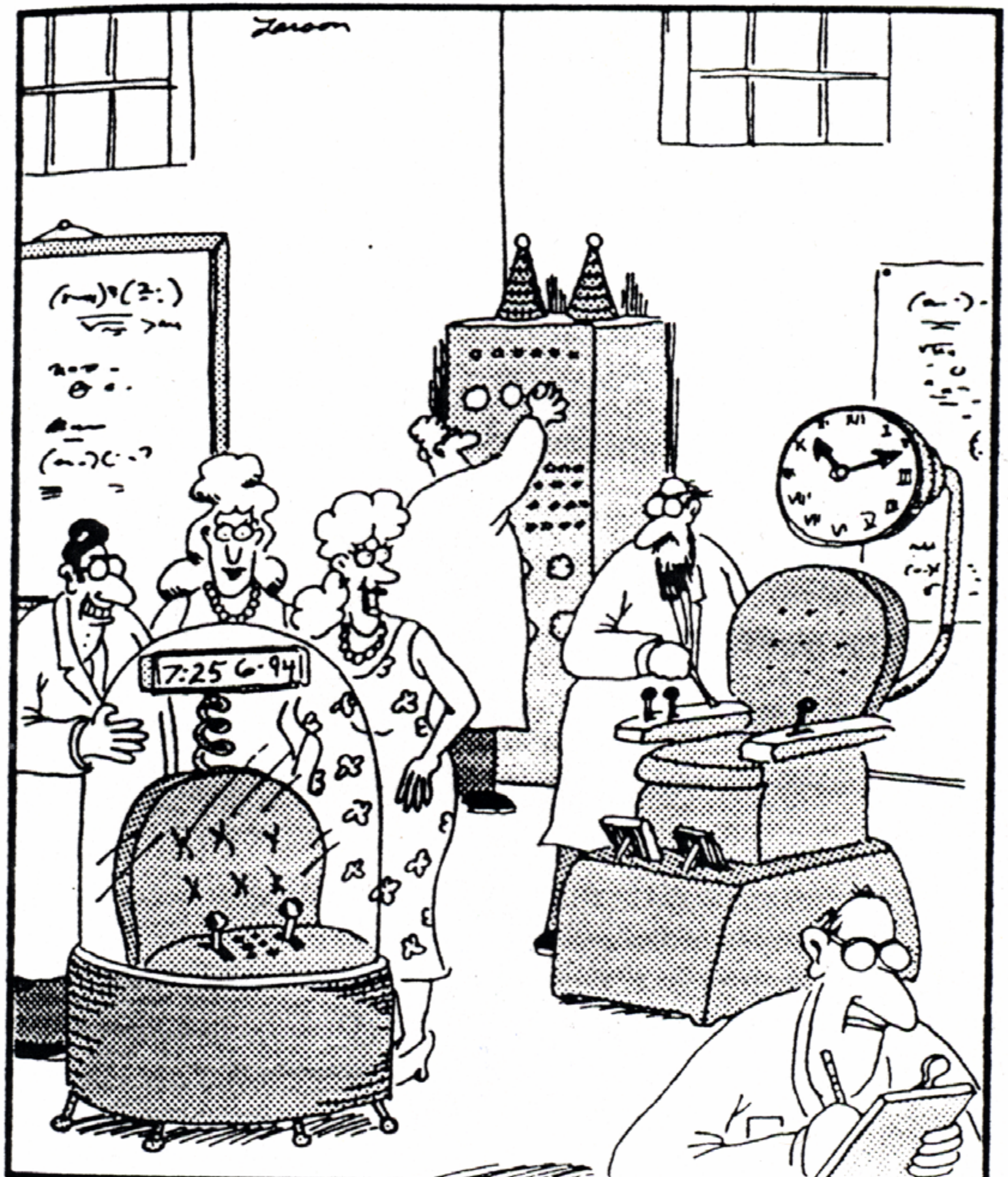
other canons emerge!

Ordonez + v.K. '92
v.K. '94
...



similar to phenomenological
potential models,
e.g. AV18 – (OPE)^2 + non-local terms

Stoks, Wiringa + Pieper '94

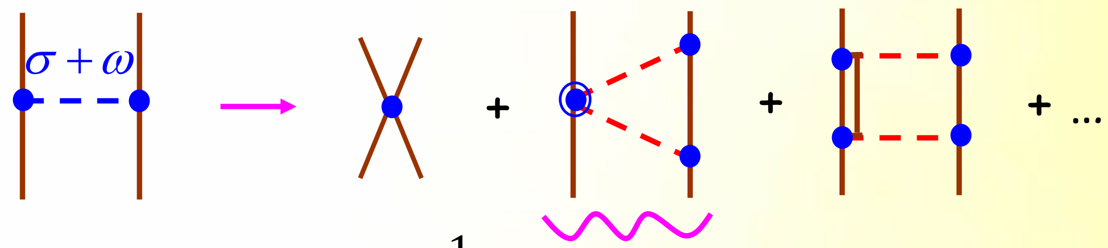


"Oh, Professor DeWitt! Have you seen Professor *Weinberg's* time machine? ... It's digital!"

But: NOT your usual potential!

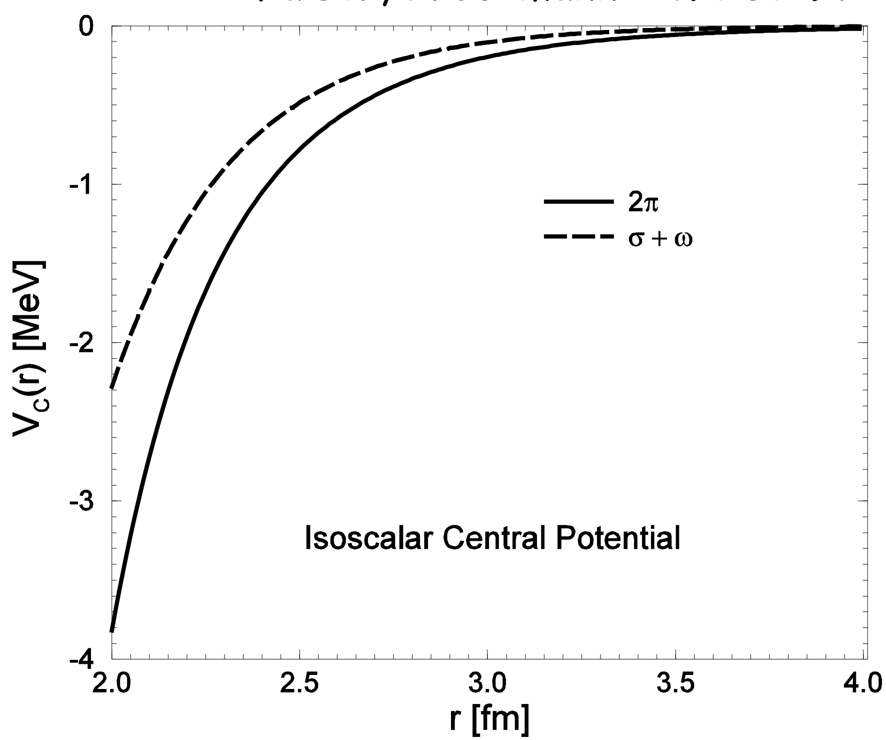
Ordonez + v.K. '92
(*cf.* Stony Brook TPE)

e.g.,



chiral v.d. Waals force $\sim \frac{1}{r^6}$ for $m_\pi^2 \to 0$

Kaiser, Brockmann + Weise '97



Rentmeester et al. '01, '03

Nijmegen PSA of 1,951 *pp* data

long-range pot	#bc	χ^2_{min}
OPE	31	2026.2
OPE + TPE (*lo*)	28	1984.7
OPE + TPE (*nlo*)	23	1934.5
Nijm78	19	1968.7

parameters found consistent with πN data!

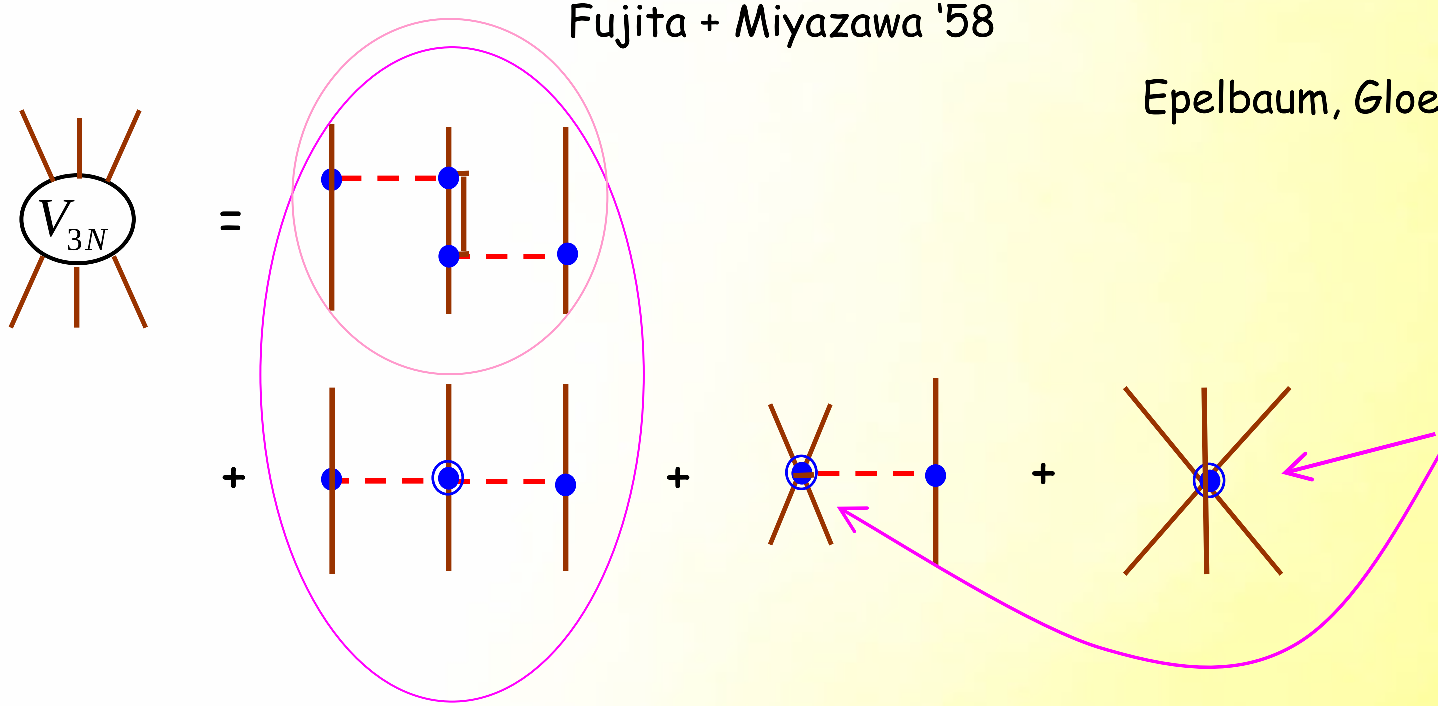
at least as good!

Similar results in other channels, *e.g.* spin-orbit force!

models with σ, ω, ... might be misleading...

v.K. '94
Friar, Hueber + v.K. '99
Coon + Han '99
Epelbaum, Gloeckle+ Meissner '02
...

Fujita + Miyazawa '58



two unknown parameters

+ ...

Tucson-Melbourne pot with

Coon *et al.* '78

$$\begin{cases} a \to a - 2m_\pi^2 c \\ c \to 0 \end{cases}$$

TM' potential

$$\left(t_{\pi N}\left(\vec{q},\vec{q}\,'\right)\right)_{\alpha\beta} = \delta_{\alpha\beta}\left[a + b\vec{q}\cdot\vec{q}\,' + c\left(\vec{q}^{\,2} + \vec{q}\,'^{2}\right)\right] - d\,\varepsilon_{\alpha\beta\gamma}\tau_{3\gamma}\vec{\sigma}\cdot\vec{q}\times\vec{q}\,' + \dots$$

Many successes of Weinberg's counting, *e.g.*,

Ordonez, Ray + v.K. '96
...
Epelbaum, Gloeckle + Meissner '02
Entem + Machleidt '03
...

✓ To NNNNLO (w/o deltas), fit to NN phase shifts comparable to those of "realistic" phenomenological potentials

Entem + Machleidt '03

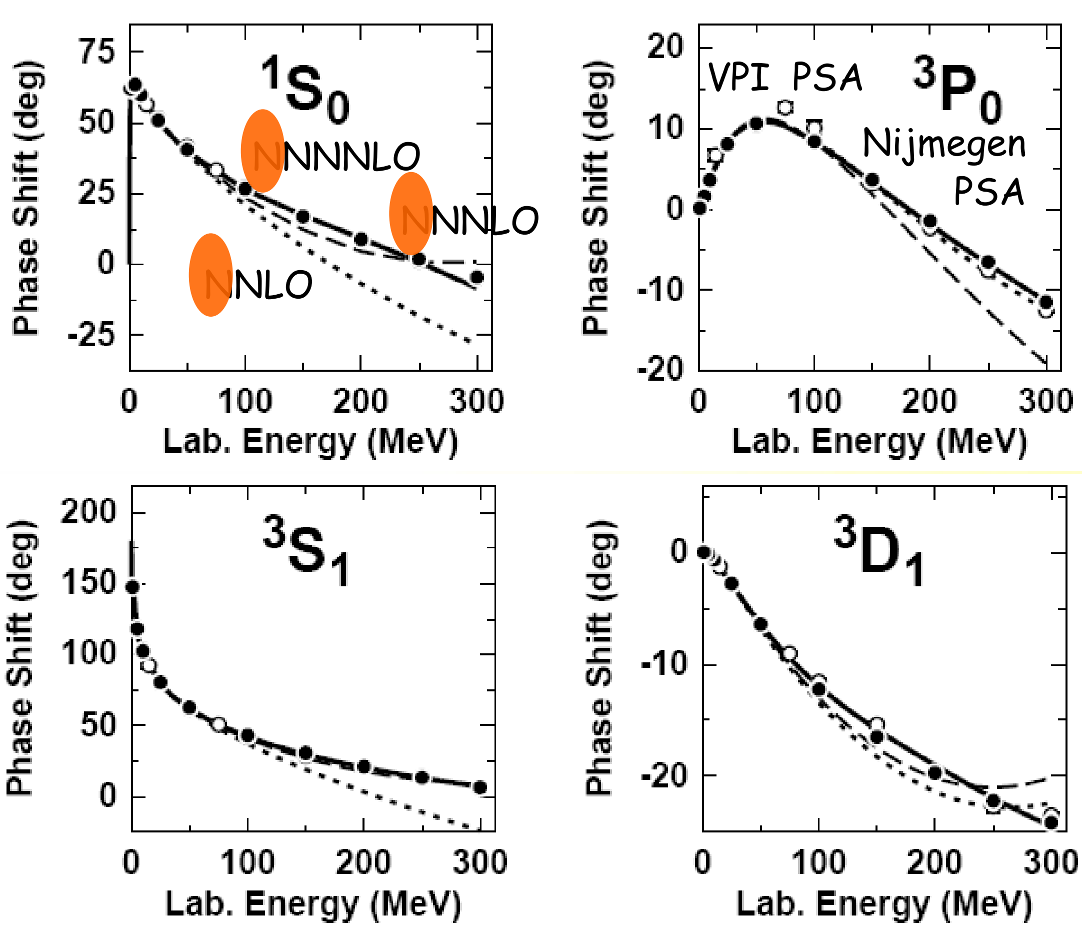


TABLE II. χ^2/datum for the reproduction of the 1999 *np* database [40] below 290 MeV by various *np* potentials.

Bin (MeV)	No. of data	N^3LO[a]	NNLO[b]	NLO[b]	AV18[c]
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

TABLE V. Two- and three-nucleon bound-state properties. (Deuteron binding energy B_d; asymptotic S state A_S; asymptotic D/S state η; deuteron radius r_d; quadrupole moment Q; D-state probability P_D; triton binding energy B_t.)

	N^3LO[a]	CD-Bonn [10]	AV18 [22]	Empirical[b]
Deuteron				
B_d(MeV)	2.224575	2.224575	2.224575	2.224575(9)
A_S(fm$^{-1/2}$)	0.8843	0.8846	0.8850	0.8846(9)
η	0.0256	0.0256	0.0250	0.0256(4)
r_d(fm)	1.978[c]	1.970[c]	1.971[c]	1.97535(85)
Q(fm^2)	0.285[d]	0.280[d]	0.280[d]	0.2859(3)
P_D(%)	4.51	4.85	5.76	
Triton				
B_t(MeV)[e]	7.855	8.00	7.62	8.48

- ✓ With NNNNLO 2N and NNNLO 3N potentials (w/o deltas), good description of
- 3N observables and 4N binding energy
- levels of p-shell nuclei

Epelbaum et al. '02

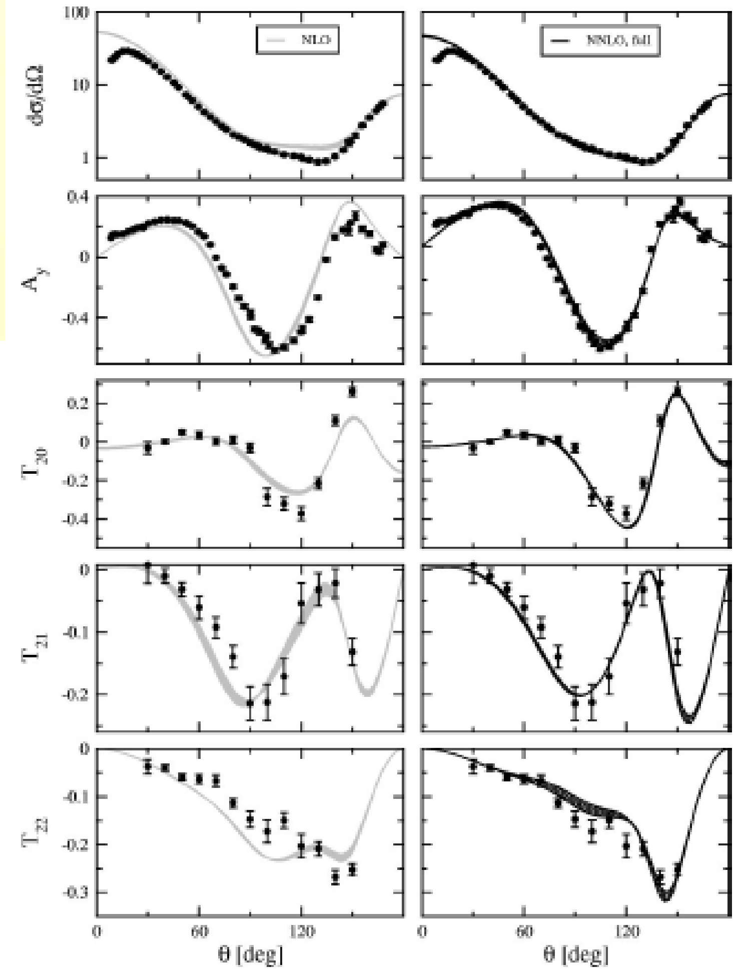


FIG. 6. *nd* elastic scattering observables at 65 MeV at NLO (left column) and NNLO (right column). The filled circles are *pd* data [63,69]. The bands correspond to the cutoff variation between 500 and 600 MeV. The unit of the cross section is mb/sr.

Gueorguiev, Navratil, Nogga, Ormand + Vary '07

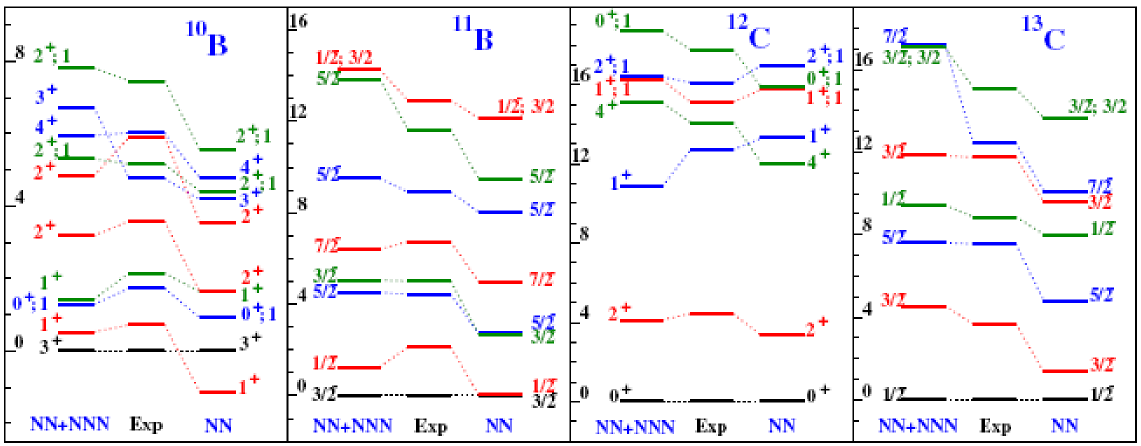


FIG. 4 (color online). States dominated by *p*-shell configurations for ^{10}B, ^{11}B, ^{12}C, and ^{13}C calculated at $N_{max} = 6$ using $\hbar\Omega = 15$ MeV (14 MeV for ^{10}B). Most of the eigenstates are isospin $T = 0$ or $1/2$, the isospin label is explicitly shown only for states with $T = 1$ or $3/2$. The excitation energy scales are in MeV.

Many reactions:

$\gamma d \to d\gamma$

measured: Illinois '94, SAL '00, Lund '03

extracted nucleon polarizabilities: Beane, Malheiro, McGovern, Phillips + v.K. '04

$\gamma d \to d\pi^0$

threshold amplitude <u>predicted</u>: Beane, Bernard, Lee, Meissner + v.K. '97

<u>confirmed</u>: SAL '98, Mainz '01

$pp \to pp\pi^0$

$pp \to pn\pi^+$

$pp \to d\pi^+$

$pn \to d\pi^0$

measured: IUCF '90-…, TRIUMF '91-…, Uppsala '95-…

S waves sensitive to high orders: Miller, Riska + v.K. '96

P waves converge, fix 3BF LEC: Hanhart, Miller + v.K. '00

CSB asymmetry sign <u>predicted</u>: Miller, Niskanen + v.K. '00

<u>confirmed</u>: TRIUMF '03

$dd \to \alpha\pi^0$

measured: IUCF '03

mechanisms surveyed: Fonseca, Gardestig, Hanhart, Horowitz, Miller, Niskanen, Nogga +v.K. '04 '06

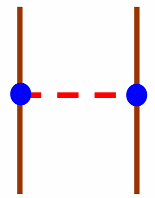
+ **PARITY, TIME-REVERSAL VIOLATION**, *etc.*

Chiral EFT has been recognized as
the basis for nuclear physics.
Now it is the favorite input for
the blossoming *ab initio* methods
that are revolutionizing
nuclear structure/reaction physics

BUT

Is Weinberg's power counting consistent?

No!



$$\sim \left(\frac{g_A}{2f_\pi}\right)^2 \frac{m_\pi^3}{4\pi}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left\{ \frac{S_{12}(\hat{r})}{(m_\pi r)^3} + \ldots \right\} e^{-m_\pi r}$$

attractive in some channels

singular potential

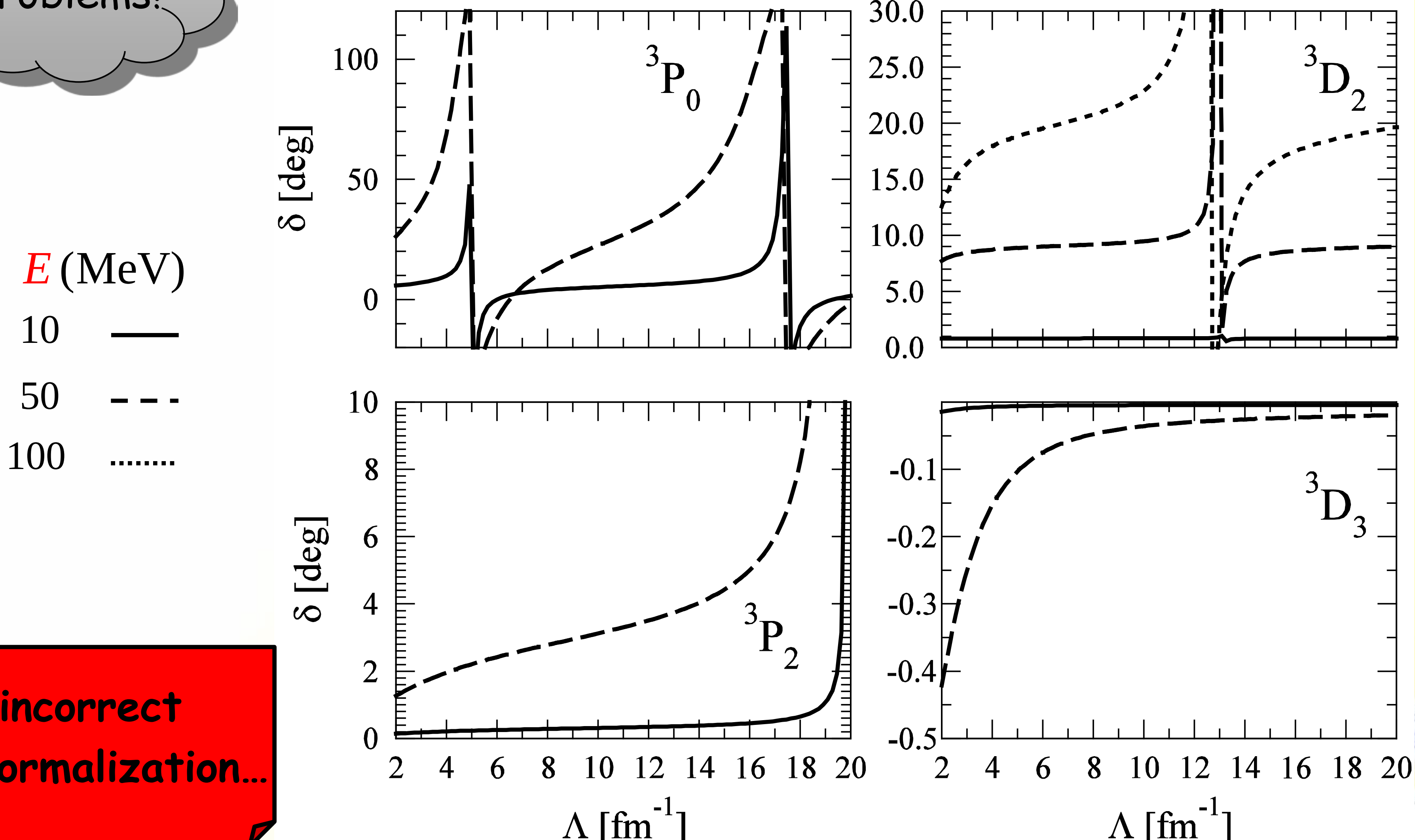
not enough contact interactions

for renormalization-group invariance even at LO

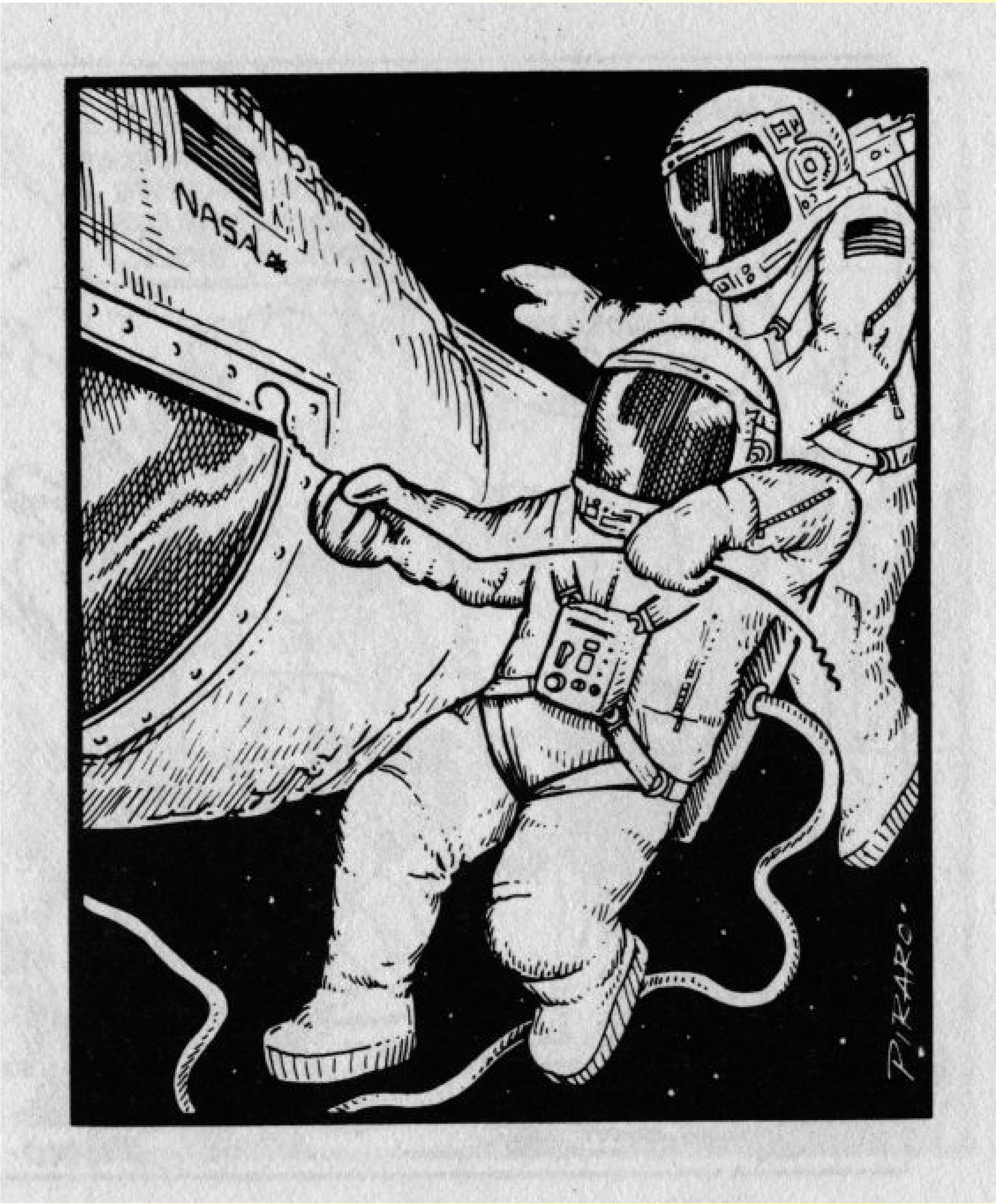
Nogga, Timmermans + v.K. '05
Pavon-Valderrama + Ruiz-Arriola '06

Problems!

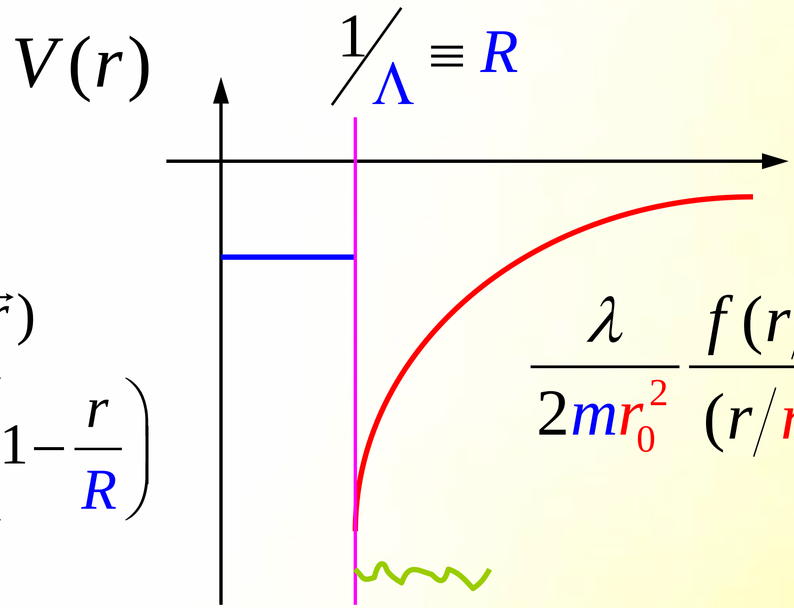
Attractive-tensor channels:



incorrect renormalization…


NASA
PIRARO

Renormalization of the $1/r^n$ potential



$$C_0(\Lambda)\,\delta^{(3)}_{\Lambda}(\vec{r})$$

$$\rightarrow V_0(R)\,\theta\left(1-\frac{r}{R}\right)$$

$$\frac{\lambda}{2mr_0^2}\,\frac{f(r/r_0)}{(r/r_0)^n}$$

OPE:

$$\begin{cases} m = m_N/2 \\ r_0 = 1/m_\pi \\ \lambda = m_\pi/M_{NN} \\ f(r/r_0) = \exp(-r/r_0) \end{cases}$$

s wave

$$\psi_n(r \sim R \ll r_0) \equiv \frac{u_n(r)}{r}$$

matching

$$\sqrt{-2mR^2V_0}\,\cot\sqrt{-2mR^2V_0} = F_n(\lambda, r_0, R)$$

so that

$$\frac{R}{T_s}\frac{\partial T_s}{\partial R}(k \sim 1/r_0) = \mathcal{O}\left(\frac{R}{r_0}\right)$$

$n \geq 2$

Beane, Bedaque, Childress, Kryjevski, McGuire + v.K. '02

Two **regular** solutions that oscillate!

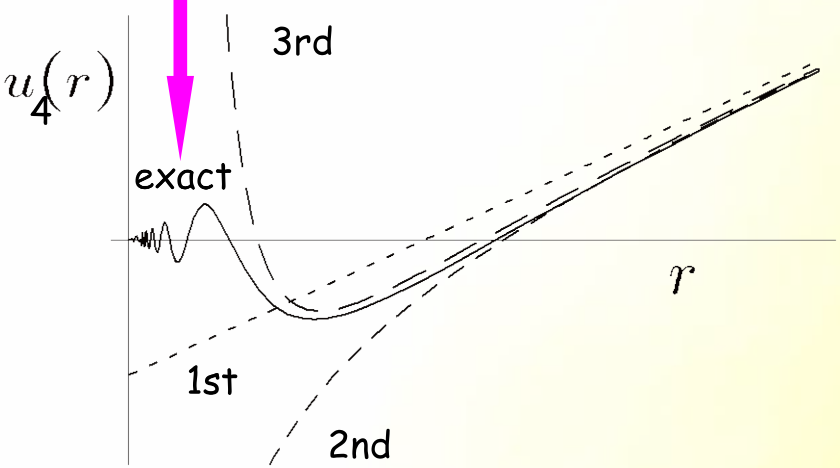
$$u_n(r \ll r_0) = \left(\frac{\lambda}{(r/r_0)^n}\right)^{-\frac{1}{4}} \cos\left(\frac{\sqrt{\lambda}}{(n/2-1)(r/r_0)^{n/2-1}} + \delta_n\right) + \ldots$$

if no counterterm, will depend on cutoff R

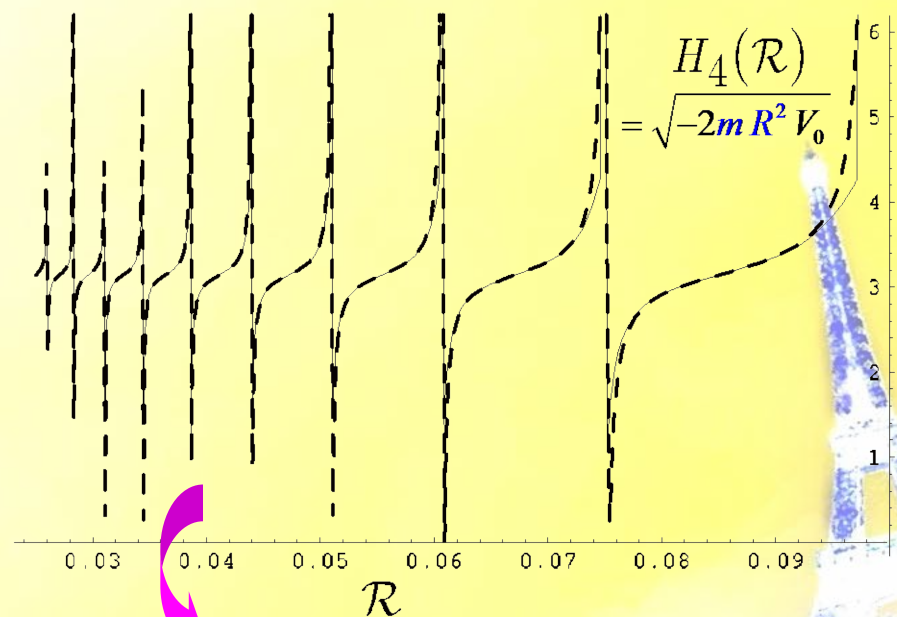
➡ model dependence

determined by low-energy data

$$F_n(\lambda, r_0, R) = \frac{n}{4} - \sqrt{\lambda}\left(\frac{R}{r_0}\right)^{1-n/2} \tan\left(\frac{\sqrt{\lambda}}{(n/2-1)(R/r_0)^{n/2-1}} + \delta_n\right) + \ldots$$

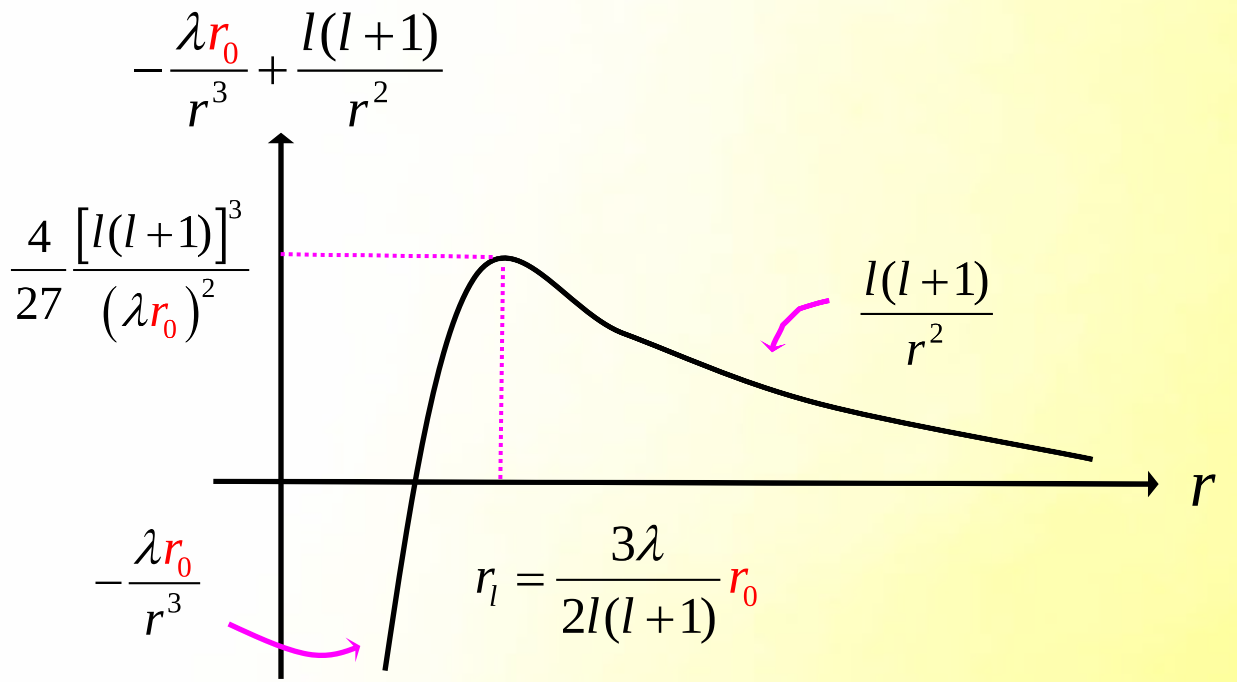


exact vs perturbation th



limit-cycle-like behavior

Same is true in all channels where attractive singular potential is iterated



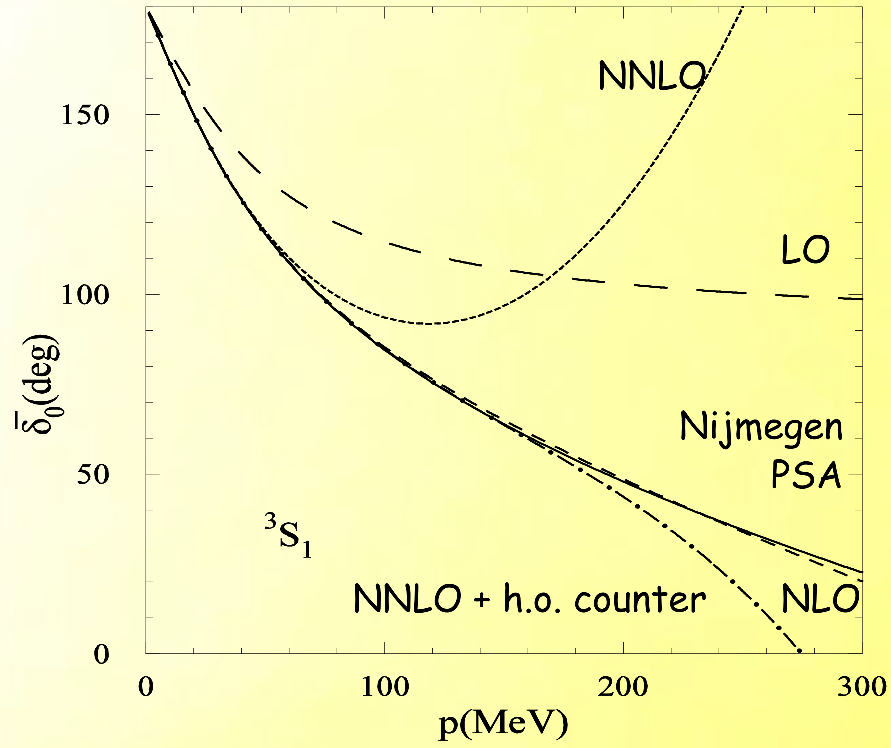
but $\quad r_l \sim \frac{1}{M} \ll r_0 \quad$ for $\quad l(l+1) \gg \lambda$

➡ singular potential only needs to be iterated in a few waves, where counterterms are needed

"Perturbative pions" $\lambda = \frac{m_\pi}{M_{NN}} \ll 1$

Kaplan, Savage + Wise '98

Fleming, Mehen + Stewart '01



➡ $M_{NN} \sim f_\pi$ indeed

Non-peturbative pions

$$l(l+1) \lesssim \frac{3M_{QCD}}{2M_{NN}} \sim 5 \quad ➡ \quad l \lesssim 2$$

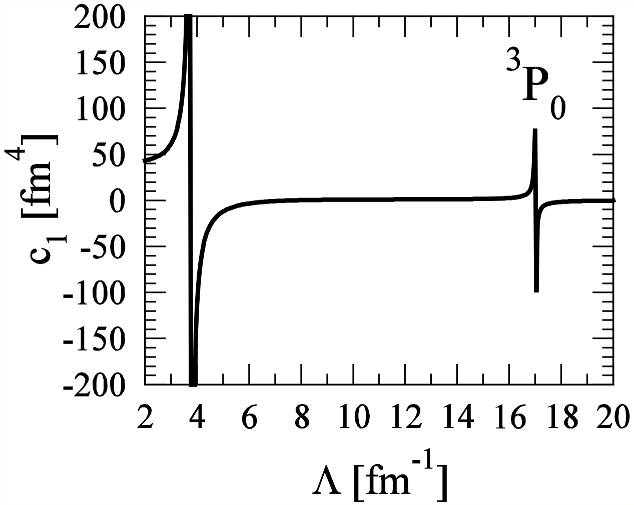
Beane, Bedaque, Savage + v.K. '02
Nogga, Timmermans + v.K. '05
Pavon Valderrama + Ruiz-Arriola '06
Birse, '06, '07
Long + v.K. '07
Pavon Valderrama '10
...

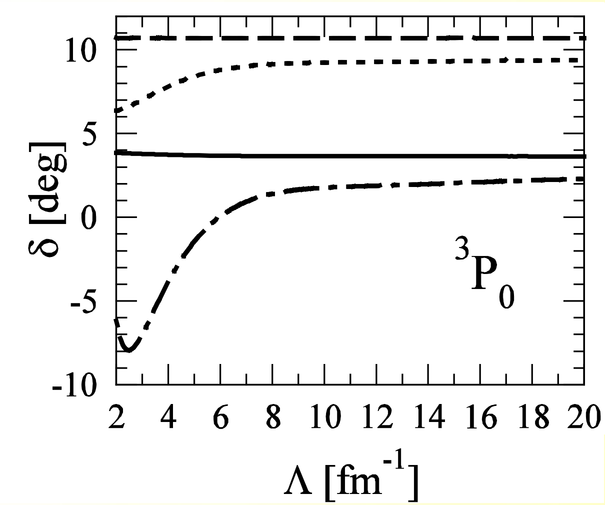
+ subleading orders: in perturbation theory, as in NRQED

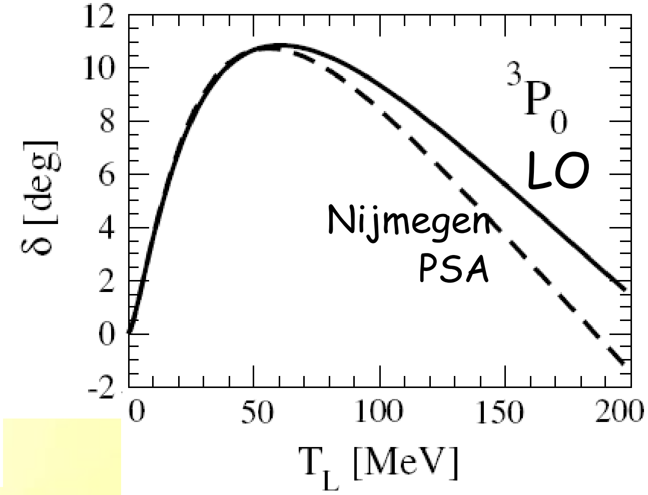
Nogga, Timmermans + v.K. '05
(cf. Pavon Valderrama + Ruiz-Arriola, '06)

Add counterterms

$$V_{l=1,\,j=0} = \frac{c_1}{(2\pi)^3}\, pp'$$



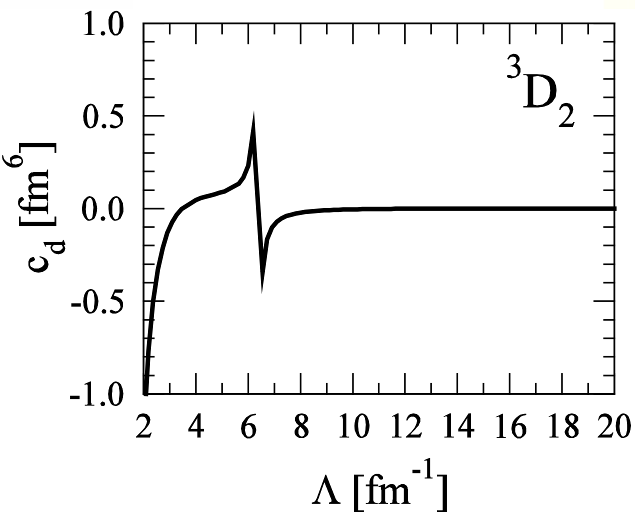


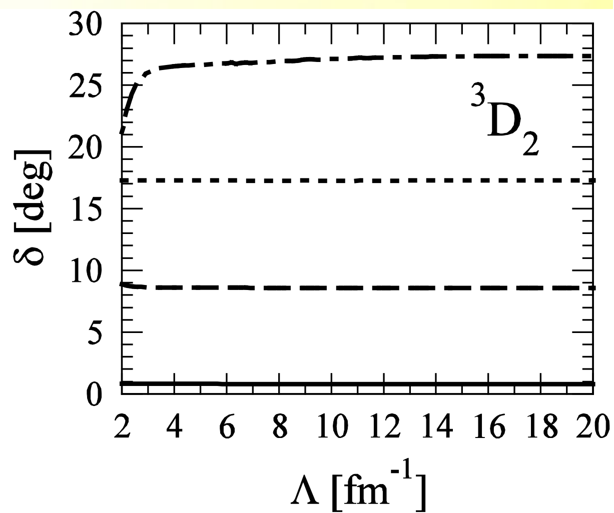


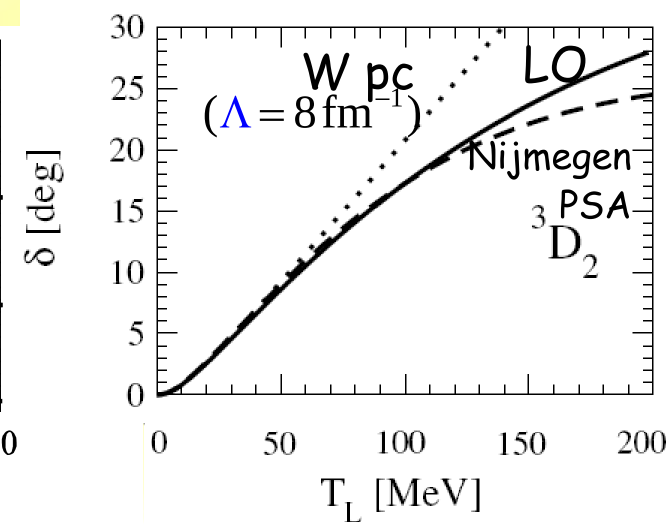
E (MeV)

10
50
100
190

$$V_{l=2,\,j=2} = \frac{c_d}{(2\pi)^3}\, p^2 p'^2$$







certain counterterms that in Weinberg's counting

were assumed suppressed by powers of $\frac{Q}{M_{QCD}}$

are in fact suppressed by powers of $\frac{Q}{lf_{\pi}}$

$\Downarrow$

short-range physics more important than assumed by Weinberg's;

most qualitative conclusions unchanged,

but quantitative results need improvement

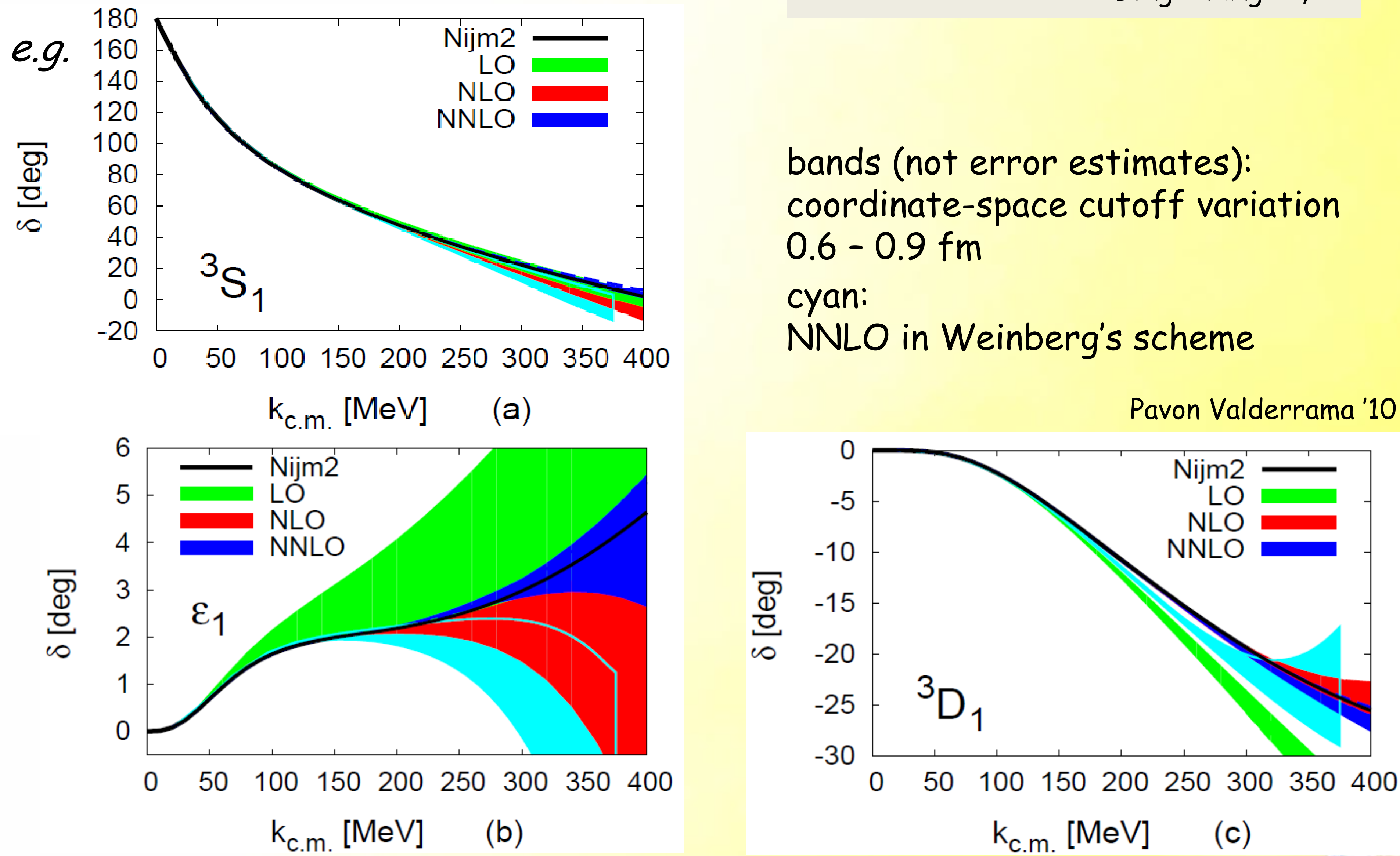
ACTIVE RESEARCH AREA

new PC

Fits to data

Pavon Valderrama '10, '11
Long + Yang '11, 12

e.g.



bands (not error estimates):
coordinate-space cutoff variation
0.6 – 0.9 fm

cyan:
NNLO in Weinberg's scheme

Pavon Valderrama '10

Summary

- A low-energy EFT of QCD has been constructed and used to describe nuclear systems
- Chiral symmetry plays an important role,
  in particular setting the scale for nuclear bound states
- Nuclear physics canons emerge from chiral potential
- A new power counting has been formulated:
  more counterterms at each order relative to Weinberg's;
  expect even better description of observables

Stay tuned:
next, how to extend EFT to larger systems