DOUBLE BETA DECAY AND NEUTRINO MASSES

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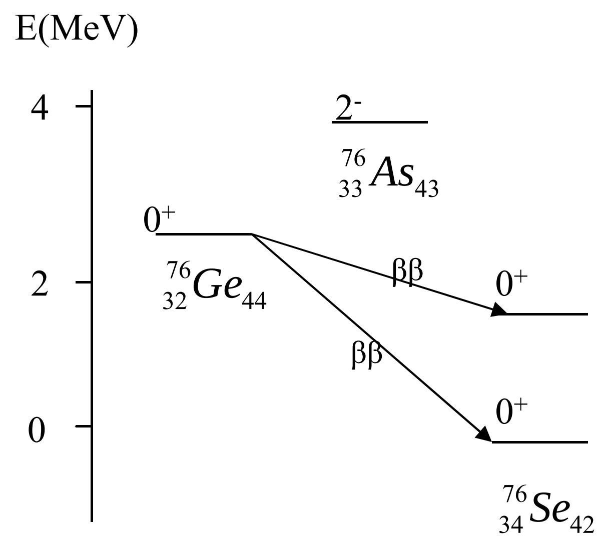
Lecture 3

EVALUATION OF PSF: 0νββ

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z\pm2}Y_{N\mp2} + 2e^{\mp} $$



Half-life for the process:

$$ \left[\tau_{1/2}^{0\nu\beta\beta}(0^+ \rightarrow 0^+)\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2 \left|f(m_i, U_{ei})\right|^2 $$

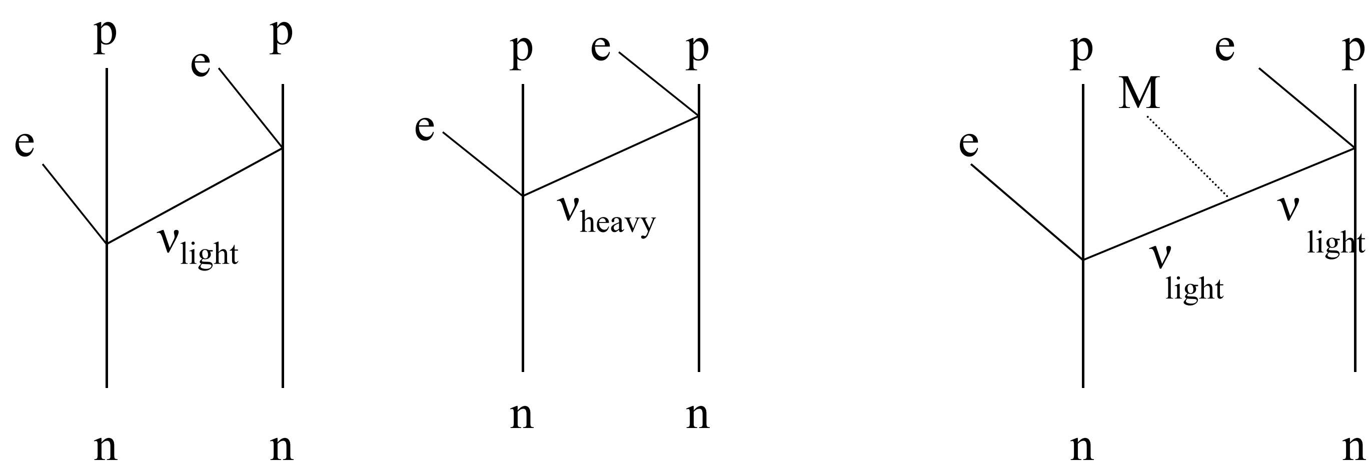
Phase-space factor
(Atomic physics)

Matrix elements
(Nuclear physics)

Beyond the standard model
(Particle physics)

Three scenarios have been considered [¶,§,#].

1 2 3



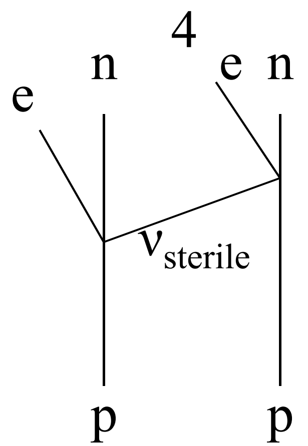
$M_{\nu light} \ll 1MeV$ $M_{\nu heavy} \gg 1GeV$

M. Doi *et al.*, Prog. Theor. Phys. 66, 1739 (`1981); 69, 602 (1983).

[¶] T.Tomoda, Rep. Prog. Phys. 54, 53 (1991).

[§] F.Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999).

In recent years, a fourth scenario is being considered



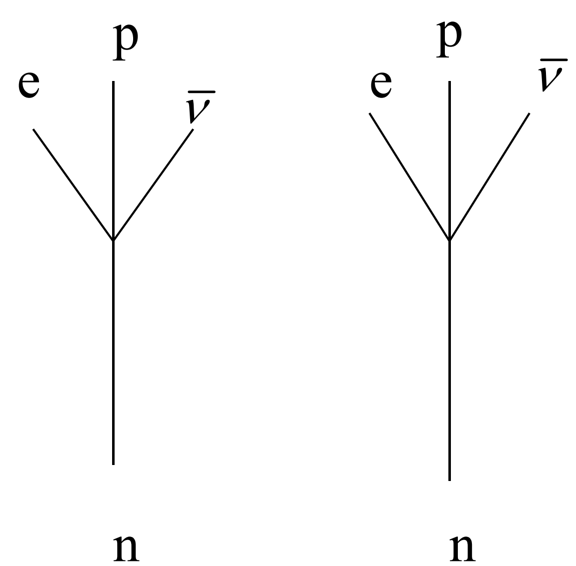
The phase space factors for scenarios 1,2 and 4 are all the same.

The phase space factor for Majoron emission is different and needs to be calculated separately.

Also the phase space factor for the process R0νββ needs separate attention.

EVALUATION OF PSF: 2νββ

The phase space factor for this process is different from 0νββ and needs to be calculated separately.



PHASE SPACE FACTORS (PSF)

PSF were calculated in the 1980's by Doi *et al.* [*]. Also, a calculation of phase-space factors is reported in the book of Boehm and Vogel [§]. These calculations use an approximate expression for the electron wave functions at the nucleus.

PSF have been recently recalculated [**] with exact Dirac electron wave functions and including screening by the electron cloud.

These new PSF are available from jenni.kotila@yale.edu and are on the webpage www.nucleartheory.yale.edu

[*] M. Doi, T. Kotani, N. Nishiura, K. Okuda and E. Takasugi, Prog. Theor. Phys. 66 (1981) 1739.

[§] F. Bohm and P. Vogel, *Physics of massive neutrinos*, Cambridge University Press, 1987.

[**] J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

ELECTRON DECAYS ¶

Two neutrino double electron decay ($2\nu\beta^-\beta^-$)

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}$$

Neutrinoless double beta decay ($0\nu\beta^-\beta^-$)

$$(A,Z) \rightarrow (A,Z+2) + 2e^-$$

¶ J. Kotila and F. Iachello, Phys. Rev. C85, 034316 (2012).

The wave functions are obtained by solving numerically ¶ the Dirac equation with potential

$$V(r) = \begin{bmatrix} -\frac{Z_d}{r} \\ -Z_d\left(\frac{3-(r/R)^2}{2R}\right) \end{bmatrix} \times \varphi(r) \qquad \begin{matrix} r \geq R \\ r < R \end{matrix}$$

d=daughter nucleus

The function φ(r) is obtained numerically § by solving the Thomas-Fermi equation

$$\frac{d^2\varphi}{dx^2} = \frac{\varphi^{3/2}}{\sqrt{x}} \qquad x = r/b \qquad b \simeq 0.885 a_0 Z_d^{-1/3}$$

with boundary conditions (final nucleus positive ion with charge +2)

$$\varphi(0) = 1 \qquad \varphi(\infty) = \frac{2}{Z_d}$$

¶ F. Salvat, J.M. Fernadez-Varea, and W. Williamson Jr., Comp. Phys. Comm. 90 (1995) 151.

§ S. Esposito, Am. J. Phys. 70 (2002) 852. Method of solution suggested by Ettore Majorana.

The wave functions are positive energy Dirac central field wave functions

$$\psi_{\varepsilon\kappa\mu}(\vec{r}) = \begin{pmatrix} g_\kappa(\varepsilon, r)\chi_\kappa^\mu \\ if_\kappa(\varepsilon, r)\chi_{-\kappa}^\mu \end{pmatrix}$$

where χ are spherical spinors and g, f radial functions with energy ε depending on κ

$$\kappa = (l - j)(2j + 1)$$

These functions satisfy

$$\frac{dg_\kappa(\varepsilon, r)}{dr} = -\frac{\kappa}{r} g_\kappa(\varepsilon, r) + \frac{\varepsilon - V + m_e c^2}{c\hbar} f_\kappa(\varepsilon, r)$$

$$\frac{df_\kappa(\varepsilon, r)}{dr} = -\frac{\varepsilon - V - m_e c^2}{c\hbar} g_\kappa(\varepsilon, r) + \frac{\kappa}{r} f_\kappa(\varepsilon, r)$$

2νββ $0_1^+ \rightarrow 0_1^+$

The differential decay rate is given by

$$dW_{2\nu} = \left(a^{(0)} + a^{(1)} \cos\theta_{12}\right) w_{2\nu} d\omega_1 d\varepsilon_1 d\varepsilon_2 d\left(\cos\theta_{12}\right)$$

Neutrino energies (→ $d\omega_1$) Electron energies (→ $d\varepsilon_2$)

Fermi constant (→ G) Cabibbo angle (→ θ_C)

$$w_{2\nu} = \frac{g_A^4 \left(G\cos\theta_C\right)^4}{64\pi^4\hbar}\omega_1^2\omega_2^2(p_1c)(p_2c)\varepsilon_1\varepsilon_2$$

Through a complicated derivation, one can obtain the phase space factors

$G^{(0)}_{2\nu}$ $G^{(1)}_{2\nu}$ [units yr^{-1}]

From these, one has, with

$$N_{2\nu} = g_A^4 \left| m_e c^2 M^{2\nu} \right|^2 = \left| M_{2\nu} \right|^2$$

← NME dimensionless

Half-life

$$\left[\tau^{2\nu}_{1/2} \right]^{-1} = N_{2\nu} G^{(0)}_{2\nu}$$

Differential decay rate

$$\frac{dW_{2\nu}}{d\varepsilon_1} = N_{2\nu} \frac{dG_{2\nu}^{(0)}}{d\varepsilon_1}$$

Summed energy spectrum of the two electrons

$$\frac{dW_{2\nu}}{d\left(\varepsilon_1 + \varepsilon_2 - 2m_e c^2\right)} = N_{2\nu} \frac{dG_{2\nu}^{(0)}}{d\left(\varepsilon_1 + \varepsilon_2 - 2m_e c^2\right)}$$

Angular correlation between the two electrons

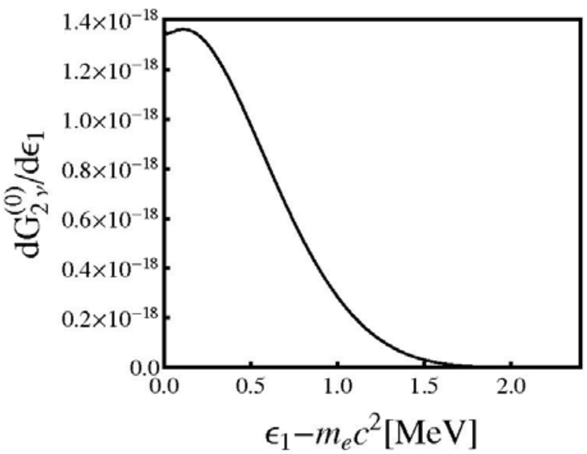
$$\alpha(\varepsilon_1) = \frac{dG_{2\nu}^{(1)} / d\varepsilon_1}{dG_{2\nu}^{(0)} / d\varepsilon_1}$$

Double differential rate

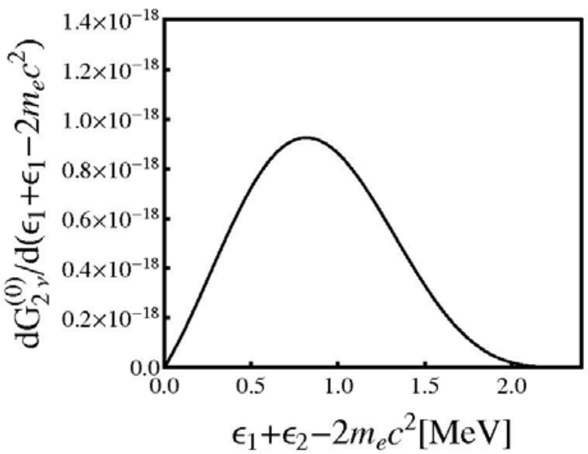
$$\frac{dW_{2\nu}}{d\varepsilon_1 d\left(\cos\theta_{12}\right)} = N_{2\nu} \frac{dG_{2\nu}^{(0)}}{d\varepsilon_1}\left[1 + \alpha(\varepsilon_1)\cos\theta_{12}\right]$$

Example $^{136}Xe \rightarrow {}^{136}Ba$ (EXO)

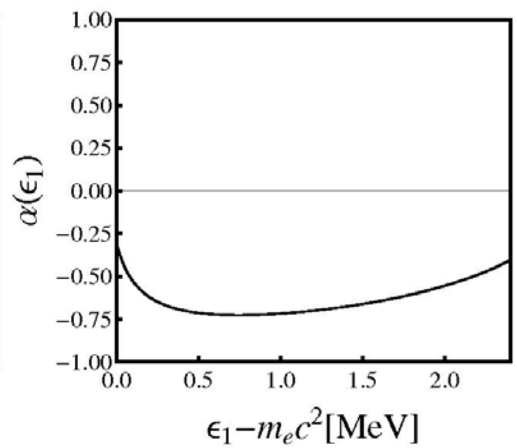
Differential rate



Summed electron spectrum

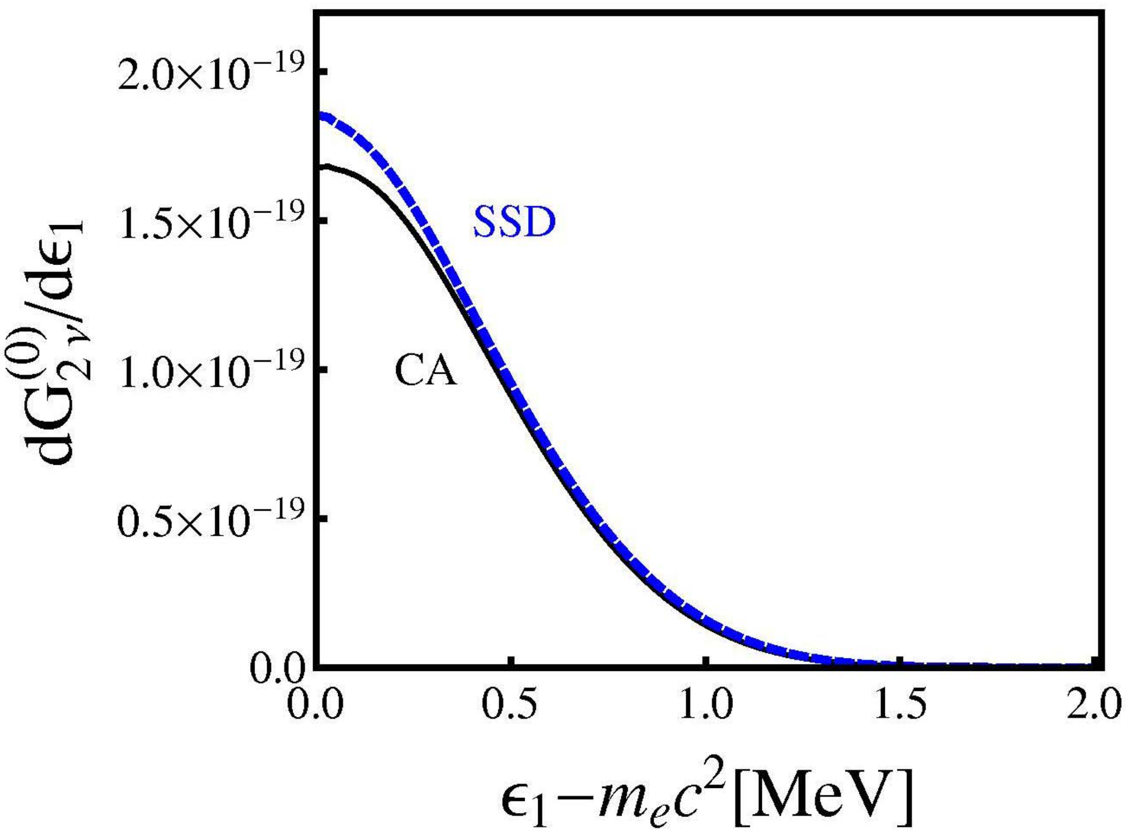


Angular correlation



If accurately measured one can distinguish between single state dominance (SSD) and closure approximation (CA).

Example: $^{110}Pd \rightarrow ^{110}Cd$ (COBRA)



$0\nu\beta\beta$ $\quad 0_1^+ \rightarrow 0_1^+$

The differential rate is given by

$$dW_{0\nu} = \left(a^{(0)} + a^{(1)} \cos\theta_{12}\right) w_{0\nu} d\varepsilon_1 d(\cos\theta_{12})$$

where

$$w_{0\nu} = \frac{g_A^4 \left(G\cos\theta_C\right)^4}{16\pi^5} (m_e c^2)^2 (\hbar c^2)(p_1 c)(p_2 c)\varepsilon_1 \varepsilon_2$$

Again from a complicated derivation one obtains the PSF

$$G_{0\nu}^{(0)} \qquad G_{0\nu}^{(1)} \qquad [\text{Units yr}^{-1}]$$

From these one has, with

$$f = \frac{\langle m_\nu \rangle}{m_e}$$ ← Light neutrino exchange

$$f_h = \frac{m_p}{\langle m_{\nu_h} \rangle}$$ ← Heavy neutrino exchange

$$N_{0\nu} = g_A^4 \left| M^{0\nu} \right|^2 \left| f \right|^2$$

$$g_A^4 \left| M^{0\nu} \right|^2 = \left| M_{0\nu} \right|^2$$ ← NME

Half-life

$$\left[\tau_{1/2}^{0\nu} \right]^{-1} = N_{0\nu} G_{0\nu}^{(0)}$$

Single electron spectrum

$$\frac{dW_{0\nu}}{d\varepsilon_1} = N_{0\nu} \frac{dG_{0\nu}^{(0)}}{d\varepsilon_1}$$

Angular correlation between the two electrons

$$\alpha(\varepsilon_1) = \frac{dG_{0\nu}^{(1)} / d\varepsilon_1}{dG_{0\nu}^{(0)} / d\varepsilon_1}$$

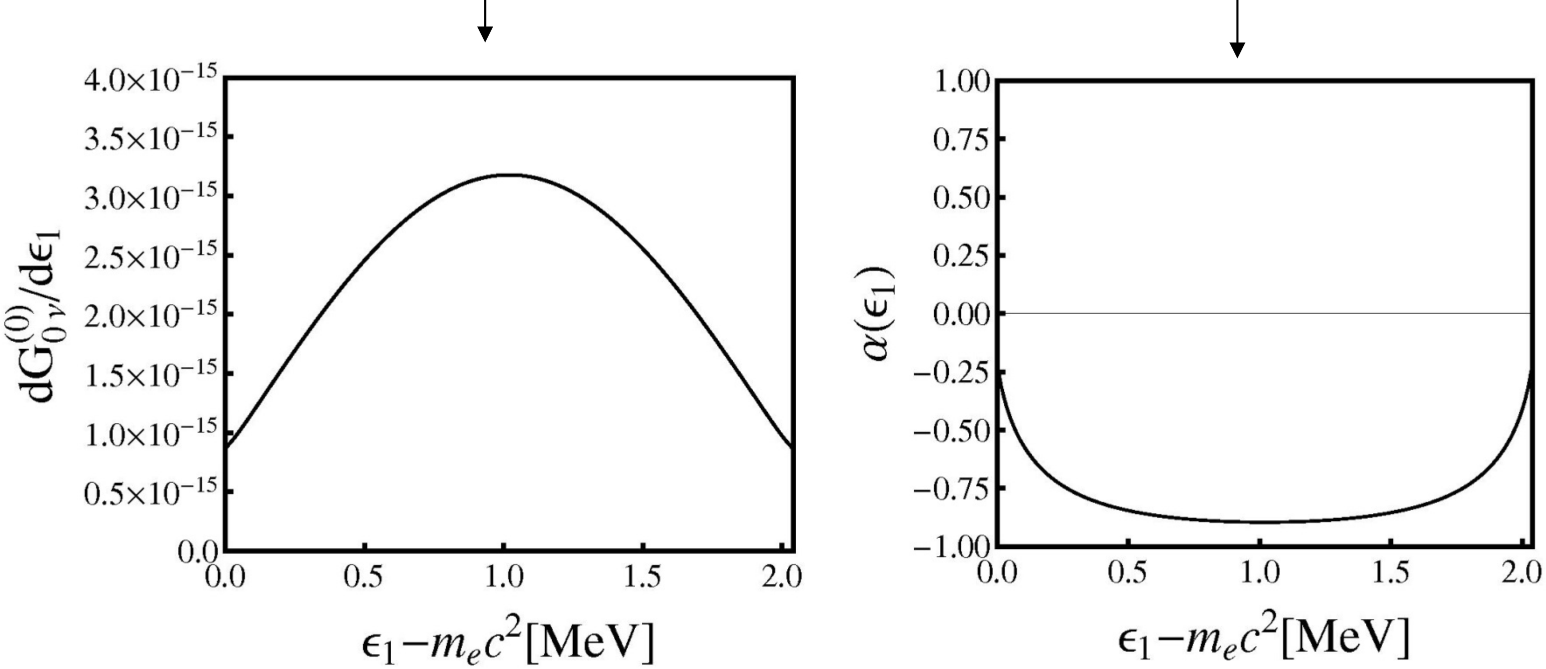
Double differential rate

$$\frac{dW_{0\nu}}{d\varepsilon_1 d(\cos\theta_{12})} = N_{0\nu} \frac{dG_{0\nu}^{(0)}}{d\varepsilon_1}\left[1 + \alpha(\varepsilon_1)\cos\theta_{12}\right]$$

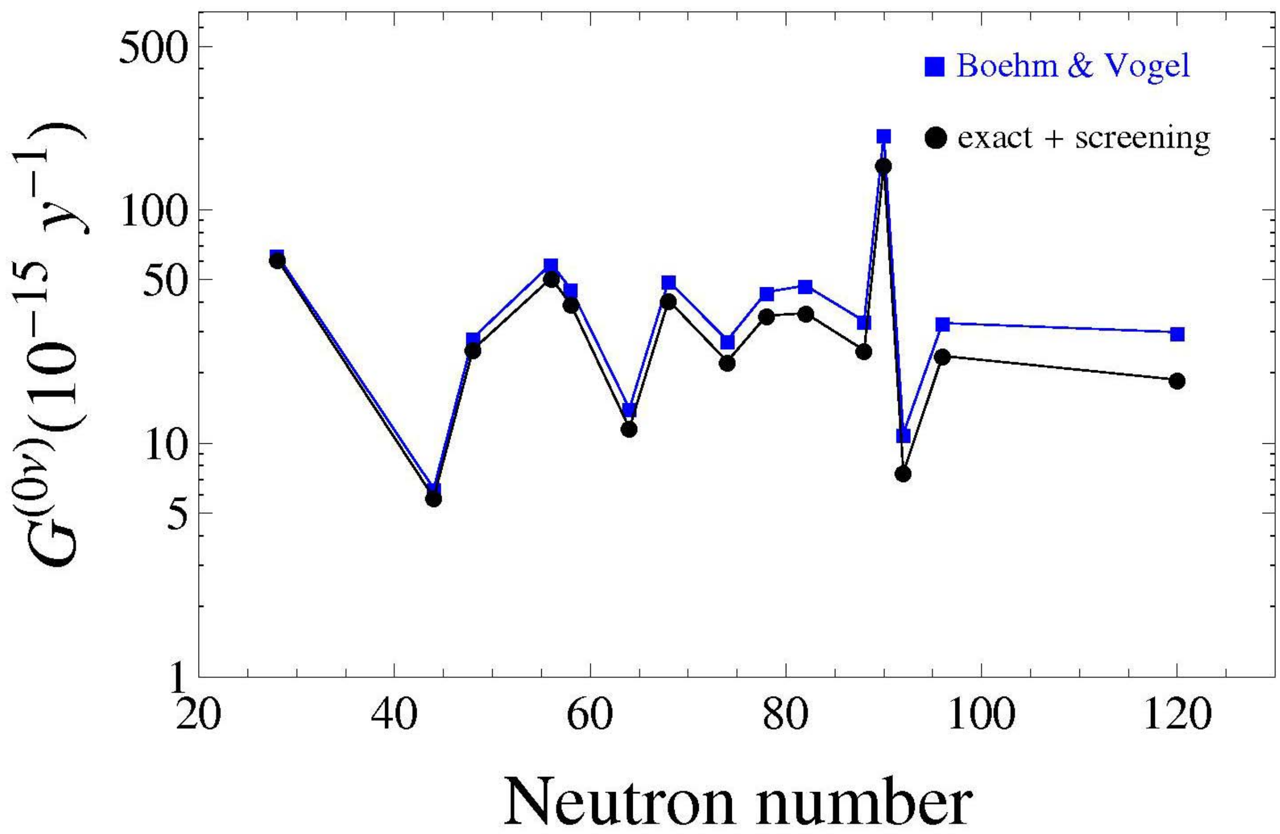
Example $^{76}Ge \rightarrow {}^{76}Se$ (GERDA)

Single electron spectrum

Angular correlation



Comparison between approximate § and exact + screening ¶ phase space factors



§ F. Böhm and P. Vogel, *loc. cit.*

¶ J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

POSITRON DECAYS ¶

Two neutrino double-positron decay ($2\nu\beta^+\beta^+$)

$$\left(A,Z\right)\rightarrow\left(A,Z-2\right)+2e^{+}+2\nu$$

Positron emitting two neutrino electron capture ($2\nu\beta^+EC$)

$$\left(A,Z\right)+e^{-}\rightarrow\left(A,Z-2\right)+e^{+}+2\nu$$

Two neutrino double electron capture ($2\nu ECEC$)

$$\left(A,Z\right)+2e^{-}\rightarrow\left(A,Z-2\right)+2\nu$$

Neutrinoless double positron decay ($0\nu\beta^+\beta^+$)

$$\left(A,Z\right)\rightarrow\left(A,Z-2\right)+2e^{+}$$

Positron emitting neutrinoless electron capture ($0\nu\beta^+EC$)

$$\left(A,Z\right)+e^{-}\rightarrow\left(A,Z-2\right)+e^{+}$$

¶ J. Kotila and F. Iachello, Phys. Rev. C87, 024313 (2013).

Wave functions

β^+ decay

Negative energy Dirac central field scattering state wave functions

$$\psi_{\varepsilon\kappa\mu}(\vec{r}) = \begin{pmatrix} i f_\kappa(\varepsilon, r)\chi_{-\kappa}^{-\mu} \\ -g_\kappa(\varepsilon, r)\chi_{\kappa}^{-\mu} \end{pmatrix}$$

Where χ are spherical spinors and f, g are radial functions, with energy ε, depending on the relativistic quantum number κ

$$\kappa = (\ell - j)(2j+1)$$

[Same as for β^- decay but for a change in sign of the Sommerfield parameter

$$\eta = Ze^2 / \hbar v \qquad].$$

Electron Capture EC

Positive energy Dirac central field bound state wave functions

$$\psi_{n'\kappa\mu}(\vec{r}) = \begin{pmatrix} g^b_{n',\kappa}(r)\chi^\mu_\kappa \\ if^b_{n',\kappa}(r)\chi^\mu_{-\kappa} \end{pmatrix}$$

Where n' denotes the radial quantum number and the quantum number κ is related to the total angular momentum

$$j_\kappa = |\kappa| - 1/2$$

K-shell electrons $n' = 0, \kappa = -1, 1S_{1/2}$

L_I–shell electrons $n' = 1, \kappa = -1, 2S_{1/2}$

L_{II} and L_{III} capture neglected because it is suppressed by the nonzero angular momentum

Obtained numerically as in $\beta^- \beta^-$ decay but with potentials

$$V(r) = \begin{bmatrix} \pm \dfrac{Z_i(\alpha\hbar c)}{r} \\ \pm Z_i(\alpha\hbar c)\left(\dfrac{3-(r/R)^2}{2R}\right) \end{bmatrix} \times \varphi(r) \qquad \begin{matrix} r \geq R \\ r < R \end{matrix}$$

i=d,m

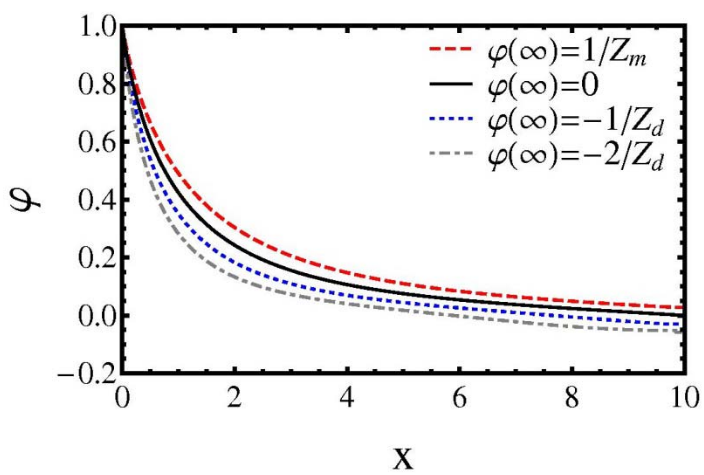
d=daughter nucleus
m=mother nucleus

Thomas-Fermi equation with boundary conditions

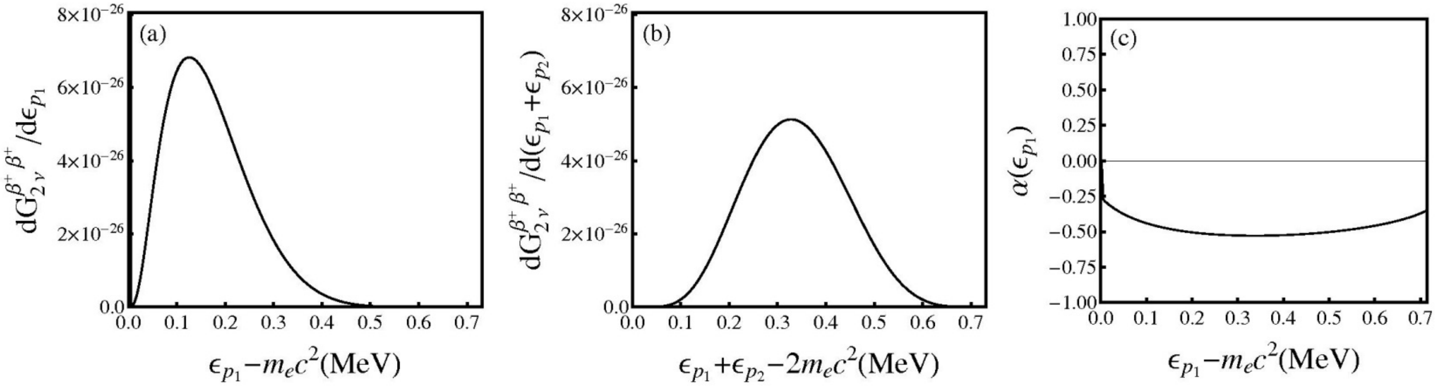
$$\varphi(0) = 1, \varphi(\infty) = -\frac{2}{Z_d} \qquad (\beta^+\beta^+)$$

$$\varphi(0) = 1, \varphi(\infty) = \frac{1}{Z_m} \qquad (EC)$$

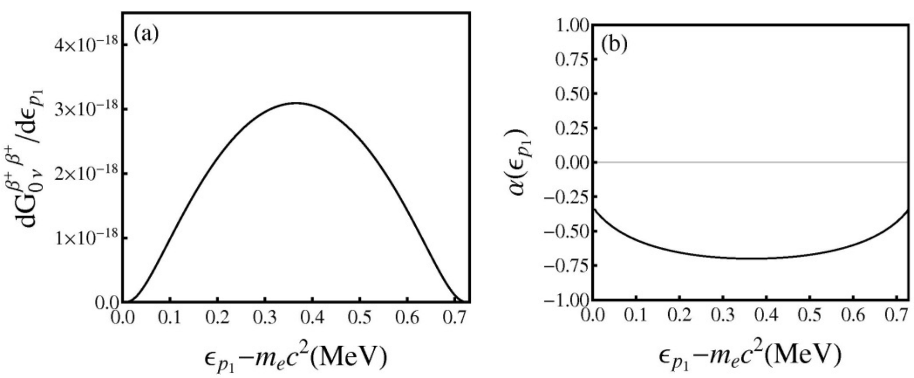
$$\varphi(0) = 1, \varphi(\infty) = -\frac{1}{Z_d} \qquad (\beta^+)$$



Results $2\nu\beta^+\beta^+$:



Results $0\nu\beta^+\beta^+$.



RESONANT ECEC: R0νECEC ¶

$$\left(A,Z\right)+2e^{-}\rightarrow\left(A,Z-2\right)$$

$$\left[\tau_{1/2}^{ECEC}\right]^{-1}=g_A^4 G_{0\nu}^{ECEC}\left|M_{ECEC}^{0\nu}\right|^2\left|f\left(m_i,U_{ei}\right)\right|^2\frac{\left(m_e c^2\right)\Gamma}{\Delta^2+\Gamma^2/4}$$

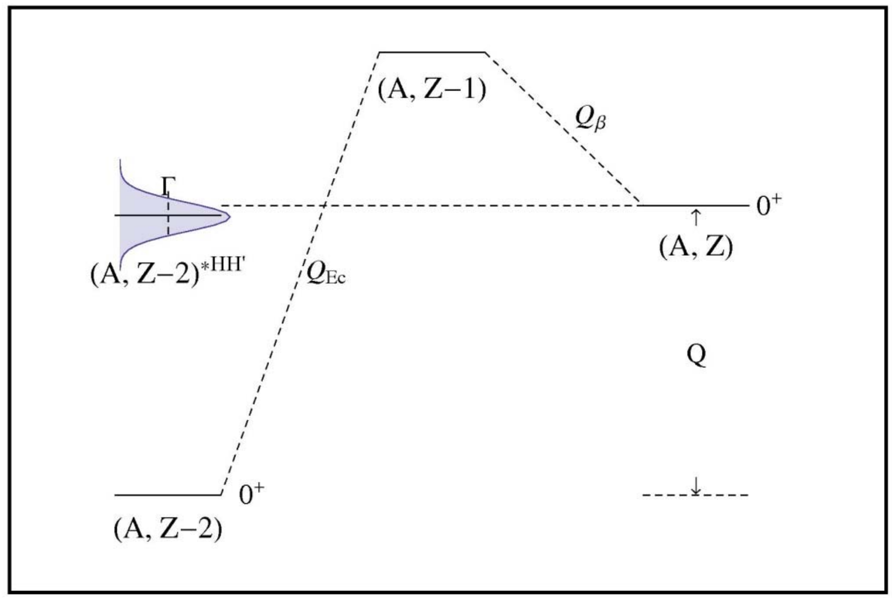
Degeneracy parameter

Energy of double electron hole §

$$\Delta=\left|Q-B_{2h}-E\right|$$

Two-hole width

$$\Gamma=\Gamma_{e_1}+\Gamma_{e_2}$$



¶ J. Kotila, J. Barea and F. Iachello, Phys. Rev. C89, 064319 (2014)

§ N.I. Kivoruchenko, F. Šimkovic, D. Frekers, and A. Faessler, Nucl. Phys. A859, 140 (2011).

PREFACTOR

$$PF \equiv G_{0\nu}^{ECEC}$$

Wave functions

Positive energy Dirac central field bound state wave functions

$$\psi_{n'\kappa\mu}(\vec{r}) = \begin{pmatrix} g^{b}_{n',\kappa}(r)\chi^{\mu}_{\kappa} \\ if^{b}_{n',\kappa}(r)\chi^{\mu}_{-\kappa} \end{pmatrix}$$

Probability that an electron is found at the nucleus

$$B_{n',\kappa}^2 = \frac{1}{4\pi\left(m_e c^2\right)^3}\left(\frac{\hbar c}{a_0}\right)^3\left(\frac{a_0}{R}\right)^2\left[\left(g_{n',\kappa}^b(R)\right)^2 + \left(f_{n',\kappa}^b(R)\right)^2\right]$$

K-capture $\quad n' = 0, \kappa = -1, 1S_{1/2}$

L_I-capture $\quad n' = 1, \kappa = -1, 2S_{1/2}$

The PF is given by

$$G_{0\nu}^{ECEC} = \frac{1}{4R^2 \ln 2}\frac{\left(G\cos\theta_C\right)^4}{2\pi^2}(\hbar c)^2(m_e c^2)^7 B_{n'_{e_1}-1}^2 B_{n'_{e_2}-1}^2$$

Resonant enhancement factors

TABLE I. Double electron captures considered, the Q-value of the decay, energy of the resonant state in the daughter nucleus, capture shells, two-hole width, and resonance enhancement factor.

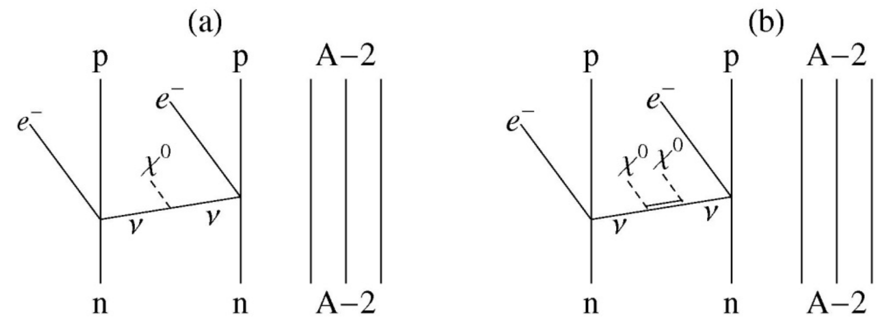
Decay	Q-value(keV)	E(keV)	$Q-E$(keV)	Shells	$E_{b_1+b_2}$(keV)[f]	Δ(keV)	Γ(keV)[g]	$(m_e c^2)$F
${}^{124}_{54}\mathrm{Xe}_{70} \to {}^{124}_{52}\mathrm{Te}^*_{72}$	2856.73 ± 0.12[a]	2790.41 ± 0.09	66.32	$K-K$	64.46	1.86	0.0198	1.40
${}^{152}_{64}\mathrm{Gd}_{88} \to {}^{152}_{62}\mathrm{Sm}_{90}$	55.70 ± 0.18 [b]	0.0	55.70	$K-L_1$	54.57	1.13	0.023	9.26
${}^{156}_{66}\mathrm{Dy}_{90} \to {}^{156}_{64}\mathrm{Gd}^*_{92}$	2005.95 ± 0.10[c]	1988.5 ± 0.2	17.45	L_1-L_1	16.76	0.69	0.0076	7.97
${}^{164}_{68}\mathrm{Er}_{96} \to {}^{164}_{66}\mathrm{Dy}_{98}$	25.07 ± 0.12[d]	0.0	25.07	L_1-L_1	18.1	6.97	0.0086	0.090
${}^{180}_{74}\mathrm{W}_{106} \to {}^{180}_{72}\mathrm{Hf}_{108}$	143.20 ± 0.27[e]	0.0	143.20	$K-K$	130.7	12.50	0.072	0.234

[a] D. A. Nesterenko *et al.*, Phys. Rev. C **86**, 044313 (2012).
[b] S. Eliseev *et al.*, Phys. Rev. Lett. **106**, 052504 (2011).
[c] S. Eliseev *et al.*, Phys. Rev. C **84**, 012501(R) (2011).
[d] S. Eliseev *et al.*, Phys. Rev. Lett. **107**, 152501 (2011).
[e] C. Droese *et al.*, Nucl. Phys. A **875**, 1 (2012).
[f] F. B. Larkins, At. Data Nucl. Data Tables **20**, 313 (1977).
[g] J. L. Campbell and T. Papp, At. Data Nucl. Data Tables **77**, 1 (2001).

MAJORON EMISSION ¶

$$\left(A,Z\right)\rightarrow\left(A,Z+2\right)+2e^{-}+\chi_0$$

$$\left(A,Z\right)\rightarrow\left(A,Z+2\right)+2e^{-}+2\chi_0$$



Differential rate

$$dW_{m\chi_0 n}=\left(a^{(0)}+a^{(1)}\cos\theta_{12}\right)w_{m\chi_0 n}d\varepsilon_1 d\varepsilon_2 d(\cos\theta_{12})$$

Through a complicated calculation one obtains the PSF

$$G^{(0)}_{m\chi_0 n},G^{(1)}_{m\chi_0 n}$$

From which one can obtain the half-lives

$$\left[\tau^{0\nu}_{1/2}\right]^{-1}=G^{(0)}_{m\chi_0 n}\left|\left\langle g_{\chi^M_{ee}}\right\rangle\right|^{2m}\left|M^{(m,n)}_{0\nu M}\right|^2$$

¶ J. Kotila, J. Barea and F. Iachello, Phys. Rev. C91, 064310 (2015).

Results

Nucleus	$G^{(0)}_{m\chi_0 n}(10^{-18}\ \mathrm{yr}^{-1})$			
	m=1,n=1	m=1,n=3	m=2,n=3	m=2,n=7
^{48}Ca	1540	17.1	73.6	690
^{76}Ge	44.2	0.073	0.22	0.420
^{82}Se	361	1.22	3.54	26.9
^{96}Zr	905	4.21	11.0	128.
^{100}Mo	598	2.42	6.15	50.8
^{110}Pd	94.1	0.205	0.487	0.946
^{116}Cd	569	2.28	5.23	33.9
^{124}Sn	209	0.653	1.45	4.45
^{128}Te	3.06	0.001	0.003	0.0003
^{130}Te	413	1.51	3.21	14.4
^{134}Xe	2.92	0.002	0.003	0.0002
^{136}Xe	409	1.47	3.05	12.5
^{148}Nd	197	0.505	0.986	1.72
^{150}Nd	3100	21.1	40.8	538
^{154}Sm	28.2	0.034	0.064	0.021
^{160}Gd	1590	0.361	0.672	0.899
^{198}Pt	60.7	0.068	0.110	0.021
^{232}Th	82.4	0.073	0.105	0.009
^{238}U	337	0.532	0.756	0.213

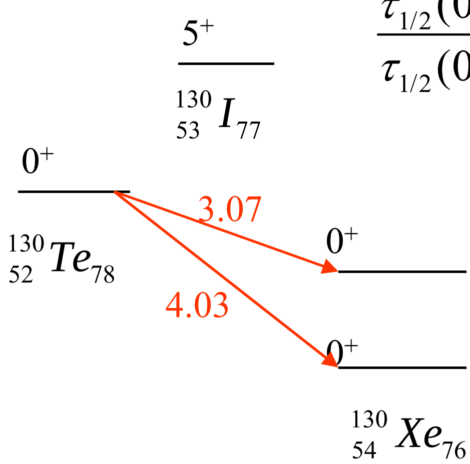
EXCITED STATES

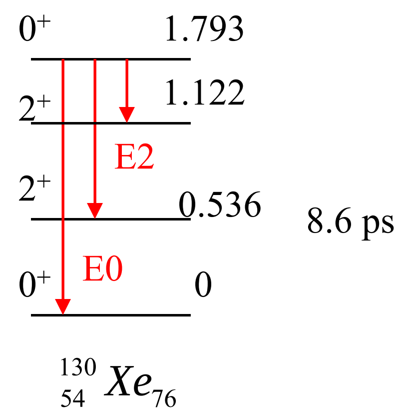
Calculations of PSF for 0^+_2 states have also been completed for all processes described previously.

In some cases, the matrix elements to the first excited 0^+ state are large. Although the kinematical factor hinders the decay to the excited state, large matrix elements offer the possibility of a direct detection, by looking at the γ-ray de-exciting the 0^+ level.

[On the contrary, matrix elements 0ν to the excited 2^+ state are zero in lowest order since with two leptons in the final state we cannot form angular momentum 2.]

$$\frac{\tau_{1/2}(0^+ \rightarrow 0^+_2)}{\tau_{1/2}(0^+ \rightarrow 0^+_1)} = 79.4$$

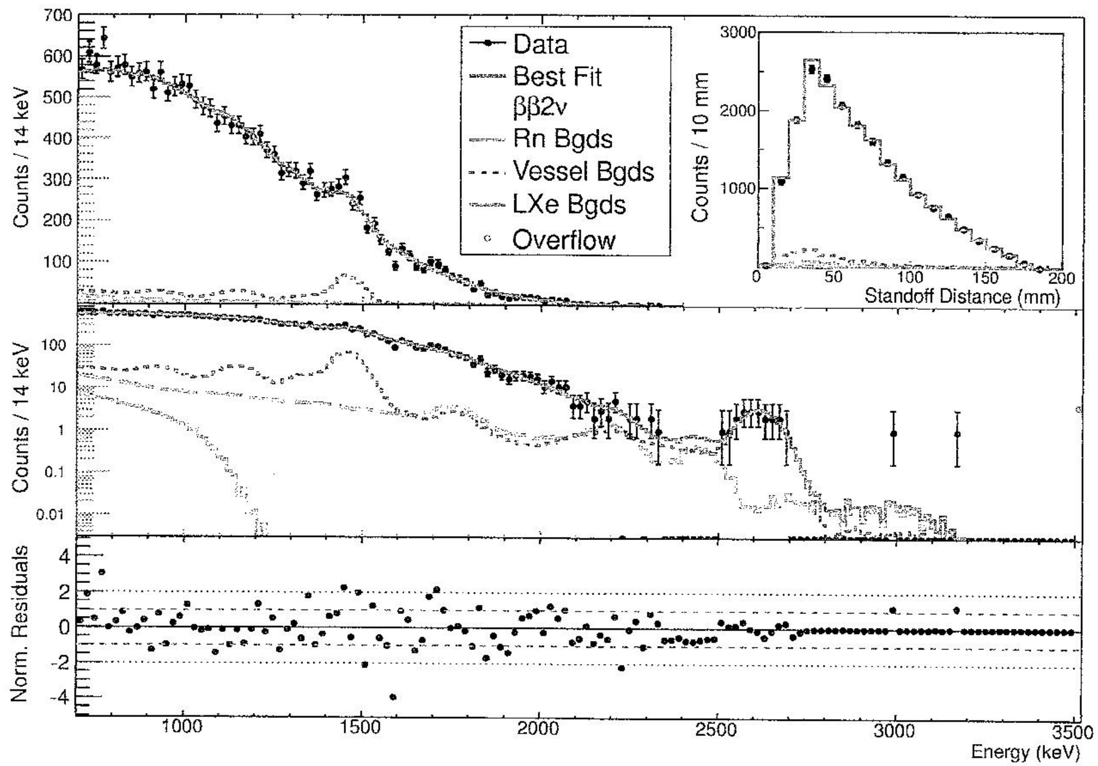




USE OF PSF

Simulations of expected spectra and fit to observed spectra
EXO
GERDA
SUPERNEMO
CUORE

J.B. Albert *et al*.
(EXO-200)



[For simulations, one may also need the
Triple differential rate

$$\frac{d^3W_{2\nu}}{d\varepsilon_1 d\varepsilon_2 d(\cos\theta_{12})} = N_{2\nu}\left[\frac{d^2G_{2\nu}^{(0)}}{d\varepsilon_1 d\varepsilon_2} + \frac{d^2G_{2\nu}^{(1)}}{d\varepsilon_1 d\varepsilon_2}\cos\theta_{12}\right] \equiv T\left(\varepsilon_1, \varepsilon_2, \theta_{12}\right)$$

This was not reported in KI but can be extracted easily from the program and is now (2015) available.]

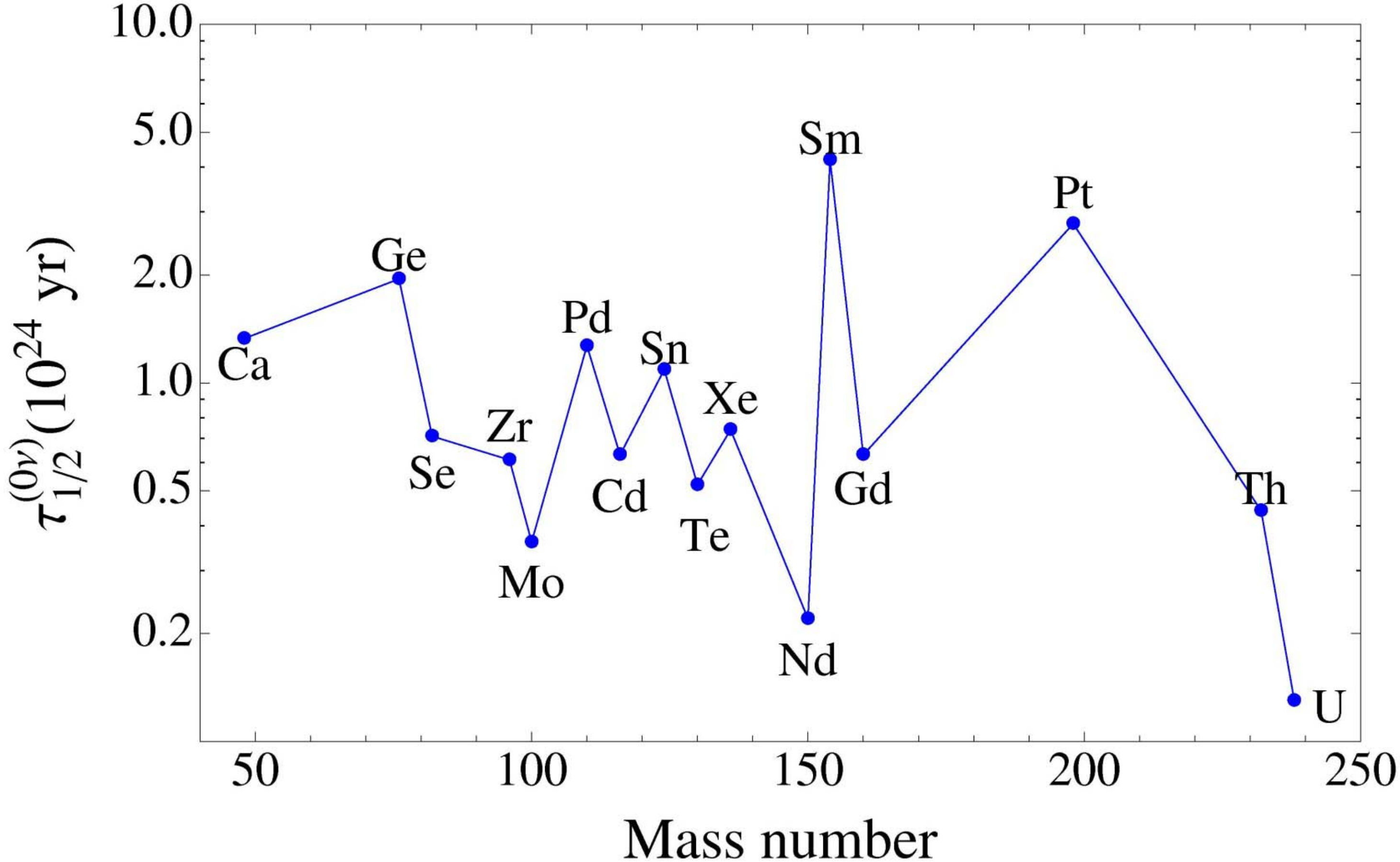
HALF-LIVES

From the calculated NME (lecture 2) and PSF (lecture 3), and using the formulas of lecture 1, one can calculate the expected half-lives for neutrinoless double beta decay, double positron decay and double resonant electron capture.

0νββ

$$\left[\tau_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^{2}\left|f(m_i, U_{ei})\right|^{2}$$

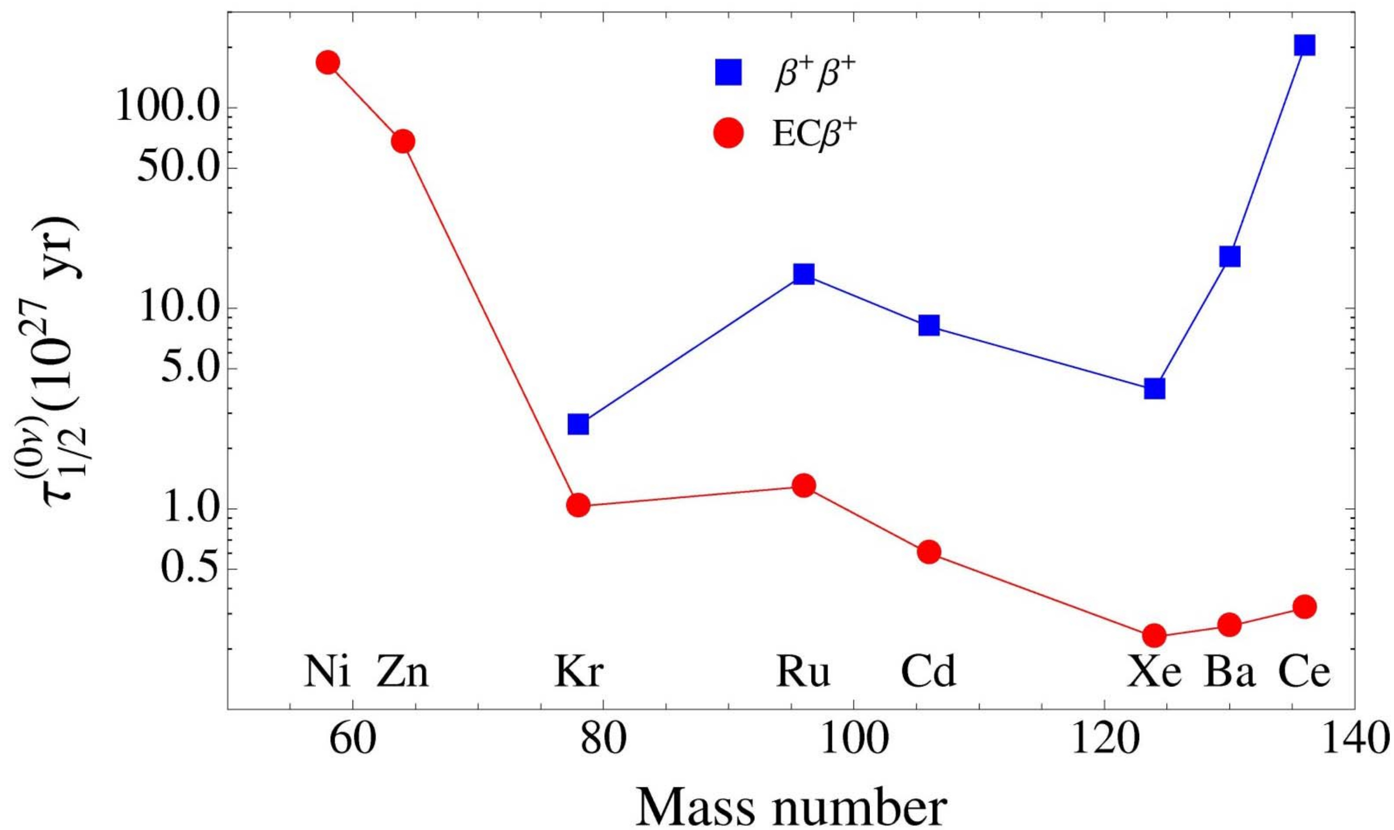
EXPECTED HALF-LIVES (2015) $0\nu\beta^-\beta^-$



$$\langle m_\nu \rangle = 1.0 eV$$

$$g_A = 1.269$$

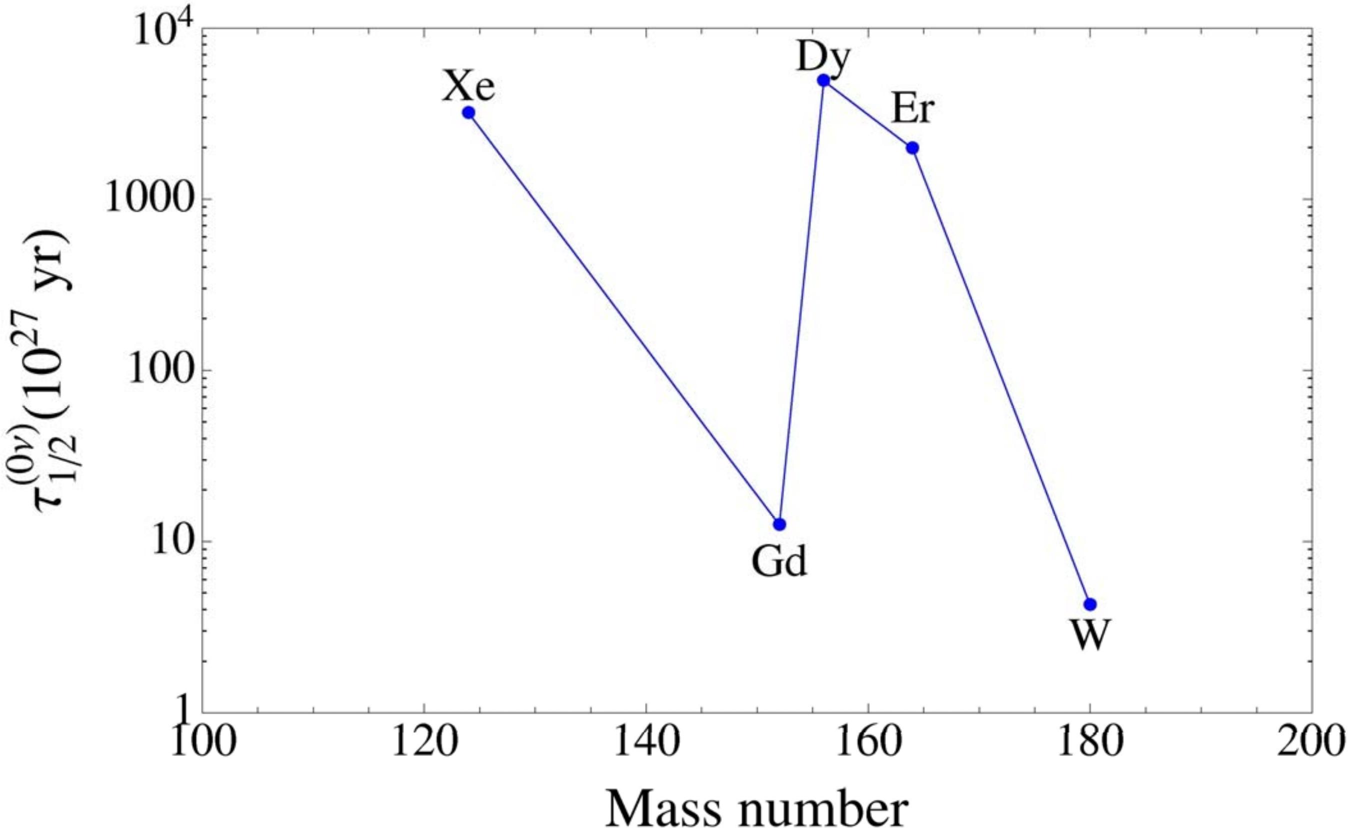
EXPECTED HALF-LIVES (2015) $0\nu\beta^+\beta^+/0\nu\beta^+EC$



$$\langle m_\nu \rangle = 1.0 eV$$

$$g_A = 1.269$$

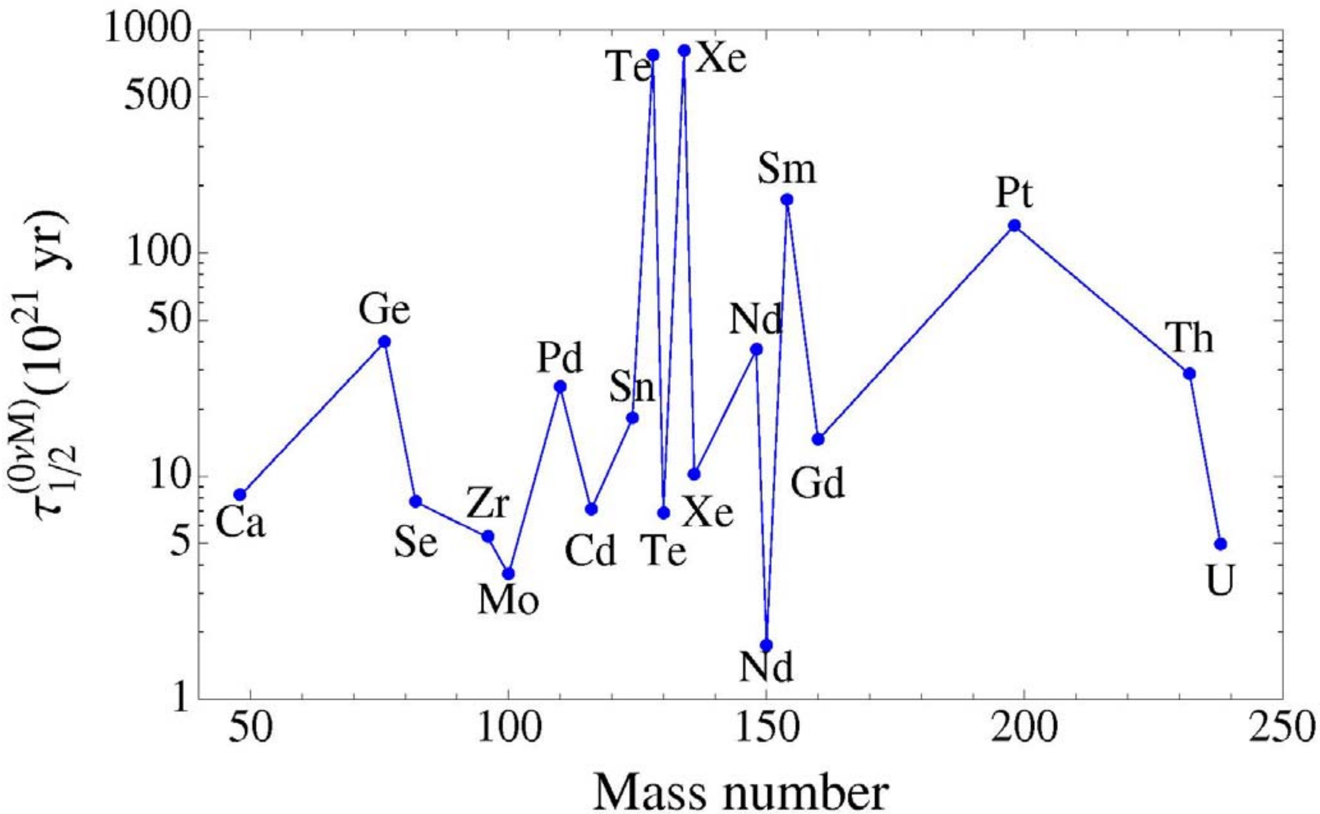
EXPECTED HALF-LIVES (2015) R0νECEC



$$\langle m_\nu \rangle = 1eV$$

$$g_A = 1.269$$

EXPECTED HALF-LIVES (2015) MAJORON EMISSION



$$\langle m_\nu \rangle = 1eV$$

$$g_A = 1.269$$

LIMITS ON NEUTRINO MASSES

From the expected half-lives and the experimental limits on them, one can set limits on neutrino masses.

LIMITS ON NEUTRINO MASSES (2015)

g_A=1.269

Decay	$\tau^{0\nu}_{1/2}(10^{24}\text{yr})$	$\tau^{0\nu}_{1/2,exp}(\text{yr})$	$\langle m_\nu \rangle$ (eV)
^{48}Ca→^{48}Ti	1.33	$> 5.8 \times 10^{22}$	< 4.8
^{76}Ge→^{76}Se	1.95	$> 1.9 \times 10^{25}$	< 0.32
		$1.2 \times 10^{25\,a}$	0.40
		$> 1.6 \times 10^{25\,b}$	< 0.35
		$> 2.1 \times 10^{25\,c}$	< 0.30
^{82}Se→^{82}Kr	0.71	$> 3.6 \times 10^{23}$	< 1.4
^{96}Zr→^{96}Mo	0.61	$> 9.2 \times 10^{21}$	< 8.1
^{100}Mo→^{100}Ru	0.36	$> 1.1 \times 10^{24}$	< 0.57
^{110}Pd→^{110}Cd	1.27		
^{116}Cd→^{116}Sn	0.63	$> 1.7 \times 10^{23}$	< 1.9
^{124}Sn→^{124}Te	1.09		
^{128}Te→^{128}Xe	10.19	$> 1.5 \times 10^{24}$	< 2.6
^{130}Te→^{130}Xe	0.52	$> 2.8 \times 10^{24}$	< 0.43
^{134}Xe→^{124}Ba	10.23		
^{136}Xe→^{136}Ba	0.74	$> 1.9 \times 10^{25\,d}$	< 0.20
		$> 1.6 \times 10^{25\,e}$	< 0.22
^{148}Nd→^{148}Sm	1.87		
^{150}Nd→^{150}Sm	0.22	$> 1.8 \times 10^{22}$	< 3.5
^{154}Sm→^{154}Gd	4.19		
^{160}Gd→^{160}Dy	0.63		
^{198}Pt→^{198}Hg	2.77		
^{232}Th→^{232}U	0.44		
^{238}U→^{238}Pu	0.13		

[a] H.V. Klapdor-Kleingrothaus *et al.*, Phys. Lett. B **586**, 198 (2004).
[b] C. E. Aalseth *et al.* (IGEX collaboration), Phys. Rev. D **65**, 092007 (2002).
[c] M. Agostini *et al.* (GERDA Collaboration), Phys. Rev. Lett. **111**, 122503 (2013).
[d] A. Gando *et al.* (KamLAND-Zen collaboration), Phys. Rev. Lett. **110**, 062502 (2013)
[e] M. Auger *et al.* (EXO collaboration) Phys. Rev. Lett. **109**, 032505 (2012).

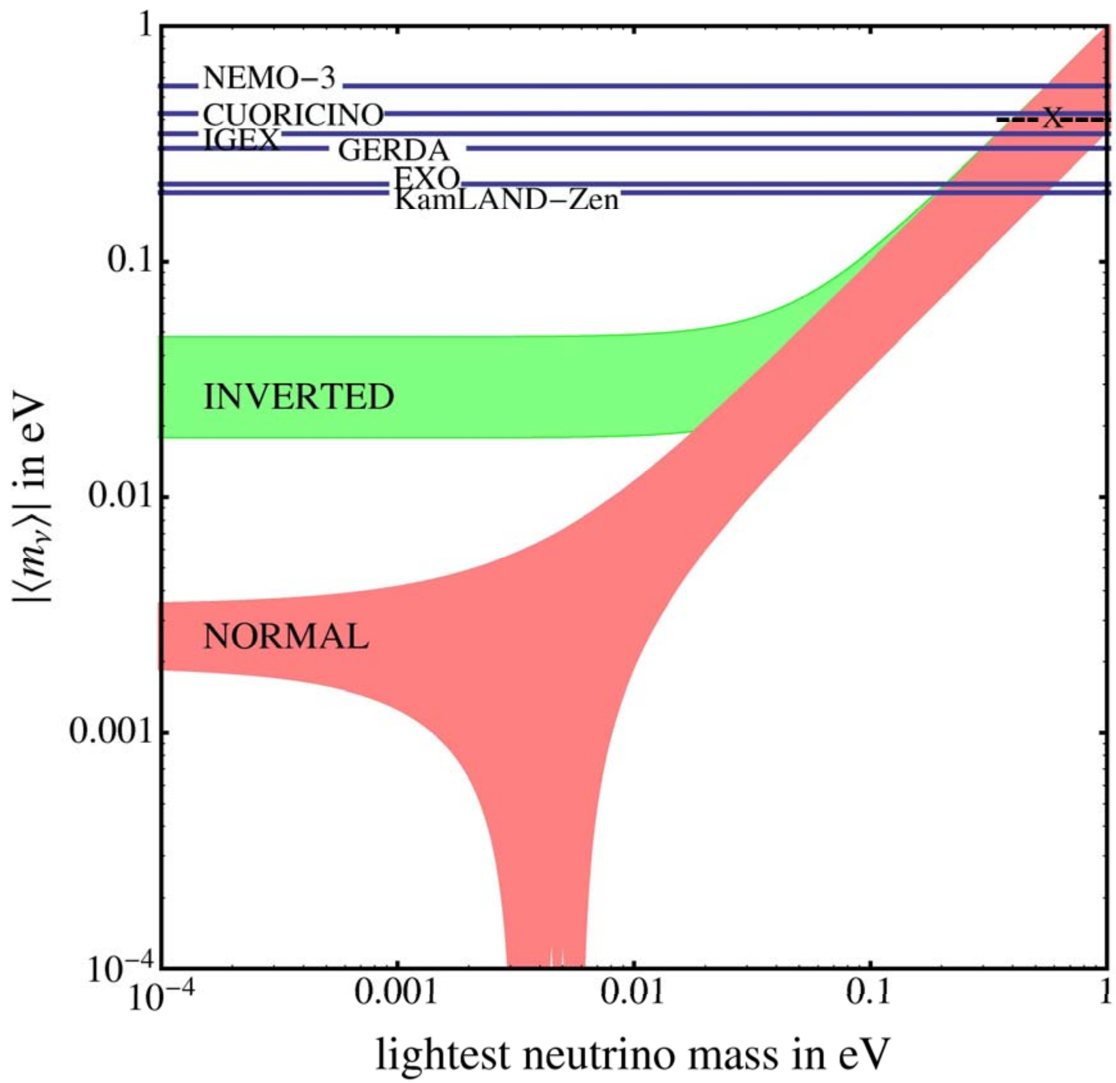
LIMITS ON NEUTRINO MASSES (HEAVY EXCHANGE) 2015

Decay	$\tau_{1/2}^{0\nu_h}(10^{24}\mathrm{yr})$	$\tau_{1/2,exp}^{0\nu_h}(\mathrm{yr})$	$\lvert\eta\rvert(10^{-6})$	$\langle m_{\nu_h}\rangle(\mathrm{GeV})$
$^{48}\mathrm{Ca}\rightarrow^{48}\mathrm{Ti}$	0.72	$> 5.8 \times 10^{22}$	<0.36	>11.9
$^{76}\mathrm{Ge}\rightarrow^{76}\mathrm{Se}$	1.51	$> 1.9 \times 10^{25}$	<0.028	>148
		$1.2 \times 10^{25\mathrm{a}}$	0.035	118
		$> 1.6 \times 10^{25\mathrm{b}}$	<0.031	>136
		$> 2.1 \times 10^{25\mathrm{c}}$	<0.027	>156
$^{82}\mathrm{Se}\rightarrow^{82}\mathrm{Kr}$	0.55	$> 3.6 \times 10^{23}$	<0.12	>34
$^{96}\mathrm{Zr}\rightarrow^{96}\mathrm{Mo}$	0.19	$> 9.2 \times 10^{21}$	<0.46	>9.15
$^{100}\mathrm{Mo}\rightarrow^{100}\mathrm{Ru}$	0.09	$> 1.1 \times 10^{24}$	<0.028	>146
$^{110}\mathrm{Pd}\rightarrow^{110}\mathrm{Cd}$	0.33			
$^{116}\mathrm{Cd}\rightarrow^{116}\mathrm{Sn}$	0.19	$> 1.7 \times 10^{23}$	<0.11	>39.5
$^{124}\mathrm{Sn}\rightarrow^{124}\mathrm{Te}$	0.67			
$^{128}\mathrm{Te}\rightarrow^{128}\mathrm{Xe}$	6.43	$> 1.5 \times 10^{24}$	<0.21	>20.2
$^{130}\mathrm{Te}\rightarrow^{130}\mathrm{Xe}$	0.32	$> 2.8 \times 10^{24}$	<0.034	>123
$^{134}\mathrm{Xe}\rightarrow^{134}\mathrm{Ba}$	8.57			
$^{136}\mathrm{Xe}\rightarrow^{136}\mathrm{Ba}$	0.50	$> 1.9 \times 10^{25\mathrm{d}}$	<0.016	>257
		$> 1.6 \times 10^{25\mathrm{e}}$	<0.018	>236
$^{148}\mathrm{Nd}\rightarrow^{148}\mathrm{Sm}$	0.36			
$^{150}\mathrm{Nd}\rightarrow^{150}\mathrm{Sm}$	0.05	$> 1.8 \times 10^{22}$	<0.16	>26.3
$^{154}\mathrm{Sm}\rightarrow^{154}\mathrm{Gd}$	1.00			
$^{160}\mathrm{Gd}\rightarrow^{160}\mathrm{Dy}$	0.17			
$^{198}\mathrm{Pt}\rightarrow^{198}\mathrm{Hg}$	0.48			
$^{232}\mathrm{Th}\rightarrow^{232}\mathrm{U}$	0.11			
$^{238}\mathrm{U}\rightarrow^{238}\mathrm{Pu}$	0.03			

[a] H.V. Klapdor-Kleingrothaus *et al.*, Phys. Lett. B **586**, 198 (2004).
[b] C. E. Aalseth *et al.* (IGEX collaboration), Phys. Rev. D **65**, 092007 (2002).
[c] M. Agostini *et al.* (GERDA Collaboration), Phys. Rev. Lett. **111**, 122503 (2013).
[d] A. Gando *et al.* (KamLAND-Zen collaboration), Phys. Rev. Lett. **110**, 062502 (2013)
[e] M. Auger *et al.* (EXO collaboration) Phys. Rev. Lett. **109**, 032505 (2012).

g_A=1.269

SUMMARY OF RESULTS (LIGHT NEUTRINOS) 2015



$g_A = 1.269$

CONCLUSIONS

Major progress has been made in the last few years to narrow down predictions for NME in all nuclei of interest.

PSF for all processes have been calculated.

Expected half-lives have been obtained for all processes, $0\nu\beta\beta$, $0\nu\beta EC$, $R0\nu ECEC$; $2\nu\beta\beta$, $2\nu\beta EC$, $2\nu ECEC$; $0\nu\chi\beta\beta$.

With current estimates and g_A=1.269:

For light neutrino exchange, only the degenerate region can be tested in the immediate future. The current best limit (with g_A=1.269) is from KamLAND-Zen, $m_\nu < 0.20$ eV.
Exploration of the inverted region >1 ton
Exploration of the normal region >>1 ton

For heavy neutrino exchange, the limit is model dependent. In the model of Tello *et al.* ¶, the current best limit from KamLAND-Zen is $m_{\nu h} > 257$ GeV$(3.5/M_{WR})^4$.

¶ V. Tello, M. Nemevšek, F. Nesti, O. Senjanovic, and F. Vissani, Phys. Rev. Lett. 106, 151801 (2011).

The major remaining question is the value of g_A.

Three scenarios are[¶,§] :

$$g_A = 1.269$$ ← Free value

$$g_A = 1$$ ← Quark value

$$g_A = 1.269 A^{-0.18}$$ ← Maximal quenching

[¶] J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C 87, 014315 (2013).

[§] S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D90, 033005 (2014).

If g_A is renormalized to 0.8-0.6, as in single β/EC, and 2νββ (discussed in lecture 2), maximal quenching, all estimates should be increased by a factor of 4-16 making it impossible to reach, in the foreseeable future, even the inverted region.

Possibilities to escape this negative conclusion are:
(1) The neutrino masses are degenerate and large.

This possibility will be in tension with the cosmological bound on the sum of the neutrino masses (lecture 1)

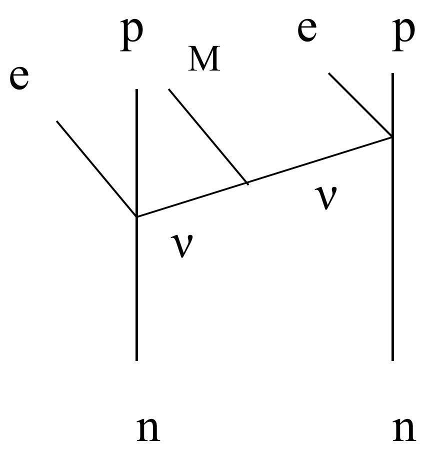
(2) Both mechanisms, light and heavy contribute simultaneously, are of the same order of magnitude, and interfere constructively.

$$[\tau_{1/2}^{0\nu\beta\beta}(0^+ \to 0^+)] = G_{0\nu} \left| M_{0\nu,light} \frac{\langle m_\nu \rangle}{m_e} + M_{0\nu,heavy} \frac{m_p}{\langle m_{\nu_h} \rangle} \right|^2$$

This possibility requires a fine tuning which is quite unlikely.

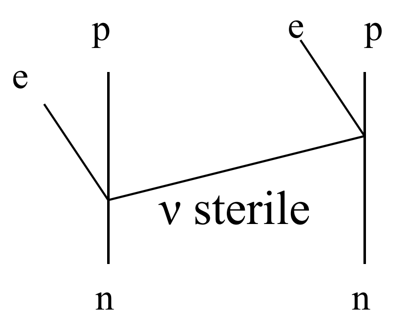
(3) Other scenarios (Majoron emission, …) and/or new mechanisms (sterile neutrinos, …) must be considered.

3



Majoron means a massless neutral boson

4



Sterile means no standard model interactions §

§B. Pontecorvo, Phys. Lett. B26, 630 (1968).

Scenario 3: MAJORON EMISSION ¶

The inverse half-life for this scenario (0νββχ decay) is given by

$$\left[\tau_{1/2}^{0\nu\beta\beta\chi}\right]^{-1} = G_{0\nu\varphi}\left|M_{0\nu}\right|^2\left\langle g\right\rangle^2$$

effective Majoron coupling constant

NME are the same as for scenario 1 and 2.
PSF have been recalculated recently.

This scenario was suggested by H.M. Georgi, S.L. Glashow, and S. Nussinov, Nucl. Phys. B193, 297 (1981).

¶ J. Kotila, J. Barea and F. Iachello, Phys. Rev. C91, 064310 (2015).

From the expected half-lives and experimental limits one can set limits on the Majoron coupling constant

Decay	$\tau_{1/2}^{0\nu M}$ (10^{21}yr)	$\tau_{1/2,exp}^{0\nu M}$(yr)	$\langle g_{ee}^{M} \rangle$ (eV)
$^{48}Ca \to ^{48}Ti$	8.19	$> 7.2 \times 10^{20}$[a]	$< 3.4 \times 10^{-3}$
$^{76}Ge \to ^{76}Se$	39.8	$> 6.4 \times 10^{22}$[b]	$< 7.9 \times 10^{-4}$
$^{82}Se \to ^{82}Kr$	7.68	$> 1.5 \times 10^{22}$[c]	$< 7.2 \times 10^{-4}$
$^{96}Zr \to ^{96}Mo$	5.32	$> 1.9 \times 10^{21}$[c]	$< 1.7 \times 10^{-3}$
$^{100}Mo \to ^{100}Ru$	3.62	$> 3.9 \times 10^{22}$[rr]	$< 3.0 \times 10^{-4}$
$^{110}Pd \to ^{110}Cd$	25.0		
$^{116}Cd \to ^{116}Sn$	7.06	$> 8 \times 10^{21}$[e]	9.4×10^{-4}
$^{124}Sn \to ^{124}Te$	18.1		
$^{128}Te \to ^{128}Xe$	765	$> 2 \times 10^{24}$[f]	$< 6.2 \times 10^{-4}$
$^{130}Te \to ^{130}Xe$	6.82	$> 1.6 \times 10^{22}$[g]	$< 6.5 \times 10^{-4}$
$^{134}Xe \to ^{134}Ba$	805		
$^{136}Xe \to ^{136}Ba$	10.1	$> 2.6 \times 10^{24}$[h]	$< 6.2 \times 10^{-5}$
		$> 1.2 \times 10^{24}$[i]	$< 9.2 \times 10^{-5}$
$^{148}Nd \to ^{148}Sm$	36.8		
$^{150}Nd \to ^{150}Sm$	1.74	$> 1.5 \times 10^{21}$[c]	$< 1.1 \times 10^{-3}$
$^{154}Sm \to ^{154}Gd$	173		
$^{160}Gd \to ^{160}Dy$	14.5		
$^{198}Pt \to ^{198}Hg$	132		
$^{232}Th \to ^{232}U$	28.7		
$^{238}U \to ^{238}Pu$	4.94		

[a] A. S. Barabash, Phys. Lett. B **216**, 257 (1989).
[b] H. V. Klapdor-Kleingrothaus *et al.* , Eur. Phys. J. A, **12**, 147 (2001).
[c] A. S. Barabash and V. B. Brudanin (NEMO Collaboration), Phys. At. Nucl. **74**, 312 (2011).
[d] R. Arnold *et al.* (NEMO-3 collabaration), Phys. Rev. D, **89**, 111101 (2014).
[e] F. A. Danevich, Phys. Rev. C **68**, 035501 (12003).
[f] O. K. Manuel , J. Phys. G, **17**, 221 (1991). Geochemical.
[g] R. Arnold *et al.* (NEMO-3 Collaboration), Phys. Rev. Lett., **107**, 062504 (2011).
[h] A. Gando *et al.* (KamLAND-Zen Collaboration), Phys. Rev. C **86**, 021601(R) (2012).
[i] J. B. Albert *et al.* (EXO-200 Collaboration), Phys. Rev. D **90**, 092004 (2014).

Scenario 4: STERILE NEUTRINOS ¶

Another scenario is currently being discussed, namely the mixing of additional "sterile" neutrinos.
[The question on whether or not "sterile" neutrinos exist is an active areas of research at the present time with experiments planned at FERMILAB and CERN-LHC.]

IBM-2 NME for this scenario have just been calculated.
PSF are the same as in scenarios 1 and 2.

¶ J. Barea, J. Kotila, and F. Iachello, Phys. Rev. D92, 093001 (2015).

The half-lives can be written in terms of a simple formula

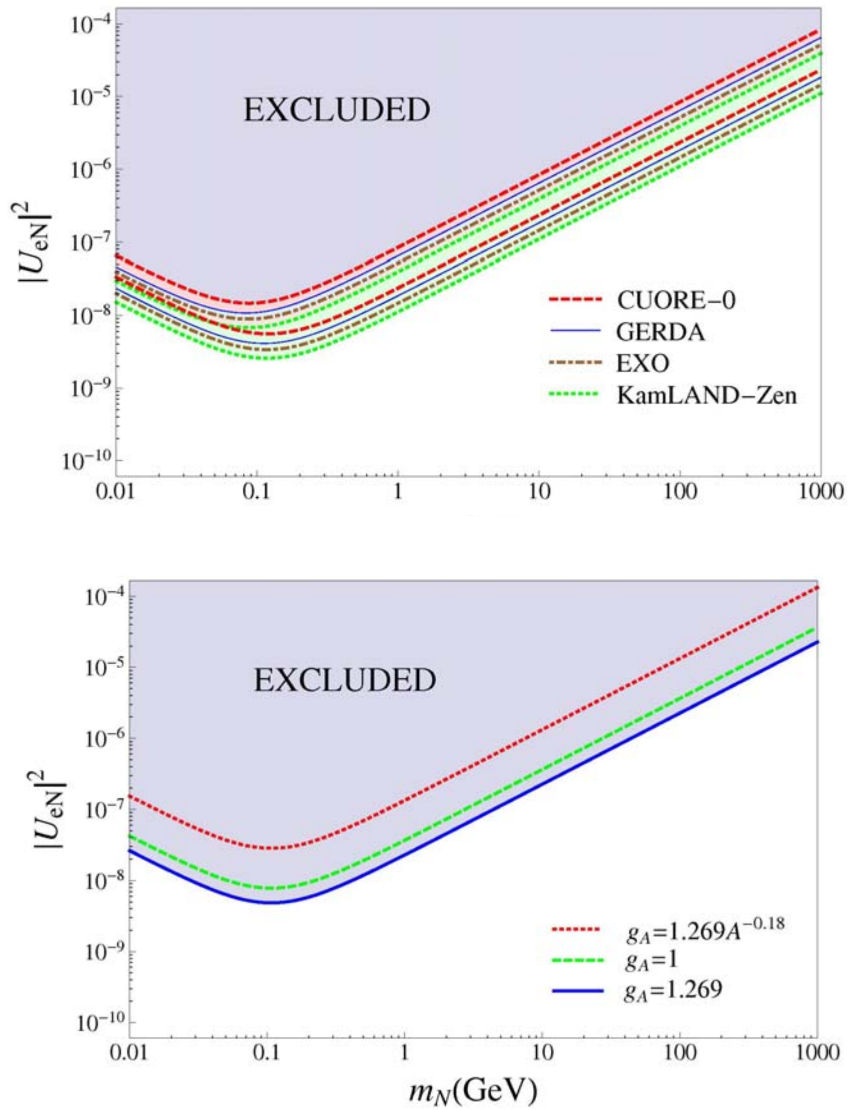
$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = g_A^4 G_{0\nu} \left|M^{(0\nu_h)}\right|^2 \left| m_p \sum_N (U_{eN})^2 \frac{m_N}{\langle p\rangle^2 + m_N^2} \right|^2$$

with

$$\langle p^2 \rangle = \frac{M^{(0\nu_h)}}{M^{(0\nu)}} m_p m_e$$

[For a single sterile neutrino of mass m_N and coupling U_{eN} , there is no sum.]

From experimental limits one can construct an exclusion plot (2015)



Several types of sterile neutrinos have been suggested.

Scenario 4a: LIGHT STERILE NEUTRINOS

Sterile neutrinos with masses $m_N \sim 1eV$

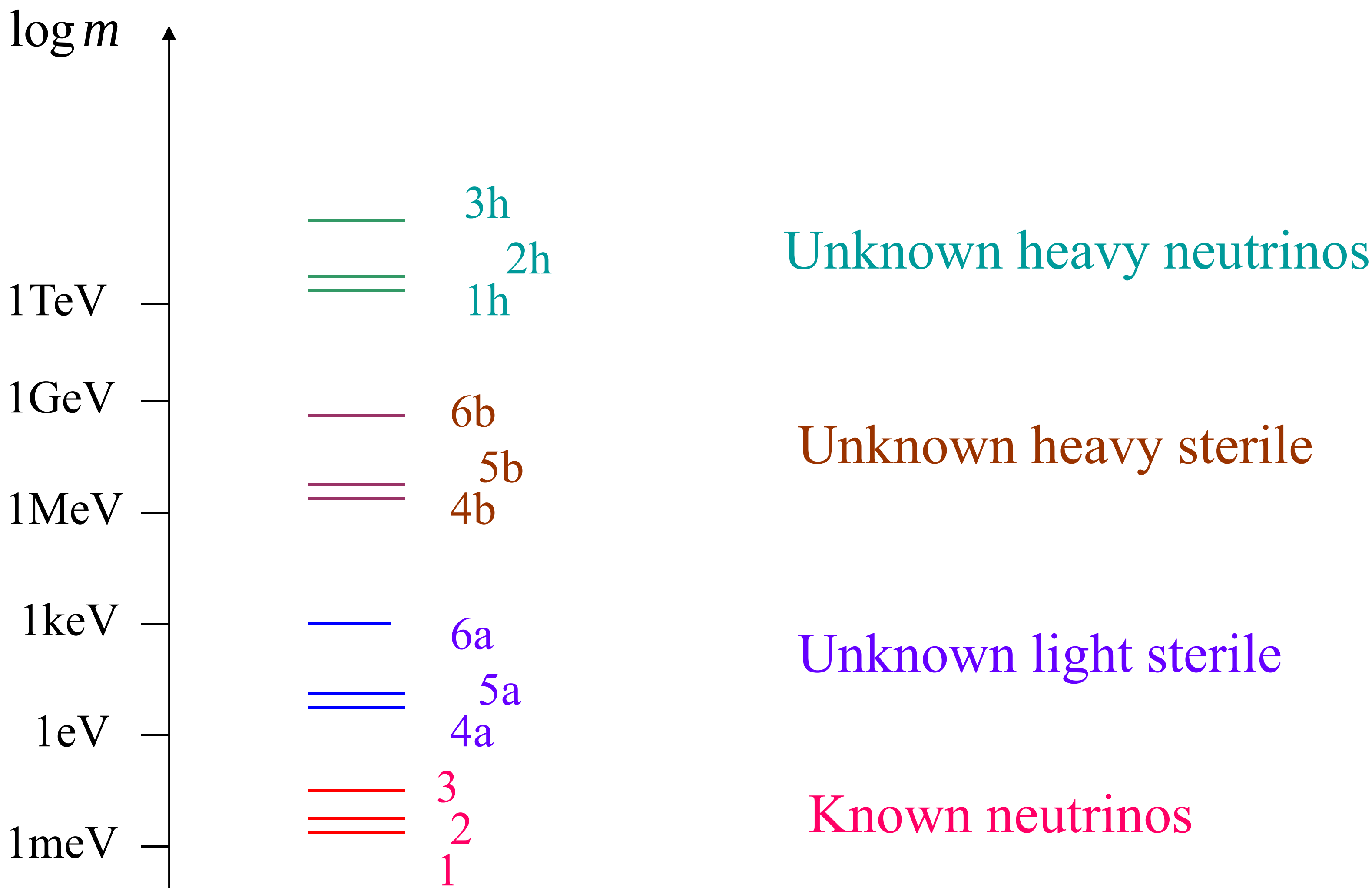
Very recently C. Giunti and M. Laveder have suggested sterile neutrinos, 4a,…, with masses in the eV range to account for the reactor anomaly in oscillation experiments, G. Giunti, XVI International Workshop on Neutrino Telescopes, Venice, Italy, March 4, 2015.
C. Giunti and M. Laveder, Phys. Rev. D82, 053005 (2010).

Scenario 4b: HEAVY STERILE NEUTRINOS

Sterile neutrinos with masses $m_N \gg 1eV$

Possible values of the sterile neutrino, 4b,5b, 6b,…, masses in the keV-GeV range have been suggested by T. Asaka and M. Shaposhnikov, Phys. Lett. B620, 17 (2005) and T. Asaka, S. Blanchet, and M. Shaposhnikov, Phys. Lett. B631, 151 (2005).

HYPOTHETICAL NEUTRINO MASS SPECTRUM



CONTRIBUTIONS OF HYPOTHETICAL NEUTRINOS ALL

$$\left[\tau_{1/2}^{0\nu\beta\beta}\right]^{-1} = g_A^4 G_{0\nu} M \left| \frac{m_p}{B}\sum_{k=1}^{3} U_{ek}^2 m_k + m_p \sum_{i_b} \frac{U_{ei_b}^2}{m_{i_b}} + \frac{m_p}{B}\sum_{i_a} U_{ei_a}^2 m_{i_a} + m_p \sum_{h} \frac{U_{eh}^2 m_h}{B + m_h^2} \right|^2$$

Known neutrinos

Unknown heavy sterile

Unknown light sterile

Unknown heavy neutrinos

The values of M and B in IBM-2 have been just calculated ¶

¶ J. Barea, J. Kotila and F. Iachello, Phys. Rev. D92, 093001 (2015).

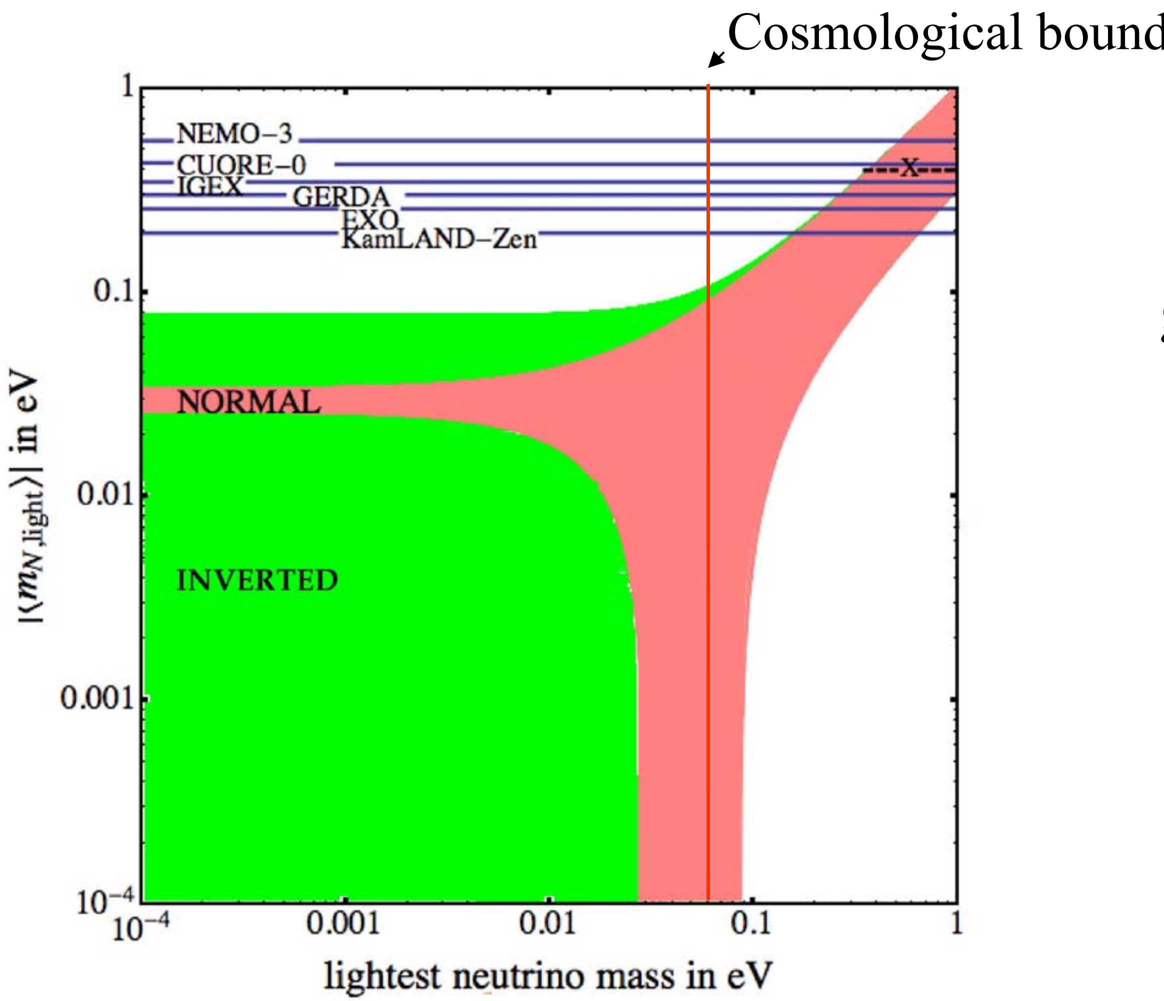
Of particular interest is scenario 4a for which

$$\left[\tau_{1/2}^{0\nu\beta\beta}\right]^{-1} = g_A^4 G_{0\nu} \left|M^{(0\nu)}\right|^2 \left(\frac{\langle m_{N,light}\rangle}{m_e}\right)^2$$

If there is a 4th neutrino $\left|U_{e4}\right|^2 = 0.03$

$$\langle m_{N,light}\rangle = \sum_{k=1}^{3} U_{ek}^2 m_k + \left|U_{e4}\right|^2 e^{i\alpha_4} m_4$$

The presence of a sterile fourth neutrino changes dramatically the situation for DBD!



g_A=1.269

$m_4 = 1eV$

$|U_{e4}|^2 = 0.03$

With sterile neutrinos, neutrinoless DBD is within reach of the next generation of experiments (GERDA-II,CUORE,…).

FINAL CONCLUSION

No matter what the mechanism of neutrinoless DBD is, its observation will answer the fundamental questions:

- What is the absolute neutrino mass scale?
- Are neutrinos Dirac or Majorana particles?
- How many neutrino species are there?